Improving Adaptive Quantum Entanglement Witnessing

Arianna Meinking
Advisor: Theresa W. Lynn

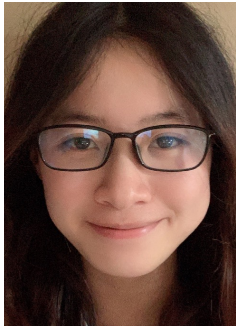
HARVEY MUDD COLLEGE
Department of Physics

Acknowledgements

Ben Hartley
W Characterization, W’ Proposal

Oscar Scholin
W’ Choice Optimization

Paco Navarro
Current lab



Eritas Yang
W Characterization, W’ Proposal

Becca Verghese
W Characterization, W’ Proposal

Theresa W. Lynn
Advisor

Work accompanied by experimental tests by:
Alec Roberson
Richard Cheng
Lev Gruber

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HMC Physics Summer Research Fund
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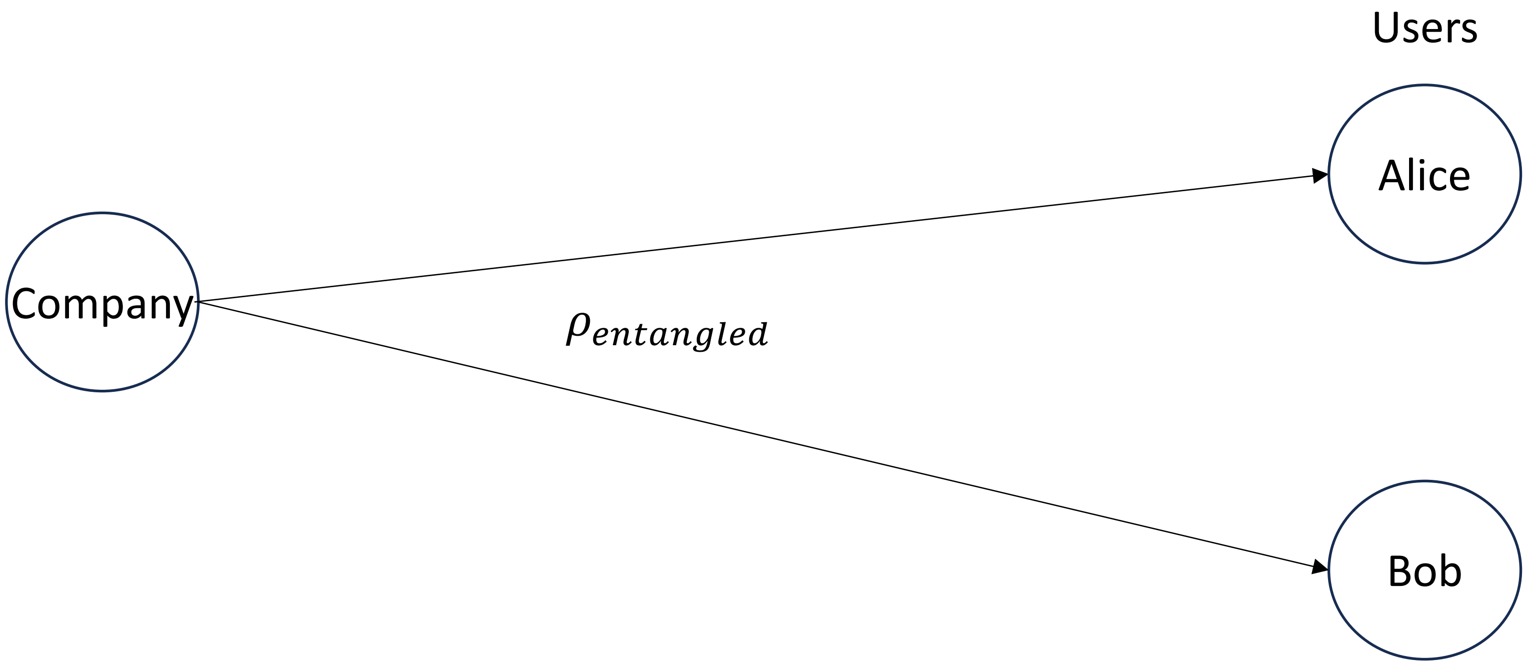
Goal: high chance of witnessing two-qubit entanglement if present from a small fraction of measurements required for full state tomography

Quantum Communication via … Entanglement!



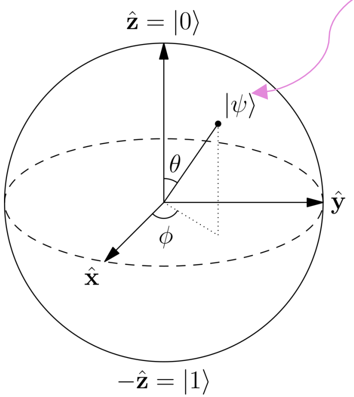
A Definition of Entanglement

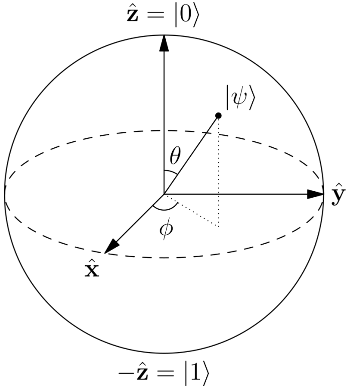
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi|$$





They're correlated across ANY bases!

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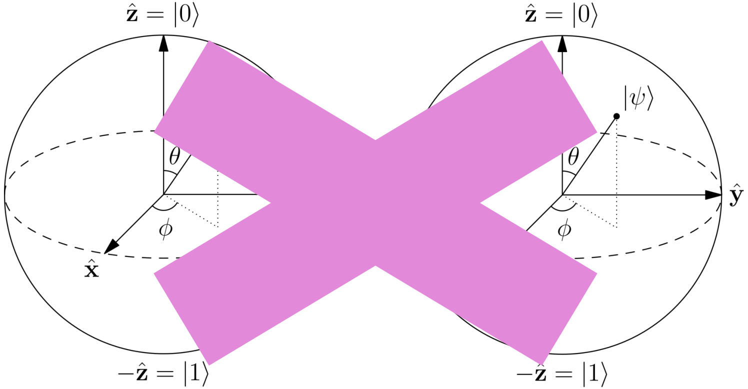

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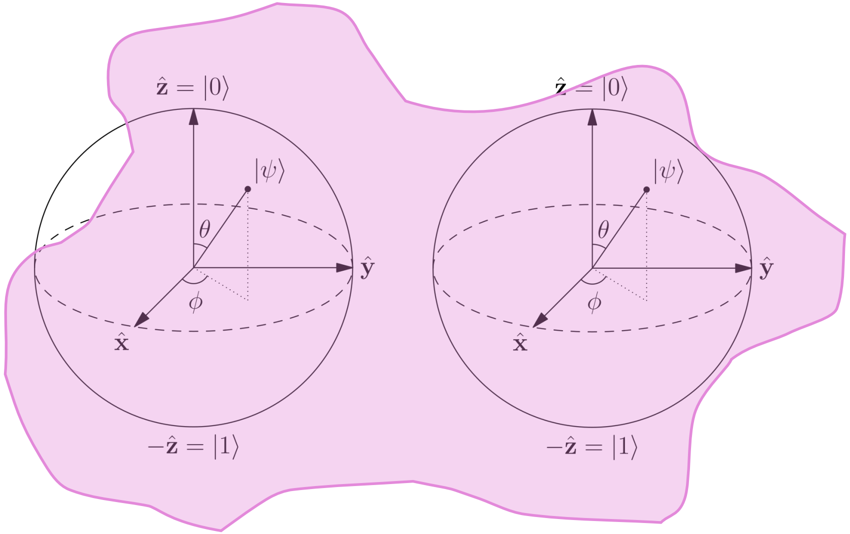


Entangled system!

They're correlated across ANY bases!

Check ALL the Measurements

Quantum Tomography

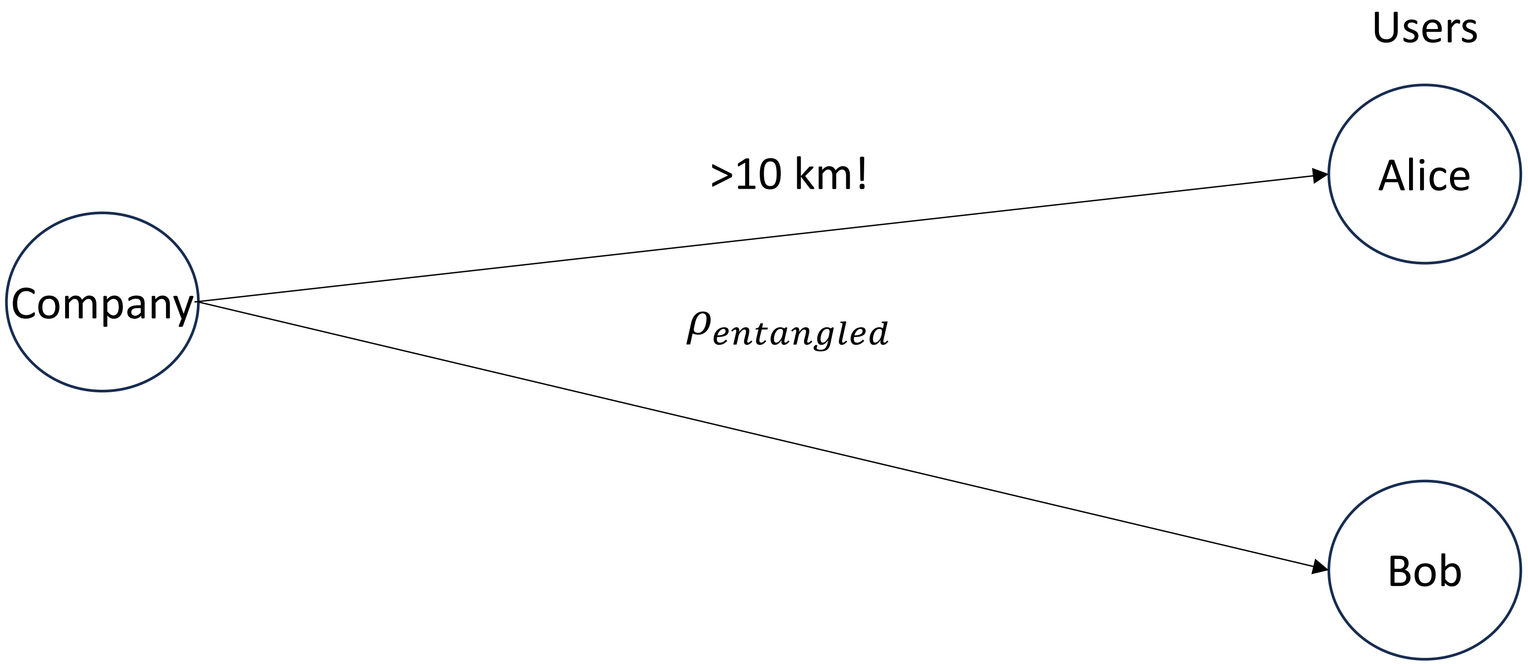
$\boldsymbol{\sigma_x} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_y} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_z} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_z}$

4 measurements each!

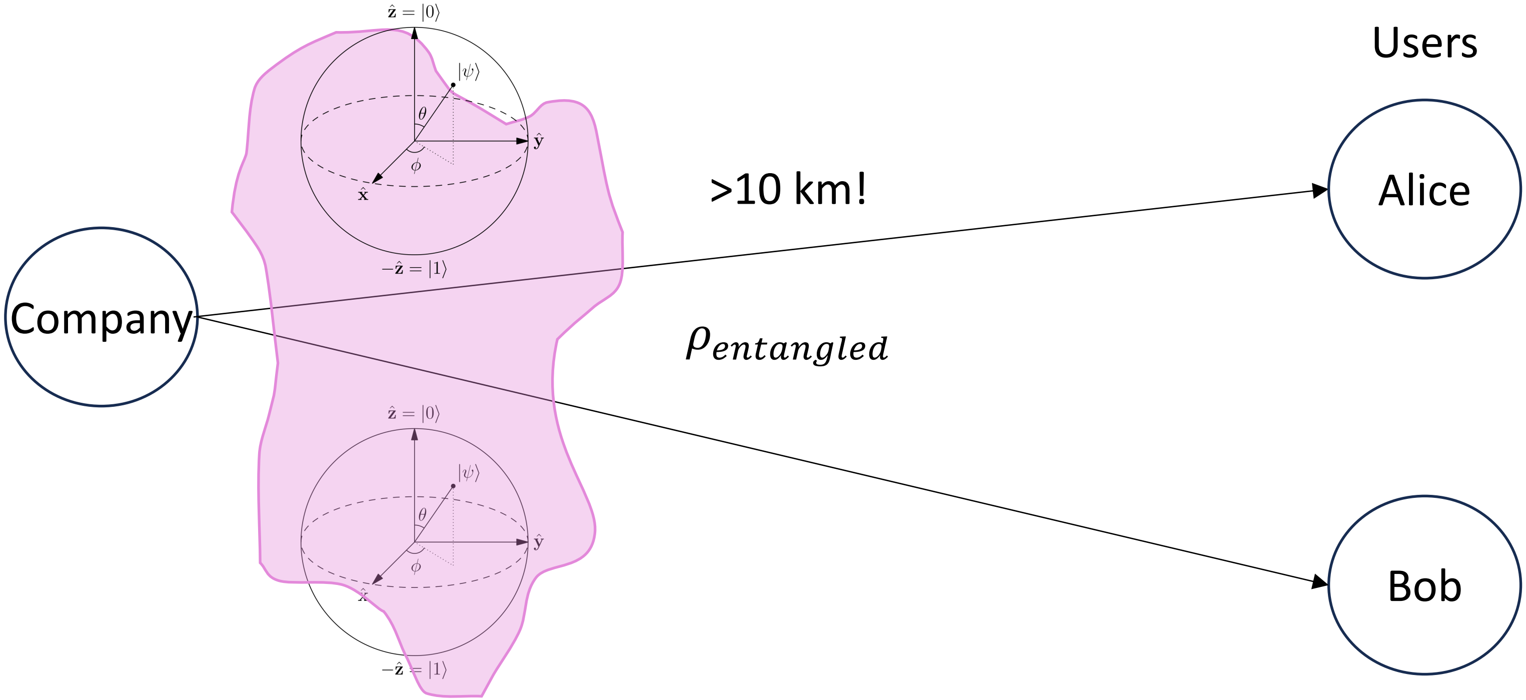
9 Pauli combos → 36 measurements!

included

Quantum Communication via … Entanglement!



Quantum Communication via … Entanglement!



Liu, X., Hu, J., Li, ZF, et al. *Nature* 10.1038 (2021)

A Solution: Entanglement Witnessing

Entanglement Witnesses

$\boldsymbol{\sigma_x} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_y} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_z} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_z}$

- $\langle W \rangle < 0$ means entanglement
- Decomposable
  $W = |\varphi_k\rangle\langle\varphi_k|^{\Gamma}$

Riccardi et al., PRA 101, 062319 (2020)

A Solution: Entanglement Witnessing

Entanglement Witnesses

$\boldsymbol{\sigma_x} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_y} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_z} \otimes \mathbb{I}$	$\mathbb{I} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_z}$

$$W = |\varphi_k\rangle\langle\varphi_k|^{\Gamma}$$

How about…

$|\varphi_k\rangle$ = $\frac{|01\rangle - |10\rangle}{\sqrt{2}}$

Then

$$|\varphi_k\rangle\langle\varphi_k| = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|\varphi_k\rangle\langle\varphi_k|^{\Gamma} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

Riccardi et al., PRA 101, 062319 (2020)

The W Witness Group

Measurements

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

$$W_1 = \frac{1}{4}\left[\mathbb{I}\otimes\mathbb{I} + \sigma_z\otimes\sigma_z + (a^2 - b^2)\sigma_x\otimes\sigma_x + (a^2 - b^2)\sigma_y\otimes\sigma_y + 2ab(\mathbb{I}\otimes\sigma_z + \sigma_z\otimes\mathbb{I})\right]$$

- Minimize a and b $\rightarrow$ going from a family to just one W
- Riccardi et al. proposed 6 Ws – only include $\sigma_z\otimes\sigma_z, \sigma_x\otimes\sigma_x, \sigma_y\otimes\sigma_y$ matrices
- Computationally generate random entangled mixed states*
- W_{1-6} detect 33% miss 67% of those states

Riccardi et al., PRAppl. 101, 062319 (2020)

*Random state generation following method used in Roik et al., PRAppl. 15.054006(2021)

The W's

Measurements

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

$$\begin{aligned} W'_1 = \frac{1}{4}\Big[& \mathbb{I}\otimes\mathbb{I} + \sigma_z\otimes\sigma_z + 2\cos 2\theta\,(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y) \\ & + 2\sin 2\theta\cos\alpha\,\,(\mathbb{I}\otimes\sigma_z + \sigma_z\otimes\mathbb{I}) \\ & + 2\sin 2\theta\sin\alpha\,\,(\sigma_x\otimes\sigma_y + \sigma_y\otimes\sigma_x)\Big] \end{aligned}$$

- Now, minimize θ and α
- Mixed Pauli pairs!
- Subgroups
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}

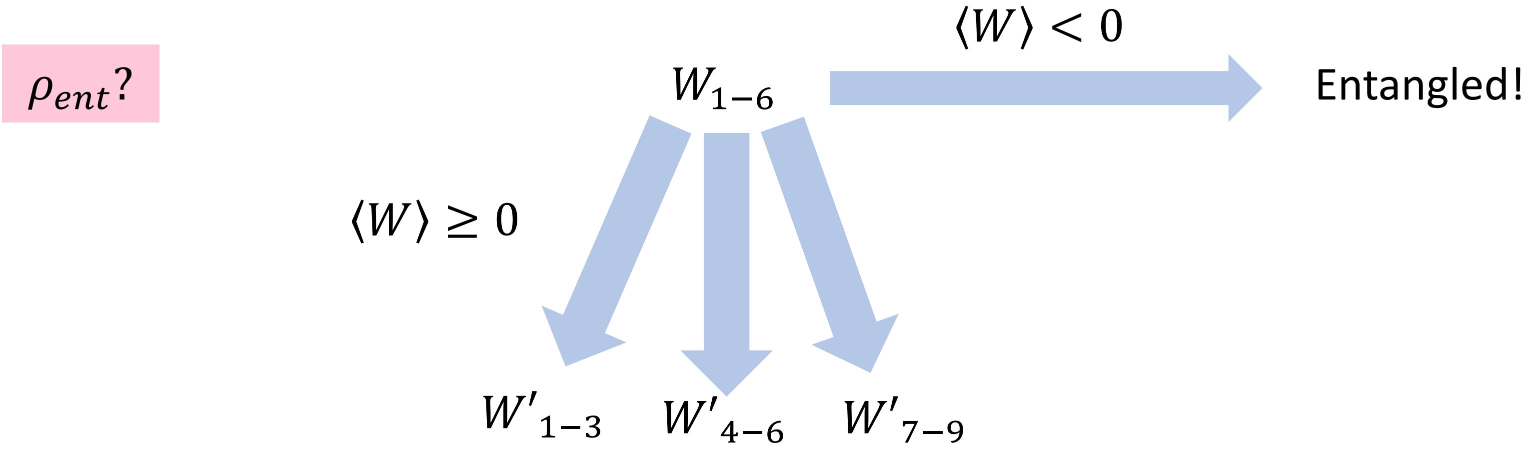
Yang, Verghese, Hartley. HMC Quantum Optics unpublished, https://github.com/Lynn-Quantum-Optics/Summer-Spring-2022-3/blob/main/Summer2022/ summer- 2022- QO_write_up.pdf

The Two-Step Process

ρ_{ent}?

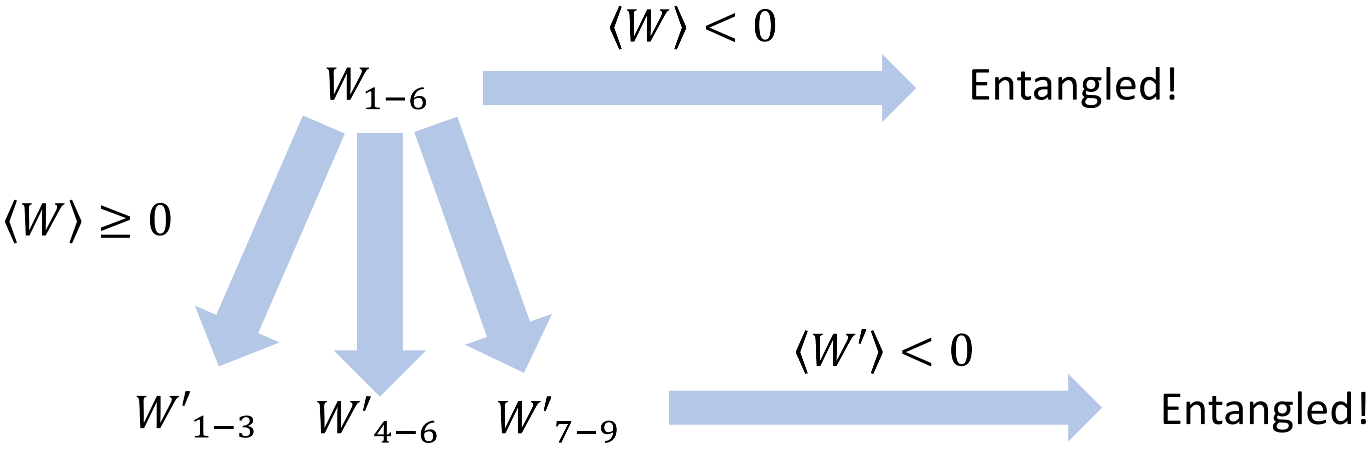
$W_{1-6} \xrightarrow{\langle W \rangle < 0}$ Entangled!

The Two-Step Process



The Two-Step Process

ρ_{ent}?



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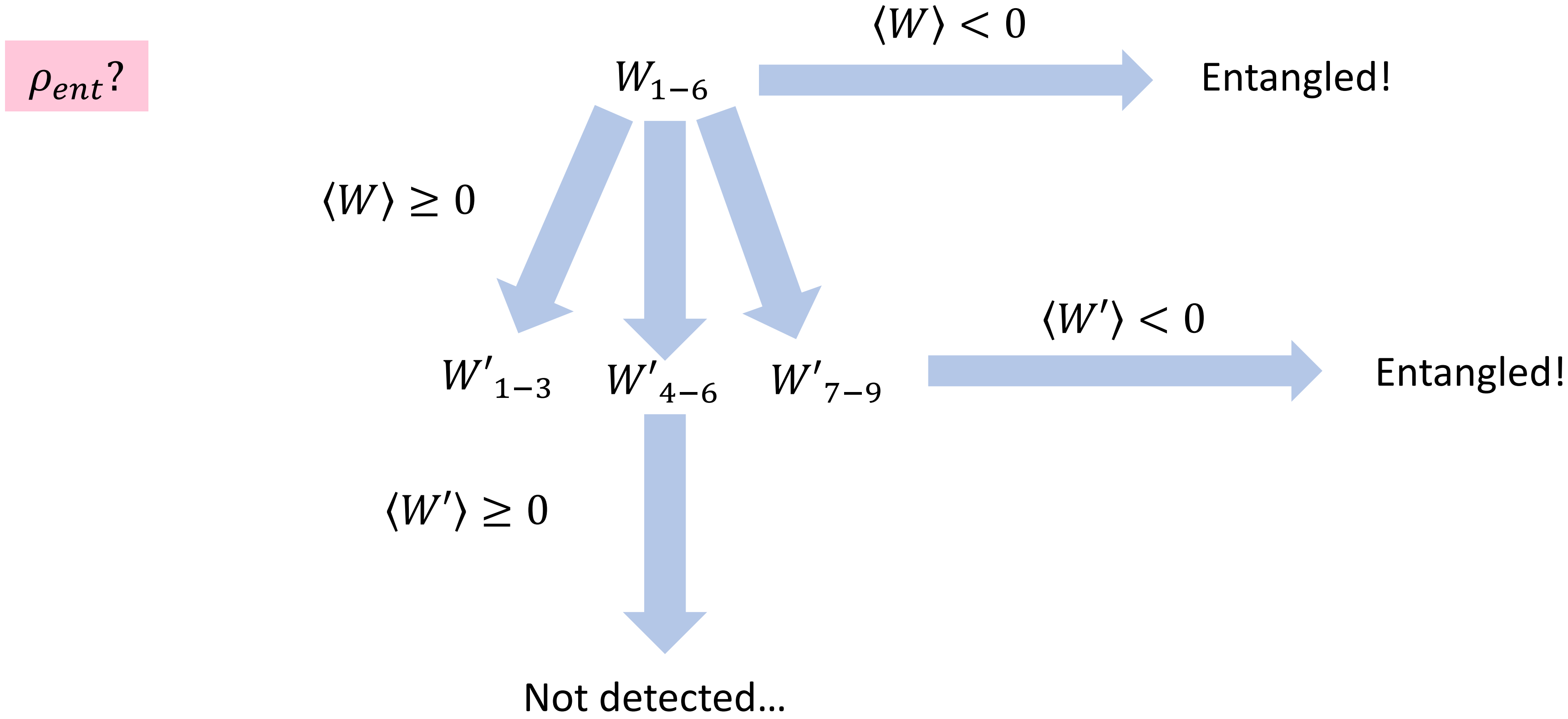


The Two-Step Process

ρ_{ent}?

W_{1-6}

$\langle W \rangle < 0$

Entangled!

$\langle W \rangle \geq 0$

(neural network) W'_{1-3} W'_{4-6} W'_{7-9}

$\langle W' \rangle < 0$

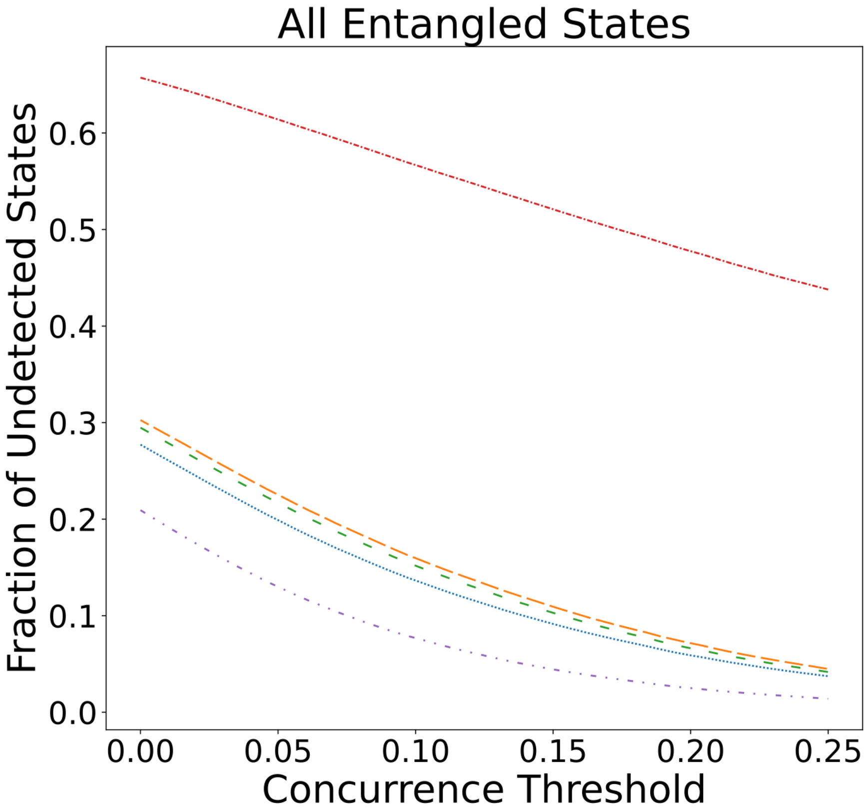
Entangled!

$\langle W' \rangle \geq 0$

Not detected...

Yang, Verghese, Scholin, Unpublished

Two-Step Process Performance



Scholin, Zheng, Roberson, Lynn, Unpublished

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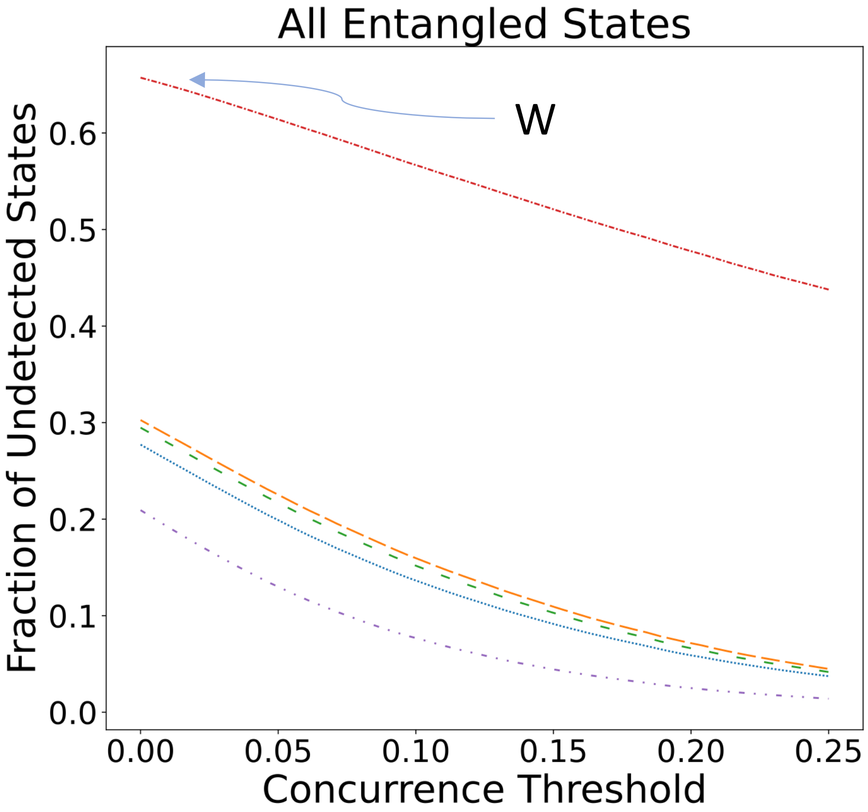

Scholin, Zheng, Roberson, Lynn, Unpublished

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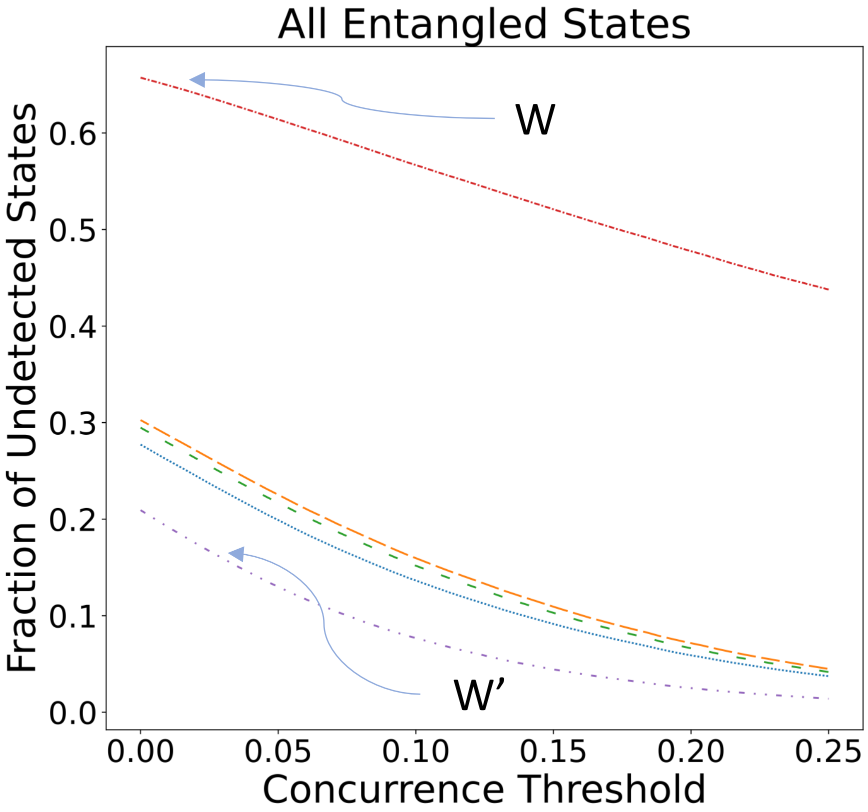

Scholin, Zheng, Roberson, Lynn, Unpublished

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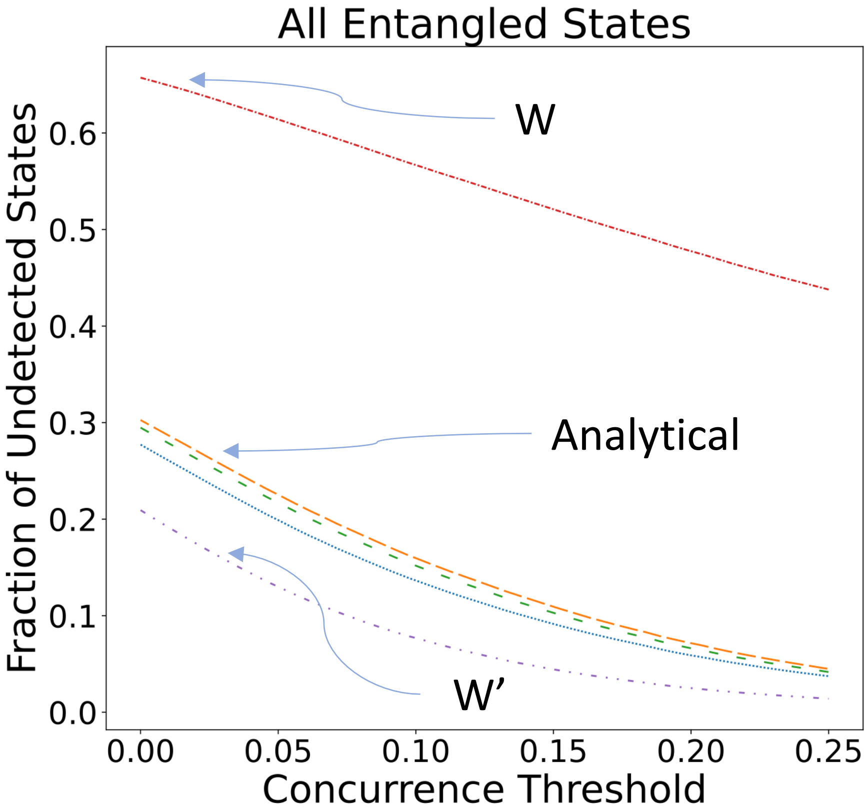

Scholin, Zheng, Roberson, Lynn, Unpublished

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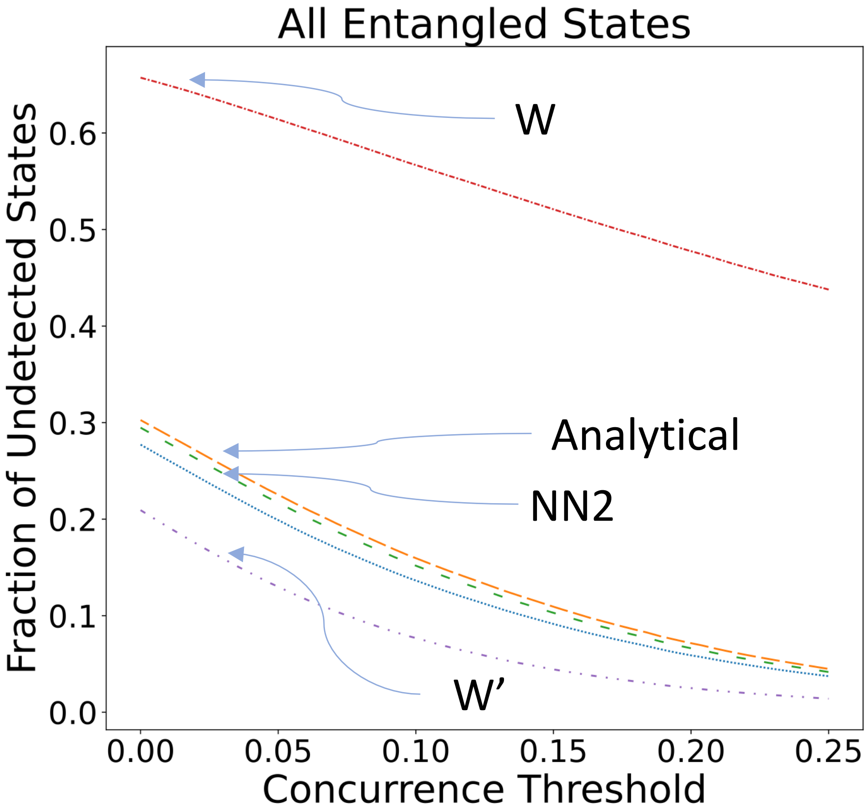

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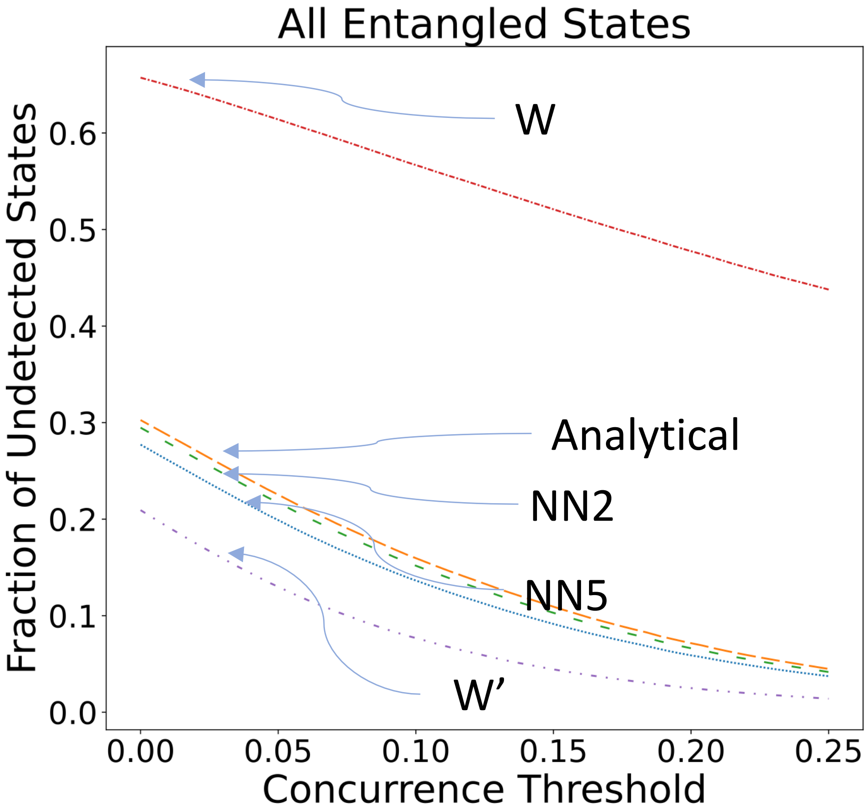

Scholin, Zheng, Roberson, Lynn, Unpublished

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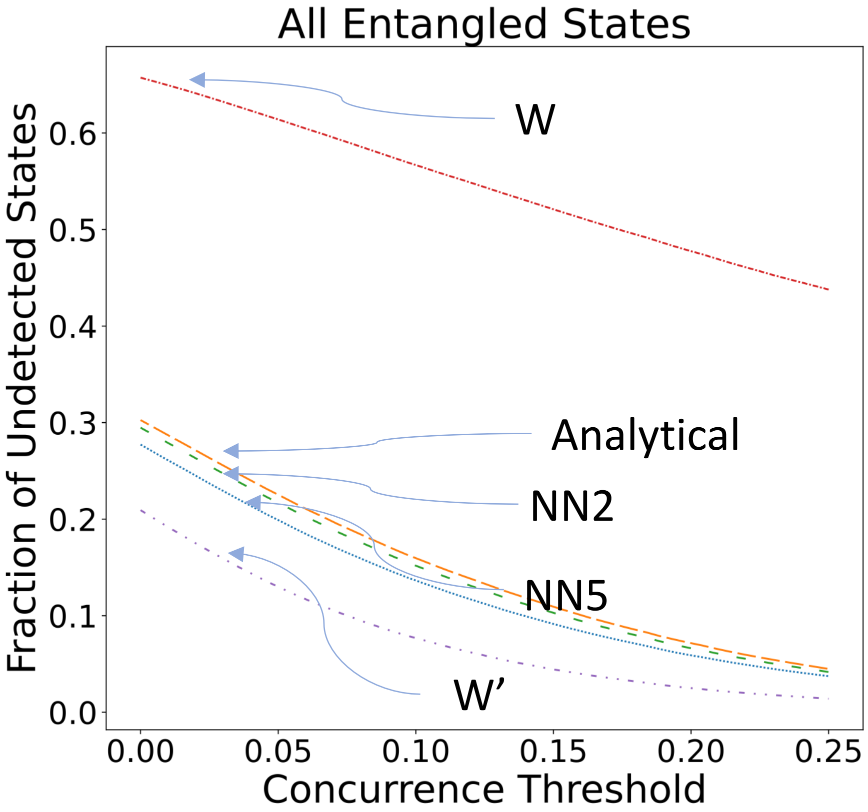
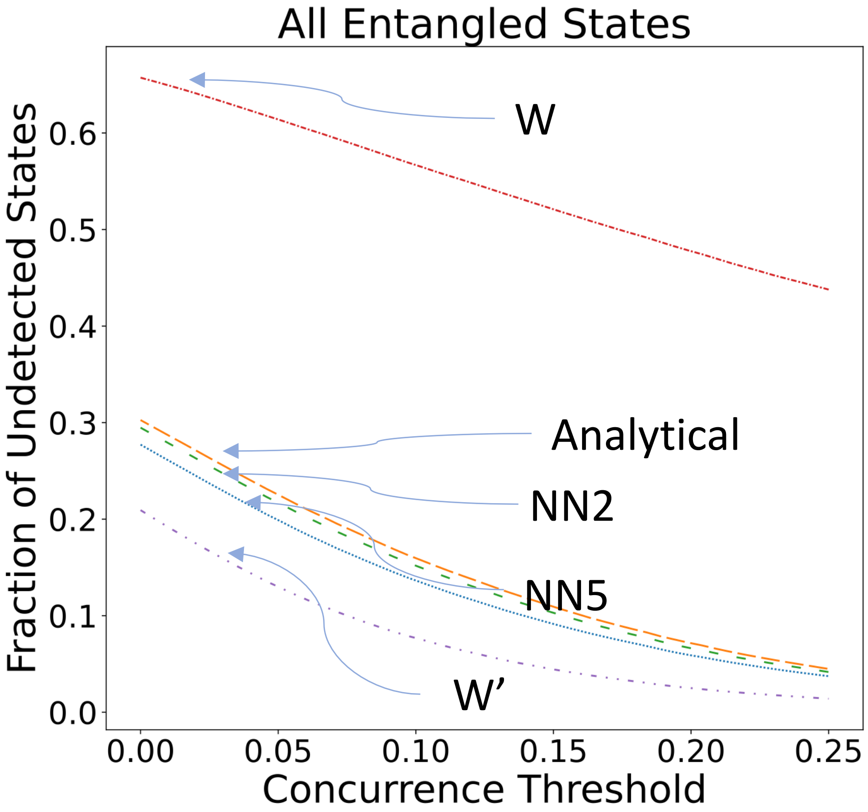


5/9 of the measurements witness 2/3 of the states!

Scholin, Zheng, Roberson, Lynn, Unpublished

Two-Step Process Performance



5/9 of the measurements witness 2/3 of the states!

More work to be done…

Scholin, Zheng, Roberson, Lynn, Unpublished

My Thesis: Expand the W’ Options?

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

- Another subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
  - W'_{10-12}?

Expanding the W’ Options... Impossible!

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\cancel{\boldsymbol{\sigma_x \otimes \sigma_z}}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\cancel{\boldsymbol{\sigma_y \otimes \sigma_z}}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

- Another subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
  - W'_{10-12}?
- The $\boldsymbol{\sigma_i \otimes \sigma_j}$ groups must come with a paired $\boldsymbol{\sigma_j \otimes \sigma_i}$ or some **cross-terms …**
- $W = |\varphi_k\rangle\langle\varphi_k|^{\Gamma}$

The V Witnesses: Include Cross-Terms

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

- Different Subgroups!
  - *Include* a cross-term!
  - $\boldsymbol{\sigma_i \otimes \sigma_j, \sigma_j \otimes \sigma_k \; and \; \sigma_i \otimes \sigma_k}$
- Key Points:
  - Each Pauli pair adds restrictions
  - The groups we *exclude* is how we get constraints!

The V Witnesses: Include Cross-Terms

V_1

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

V_2

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

V_3

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

V_4

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

- Different Subgroups!
  - $V_1 - V_4$
- Four groups of $\boldsymbol{\sigma_i \otimes \sigma_j, \sigma_j \otimes \sigma_k, \sigma_i \otimes \sigma_k}$
- Key Points:
  - Each Pauli pair adds restrictions
  - The groups we *exclude* is how we get constraints!

The W”: Do a W’ then ONE More

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

- One subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}

The W”: Do a W’ then ONE More

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$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

- One subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
- Now W”, just one additional measurement!
  - E.g. $\sigma_y \otimes \sigma_z + W'_{7-9} = W”_{\alpha}$
  - One extra measurement, or 3 measurements together, may be useful

The W”: Do a W’ then ONE More

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

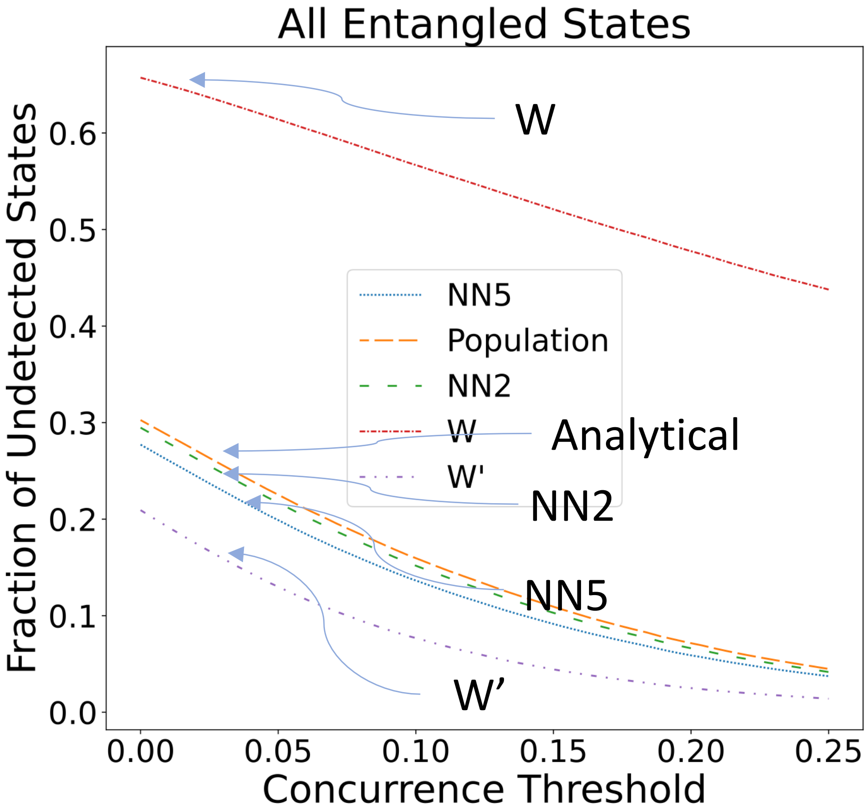
W''_α

- One subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
- Now W'', just one additional measurement!
  - E.g. $\sigma_y \otimes \sigma_z + W'_{7-9} = W''_\alpha$
  - One extra measurement, or 3 measurements together, may be useful

W''_β

$\sigma_x \otimes \sigma_x$	$\sigma_x \otimes \sigma_y$	$\sigma_x \otimes \sigma_z$
$\sigma_y \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$	$\sigma_y \otimes \sigma_z$
$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$

And the V and W” Witness New States!

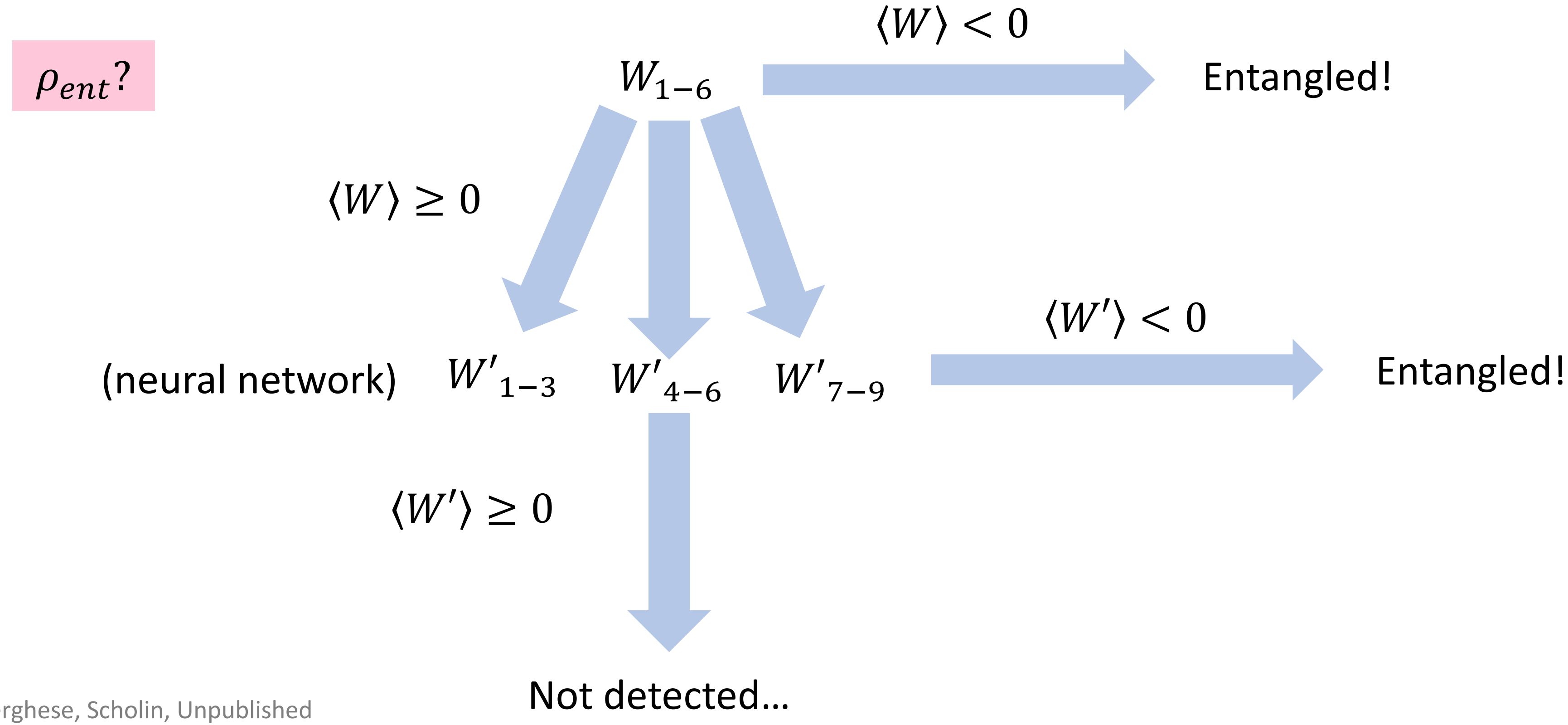


Scholin, Zheng, Roberson, Lynn, Unpublished

Without Optimization …

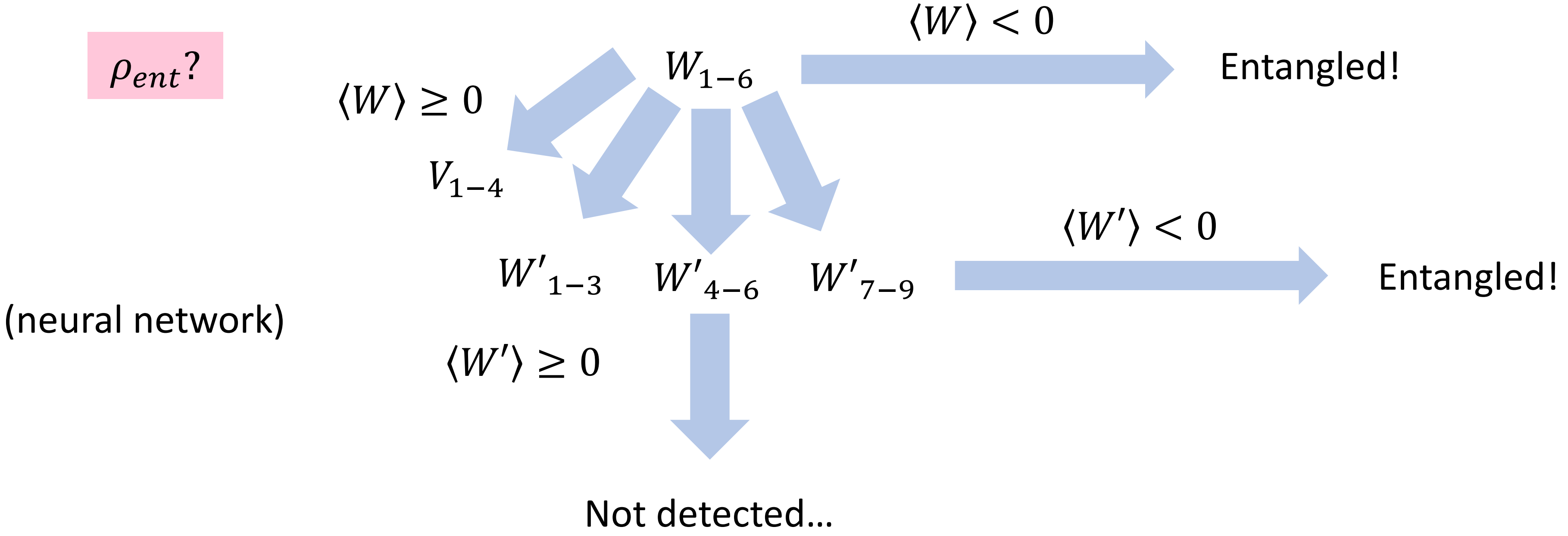
- V witness 0.5% states that are **undetected by all W, W’**
- W” witness 0.01% states that are **undetected by all W, W’**

The Two-Step Process

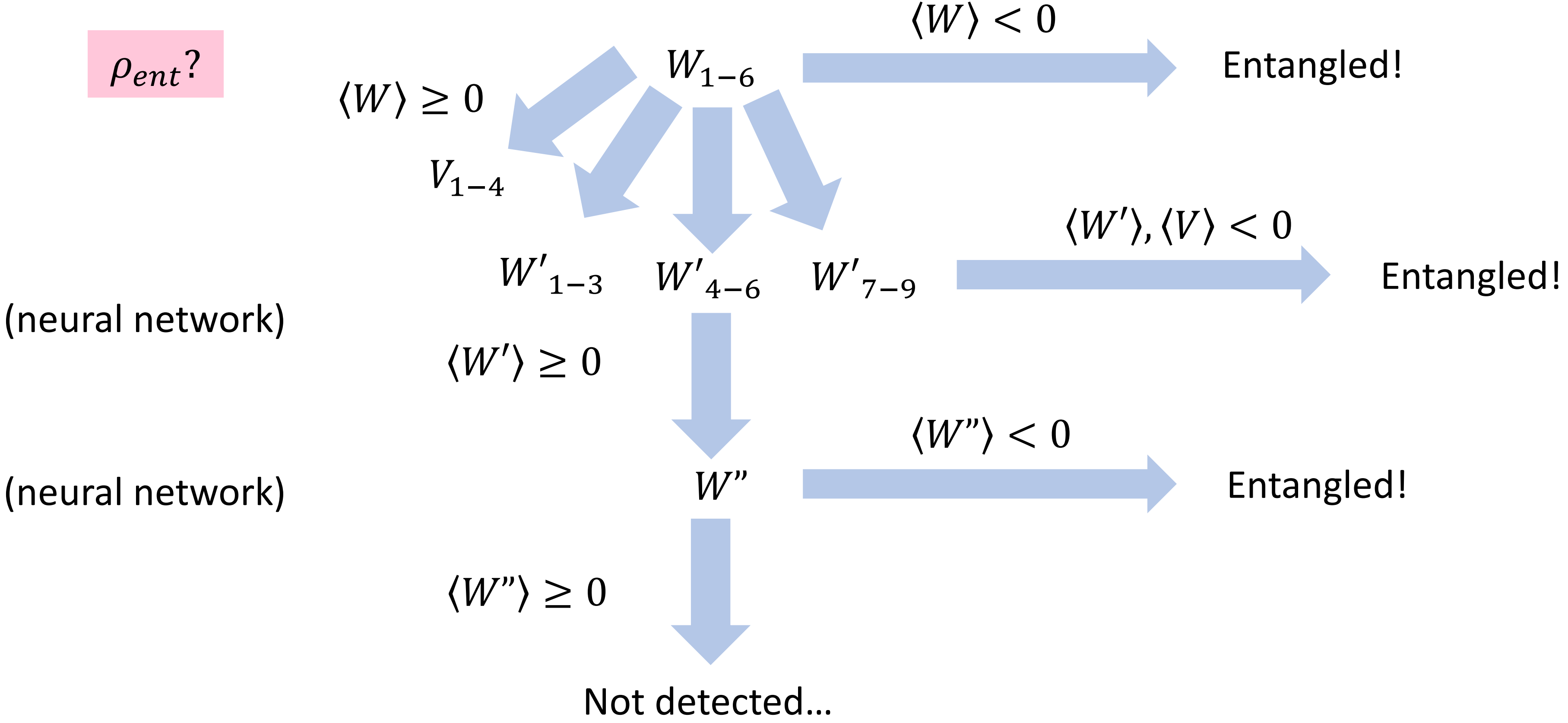


Yang, Verghese, Scholin, Unpublished

The Three-Step V-Step Process



The Three-Step V-Step Process



Key Takeaways / Questions?

Remember…

Goal: high chance of witnessing two-qubit entanglement if present from a small fraction of measurements required for full state tomography

My work!

V and W" witnesses find new states!

Future work

- Minimize the V and W"!
- New way to parameterize witnesses?

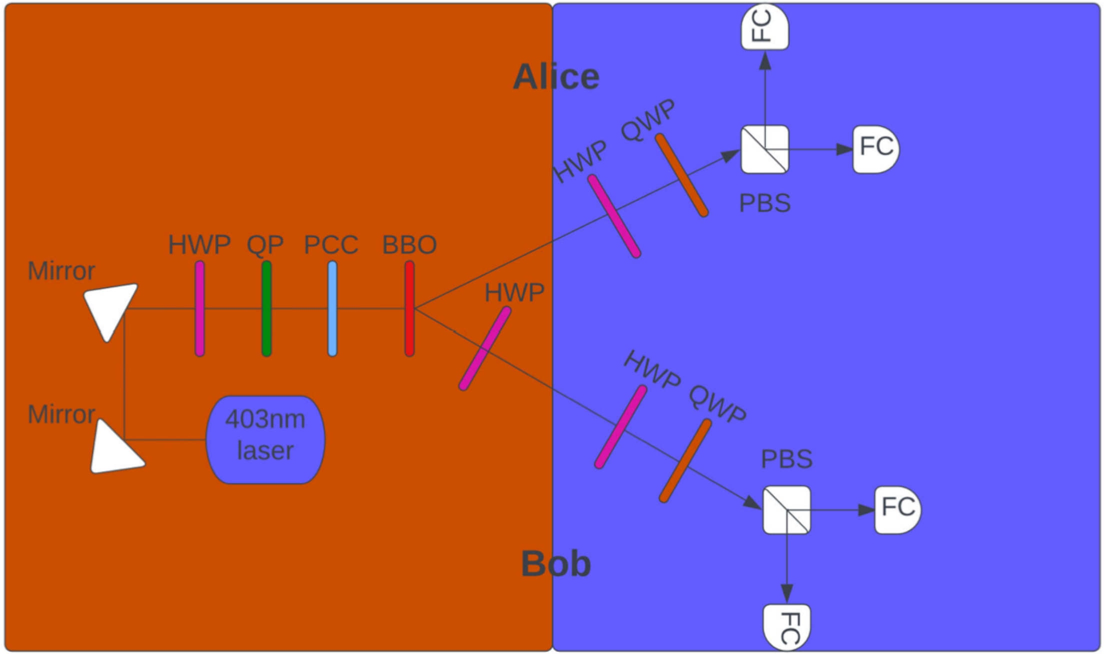
BACKUP SLIDES

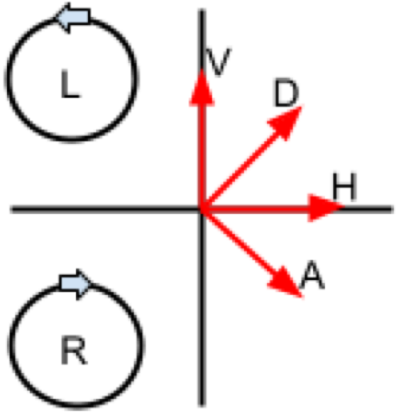
W’ Extension impossible (backup)

$$\begin{bmatrix} 1 & e^{-i\varphi} & e^{-i\alpha} & e^{-i\beta} \\ e^{i\varphi} & 1 & e^{i(\varphi-\alpha)} & e^{i(\varphi-\beta)} \\ e^{i\alpha} & e^{i(\alpha-\varphi)} & 1 & e^{i(\alpha-\beta)} \\ e^{i\beta} & e^{i(\beta-\varphi)} & e^{i(\beta-\alpha)} & 1 \end{bmatrix}$$

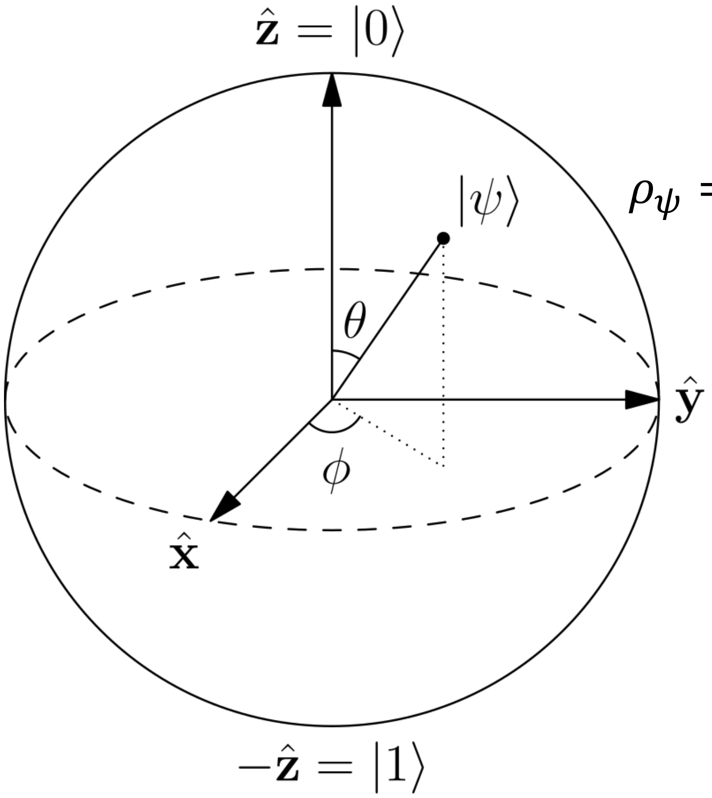
$$|HH\rangle + e^{i\varphi}|HV\rangle + e^{i\alpha}|VH\rangle + e^{i\beta}|VV\rangle$$

Experimental Apparatus (backup)





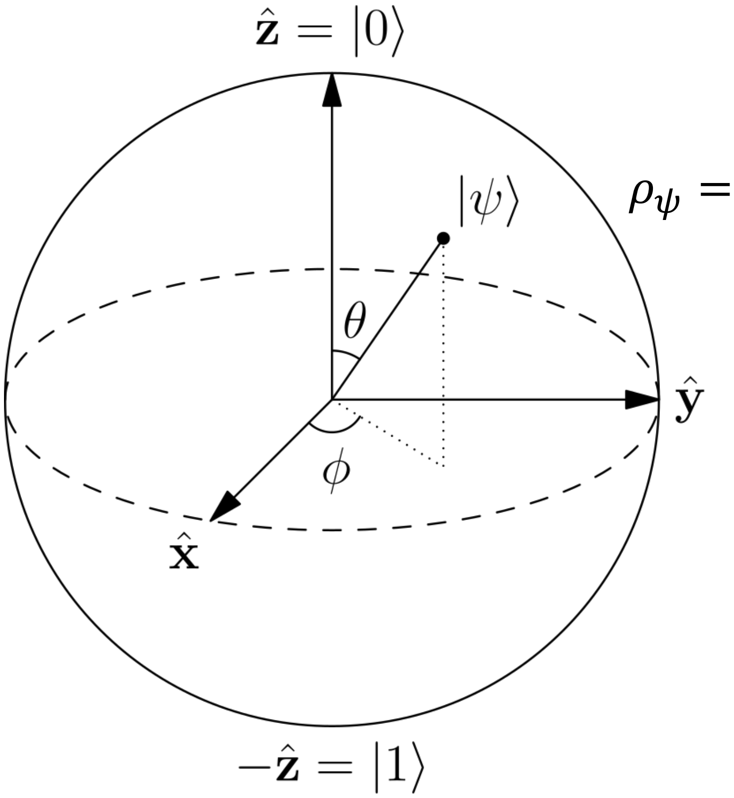
We start with … a single qubit!



$$|\psi\rangle \ = \ \cos\theta\,|0\rangle + \ e^{i\phi}\sin\theta\,|1\rangle$$

$$\rho_\psi = |\psi\rangle\,\langle\psi| \xrightarrow{\{|0\rangle,|1\rangle\}} \begin{pmatrix} \cos^2\theta & e^{-i\phi}\cos\theta\sin\theta \\ e^{i\phi}\cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$$

We start with … a single qubit!



$$|\psi\rangle = \cos\theta\,|0\rangle + e^{i\phi}\sin\theta\,|1\rangle$$

$$\rho_\psi = |\psi\rangle\langle\psi| \xrightarrow{\{|0\rangle,|1\rangle\}} \begin{pmatrix} \cos^2\theta & e^{-i\phi}\cos\theta\sin\theta \\ e^{i\phi}\cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$$

Measurements?

$$\langle\sigma_z\rangle = tr(\rho_\psi\sigma_z)$$

Where..

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

References

Alberto Riccardi, Dariusz Chruściński, and Chiara Macchiavello. "Optimal entanglement wit- nesses from limited local measurements". en. In: Physical Review A 101.6 (June 2020), p. 062319. ISSN: 2469-9926, 2469-9934. DOI: 10.1103/PhysRevA.101.062319. URL: https://link.aps.org/doi/10.1103/PhysRevA.101.062319 (visited on 05/24/2023).

Eritas Yang, Becca Verghese, and Ben Hartley. "Entanglement Witness Writeup". In: Unpublished (July 2022). URL: https://github.com/Lynn-Quantum-Optics/Summer-Spring-2022-3/blob/main/Summer2022/summer- 2022- QO_write_up.pdf.

Jan Roik et al. "Accuracy of Entanglement Detection via Artificial Neural Networks and Human- Designed Entanglement Witnesses". In: Physical Review Applied 15 (May 2021). Publisher: American Physical Society, p. 054006. DOI: 10.1103/PhysRevApplied.15.054006. URL: https://link.aps.org/doi/10.1103/PhysRevApplied.15.054006 (visited on 05/30/2023).

Oscar Scholin, Richard Zheng, Alec Roberson, Theresa W. Lynn, "Entanglement Witnessing: Neural Network Optimization and Experimental Realization". Presented at SQUINT 2023.