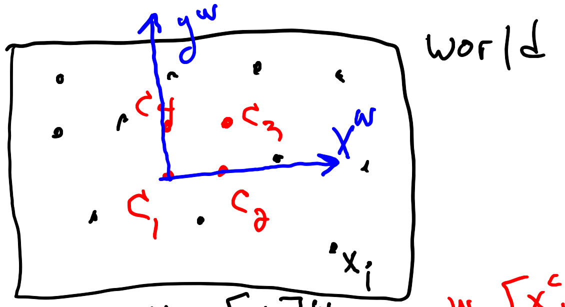
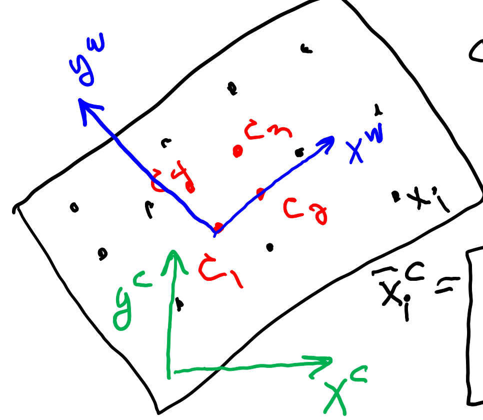


$\bar{x}_i^w = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^w \quad i = 1 \ldots N$

$\bar{c}_j^w = \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}^w \quad j = 1..4$



Camera is twisted

$\bar{x}_i^c = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^c \quad \bar{c}_j^c = \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}^c$

1. Pick $\bar{c}_j^w$
2. Know $\bar{x}_i^w$
3. Compute α_{ij}. Think of $\bar{c}_j^w$ as basis vectors (like $\bar{i}, \bar{j}, \bar{k}$)

   Any point $\bar{x}_i^w$ can be written as function of c_j^w as long as C are not coplanar. So $\bar{x}_i^w = \sum_j \bar{\alpha}_{ij} \bar{c}_j^w$

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^w = \begin{bmatrix} x_1^c & x_2^c & \cdots & x_4^c \\ y_1^c & y_2^c & \cdots & y_4^c \\ z_1^c & z_2^c & & z_4^c \end{bmatrix}^w \begin{bmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \\ \alpha_{i4} \end{bmatrix}$$

But don't need 4 only 3 so there is ∞ solutions so:

$$\alpha_{i1} + \alpha_{i2} + \alpha_{i3} + \alpha_{i4} = 1$$

there are $4 \times N$ α_s

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}^w = \begin{bmatrix} \bar{c}_1^w & \bar{c}_2^w & \bar{c}_3^w & \bar{c}_4^w \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \\ \alpha_{i4} \end{bmatrix} = \begin{bmatrix} [\bar{c}_j]^w \\ 1\,1\,1\,1 \end{bmatrix} \begin{bmatrix} \bar{\alpha}_i \end{bmatrix}$$

4. α_s are Same in W and C so we want to find C in Camera
5. $\bar{C}^w = {}^wT^c\, C^c = D_{xyz}\, R_{\psi,\phi,\theta}\, \bar{C}^c$

Start with equation 4

$$\forall i,\quad w_i \underbrace{\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}}_{3\times1} = \underbrace{\begin{bmatrix} f_u & 0 & u_c \\ 0 & f_v & v_c \\ 0 & 0 & 1 \end{bmatrix}}_{3\times3} \underbrace{\sum_{j=1}^{4} \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}}_{3\times1}. \qquad (4)$$

unknowns, locations of $\vec{C}_j$ in camera cosys

α are known, see note 4.

this is camera lens projection, known

$u_i\ v_i$ are known

w_i are "depth" and unknown

$$\sum_j \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix} = \alpha_{i1} \begin{bmatrix} x_1^c \\ y_1^c \\ z_1^c \end{bmatrix} + \alpha_{i2} \begin{bmatrix} x_2^c \\ y_2^c \\ z_2^c \end{bmatrix} + \dots + \alpha_{i4} \begin{bmatrix} x_4^c \\ y_4^c \\ z_4^c \end{bmatrix} \quad (3\times1)$$

$$\begin{bmatrix} F_{23} \\ 0\ 0\ 1 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

B used below

Row 3 of ~~right~~ hand side: $\alpha_{i1} z_1^c + \alpha_{i2} z_2^c + \dots \alpha_{i4} z_4^c$

$$\begin{bmatrix} w_i u_i \\ w_i v_i \\ w_i \end{bmatrix} \quad \text{so} \quad w_i = \sum_j \alpha_{ij} z_j^c \quad (1\times1)$$

$$\underbrace{w_i}_{1\times1} \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{2\times1} = \underbrace{F_{23}}_{2\times3} \sum_j \alpha_{ij} \underbrace{C_j}_{3\times1}$$

$$d_i = \sum_j \alpha_{ij} \underbrace{z_j^c}_{(1\times1)} \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{2\times1} - F_{23} \sum_j \alpha_{ij} C_j \qquad C_j = \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}$$

$$d_i = w_i \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{2\times1} - \underbrace{F_{23}}_{2\times3} \sum_j \alpha_{ij} \underbrace{C_j}_{3\times1}$$

$$d_i = \sum_j \alpha_{ij} \underbrace{z_j^c}_{(1\times1)} \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{2\times1} - F_{23} \sum_j \alpha_{ij} C_j \qquad C_j = \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}$$

this is 2×1

$$e_i = \underbrace{d_i^T}_{1\times2} \underbrace{d_i}_{2\times1} = \text{square of error}$$

$$E = \sum_{i=1}^{N} e_i$$

$$d_i = \sum_j \alpha_{ij} \underbrace{z_j^c}_{(1\times1)} \begin{bmatrix} u_i \\ v_i \end{bmatrix}_{2\times1} - F_{23} \sum_j \alpha_{ij} C_j \qquad C_j = \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}$$

Note all terms contain unknowns this yields homogenious equation

To minimize E let $\dfrac{\partial E}{\partial C_k} = 0$ for all 12 unknown C elements

call this (D) $\quad X = \begin{bmatrix} x_1^c \\ x_2^c \\ x_3^c \\ x_4^c \\ y_1^c \\ \vdots \\ z_1^c \\ \vdots \\ z_4^c \end{bmatrix}_{12\times1} = \begin{bmatrix} \underline{x}^c \\ \underline{y}^c \\ \underline{z}^c \end{bmatrix}$

$$\frac{\partial E}{\partial C_k} = \sum_{i=1}^{N} \frac{\partial e_i}{\partial C_k} = \sum_{i=1}^{N} 2 d_i^T \frac{\partial d_i}{\partial C_k}$$

Also $\dfrac{\partial E}{\partial C_k} = \sum_i 2 \dfrac{\partial d_i^T}{\partial C_k} d_i \qquad \dfrac{\partial d_i^T}{\partial C_k} = \left(\dfrac{\partial d_i}{\partial C_k}\right)^T$

find $\dfrac{\partial d_i}{\partial C_k}$

notice no unknowns in partials

$$\frac{\partial d_i}{\partial x_j^c} = -F_{23} \alpha_{ij} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2\times1) = -\alpha_{ij} \begin{bmatrix} f_{11} \\ f_{21} \end{bmatrix} \quad \text{see (B)}$$

$$\frac{\partial d_i}{\partial y_j^c} = -F_{23} \alpha_{ij} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (2\times1) = -\alpha_{ij} \begin{bmatrix} f_{12} \\ f_{22} \end{bmatrix}$$

$$\frac{\partial d_i}{\partial z_j^c} = \alpha_{ij} \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{(2\times1)} - F_{23} \alpha_{ij} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{(2\times1)} = \alpha_{ij} \begin{bmatrix} u_i \\ v_i \end{bmatrix} - \alpha_{ij} \begin{bmatrix} f_{13} \\ f_{23} \end{bmatrix}$$

$$\sum_j \alpha_{ij} C_j = \sum_j \alpha_{ij} \begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}_{3\times1} = \sum_j \underbrace{\alpha_{ij}}_{1\times1} \begin{bmatrix} S_j & O_4 & O_4 \\ O_4 & S_j & O_4 \\ O_4 & O_4 & S_j \end{bmatrix}_{3\times12} \underbrace{X}_{12\times1} \quad \text{see D}$$

$S_j = \underbrace{[0 \ldots 1 \ldots 0]}_{1\times4}$ (jth column) $\qquad O_4 = [0\,0\,0\,0] \qquad \cancel{S_j}$

$$\sum_j \alpha_{ij} \underbrace{z_j^c}_{(1\times1)} \begin{bmatrix} u_i \\ v_i \end{bmatrix}_{2\times1} = \sum_j \underbrace{\alpha_{ij}}_{1\times1} \begin{bmatrix} u_i \\ v_i \end{bmatrix}_{2\times1} \begin{bmatrix} O_4 & O_4 & S_j \end{bmatrix}_{1\times12} X_{12\times1}$$

$$d_i = \sum_j \alpha_{ij} \underbrace{z_j^c}_{(1\times 1)} \underbrace{\begin{bmatrix} u_i \\ v_i \end{bmatrix}}_{2\times 1} - F_{23} \sum_j \alpha_{ij} c_j$$

$$d_i = \left[\sum_j \alpha_{ij} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \begin{bmatrix} 0_4 & 0_4 & S_j \end{bmatrix} - F_{23} \sum_j \alpha_{ij} \begin{bmatrix} \$_j \end{bmatrix} \right] X$$

Similifies to something like $\begin{bmatrix} 0_4 & 0_4 & \alpha_{i1} & \alpha_{i2} & \alpha_{i3} & \alpha_{i4} \end{bmatrix}$

4 eqns, $j = 1..4$

$$\frac{\partial E}{\partial x_j^c} = \sum_i \left(-\alpha_{ij} \begin{bmatrix} f_{11} & f_{21} \end{bmatrix} \left[\sum_k \alpha_{ik} \begin{bmatrix} u_i \\ v_i \end{bmatrix} \begin{bmatrix} 0_4 & 0_4 & S_k \end{bmatrix} - F_{23} \sum_k \underbrace{\alpha_{ik} \$_k}_{\text{similarly simplifies}} \right] \right) X$$

4 eqns, $j = 1..4$

$$\frac{\partial E}{\partial y_j^c} = \sum_i \left(-\alpha_{ij} \begin{bmatrix} f_{12} & f_{22} \end{bmatrix} \left[\quad \text{same} \quad \right] \right) X$$

4 eqns, $j = 1..4$

$$\frac{\partial E}{\partial z_j^c} = \sum_i \left(-\alpha_{ij} \left(\begin{bmatrix} f_{13} & f_{23} \end{bmatrix} - \begin{bmatrix} u_i & v_i \end{bmatrix} \right) \left[\quad \text{same} \quad \right] \right) X$$

has this form

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} X = \begin{bmatrix} \\ \emptyset \\ \\ \end{bmatrix}$$

solve for X to get $\begin{bmatrix} x_j^c \\ y_j^c \\ z_j^c \end{bmatrix}$