1.1. Examples of Two Stage Stochastic Linear Programs

In order to illustrate the wide range of applications of this important class of mathematical programming models, we will summarize a number of models described in the literature. In particular, we will describe examples that arise in production capacity planning, power generation planning, air freight scheduling, and telecommunciations network planning. These models will be referred to as CEP1, PGP2, STORM, and SSN, respectively, and will be used throughout this book as we report on computational experience with our methods.

1.1.1 Capacity Expansion Planning: CEP1

Consider the task of planning for the expansion of productive capacity in a facility that produces m parts. Each of the machines under consideration is flexible in nature, and thus each part can be produced on any one of n machines. Machine j is currently available for h_j hours of operation per week, and additional hours may be acquired at an amortized weekly cost of c_j per hour. Total usage of machine j is constrained by an upper limit of u_j. Additionally, machine j is required to undergo t_j hours of scheduled maintenance for each hour of operation, and the total scheduled maintenance cannot exceed T hours. Part type i may be produced on machine j at rate a_{ij}, with an associated cost of g_{ij} per hour.

In production systems, demand forecasts are often critical to the planning process. When demand is assumed to be known with certainty, one can easily obtain an optimal production plan, which in turn leads to an optimal capacity plan. In reality, demand is rarely known with certainty. Hence, production plans are usually postponed until better information is available. However, capacity plans cannot be postponed, and hence cannot rely on the production plan. Indeed, as demand varies from week to week, there may not be a unique production plan. Thus, the two stage nature of this planning process is apparent.

The weekly production plans are determined by the demand for parts, which varies from week to week. Thus, the demand for part i in any given week is represented by the random variable $\tilde{\omega}_i$. Note that weekly demands are independent and identically distributed random variables. Upon learning the demand profile for a given week, the allocation of parts to machines is done on a least cost basis. Management has recommended that inventories should not be built, and furthermore, there is a penalty p_i for each unit of unsatisfied demand for part type i (which may be thought of as either a penalty for a 'lost sale', or the additional cost required to satisfy the demand via a subcontractor). The objective involves the minimization of the amortized expansion cost plus the expected weekly production costs.

This problem may be formulated as a two stage stochastic linear program with recourse. Let $J = \{1, \ldots, n\}$ and $I = \{1, \ldots, m\}$. Let x_j represent the number of hours (per week) of new capacity that is acquired for machine $j \in J$, and y_{ij} represent the number of hours in a given week that machine j is devoted to the production of part $i \in I$. Note that for a fixed set of capacity values, $(x_1, \ldots, x_n)$, the utilization y_{ij} will vary from week to week according to the demand realizations. This capacity expansion planning problem may be modeled as follows, and data for a specific instance of the model is provided in the appendix of this chapter.

z_j: allowable hours of operation per week of machine j.

$$\begin{aligned}
\text{Min } & \sum_{j=1}^{n} c_j x_j + E[h(z,\tilde{\omega})] && (CEP1)\\
\text{s.t. } & -x_j + z_j \le h_j \quad \forall j \in J \\
& \sum_{j=1}^{n} t_j z_j \le T \\
& 0 \le z_j \le u_j \quad \forall j \in J \\
& 0 \le x_j \quad \forall j \in J
\end{aligned}$$

where

$$\begin{aligned}
h(z,\omega) = \text{Min } & \sum_{i\in I}\sum_{j\in J} g_{ij} y_{ij} + \sum_{i\in I} p_i s_i \\
\text{s.t. } & \sum_{j=1}^{n} a_{ij} y_{ij} + s_i \ge \omega_i \quad \forall i \in I \\
& \sum_{i=1}^{m} y_{ij} \le z_j \quad \forall j \in J && (1.1)\\
& y_{ij} \ge 0 \quad \forall i \in I,\ j \in J \\
& s_i \ge 0 \quad \forall i \in I
\end{aligned}$$

In this example, the random variables correspond to the demands for each of the m parts. It is easy to envision examples in which the machines themselves might fail. In such a case, the right hand side of (1.1) might be replaced by $\theta_j z_j$, where θ_j denotes an outcome of a random variable representing the availability of machine j. A somewhat more general interpretation of the above model is also possible. For instance, if each 'machine' represents a particular configuration of a flexible manufacturing system (FMS), then the right hand side of (1.1) would be replaced by the production capacity of the FMS in its j^{th} configuration (see e.g., Askin, Ritchie and Krisht [1989]). In the context of such FMS models, it would be natural to obtain observations of random variables like production capacity by using a simulation model.

1.1.2 Power Generation Planning: PGP2

Next, we consider the task of selecting power generators that minimize the total cost of supplying enough electricity to satisfy regional demand. Deterministic power generation planning models have a long history dating back to Masse and Gibrat [1957]. More recently, two stage stochastic programming formulations have been proposed in Murphy, Sen and Soyster [1982] and a multi-stage model appears in Louveaux and Smeers [1988].

To begin, suppose that we wish to select the optimal sizes of three different types of generators: gas fired, coal fired, and nuclear. The annualized capital cost (\$/kw) for the acquisition of a generator of type j is given by c_j, while the cost of producing a unit of energy (\$/kw-hr) is given by f_j. Note that while decisions regarding the generation of electricity may be postponed until regional demand is known, decisions regarding the acquisition of generation capacity cannot be postponed. Thus, we see a natural two stage progression of the decisions being undertaken. That is, the first stage decision identifies the generating capacity to be acquired, while the second stage decision identifies a more detailed operating plan undertaken in response to the imminent regional demand.

Regional demand for electricity is usually cyclic and the chronological load may be represented as shown in Figure 1.1. For the purposes of capacity planning, the chronology is not as important as the duration of the load. A load duration curve (LDC, see Figure 1.2) is a reordering of the chronological load curve. For a given load level h, the horizontal axis provides the number of hours during the year

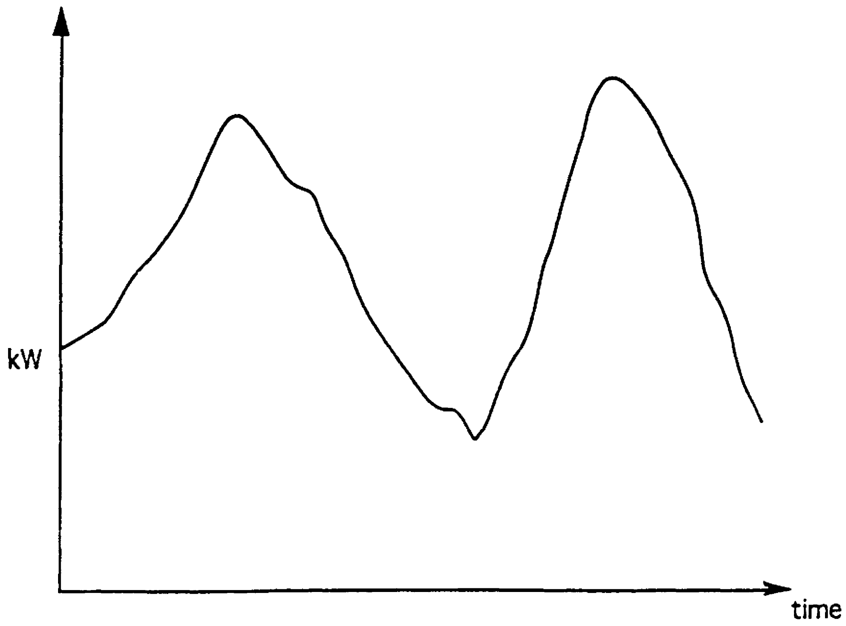

Figure 1.1: Chronological load curve

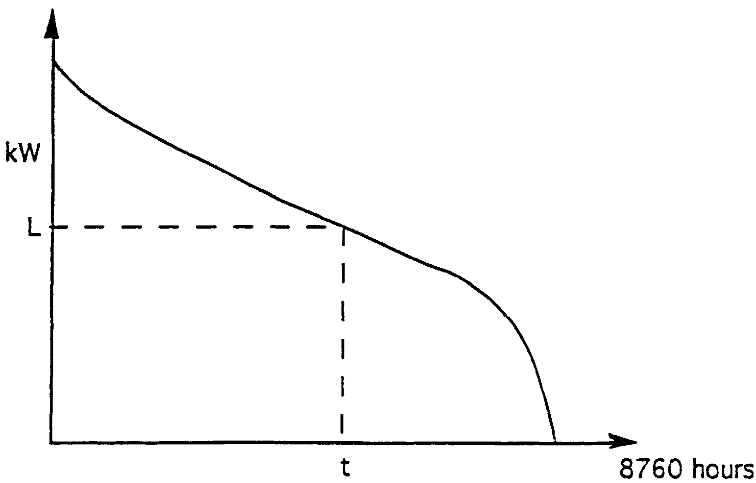

Figure 1.2: Load duration curve

that the load exceeds h. The area under the LDC gives the total energy consumption during the year. For planning purposes, the LDC is discretized into several steps, as shown in Figure 1.3.

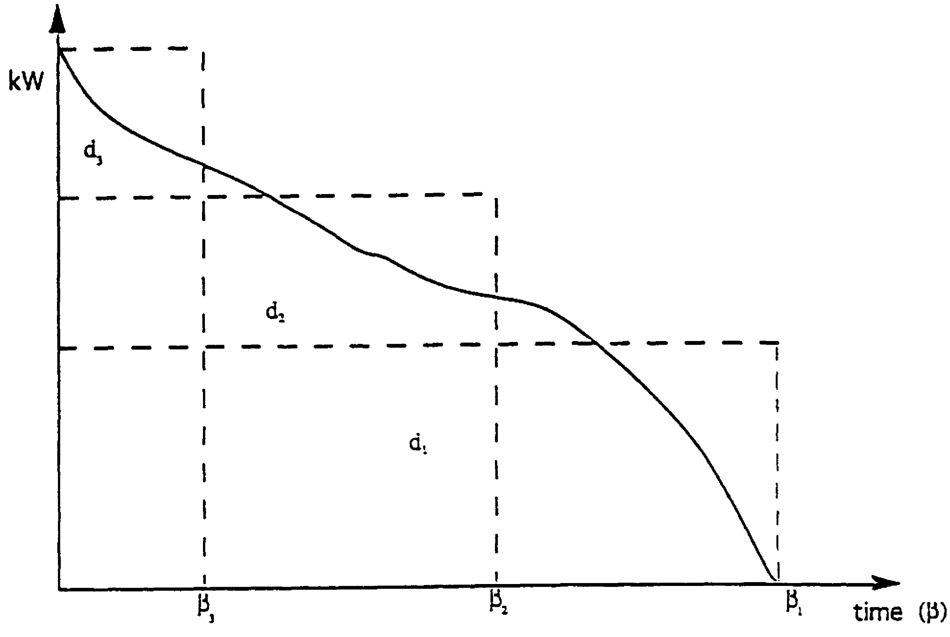

Figure 1.3: Discretized load duration curve

To formulate a model of the power generation planning problem, let $J = \{1, \ldots, n\}$ denote the set of possible generation equipment, and $I = \{1, \ldots, m\}$ the set of steps of the discretized LDC. For $j \in J$, we are given the costs c_j and f_j described above. For $i \in I$, we require the step sizes, ω_i, and the load duration β_i. We assume that the durations of the steps, $\{\beta_i\}_{i \in I}$, which depend on characteristics such as the number of workdays vs. weekend days, etc. during the planning period are predictable. However, the step sizes, which correspond to the actual demand for power during the i^{th} load segment will vary in response to several unpredictable factors (e.g., weather). Thus, these demands for electrical power are treated as random variables, $\tilde{\omega}_i$. The decision variables are identified as

x_j = kw of generation capacity type j

y_{ij} = kw of demand segment i served using generator type j.

Finally, the model presented below includes two first stage constraints: a minimum capacity of M and a budgetary restriction of \$$b$ are imposed. If the acquired generating capacity is insufficient to satisfy the power requirement, additional generating capacity of type j need to be subcontracted, denoted as z_j, at a cost of p_j per KW capacity.

Thus, the power generation planning problem may be written as

$$\begin{aligned} \text{Min} \quad & \sum_{j \in J} c_j x_j + E[h(x, \tilde{\omega})] && (PGP2) \\ \text{s.t.} \quad & \sum_{j \in J} c_j x_j \leq b \\ & \sum_{j \in J} x_j \geq M \\ & x_j \geq 0 \qquad \forall j \in J \end{aligned}$$

where

$$\begin{aligned} h(x, d) = \text{Min} \quad & \sum_{i \in I} \sum_{j \in J} f_j \beta_i y_{ij} + \sum_{j \in J} p_j z_j \\ \text{s.t.} \quad & -z_j - x_j + \sum_{i \in I} y_{ij} \leq 0 \qquad \forall\, j \in J \\ & \sum_{j \in J} y_{ij} = \omega_i \qquad \forall\, i \in I \\ & y_{ij} \geq 0 \qquad \forall\, i \in I,\ j \in J \end{aligned}$$

The first stage constraints require that the total cost of the acquired capacity satisfies the budget restriction as well as the minimum capacity restriction. In the second stage, the first set of constraints restricts the total power that can be obtained from a generator with capacity x_j, while the second set of constraints ensures that the power requirement of each step of the LDC is satisfied. Finally, as with CEP1, the data for PGP2 can be found in the appendix of this chapter.

1.1.3 Air Freight Scheduling: STORM

In this section, we consider a model that was used by the US Military to plan the allocation of aircraft to routes during the Gulf War of 1991, and is known as STORM. This model appears in Mulvey and Ruszczyński [1992]. For the sake of exposition, we will avoid the use of military jargon and present the model within a commercial setting.

STORM models a problem commonly encountered by an air-cargo outfit that serves a network of cities. That is, the problem involves the scheduling of flights between cities and the assignment of cargo to flights in order to minimize the cost of satisfying customer freight demands. Note that cargo can be efficiently assigned to flights once the freight demand is known. However, flight schedules must typically be determined well in advance of these demands. Thus, once again we see the natural two stage progression of the decisions being undertaken. To facilitate the reader's understanding of the model, we present the data and decision variables below, grouped by the stage in which they appear.

In the first stage, flight schedules must be determined. In this stage, the constraints can be classified as frequency requirements, upper bounds on the number of landings, restrictions on flying time, and balance constraints. In the first stage the sets and parameters used in the problem description are as follows.