MEE 3017 System Modelling and Analysis

L04 Modelling Electrical Systems

The fundamental variables

Electricity is the physical phenomenon that is associated with the presence and flow of **electric charge.**
Then the electrical systems are the systems that operates by utilizing their components behaivors under the effect the flow of the electrical charge.

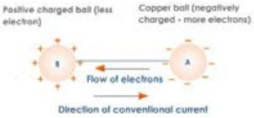
The Electrical **Charge**

The electric charge is a characteristic of a unit of matter that expresses the extent to which it has more or fewer electrons than protons. In an atom of matter, an electrical charge occurs whenever the number of protons in the nucleus differs from the number of electrons surrounding that nucleus.

Unit of the charge is Coulombs [C] ant its symbol is **Q**.

An electric field, also called an electrical potential field surrounds any object that has charge. Therefore, when two objects having electric charge are brought into each other's vicinity, an electrostatic force is manifested between them.

The accumulation of charges in seperated regions result in two terminal system that has a potetial to effect their charges. If the media beetwen them is conductive then this potential causes the motion of the charges.



Dr. Levent ÇETİN

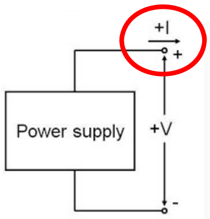
The fundamental variables

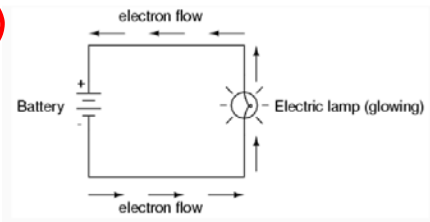
The Electrical Potential Difference or **Voltage**

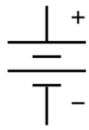
Voltage, also called electromotive force, is a quantitative expression of the potential difference in charge between two points in an electrical field. This potential difference causes a effort on the charges to move from higher potential to lower potential.

Therefore, voltage is a measure of the force that causes charge (electron) motion.

Unit of the voltage is Volts [V] and its symbols are V, U or E.







Electromotive force, also called emf (denoted and measured in volts), refers to voltage generated by a battery or by the magnetic force according to Faraday's Law, which states that a time varying magnetic field induces an electric current

The fundamental variables

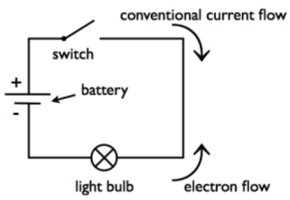
The Electric Current

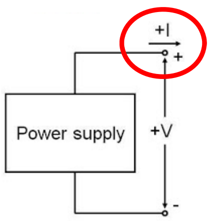
Electric charge flows through a conductor when there is voltage present across a conductor.

The current is a flow of electrical charge carriers, usually electrons or electron-deficient atoms (ions).

Unit of the current is Amperes [A] and its symbol is **I**.

$$I = \frac{dQ}{dt} [A]$$





Although the considered motion is the motion of the negative charges, The conventional direction current flow is from positive to negative.

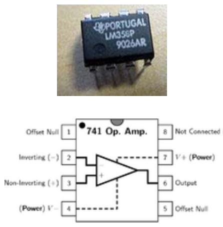
The effort and flow variables defines the power of any physical system together. So the electrical power is:

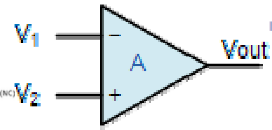
$$P = VI \quad [W]$$

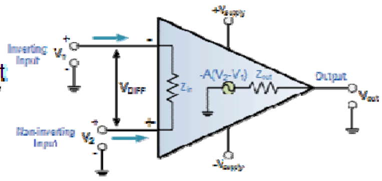
The components of electrical systems

The electrical systems may be composed of different types of complex electrical components (Transistors, OpAmps, diodes etc.)

From the analysis point of view, all components exhibit some sort of relationship between the current flowing through them and the voltage drop accross them







The relationship between voltage and current can be defined via three basic property of components referring to three constitutional laws.

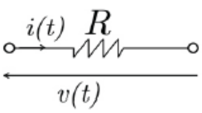
1. Resistive Components
2. Capacitive Components
3. Inductive Components

The components of electrical systems

The Resistive Components

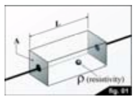
Elements that have an algebraic relationship between the voltage across them and the current through them are resistive elements.

The characteristic value that describes this type of voltage current relationship is the **R**–resistance (which has the unit of Ω, ohms)



$$v(t) = Ri(t)$$

$$R = \rho \frac{L}{A}$$



ρ: specific conductivity

Purely resistive elements are defined as **resistors.**

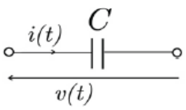
Resistive elements are energy dissipative elements (i.e., the energy put into these elements can never be recovered). So the virtual work done by resistive component can be writen as

$$\delta w = -Ri\delta q = -R\dot{q}dq$$

v(t): voltage across the component
i(t): current through the component
R: resistance

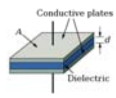
The components of electrical systems
The Capacitive Components

Elements that have an algebraic relationship between the current through them and the time derivative of voltage across them are capacitive elements.
The characteristic value that describes this type of voltage current relationship is the **C**–capacitance (which has the unit of F, Farads)



$$i(t)=C\frac{dV_c(t)}{dt}$$

$$C=\varepsilon\frac{A}{d}$$



ε: dielectric constant

Purely capacitive elements are defined as **capacitors.**

$$V_c(t)=\frac{1}{C}\int i(t)dt$$

Using i(t)=dq(t)/dt

$$V_c(t)=\frac{1}{C}\int\frac{dq(t)}{dt}dt=\frac{1}{C}\int dq(t)\Rightarrow V_c(t)=\frac{q(t)}{C}$$

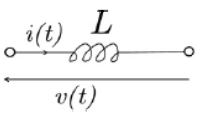
Capacitive elements are electrical energy storage elements (i.e., the energy put into these elements can be recovered). So the electrical potential energy stored in a capacitor can be writen as

$$E_p=\frac{1}{2}CV^2=\frac{1}{2}C\frac{q^2}{C^2}=\frac{1}{2}\frac{1}{C}q^2$$

v(t): voltage across the component
i(t): current through the component
C: capacitance

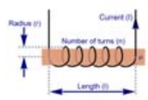
The components of electrical systems
The Inductive Components

Elements that have an algebraic relationship between the voltage across them and the time derivative of current through them are inductive elements.
The characteristic value that describes this type of voltage current relationship is the **L**–Inductance (which has the unit of H, Henry)



$$V_L(t)=L\frac{di(t)}{dt}$$

$$L=\frac{\phi}{i}$$



ϕ: magnetic flux

Purely inductive elements are defined as **capacitors.**

Using i(t)=dq(t)/dt

$$V_L(t)=L\frac{d}{dt}\frac{dq(t)}{dt}\Rightarrow V_L(t)=L\frac{d^2q(t)}{dt^2}$$

Inductive elements are magnetic energy storage elements (i.e., the energy put into these elements can be recovered). So the magnetic energy stored in a inductor can be writen as

$$E_p=\frac{1}{2}Li^2=\frac{1}{2}L\dot{q}^2$$

v(t): voltage across the component
i(t): current through the component
L: inductance

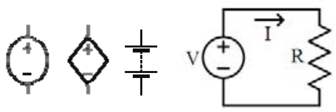
The excitations (inputs) of electrical systems

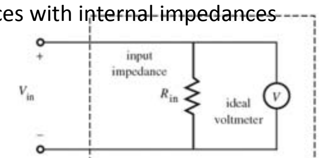
There are two type of devices which can be used externally to supply power to electrical systems.

Voltage source is a two terminal device which can maintain a fixed voltage. An ideal voltage source can maintain the fixed voltage independent of the load resistance or the output current.

$$\delta w_V = V_s \delta q_s$$

Real-world sources of electrical energy, such as batteries, generators, and power systems, can be modeled as voltage sources with internal impedances

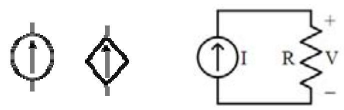


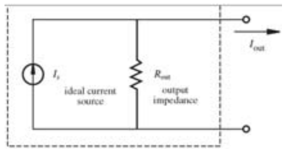


Current Source is an electronic device that delivers or absorbs an electric current which is independent of the voltage across it.

In real-world sources of electrical current are electrical systems that also can be modeled as current sources with internal impedances

$$\delta w_I = 0$$



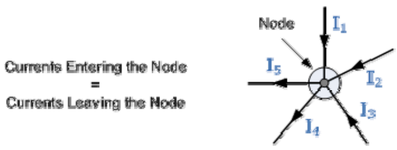


L04 Modelling Electrical Systems

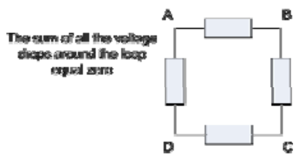
The components of electrical systems

There are two basic equations that describes the voltage -current relationship : Kirchhoff's Laws rules which deal with the conservation of current and energy within electrical circuits.

1. **Kirchoffs Current Law** or KCL, states that the "*total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node*".
2. **Kirchoffs Voltage Law** or KVL, states that "*in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop*" which is also equal to zero.



$$I_1 + I_2 + I_3 = I_4 + I_5 = 0$$



$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

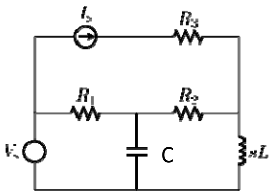
An important facts to keep in the mind during electrical system analysis are:

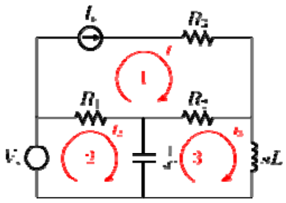
The voltage drop on parallel branches are the same

The currents flow through components in series are thesame

DoF : Number of loops in the circuit that does not contain any other loop.

Examples of electrical systems





Number of equations?

3

Current of first loop?

$I_1 = I_s$

So the current input is similar to position input in mechanical systems