# Dimensional Analysis

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**Abstract:** This lecture deals with definition and usage of dimensional analysis

## 1.Introduction

Understanding dimensions is of utmost importance as it helps us in studying the nature of physical quantities mathematically.
The study of relationships between physical quantities with the help of their dimensions and units of measurements is called dimensional analysis.

Dimensional Analysis is a very basic aspect of measurement and has many applications in engineering. The dimensional analysis is used for three prominent reasons:

- Consistency of a dimensional equation
- Derive relation between physical quantities in physical phenomena
- To change units from one system to another

The dimension of any physical quantity expresses its dependence on the base quantities as a product of symbols (or powers of symbols) representing the base quantities.
Most physical quantities (dimensions) can be expressed in terms of five basic dimensions, listed in table with their SI units. For example, acceleration is length per unit time per unit time or $LT^{-2}$.

Table 1. Primary Dimensions

| Name | Symbol of quantity | Symbol of dimension | SI base unit |
|---|---|---|---|
| Length | $l$ | L | meter (m) |
| Time | $t$ | T | second (s) |
| Mass | $m$ | M | kilogram (kg) |
| Electrical charge | $Q$ | Q | Coulomb (C) |
| Temperature | $T$ | θ | Kelvin (K) |

Only quantities with like dimensions may be added(+), subtracted(−) or compared (=,<,>). A measurement of length is said to have dimension L or $L^1$, a measurement of mass has dimension M or $M^1$, and a measurement of time has dimension T or $T^1$.

$$[Velocity] = \frac{[displacement]}{[time]} = \frac{L}{T} = LT^{-1}$$

Unit for force is kg $ms^{-2}$ or N

$$[Force] = [mass]\ x\ [acceleration]$$

$$[mass]\ x \frac{[change\ of\ velocity]}{[time]} = M\frac{LT^{-1}}{T} = MLT^{-2} =$$

$$[Frequency] = \frac{1}{[period]} = \frac{1}{T} = T^{-1}$$

Like units, dimensions obey the rules of algebra. Thus, area is the product of two lengths and so has dimension $L^2$, or length squared. Similarly, volume is the product of three lengths and has dimension $L^3$, or length cubed. Speed has dimension length over time, L/T or $LT^{-1}$. Volumetric mass density has dimension M/$L^3$ or $ML^{-3}$, or mass over length cubed.

## 2.Dimensional Consistency

The importance of the concept of dimension arises from the fact that any mathematical equation relating physical quantities must be **dimensionally consistent**, which means the equation must obey the following rules:

- Every term in an expression must have the same dimensions; it does not make sense to add or subtract quantities of differing dimension (think of the old saying: "You can't add apples and oranges"). In particular, the expressions on each side of the equality in an equation must have the same dimensions.

- The arguments of any of the standard mathematical functions such as trigonometric functions (such as sine and cosine), logarithms, or exponential functions that appear in the equation must be dimensionless. These functions require pure numbers as inputs and give pure numbers as outputs.

If either of these rules is violated, an equation is not dimensionally consistent and cannot possibly be a correct statement of physical law.

## 2.1 Example of checking for dimensional consistency

Consider one of the equations of constant acceleration,

$$s = ut + \frac{1}{2}at^2 \qquad (1)$$

The equation contains three terms: s, ut and $1/2at^2$. All three terms must have the same dimensions.

- s: displacement = a unit of length, L
- ut: velocity x time = $LT^{-1}$ x T = L
- $1/2at^2$ = acceleration x time = $LT^{-2}$ x $T^2$ = L

All three terms have units of length and hence this equation is dimensionally valid. Of course this does not tell us if the equation is physically correct, nor does it tell us whether the constant 1/2 is correct or not.

Example 1:Check consistency of dimensional equation of speed.

Speed = Distance/Time

$[LT^{-1}] = L/T$

$[LT^{-1}] = [LT^{-1}]$

The equation is dimensionally correct, as the dimension of speed is same on both sides. Dimensional Analysis is a basic test to find out the consistency of equation and doesn't guarantee the correctness of equation. One drawback of this method is that we can't predict constants of many physical quantities. Also, the logarithmic, trigonometric and exponential function is dimensionless. For Example, take Sine of angle = Length/Length is unitless; therefore it is a dimensionless quantity.

Example 2

Check whether the given equation is dimensionally correct.

$W = 1/2\ mv^2 - mgh$

where W stands for work done, m means mass, g stands for gravity, v for velocity and h for height.

To check the above equation as dimensionally correct, we first write dimensions of all the physical quantities mentioned in the equation.

W = Work done = Force × Displacement = $[MLT^{-2}] \times [L] = [ML^2T^{-2}]$

$1/2\ mv^2$ = Kinetic Energy = $[M] \times [L^2T^{-2}] = [ML^2T^{-2}]$

mgh = Potential Energy = $[M] \times [LT^{-2}] \times [L] = [ML^2T^{-2}]$

Since all the dimensions on left and right sides are equal it is a dimensionally correct equation.

Denklemi buraya yazın.

## 3. Investigation of relationship between physical parameters

The method of dimension analysis can also be useful in finding out relations between physical quantities. The related physical dimensions can be organized to give an equation which can be utilized to calculate desired quantitiy with given dimensions

## 3.1 Example of generating equations

We want to know how the speed of waves, v, on a string depends its mass m, length l, and tension Q? We can solve this problem using dimensional analysis.

First work out the dimensions of all the terms:

- Speed, v : $LT^{-1}$
- Mass, m : M
- Length, l : L
- Tension, Q : A force = mass x acceleration: $MLT^{-2}$

Our equation is going to take the form $v = m^a l^b Q^c$ where the power constants a,b and c are unknown.

Re-writing our equation using dimensions: $(LT^{-1}) = (M^a)\ (L^b)\ (M^c\ L^c\ T^{-2c})$

To be dimensionally consistent, each dimension must appear to the same power on each side. Hence:

- For L: 1 = b + c
- For M: 0 = a + c
- For T: -1 = -2c

Solving these equations, we get: a = -1/2, b=1/2 and c = 1/2.

Hence, $v = k\ m^{-1/2}\ l^{1/2}\ Q^{1/2}$ where k is an arbitrary constant.

## 4. Conversion Factor for Units

Dimensional analysis is also used in obtaining the value of the physical quantity in another system. For Example, if we want to convert a physical quantity from S.I or metric system to C.G.S system we can easily do that with the help of dimensional analysis.

A physical quantity has two parts; one is the numerical or magnitude part and the other part is the unit part. Suppose there's a physical quantity X, which has unit "U" and magnitude "N", then it will be expressed as:

X = NU

To convert a physical quantity from one unit to another we use below relation:

$N_1U_1 = N_2U_2$

where $N_1$ and $N_2$ are numerical parts and $U_1$ and $U_2$ are dimensions or units of both quantities.

### 4.1 Example:

How many feet are there in 360 inches?

In order to answer the question, I need to know a"conversion fact." In this case, the fact is:

1 foot = 12 inches.

$$\frac{360\cancel{in}}{1} \bullet \frac{1ft}{12\cancel{in}} = \frac{360ft}{12} = 30ft$$

Convert 60 mph to kilometers per hour; round to the nearest hundredth.

Conversion facts needed

1mi=1760yd 1yd=36in 1m=39in

1km=1000m

$$\frac{60mi}{1hr}\frac{1760yd}{1mi}\frac{36in}{1yd}\frac{1m}{39in}\frac{1km}{1000m}=97.48kmph$$