MEE303 Sensor Systems

W01
Sensor Characteristics

Mechatronics System

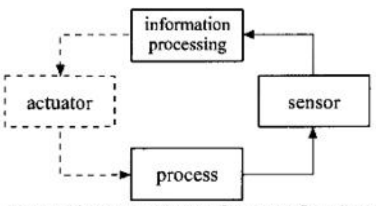


Figure 1. Mechatronics system information flow diagram

sensors and the associated measuring systems provide the required measurable information about the process in mechatronic systems

Sensors that measure <u>mechanical or thermal</u> quantities and transform them into an electrical signal are of special importance for mechatronic systems

Measuring System

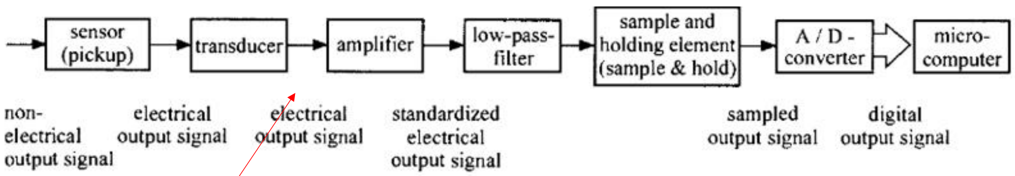


Figure 2. Measuring system components

0...20 mA or 4...20 mA or 0...10 V

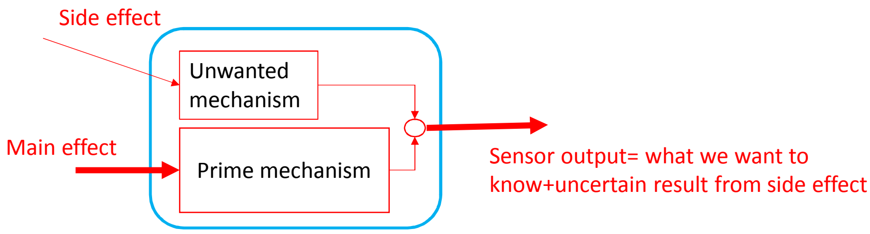
Classification Of Sensors

- Mechanical quantities;
- Thermal/caloric quantities;
- Electrical quantities;
- Chemical and physical quantities.

Sensor Properties

The transformation of non-electrical quantities into electrical ones depends on physical or chemical effects. These may be divided into main and side effects

- The main effect is responsible for generating the desired measuring signal,e.g., the electrical voltage of a piezoelectric pressure sensor.
- The disturbing side effects are frequently superimposed, e.g., the influence of temperature changes. The design process for sensors needs to take these side effects (sometimes called "cross sensitivity") into account. Their influence should have only little effector should be compensated by appropriate measures

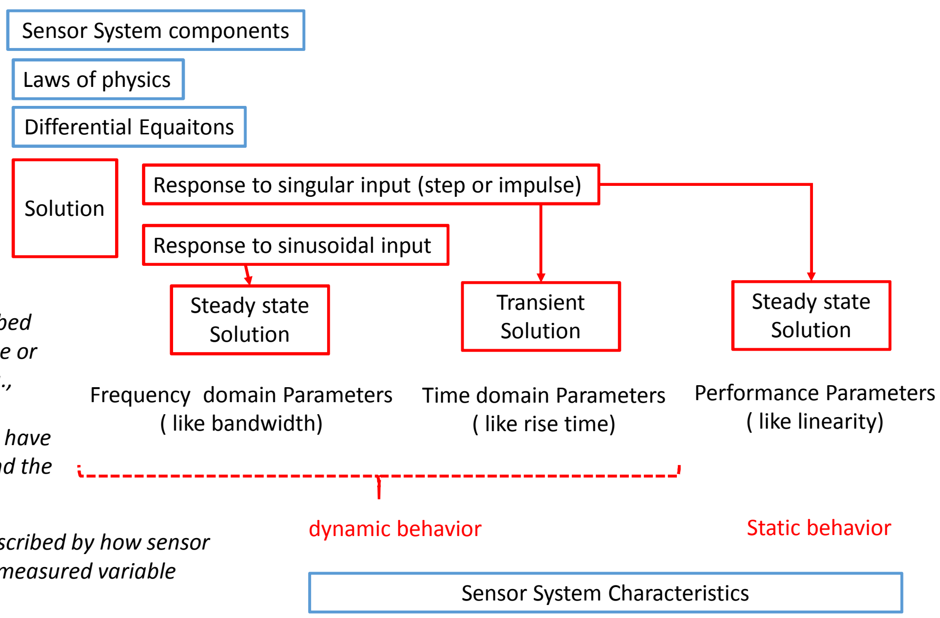


Sensor properites

- static behavior;
- dynamic behavior;
- quality class,
- measuring range;
- overload capacity;
- compatibility with associated components;
- environmental influences;
- reliability

*The **dynamic behavior** is described by a sensor's frequency response or simple characteristic values, e.g., cut-off frequencies or time constants. The sensor dynamics have to be adjusted to the process and the measuring task*

*A **sensor's static behavior** is described by how sensor output signal changes wrt the measured variable (input)*



Sensor Characteristics

Concept of transfer function

The transfer function of a control system is defined as the ratio of the Laplace transform of the output variable to Laplace transform of the input variable assuming all initial conditions to be zero.

$$\underbrace{a_n x^{(n)} + \dots + a_1 \dot{x} + a_0 = b_n u^{(n)} + \dots + b_1 u + b_0}_{\text{differential equation}} \Rightarrow \sum_{i=0}^{n} a_i x^{(i)} = \sum_{j=0}^{m} b_j u^{(j)}$$

$$L\left\{\sum_{i=0}^{n} a_i x^{(i)} = \sum_{j=0}^{m} b_j u^{(j)}\right\} \Rightarrow \frac{x(s)}{u(s)} = \overbrace{\frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}}^{\text{transfer function}}$$

Transfer function corresponds to differential equation in Laplace Domain.

Time Response

The sensor characteristics are dependent on the order of the differential equaiton. Hence, they are analyzed seperately for unit step input which is used to characterize a system's response to sudden changes in its input

Poles, Zeros and Order of the Sytems

$$\frac{x(s)}{u(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0} = \frac{b_m}{a_n} \frac{\prod_{j=1}^{m} (s - z_j)}{\prod_{j=1}^{n} (s - p_i)}$$

$\forall p_i$: poles of transfer function $\quad n \geq m$ Casuality

$\forall z_j$: zeros of transfer function $\quad n$: order of the system

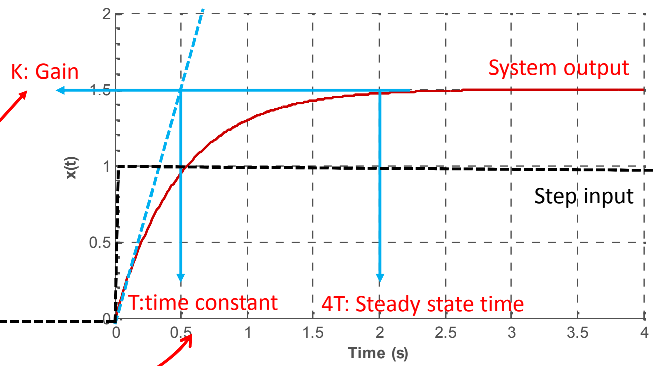
Sensor Characteristics

Time Response ctd

Time response of the first order systems

$$a_1 \frac{dx}{dt} + a_0 x = b_0 u \quad x(t) = K(1 - e^{-\frac{t}{\tau}})$$

$$\underbrace{\boxed{\frac{a_1}{a_0}}}_{\tau} \frac{dx}{dt} + x = \underbrace{\boxed{\frac{b_0}{a_0}}}_{K} u \Rightarrow \tau \frac{dx}{dt} + x = Ku$$



Gain of the system is defined as the ratio of input quantity to output quantity and it is also named as sensitivity.

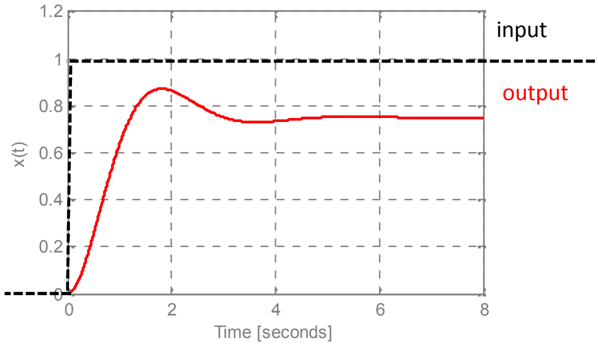
The time constant of the system, which has units of time, is the system parameter that establishes the time scale of system responses. It is defined as the duration from initial time to the time instant where the output reaches 63% of its final value

It also defines the steady state time of the system with a multiplication factor of 4τ(for 98%) and 5τ(for 99.3%).

Sensor Characteristics

Time Response ctd

Time response of the second order systems



$$a_2\frac{d^2x}{dt^2}+a_1\frac{dx}{dt}+a_0x=b_0u \qquad \frac{d^2x}{dt^2}+\frac{a_1}{a_2}\frac{dx}{dt}+\frac{a_0}{a_2}x=\frac{b_0}{a_2}u$$

$$\frac{d^2x}{dt^2}+\frac{a_1}{a_2}\frac{dx}{dt}+\frac{a_0}{a_2}x=\frac{b_0}{a_2}\frac{a_0}{a_0}u=\frac{b_0}{a_o}\frac{a_0}{a_2}u$$

$$s_{1,2}=-\frac{a_1}{2a_2}\pm\frac{\sqrt{(a_1)^2-4a_2a_0}}{2a_2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$$

The **natural frequency** defined as the frequency of the hypothetical oscillations that system represents when no input applied and there is no damping

$$\omega_n=\sqrt{\frac{a_0}{a_2}}$$

The **damping ratio** is defined as a ratio of the damping factor (generally represented by the coefficient of the first order term a_1) to the critical damping factor (a_1 value) that makes polynomial full square (double root).

$$\xi=\frac{a_1}{2\sqrt{a_0a_2}} \qquad C_{cr}=2\sqrt{a_0a_2}$$

Solution for underdamped case

$$x(t)=K\left(1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}t+\phi)\right)$$

$$\phi=\mathrm{Arccos}(\xi)\Rightarrow\phi=\tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

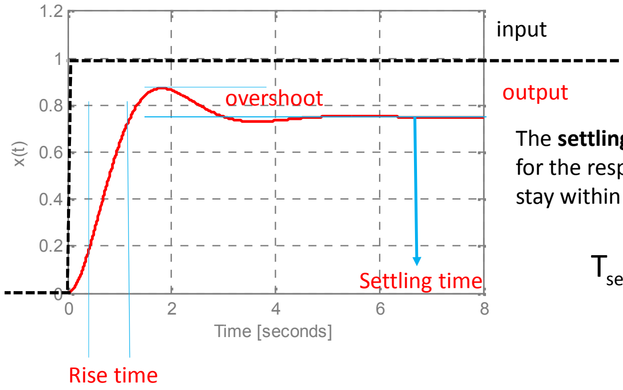
Sensor Characteristics

Time Response ctd

$$s_{1,2}=-\frac{a_1}{2a_2}\pm\frac{\sqrt{(a_1)^2-4a_2a_0}}{2a_2}=-\xi\omega_n\pm\omega_n\sqrt{\xi^2-1}$$

Time response of the second order systems

The definition of **rise time** is "that time taken for a system output to rise from 10% to 90% of its final value when stimulated by a step input". The rise time is inversely proportional to the system bandwidth and it represents pace of the system



The **settling time** is the time required for the response curve to reach and stay within 2% of the final value.

$$T_{settling}=\frac{4}{\xi w_n}$$

The **overshoot** is the maximum peak value of the response curve measured from steady state value of the response.

$$OV\%=\frac{\textit{Maximum Value}-\textit{Steady State Value}}{\textit{Steady State Value}}\times 100 \qquad OV\%=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\times 100$$

Sensor Characteristics

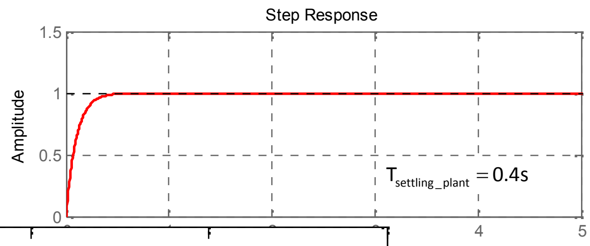
Time Response ctd

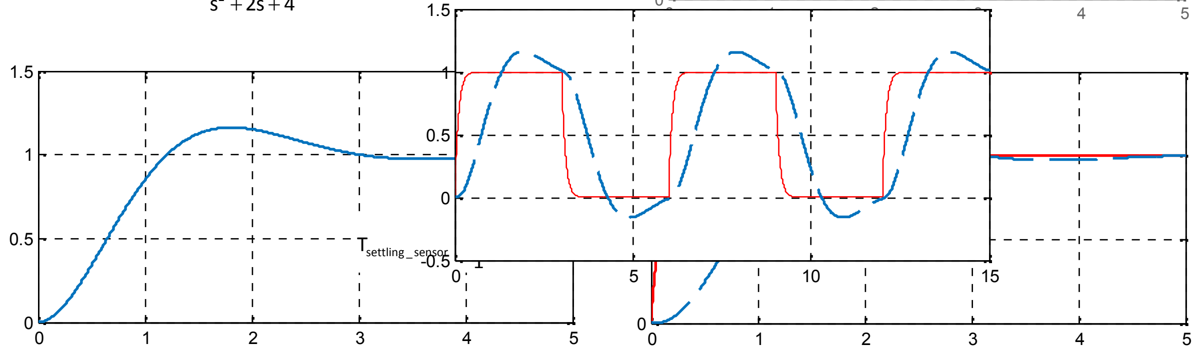
Plant Dynamics (A first order electric machine):

$$G_{Plant}(s)=\frac{1}{0.1s+1}$$

Sensor Dynamics (A second order OPamp circuit):

$$G_{sensor}(s)=\frac{4}{s^2+2s+4}$$





Sensor Characteristics

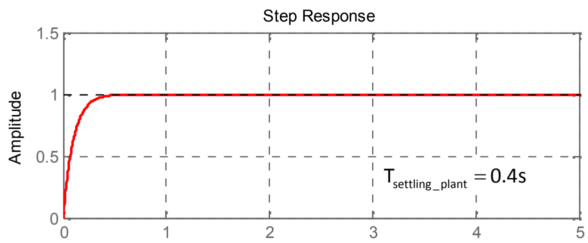
Time Response ctd

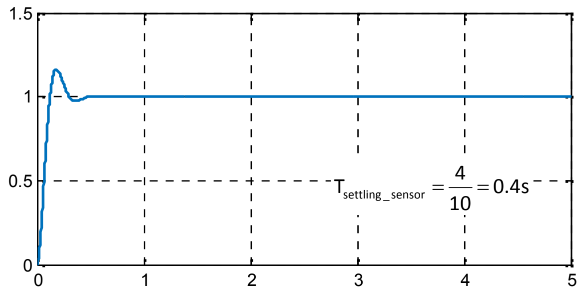
Plant Dynamics (A first order electric machine):

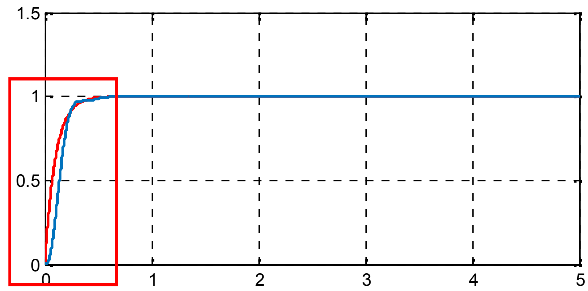
$$G_{Plant}(s)=\frac{1}{0.1s+1}$$

Sensor Dynamics (A second order OPamp circuit):

$$G_{sensor}(s)=\frac{400}{s^2+20s+400}$$







Sensor Characteristics

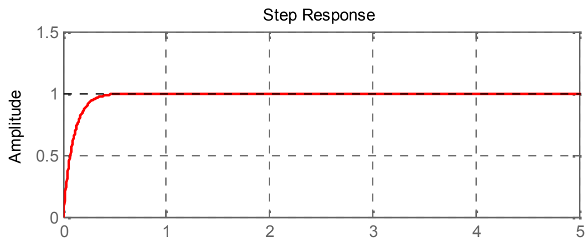
Time Response ctd

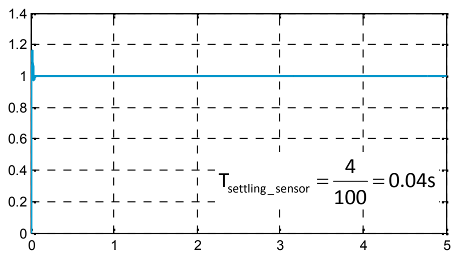
Plant Dynamics (A first order electric machine):

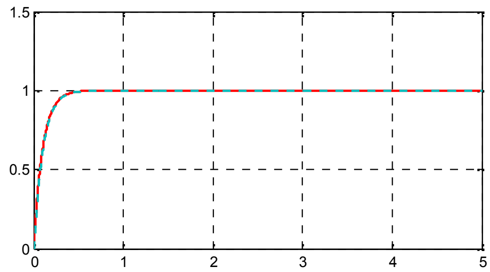
$$G_{Plant}(s) = \frac{1}{0.01s + 1}$$

Sensor Dynamics (A second order OPamp circuit):

$$G_{sensor}(s) = \frac{40000}{s^2 + 200s + 40000}$$





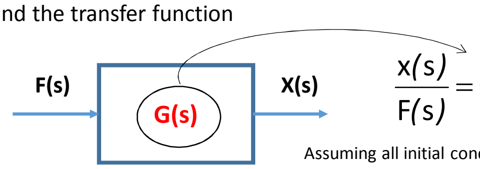


Sensor Characteristics

Concept of transfer function

Frequency response of the systems

- Find the transfer function



$$\frac{x(s)}{F(s)} = G(s)$$

Assuming all initial conditions are zero

- Set s=jw and find F(jw) which is complex function

$$G(s) \xrightarrow{s=jw} H(jw) \qquad H(jw) = Re[H(jw)] + Im[H(jw)]$$

- Convert Cartesian form to polar form using formulas

$$H(w) = |H(jw)| = \sqrt{Re[H(jw)]^2 + Re[H(jw)]^2} \qquad \tan\varphi = \frac{Im[H(jw)]}{Re[H(jw)]} \Rightarrow \varphi = \tan^{-1}\left(\frac{Im[H(jw)]}{Re[H(jw)]}\right)$$

- Find the steady state sinusoidal function with given formula

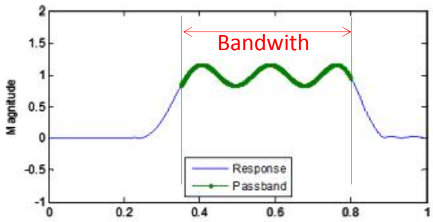
$$x_{steadystate}(t) = x_0 |H(jw)| \sin(wt + \varphi)$$

Sensor Characteristics

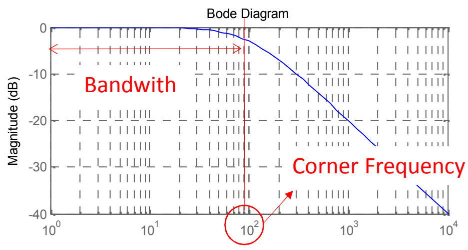
Concept of transfer function

Frequency response characteristics of the systems

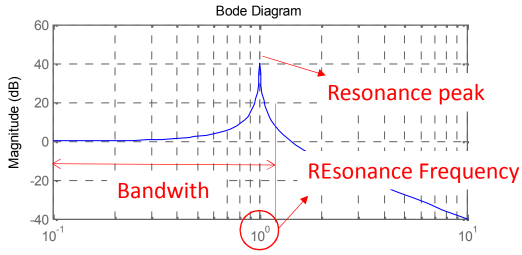
The width of the quasi flat region on frequency response in frequency space is called **bandwidth.**
Frequency points ƒL and ƒH relate to the lower corner or cut-off frequency and the upper corner or cut-off frequency points



These points on a frequency response curve are known commonly as the -3dB (decibel) points.
These -3dB corner frequency points define the frequency at which the output gain is reduced to 70.71% of its maximum value.



Frequency response of first order systems



Frequency response of second order systems

For second underdamped systems, at a specific frequency, the gain coming from frequency response becomes maximum.

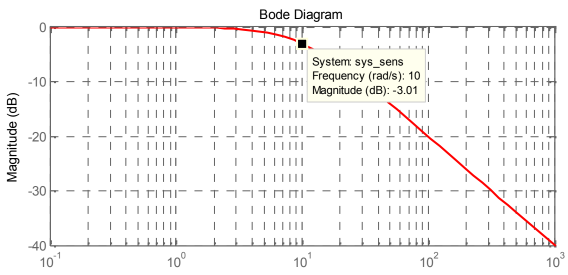
Sensor Characteristics

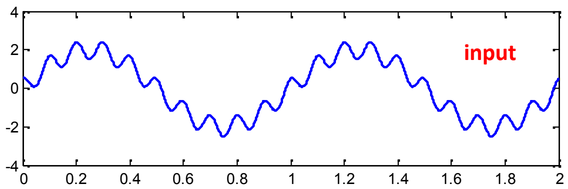
Plant Dynamics (A first order electric machine):

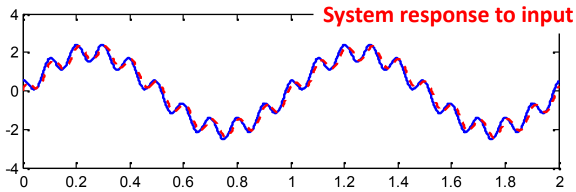
$$G_{Plant}(s) = \frac{1}{0.01s + 1}$$

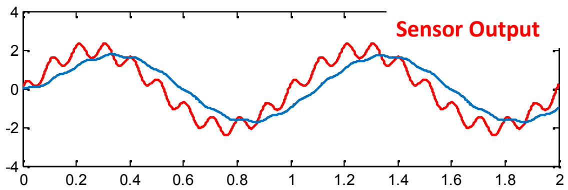
Sensor Dynamics (A first order instrument model):

$$G_{sensor}(s) = \frac{1}{0.1s + 1}$$









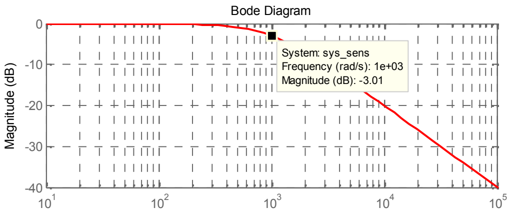
Sensor Characteristics

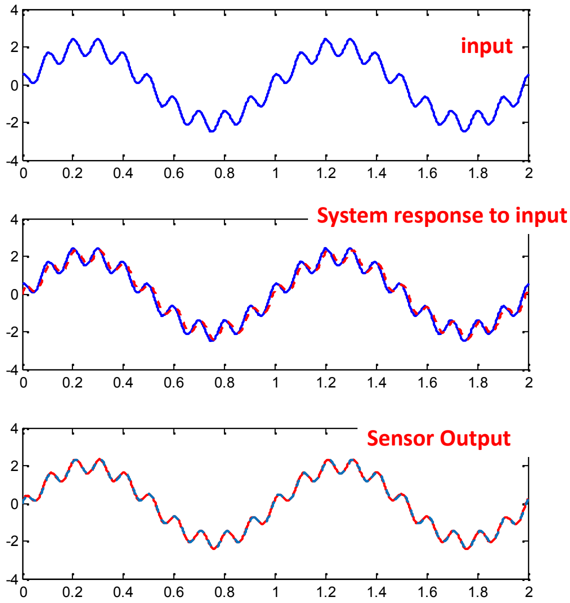
Plant Dynamics (A first order electric machine):

$$G_{Plant}(s) = \frac{1}{0.01s+1}$$

Sensor Dynamics (A first order instrument model):

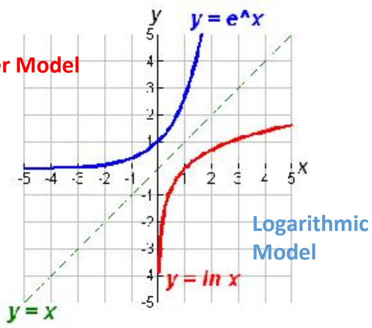
$$G_{sensor}(s) = \frac{1}{0.001s+1}$$

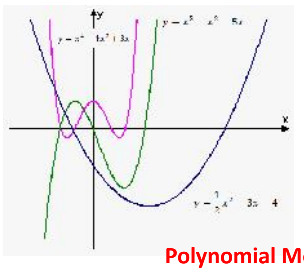




Sensor Characteristics

Power Model

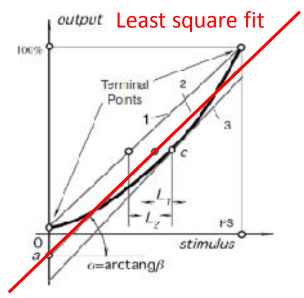




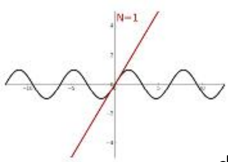
Polynomial Model

Nonlinear Input/Output realtionships

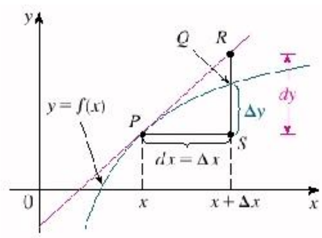
Linear Model



Linearization via Taylor Series

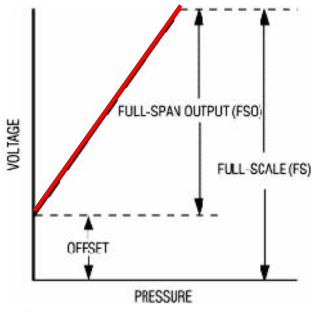


$$F(x = x_0) = f(x_0) + \frac{df(x)}{dx}(x - x_0)$$



Sensor Characteristics

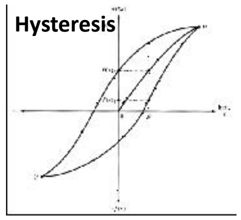
Linear Relationship

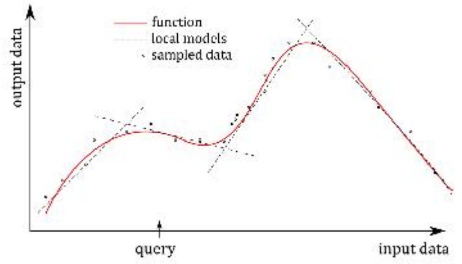


$$V(p) = mP + c$$

m : slope: Sensitivity
c : constant: Offset

Range or Span (Full-Scale Input)
Full-scale output (FSO)
Calibration





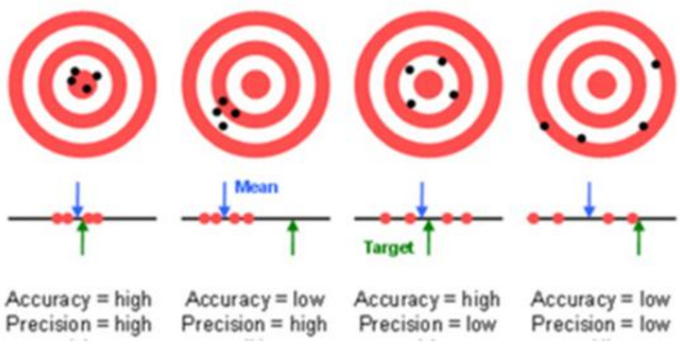
In piecewise linear approximation, the sensitivity and offset values change with operating (linearization) points.

Sensor Characteristics

Accuracy of an instrument indicates the deviation of the reading from a known input
Precision of an instrument describes the spread of these measurements when repeated under same conditions.

Tolerance is a term that is closely related to accuracy and defines the maximum error that is to be expected in some value.

Repeatability assesses whether the same observer can measure the same part/sample multiple times with the same measurement device and get the same value.
Reproducibility assesses whether different observers can measure the same part/sample with the same measurement device and get the same value



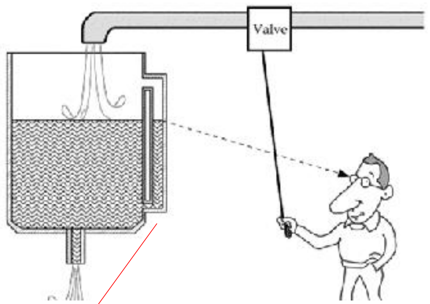
Sensor Redefinition

A sensor is a device that receives a stimulus and responds with an electrical signal

Voltage
Current
Charge

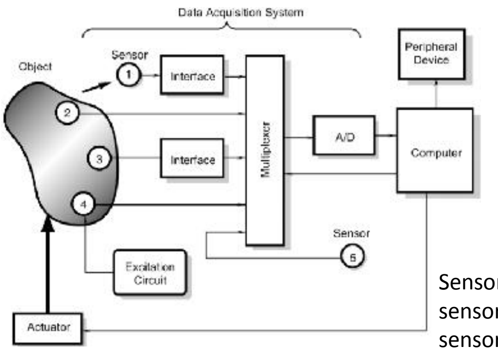
These may be further described in terms of:
- Amplitude
- Frequency
- Digital code

The stimulus is the quantity, property, or condition that is sensed.



In a human operator controlled level-control system, a sight tube and operator's eye form a sensor (a device which converts information into electrical signal)

Sensors and data acquisition system



Sensor 1 is noncontact, sensors 2and 3 are passive, sensor 4 is active, and sensor 5 is internal to a data acquisition system.