MEE 3017 System Modelling and Analysis

Modelling Fluid Systems

Introduction

A fluid system uses one or more fluids to achieve its purpose. Because the fluids (gas or liquid) are the most versatile medium for transmitting signals and power, they have wide usage in industry.

There two major types of fluid systems that are important in mechanical engineering:

- •hydraulic describes fluid systems that use liquids (e.g., oil or water)
- •pneumatic describes those using air or other gases.

Hydraulics is the study of systems in which the fluid is incompressible, that is, its density stays approximately constant over a range of pressures

Pneumatics is the study of systems in which the fluid is compressible.

Hydraulics and pneumatics share a common modeling principle: **conservation of mass**. It will form the basis of all our models of such systems.

Modeling pneumatic systems also requires application of thermodynamics, because the temperature and density of a gas can change when its pressure changes.

The major complexity of the fluid (and thermal) systems is that the characteristics of the system components cannot be defined easiliy using some formulation in most cases they are determined statictically as a result of some experiments.

We will avoid complex system models by describing only the gross system behavior instead of the details of the fluid motion patterns. The study of such motion belongs to the specialized subject of fluid mechanics and will not be treated here.

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Fluid System Variables

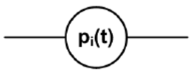
For incompressible fluids, conservation of mass is equivalent to conservation of volume, because the fluid density is constant. If we know the mass density *ρ and the* volume flow rate q_v we can compute the mass flow rate q_m.

$$q_m = \rho q_v$$

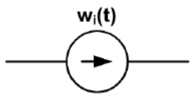
Therefore the flow variable is the *volumetric flow rate* for fluid systems

The effort variable is the one that causes the flow so for the fluid systems, it is the pressure. The across variable is the *pressure.*

The inputs will be ideal pressure sources and ideal flow rate sources.



ideal pressure source



ideal flow rate source

Capacitance

Elements that have an algebraic relationship between *the flow rate through the element and the time derivative of the pressure across the element are capacitive elements.*

$$q(t) = C_f \frac{dp(t)}{dt}$$

Fluid capacitance is the relation between pressure and stored volume or mass. Volume of liquid stored in an open vessel

$$V = \int_0^T \sum q(t)dt = \int_{p_1}^{p_2} C_f dp$$

Capacitive elements are energy storage elements (i.e., the energy put into these elements can be recovered).

Cf–fluid capacitance (m^5/N)

Capacitance

Fluid capacitance is modeled by a relationship between the derivative of the effort and the flow. Examples of fluid capacitance are tanks (like a water tower), fluid compressibility, fluid line compliance,and accumulators.

Tanks:

The pressure at the bottom of a tank is given by

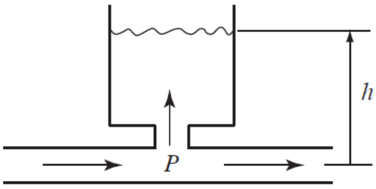
$$p(t) = \rho g h(t) + p_a$$

where g is the acceleration due to gravity, and h is the height of the fluid in the tank. Taking the derivative of this expression gives:

$$\frac{dp(t)}{dt} = \frac{d(\rho g h(t) + p_a)}{dt} = \rho g \frac{dh(t)}{dt}$$

for a tank with constant cross section and with net mass flow rate

$$\frac{dp(t)}{dt} = \rho g \frac{q_v}{A} \qquad C_f = \frac{A}{\rho g}$$



$$\frac{dV(t)}{dt} = q_{vi} - q_{vo}$$

$$V(t) = Ah(t)$$

$$A\frac{dh(t)}{dt} = q_{vi} - q_{vo}$$

$$q_v = q_{vi} - q_{vo}$$

$$\frac{dh(t)}{dt} = \frac{q_v}{A}$$

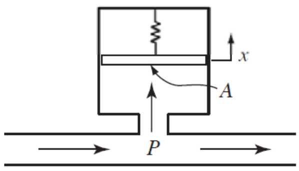
Capacitance

Accumulators

A spring-type accumulator works by having fluid push against a spring-loaded piston to store energy. If the mass of the piston can be neglected, then we can write

$$P = \frac{F}{A} = \frac{kx}{A}$$

where F is the force applied to the piston,
A is the cross-sectional area of the piston,
k is the spring constant, and x is the spring deflection



The total volume change may be written:

$$\Delta P = \frac{k\Delta V}{A^2}$$

$$\frac{\Delta P}{\Delta t} = \frac{k}{A^2}\frac{\Delta V}{\Delta t} \Rightarrow \frac{dP}{dt} = \frac{k}{A^2} q_v$$

$$C = \frac{A^2}{k}$$

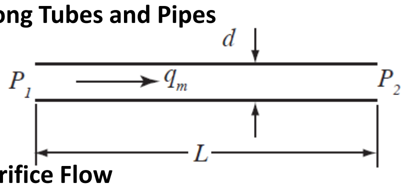
Resistance

Elements that have an algebraic relationship between *flow rate* through the element and *pressure difference* across the element are resistive elements.

$$\Delta p(t) = R_f q(t)$$

Examples of fluid resistors include long tubes or pipes, orifices, and valves. In each of these cases, the pressure drop across the device is related to the flow through the device.
Resistive elements are energy dissipative elements (i.e., the energy put into these elements can never be recovered.

Long Tubes and Pipes



$$R_f = 128\frac{\mu L}{A}$$

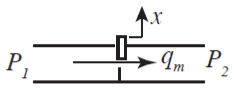
μ is the viscosity of the fluid

Orifice Flow

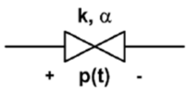
$$P_1 - P_2 = \frac{1}{2\rho C_d^2 A_0^2} q_m |q_m|$$

C_d is the discharge coefficient,,orifice with cross-sectional area A_0

Valves



$$P_1 - P_2 = \frac{1}{2\rho C_d^2(x) A^2(x)} q_m |q_m|$$

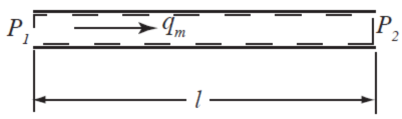


N

Inertance

For fluid flowing through a pipe, Newton's second law holds:

$$\sum F = ma$$



$$A(P_1 - P_2) = m\dot{v}$$

$$v = \frac{q_v}{A} \Rightarrow \dot{v} = \frac{1}{A}\dot{q}_v$$

$$A(P_1 - P_2) = m\frac{1}{A}\dot{q}_v$$

$$m = \rho AL$$

$$A(P_1 - P_2) = \rho AL\frac{1}{A}\dot{q}_v$$

$$P = \frac{\rho L}{A}\frac{dq_v}{dt} \qquad I_f = \frac{\rho L}{A}$$

The total pressure drop in the pipe is P. L: length of pipe, A: pipe cross section

Modelling Fluid Systems

Conservation of Mass

For incompressible fluids conservation of mass turns to conservation of volume

$$\frac{dV(t)}{dt} = q_{vi} - q_{vo}$$

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Mekanik (Öteleme, Dönme), Elektrik, Akış, Isıl Sistemlerde Analoji

x Yer değ. (m)	$\dot{x}$ Hız (m/s)	$\ddot{x}$ İvme (m/s^2)	m Kütle (kg)	k Yay sabiti (N/m)	c Sönüm sab. (Ns/m)	f Kuvvet (N)
θ Açısal konum (rad)	$\dot{\theta}$ Açısal hız (rad/s)	$\ddot{\theta}$ Açısal ivme (rad/s^2)	I_G Atalet momenti (kg-m^2)	K_r Dönel yay sab. (Nm/rad)	C_r Dönel sönüm sab. Nm/(rad/s)	M Moment (Nm)
q Elek. yükü (Coulomb)	i Akım (Amper)		L İndüktör (Henry)	1/C C:Kapasitör (Farad)	R Direnç (Ohm)	V Gerilim (Volt)
V_f Hacim (m^3)	Q_f Debi (m^3/s)		I_f Akışkan ataleti (kg/m^4)	$1/C_f$ C_f:Akış kapasitesi	R_f Akış direnci	P Basınç (N/m^2)
H_t Isı (Joule)	Q_t Isıl debi (J/s)			$1/C_t$ C_t:Isıl kapasite	R_t Isıl direnç	T Sıcaklık (^{o}C)

V_f Hacim (m^3)	Q_f Debi (m^3/s)		I_f Akışkan ataleti (kg/m^4)	$1/C_f$ C_f:Akış kapasitesi	R_f Akış direnci	P Basınç (N/m^2)

D.Rowell & D.N.Wormley, System Dynamics:An Introduction, Prentice Hall, 1997

Boruda akışkan ataleti:

$$I_f = \frac{\rho L}{A}$$ I_f:Akışkan Ataleti, ρ: Yoğunluk, L: Boru uzunluğu, A: Boru kesiti

Akışkan depo kapasitesi:

$$C_f = \frac{A_d}{\rho g}$$ C_f: Akışkan depo kapasitesi, A_d:Depo kesit alanı, g=9.81 m/s^2

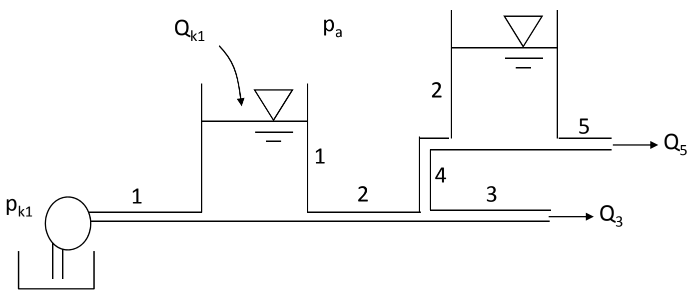
Laminar akışta boruda akışkan direnci:

$$R_f = \frac{128\mu L}{\pi d^4}$$ R_f:Direnç, μ:Viskozite, L:Boru boyu, d:Boru çapı

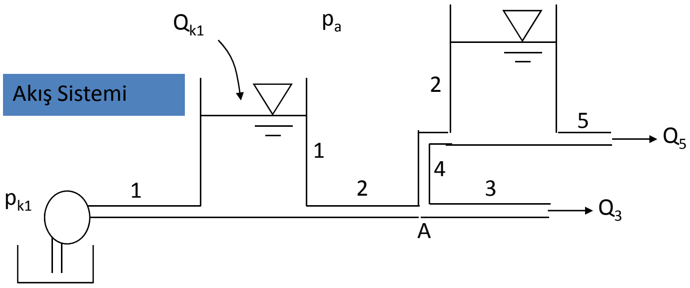
Laminar akışta Reynold Sayısı<2000

Reynolds Sayısı= $\frac{4\rho Q_f}{\pi d \mu}$

Akış Sistemlerinin Benzer Elektrik Devreleri

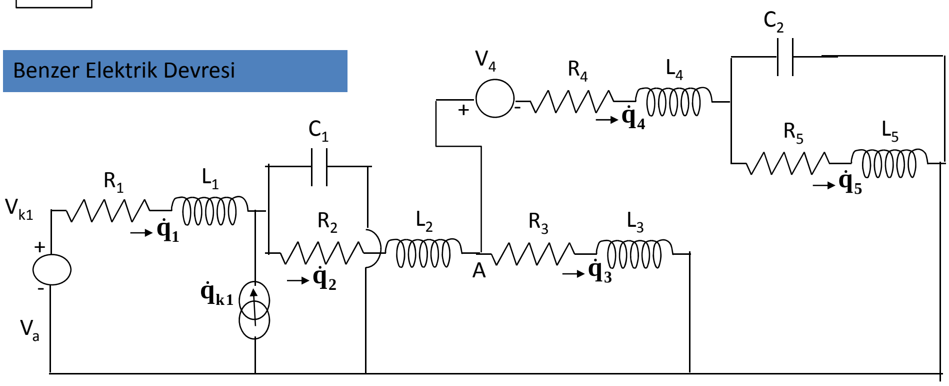


Akış Sistemi



Akış	Elek.
p_n	V_n
R_n	R_n
I_n	L_n
Q_n	$\dot{q}_n$
C_n	C_n

Benzer Elektrik Devresi



Akış sistemleri yerine benzer elektrik devreleri incelenebilir

Dinamik (Transient)

Düzgün rejim (Steady-state)

Günümüzde bilgisayar destekli mühendislik (CAD/CAE)