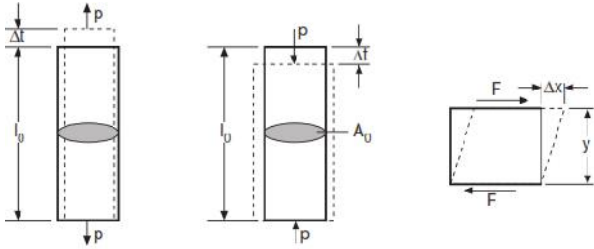


## 11. Force And Torque Measurement

Force sensors are required for a basic understanding of the response of a system.

- cutting forces generated by a machining process.
- Force sensors are used to monitor impact forces in the automotive industry.
- Robotic handling and assembly tasks are controlled by detecting the forces generated at the end effector.

Direct measurement of forces is useful in controlling many mechanical systems. Some types of force sensors are based on measuring a deflection caused by the force. Relatively high deflections. The relation between force and deflection in the elastic region is demonstrated by Hooke's law. Force sensors that employ strain gage elements or piezoelectric (quartz) crystals with built-in microelectronics are common.



$$\sigma = \frac{P}{A_0} \quad \epsilon = \frac{\Delta l}{l_0} \quad \tau = \frac{F}{A} \quad \gamma = \frac{\Delta x}{y}$$

When a rigid body is loaded in uniaxial tension, uniaxial compression, or simple shear, it will behave elastically until a critical value of normal stress ( $\sigma$ ) or shear stress ( $\tau$ ) is reached, and then it will deform plastically. In the elastic region, the atoms are temporarily displaced but return to their equilibrium positions when the load is removed. The relation between stress and strain in the elastic region is given by Hooke's law:

$$\sigma = \epsilon E \Rightarrow F = \epsilon EA$$

$$\tau = \gamma G \Rightarrow F = \gamma GA$$

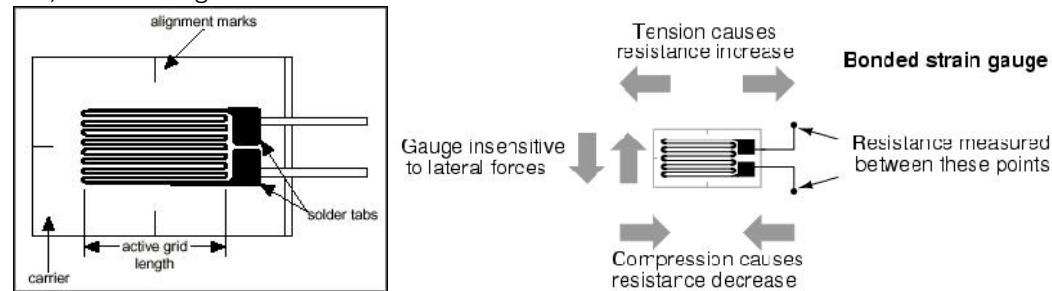
Strain is a normalized linear deformation of the material. Stress is a measure of elasticity of the material. Stress and strain values are representatives of the force applied in micro scale.

### 11.1 Pick ups for effort sensing

Strain gauges and piezoelectric elements are functional in effort measurement systems as pick up elements.

Strain gauge

Strain gauge is a sensing element whose resistance varies with applied force; It converts force, pressure, tension, weight, etc., into a change in electrical resistance which can then be measured.



If a strip of conductive metal is stretched, it will become skinnier and longer, both changes resulting in an increase of electrical resistance end-to-end. Conversely, if a strip of conductive metal is placed under compressive force (without buckling), it will broaden and shorten. If these stresses are kept within the elastic limit of the metal strip (so that the strip does not permanently deform), the strip can be used as a measuring element for physical force, the amount of applied force inferred from measuring its resistance.

$$R = \rho \frac{l}{A}$$

Change of relative dimensions, as the resistance is related to length and cross-sectional area :

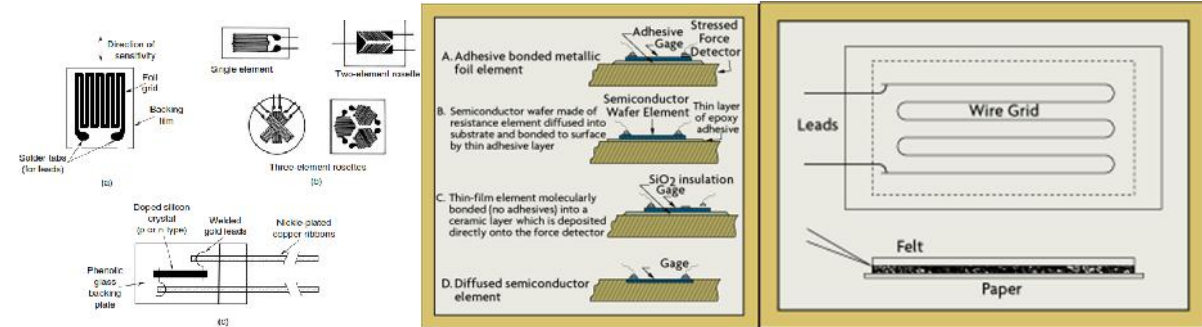
$$dR = \frac{\rho}{A} dL + \frac{L}{A} d\rho - \frac{\rho L}{A^2} dA$$

$$\frac{dR}{R} = \frac{dL}{L} + \frac{d\rho}{\rho} - \frac{dA}{A}$$

Gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

$$GF = \frac{dR/R}{dL/L} = 1 + \frac{\rho}{\rho} \frac{L}{dL} + 2\nu$$

The unity term, two times poisson ratio and resistivity related term show the resistance change due to change in length, change in area and piezoresistive effect on material, respectively.



Typical foil-type strain gages: The gage consists of a grid of very fine metallic wire, foil, material bonded to the strained surface or carrier matrix by a thin insulated layer of epoxy. When the carrier matrix is strained, the strain is transmitted to the grid material through the adhesive. The variations in the electrical resistance of the grid are measured as an indication of strain.

Semiconductor Strain Gauges: For a high sensitivity, a high value of gauge factor is desirable. A high gauge factor means a relatively higher change in resistance which can be easily measured with a good degree of accuracy. Semiconductor strain gauges are used where a very high gauge factor and a small envelope are required. They depend for their action upon piezo-resistive effect i.e. the change in the value of the resistance due to change in resistivity.

Favorable circuit for use in strain-gage measurements is the Wheatstone bridge. One or more of the four resistors  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  in the bridge may represent strain gages.

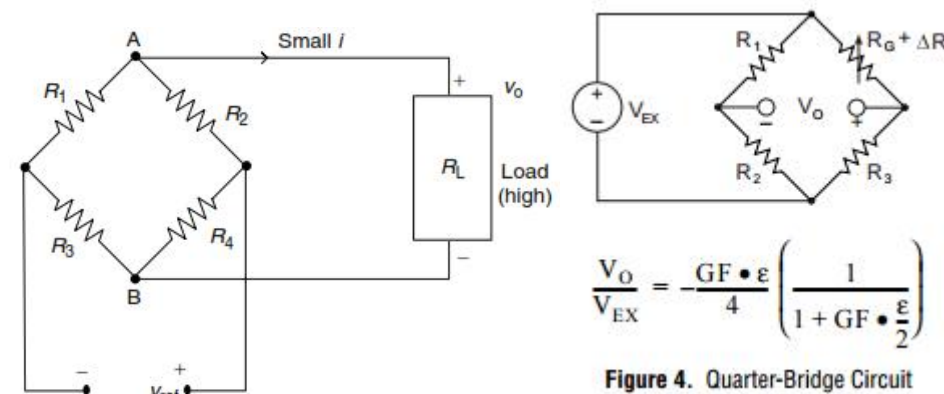
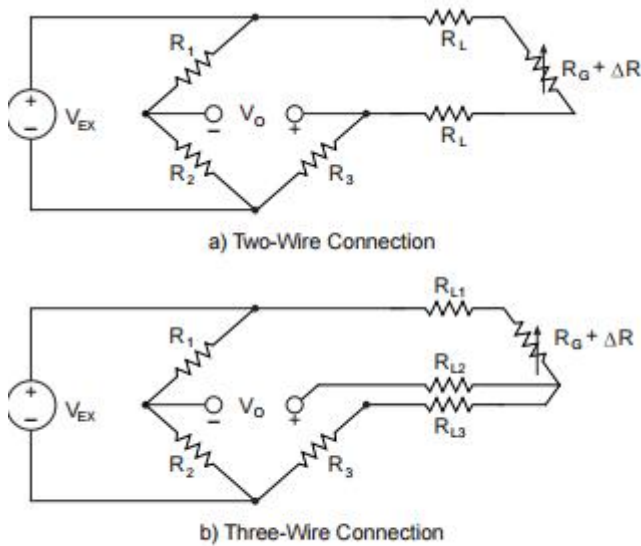


Figure 4. Quarter-Bridge Circuit

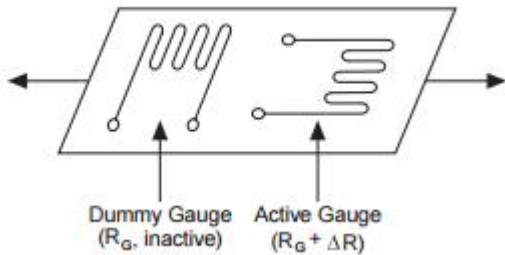
An active strain gauge, any changes in the strain gauge resistance will unbalance the bridge and produce a nonzero output voltage. If the nominal resistance of the strain gauge is designated as  $R_G$ , then the strain-induced change in resistance,  $\Delta R$ , can be expressed as  $\Delta R = R_G \cdot GF \cdot \epsilon$ . Assuming that  $R_1 = R_2$  and  $R_3 = R_G$ , the bridge equation above can be rewritten to express  $V_O/V_{EX}$  as a function of strain. Note the presence of the  $1/(1+GF \cdot \epsilon/2)$  term that indicates the nonlinearity of the quarter-bridge output with respect to strain.



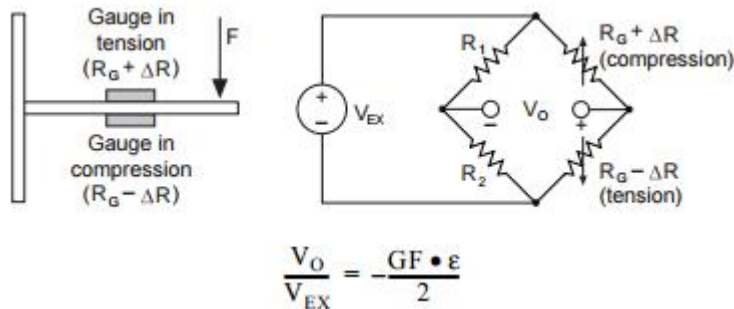
**Figure 8. Two-Wire and Three-Wire Connections of Quarter-Bridge Circuit**

The figures and equations in the previous section ignore the resistance in the lead wires of the strain gauge. While ignoring the lead resistances may be beneficial to understanding the basics of strain gauge measurements, doing so in practice can be very dangerous. For example, consider the two-wire connection of a strain gauge shown in Figure. Suppose each lead wire connected to the strain gauge is 15 m long with lead resistance  $R_L$  equal to 1  $\Omega$ . Therefore, the lead resistance adds 2  $\Omega$  of resistance to that arm of the bridge.

You can compensate for this error by measuring the lead resistance  $R_L$  and using the measured value in the strain equations. However, a more difficult problem arises from changes in the lead resistance due to temperature changes. Given typical temperature coefficients for copper wire, a slight change in temperature can generate a measurement error of several  $\mu\epsilon$ . Therefore, the preferred connection scheme for quarter-bridge strain gauges is the three-wire connection, shown in Figure. In this configuration,  $R_{L1}$  and  $R_{L3}$  appear in adjacent arms of the bridge. Therefore, any changes in resistance due to temperature cancel each other. The lead resistance in the third lead,  $R_{L2}$ , is connected to the measurement input. Therefore, this lead carries very little current and the effect of its lead resistance is negligible.

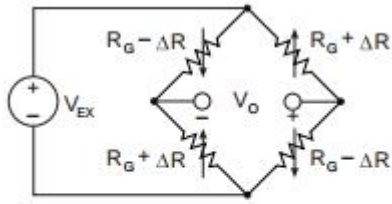


By using two strain gauges in the bridge, the effect of temperature can be avoided. For example, Figure illustrates a strain gauge configuration where one gauge is active ( $R_G + \Delta R$ ), and a second gauge is placed transverse to the applied strain. Therefore, the strain has little effect on the second gauge, called the dummy gauge. However, any changes in temperature will affect both gauges in the same way. Because the temperature changes are identical in the two gauges, the ratio of their resistance does not change, the voltage  $V_O$  does not change, and the effects of the temperature change are minimized.



Alternatively, one can double the sensitivity of the bridge to strain by making both gauges active, although in different directions. For example, Figure illustrates a bending beam application with one bridge mounted in tension ( $R_G + \Delta R$ ) and

the other mounted in compression ( $R_G - \Delta R$ ). This half-bridge configuration, whose circuit diagram is also illustrated in Figure 6, yields an output voltage that is linear and approximately doubles the output of the quarter-bridge circuit.



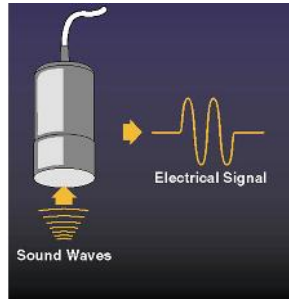
$$\frac{V_O}{V_{EX}} = -GF \cdot \epsilon$$

**Figure 7. Full-Bridge Circuit**

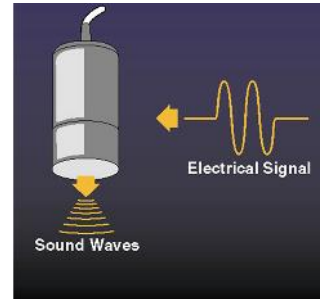
Finally, you can further increase the sensitivity of the circuit by making all four of the arms of the bridge active strain gauges, and mounting two gauges in tension and two gauges in compression.

#### Piezoelectric element

Piezoelectricity is understood as the electricity which is the by product of electromechanical interactions, primarily electrical and mechanical oscillations.

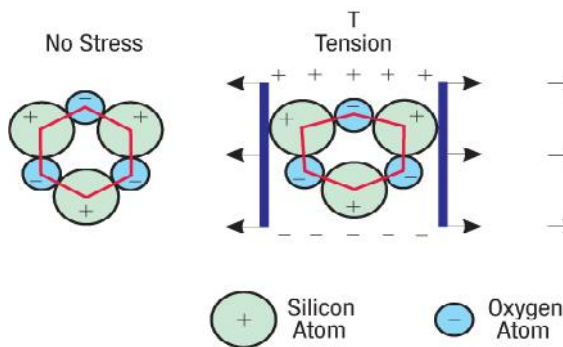


These engagements create a transducer effect as per the application of mechanical stress on materials which have no isotropic symmetry such as barium titanate, single-crystal quartz, and lead zirconatetitanate (PZT). As a result of this centro-symmetric attribute, the reverse piezoelectric effect is plausible; this is, a substance can yield mechanical pressure when subjected to an electric field. In particular, when an electric field is applied to the material to change the ionic polarization, the material will shed the strain and regain its original shape.



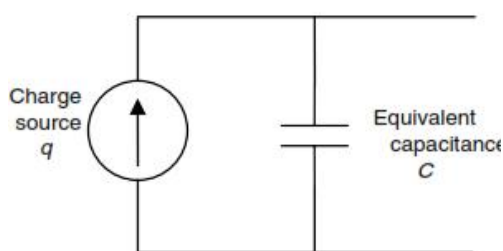
The electrical response to mechanical stimulation is called the direct piezoelectric effect, and the mechanical response to electrical stimulation is called the inverse piezoelectric effect.

#### Piezoelectric Effect in Quartz



As shown above, the piezoelectric effect works because of the movement on atoms in the crystal's molecules. All piezoelectric compounds are made of ions, atoms that have either gained or lost electrons and hence accumulated electric charge. Piezoelectric crystals are composed of positive and negative ions in an alternating fashion. Tension, or pulling, and compression, or squeezing, push and pull these positive and negative away from either other, creating an energy gradient across the crystal and allowing an electric current to flow.

Piezoelectricity is mathematically expressed as a linear relationship between the mechanical factors of strain, compliance and stress, and the electrical factors of electric displacement, permittivity, and the strength of the electric field. Effectively, selection of the right crystal class (polar) and the correct polymer to maximize mechanical facets, will together produce versatile results in engineering design.



A piezoelectric crystal can be considered to be in the form of a disk with two electrodes plated on the two opposite faces. Since the crystal is a dielectric medium, this device is essentially a capacitor, which may be modeled by a capacitance  $C$ . According to this fact, Equivalent circuit representation of a piezoelectric sensor is as follows:

A quartz crystal may present an impedance of several megaohms at 100 Hz, increasing hyperbolically with decreasing frequencies.

$$Z = \frac{1}{2\pi fC}$$

This is one reason why piezoelectric sensors have a limitation on the useful lower frequency. The other reason is the charge leakage. The sensitivity of a piezoelectric crystal may be represented either by its charge sensitivity or by its voltage sensitivity which are defined as:

$$s_q = \frac{dq}{dF} = \frac{1}{A} \frac{dq}{dP}$$

$$s_v = \frac{1}{d} \frac{dV}{dP}$$

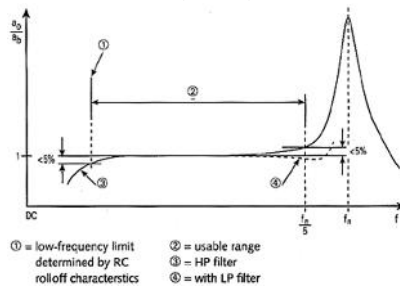
where  $d$  denotes the crystal thickness. The following relationship between charge sensitivity and voltage sensitivity is obvious:

$$s_q = k s_v$$

$k$  is the dielectric constant (permittivity) of the crystal capacitor.

### Dynamic Behavior

Piezoelectric sensors for measuring pressure, force, and acceleration may be regarded as underdamped, spring mass systems with a single degree of freedom. They are modeled by the classical second-order differential equation whose solution in frequency domain is:



In this typical response curve,

1 low-frequency limit determined by RC rolloff characteristics;

2 usable range;

3 high-pass filter

4 with low-pass filter, which can be used to attenuate the effects of the amplitude rise at  $\sim 1/5$  the resonance frequency.

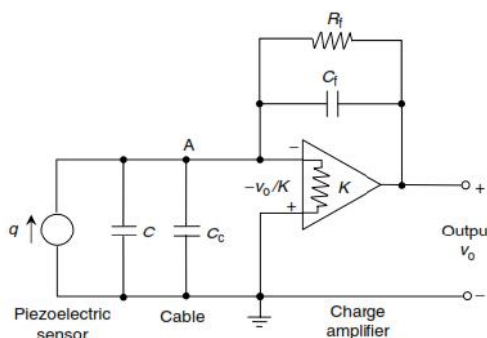
Low-pass filtering can be used to attenuate the effects of this..

Piezoelectric signals cannot be read using low-impedance devices. The two primary reasons for this are:

1. High output impedance in the sensor results in small output signal levels and large loading errors.
2. The charge can quickly leak out through the load.

A charge amplifier is commonly used as the signal-conditioning device for piezoelectric sensors, in order to overcome these problems to a great extent. Because of impedance transformation, the impedance at the output of the charge amplifier becomes much smaller than the output impedance of the piezoelectric sensor.

Basically the charge amplifier consists of a high-gain inverting voltage amplifier with a MOSFET or JFET at its input to achieve high insulation resistance



Owing to very high input impedance of the opamp, the currents through its input leads are negligible. Current balance at point A gives:

$$\dot{q} + C \frac{\dot{v}_o}{K} + C_c \frac{\dot{v}_o}{K} + C_f \left( \dot{v}_o + \frac{\dot{v}_o}{K} \right) + \frac{v_o + v_o/K}{R_f} = 0.$$

Since gain  $K$  is very large compared to unity, this differential equation may be approximated as:

$$R_f C_f \frac{dv_o}{dt} + v_o = -R_f \frac{dq}{dt}$$

The corresponding transfer function is



$$\frac{v_o(j\omega)}{q(j\omega)} = -\frac{R_f j\omega}{[R_f C_f j\omega + 1]}$$

$$\tau_c = R_f C_f.$$

$$G(j\omega) = \frac{j\tau_c \omega}{[j\tau_c \omega + 1]}$$

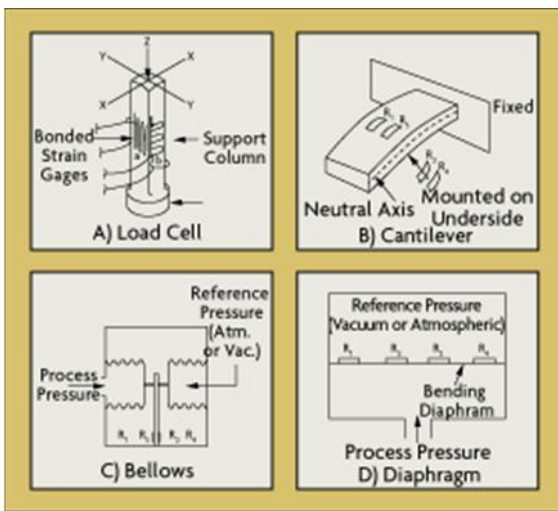
$$M = \frac{\tau_c \omega}{\sqrt{\tau_c^2 \omega^2 + 1}}$$

Measurement accuracy depends on the closeness of M to 1. If the required lower frequency limit is  $\omega_{min}$ . The time constant requirement is:

$$R_f C_f > \frac{M_o}{\omega_{min} \sqrt{1 - M_o^2}}.$$

For a specified level of accuracy, a specified lower limit on frequency of operation may be achieved by increasing the time constant

#### Transducer Designs



Strain gages are used to measure displacement, force, load, pressure, torque or weight. Modern strain-gage transducers usually employ a grid of four strain elements electrically connected to form a Wheatstone bridge measuring circuit.

The strain-gage sensor is one of the most widely used means of load, weight, and force detection. In Figure, a vertical beam is subjected to a force acting on the vertical axis. As the force is applied, the support column experiences elastic deformation and changes the electrical resistance of each strain gage. By the use of a Wheatstone bridge, the value of the load can be measured. Load cells are popular weighing elements for tanks and silos and have proven accurate in many other weighing applications.

Strain gages may be bonded to cantilever springs to measure the force of bending. The strain gages mounted on the top of the beam experience tension, while the strain gages on the bottom experience compression.

The transducers are wired in a Wheatstone circuit and are used to determine the amount of force applied to the beam.

Strain-gage elements also are used widely in the design of industrial pressure transmitters. Figure C shows a bellows type pressure sensor in which the reference pressure is sealed inside the bellows on the right, while the other bellows is exposed to the process pressure. When there is a difference between the two pressures, the strain detector elements bonded to the cantilever beam measure the resulting compressive or tensile forces.

A diaphragm-type pressure transducer is created when four strain gages are attached to a diaphragm (Figure D). When the process pressure is applied to the diaphragm, the two central gage elements are subjected to tension, while the two gages at the edges are subjected to compression. The corresponding changes in resistance are a measure of the process pressure. When all of the strain gages are subjected to the same temperature, such as in this design, errors due to operating temperature variations are reduced.