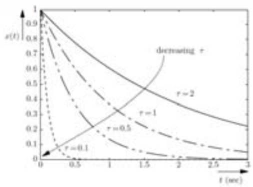
The Eigenvalues

Eigenvalues of system matrice determines the systems response. For the secon order systems, they are poles of the "s" polynomial.
They can be real or complex numbers.
From the engineering point of view, it is important that they are negative or complex number with negative real part so that system is defined to be stable.

The Negative Real Eigenvalues



Suppose that i^{th} eigen value is negative real.

$$s_i = -\sigma \quad \text{(where } \sigma>0\text{)}$$

The coresponding solution is in the following form.

$$A_i e^{-\sigma t} \quad \text{(where } A_i \text{ is calculated using initial values.)}$$

Time constant of the system is defined as the reciprocal of absolute value of the eigenvalue

$$\tau = \frac{1}{\sigma}$$

Time constant determines the rate of the system (i.e how fast the exponential function will approch to zero)

$$A_i e^{-\frac{t}{\tau}}$$

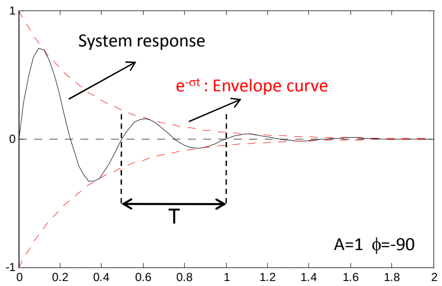
The Complex Eigenvalues with Negative Real Part

Suppose that i^{th} and $(i+1)^{th}$ eigenvalues are complex conjugate and theri real part is negative

$$s_{i,i+1} = -\sigma \mp i\omega \quad \text{(where } \sigma>0\text{)}$$

The coresponding solution is in the following form.

$$A_i e^{-\sigma t}\cos(\omega t + \phi) \quad \text{(where } A_i \text{ and } \phi_i \text{ is calculated using initial values.)}$$



$$T = \frac{2\pi}{\omega}$$

The Complex Eigenvalues with Negative Real Part

There are two characteristics in time response of the complex conjugate pair:

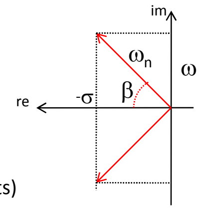
The natural frequency is defined as the frequency of the oscilations that the system would do if there were no damping and no external force, applied.

The damping ratio is the measure of the system dissipative action that shows the ability of the system to reduce magnitude of the oscilations.

Both can be calculated using following equations:

$$\omega_n = \sqrt{\sigma^2 + \omega^2}$$

$$\xi = \cos\beta$$



ω: Damped frequency (observed)

ω_n: natural (undamped) frequency (no damping= no dissipative components)

ξ: Damping Ratio

The complex conjugate pole can be writen in terms of these characteristics:

$$s_{i,i+1} = -\xi\omega_n \mp i\omega_n\sqrt{1-\xi^2}$$

L04 Solving set of linear Equations

The Complex Eigenvalues with Negative Real Part

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$$s_{i,i+1} = -\xi\omega_n \mp i\omega_n\sqrt{1-\xi^2}$$