MEE 3017 System Modelling and Analysis

L02 Mechanical Systems
Lagrange Equations

Lagrange Equations

takes roots on from principle of least action and gives same results with Free Body Diagram method.

- It is advantageous in multibody systems because it minimize the number of equations that describe overall system behaviour.
- It uses the kinematic constraints to find out generalized coordinates.

Generalized Coordinates

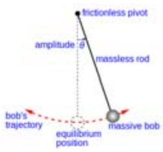
These parameters define the configuration of the system relative to the reference configuration.

If these parameters are <u>independent of one another</u>, then number of independent generalized coordinates is defined by the <u>*number of degrees of freedom of the system.*</u> (Minimum number of equations)

Kinematic Constraints

Defined as the equations that limits the free motion of the sytem components.

Pendulum Example



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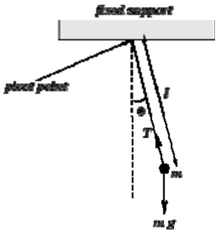
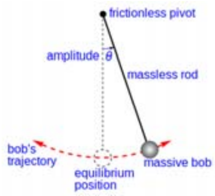
Lagrange Equations

Pendulum Example

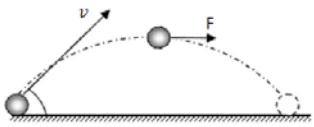




The cord does not stretch

Lagrange Equations

Projectile Motion



$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

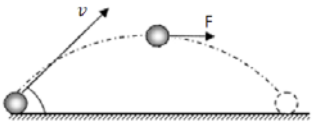
$$F = ma \qquad KE = \frac{1}{2}mv^2 \qquad PE = mgh$$

If F changes with time

$$a = \frac{F}{m} \qquad v^2 = u^2 + 2\frac{F}{m}s \qquad \frac{mv^2}{2} = \frac{mu^2}{2} + Fs$$

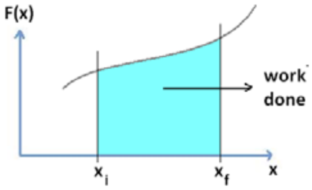
Lagrange Equations

Projectile Motion



$$KE_f - KE_i = Fs$$

$$W = \int_{1=i}^{2=f} F dx$$



$$W = \underbrace{\int F dx \Big|_f}_{B_f} - \underbrace{\int F dx \Big|_i}_{B_i}$$

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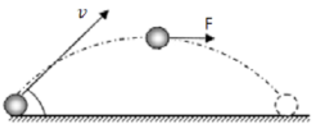

$$KE_f - KE_i = B_f - B_i$$

$$KE_f - B_f = KE_i - B_i$$

$$B = -u$$

$$KE_f + \underbrace{u_f}_{\text{Potential Energy}} = KE_i + u_i$$

This new equality states that energy is conserved but here key assumption is that force acting on the ball is the gravitaional force

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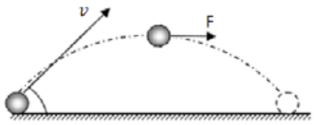
Lagrange Equations

Projectile Motion



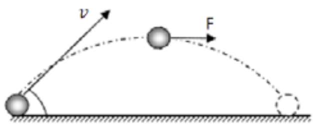
$$u = -\int F dx$$ Definition of **conservative forces** $$\frac{du}{dx} = -F$$

In projectile motion case:

$$u = mgh \qquad \frac{du}{dh} = -mg$$

Lagrange Equations

Projectile Motion



Why does ball follows this particular trajectory?

The difference between the kinetic and potential energy, at any instant during motion gives, the work done at that particular instant.

The sum or the integration of this difference gives the “action” during entire motion.

$$\sum (KE - PE) = A \qquad A = \int (KE - PE) dt$$

Action (physics), an attribute of the development of a system over a period of time

The difference between kinetic energy and potential energy is defined as LANGRANGIAN

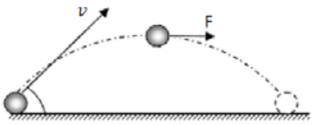
$$L = KE - PE$$

$$L = T - V$$

T: Kinetic energy of entire system (E_1)
V:Potential Energy of entire system (E_2)

Lagrange Equations

Projectile Motion



LANGRANGIAN

$$L = KE - PE$$

$$L = T - V$$

T: Kinetic energy of entire system (E_1)
V:Potential Energy of entire system (E_2)

Why does ball follows this particular trajectory?
So the action can be written in terms of Langrangian:

$$A = \int L dt$$

The trajectory of the system occurs in harmony with the nature obeying an optimization: Principle of Least Action
Principle of Least Action states that the trajectrory (or time evolution) of a system is the one that minimizes the action integral

$$\frac{dA}{dx} = 0$$

The further deviation of the equations can be done via calculus of variations (not considered in this course). But the result is important

Lagrange Equations

Equations of motion (Lagrange Equation)
The result of principle least action is Euler Lagrange Equations which does not show the effect of non conservative forces such as friciton forces.

$$\frac{dL}{dx} = \frac{d}{dt}\frac{dL}{d\dot{x}}$$

So the work terms, done by external forces, should be addded. The principle of virtual work helps to compensate missing term.
Generalized forces is defined as the forces that does work in generalized coordinates they can be found by calculating virtual work in generalized coordinates.
Eventually we obtain Langrange equations defining system dynamics as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_k$$

$$L = KE - PE$$

$$L = T - V$$

T: Kinetic energy of entire system (E_1)
V:Potential Energy of entire system (E_2)

Q_k:Generalized forces

q_k:Generalized coordinates