

## Homework 10 – Game Theory

**Due:**            **Self-exercise only. No submissions required.**

1. Winston (2004), Page 192, Section 4.1-3, Prob. 1

A soldier can hide in one of five foxholes (1, 2, 3, 4, or 5) (see Figure 1). A gunner has a single shot and may fire at any of the four spots A, B, C, or D. A shot will kill a soldier if the soldier is in a foxhole adjacent to the spot where the shot was fired. For example, a shot fired at spot B will kill the soldier if he is in foxhole 2 or 3, while a shot fired at spot D will kill the soldier if he is in foxhole 4 or 5. Suppose the gunner receives a reward of 1 if the soldier is killed and a reward of 0 if the soldier survives the shot.



Figure 1 Foxholes and spots

- a) Assuming this to be a zero-sum game, construct the reward matrix.
  - b) Find and eliminate all dominated strategies.
  - c) We are given that an optimal strategy for the soldier is to hide  $1/3$  of the time in foxholes 1, 3, and 5. We are also told that for the gunner, an optimal strategy is to shoot  $1/3$  of the time at A,  $1/3$  of the time at D, and  $1/3$  of the time at B or C. Determine the value of the game to the gunner.
  - d) Suppose the soldier chooses the following nonoptimal strategy:  $1/2$  of the time, hide in foxhole 1;  $1/4$  of the time, hide in foxhole 3; and  $1/4$  of the time, hide in foxhole 5. Find a strategy for the gunner that ensures that his expected reward will exceed the value of the game.
  - e) Write down each player's LP and verify that the strategies given in part (c) are optimal strategies.
2. Winston (2004), Page 197, Section 4.4, Prob. 3

The New York City Council is ready to vote on two bills that authorize the construction of new roads in Manhattan and Brooklyn. If the two boroughs join forces, they can pass both bills, but neither borough by itself has enough power to pass a bill. If a bill is passed, it will cost the taxpayers of each borough \$1 million, but if roads are built in a borough, the benefits to the borough are estimated to be \$10 million. The council votes on both bills simultaneously, and each councilperson must vote on the bills without knowing how anybody else will vote. Assuming that each borough supports its own bill, determine whether this game has any equilibrium points. Is this game analogous to the Prisoner's Dilemma? Explain why or why not.