$$\forall i, \quad w_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} \sum_{j=1}^{4} \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}.$$

$$w_i \begin{bmatrix} \bar{U}_i \\ 1 \end{bmatrix} = A \begin{bmatrix} x_1^c & x_2^c & x_3^c & x_4^c \\ y_1^c & y_2^c & & \\ z_1^c & z_2^c & & \end{bmatrix} \begin{bmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \\ \alpha_{i4} \end{bmatrix}$$

$$w_i \begin{bmatrix} \bar{U}_i \\ 1 \end{bmatrix} = A C \alpha_i$$

$$e_i = A C \alpha_i - \begin{bmatrix} U_i \\ 1 \end{bmatrix} w_i$$

$$s_i = e_i^T e_i$$

$$E = \sum_i s_i$$

$$\frac{\partial E}{\partial x_i^c} = \frac{\partial e_i^T}{\partial x_i^c} e_i + e_i^T \frac{\partial e_i}{\partial x_i^c}$$

$$\frac{\partial e_i}{\partial x_i^c} = A \frac{\partial C}{\partial x_i^c} \alpha_i - \cancel{\frac{\partial u_i}{\partial y_i^c}}$$

$$\frac{\partial C}{\partial x_1^c} = \begin{bmatrix} 1 & \\ 0 & 0 \\ 0 & \end{bmatrix} \qquad \frac{\partial C}{\partial y_1^c} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial C}{\partial x_2} = \begin{bmatrix} 0 & \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} & 0 \end{bmatrix} \qquad \text{etc.}$$

$$\frac{\partial C}{\partial x_3} = \begin{bmatrix} 0 & \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \end{bmatrix}$$

$$\forall i, \quad w_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} \sum_{j=1}^{4} \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}. = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \alpha_i \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \end{bmatrix} C \alpha_i$$

$$z_1^c \alpha_{i1} + z_2^c \alpha_{i2} + z_3^c \alpha_{i3} + z_4^c \alpha_{i4}$$

$$C = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}_{3\times 4} \qquad \underset{1\times 4}{C_z} \; \underset{4\times 1}{\alpha_i} \begin{bmatrix} U_i \end{bmatrix} = A_2 C \alpha_i$$

$$e_i = A_2 C \alpha_i - C_z \alpha_i u_i \qquad s_i = e_i^T e_i$$

$$E = \sum_i e_i^T e_i \qquad \frac{\partial E}{\partial x_j^c} = \sum_i \frac{\partial e_i^T}{\partial x_i^c} e_i + e_i^T \frac{\partial e_i}{\partial x_i^c} \quad \Big| \quad \frac{\partial E}{\partial x_j^c} = 2 \sum_i \frac{\partial e_i^T}{\partial x_j^c} e_i$$

$$\frac{\partial E}{\partial y_j^c} = 2 \sum_i \frac{\partial e_i^T}{\partial y_j^c} e_i$$

$$\frac{\partial E}{\partial z_j^c} = 2 \sum_i \frac{\partial e_i^T}{\partial z_j^c} e_i$$

$$\frac{\partial e_i}{\partial z_i^c} = A_2 \frac{\partial C}{\partial z_i^c} \alpha_i - \frac{\partial C_z}{\partial z_i^c} \alpha_i U_i$$

$$\forall i, \quad w_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix} \sum_{j=1}^{4} \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}.$$

$$w_i \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} f_v & 0 & u_c \\ 0 & f_v & v_c \end{bmatrix} \begin{bmatrix} c_x^1 & c_x^2 & & \\ c_{y_1} & & & \\ c_z & & & c_z^4 \end{bmatrix} \begin{bmatrix} \alpha_{ji} \\ \vdots \\ \alpha_{4i} \end{bmatrix}$$

$$w_i = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C \end{bmatrix} \alpha_i$$

$$w_i = \begin{bmatrix} c_z^1 & \cdots & c_z^4 \end{bmatrix} \alpha_i = \overset{1\times 1}{C_z \alpha_i}$$

$$\begin{bmatrix} \overset{1\times 4}{[C_z]} \overset{4\times n}{[\alpha_i]} & \overset{1\times n}{U_i} \\ \underset{1\times 4}{[C_z]} \underset{4\times n}{[\alpha_i]} & \overset{1\times n}{v_i} \end{bmatrix} = \overset{2\times 3}{F_2} \overset{3\times 4}{C} \overset{4\times n}{\alpha_i}$$

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^i = \begin{bmatrix} F_2 C \alpha_i \end{bmatrix} - \begin{bmatrix} c_z \alpha_i & U_i \\ c_z \alpha_i & v_i \end{bmatrix}$$

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix}^i = e^{iT} e^i \qquad E = \sum_{i=1}^{n} e^{iT} e^i$$

$$\underset{y}{\frac{\partial E}{\partial C_{xj}}} = \sum_{i=1}^{n} \underset{y}{\frac{\partial e^{iT}}{\partial C_{xj}}} e^i + e^{iT} \underset{y}{\frac{\partial e^i}{\partial C_{xj}}}$$

$$\frac{\partial e^i}{\partial s_{xj}} = F_2 \frac{\partial C}{\partial C_{xj}} \alpha_i$$

$$\frac{\partial e^i}{\partial s_{yj}} = F_2 \frac{\partial C}{\partial C_{yj}} \alpha_i$$

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^i = \begin{bmatrix} F_2 C \alpha_i \end{bmatrix} - \begin{bmatrix} c_z \alpha_i & U_i \\ c_z \alpha_i & v_i \end{bmatrix}$$

$$\frac{\partial e^i}{\partial C_{zj}} = F_2 \frac{\partial C}{\partial C_{zj}} \alpha_i - \frac{\partial C_z}{\partial C_{zj}} \alpha_i \begin{bmatrix} U_i \\ v_i \end{bmatrix}$$

$$\frac{\partial E}{\partial C_{xj}} = e^{iT} F_2 \frac{\partial C}{\partial C_{xj}} \alpha_i = \left(F_2 \underline{C} \alpha_i - \underline{C_z} \alpha_i \begin{bmatrix} U \\ v \end{bmatrix}^i \right)^T$$

$$F_2 \frac{\partial C}{\partial x_j} \alpha_i$$

$$\frac{\partial E}{\partial c_{y_j}} = \left(F_2 C \alpha_i - C_z \alpha_i \begin{bmatrix} u \\ v \end{bmatrix}^i \right)^T F_2 \frac{\partial c}{\partial y_j} \alpha_i$$

$$\frac{\partial E}{\partial c_{z_j}} = \left(F_2 C \alpha_i - C_z \alpha_i \begin{bmatrix} u \\ v \end{bmatrix}^i \right)^T \left(F_2 \frac{\partial c}{\partial c_{z_j}} \alpha_i - \frac{\partial c_z}{\partial c_{z_j}} \alpha_i \begin{bmatrix} u \\ v \end{bmatrix}^i \right)$$

$$F_2 C \alpha_i - C_z \alpha_i \begin{pmatrix} u \\ v \end{pmatrix}^i$$

$$F_2 \left(\alpha_i^T C^T \right)^T - \begin{pmatrix} u \\ v \end{pmatrix}^i \left(\alpha_i^T C_z^T \right)^T$$

$$\underset{2\times 3}{F_2} \left(\underset{1\times 4}{(\alpha_1 \; \alpha_2 \; \alpha_3 \; \alpha_4)^i} \overset{4\times 3}{\begin{bmatrix} c_x^1 & \text{---} & c_z \\ c^2 & & \\ c^3 & & \\ c^4 & & \end{bmatrix}} \right)^T - \underset{2\times 1}{\begin{pmatrix} u \\ v \end{pmatrix}^i} \underset{1\times 4}{(\alpha_1 \alpha_2 \alpha_3 \alpha_4)} \underset{4\times 1}{\begin{bmatrix} c_z^1 \\ c_z^2 \\ c_z^3 \\ c_z^4 \end{bmatrix}}$$

$$F_2 C \alpha_i - C_z \alpha_i \begin{bmatrix} u \\ v \end{bmatrix}^i$$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \end{bmatrix} \begin{bmatrix} c_x^1 & c_x^2 & c_x^3 & c_x^4 \\ c_y & & & \\ c_z & & & \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}^i$$

$$- \left(c_z^1 \alpha_1^i + c_z^2 \alpha_2^i + c_z^3 \alpha_3^i + c_z^4 \alpha_4^i \right) \begin{bmatrix} u \\ v \end{bmatrix}^i$$

$$\underset{2\times 3}{\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \end{bmatrix}} \left[\underset{1\times 4}{(\alpha_1 \ldots \alpha_4)} \overset{4\times 3}{\begin{bmatrix} c_x & c_z \\ \vdots & \\ c_x^4 & c_z^4 \end{bmatrix}} \right]^T -$$

$$\begin{bmatrix} c_x^1 & \ldots & c_x^4 \\ \vdots & & \\ c_z^1 & \ldots & c_z^4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_4 \end{bmatrix}^i$$

$$c_x^1$$