ECO 384 Intermediate Microeconomics II
Fall 2019
Problem Set 1

Due Date: October 17th. No Late Problem Sets Will Be Accepted.

1. Suppose you are faced with the following decision under uncertainty. You are considering buying a signed Connor McDavid Jersey. Your purchase is based solely on the hope of reselling the jersey in one year after McDavid has another season to show off his abilities. You currently have \$200. The strength of the resale market for Edmonton Oilers Memorabilia varies dramatically with the performance of the team and Connor McDavid.

   Two outcomes can arise, the Oilers and McDavid can have a bad season (w_1) or a good season (w_2). If the Oilers and McDavid have a bad season, you can resell the jersey for only w_1 =\$50. If the Oilers and McDavid have a good season, you can resell the jersey for w_2=\$650. Suppose you believe that the Oilers will have a bad season with probability $p_1 = 0.75$ and the probability of a good season is $p_2 = 1 - p_1 = 0.25$.

   Suppose there is no time discounting associated with payoffs (i.e., \$1 today equals \$1 in one year).

   (a). What is the expected income and variance of this lottery?

   (b). Suppose your utility function for receiving income x is given by: $u(x) = \alpha\sqrt{x}$ where $\alpha > 0$ is a constant.

   (i). Should you stick with the \$200 with certainty or should you take the potential lottery? Does this answer depend on the value of α? Explain.

   (ii). Illustrate this problem graphically. Does Jensen's Inequality hold? Illustrate why or why not.

   (iii). How would your answer to part (i) change if $w_2 = 1400$ and $\alpha = 2$?

   (iv). Suppose $\alpha = 2$ and $w_2 = 650$ (the original problem). Compute the risk-premium of this lottery. Illustrate the Risk Premium Graphically.

   (v). Now treat the probability p_1 as an arbitrary constant. Suppose $\alpha = 2$ and $w_2 = 650$ (the original problem). Is there a feasible value of p_1 where you would accept this lottery over the certain payoff of \$200? (Recall $p_2 = 1 - p_1$).

2. Jack's income from renting his old house is \$40,000 and he is risk averse. The probability of someone slipping on his stairs is $p = \frac{1}{8}$. If this happens, he will be sued for \$30,000 in damages and will have to pay that amount. He can purchase renters insurance at a price of \$$C$. This insurance covers any damages that arise in the bad "state of nature" (i.e., is full coverage).

   Suppose Jack has the following utility function $U(w) = \sqrt{w}$ that represents his payoff for any income level w.

   (a). Suppose $C = 4500$. Is this the Actuarially fair rate? Will Jack Accept this offer and purchase insurance?

   (b). What is the highest value on C that Jack is willing to pay before he decides to reject the insurance offer?

   (c). How is your answer to part (b) affected by the probability that an accident will happen (p)? Show this result mathematically.

3. Suppose you are faced with the following decision under uncertainty. You are considering buying stock in an Oil and Gas Company. You currently have \$200. The current stock price is \$10 per share. Assume that you have two decisions to make (i) do not purchase any stock or (ii) spend all \$200 on the stock of the company (20 shares) and sell all shares in one year.

   The value of the stock varies dramatically with global price of oil. Two outcomes can arise, the oil markets can have a "bad" or "good" year. If the global oil markets have a bad year, you can resell the stock for only \$2.50 per share (resulting in income w_1 =\$50). If the oil market has a good year, you can resell the stock for \$32.50 per share (resulting in income w_2=\$650). Suppose you believe that the oil market will have a bad year with probability $p_1 = 0.75$ and the probability of a good year is $p_2 = 1 - p_1 = 0.25$.

   Suppose there is no time discounting associated with payoffs (i.e., \$1 today equals \$1 in one year).

   (a). What is the expected income and variance of this lottery?

   (b). Suppose your utility function for receiving income w is given by: $u(w) = \alpha\sqrt{w}$ where $\alpha > 0$ is a constant.

      (i). Should you stick with the \$200 with certainty or should you take the potential lottery? Does this answer depend on the value of α? Explain.

      (ii). Illustrate this problem graphically. Does Jensen's Inequality hold? Illustrate why or why not.

      (iii). How would your answer to part (i) change if $w_2 = 1400$ (i.e., the good outcome yields a price per share of \$70) and $\alpha = 2$?

      (iv). Suppose $\alpha = 2$ and $w_2 = 650$ (the original problem). Compute the risk-premium of this lottery. Illustrate the Risk Premium Graphically.

      (v). Now treat the probability p_1 as an arbitrary constant. Suppose $\alpha = 2$ and $w_2 = 650$ (the original problem). Is there a feasible value of p_1 where you would accept this lottery over the certain payoff of \$200? (Recall $p_2 = 1 - p_1$).

   (c). Suppose you can also invest in stock of a downstream plastics manufacturer. Because oil is an key input in plastic manufacturing, suppose that the value of the plastics company decreases as the oil price increases. Suppose initially the stock price of the plastics company is \$5 per share. If oil has a bad year (with probability $p_1 = 0.75$), then the stock price of the plastics company increases to \$25 per share. If oil has a good year (with probability $p_2 = 0.25$), then the stock price of the plastics company decreases to \$1 per share.

      Suppose you now only have two options: (i) invest all \$200 in the Oil and Gas company and you face the lotter specified in part (a) or (ii) invest \$100 in the Oil and Gas company (i.e., buy 10 shares of the oil and gas company today at a price of \$10 per share) and invest \$100 in the Plastics Company (i.e., buy 20 shares of the plastics company today at a price of \$5 per share), then sell all stock in exactly one year. Which lottery would you choose, given your utility function is $U(w) = 5\sqrt{w}$?

4. Suppose you're an owner of a small taxi company that employs several drivers. You are considering upgrading your vehicle fleet from traditional gas powered cars to electric power Teslas. The shift requires a large upfront investment, but yields a stream of potentially positive payoffs because of the boost in demand and reduction in fuel costs.

   (a). Suppose you set your personal discount rate via the CAPM methodology. Suppose you face the following payoff (utility) stream if you make the investment:

Period	0	1	2	3	4	5	6
Payoff	-200,000	20,000	30,000	40,500	50,000	60,500	60,500

   (i). A central input to the costs of owning an electric vehicle is the price of electricity used to fuel the car. You realize that there is a large amount of uncertainty associated with the future electricity prices, the annual payoffs above are the expected payoff levels. Suppose that the risk-free interest rate is 0.04, stock market return rate is 0.12, and an econometrician has determined that your *asset beta* is 1.8. Should you undertake this large investment?

   (ii). Suppose you can sign a deal with a competitive electricity retailer that provides a fixed-price contract for your electricity purchases to "fuel" your car. The fixed-priced contract shields you from some of the non-diversifiable risk. This reduces the *asset beta* to $\beta = 0.20$. Should you undertake the investment in this setting?

   (b). Suppose instead of utilizing the CAPM methodology, we undertake the expected utility approach. The discount rate utilized is based on your personal preferences for adjusting the vehicle fleet. Suppose your utility function is defined by $u(w) = \sqrt{w}$. Unlike part (a), you explicitly model the uncertainty you face in each period.

   (i). Suppose you have already upgraded your fleet of vehicles to Telas. Assume for the next six years you face the following lottery in each period (you earn zero payoff today $t = 0$ as the purchase decision has been made):

W_1	p_1	W_2	p_2
20,500	0.50	100,000	0.50

   Suppose your annual discount factor equals $\delta = \frac{1}{1+R}$. Compute your discounted utility level from this investment when $R = 0.05$.

   (ii). Suppose you can avoid the uncertainty in each period by signing a fixed price contract with a competitive electricity retailer (shielding you from fuel cost uncertainty) and contracting your drivers out to a luxury hotel that needs a fixed number of rides for 6 years (shielding you from demand-side uncertainty). However, your annual payoff would fall to be equal to $\$60,500$ with certainty. Would you want to sign the contracts to avoid the uncertainty?

5. Suppose there are two players. Each player has the following strategy set $S_1 = \{U, M, D\}$ and $S_2 = \{L, C, R\}$. The players' preferences are well-defined over the set of outcomes. This game is represented by the matrix below.

		Player 2		
		L	C	R
	U	20, 0	12, 5	7, 7
Player 1	M	25, 10	15, 15	5, 12
	D	20, 20	10, 25	0, 20

(a). Characterize the pure strategies that survive the Iterative Deletion of Strictly Dominated Strategies (IDSDS).

(b). Identify all Nash Equilibrium of this game (Pure and Mixed!).

6. Consider the following sequential move game between three players. Suppose two entrepreneurs (players 1 and 2) are working on a joint project and a potential investor (player 3) is trying to decide if she wants to fund the project. First, player 1 decides to devote either a *High* or *Low* level of effort to the project. Second, after observing player 1's effort level, player 2 chooses either a *High* or *Low* level of effort. In a third-stage, after observing the effort levels of both players (e.g., player 3 observes the quality of the project proposal), the potential investor (player 3) chooses to either *Invest* or *Do Not Invest.*

(*Note: The only terminal nodes in the game arise after player 3 makes its final decision to invest or do not invest*)

The payoffs are as follows. Each entrepreneur gets a payoff of 5 if the project is funded and 0 otherwise. In addition, choosing high effort costs an entrepreneur $e_H = 1$, while choosing low effort costs $e_L = 0$. The total payoff of each entrepreneur is the payoff from the project, minus their cost of effort.

The potential investor (player 3) earns a payoff of 3 for each entrepreneur who chooses a high level of effort, and 0 for each entrepreneur who chooses a low level of effort. Investing in the project costs the potential investor 2. If the investor chooses *Do Not Invest*, its payoff equals zero.

(a). Draw this extensive form game.

(b). Find the SPNE of the game. Illustrate which terminal node is reached in your game through the "path of play". **Demonstrate the behavior of each player clearly at every decision node in your answer**.

7. Consider the following game. Suppose there are $N \geq 2$ countries. There are rising concerns that a virus will spread across counties all over the world. Each country can choose its level of effort to prevent the spreading of this virus. Suppose each country i can choose an effort level $S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ for all $i = 1, 2, ..., N$. A higher effort level reflects a higher level of time spent on preventative measures to prevent the spreading of this virus. However, effort comes at some cost $b > 0$ for each unit of effort.

   Suppose each player *simultaneously* and *independently* chooses their effort level s_i in the set S_i. Countries are highly connected and so, the strength of virus prevention is only as strong as the weakest link: the minimum level of effort across the entire set of countries. Therefore, suppose each country receives the following payoff from choosing s_i:

$$\pi_i(s_1, s_2, ..., s_N) = 100 + a \min\{s_1, s_2, ..., s_N\} - bs_i$$

   where a, b are positive constants and s_i is firm i's effort level.

   (a). Assume $a = 40$ and $b = 10$. Find all Pure Strategy Nash Equilibrium of this game. Explain your solution and show your work.

   (b). Assume $a = 10$ and $b = 40$. Find all Pure Strategy Nash Equilibrium of this game. Explain your solution and show your work. Explain why this result is intuitive given the payoff function and the updated assumptions on the values of a and b.

   (c). Suppose we adjusted the preferences of players. Rather than focusing on the weakest-link (i.e., $\min\{s_1, s_2, ..., s_N\}$), what if players only cared about the maximum value. That is, suppose each country receives the following payoff from choosing s_i:

$$\pi_i(s_1, s_2, ..., s_N) = 100 + 40 \max\{s_1, s_2, ..., s_N\} - 10s_i$$

   Find all PSNE in this setting. Explain your solution and show your work.