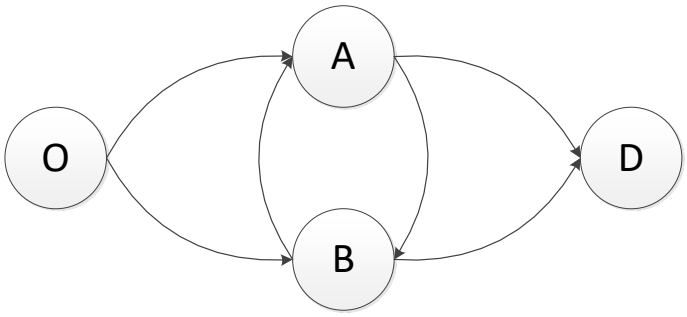
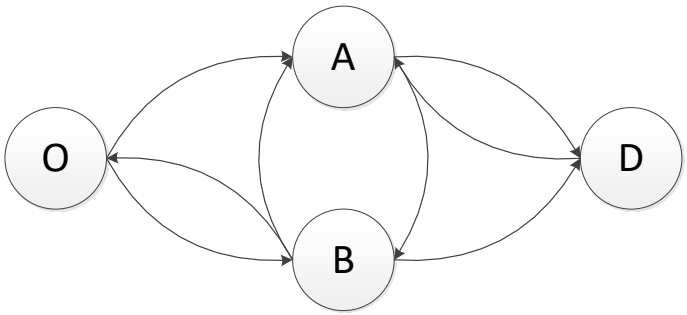
Homework 3 – Maximum Flow Problem

Due: 1:30 pm, March 19, 2025

1. For the two directed networks in Figure 1, write down the **specific** (primal) models of the maximum flow problems from node O to node D, and the corresponding dual models, using the **same notation** as in the lecture.



(a)



(b)

Figure 1 The networks in Problem 1

Notes:

1) The capacity u_{ij} for each arc $(i,j) \in A$ is assumed to be given as in the lecture.

2) The primal and dual models **must be specific to the networks in Figure 1**, but **NOT** the generic or equivalently transformed models as in the lectures.

2. Find the maximum flow from node 1 to node 8 in the network in Figure 2, using the Ford-Fulkerson algorithm. Identify the associated minimum cut set.

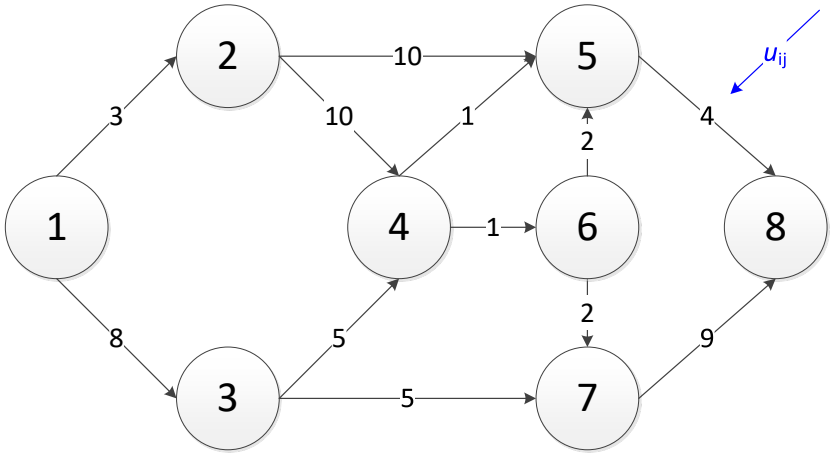


Figure 2 The network in Problem 2

3. Winston (2004), Section 1.3, Problem 16 on Page 21.

   During the next four months, a construction firm must complete three projects. Project 1 must be completed within three months and requires 8 months of labor. Project 2 must be completed within four months and requires 10 months of labor. Project 3 must be completed within two months and requires 12 months of labor. Each month, 8 workers (note: worker = labor) are available. During a given month, no more than 6 workers can work on a single project.

   1) Formulate a maximum flow problem that could be used to determine whether all three projects can be completed on time. (Hint: If the maximum flow in the network is 30, all projects can be completed on time.)

   2) Solve the problem using Gurobi in Python.

   Notes:

   1) You are encouraged to design an Excel file to read the input data from and write the output solution into. Please include the Excel file in your Python package to make sure that your Python files are executable.

   2) However, you are NOT required to implement the I/O part as in 1). That is, generating the input data manually is fine.

Submission requirements:

1. For each (sub)problem, name the solution file as "*problem_x.ext*," where "*x*" represents the (sub)problem number (x = 1, 2, 3 or x = 1a, 1b, 1c) and the file extension "*ext*" depends on the file type (Word, Excel, PDF, etc.). If the solution to a (sub)problem contains multiple files (e.g., a Python package), organize the file(s) into a folder and name the folder as "*problem_x*."

2. Note that your Python files must be able to be executed directly. So use relative paths instead of absolute paths. If necessary, you may provide a short "user manual" of instructions on how to execute your codes. Warning: If the TAs have to manipulate your Python package to verify your solutions, you will be deducted points from your grade.

3. Pack all the "(sub)problem" folders in a zip file and name the zipped file "*hw_##_Chinese name.zip*," where "##" (two digits) represents the homework number, for example, "*hw_03_赵磊.zip*."