Homework 9 – Probabilistic Dynamic Programming

Due: 1:30pm, June 4, 2024

1. Winston (2004), Page 220, Section 5.2, Prob. 3

   In Example 2 (a stochastic inventory model), suppose that shortages are allowed, and each shortage results in a lost sale and a cost incurred of \$3. Now re-solve Example 2.

   Recall Example 2 in the textbook:

   Consider the following three-period inventory problem. At the beginning of each period, a firm must determine how many units should be produced during the current period. During a period in which x units are produced, a production cost $c(x)$ is incurred, where $c(0) = 0$, and for $x > 0$, $c(x) = 3 + 2x$. Production during each period is limited to at most 4 units. After production occurs, the period's random demand is observed. Each period's demand is equally likely to be 1 or 2 units. After meeting the current period's demand out of current production and inventory, the firm's end-of-period inventory is evaluated, and a holding cost of \$1 per unit is assessed. Because of limited capacity, the inventory at the end of each period cannot exceed 3 units. It is required that all demand be met on time. Any inventory on hand at the end of period 3 can be sold at \$2 per unit. At the beginning of period 1, the firm has 1 unit of inventory. Use dynamic programming to determine a production policy that minimizes the expected net cost incurred during the three periods.

2. A company has five sales representatives available for assignment to three sales districts. The sales in each district during the current year depend on the number of sales representatives assigned to the district and on whether the national economy has a bad or good year (see Table 1). In the Sales column for each district, the *first* number represents sales if the national economy had a *bad* year, and the *second* number represents sales if the economy had a *good* year.

   Table 1 Sales in the districts

No. of sales rep. assigned to district	Sales (millions)		
	District 1	District 2	District 3
0	\$1, \$4	\$2, \$5	\$3, \$4
1	\$2, \$6	\$4, \$6	\$5, \$5
2	\$3, \$7	\$5, \$6	\$6, \$7
3	\$4, \$8	\$6, \$6	\$7, \$7

   There is a 30% chance that the national economy will have a good year and a 70% chance that the national economy will have a bad year. Use dynamic programming to determine an assignment of sales representatives to districts that maximizes the company's expected sales.

   a) Clearly define and denote the key elements (state, decision, exogenous information, transition function, one-period contribution/cost function, and objective function) of

this finite horizon sequential decision problem.

b) Determine the optimal policy and the company's expected sales.

Submission requirements:

1. For each (sub)problem, name the solution file as "*problem_x.ext*," where "*x*" represents the (sub)problem number (x = 1, 2, 3 or x = 1a, 1b, 1c) and the file extension "*ext*" depends on the file type (Word, Excel, PDF, etc.). If the solution to a (sub)problem contains multiple files (e.g., a Python package), organize the file(s) into a folder and name the folder as "*problem_x*."

2. Note that your Python files must be able to be executed directly. So use relative paths instead of absolute paths. If necessary, you may provide a short "user manual" of instructions on how to execute your codes.

   Warning: If the TAs have to manipulate your Python package to verify your solutions, you will be deducted points from your grade.

3. Pack all the "(sub)problem" folders in a zip file and name the zipped file "*hw_##_Chinese name.zip*," where "##" (two digits) represents the homework number, for example, "*hw_09_赵磊.zip*."