Introduction to Physical Systems

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Abstract: This lecture deals with definition of physical systems and common exmaples in mechatronics engineering.

1.Introduction

Mechatronics engineering is well known for its multidisciplinary nature. Hence, the related design procedures involves in knowledge of physical systems in different domains.

A physical system is a bounded portion of the physical universe in which components of the system interacts. The energy storage and power flow in a system defines the domain of physics. In mechatronics, the most common physical domains are mechanical, electrical, thermal and fluidic domains.

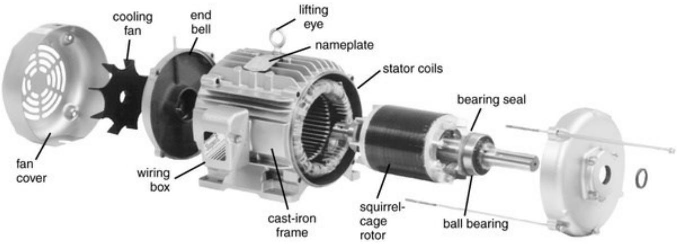

Figure 1. An Electric Motor

A simple electrical motor is considered as a part of mechatronics system. The motor simply has an stator electrical circuit. The coil resistance and inductance defines magnitudes of domain variables. The magnetic energy stored in inductance interacts with rotor conductors and creates a torque in rotating shaft of the rotor. The torquw interacts with load inertia and bearing frictions and creates an observable motion. In the meantime, the bearing friction and resistance of the stator circuit causes a power dissipation in the form of heat. The heat transfer to coil windings increases the temperature of the copper wires. The changing resistance effects electrical circuit. The over heating of the coil may cause a burn out in motor which is an unwanted situation. To prevent the fail of whole system a flow setup is included in motor to transfer dissipated heat to environment. Hence, The care should be taken when considering mutliphysics nature of mechatronics system.

2. A Generic Physical System

The energy based mathematical modeling of systems concerns the energy conversation law to represent the relationship between the components.

In any physical domain, there are three types of elements:

- The first type of element is a <u>potential energy</u> (T) storage element,
- The second type of the elements is a <u>kinetic energy</u> (V) storage element and
- The last type of the elements is a <u>power dissipation (P_d)</u> element.

The relationship between components can be expressed according to energy conversion principle as follows

$$H = T + V \qquad (1)$$

$$\frac{dH}{dt} = P + P_d \qquad (2)$$

The first equation represents totoal energy stored in a system and the second equation states that the rate of change of total energy in a system is equal to sun of input power and dissipated power. (<u>Also note the consistency of units, the unit of energy is Joule (J) and the unit of the power is Watt (W), which is actually time derivative of Joule ($\frac{J}{s}$)</u>).

As the system operates under the effect of an effort variable (e), the changes representing the dynamical relationship between components are observed in a flow variable (f). Power input to system is defined as

$$P = f \cdot e \qquad (3)$$

Each component of the system <u>constitutes a relationship</u> between effort and flow as follows:

$$f = Ce \qquad (4)$$

The parameter C is a structural constant which refers to domain specific attribute of the components. For example, The stiffness of a copper rod should be used in mechanical system and its electrical conductivity should be used when electrical system is in concern.

From measurement point of view for any physical system, one can consider measuring flow and effort variables or structural constants. It should be noted that not only the measurement setups for each measurand is different but also their evaluation should be different referring to the fact that the effort and flow variables are time dependent (transient) whereas the structural constants are time independent.

3. Mechanical Systems

The mechanical systems are formed to generate a desired dynamical relationship between force/torque and the motion. The type of the motion defines characteristics of a mechanical system as

- Translational motion in which bodies translates in space under the effect of forces,
- Rotational Motion in which bodies rotates around an arbitrary axis in space under the effect of torques

3.1Translational mechanical systems

The components of mechanical systems are matter, spring and fricitional elelments. The effort variable is force (unit: Newton [N]) and flow variable is velocity (unit: meter per second [m/s])

Matter is any substance that has mass and takes up space by having volume. In terms of kinematics, the matter reveals itself as mass (unit: Kg), which is a measure of its resistance to acceleration (a change in its state of motion) when a net force is applied. The mass is a kinetic energy storage element. The kinetic energy stored in a mass (m) moving with the velocity (v) is calculated as follows:

$$T = \frac{1}{2}mv^2 \qquad (5)$$

The inertial force resulting from the reaction of the mass is calculated as follows:

$$F_a = m\frac{dv}{dt} \qquad (6)$$

The spring is a representative value of the elasticity of a body. It is a potential energy storage element. The potential energy stored in a spring (k) when it is subjected to relative displacement (x) from its rest position is calculated as follows:

$$V = \frac{1}{2}kx^2 \qquad (7)$$

The spring force resulting from the reaction of the spring is calculated as follows:

$$F_s = kx \qquad (8)$$

Frictional effects arise from mechanical interaction of the surfaces. In most mechanical engineering setups, the surfaces are lubricated (covered with oil) hence the fricitonal effect can be modelled as viscous friction and the force is named drag force. The drag force can be calculated as follows:

$$F_d = Cv \qquad (9)$$

If the velocity of the body moving on frictional surface is v then the power dissipation is

$$P_d = \frac{1}{2}Cv^2 \qquad (10)$$

3.2 Rotational mechanical systems

The components of mechanical systems are matter, rotational spring and rotational fricitional elelments. The effort variable is torque (unit: Newton [Nm]) and flow variable is angular velocity (unit: radian per second [r/s])

The matter reveals itself as mass moment of inertia (unit: Kg-m²), which is a measure of its resistance to angular acceleration (a change in its state of motion) when a net torque is applied. The rotational springs and frictiional effects are arisen from the same concepts like their translational correspondences. The governing formulas for rotational motion is given in table 1.

Table 1. Governing formulas for rotational motion

Component	Force	Energy/Power
Inertia	$\tau_a = I\frac{d\omega}{dt}$	$T = I\omega^2$
Spring	$\tau_s = K\theta$	$V = K\theta^2$
Friction	$\tau_d = C\omega$	$P_d = C\omega^2$

4. Electrical Systems

The electrical systems are formed to generate a desired dynamical relationship between voltage and the current. The components of electrical systems are inductive, capacitive and resistive elements. The effort variable is voltage (unit: Volt [V]) and flow variable is current (unit: Amperé [A]). It should also be noted that current is a representative value corresponding to number of electric charges (unit: [C]oulomb) passing from the cross section of the conductor at an instant.

$I = \frac{Q}{\Delta t}$ $[A = \frac{C}{s}]$ (11)

The inductive elements are coils and when a current flows through them, they become magnetized. Hence, the energy stored on coils is magnetic energy (it corresponds to kinetic energy in mechanical systems). The component characteristics for coil is the inductance (L, unit is Henry) and it also functions as a current storage. The magnetic energy stored in a coil can be calculated with given formula:

$$T = \frac{1}{2} L i^2 \qquad (12)$$

The voltage drop on a coil is proportional to time derivative of the current flow through it.

$$v_L = L \frac{di_L}{dt} \qquad (13)$$

The capacitive elements are capacitors and when a current flows through them, they become charged. Hence, the energy stored in capacitors is electrical potential energy (it corresponds to potential energy in mechanical systems). The component characteristics for capacitor is the capacitance (C, unit is [F]arad) and it also functions as a voltage storage. The electrical potential energy stored in a capacitor can be calculated with given formula:

$$V = \frac{1}{2C} v_c^2 \qquad (14)$$

The current flow through a capacitor is proportional to time derivative of the voltage drop on it.

$$i_c = C \frac{dV_C}{dt} \qquad (15)$$

The resistive elements arise from loses due to collisions in atomic level. The representative value for resistive effects is resistance R (Unit: Ohm Ω). The power dissipated in the form of heat can be calculated as follows:

$$P_d = \frac{1}{2} R i^2 \qquad (16)$$

The voltage drop on a resistive element is proportional to current flow through:

$$v_R = Ri \qquad (17)$$

5. Thermal Systems

The thermal systems are formed to control heat transfer for protecting undesired operating conditions. The main difference between thermal systems and other system is that the concerned flow is not a matter flow but energy flow. Two domain variables are heat flow rate (Unit: [W]att) and temperature. In practice temperature is defined using degrees Celsius (°C), while SI unit for temperature is to use Kelvin (0°K = −273.15°C). However, temperature differences are taken as variable of concern. Using electrical analogy, thermal system can be considered to have two components: Thermal capacitance and resistance.

The thermal capacitance of an object is a measure of how much heat it can store. The heat flow rate to a thermal capacitance is proportional to rate of temperature change of the material. The governing relationship in capacitive systems is as follows:

$$q_c = C \frac{dT_c}{dt} \qquad (18)$$

Thermal resistance of an object is a mathematical relationship relating temperature difference to heat flow through it. Although there are three different mechanisms for heat transfer (conduction, convection and radiation), each of them can be described by domain specific parameters.

$$q_R = \frac{\Delta T_R}{R_T} \qquad (18)$$

6. Flow Systems

The flow systems are formed to control fluid transfer for useful work done (for eg. deport undesired operating conditions). The flow systems are basically mechanical systems but their motion is dominating parameter so they define with a velocity like term as flow variable that is flowrate (unit: [mm³/s]). The flow arises from the external forces which is defined per area that is pressure (unit: [Pa]scal) and in total pressure difference between input output terminals.

Table 2. Flow system equations

Component	**Force**
Inductance	$\Delta P = I_f \frac{dq}{dt}$
Capacitance	$q = C \frac{d\Delta P}{dt}$
REsistance	$q = \frac{\Delta P}{R}$