- 1. An individual (Jeff) is on a game show: Who Wants to be a Millionaire. Jeff has a current wealth equal to 9,000. The next round of questioning is coming up and he believes that he has a (.40) probability of getting the question correct and (.60) probability of getting it wrong. If he gets the question wrong his wealth falls to zero, if he answers it correctly his wealth rises to 25,000. He can decide to either take the lottery (and move on to the next question), or he can take the 9,000 and quit the game. Answer the following questions.
 - (a). Find the expected value of his income from participating in this lottery (gamble).
 - (b). Suppose his utility u(x) = 10x. Will he accept the lottery? Calculate the Risk Premium. Does Jensen's inequality hold in this setting? Explain your results for each of these components.
 - (c). Suppose his utility $u(x) = 10\sqrt{x}$. Will he accept the lottery? Calculate the Risk Premium. Does Jensen's inequality hold in this setting? Explain your results for each of these components. Further, represent this environment graphically.
 - (d). Suppose an independent insurance company is willing to offer insurance to Jeff to cover his potential losses from the lottery (that is, if he purchases insurance Jeff is guaranteed to receive \$25,000 from the next question). Further, suppose Jeff has to accept the gamble and move onto the next question regardless if he accepts the insurance or not. Suppose Jeff's preferences are according to those characterized in part (c). Derive how much Jeff is willing to pay to accept the insurance program.
- 2. Consider the following normal-form game.

Player 2 | X | Y | Z | | a | 2,3 | 1,4 | 3,2 | | b | 5,1 | 2,3 | 1,2 | | c | 3,7 | 4,6 | 5,4 | | d | 4,2 | 1,3 | 6,1 |

- (i). Find all Strictly Dominated Strategies of this Game.
- (ii). Identify all Nash Equilibria.

- 3. Victoria currently holds 1 share of a stock, Apple Inc, that is currently trading at \$100. Over the next year, the stock will either appreciate to \$144 or depreciate to \$72 with equal probability. Victorias utility of income x function is $U(x) = x^{0.5} = \sqrt{x}$. Suppose (for now) Victoria does not discount her future income (i.e., R = 0).
 - (a). Should Victoria hold the stock for the following year, or sell it now? Explain.
 - (b). Compute Victoria's Risk-Premium for this lottery.
 - (c). (Challenging) Suppose now Victoria discounts future payoffs. Suppose her discount rate is 20% (R = 0.20).
 - (i). Should Victoria hold the stock for the following year, or sell it now in this new setting?
 - (ii). Suppose Victoria can purchase insurance that pays her \$72 if the stock depreciates. Suppose the cost of the insurance is equal to C. When should Victoria purchase the insurance? Explain.
- 4. Suppose I am offering you two contracts for your new position at Brown Consulting Company. In each offer, if you succeed at the job you receive a high pay and if you fail you receive a low pay. Suppose the probability of success and failure is know. The two offers are given below.

Example:	p_1	x_1	p_2	x_2
Contract 1 (X_1)	0.60	100,000	0.40	0
Contract 2 (X_2)	0.60	50,000	0.40	30,000

Suppose you have a utility function that depends solely on income, $u(I) = 5\sqrt{I}$. What is your expected income under both job offers? What is your expected utility under each job offer? Which job would you choose? Graphically represent the utility functions and the two lotteries (contract offers). Provide the Jensen's inequality for both of these contracts.

- 5. Brian won the Alberta lottery grand prize of \$5 million. The lottery offers him two options: five annual payments of \$1 million (starting at time zero) or one lump sum payment of \$4.5 million right away (at time zero).
 - (a). If the interest rate (discount rate) is R=5%, what is the present value of both options? Which option should he choose?
 - (b). If the interest rate (discount rate) is R = 10%, what is the present value of both options? Which option should he choose?
 - (c). If your results vary across (a) and (b), explain why intuitively.

6. Consider the following game. Drivers in two different vehicles are approaching an intersection at the same time. The drivers are approaching at right angles, and each desires to drive straight through. There are no traffic rules or signs.

Each driver must decide whether or not to stop to let the other driver through the intersection first. Each driver would prefer to go through first; however, stopping to let the other driver go first is preferred to having an accident, which is what happens if each goes through without stopping. In particular, suppose that if one stops and the other does not, the one who stops gets a payoff of 2, while the other gets a payoff of 3. If both stop, each ends up waiting even longer, and each gets a payoff of 1. However, if neither stop then there is an accident, with each getting a payoff of -5. The drivers make their decisions once and simultaneously.

- (a). Draw a table representing this game in Strategic form.
- (b). Identify all Nash Equilibria in pure strategies (that is, where each drivers strategy is a decision of whether or not to stop). Given your answer, what do you think will happen when this game is played?
- (c). Now suppose that stop signs are installed in one direction (player 1 only). If the player traveling in that direction fails to stop, then in addition to the payoffs described above, he suffers a disutility from the expected punishment of -4 (since with some probability he will be caught and fined). So for example, if neither driver stops and there is an accident, his payoff is now -5 4 = -9. Redo parts (a) and (b).
- 7. Each of a group of hunters has two options: she may remain attentive to the pursuit of the stag, or she may catch the hare (i.e., $S_i = \{Stag, Hare\}$). If all hunters pursue the stag, they catch it and share it equally; if any hunter devotes energy to catching a hare, the stage escapes, and the hare belongs to the defecting hunter alone. Each hunter prefers a share of the stag to a hare.

For each player, the preference ranking is as follows: (1) the outcome in which all players choose Stag is ranked highest, (2) followed by any profile where the player chooses Hare, (3) followed by a strategy profile in which she chooses Stag alone while the other player chooses Hare.

Setup the normal-form representation of this game and find all Nash Equilibria (pure or mixed). For any Nash Equilibria you identify, characterize the best-reply functions.

8. Consider the following two player game. There is an island of people. In the first stage a group of invaders decide to either *Invade* or *Do Not Invade*. If they *Do Not Invade* the game is over. If they choose to *Invade* the islanders can decide to *Escape* the island or stay and *Fight*.

The players' preferences are as follows. If the Invaders *Do Not Invade*, the Invaders get a payoff of 1 and the Islanders get a payoff of 2. If the Invaders choose to *Invade* and the Islanders *Fight*, both players receive a payoff of 0. If the Invaders choose to *Invade* and the Islanders *Escape*, the Invaders receive a payoff of 2 and the Islanders receive a payoff of 1.

- (a) Present the Extensive Form of this game. Find all Subgame Perfect Nash Equilibria.
- (b) Derive the strategic form representation of the game above and find all the Pure Strategy Nash Equilibria. Briefly summarize why any differences may arise in the Pure Strategy Nash Equilibrium compared to the Subgame Perfect Nash Equilibrium solution mechanism.
- 9. Suppose there are 9 players who are choosing their Winter Boots for the upcoming winter. Each player i has the following strategy set $S_i = \{UGG, Sorel\}$ for all i = 1, 2, 3, ..., 9. Suppose each player simultaneously and independently chooses their Boot brand from their strategy set.

Because it is not "cool" to have the same pair of Boots as everyone else, the players prefer to be in the Minority group. Let m denote the number of individuals with UGG boots. If m < 5, then having UGG boots is esteemed (since fewer individuals have UGGs than Sorels) and having Sorel boots is not. Alternatively, if $m \ge 5$ then having Sorel boots is esteemed and having UGGs is not. Suppose a player receives a payoff of 5 if they are in the Esteemed Minority group and 0 otherwise.

Find all Pure Strategy Nash Equilibria of this game.

10. Greenwashing occurs when a firm markets itself as environmentally friendly, but they are not actually undertaking environmentally friendly practices. To crack down on this counterfeit behavior, there are penalties if a firm is caught "greenwashing". However, if a firm goes undetected, there are benefits through increased revenues due to a rise in demand as consumers value a firm's products more if they are environmentally friendly. The following symmetric game analyzes firms' decisions to undertake such "greenwashing" behavior.

Assume there are 100 firms who are **simultaneously** and **independently** making a decision to *Greenwash* or *Do Not Greenwash*. If a firm chooses to *Greenwash* and goes undetected, it receives a payoff of W. If the firm chooses *Greenwash* and is caught, then its payoff is P. If a firm chooses *Do Not Greenwash* it earns a payoff of zero. Assume that W > 0 > P. The probability that a firm is caught "greenwashing" is equal to $\frac{1}{m}$, where m is the number of firms who choose to *Greenwash*. Thus, the probability of being caught decreases as the number of firms who choose *Greenwash* increases. This implies that for any m, the payoff function for a firm i equals:

$$\pi_i = \left\{ \begin{array}{ll} \left(\frac{m-1}{m}\right)W + \frac{1}{m}P & \text{if firm i choose $Greenwash} \\ 0 & \text{if firm i choose Do Not $Greenwash} \end{array} \right.$$

- (a) Suppose W=10 and P=-40. Find all Pure Strategy Nash Equilibria of this game. Prove that these outcome(s) are PSNE.
- (b) How do you suspect the Pure Strategy Nash Equilibrium outcome(s) to change as the magnitude of W and P varies?