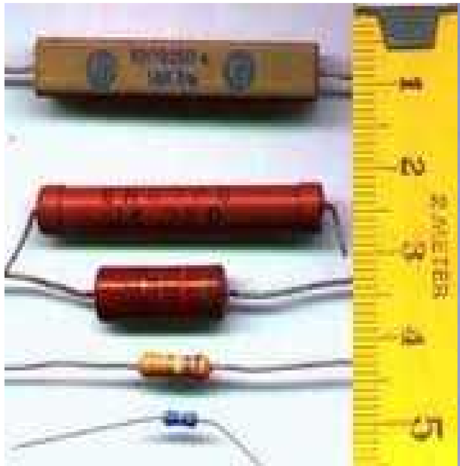
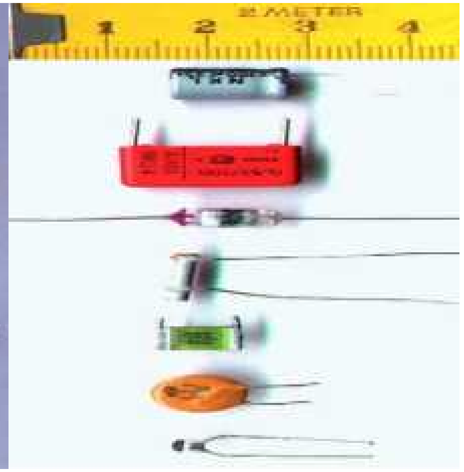
EEE 2015 ELECTRICS

# L03 AC Circuit Analysis&Power Calculations

## Behaviors of Basic Circuit Components under AC

| Resistor (R) | Coil (L) (Inductor) | Capacitor(C) (Condenser) |
|---|---|---|
| **Resistor** $100\ \Omega$ | **Inductor** $100$ mH, $159.15$ Hz | **Capacitor** $10\ \mu F$, $159.15$ Hz |
| $R = 100\ \Omega$ | $R = 0\ \Omega$ | $R = 0\ \Omega$ |
| $X = 0\ \Omega$ | $X = 100\ \Omega$ | $X = 100\ \Omega$ |
| $Z = 100\ \Omega \angle 0^{\circ}$ | $Z = 100\ \Omega \angle 90^{\circ}$ | $Z = 100\ \Omega \angle -90^{\circ}$ |





## AC circuit –RL in series

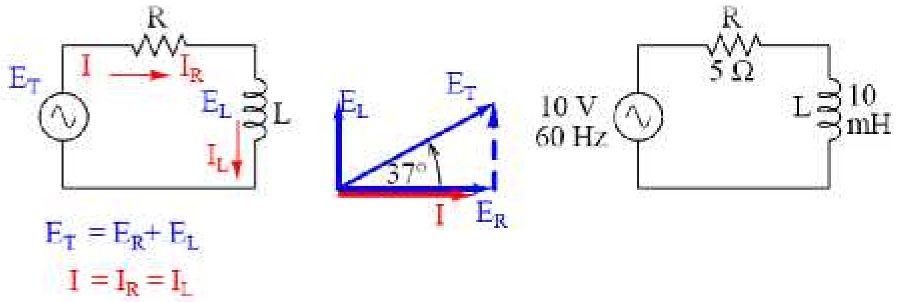



Figure 3.10: *Series resistor inductor circuit: Current lags applied voltage by 0° to 90°.*

Inductive reactance of the coil

$$X_L = 0 + 3.7699j\Omega$$

The total effect is called as **impedance**.

$$Z = R + X_L = 5 + 3.7699j\Omega = 6.262\angle 37.016\Omega$$

$$Z = \frac{V}{I}$$

$$I = \frac{10\angle 0V}{6.262\angle 37.016\Omega} = 1.597\angle -37.016A$$

# AC circuit –RL in series

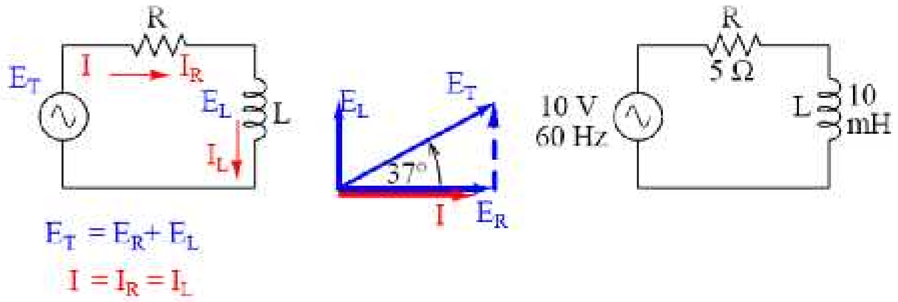



Figure 3.10: *Series resistor inductor circuit: Current lags applied voltage by 0° to 90°.*

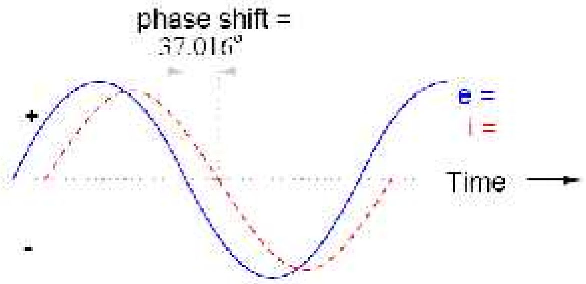



Figure 3.11: *Current lags voltage in a series L-R circuit.*

$$E = IZ$$

$$E_R = I_R Z_R$$

$$E_R = (1.597 \text{ A} \angle -37.016^\circ)(5\ \Omega \angle 0^\circ)$$

$$E_R = 7.9847 \text{ V} \angle -37.016^\circ$$

$$E = IZ$$

$$E_L = I_L Z_L$$

$$E_L = (1.597 \text{ A} \angle -37.016^\circ)(3.7699\ \Omega \angle 90^\circ)$$

$$E_L = 6.0203 \text{ V} \angle 52.984^\circ$$

*Notice that the phase angle of $E_L$ is exactly $90^o$ more than the phase angle of the current.*

$$E_{total} = E_R + E_L$$

$$E_{total} = (7.9847 \text{ V} \angle -37.016^\circ) + (6.0203 \text{ V} \angle 52.984^\circ)$$

$$E_{total} = 10 \text{ V} \angle 0^\circ$$

# AC circuit –RL in parallel

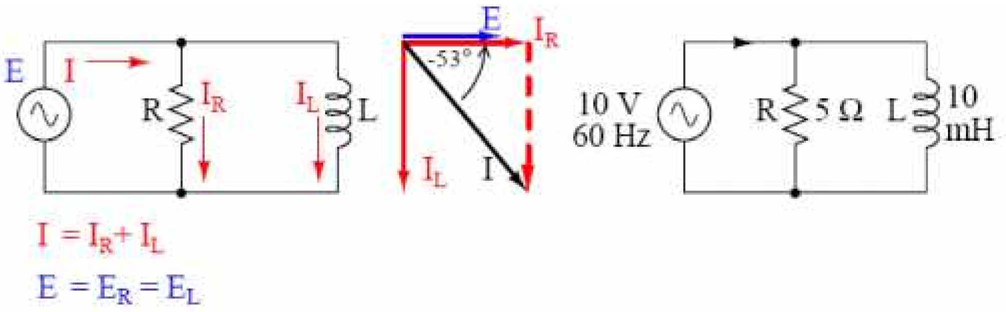
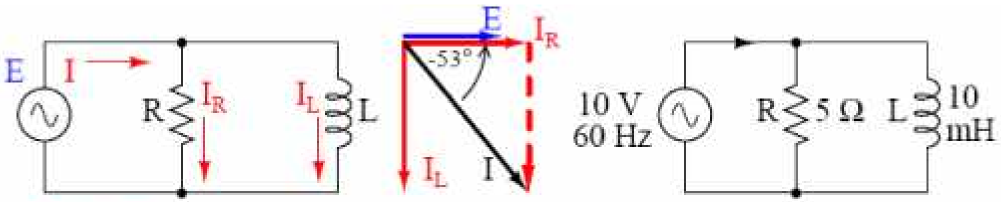



Figure 3.14: *Parallel R-L circuit.*

| | R | L | Total | |
|---|---|---|---|---|
| E | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | Volts |
| I | | | | Amps |
| Z | $5 + j0$<br>$5 \angle 0^\circ$ | $0 + j3.7699$<br>$3.7699 \angle 90^\circ$ | | Ohms |

*Rule of parallel circuits:*
$E_{total} = E_R = E_L$

# AC circuit –RL in parallel




$I = I_R + I_L$

$E = E_R = E_L$

Figure 3.14: *Parallel R-L circuit.*

| | R | L | Total | |
|---|---|---|---|---|
| E | 10 + j0<br>10 ∠ 0° | 10 + j0<br>10 ∠ 0° | 10 + j0<br>10 ∠ 0° | Volts |
| I | **2 + j0**<br>**2 ∠ 0°** | **0 - j2.6526**<br>**2.6526 ∠ -90°** | | Amps |
| Z | 5 + j0<br>5 ∠ 0° | 0 + j3.7699<br>3.7699 ∠ 90° | | Ohms |

*Ohm's Law* $I = \frac{E}{Z}$ (R column)

*Ohm's Law* $I = \frac{E}{Z}$ (L column)

# AC circuit –RL in parallel

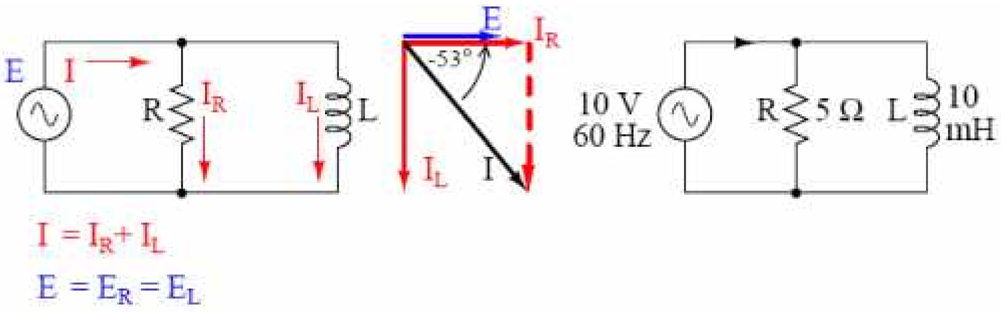


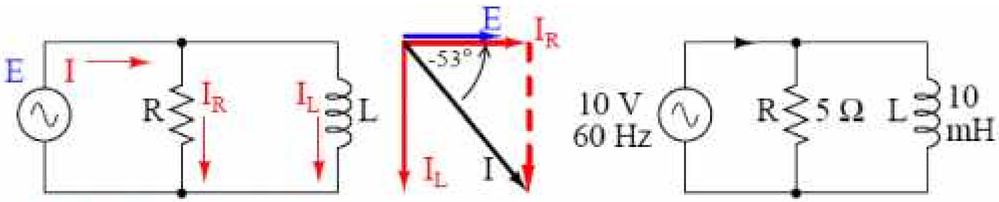

Figure 3.14: *Parallel R-L circuit.*

| | R | L | Total | |
|---|---|---|---|---|
| E | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | Volts |
| I | $2 + j0$<br>$2 \angle 0^\circ$ | $0 - j2.6526$<br>$2.6526 \angle -90^\circ$ | $\mathbf{2 - j2.6526}$<br>$\mathbf{3.3221} \angle -52.984^\circ$ | Amps |
| Z | $5 + j0$<br>$5 \angle 0^\circ$ | $0 + j3.7699$<br>$3.7699 \angle 90^\circ$ | | Ohms |

*Rule of parallel circuits:*

$I_{total} = I_R + I_L$

# AC circuit –RL in parallel




$I = I_R + I_L$

$E = E_R = E_L$

Figure 3.14: *Parallel R-L circuit.*

| | R | L | Total | |
|---|---|---|---|---|
| E | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | Volts |
| I | $2 + j0$<br>$2 \angle 0^\circ$ | $0 - j2.6526$<br>$2.6526 \angle -90^\circ$ | $2 - j2.6526$<br>$3.322 \angle -52.984^\circ$ | Amps |
| Z | $5 + j0$<br>$5 \angle 0^\circ$ | $0 + j3.7699$<br>$3.7699 \angle 90^\circ$ | **$1.8122 + j2.4035$**<br>**$3.0102 \angle 52.984^\circ$** | Ohms |

*Ohm's Law*

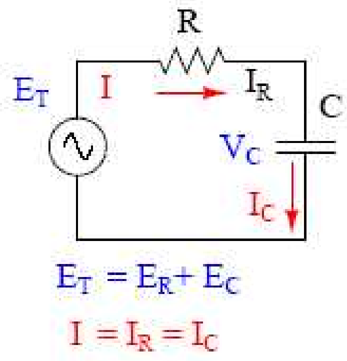
$$Z = \frac{E}{I}$$

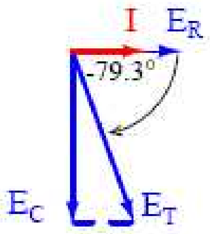
*or*

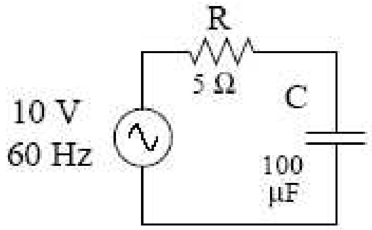
*Rule of parallel circuits:*

$$Z_{total} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L}}$$

## AC circuit –RC in series




$$X_c = 0 - 26.5258j\Omega$$

$$R = 5 + 0j\Omega$$

$$Z = R + X_c = 5 - 26.5258j\Omega = 26.993\angle -79.325$$

$$I = \frac{E}{Z}$$

$$I = \frac{10\ \text{V} \angle 0^\circ}{26.933\ \Omega \angle -79.325^\circ}$$

$$I = 370.5\ \text{mA} \angle 79.325^\circ$$

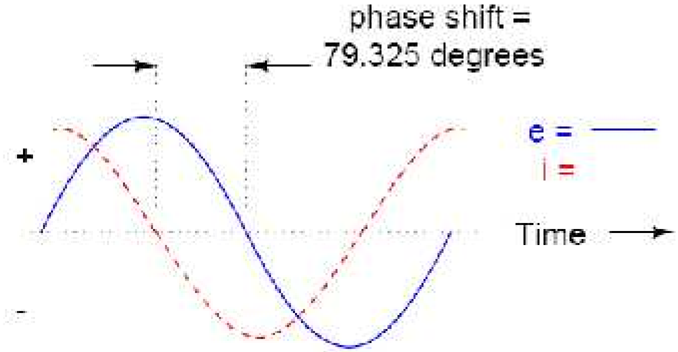
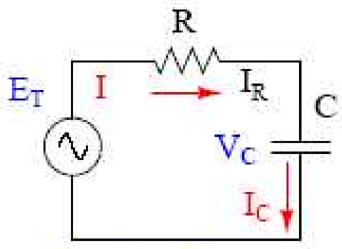
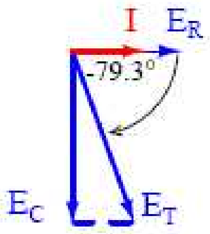
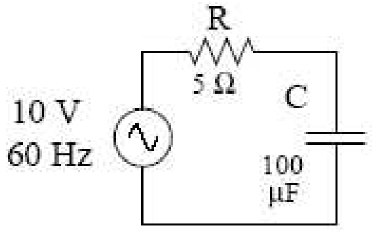



Figure 4.11: *Voltage lags current (current leads voltage)in a series R-C circuit.*

# AC circuit –RC in series




$E_T = E_R + E_C$

$I = I_R = I_C$

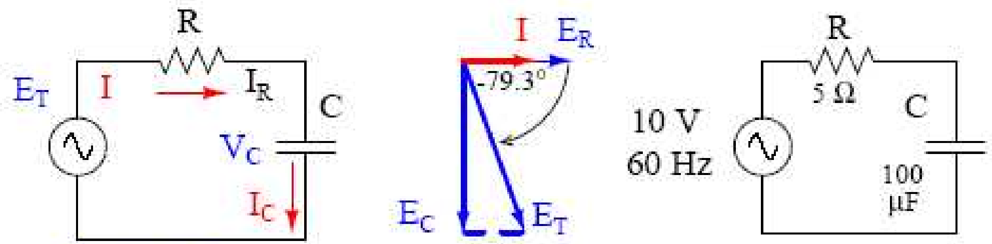
| | R | C | Total | |
|---|---|---|---|---|
| E | | | 10 + j0<br>10 ∠ 0° | Volts |
| I | | | 68.623m + j364.06m<br>370.5m ∠ 79.325° | Amps |
| Z | 5 + j0<br>5 ∠ 0° | 0 - j26.5258<br>26.5258 ∠ -90° | 5 - j26.5258<br>26.993 ∠ -79.325° | Ohms |

| | R | C | Total | |
|---|---|---|---|---|
| E | | | 10 + j0<br>10 ∠ 0° | Volts |
| I | **68.623m + j364.06m**<br>**370.5m ∠ 79.325°** | **68.623m + j364.06m**<br>**370.5m ∠ 79.325°** | 68.623m + j364.06m<br>370.5m ∠ 79.325° | Amps |
| Z | 5 + j0<br>5 ∠ 0° | 0 - j26.5258<br>26.5258 ∠ -90° | 5 - j26.5258<br>26.993 ∠ -79.325° | Ohms |

*Rule of series circuits:*

$I_{total} = I_R = I_C$

# AC circuit –RC in series




$E_T = E_R + E_C$

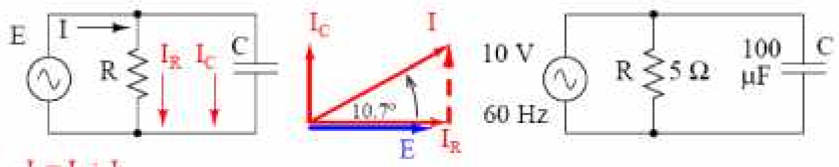
$I = I_R = I_C$

| | R | C | Total | |
|---|---|---|---|---|
| E | **343.11m + j1.8203**<br>**1.8523 ∠ 79.325°** | **9.6569 - j1.8203**<br>**9.8269 ∠ -10.675°** | 10 – j0<br>10 ∠ 0° | Volts |
| I | 68.623m + j364.06m<br>370.5m ∠ 79.325° | 68.623m + j364.06m<br>370.5m ∠ 79.325° | 68.623m + j364.06m<br>370.5m ∠ 79.325° | Amps |
| Z | 5 + j0<br>5 ∠ 0° | 0 - j26.5258<br>26.5258 ∠ -90° | 5 - j26.5258<br>26.993 ∠ -79.325° | Ohms |

Ohm's Law $E = IZ$ (R column) Ohm's Law $E = IZ$ (C column)

As it can be considered easily, the phase shift is 79.325 degrees in this circuit whereas in the circuit that has only one capacitor it was 90 degrees.
The current and the voltage on the resistor is on the same phase as it is mentioned.
However, the current on a capacitor leads voltage by 90 degrees.

# AC circuit –RC in parallel




$I = I_R + I_L$

$E = E_R = E_C$

F

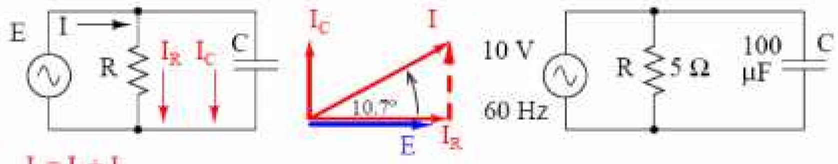
| | R | C | Total | |
|---|---|---|---|---|
| E | | | $10 + j0$<br>$10 \angle 0°$ | Volts |
| I | | | | Amps |
| Z | $5 + j0$<br>$5 \angle 0°$ | $0 - j26.5258$<br>$26.5258 \angle -90°$ | | Ohms |

| | R | C | Total | |
|---|---|---|---|---|
| E | **$10 + j0$**<br>**$10 \angle 0°$** | **$10 + j0$**<br>**$10 \angle 0°$** | $10 + j0$<br>$10 \angle 0°$ | Volts |
| I | | | | Amps |
| Z | $5 + j0$<br>$5 \angle 0°$ | $0 - j26.5258$<br>$26.5258 \angle -90°$ | | Ohms |

*Rule of parallel circuits:*

$E_{total} = E_R = E_C$

# AC circuit –RC in parallel




$I = I_R + I_L$

$E = E_R = E_C$

Figur

| | R | C | Total | |
|---|---|---|---|---|
| E | $10 + j0$<br>$10 \angle 0°$ | $10 + j0$<br>$10 \angle 0°$ | $10 + j0$<br>$10 \angle 0°$ | Volts |
| I | **$2 + j0$**<br>**$2 \angle 0°$** | **$0 + j376.99m$**<br>**$376.99m \angle 90°$** | | Amps |
| Z | $5 + j0$<br>$5 \angle 0°$ | $0 - j26.5258$<br>$26.5258 \angle -90°$ | | Ohms |

*Ohm's Law*

$I = \frac{E}{Z}$

*Ohm's Law*

$I = \frac{E}{Z}$

| | R | C | Total | |
|---|---|---|---|---|
| E | $10 + j0$<br>$10 \angle 0°$ | $10 + j0$<br>$10 \angle 0°$ | $10 + j0$<br>$10 \angle 0°$ | Volts |
| I | $2 + j0$<br>$2 \angle 0°$ | $0 + j376.99m$<br>$376.99m \angle 90°$ | **$2 + j376.99m$**<br>**$2.0352 \angle 10.675°$** | Amps |
| Z | $5 + j0$<br>$5 \angle 0°$ | $0 - j26.5258$<br>$26.5258 \angle -90°$ | | Ohms |

*Rule of parallel circuits:*

$I_{total} = I_R + I_C$

# AC circuit –RC in parallel

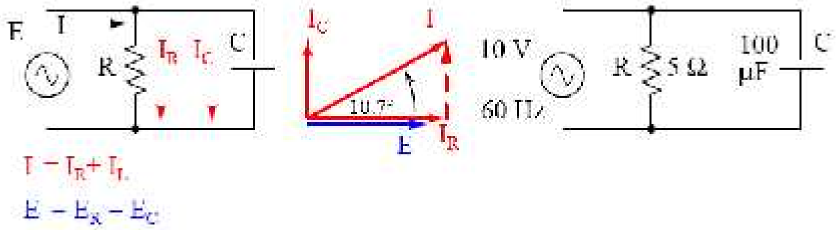



Figure 4.14: *Parallel R-C circuit.*

| | R | C | Total | |
|---|---|---|---|---|
| **E** | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | $10 + j0$<br>$10 \angle 0^\circ$ | Volts |
| **I** | $2 + j0$<br>$2 \angle 0^\circ$ | $0 + j376.99m$<br>$376.99m \angle 90^\circ$ | $2 + j376.99m$<br>$2.0352 \angle 10.675^\circ$ | Amps |
| **Z** | $5 + j0$<br>$5 \angle 0^\circ$ | $0 - j26.5258$<br>$26.5258 \angle -90^\circ$ | **4.8284 j910.14m**<br>**4.9135 ∠ -10.675°** | Ohms |

*Ohm's Law*

$$Z = \frac{E}{I}$$

*or*

*Rule of parallel circuits:*

$$Z_{total} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$

## AC circuit –RLC in series

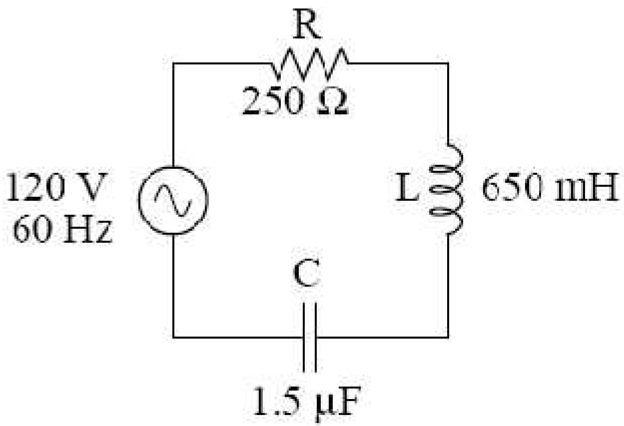



Figure 5.2: *Example series R, L, and C circuit.*

$$X_L = 2\pi fL$$

$$X_L = (2)(\pi)(60\text{ Hz})(650\text{ mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(60\text{ Hz})(1.5\ \mu\text{F})}$$

$$X_C = 1.7684\text{ k}\Omega$$

$$Z_R = 250 + j0\ \Omega \quad \textit{or} \quad 250\ \Omega \angle 0^{\circ}$$

$$Z_L = 0 + j245.04\ \Omega \quad \textit{or} \quad 245.04\ \Omega \angle 90^{\circ}$$

$$Z_C = 0 - j1.7684\text{k}\ \Omega \quad \textit{or} \quad 1.7684\text{ k}\Omega \angle -90^{\circ}$$

# AC circuit –RLC in series

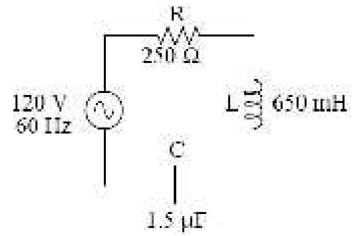



Figure 5.2: Example series R, L, and C circuit.

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | | | | **120 + j0**<br>**120 ∠ 0°** | Volts |
| I | | | | | Amps |
| Z | **250 + j0**<br>**250 ∠ 0°** | **0 + j245.04**<br>**245.04 ∠ 90°** | **0 - j1.7684k**<br>**1.7684k ∠ -90°** | | Ohms |

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | | | | | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | **250 - j1.5233k**<br>**1.5437k ∠ -80.680°** | Ohms |

*Rule of series circuits:*

$$Z_{total} = Z_R + Z_L + Z_C$$

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | | | | **12.589m + 76.708m**<br>**77.734m ∠ 80.680°** | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | 250 - j1.5233k<br>1.5437k ∠ -80.680° | Ohms |

*Ohm's Law*

$$I = \frac{E}{Z}$$

# AC circuit –RLC in series

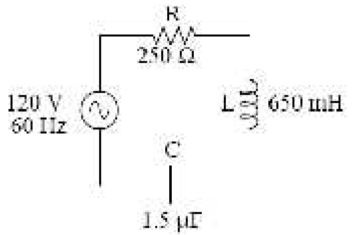



*Figure 5.2: Example series R, L, and C circuit.*

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | **12.589m + 76.708m**<br>**77.734m ∠ 80.680°** | **12.589m + 76.708m**<br>**77.734m ∠ 80.680°** | **12.589m + 76.708m**<br>**77.734m ∠ 80.680°** | 12.589m + 76.708m<br>77.734m ∠ 80.680° | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | 250 - j1.5233k<br>1.5437k ∠ -80.680° | Ohms |

*Rule of series circuits:*
$I_{total} = I_R = I_L = I_C$

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | **3.1472 + j19.177**<br>**19.434 ∠ 80.680°** | **-18.797 + j3.0848**<br>**19.048 ∠ 170.68°** | **135.65 - j22.262**<br>**137.46 ∠ -9.3199°** | 120 + j0<br>120 ∠ 0° | Volts |
| I | 12.589m + 76.708m<br>77.734m ∠ 80.680° | 12.589m + 76.708m<br>77.734m ∠ 80.680° | 12.589m + 76.708m<br>77.734m ∠ 80.680° | 12.589m + 76.708m<br>77.734m ∠ 80.680° | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | 250 - j1.5233k<br>1.5437k ∠ -80.680° | Ohms |
| | Ohm's Law<br>E = IZ | Ohm's Law<br>E = IZ | Ohm's Law<br>E = IZ | | |

$E_R + E_L + E_C$ *should equal* $E_{total}$

$$
\begin{array}{rl}
3.1472 + j19.177\ \text{V} & E_R \\
-18.797 + j3.0848\ \text{V} & E_L \\
+\ 135.65 - j22.262\ \text{V} & E_C \\
\hline
120 + j0\ \ \text{V} & E_{total}
\end{array}
$$

It should be considered that the amplitude of the voltage on the capacitor is greater than the voltage supplied to the circuit. The influence of the impedance in the whole circuit is smaller than the influence of impedance of any single component. This case causes higher voltages on single components.

# AC circuit –RLC in paralel

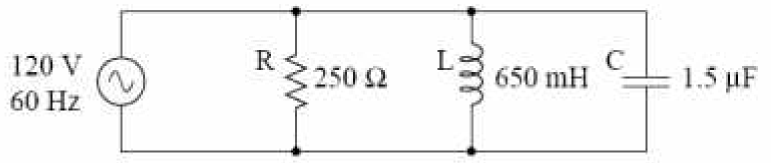
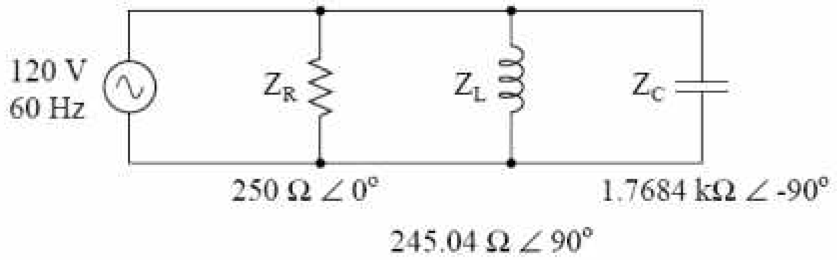



Figure 5.5: *Example R, L, and C parallel circuit.*




| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | | | | **120 + j0** <br> **120 ∠ 0°** | Volts |
| I | | | | | Amps |
| Z | **250 + j0** <br> **250 ∠ 0°** | **0 + j245.04** <br> **245.04 ∠ 90°** | **0 - j1.7684k** <br> **1.7684k ∠ -90°** | | Ohms |

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | **120 + j0** <br> **120 ∠ 0°** | **120 + j0** <br> **120 ∠ 0°** | **120 + j0** <br> **120 ∠ 0°** | 120 + j0 <br> 120 ∠ 0° | Volts |
| I | | | | | Amps |
| Z | 250 + j0 <br> 250 ∠ 0° | 0 + j245.04 <br> 245.04 ∠ 90° | 0 - j1.7684k <br> 1.7684k ∠ -90° | | Ohms |

*Rule of parallel circuits:*
$E_{total} = E_R = E_L = E_C$

# AC circuit –RLC in paralel

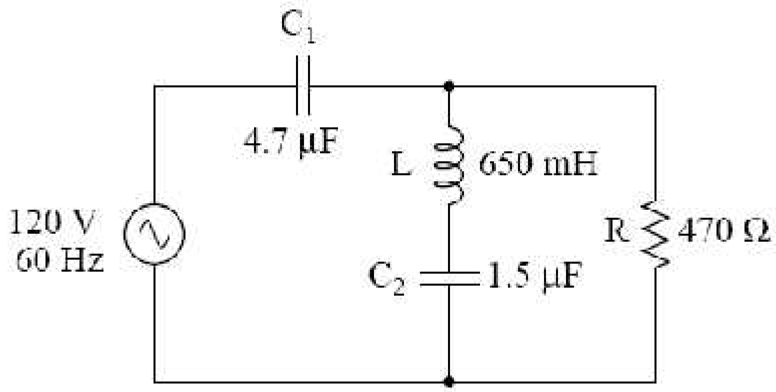

Figure 5.5: *Example R, L, anc*

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | Volts |
| I | **480m + j0**<br>**480m ∠ 0°** | **0 - j489.71m**<br>**489.71m ∠ -90°** | **0 + j67.858m**<br>**67.858m ∠ 90°** | | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | | Ohms |

| | R | L | C |
|---|---|---|---|
| | *Ohm's Law* $I = \frac{E}{Z}$ | *Ohm's Law* $I = \frac{E}{Z}$ | *Ohm's Law* $I = \frac{E}{Z}$ |

| | R | L | C | Total | |
|---|---|---|---|---|---|
| E | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | 120 + j0<br>120 ∠ 0° | Volts |
| I | 480m + j0<br>480 ∠ 0° | 0 - j489.71m<br>489.71m ∠ -90° | 0 + j67.858m<br>67.858m ∠ 90° | **480m - j421.85m**<br>**639.03m ∠ -41.311°** | Amps |
| Z | 250 + j0<br>250 ∠ 0° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | **141.05 + j123.96**<br>**187.79 ∠ 41.311°** | Ohms |

# AC circuit –Complex Circuit




*Reactances and Resistances:*

| | |
|---|---|
| $X_{C1} = \frac{1}{2\pi f C_1}$<br>$X_{C1} = \frac{1}{(2)(\pi)(60\text{ Hz})(4.7\ \mu\text{F})}$<br>$X_{C1} = 564.38\ \Omega$ | $X_L = 2\pi f L$<br>$X_L = (2)(\pi)(60\text{ Hz})(650\text{ mH})$<br>$X_L = 245.04\ \Omega$ |
| $X_{C2} = \frac{1}{2\pi f C_2}$<br>$X_{C2} = \frac{1}{(2)(\pi)(60\text{ Hz})(1.5\ \mu\text{F})}$<br>$X_{C2} = 1.7684\text{ k}\Omega$ | $R = 470\ \Omega$ |

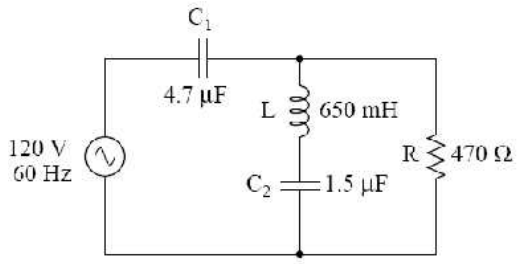
$Z_{C1} = 0 - j564.38\ \Omega$ *or* $564.38\ \Omega \angle -90^\circ$

$Z_L = 0 + j245.04\ \Omega$ *or* $245.04\ \Omega \angle 90^\circ$

$Z_{C2} = 0 - j1.7684k\ \Omega$ *or* $1.7684\text{ k}\Omega \angle -90^\circ$

$Z_R = 470 + j0\ \Omega$ *or* $470\ \Omega \angle 0^\circ$

# AC circuit –Complex Circuit




| | $C_1$ | L | $C_2$ | R | Total | |
|---|---|---|---|---|---|---|
| E | | | | | $120 + j0$<br>$120 \angle 0°$ | Volts |
| I | | | | | | Amps |
| Z | $0 - j564.38$<br>$564.38 \angle -90°$ | $0 + j245.04$<br>$245.04 \angle 90°$ | $0 - j1.7684k$<br>$1.7684k \angle -90°$ | $470 + j0$<br>$470 \angle 0°$ | | Ohms |

The calculation of impedance in this circuit should be completed step by step.
First, serial connection branch of $C_2$ and L,
afterwards the parallel branch of resistor and last the serial capacitor effects should be calculated.

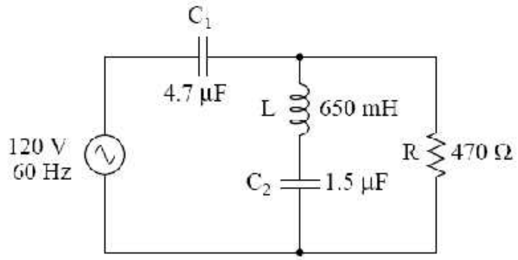
| | $L -- C_2$ | $R // (L -- C_2)$ | *Total*<br>$C_1 -- [R // (L -- C_2)]$ | |
|---|---|---|---|---|
| E | | | $120 + j0$<br>$120 \angle 0°$ | Volts |
| I | | | | Amps |
| Z | **$0 - j1.5233k$**<br>**$1.5233k \angle -90°$** | **$429.15 - j132.41$**<br>**$449.11 \angle -17.147°$** | **$429.15 - j696.79$**<br>**$818.34 \angle -58.371°$** | Ohms |

*Rule of series circuits:*
$$Z_{L--C2} = Z_L - Z_{C2}$$

*Rule of parallel circuits:*
$$Z_{R//(L--C2)} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_{L--C2}}}$$

*Rule of series circuits:*
$$Z_{total} = Z_{C1} + Z_{R//(L--C2)}$$

# AC circuit –Complex Circuit




Calculate current drawn from source and passing through $C_1$

| | L -- $C_2$ | R // (L -- $C_2$) | *Total* $C_1$ -- [R // (L -- $C_2$)] | |
|---|---|---|---|---|
| E | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | | | **76.899m + j124.86m**<br>**146.64m ∠ 58.371°** | Amps |
| Z | 0 - j1.5233k<br>1.5233k ∠ -90° | 429.15 - j132.41<br>449.11 ∠ -17.147° | 429.15 - j696.79<br>818.34 ∠ -58.371° | Ohms |

*Ohm's Law*

$$I = \frac{E}{Z}$$

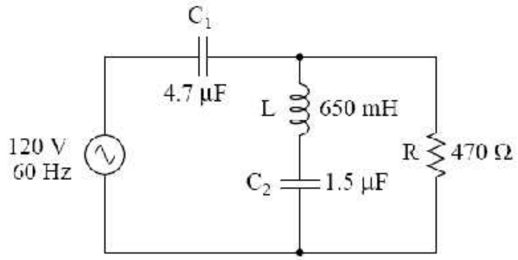
Same current is passing through L-$C_2$ branch

| | L -- $C_2$ | R // (L -- $C_2$) | *Total* $C_1$ -- [R // (L -- $C_2$)] | |
|---|---|---|---|---|
| E | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | | **76.899m + j124.86m**<br>**146.64m ∠ 58.371°** | 76.899m + j124.86m<br>146.64m ∠ 58.371° | Amps |
| Z | 0 - j1.5233k<br>1.5233k ∠ -90° | 429.15 - j132.41<br>449.11 ∠ -17.147° | 429.15 - j696.79<br>818.34 ∠ -58.371° | Ohms |

*Rule of series circuits:*

$$I_{total} = I_{C1} = I_{R//(L--C2)}$$

# AC circuit –Complex Circuit




## Voltage drop on L-$C_2$ branch

| | $L -- C_2$ | $R // (L -- C_2)$ | Total<br>$C_1 -- [R // (L -- C_2)]$ | |
|---|---|---|---|---|
| E | | **49.533 + j43.400**<br>**65.857** $\angle$ 41.225° | 120 + j0<br>120 $\angle$ 0° | Volts |
| I | | 76.899m + j124.86m<br>146.64m $\angle$ 58.371° | 76.899m + j124.86m<br>146.64m $\angle$ 58.371° | Amps |
| Z | 0 - j1.5233k<br>1.5233k $\angle$ -90° | 429.15 - j132.41<br>449.11 $\angle$ -17.147° | 429.15 - j696.79<br>818.34 $\angle$ -58.371° | Ohms |

*Ohm's Law*

E = IZ

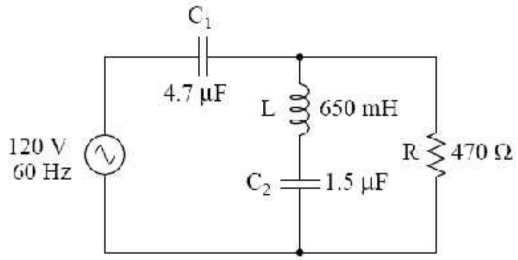
## Same Voltage drops on parallel R branch

| | $L -- C_2$ | $R // (L -- C_2)$ | Total<br>$C_1 -- [R // (L -- C_2)]$ | |
|---|---|---|---|---|
| E | **49.533 + j43.400**<br>**65.857** $\angle$ 41.225° | 49.533 + j43.400<br>65.857 $\angle$ 41.225° | 120 + j0<br>120 $\angle$ 0° | Volts |
| I | | 76.899m + j124.86m<br>146.64m $\angle$ 58.371° | 76.899m + j124.86m<br>146.64m $\angle$ 58.371° | Amps |
| Z | 0 - j1.5233k<br>1.5233k $\angle$ -90° | 429.15 - j132.41<br>449.11 $\angle$ -17.147° | 429.15 - j696.79<br>818.34 $\angle$ -58.371° | Ohms |

*Rule of parallel circuits:*

$E_{R//(L--C2)} = E_R = E_{L--C2}$

# AC circuit –Complex Circuit




| | $L -- C_2$ | $R // (L -- C_2)$ | Total $C_1 -- [R // (L -- C_2)]$ | |
|---|---|---|---|---|
| E | 49.533 + j43.400<br>65.857 ∠ 41.225° | 49.533 + j43.400<br>65.857 ∠ 41.225° | 120 + j0<br>120 ∠ 0° | Volts |
| I | **-28.490m + j32.516m**<br>**43.232m ∠ 131.22°** | 76.899m + j124.86m<br>146.64m ∠ 58.371° | 76.899m + j124.86m<br>146.64m ∠ 58.371° | Amps |
| Z | 0 - j1.5233k<br>1.5233k ∠ -90° | 429.15 - j132.41<br>449.11 ∠ -17.147° | 429.15 - j696.79<br>818.34 ∠ -58.371° | Ohms |

*Ohm's Law*

$I = \frac{E}{Z}$

| | $C_1$ | L | $C_2$ | R | Total | |
|---|---|---|---|---|---|---|
| E | | | | | 120 + j0<br>120 ∠ 0° | Volts |
| I | | | | | | Amps |
| Z | 0 - j564.38<br>564.38 ∠ -90° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | 470 + j0<br>470 ∠ 0° | | Ohms |

| | $C_1$ | L | $C_2$ | R | |
|---|---|---|---|---|---|
| E | 70.467 - j43.400<br>82.760 ∠ -31.629° | **-7.968 - j6.981**<br>**10.594 ∠ 221.22°** | **57.501 + j50.382**<br>**76.451 ∠ 41.225** | 49.533 + j43.400<br>65.857 ∠ 41.225° | Volts |
| I | 76.899m + j124.86m<br>146.64m ∠ 58.371° | -28.490m + j32.516m<br>43.232m ∠ 131.22° | -28.490m + j32.516m<br>43.232m ∠ 131.22° | 105.39m + j92.341m<br>140.12m ∠ 41.225° | Amps |
| Z | 0 - j564.38<br>564.38 ∠ -90° | 0 + j245.04<br>245.04 ∠ 90° | 0 - j1.7684k<br>1.7684k ∠ -90° | 470 + j0<br>470 ∠ 0° | Ohms |

*Ohm's Law* $E = IZ$ (L)

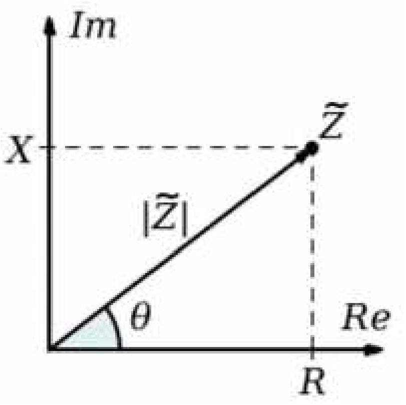
*Ohm's Law* $E = IZ$ ($C_2$)

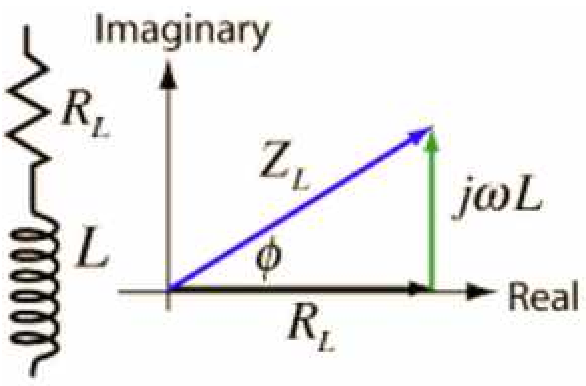
## Power in AC circuit

Since it was mentioned, there is a phase shift between current and voltage in AC circuits. The reason is the complex number impedance as it was stated.
So there are three definitions in AC circuits which are related with power.
These are:

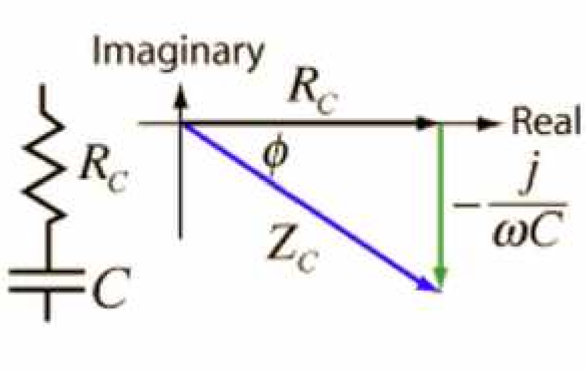
True power (active power),
Reactive power,
Apparent power.

Impedance Calculations:




Inductive Circuit




Capacitive Circuit

# Power in AC circuit

**P** = true power $P = I^2R$ $P = \frac{E^2}{R}$

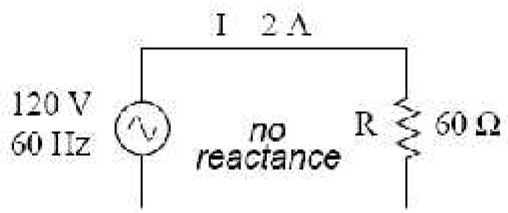
*Measured in units of* ***Watts***

**Q** = reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$

*Measured in units of* ***Volt-Amps-Reactive (VAR)***

**S** = apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$

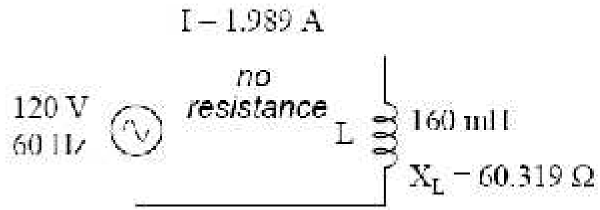
*Measured in units of* ***Volt-Amps (VA)***




P = true power = $I^2R$ = 240 W

Q = reactive power = $I^2X$ = 0 VAR

S = apparent power = $I^2Z$ = 240 VA




P = true power = $I^2R$ = 0 W

Q = reactive power = $I^2X$ = 238.73 VAR

S = apparent power = $I^2Z$ = 238.73 VA

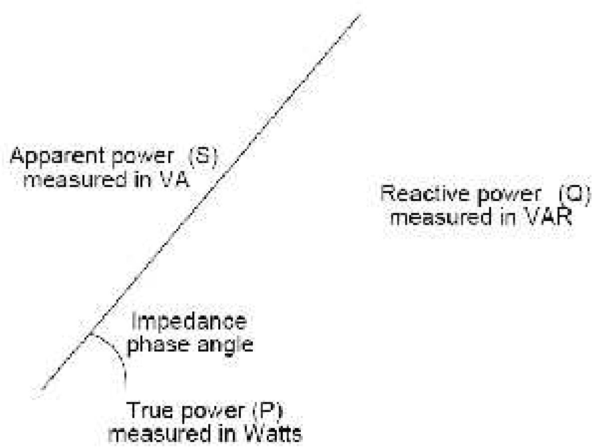
# Power in AC circuit

**P** – true power $P = I^2R$ $P = \dfrac{E^2}{R}$

*Measured in units of **Watts***

The power quantities are scalar quantities. if we consider the 90 degrees of direction angle between the resistor and the reactance and phase shift in the circuit. **This perpendicular triangle is called as 'Power Triangle**

*The 'Power Triangle'*




Q = reactive power $Q = I^2X$ $Q = \dfrac{E^2}{X}$

*Measured in units of **Volt-Amps-Reactive (VAR)***

S = apparent power $S = I^2Z$ $S = \dfrac{E^2}{Z}$ $S = IE$

*Measured in units of **Volt-Amps (VA)***

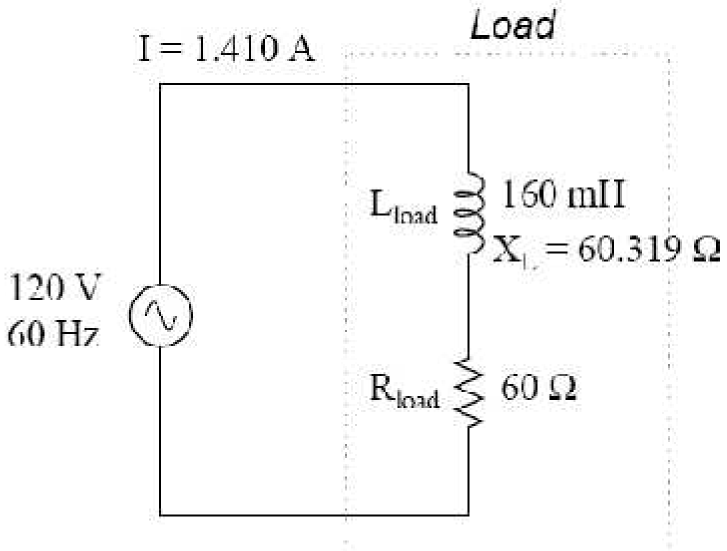
A part of the power cannot be converted to electrical work in an AC circuit.
The generated effective power is just as the true power.
**Power factor** is the cosine of the angle between the true and apparent powers ($\cos \phi$).
This value is equal to 1 in only circuits those have just resistors. But if there is a reactance, then the value is between 0 and 1.

# Power in AC circuit

**P** – true power $P = I^2R$ $P = \frac{E^2}{R}$
*Measured in units of **Watts***

**Q** – reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$
*Measured in units of **Volt-Amps-Reactive (VAR)***

**S** – apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$
*Measured in units of **Volt-Amps (VA)***




$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

$$\text{Power factor} = 0.705$$

P = true power = $I^2R$ = 119.365 W

Q = reactive power = $I^2X$ = 119.998 VAR

S = apparent power = $I^2Z$ = 169.256 VA

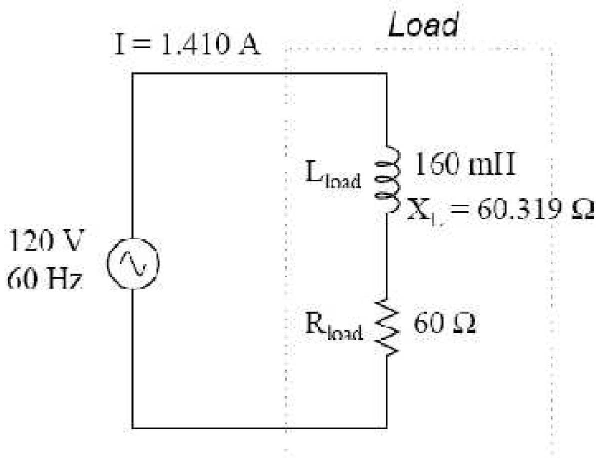
The power factor value shows that the **70.5 % of the power used from the grid is served for the purpose.**

# Compensation in AC circuit

**P** = true power $P = I^2R$ $P = \frac{E^2}{R}$
*Measured in units of **Watts***

**Q** = reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$
*Measured in units of **Volt-Amps-Reactive (VAR)***

**S** = apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$
*Measured in units of **Volt-Amps (VA)***




$P$ = true power = $I^2R$ = 119.365 W

$Q$ = reactive power = $I^2X$ = 119.998 VAR

$S$ = apparent power = $I^2Z$ = 169.256 VA

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

Power factor = 0.705

The power factor value shows that the **70.5 % of the power used from the grid is served for the purpose.**

This situation is not wanted.
So, in circuit design stage, it must be noted that the power factor is approximately equal to 1.

For this reason, the capacitive and inductive reactance values should be approximately equal to each other.

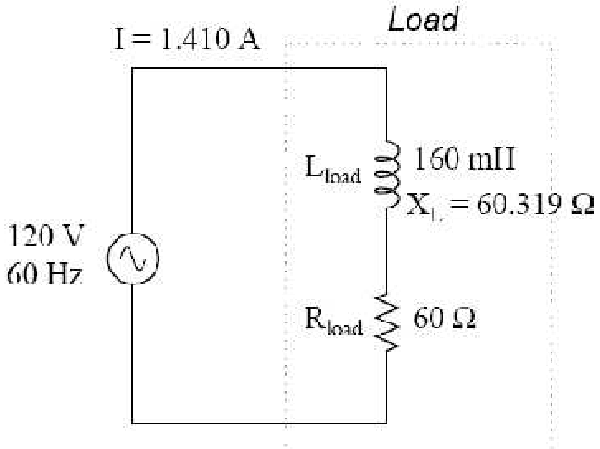
If this is not possible, a capacitor or an inductor should be externally added to the circuit. **This improvement is called as compensation.**

# Compensation in AC circuit

**P** – true power $P = I^2R$ $P = \frac{E^2}{R}$
*Measured in units of* ***Watts***

**Q** – reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$
*Measured in units of* ***Volt-Amps-Reactive (VAR)***

**S** – apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$
*Measured in units of* ***Volt-Amps (VA)***




P = true power = $I^2R$ = 119.365 W

Q = reactive power = $I^2X$ = 119.998 VAR

S = apparent power = $I^2Z$ = 169.256 VA

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

Power factor = 0.705

The circuit is inductive so a **parallel** capacitor should be added so that the total reactance of the circuit becomes approximately zero

$$Q = \frac{E^2}{X}$$

*. . . solving for X . . .*

$$X = \frac{E^2}{Q}$$

$$X = \frac{(120\ \text{V})^2}{119.998\ \text{VAR}}$$

$$X = 120.002\ \Omega$$

In parallel branches voltage is constant so the necessary reactance value can be calculated using voltage based power formula

# Compensation in AC circuit

**P** = true power $P = I^2R$ $P = \frac{E^2}{R}$

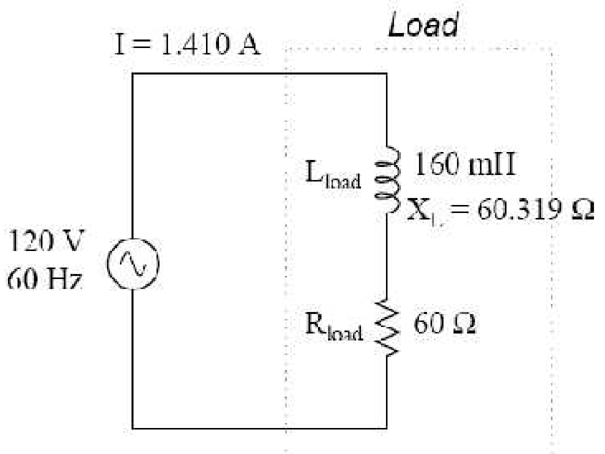
*Measured in units of* ***Watts***

**Q** = reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$

*Measured in units of* ***Volt-Amps-Reactive (VAR)***

**S** = apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$

*Measured in units of* ***Volt-Amps (VA)***




P = true power = $I^2R$ = 119.365 W

Q = reactive power = $I^2X$ = 119.998 VAR

S = apparent power = $I^2Z$ = 169.256 VA

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

Power factor = 0.705

The calculated reactance value is used to find capacitor value

$$X_C = \frac{1}{2\pi fC}$$

*. . . solving for C . . .*

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi(60\ \text{Hz})(120.002\ \Omega)}$$

$$C = 22.105\ \mu\text{F}$$

# Compensation in AC circuit

**P** – true power $P = I^2R$ $P = \frac{E^2}{R}$

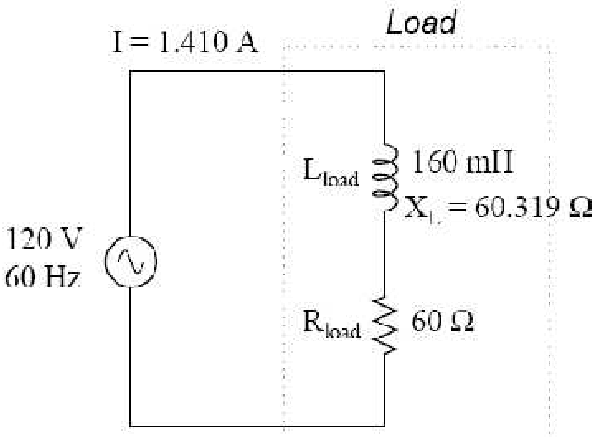
*Measured in units of **Watts***

**Q** – reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$

*Measured in units of **Volt-Amps-Reactive (VAR)***

**S** – apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$

*Measured in units of **Volt-Amps (VA)***




P = true power = $I^2R$ = 119.365 W

Q = reactive power = $I^2X$ = 119.998 VAR

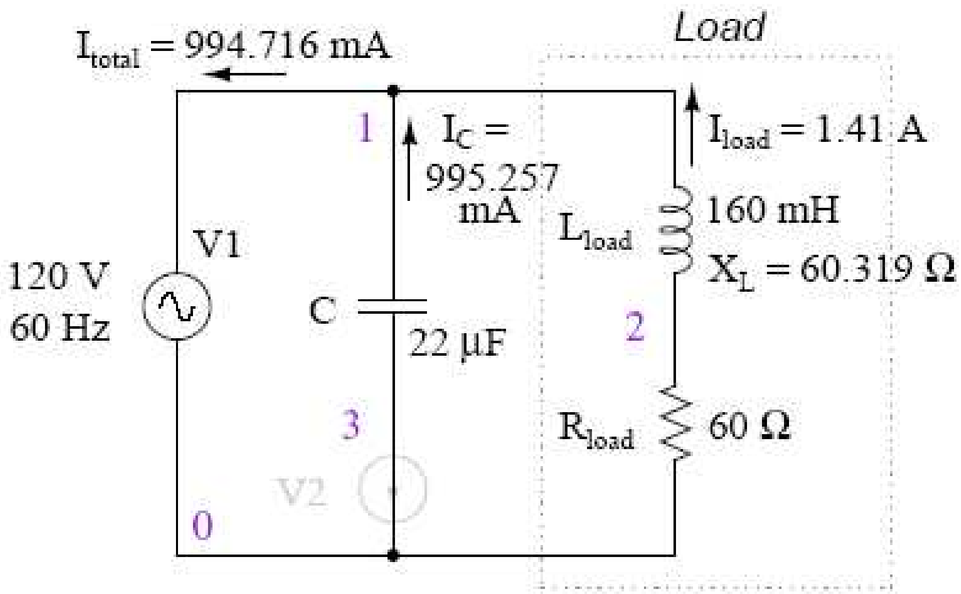
S = apparent power = $I^2Z$ = 169.256 VA

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

Power factor = 0.705

The capacitor value found is not a standart value for capacitors, so the closest standart value shold be chosen (22 µF) and connected in parallel with the circuit.



# Compensation in AC circuit

**P** – true power $P = I^2R$ $P = \frac{E^2}{R}$

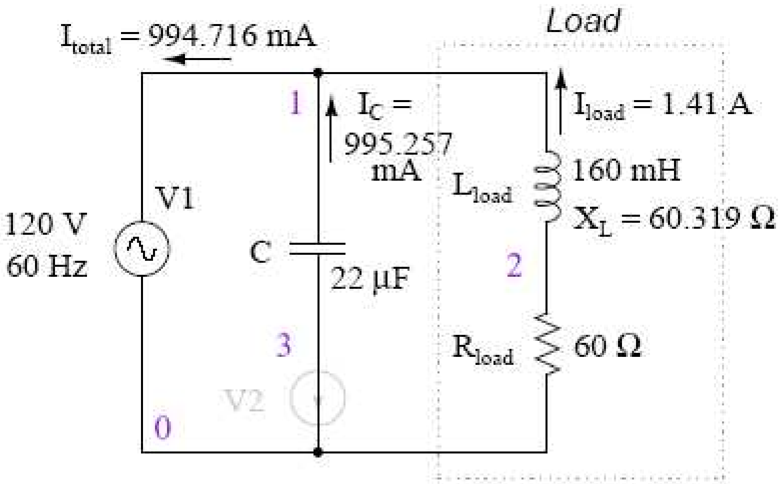
*Measured in units of* ***Watts***

**Q** – reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$

*Measured in units of* ***Volt-Amps-Reactive (VAR)***

**S** – apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$

*Measured in units of* ***Volt-Amps (VA)***




$$Z_{total} = Z_C \,//\, (Z_L -- Z_R)$$

$$Z_{total} = (120.57\ \Omega \angle -90^\circ) \,//\, (60.319\ \Omega \angle 90^\circ -- 60\ \Omega \angle 0^\circ)$$

$$Z_{total} = 120.64 - j573.58m\ \Omega \quad \textit{or} \quad 120.64\ \Omega \angle 0.2724^\circ$$

$$P = \text{true power} = I^2R = 119.365\ W$$

$$S = \text{apparent power} = I^2Z = 119.366\ VA$$

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ W}{119.366\ VA}$$

$$\text{Power factor} = 0.9999887$$

this improvement made the power factor closer to 1. Besides, the current is decreased.

# Compensation in AC circuit

**P** = true power $P = I^2R$ $P = \frac{E^2}{R}$

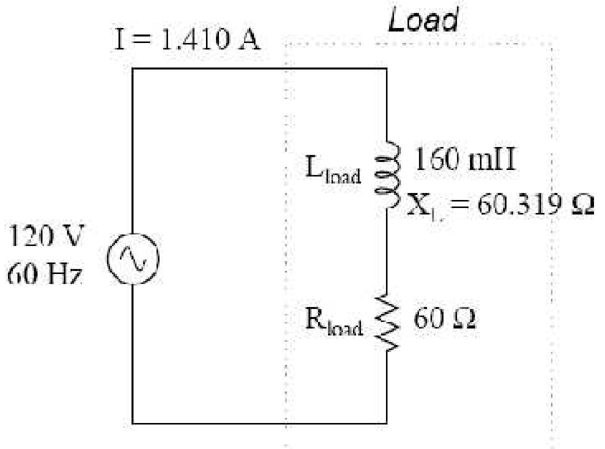
*Measured in units of **Watts***

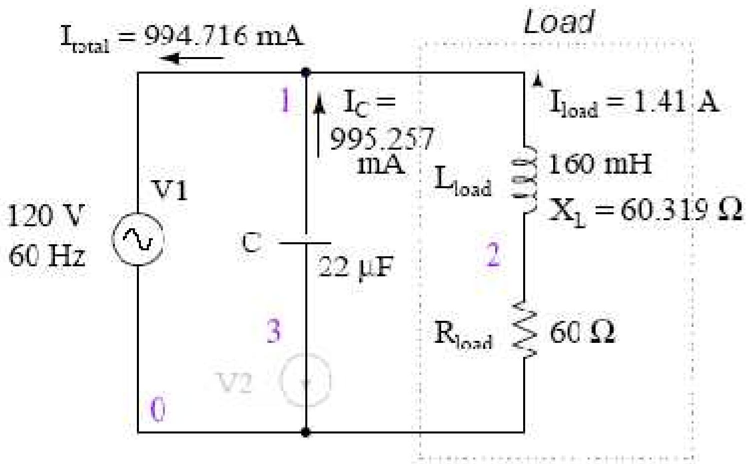
**Q** = reactive power $Q = I^2X$ $Q = \frac{E^2}{X}$

*Measured in units of **Volt-Amps-Reactive (VAR)***

**S** = apparent power $S = I^2Z$ $S = \frac{E^2}{Z}$ $S = IE$

*Measured in units of **Volt-Amps (VA)***




P = true power = $I^2R$ = 119.365 W

Q = reactive power = $I^2X$ = 119.998 VAR

S = apparent power = $I^2Z$ = 169.256 VA

Power factor = $\frac{\text{True}}{\text{Appare}}$ P = true power = $I^2R$ = 119.365 W

S = apparent power = $I^2Z$ = 119.366 VA

Power factor = $\frac{119.365\ \text{W}}{169.256\ \text{VA}}$

Power factor = 0.705

Power factor = $\frac{\text{True power}}{\text{Apparent power}}$

Power factor = $\frac{119.365\ \text{W}}{119.366\ \text{VA}}$

Power factor = 0.9999887