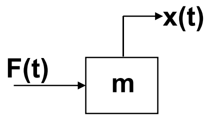
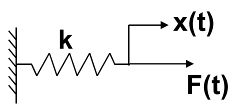
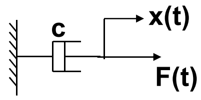
TEMEL MEKANİK SİSTEM ELEMANLARI

Ötelenen Elemanlar







$F = m\ddot{x}$ $\quad E_1 = \frac{1}{2}m\dot{x}^2$ $\quad F = kx$ $\quad E_2 = \frac{1}{2}kx^2$ $\quad F = c\dot{x}$ $\quad \delta W = -c\dot{x}\delta x$

$$\delta W = F\delta x$$

Dönel Elemanlar

$T = I_G\ddot{\theta}$ $\quad E_1 = \frac{1}{2}I_G\dot{\theta}^2$ $\quad T = K_r\theta$ $\quad E_2 = \frac{1}{2}K_r\theta^2$ $\quad T = C_r\dot{\theta}$ $\quad \delta W = -C_r\dot{\theta}\delta\theta$

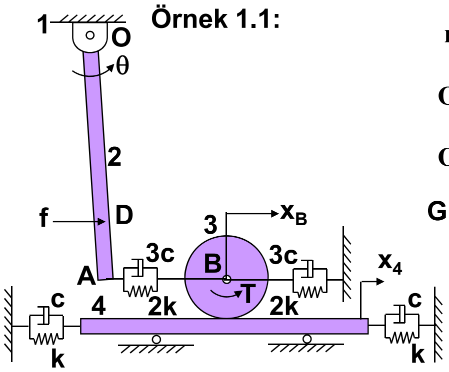
$$\delta W = T\delta\theta$$

$I_G = \frac{1}{12}mL^2$ $\quad I_G = \frac{1}{2}mR^2$

CAD : Katı Modelleme

Öteleme ve Dönme Hareketi yapan Kütle: $E_1 = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\dot{\theta}^2$

1. Kinetik enerji, potansiyel enerji, sanal iş



Örnek 1.1:

$m_2 = \frac{2m}{3}$ $\quad m_3 = \frac{m}{2}$ $\quad m_4 = m$

$OA = \frac{3L}{4}$ $\quad R_3 = \frac{L}{3}$ $\quad L_4 = L$

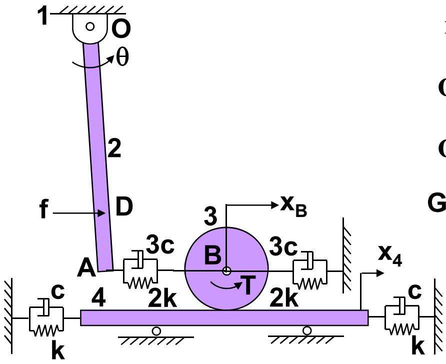
$OD = \frac{9L}{16}$ $\quad \begin{Bmatrix} \theta \\ x_B \end{Bmatrix}$

Girdiler: f, T, x_4

$\theta \ll 1 \quad \sin\theta \approx \theta \quad \cos\theta \approx 1$

$$E_1 = \frac{1}{2}\frac{2m}{3}\left(\frac{3L\dot{\theta}}{8}\right)^2 + \frac{1}{2}\frac{1}{12}\frac{2m}{3}\left(\frac{3L}{4}\right)^2\dot{\theta}^2 + \frac{1}{2}\frac{m}{2}\dot{x}_B^2 + \frac{1}{2}\frac{1}{2}\frac{m}{2}\left(\frac{L}{3}\right)^2\left[\frac{3(\dot{x}_4 - \dot{x}_B)}{L}\right]^2 + \frac{1}{2}m\dot{x}_4^2$$

$$E_2 = \frac{1}{2}2k\left(x_B - \frac{3L}{4}\theta\right)^2 + \frac{1}{2}2kx_B^2 + \frac{1}{2}kx_4^2 + \frac{1}{2}kx_4^2$$



$$m_2=\frac{2m}{3} \qquad m_3=\frac{m}{2} \qquad m_4=m$$

$$OA=\frac{3L}{4} \qquad R_3=\frac{L}{3} \qquad L_4=L$$

$$OD=\frac{9L}{16} \qquad \begin{Bmatrix}\theta\\ x_B\end{Bmatrix}$$

Girdiler: f, T, x_4

$$\theta<<1 \quad \sin\theta\approx\theta \quad \cos\theta\approx 1$$

$$\delta W=f\delta\left(\frac{9L}{16}\theta\right)+T\delta\left[\frac{3(x_4-x_B)}{L}\right]-3c\left(\dot{x}_B-\frac{3L}{4}\dot{\theta}\right)\delta\left(x_B-\frac{3L}{4}\theta\right)-3c\dot{x}_B\delta x_B$$
$$-\cancelto{0}{c\dot{x}_4\delta x_4}-\cancelto{0}{c\dot{x}_4\delta x_4}$$

$$\delta W=\frac{9L}{16}f\delta\theta+\frac{3}{L}\cancelto{0}{T\delta x_4}-\frac{3}{L}T\delta x_B-3c\dot{x}_B\delta x_B+\frac{9cL}{4}\dot{x}_B\delta\theta+\frac{9cL}{4}\dot{\theta}\delta x_B-\frac{27cL^2}{16}\dot{\theta}\delta\theta$$
$$-3c\dot{x}_B\delta x_B$$

$$\delta W=\underbrace{\left(\frac{9L}{16}f+\frac{9cL}{4}\dot{x}_B-\frac{27cL^2}{16}\dot{\theta}\right)}_{Q_\theta}\delta\theta+\underbrace{\left(-\frac{3}{L}T-6c\dot{x}_B+\frac{9cL}{4}\dot{\theta}\right)}_{Q_{XB}}\delta x_B$$

ÖRNEK PROBLEM İÇİN BULUNAN KİNETİK ENERJİ, POTANSİYEL ENERJİ VE SANAL İŞ:

$$E_1=\frac{1}{2}\frac{2m}{3}\left(\frac{3L\dot{\theta}}{8}\right)^2+\frac{1}{2}\frac{1}{12}\frac{2m}{3}\left(\frac{3L}{4}\right)^2\dot{\theta}^2+\frac{1}{2}\frac{m}{2}\dot{x}_B^2+\frac{1}{2}\frac{1}{2}\frac{m}{2}\left(\frac{L}{3}\right)^2\left[\frac{3(\dot{x}_4-\dot{x}_B)}{L}\right]^2+\frac{1}{2}m\dot{x}_4^2$$

$$E_2=\frac{1}{2}2k\left(x_B-\frac{3L}{4}\theta\right)^2+\frac{1}{2}2kx_B^2+\frac{1}{2}kx_4^2+\frac{1}{2}kx_4^2$$

$$\delta W=\underbrace{\left(\frac{9L}{16}f+\frac{9cL}{4}\dot{x}_B-\frac{27cL^2}{16}\dot{\theta}\right)}_{Q_\theta}\delta\theta+\underbrace{\left(-\frac{3}{L}T-6c\dot{x}_B+\frac{9cL}{4}\dot{\theta}\right)}_{Q_{XB}}\delta x_B$$

LAGRANGE DENKLEMİ

$$L = E_1 - E_2 \qquad \boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = Q_i}$$

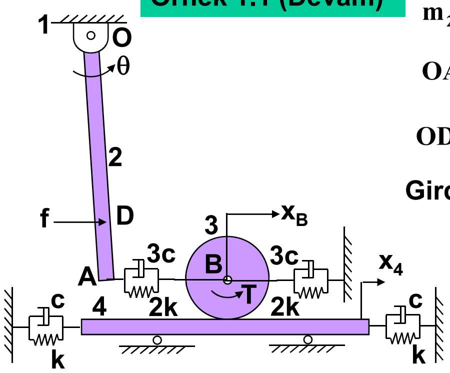
x_i : Genel koordinat

Q_i : Genel kuvvet

i=1,2,....,n

J.H. Williams, Jr., Fundamentals of Applied Dynamics, John Wiley and Sons,Inc., 1996

Örnek 1.1 (Devam)



$$m_2 = \frac{2m}{3} \qquad m_3 = \frac{m}{2} \qquad m_4 = m$$

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$$E_1 = \frac{1}{2}\frac{2m}{3}\left(\frac{3L\dot{\theta}}{8}\right)^2 + \frac{1}{2}\frac{1}{12}\frac{2m}{3}\left(\frac{3L}{4}\right)^2\dot{\theta}^2 + \frac{1}{2}\frac{m}{2}\dot{x}_B^2 + \frac{1}{2}\frac{1}{2}\frac{m}{2}\left(\frac{L}{3}\right)^2\left[\frac{3(\dot{x}_4 - \dot{x}_B)}{L}\right]^2 + \frac{1}{2}m\dot{x}_4^2$$

$$E_2 = \frac{1}{2}2k\left(x_B - \frac{3L}{4}\theta\right)^2 + \frac{1}{2}2kx_B^2 + \frac{1}{2}kx_4^2 + \frac{1}{2}kx_4^2$$

$$\delta W = \underbrace{\left(\frac{9L}{16}f + \frac{9cL}{4}\dot{x}_B - \frac{27cL^2}{16}\dot{\theta}\right)}_{Q_\theta}\delta\theta + \underbrace{\left(-\frac{3}{L}T - 6c\dot{x}_B + \frac{9cL}{4}\dot{\theta}\right)}_{Q_{XB}}\delta x_B$$

$$L = E_1 - E_2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = Q_\theta \qquad \boxed{\frac{d}{dt}\left(\frac{\partial E_1}{\partial \dot{\theta}}\right) + \frac{\partial E_2}{\partial \theta} = Q_\theta} \qquad \boxed{\frac{d}{dt}\left(\frac{\partial E_1}{\partial \dot{x}_B}\right) + \frac{\partial E_2}{\partial x_B} = Q_{XB}}$$

$$\frac{d}{dt}\left(\frac{18mL^2}{192}\dot{\theta} + \frac{18mL^2}{576}\dot{\theta}\right) - \frac{6kL}{4}\left(x_B - \frac{3L}{4}\theta\right) = \frac{9L}{16}f + \frac{9cL}{4}\dot{x}_B - \frac{27cL^2}{16}\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{m}{2}\dot{x}_B - \frac{9mL^2}{36L^2}(\dot{x}_4 - \dot{x}_B)\right) + 2k\left(x_B - \frac{3L}{4}\theta\right) + 2kx_B = -\frac{3}{L}T - 6c\dot{x}_B + \frac{9cL}{4}\dot{\theta}$$

$$\frac{mL^2}{8}\ddot{\theta} + \frac{27cL^2}{16}\dot{\theta} - \frac{9cL}{4}\dot{x}_B + \frac{9kL^2}{8}\theta - \frac{3kL}{2}x_B = \frac{9L}{16}f$$

$$\frac{3m}{4}\ddot{x}_B - \frac{9cL}{4}\dot{\theta} + 6c\dot{x}_B - \frac{3kL}{2}\theta + 4kx_B = -\frac{3}{L}T + \frac{m}{4}\ddot{x}_4$$

$$\frac{mL^2}{8}\ddot{\theta}+\frac{27cL^2}{16}\dot{\theta}-\frac{9cL}{4}\dot{x}_B+\frac{9kL^2}{8}\theta-\frac{3kL}{2}x_B=\frac{9L}{16}f$$

$$\frac{3m}{4}\ddot{x}_B-\frac{9cL}{4}\dot{\theta}+6c\dot{x}_B-\frac{3kL}{2}\theta+4kx_B=-\frac{3}{L}T+\frac{m}{4}\ddot{x}_4$$

$$\underbrace{\begin{bmatrix}\frac{mL^2}{8} & 0\\ 0 & \frac{3m}{4}\end{bmatrix}}_{M}\underbrace{\begin{Bmatrix}\ddot{\theta}\\ \ddot{x}_B\end{Bmatrix}}_{\ddot{X}}+\underbrace{\begin{bmatrix}\frac{27cL^2}{16} & -\frac{9cL}{4}\\ -\frac{9cL}{4} & 6c\end{bmatrix}}_{C}\underbrace{\begin{Bmatrix}\dot{\theta}\\ \dot{x}_B\end{Bmatrix}}_{\dot{X}}+\underbrace{\begin{bmatrix}\frac{9kL^2}{8} & -\frac{3kL}{2}\\ -\frac{3kL}{2} & 4k\end{bmatrix}}_{K}\underbrace{\begin{Bmatrix}\theta\\ x_B\end{Bmatrix}}_{X}=\underbrace{\begin{Bmatrix}\frac{9L}{16}f\\ -\frac{3}{L}T+\frac{m}{4}\ddot{x}_4\end{Bmatrix}}_{F}$$

$$\boxed{M\ddot{X}+C\dot{X}+KX=F}$$

Doğrusal diferansiyel denklem takımı

Çok serbestlik dereceli titreşim denklemi

ÖZDEĞER DENKLEMİ

$M\ddot{X}+C\dot{X}+KX=F$ $\quad F=0$, **serbest titreşim**

$X=Ae^{st}$

$[s^2M+sC+K]A\cancel{e^{st}}=0 \quad [s^2M+sC+K]A=0 \quad \underbrace{\det[s^2M+sC+K]=0}_{\text{Özdeğer denklemi}}$

Örnek 1.1 (Devam):

$$\underbrace{\begin{bmatrix}\frac{mL^2}{8} & 0\\ 0 & \frac{3m}{4}\end{bmatrix}}_{M}\underbrace{\begin{Bmatrix}\ddot{\theta}\\ \ddot{x}_B\end{Bmatrix}}_{\ddot{X}}+\underbrace{\begin{bmatrix}\frac{27cL^2}{16} & -\frac{9cL}{4}\\ -\frac{9cL}{4} & 6c\end{bmatrix}}_{C}\underbrace{\begin{Bmatrix}\dot{\theta}\\ \dot{x}_B\end{Bmatrix}}_{\dot{X}}+\underbrace{\begin{bmatrix}\frac{9kL^2}{8} & -\frac{3kL}{2}\\ -\frac{3kL}{2} & 4k\end{bmatrix}}_{K}\underbrace{\begin{Bmatrix}\theta\\ x_B\end{Bmatrix}}_{X}=0$$

Özdeğer Denklemi:
$$\begin{vmatrix}\frac{mL^2}{8}s^2+\frac{27cL^2}{16}s+\frac{9kL^2}{8} & -\frac{9cL}{4}s-\frac{3kL}{2}\\ -\frac{9cL}{4}s-\frac{3kL}{2} & \frac{3m}{4}s^2+6cs+4k\end{vmatrix}=0$$

Özdeğer Denklemi:

$$\begin{vmatrix} \frac{mL^2}{8}s^2+\frac{27cL^2}{16}s+\frac{9kL^2}{8} & -\frac{9cL}{4}s-\frac{3kL}{2} \\ -\frac{9cL}{4}s-\frac{3kL}{2} & \frac{3m}{4}s^2+6cs+4k \end{vmatrix}=0$$

m=0.85 kg, L =0.24 m, k=1200 N/m, c=38 Ns/m

$$\begin{vmatrix} 0.0061s^2+3.7s+622.08 & -20.5s-432 \\ -20.5s-432 & 0.6375s^2+228s+4800 \end{vmatrix}=0$$

$$0.0039s^4+3.75s^3+847s^2+141834s+2799360=0$$

MatLAB kodu:

```
a=[0.0039,3.75,847,141834,2799360];p=roots(a);vpa(p,4)
```

```
clc;clear
m0=0.85;l0=0.24;k0=1200;c0=38;
m=[m0*l0^2/8,0;0,3*m0/4];
c=[27*c0*l0^2/16,-9*c0*l0/4;-9*c0*l0/4,6*c0];
k=[9*k0*l0^2,-3*k0*l0/2;-3*k0*l0/2,4*k0];
syms s;p=solve(det(m*s^2+c*s+k));vpa(p,4)
```

Özdeğerler: -104.3+181.4i, -104.3-181.4i, -22.4, -730.2

Özdeğerler: -104.3+181.4i, -104.3-181.4i, -22.4, -730.2

Serbest titreşim cevabının formu:

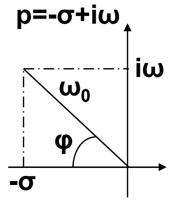
$$\theta(t)=A_1e^{-104.3t}\cos(181.4t-\varphi_1)+A_2e^{-22.4t}+A_3e^{-730.2t}$$

A_1, φ_1, A_2 ve A_3 değerlerini ilk şartlar belirler.

t sonsuza yaklaştığında cevap sıfıra yaklaşır. Düzgün rejim cevabı sıfır olur.

Özdeğerinin tümünün reel kısımlarının negatif olduğu sistem kararlıdır.

p=-σ+iω



$\omega_0=\sqrt{\sigma^2+\omega^2}$ $\Rightarrow \omega_0=\sqrt{104.3^2+181.4^2}=209.25$ rad/s

$\xi=\cos\varphi$ $\Rightarrow \xi=\frac{104.3}{209.25}=0.5$

$\Delta t=\frac{T_0}{20}$ $\Rightarrow \Delta t=0.0015$

$\sigma=\xi\omega_0$ $\omega=\omega_0\sqrt{1-\xi^2}$

$t_\infty\approx\frac{T_0}{\xi}$ $\Rightarrow t_\infty=0.06$

$\omega_0T_0=2\pi$ $\Rightarrow T_0=0.03$

$f_0=1/T_0$

---

p=-σ

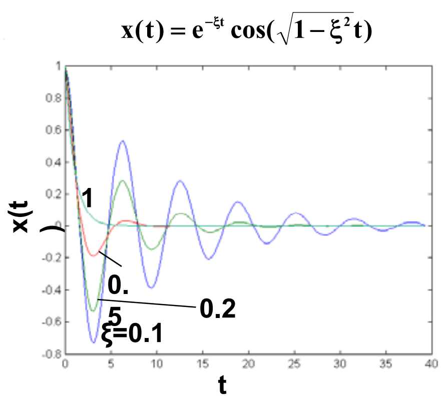
$\tau=\frac{1}{\sigma}$ $\Delta t=\frac{\tau}{\pi}$

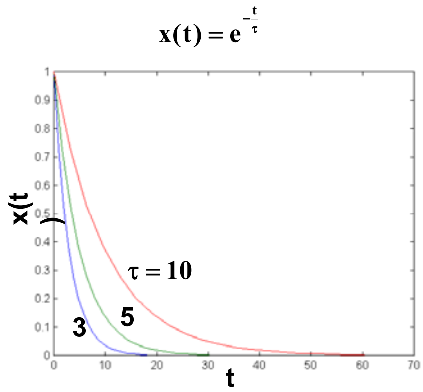
$t_\infty=2\pi\tau$

-22.4 için Δt=0.0142 , t_∞=0.28

-730.2 için Δt=0.000436 , t_∞=0.0086

Sistem için Δt=0.000436 , t_∞=0.28





Bu slaytlar Prof. Dr. Hira Karagülle nin sitesinden alınmıştır.
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