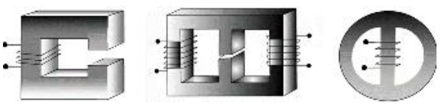
MEE303 Sensor Systems

W05
Displacement Sensing

Inductive displacement Sensing

Magnetic Circuit Analysis

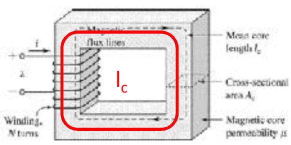
a magnetic structure of regular geometric shape called ***core*** and an exciting coil having a number of turns (= N) of conducting material are wound over the core.



the relationship of current on exiciting coil and generated **magnetic field** is given by Ampere's Law

$$\oint H dl = I_{net}$$

$$H l_c = Ni$$

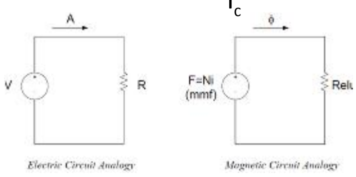


magnetic flux density due to exciting coil is

$$H = \frac{Ni}{l_c} \Rightarrow B = \mu H \Rightarrow B = \frac{\mu Ni}{l_c}$$

magnetic flux

$$\phi = BA_c \Rightarrow B = \frac{\mu Ni}{l_c} \Rightarrow \phi = \frac{\mu NiA_c}{l_c}$$



magnetomotive force $mmF = ni$

Reluctance $R_{mag} = \frac{l_c}{\mu A_c}$

magnetic flux $\phi = \frac{\mu NiA}{l} = \frac{Ni}{R_{mag}}$

l= the total length of the flux path
μ = the permeability of the magnetic circuit material
A = the cross-sectional area of the flux path

These quantities cannot be sensed directly, the effect of magnetic structure reflects to electric circuit that is formed via coil,namely via **inductance** $V = -L\frac{di}{dt}$

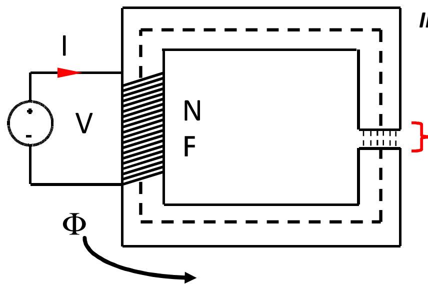
$$V = -N\frac{d\phi}{dt} \Rightarrow \phi = \frac{F_{mmf}}{R_{mag}} = \frac{ANi\mu}{l} \Rightarrow V_{emf} = -N\frac{d}{dt}\left(\frac{ANi\mu}{l}\right) \Rightarrow V_{emf} = -\left(N^2\frac{A\mu}{l}\right)\frac{di}{dt}$$

$$L = N^2\frac{A\mu}{l} = \frac{N^2}{R_{mag}}[H]$$

N: number of turns in coil

Inductive displacement Sensing

Magnetic Circuit Analysis



Pictorial representation

Iron Core:*Iron in magnetic structure has reluctance*

Length of Iron=L_{iron} $Rmag_{iron}$: Reluctance of iron core

$$Rmag_{iron} = \frac{L_{iron}}{\mu_{iron} \cdot A}$$

Air Gap *has a reluctance*

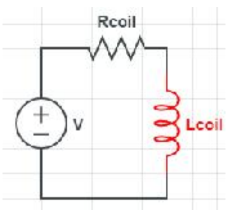
$$\mu_{iron} = \mu_r \mu_{air} = 2000 \times 4\pi 10^{-7}$$

Length of air gap=L_{ag} $Rmag_{air}$: Reluctance of air gap

$$Rmag_{air} = \frac{L_{ag}}{\mu_0 \cdot A}$$

$$Rmag_{air} >> Rmag_{iron} \qquad \mu_{air} = 4\pi 10^{-7}$$

Voltage source and coil form MMF source $mmF = ni$

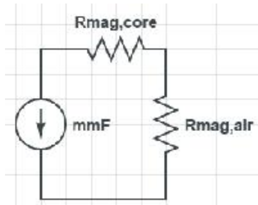


Electric Circuit

$$R_{coil} = \rho \frac{L}{A} [ohm]$$

$$L_{coil} = \frac{\Psi}{i} = \frac{N^2}{R_{mag,total}} [Henry]$$

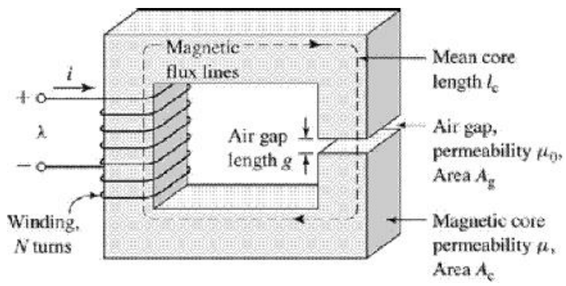
Equivalent Magnetic Circuit



$$Rmag_{total} = Rmag_{air} + Rmag_{iron}$$

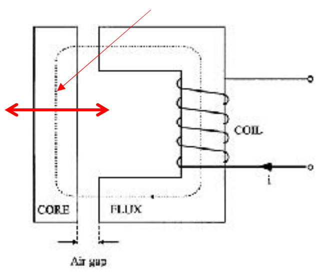
Inductive displacement Sensing

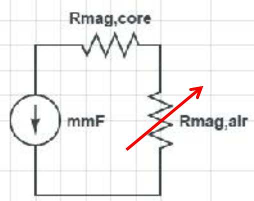
Magnetic Circuit Analysis



Electric Circuit	Magnetic Circuit
Ohm's law, *I = V/R*	= $\mathcal{F}/\mathcal{R}$
Resistance, *R = l/σA*	Reluctance, $\mathcal{R}$ = l/ A
Current, I	Flux,
Voltage, V	mmf, $\mathcal{F}$
Conductivity, *σ*	Permeability,
Conductance, G	Permeance, $\mathcal{P}$

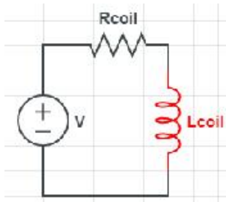
Inductive Displacement Sensor Principle





$$Rmag_{total} = Rmag_{air} + Rmag_{iron}$$

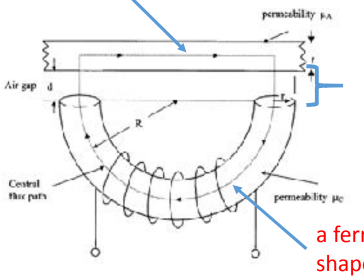
$$L_{coil} = \frac{\Psi}{i} = \frac{N^2}{R_{mag,total}} [Henry]$$



Inductive displacement Sensing

The Single-Coil Linear Variable-Reluctance Sensor:

An armature: a ferromagnetic moving plate



a variable air gap

a ferromagnetic core in the shape of a semicircular ring

the relationship between L and α is nonlinear.

the reluctances of the core, air gap, and armature

$$\Re_T = \Re_C + \Re_G + \Re_A$$

- length of flux path in the core is taken as πR
- And its cross-sectional area πr^2
- total length of the flux path in air is 2d
- length of an average central flux path in the armature is 2R
- cross section area of the armature approximately to 2rt where t is thickness.

$$\Re_T = R/\mu_C\mu_0 r^2 + 2d/\mu_0\pi r^2 + R/\mu_A\mu_0 rt$$

all of the parameters are fixed except for the one independent variable the air gap. Hence, it can be simplified as

$$\Re_T = \Re_0 + kd \quad \text{where } \Re_0 = R/\mu_0 r\,[1/\mu_C r + 1/\mu_A t], \text{ and } k = 2/\mu_0\pi r^2$$

the inductance can be written as

$$L = n^2/(\Re_0 + kd) = L_0/(1 + \alpha d)$$

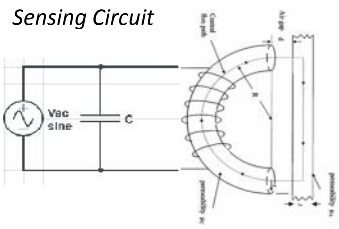
L_0 = the inductance at zero air gap

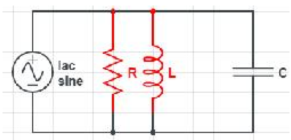
$\alpha = k/\Re_0$

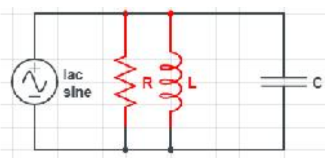
Inductive displacement Sensing

The Single-Coil Linear Variable-Reluctance Sensor:

Sensing Circuit







$$R_{mag,t} = R_{mag,0} + kd$$

$$R_{mag,0} = \frac{R}{\mu_0 r}\left[\frac{1}{\mu_c r} + \frac{1}{\mu_A r}\right] \qquad k = \frac{2}{\mu_o \pi r^2} \qquad \alpha = \frac{k}{R_{mag,0}}$$

$$R = \rho\frac{l_w}{A_w}$$

$$L = \frac{N^2}{R_{mag,0} + kd} = \frac{L_0}{1 + \alpha d}$$

$$i = i_R + i_L + i_C \qquad V = V_R = V_L = V_C$$

$$i = \frac{V}{R} + i_L + C\frac{dV}{dt} \qquad i_L = \frac{1}{L}\int Vdt$$

$$i = \frac{V}{R} + \frac{1}{L}\int Vdt + C\frac{dV}{dt}$$

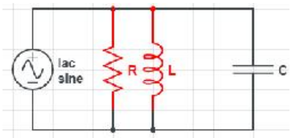
$$i(s) = v(s)\left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)$$

$$\frac{v(s)}{i(s)} = \frac{RLs}{RLCs^2 + Ls + R}$$

$$\frac{v(s)}{i(s)} = \frac{1}{C}\frac{s}{(s^2 + \frac{1}{RC}s + \frac{1}{LC})}$$

Inductive displacement Sensing

The Single-Coil Linear Variable-Reluctance Sensor:



$$\frac{v(s)}{i(s)}=\frac{1}{C}\frac{s}{(s^2+\frac{1}{RC}s+\frac{1}{LC})}$$

$$G(s)=\frac{1}{C}\frac{s}{(s^2+\frac{1}{RC}s+\frac{1}{LC})}\Rightarrow F(jw)=\frac{1}{C}\frac{jw}{((jw)^2+\frac{1}{RC}jw+\frac{1}{LC})}=\frac{1}{C}\frac{jw}{j\frac{1}{RC}w+(\frac{1}{LC}-w^2)}$$

$$F(jw)=\frac{1}{C}\frac{jw}{j\frac{1}{RC}w+(\frac{1}{LC}-w^2)}=\frac{1}{C}\frac{w}{\sqrt{(\frac{1}{LC}-w^2)^2+(\frac{1}{RC}w)^2}}\angle 90-tg^{1}\left(\frac{\frac{1}{RC}w}{\frac{1}{LC}-w^2}\right)$$

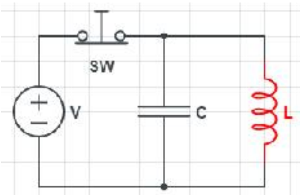
$$i(t)=I_0\sin(w_i t)$$

$$V(t)=I_0\left(\frac{1}{C}\frac{w_i}{\sqrt{(\frac{1}{LC}-w_i^{\,2})^2+(\frac{1}{RC}w_i)^2}}\right)\sin(w_i t+90-\left(tg^{1}\left(\frac{\frac{1}{RC}w}{\frac{1}{LC}-w^2}\right)\right))$$

Inductive displacement Sensing

The Single-Coil Linear Variable-Reluctance Sensor:

$$V_c(t=0)=V_0$$



$$0=L\frac{di(t)}{dt}+V_c(t)\qquad i(t)=C\frac{dV_c(t)}{dt}\qquad LC\frac{d^2V_c(t)}{dt^2}+V_c(t)=0$$

$$LC\frac{dV_c(t)}{dt}+V_c(t)=0\Rightarrow LC\left[s^2V_c(s)-sV_c(t=0)-\dot{V}_c(t=0)\right]+V_c(s)=0$$

$$V_c(s)=V_0\frac{LCs}{LCs^2+1}\qquad V_c(s)=V_0\frac{s}{s^2+\frac{1}{LC}}\qquad s_{1,2}=\pm j\sqrt{\frac{1}{LC}}=\pm jw$$

$$V_c(s)=V_0\frac{s}{(s-jw)(s+jw)}$$

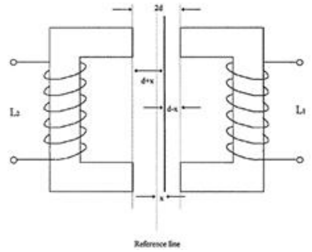
$$c_1=V_0\frac{s}{(s-jw)(s+jw)}(s-jw)\Big|_{s=jw}=V_0\frac{s}{(s+jw)}\Big|_{s=jw}=V_0\frac{jw}{(jw+jw)}=\frac{V_0}{2}$$

$$V_c(t)=V_0\cos(wt)=V_0\cos(\sqrt{\frac{1}{LC}}t)$$

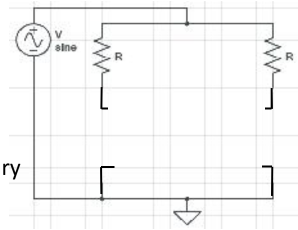
Inductive displacement Sensing

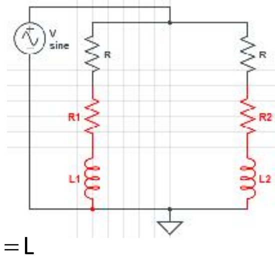
The Variable-Differential Reluctance Sensor

sensor consists of an armature moving between two identical cores, and separated by a fixed distance of 2d.



The flux generated by primary coils depends on the reluctance of the magnetic path, the main reluctance being the air gap





at $t=0$ $R_1 = R_2 = R$ and $L_1 = L_2 = L$

$$V_{out}(0) = V_{ref}\left(\frac{R+wL}{R+R+wL} - \frac{R+wL}{R+R+wL}\right) = 0$$

Any motion of the core increases the air gap on one side and decreases it on the other side, thus causing reluctance to change, in accordance

$$L_1 = L_{01} / \left[1+\alpha(d-x)\right] \qquad L_2 = L_{02} / \left[1+\alpha(d+x)\right]$$

$$X_{L1} = 2\pi f(L+\Delta L) = w(L+\Delta L) \quad X_{L2} = 2\pi f(L-\Delta L) = w(L-\Delta L)$$

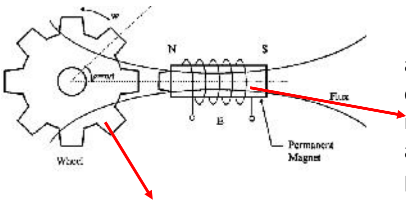
$$V_{out} = V_{ref}\left(\frac{R+wL+w\Delta L}{R+R+wL+w\Delta L} - \frac{R+wL-w\Delta L}{R+R+wL-w\Delta L}\right)$$

$$V_{out} \cong V_{ref}\frac{2w\Delta L}{2R+wL}$$

Inductive displacement Sensing

Variable-Reluctance Tachogenerators

The induced emf in the sensor depends on the linear or angular velocity of the motion.



and a coil wound onto a permanent magnet,extended by a soft iron pole piece.

The variable-reluctance tachogenerator consists of a ferromagnetic, toothed wheel attached to a rotating shaft,

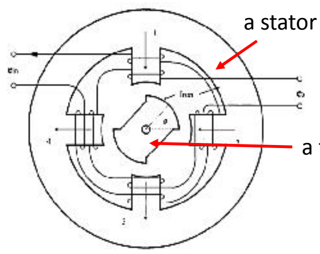
The wheel moves in close proximity to the pole piece, causing the flux linked by the coil to change, thus inducing an emf in the coil.

The reluctance of the circuit depends on the width of the air gap between the rotating wheel and the pole piece.

When the wheel rotates with a velocity ω, the induced emf is given by

$$E = bm\omega \sin m\omega t$$

Microsyn (giving output signals as low as 0.01°)



a stator carrying four coils

a ferromagnetic rotor

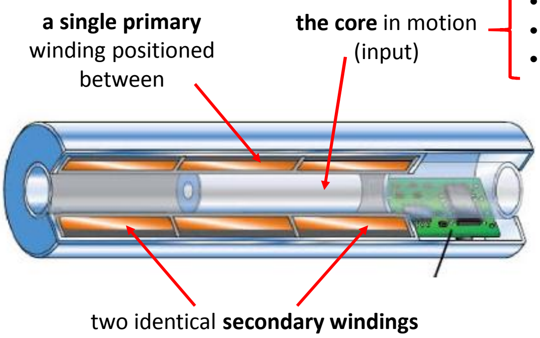
null position: the voltages induced in coils 1 and 3 are balanced by voltages induced in coils 2 and 4.

The motion of the rotor in one direction increases the reluctance of two opposite coils while decreasing the reluctance in others, resulting in a net output voltage e0 .

The movement in the opposite direction reverses this effect with a 180° phase shift

Inductive displacement Sensing

Linear Variable Differential Transformer



- magnetically permeable material
- free to move axially within the coil's hollow bore
- coupled to the object whose position is being measured

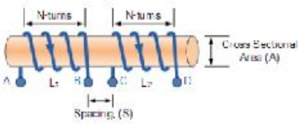
Operating Principle

When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil. The coils are said to have **mutual inductance M**, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.

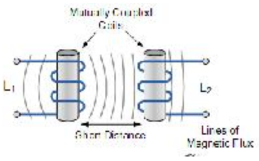
The primary winding is energized by a high-frequency 50 Hz to 20 kHz ac voltage

two secondary windings are made identical by having an equal number of turns and similar geometry

They are connected in series opposition so that the induced output voltages oppose each other



Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor.



Magnetic Coupling between coils can be effected via structure and/or material properties.

Inductive displacement Sensing

Linear Variable Differential Transformer

Mutual Inductance is the basic operating principle of any electrical component that interacts with another magnetic field.

The mutual inductance that exists between the two coils can be greatly increased by positioning them on a common soft iron core or by increasing the number of turns of either coil as would be found in a transformer.

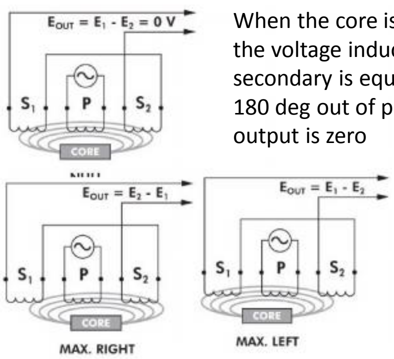
$$M_{12} = N_2 \frac{\Psi_1}{i_2} = \frac{\mu_0 \mu_r N_1 N_2 A}{l} [\text{Henry}]$$

Assuming a perfect flux linkage between the two coils the mutual inductance that exists between them can be given as

$$M_{12} = M_{21} = \sqrt{L_1 L_2} [\text{Henry}] \quad M_{12} = k\sqrt{L_1 L_2} [\text{Henry}]$$

<u>Coupling Factor Between Coils (k):</u> The amount of inductive coupling that exists between the two coils is expressed as a fractional number between 0 and 1, where 0 indicates zero or no inductive coupling, and 1 indicating full or maximum inductive coupling.

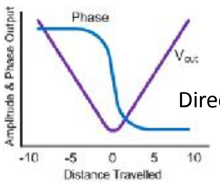
The operation principle of LVDT is closely related to this coupling factor as the flux linkages are directly modified via motion of the core



When the core is centered perfectly the voltage induced in each secondary is equal in amplitude and 180 deg out of phase. Thus the LVDT output is zero

As the core inside the former moves, the magnetic paths between primary and secondaries change, thus giving

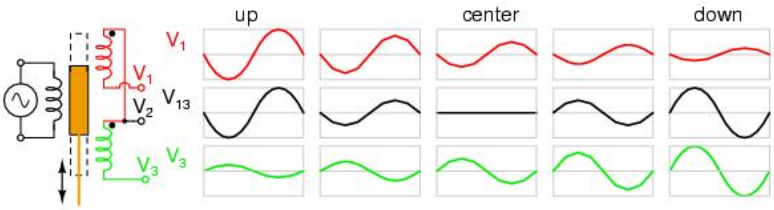
The magnitude of the output voltage is proportional to the distance moved by the core,

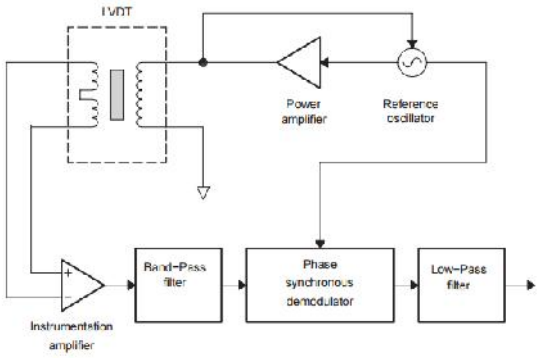


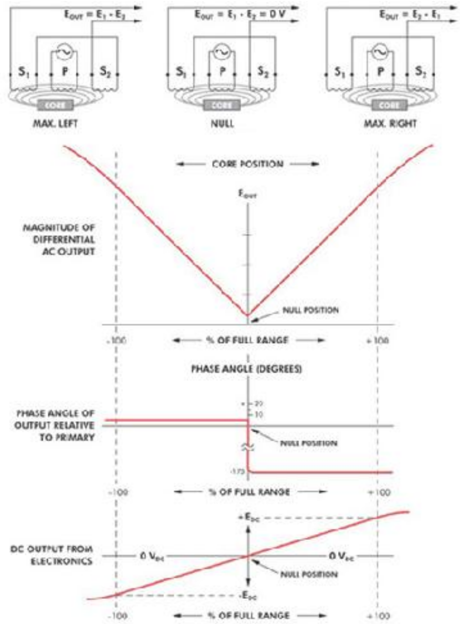
Direction:phase

Inductive displacement Sensing

Linear Variable Differential Transformer





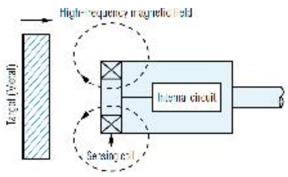


Inductive displacement Sensing

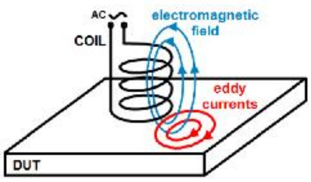
http://www.keyence.com/ss/products/measure/measurement_library/type/inductive/

Eddy-current displacement sensors

Eddy-current sensors use high-frequency magnetic fields to sense the distance to a metal target.

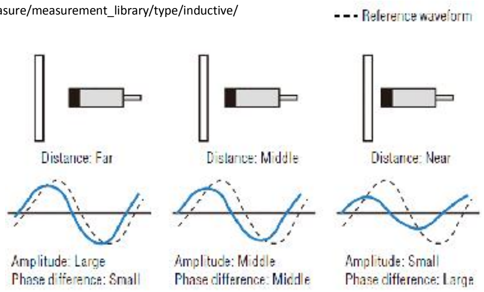


Sensing begins by passing alternating current through the sensing coil. This creates an alternating magnetic field around the coil.



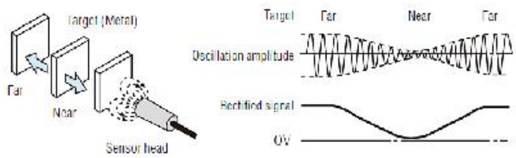
When this alternating magnetic field interacts with the conductive target, it induces a current in the target material called an eddy current.

This eddy current produces its own magnetic field which oppose the sensing coil's field As the eddy currents in the target oppose the sensing field, the impedance of the sensing coil will change.



The amount of impedance change is dependent on the distance between the target and the sensing coil in the probe

Current flow in the sensing coil, which is impedance dependent, is processed to create the output voltage which is an indication of the position of the target relative to the probe

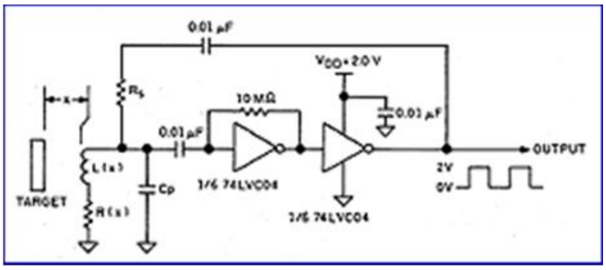


these sensors only detect metal objects

Inductive displacement Sensing

Eddy-current displacement sensors

An LC gate-oscillator circuit with an eddy current sensor generates a frequency output that depends on the target standoff.

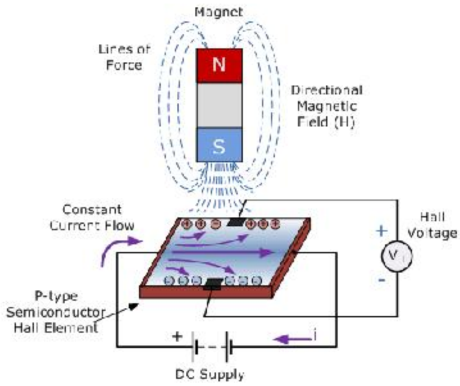


A microcontroller can directly digitize the frequency by counting the output pulses.

Magnetic sensors for displacement sensing

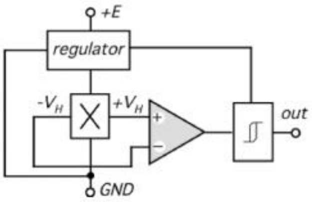
Magnetoresistive and integrated Hall sensors measure the strength of magnetic field components. With different ways, this can directly affect their successful use in a position sensing application

Hall Effect Sensor Principals

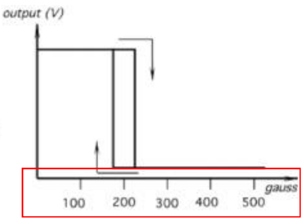


Magnetic sensors for displacement sensing

Threshold Hall Effect Sensors

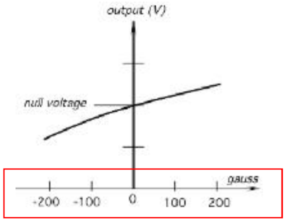


sensor + amplifier+Schmitt Triger

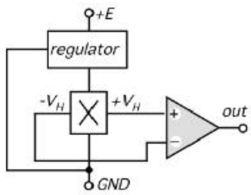


They respond to magnetic field density

Continous Hall Effect Sensors

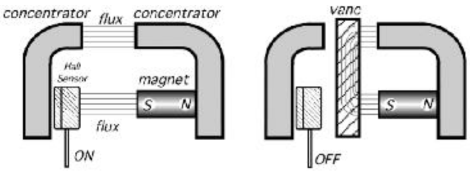


They respond to magnetic field density



sensor + amplifier

The magnetic field strength is not linear with distance $B = B(x)$ depends on the magnet shape, the magnetic circuit, and the path traveled by the magnet.



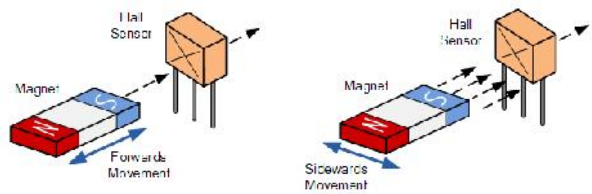
Hall sensors can be used for interrupter switching with a moving object

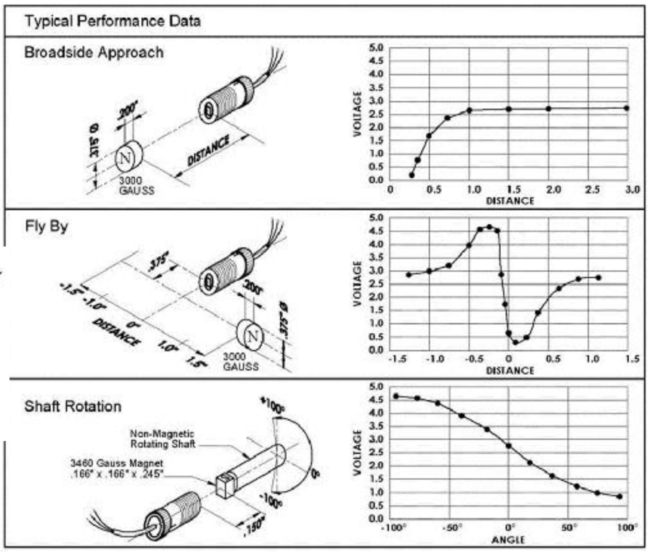
Designing a position detector with a Hall sensor, an overall analysis should be performed. First, the field strength of the magnet should be investigated. The strength will be the greatest at the pole face and will decrease with increasing distance from the magnet.
The field may be measured by a gaussmeter or a calibrated Hall sensor

Magnetic sensors for displacement sensing

Continous Hall Effect Sensors

This broadside approach generates an output signal, VH which in the linear devices represents the strength of the magnetic field, the magnetic flux density, as a function of distance away from the hall effect sensor. The nearer and therefore the stronger the magnetic field, the greater the output voltage





Sideways or slide-by detection is useful for detecting the presence of a magnetic field as it moves across the face of the Hall element within a fixed air gap distance for example, counting rotational magnets or the speed of rotation of motors.