EEE 2015 ELECTRICS

# L02 Alternating Voltage and Current

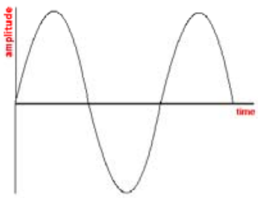


## Alternating Voltage

A DC voltage or current has a fixed magnitude (amplitude) and a definite direction associated with it. For example, +12V represents 12 volts in the positive direction, or -5V represents 5 volts in the negative direction.

An alternating function or AC Waveform on the other hand is defined as one that varies in both **magnitude and direction** in more or less an even manner with respect to time making it a **"Bi-directional" waveform**.

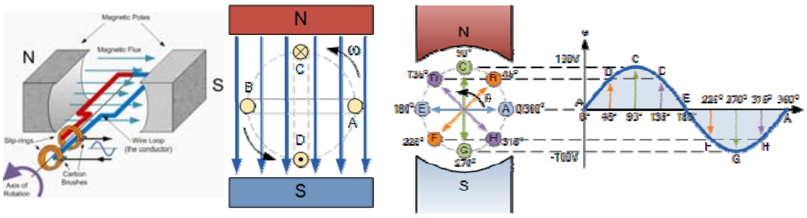
An AC waveform is constantly changing its polarity every half cycle alternating between a positive maximum value and a negative maximum value respectively with regards to time




The shape of an oscillating voltage

## Why it is an Alternating Voltage?

The periodic or AC waveform is the resulting product of a rotating electrical generator.




As the coil rotates anticlockwise around the central axis which is perpendicular to the magnetic field, the wire loop **cuts the lines of magnetic force** set up between the north and south poles **at different angles as the loop rotates**.
**The amount of induced EMF** in the loop at any instant of time is proportional to <u>the angle of rotation of the wire loop</u>.
As this wire loop rotates, electrons in the wire flow in one direction around the loop.
Now when the wire loop has rotated past the 180º point and moves across the magnetic lines of force in the opposite direction, the electrons in the wire loop change and flow in the opposite direction.

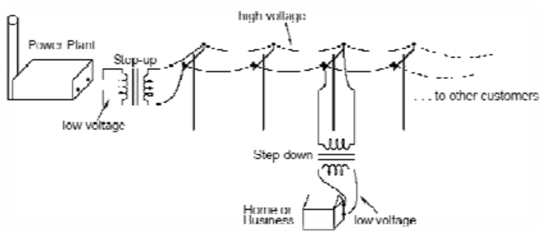




## Why Using Alternating Voltage is advantegous

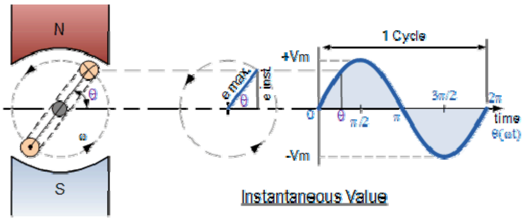
Why it is better:

- •The generation of A.C. is cheaper than that of D.C.
- •A.C. machines are simple , robust and do mot require much attention for their repairs and maintainance during their use.
- •Wide range of voltages are obtained by the use of transformer.
- •The magnitude of current can be reduced by using an inductance or a conductor without any appreciable loss of energy
- •A.C. can easily be converted into D.C. with the help of rectifiers.
- •When A.C. is supplied at higher voltages in long distance transmission , the line losses are small compared to a D.C. transmission



## Alternating Voltage Concepts

Instantaneous Voltage : Terminal voltage at any time instant t after any zero crossing




To estimate the voltage value, the t ime instant should be converted to angle. It can be done using angular frequency (or rotating speed of rotor).

$$\theta = \omega t = \frac{2\pi}{T} t$$

T: period: the time required to complete one revolution

$$V_{inst} = V_{max} \sin\theta$$

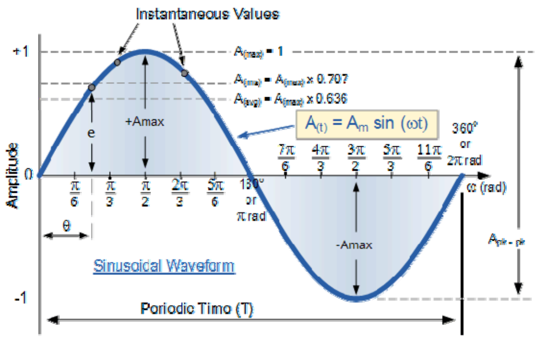
$V_{max}$ is the maximum voltage induced in the coil





## The Normalized Sinusoidal Waveform

Nominal Sinusoidal Signal Charactersitics




$$f = \frac{1}{T}$$

$$V_{avg} = \frac{2V_{max}}{\pi} = 0.663V_{max}$$

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = 0.707V_{max}$$

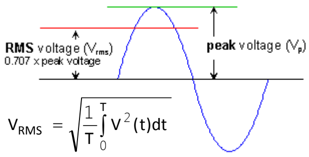
f: Frequency or oscillation of a signal is the value of repetition observed in a changing signal in unit time (per second) [Hz]
The Amplitude (A)=$V_{max}$ [V]
$V_{avg}$ : (Full Rectified) Average Voltage, average value of the signal assuming that both of the peaks are positive sided. [V]
$V_{rms}$ : Effective Value of the signal [V]

## The Normalized Sinusoidal Waveform

The RMS value of a signal

The value of an AC voltage is continually changing from zero up to the positive peak, through zero to the negative peak and back to zero again. Clearly for most of the time it is less than the peak voltage, so this is not a good measure of its real effect

The RMS value is the effective value of a varying voltage or current. It is the equivalent steady DC (constant) value which gives the same effect.




$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = 0.707 V_{max}$$

AC voltmeters and ammeters show the RMS value of the voltage or current.

## The Sinusoidal Waveform of AC Network of Turkey

f: Frequency = 50 [Hz]

The Amplitude (A)=$V_{max}$ [V]=220 $\sqrt{2}$ =311 [V]

$V_{avg}$ : (Full Rectified) Average Voltage= 198 [V]

$V_{rms}$ : Effective Value of the signal [V]=220

Function:

$$V_{AC}(t) = V_{max}\ \sin(2\pi f t) = 220\ \sqrt{2}\ \sin(2\pi 50\ t)$$



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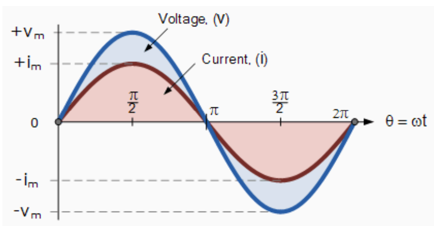
## Phase

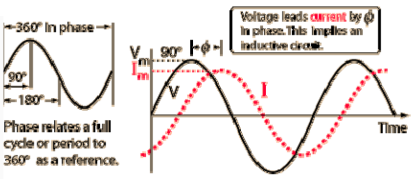
An important point in operations of two time dependent signals is whether they are synchronized or not.

For electrical definitions, two voltage signals or two corrent signals or one voltage with one current signals can be either synchronized or with phase diference.

If two signals are synchronized, they both pass the zero points and the maximum value points at the same time.

Synchronized (in phase)




Dr. Levent ÇETİN

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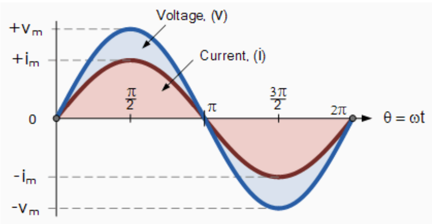


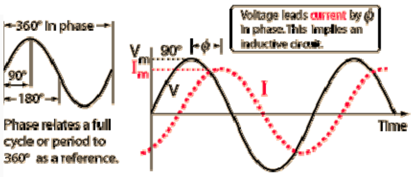




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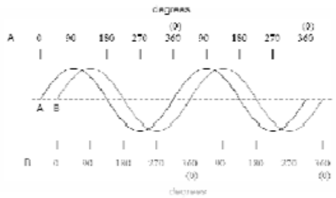
## Phase

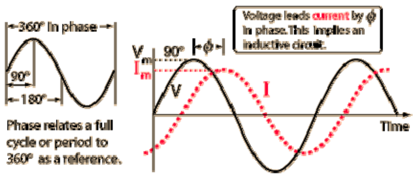
If two signals pass the zero points and maximum value points in different moments then a phase shift occurs.

**Phase or phase shift is the measure of the difference between two signals in time scale.**

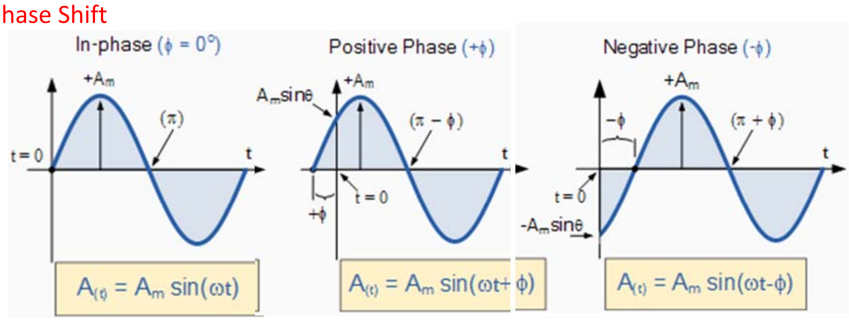
The phase shift or phase is denoted with degree in sine functions.

If one of the two sine functions is accepted as reference signal; the value of angle of the signal that is not the reference signal when the reference signal reaches zero is the phase value. Accordingly, +45 +90 +180 and 0 degrees of phase shifts are shown in the figure below



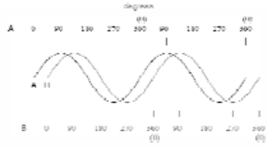


## Phase Shift




The amount of phase shift between two waves can be expressed in terms of degrees, as defined by the degree units on the horizontal axis of the waveform graph used in plotting the trigonometric sine function.

A leading waveform is defined as one waveform that is ahead of another in its evolution.

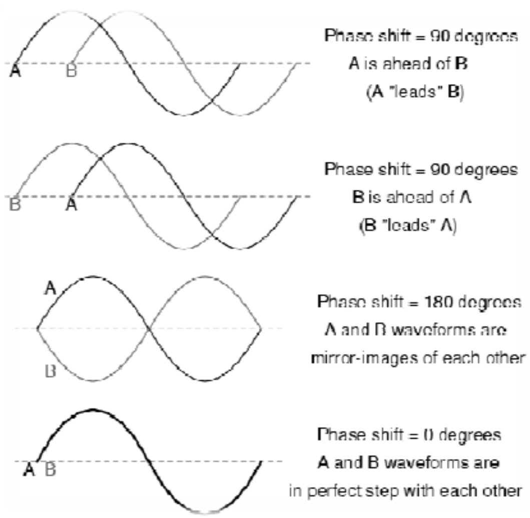


A lagging waveform is one that is behind another.



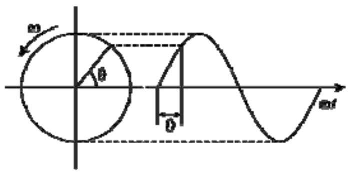


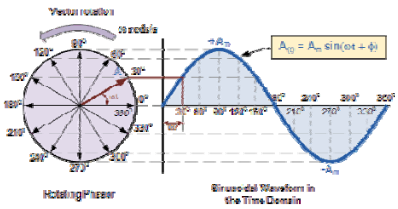
## Phase shift examples



## Phasor Representaion of AC

In order to define the affects of the alternating current to a circuit, frequency, amplitude and phase need to be known.
Frequency depends on the electrical grid, so the country or region. So, the voltage/current functions can be defined depending on two parameters.
One of the choice in modeling alternating current / voltage is to represent them using rotating vectors (namely, phasors).



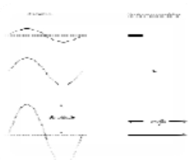



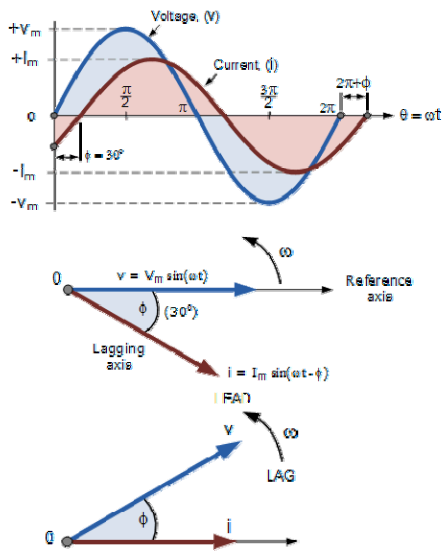

The projection of a rotating vector around origin in cartesian coordinate system is a sine function as can be seen in figure above.
The length (or the radius) of the rotating vector is the amplitude of alternating voltage in this representaion.
The angle between the vector and the horizantal axis is the phase value (θ).
The angular velocity of this rotating vector is the frequency of alternating voltage.



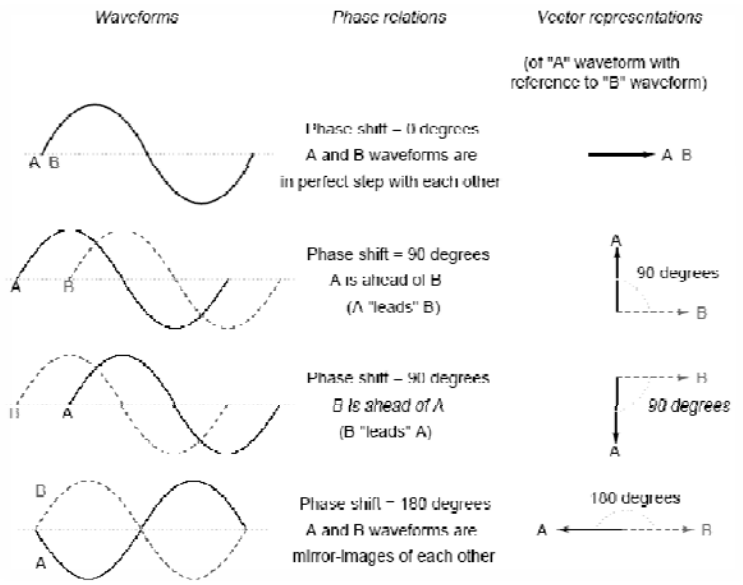


## Phasor Representaion of AC





## Phasor Representaion of AC

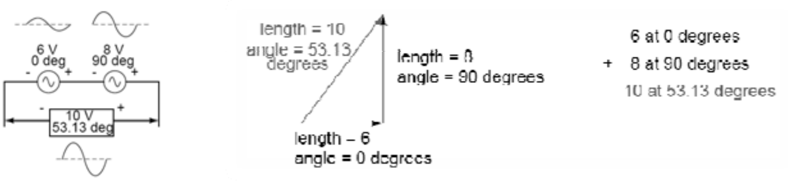







## Phasor as Complex Numbers

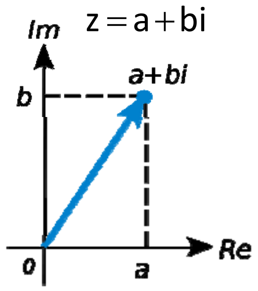
Created voltage by a combination of AC current/voltage sources connected to the same circuit is determined by vector addition and subtraction operations.

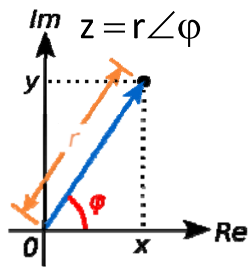



Two dimensional vectors can also be represented via complex numbers

## Phasor as Complex Numbers

A complex number is a mathematical quantity representing two dimensions of magnitude and direction. Representation of alternating voltage as a rotating vector indicates a complex number in means of mathematics. As known, complex numbers has two parts called real and imaginary which are represented in complex plane.
The common representation of a complex number in cartesian form is equation (1) whereas phasor representation that is representation of a complex number as a rotating vector and more used in electrical circuit analysis is the equation (2) below.




$$r = \sqrt{a^2 + b^2}$$

$$\varphi = tg^{-1}\left(\frac{b}{a}\right)$$





## Phasor as Complex Numbers

Four basic operations in complex numbers can be seen below.

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)i$$

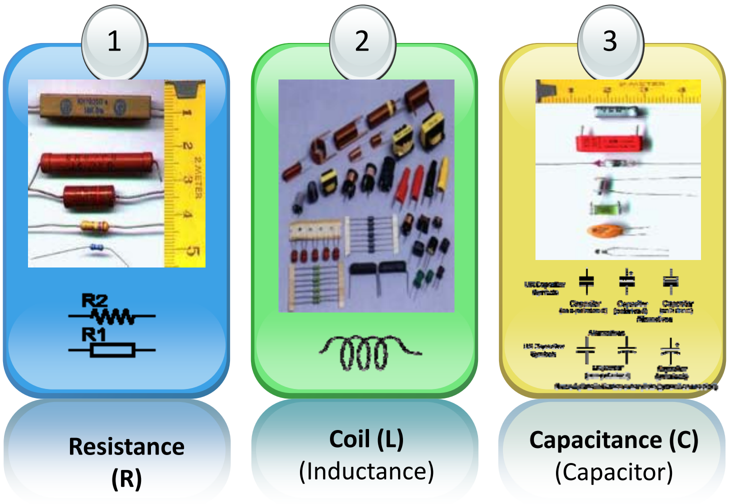
$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)i$$

$$z_1 \times z_2 = r_1 \times r_2 \angle \varphi_1 + \varphi_2$$

$$z_1 \div z_2 = r_1 \div r_2 \angle \varphi_1 - \varphi_2$$

Implementation of complex number arithmetics to voltage/current signals is examination of total effects of voltage/current sources which have different phases.
Before calculating this effect, the state of the two rotating vectors, which have different phases, according to each other should be understood.
This relation might be described as vectors which have the same starting point but have different angles respect to the horizontal line.

Behaviors of Basic Circuit Components under AC







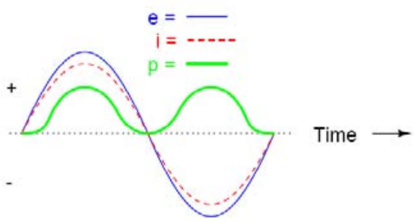

Behaviors of Basic Circuit Components under AC

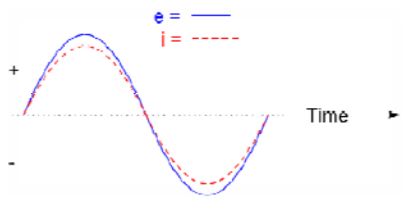
**Resistance (R)**

Ohm's Law can be used for resistance under the influence of alternating voltage.

$$V = V_{max}\sin(wt) \qquad R = \frac{V}{I} \qquad I = \frac{V_{max}}{R}\sin(wt)$$

According to equations above, there is no phase shift between current and voltage on a resistor. Nevertheles, the amplitude changes due to Ohm's Law.



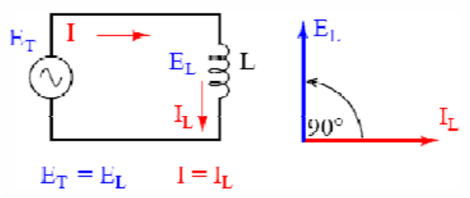
## Behaviors of Basic Circuit Components under AC

**Coil (L)** (Inductor)

In contrast with resistors, coils under alternating voltage resists against alternating current. The voltage on a coil (the voltage measured between two terminals) can be calculated using Lenz Law.

$$V(t) = L\frac{di(t)}{dt}$$

If this equation is studied considering the alternating current, the relationship between the current and voltage might be predicted.




$$I(t) = I_{max}\sin(wt)$$

$$V(t) = L\frac{di(t)}{dt} = I_{max}Lw\cos(wt)$$





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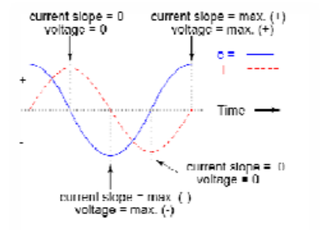
## Behaviors of Basic Circuit Components under AC

**Coil (L)** (Inductor)




$$I(t) = I_{max}\sin(wt)$$

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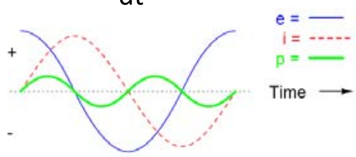



This result shows us that there is a 90 degrees of phase shift between voltage and current on a coil under AC. **The voltage leads current by phase angle of 90 degree**.

The phase shift results with negative electrical power. Negative power denotes that the coil transfer power to the circuit.

**The 'resistance' of coils changes due to frequency. This is called as reactance (inductive reactance $X_L$) for this reason.**

$$X_L = wL = 2\pi fL$$

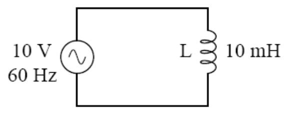
Table 3.1: *Reactance of a 10 mH inductor:*

| Frequency (Hertz) | Reactance (Ohms) |
|---|---|
| 60 | 3.7699 |
| 120 | 7.5398 |
| 2500 | 157.0796 |

## Behaviors of Basic Circuit Components under AC

**Coil (L)** (Inductor)

***Ohm's Law*** might be implemented easily to alternating current circuits using quantity, the reactance. In that case, the calculations should be made using complex numbers instead of scalars.




$$X = \frac{V}{I}$$

$$X_L = 2\pi 60 \times 10^{-2} = 3.7699\,\Omega$$

$$I = \frac{V}{X} = \frac{10}{3.7699} = 2.6526\,A$$





## Behaviors of Basic Circuit Components under AC

**Capacitor (C)** (Capacitance)

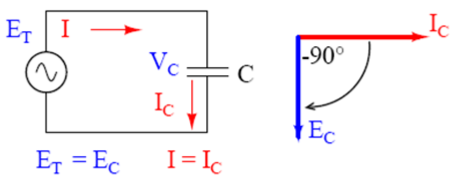
Capacitors react different due to the voltage level applied to them under alternating current:

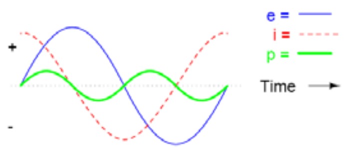
If the voltage level applied is greater than the voltage on a capacitor, the source charges the capacitor;

in opposite case, capacitor behaves like a source. And discharges through the circuit elements

The current equation for a capacitor is:

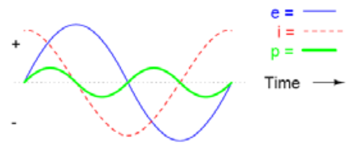
$$i(t) = C\frac{dv(t)}{dt}$$



## Behaviors of Basic Circuit Components under AC

**Capacitor (C)** (Capacitance)




The phase difference between voltage and current is 90 degrees on a capacitor or in other words, **current leads voltage 90 degrees.**
This case results with negative electrical power which means that capacitor transfers power to the circuit (i.e. Capacitor discharges its electrical charge).
The 'resistance' of the capacitors change due to the frequency of the alternating voltage. The higher frequency of the AC signal, the more easily that signal will pass through the capacitor. **Thus, this is called as capacitive reactance, $X_c$.**

Table 4.1: *Reactance of a 100 uF capacitor:*

| Frequency (Hertz) | Reactance (Ohms) |
|---|---|
| 60 | 26.5258 |
| 120 | 13.2629 |
| 2500 | 0.6366 |

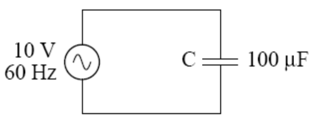
$$X_c = \frac{1}{wC} \qquad X_C = \frac{1}{2\pi f C}$$

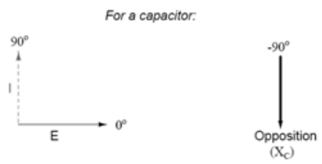




## Behaviors of Basic Circuit Components under AC

**Capacitor (C)** (Capacitance)




$$X_C = 26.5258\ \Omega$$

$$I = \frac{E}{X}$$

$$I = \frac{10\ V}{26.5258\ \Omega}$$

$$I = 0.3770\ A$$

$$\text{Opposition} = \frac{\text{Voltage}}{\text{Current}}$$

$$\text{Opposition} = \frac{10\ V \angle 0^\circ}{0.3770\ A \angle 90^\circ}$$

$$\text{Opposition} = 26.5258\ \Omega \angle -90^\circ$$

The current of the AC source leads the voltage of the source 90 degrees. The resistance effect of the capacitor to AC source is calculated considering this.

## Behaviors of Basic Circuit Components under AC

**Resistor, Reactance and Impedance**

The resistance against the current can be in three types:

**Resistance**: It is the friction of electrons during motion. Its symbol is “R” and unit is [Ω] (i.e. [Ohm]). It does not form any phase shift.

**Reactance**: It is the inertia of electrons. It occurs if there is a change in voltage or current values (if an electric or magnetic field occurs). The capacitor and inductor are the main circuit components which this influence is highly distinct.
If there is a reactance effect in a circuit, there is also phase shift. If the component is a capacitor, the current leads voltage by 90 degrees whereas if it is an inductance, the current lags voltage by 90 degrees.

**Impedance**, is the strain against the current in an electrical circuit. Impedance is the total resistance and reactance effects of all components. The resistance in DC circuits is the impedance in AC’s





## Behaviors of Basic Circuit Components under AC

**Ohm’s Law via Impedance**

The AC implemented Ohm’s Law can be seen as below. Please consider that all the quantities are in complex number form in the equation below :

$$Z = R + iX_{total}$$

$$Z = \frac{V}{I}$$

Like Ohm’s Law, other laws (Kirrschoff’s, grid theorems, etc.) used in circuit analysis can be also implemented in AC in condition of using complex numbers.