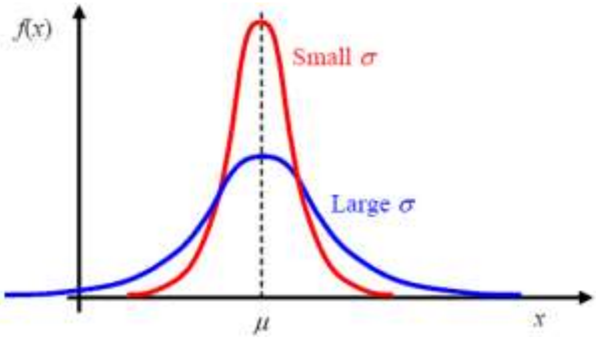
# 3. Graphical Representation of Experimental Data

## D. The Gaussian or Normal Probability Density Function

The ***Gaussian probability density function*** (also called the ***normal probability density function*** or simply the ***normal PDF***) is ***the vertically normalized PDF that is produced from a signal or measurement that has purely random errors.*** The normal probability density function is

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

This special distribution is symmetric about the mean.



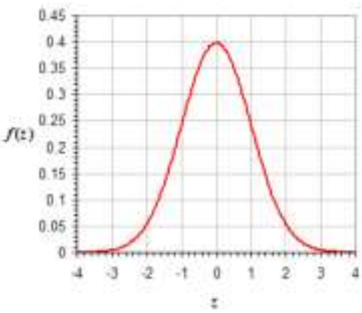

The mean and median are both equal to μ the expected value (at the peak of the distribution). The mode is undefined for a smooth, continuous distribution.
Its plot is commonly called a "bell curve" because of its shape. The actual shape depends on the magnitude of the standard deviation. Namely, if σ is small, the bell will be tall and skinny, while if σ is large, the bell will be short and fat, as sketched.

## Standard normal density function

All of the Gaussian PDF cases, for any mean value and for any standard deviation, can be collapsed into one normalized curve called the standard normal density function by using pre defined z transformation.

$$z=\frac{x-\mu}{\sigma}$$

$$f(z)=\sigma f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$




This standard normal density function is valid for *any* signal measurement, with *any* mean, and with *any* standard deviation, provided that the errors (deviations) are *purely random. It turns out that the probability that variable x lies between some range x1 and x2 is the same as the probability that the transformed variable z lies between the corresponding range z1 and z2, where z is the transformed variable defined above.*

Topic 3.Measurement REsults

***Confidence level and level of significance***

In this standard normal density function z = 1 represents a value of x exactly one standard deviation greater than the mean. Lets define integral equation for calculating probability in standard normal density function as follows

$$A(z) = \int_0^z f(z)dz$$

A(1)=0.3413

for z = –1, Thus, z = –1 represents a value of x exactly one standard deviation less than the mean, by symmetry:

A(1)=0.3413

There is a 68.26% (=2xA(1)x100) probability that for some measurement, the transformed variable z lies within one standard deviation from the mean.

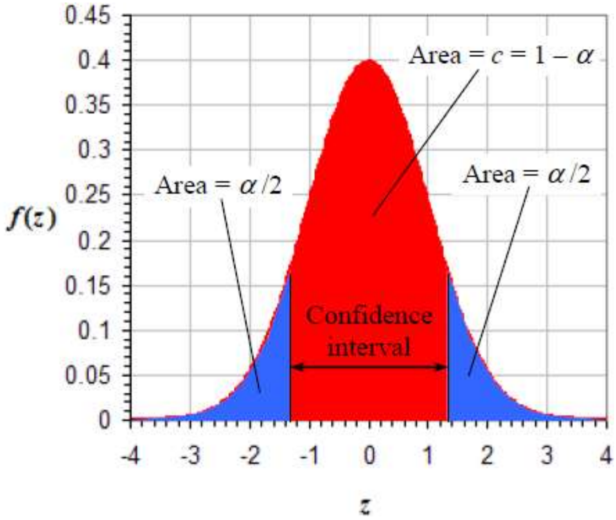
The above example leads to an important concept called ***confidence level***. For the above case, we are 68.26% confident that ***any random measurement of x will lie within*** +/- ***one standard deviation from the mean value***.

A higher confidence level is obtained by choosing a larger z value. For example, for z = 2 (two standard deviations away from the mean), we are 95.44% confident. 95% confidence level that x lies within +/- two standard deviations from the mean.

This is in fact the engineering standard, called the “two sigma confidence level” or the “95% confidence level.

***Confidence level***, ***c***, is defined as ***the probability that a random variable lies within a specified range of values***. The range of values itself is called the ***confidence interval***. For example, as discussed above, we are 95.44% confident that a purely random variable lies within +/- two standard deviations from the mean. We state this as a confidence level of *c* = 95.44%, which we usually round off to 95% for practical engineering statistical analysis.

***Level of significance***, $\alpha$, is defined as ***the probability that a random variable lies outside of a specified range of values***. In the above example, we are 100 – 95.44 = 4.56% confident that a purely random variable lies either *below* or *above* two standard deviations from the mean.



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