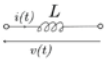
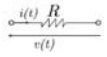
MEE 3017 System Modelling and Analysis

L04 Modelling Electrical Systems

Energy Equations in electrical systems

	voltage vs curent	voltage vs charge	Energy	in terms of charge
 Inductor	$V_L(t)=L\frac{di(t)}{dt}$	$V_L(t)=L\frac{d^2q(t)}{dt^2}$	$T=E_1=\frac{1}{2}Li^2$	$T=E_1=\frac{1}{2}L\dot{q}^2$
 Capacitor	$i(t)=C\frac{dV(t)}{dt}$	$V_C(t)=\frac{q(t)}{C}$	$V=E_2=\frac{1}{2}CV^2$	$V=E_2=\frac{1}{2}\frac{1}{C}q^2$
 Resistor	$V(t)=Ri(t)$	$V(t)=R\frac{dq(t)}{dt}$	$\delta w=-Ri\delta q$	$\delta w=-R\dot{q}\delta q$

Virtual Work done by a Voltage Source $\delta w=-V(t)\delta q$

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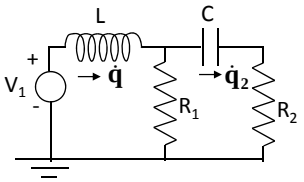
Elektrik Devrelerinin Modellenmesi:

R_1 üzerinden geçen akım = $\dot{q}-\dot{q}_2$

Örnek 3.1:

V_1: Girdi q ve q_2 : Genel yükler



$$E_1=\frac{1}{2}L\dot{q}^2 \qquad E_2=\frac{1}{2C}q_2^2$$

$$\delta W=V_1\delta q-R_1(\dot{q}-\dot{q}_2)\delta(q-q_2)-R_2\dot{q}_2\delta q_2$$

$$=\underbrace{(V_1-R_1\dot{q}+R_1\dot{q}_2)\delta q}_{Q_q}+\underbrace{(R_1\dot{q}-R_1\dot{q}_2-R_2\dot{q}_2)\delta q_2}_{Q_{q2}}$$

$$\frac{d}{dt}\left[\frac{\partial(E_1-E_2)}{\partial\dot{q}}\right]-\frac{\partial(E_1-E_2)}{\partial q}=Q_q \Rightarrow L\ddot{q}=V_1-R_1\dot{q}+R_1\dot{q}_2$$

$$\frac{d}{dt}\left[\frac{\partial(E_1-E_2)}{\partial\dot{q}_2}\right]-\frac{\partial(E_1-E_2)}{\partial q_2}=Q_{q_2} \Rightarrow \frac{1}{C}q_2=R_1\dot{q}-(R_1+R_2)\dot{q}_2$$

$$\begin{bmatrix}L & 0\\ 0 & 0\end{bmatrix}\begin{Bmatrix}\ddot{q}\\ \ddot{q}_2\end{Bmatrix}+\begin{bmatrix}R_1 & -R_1\\ -R_1 & R_1+R_2\end{bmatrix}\begin{Bmatrix}\dot{q}\\ \dot{q}_2\end{Bmatrix}+\begin{bmatrix}0 & 0\\ 0 & \frac{1}{C}\end{bmatrix}\begin{Bmatrix}q\\ q_2\end{Bmatrix}=\begin{Bmatrix}V_1\\ 0\end{Bmatrix}$$

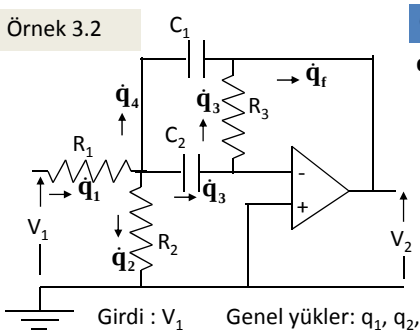
$$\begin{vmatrix}Ls^2+R_1s & -R_1s\\ -R_1s & (R_1+R_2)s+\frac{1}{C}\end{vmatrix}=0$$

L=3.4 mH, C=286 µF, R_1=3.2 Ω, R_2=4.5 Ω

D(s)=0.02618s³+26.288s²+11188.81s=0

Öz değerler: 0, -502.06±418.70i (ξ=0.768)

Örnek 3.2



Girdi : V_1 Genel yükler: q_1, q_2, q_3

Op-amp girişinde direnç çok büyüktür

$\dot{q}_1=\dot{q}_2+\dot{q}_3+\dot{q}_4 \Rightarrow \dot{q}_4=\dot{q}_1-\dot{q}_2-\dot{q}_3$

$\dot{q}_4+\dot{q}_3=\dot{q}_f \Rightarrow \dot{q}_f=\dot{q}_1-\dot{q}_2-\dot{q}_3+\dot{q}_3=\dot{q}_1-\dot{q}_2$

$$E_1=0 \qquad E_2=\frac{1}{2C_1}(q_1-q_2-q_3)^2+\frac{1}{2C_2}q_3^2$$

$$\delta W=V_1\delta q_1-V_2\delta(q_1-q_2)-R_1\dot{q}_1\delta q_1$$
$$-R_2\dot{q}_2\delta q_2-R_3\dot{q}_3\delta q_3$$

$$\frac{1}{C_1}(q_1-q_2-q_3)=V_1-V_2-R_1\dot{q}_1 \qquad -\frac{1}{C_1}(q_1-q_2-q_3)=V_2-R_2\dot{q}_2$$

$$-\frac{1}{C_1}(q_1-q_2-q_3)+\frac{1}{C_2}q_3=-R_3\dot{q}_3$$

Ayrıca Op-amp ta: $V_+=V_-=0$ $\quad -R_3\dot{q}_3=V_2$

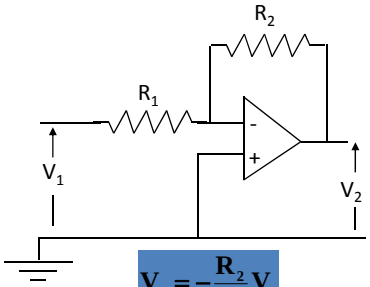
$$\begin{bmatrix}R_1 & 0 & -R_3\\ 0 & R_2 & R_3\\ 0 & 0 & R_3\end{bmatrix}\begin{Bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{Bmatrix}+\begin{bmatrix}\frac{1}{C_1} & -\frac{1}{C_1} & -\frac{1}{C_1}\\ -\frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1}\\ -\frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1}+\frac{1}{C_2}\end{bmatrix}\begin{Bmatrix}q_1\\ q_2\\ q_3\end{Bmatrix}=\begin{Bmatrix}V_1\\ 0\\ 0\end{Bmatrix}$$

$$\begin{bmatrix} R_1 & 0 & -R_3 \\ 0 & R_2 & R_3 \\ 0 & 0 & R_3 \end{bmatrix}\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix}+\begin{bmatrix} \frac{1}{C_1} & -\frac{1}{C_1} & -\frac{1}{C_1} \\ -\frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ -\frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1}+\frac{1}{C_2} \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}=\begin{Bmatrix} V_1 \\ 0 \\ 0 \end{Bmatrix}$$

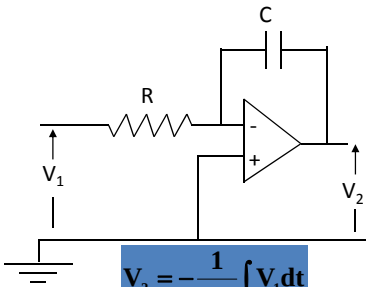
$$\begin{vmatrix} R_1 s+\frac{1}{C_1} & -\frac{1}{C_1} & -R_3 s-\frac{1}{C_1} \\ -\frac{1}{C_1} & R_2 s+\frac{1}{C_1} & R_3 s+\frac{1}{C_1} \\ -\frac{1}{C_1} & \frac{1}{C_1} & R_3 s+\frac{1}{C_1}+\frac{1}{C_2} \end{vmatrix}=0$$

R_1=15.9 kΩ, R_2=837 Ω, R_3=318 kΩ, C_1=C_2=0.005 μF

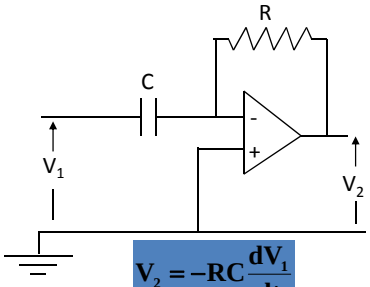
Öz değerler: 0, -628.93±12561.76i (ξ=0.05)



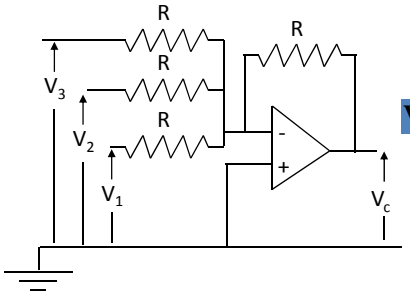
$$V_2=-\frac{R_2}{R_1}V_1$$



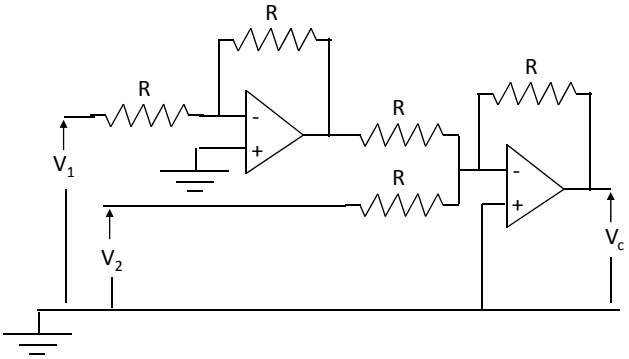
$$V_2=-\frac{1}{RC}\int V_1 dt$$



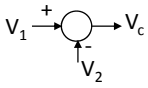
$$V_2=-RC\frac{dV_1}{dt}$$

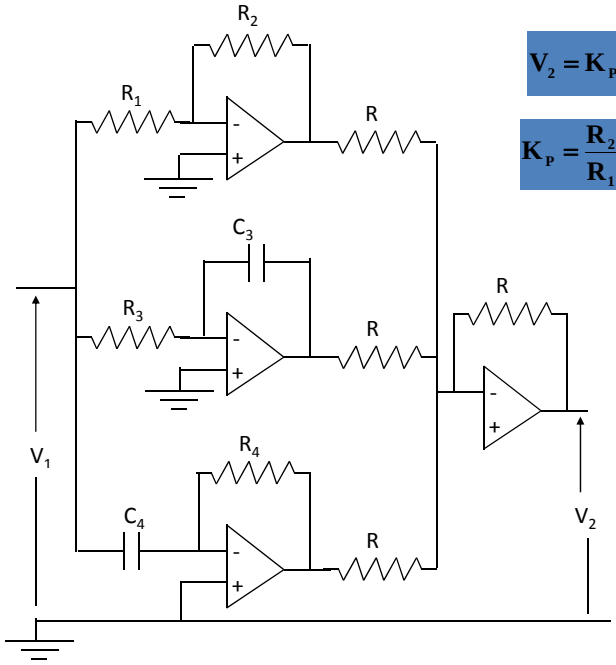


$$V_c = -(V_1 + V_2 + V_3)$$



$$V_c = V_1 - V_2$$





$$V_2 = K_P V_1 + K_I \int V_1 dt + K_D \frac{dV_1}{dt}$$

$$K_P = \frac{R_2}{R_1} \qquad K_I = \frac{1}{R_3 C_3} \qquad K_D = R_4 C_4$$

PID Kontrol Devresi