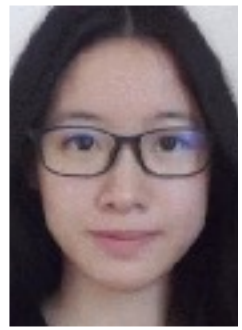
Introducing a New Group of Optimal Entanglement Witnesses

Arianna Meinking, Oscar Scholin, Eritas Yang, Becca Verghese, Ben Hartley, Laney Goldman, Paco Navarro, Theresa W. Lynn

Department of Physics

Acknowledgements



Ben Hartley
W Characterization,
W’ Proposal

Oscar Scholin
W’ Choice
Optimization

Paco Navarro
Current lab

Eritas Yang
W Characterization,
W’ Proposal

Becca Verghese
W Characterization,
W’ Proposal

Theresa W. Lynn
Advisor

Work accompanied by experimental tests by:
Alec Roberson
Richard Cheng
Lev Gruber

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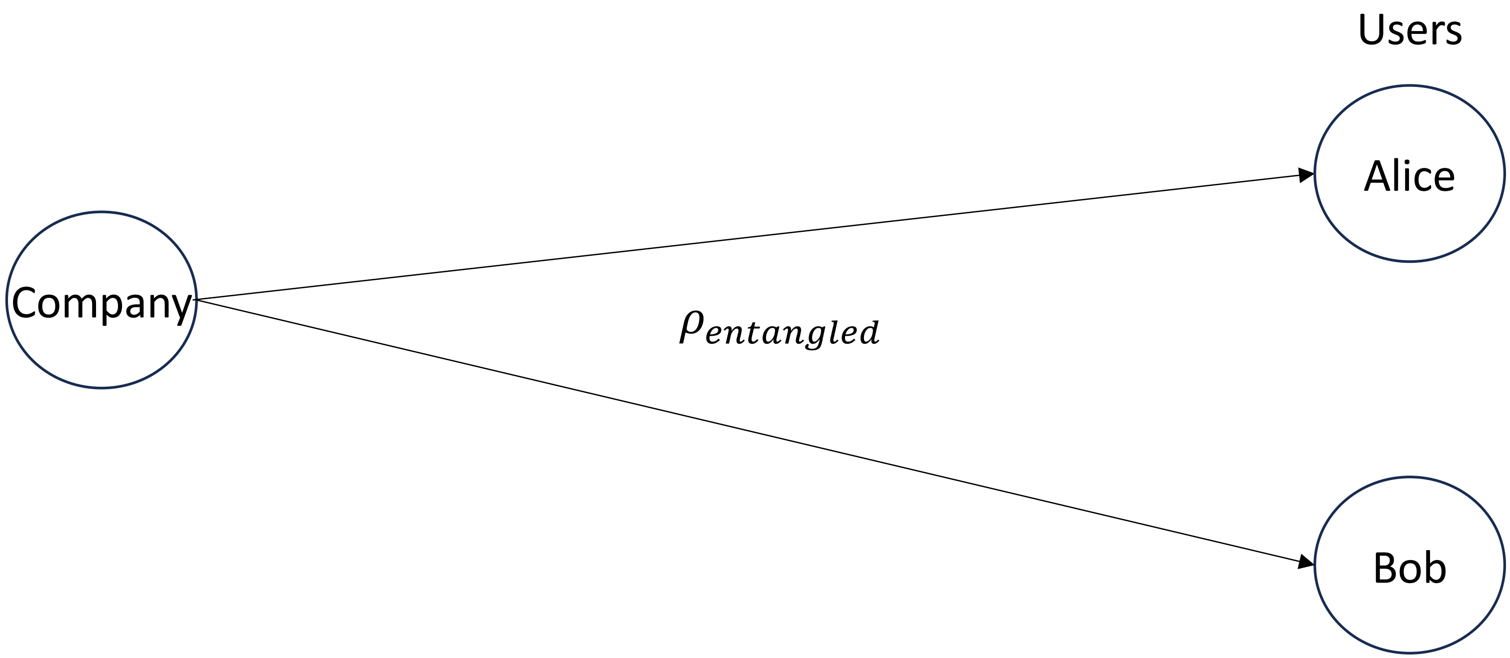
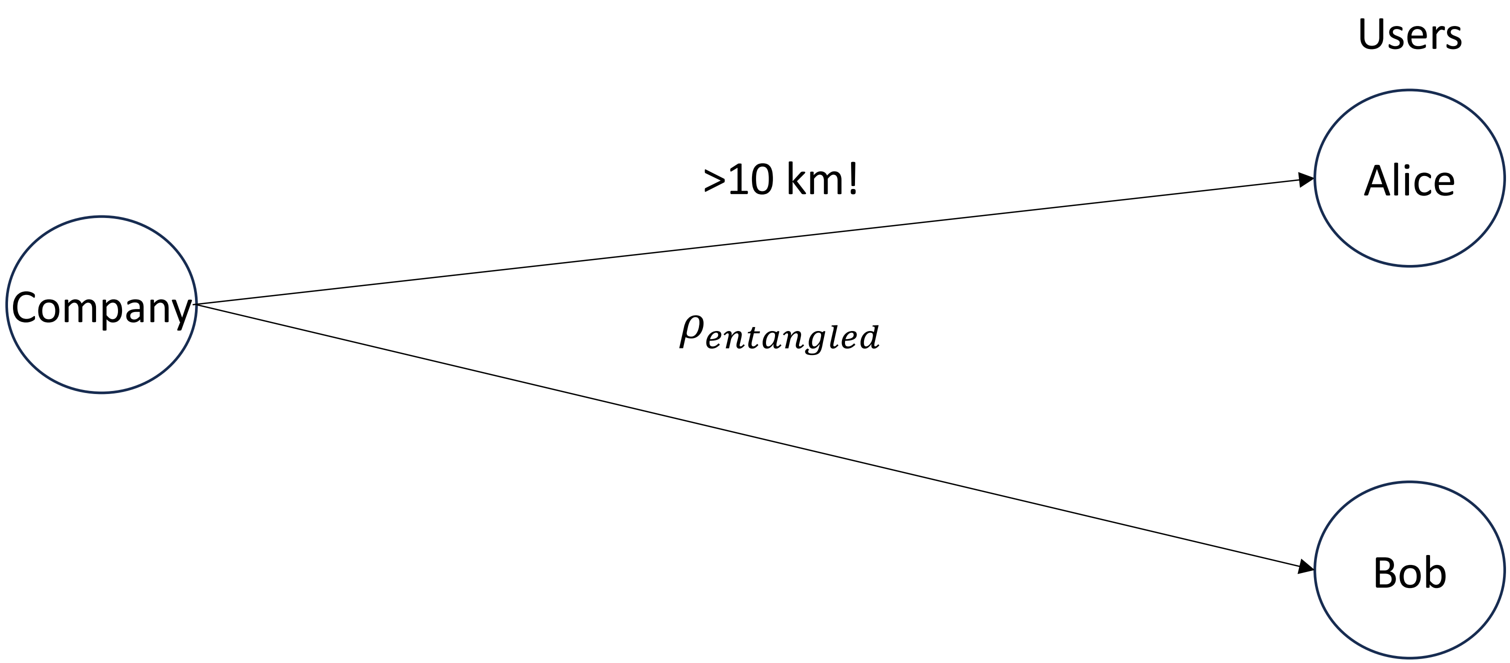
HMC Physics Summer Research Fund
Donnelly Experimental Learning Fund
Vandiver Summer Research Fund

Goal: high chance of witnessing two-qubit entanglement if present from a small fraction of measurements required for full state tomography

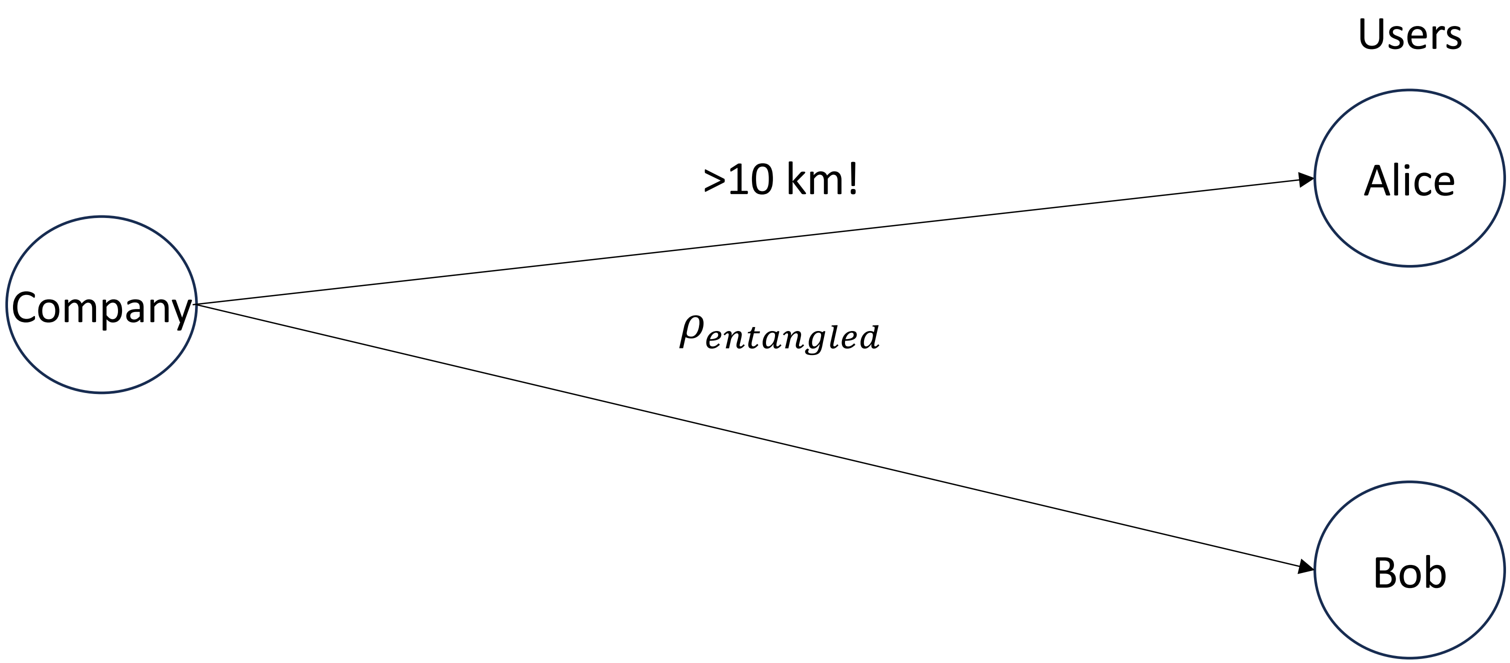
Quantum Communication via ... Entanglement!



Quantum Communication via … Entanglement!



Quantum Communication via … Entanglement!



Liu, X., Hu, J., Li, ZF, et al. *Nature* 10.1038 (2021)

A Solution: Entanglement Witnessing

Quantum Tomography

$$\boldsymbol{\sigma_x} \otimes \mathbb{I} \quad \mathbb{I} \otimes \boldsymbol{\sigma_x} \qquad \boxed{\begin{array}{ccc} \boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_x} & \boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_y} & \boldsymbol{\sigma_x} \otimes \boldsymbol{\sigma_z} \\ \boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_x} & \boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_y} & \boldsymbol{\sigma_y} \otimes \boldsymbol{\sigma_z} \\ \boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_x} & \boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_y} & \boldsymbol{\sigma_z} \otimes \boldsymbol{\sigma_z} \end{array}}$$

$$\boldsymbol{\sigma_y} \otimes \mathbb{I} \quad \mathbb{I} \otimes \boldsymbol{\sigma_y}$$

$$\boldsymbol{\sigma_z} \otimes \mathbb{I} \quad \mathbb{I} \otimes \boldsymbol{\sigma_y}$$

4 measurements each!

9 Pauli combos → 36 measurements!

included

A Solution: Entanglement Witnessing

Entanglement Witnesses

$$
\begin{array}{cc|ccc|}
\cline{3-5}
\boldsymbol{\sigma_x}\otimes\mathbb{I} & \mathbb{I}\otimes\boldsymbol{\sigma_x} & \boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_x} & \boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_y} & \boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_z} \\
\boldsymbol{\sigma_y}\otimes\mathbb{I} & \mathbb{I}\otimes\boldsymbol{\sigma_y} & \boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_x} & \boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_y} & \boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_z} \\
\boldsymbol{\sigma_z}\otimes\mathbb{I} & \mathbb{I}\otimes\boldsymbol{\sigma_y} & \boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_x} & \boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_y} & \boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_z} \\
\cline{3-5}
\end{array}
$$

- $\langle W \rangle < 0$ means entanglement
- Decomposable
  $W = |\varphi_k\rangle\langle\varphi_k|^{\Gamma}$

Riccardi et al., PRA 101, 062319 (2020)

A Solution: Entanglement Witnessing

Entanglement Witnesses

$\boldsymbol{\sigma_x}\otimes\mathbb{I}$	$\mathbb{I}\otimes\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_x}\otimes\boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_y}\otimes\mathbb{I}$	$\mathbb{I}\otimes\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_y}\otimes\boldsymbol{\sigma_z}$
$\boldsymbol{\sigma_z}\otimes\mathbb{I}$	$\mathbb{I}\otimes\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_x}$	$\boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_y}$	$\boldsymbol{\sigma_z}\otimes\boldsymbol{\sigma_z}$

$$W = |\varphi_k\rangle\langle\varphi_k|^\Gamma$$

How about…

$|\varphi_k\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

Then

$$|\varphi_k\rangle\langle\varphi_k| = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|\varphi_k\rangle\langle\varphi_k|^\Gamma = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

Riccardi et al., PRA 101, 062319 (2020)

The Ws

Measurements

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

$$W_1 = \frac{1}{4}\left[\mathbb{I}\otimes\mathbb{I} + \sigma_z\otimes\sigma_z + (a^2 - b^2)\sigma_x\otimes\sigma_x + (a^2 - b^2)\sigma_y\otimes\sigma_y + 2ab(\mathbb{I}\otimes\sigma_z + \sigma_z\otimes\mathbb{I})\right]$$

- Minimize a and b $\rightarrow$ going from a family to just one W
- Riccardi et al. proposed 6 Ws
- Computationally generate random entangled mixed states*
- W_{1-6} detect 33% miss 67% of those states

Riccardi et al., PRAppl. 101, 062319 (2020)
*Random state generation following method used in Roik et al., PRAppl. 15.054006(2021)

The W's

Measurements

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

$$
\begin{aligned}
W'_1 = \frac{1}{4}\Big[&\mathbb{I}\otimes\mathbb{I} + \sigma_z\otimes\sigma_z + 2\cos 2\theta\,(\sigma_x\otimes\sigma_x + \sigma_y\otimes\sigma_y) \\
&+ 2\sin 2\theta\cos\alpha\;(\mathbb{I}\otimes\sigma_z + \sigma_z\otimes\mathbb{I}) \\
&+ 2\sin 2\theta\sin\alpha\;(\sigma_x\otimes\sigma_y + \sigma_y\otimes\sigma_x)\Big]
\end{aligned}
$$

- Now, minimize θ and α
- Mixed Pauli pairs!
- Subgroups
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}

Yang, Verghese, Hartley. HMC Quantum Optics unpublished, https://github.com/Lynn-Quantum-Optics/Summer-Spring-2022-3/blob/main/Summer2022/ summer- 2022- QO_write_up.pdf

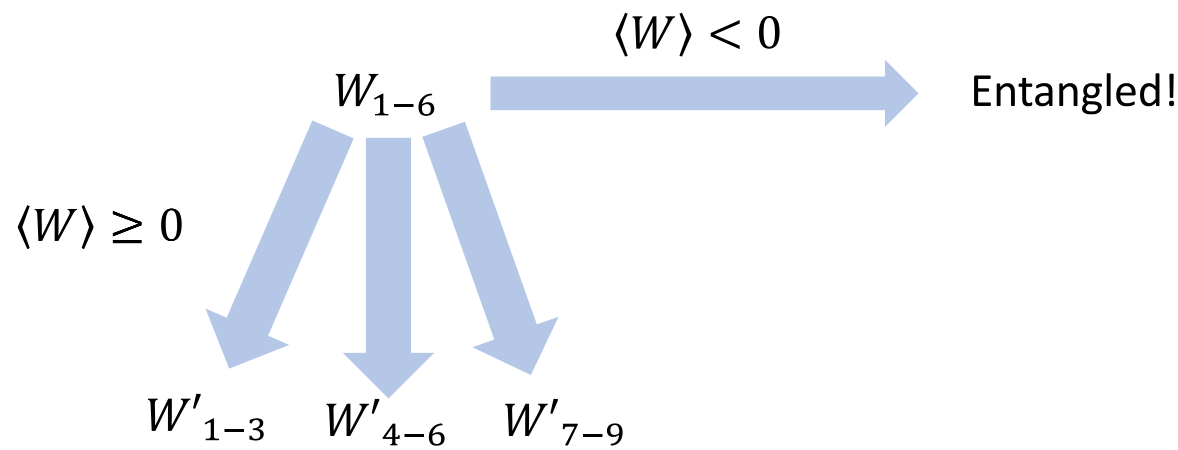
The Two-Step Process

ρ_{ent}?

$$W_{1-6} \xrightarrow{\langle W \rangle < 0} \text{Entangled!}$$

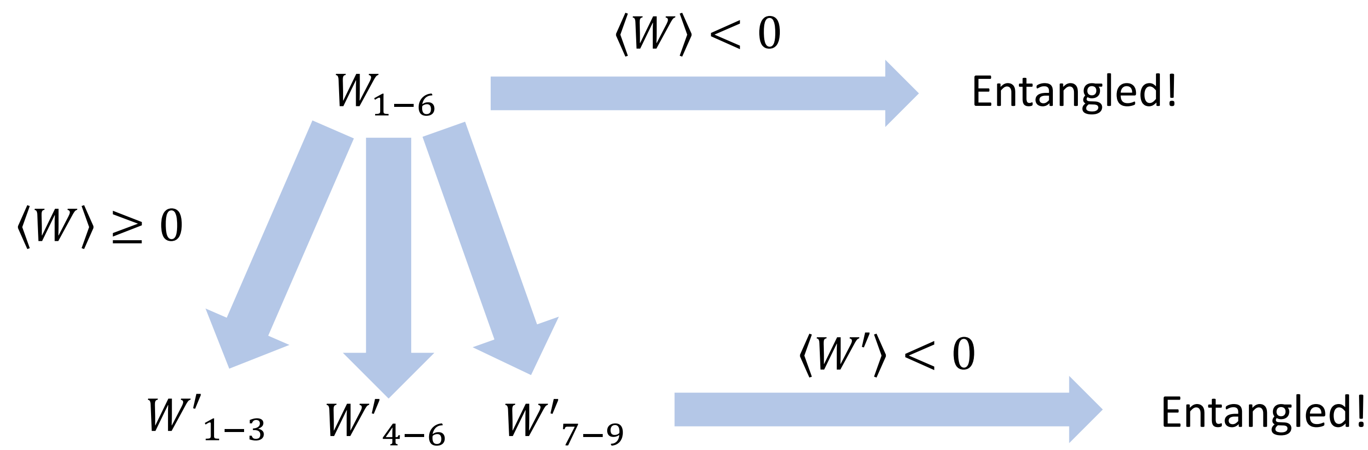
The Two-Step Process

ρ_{ent}?



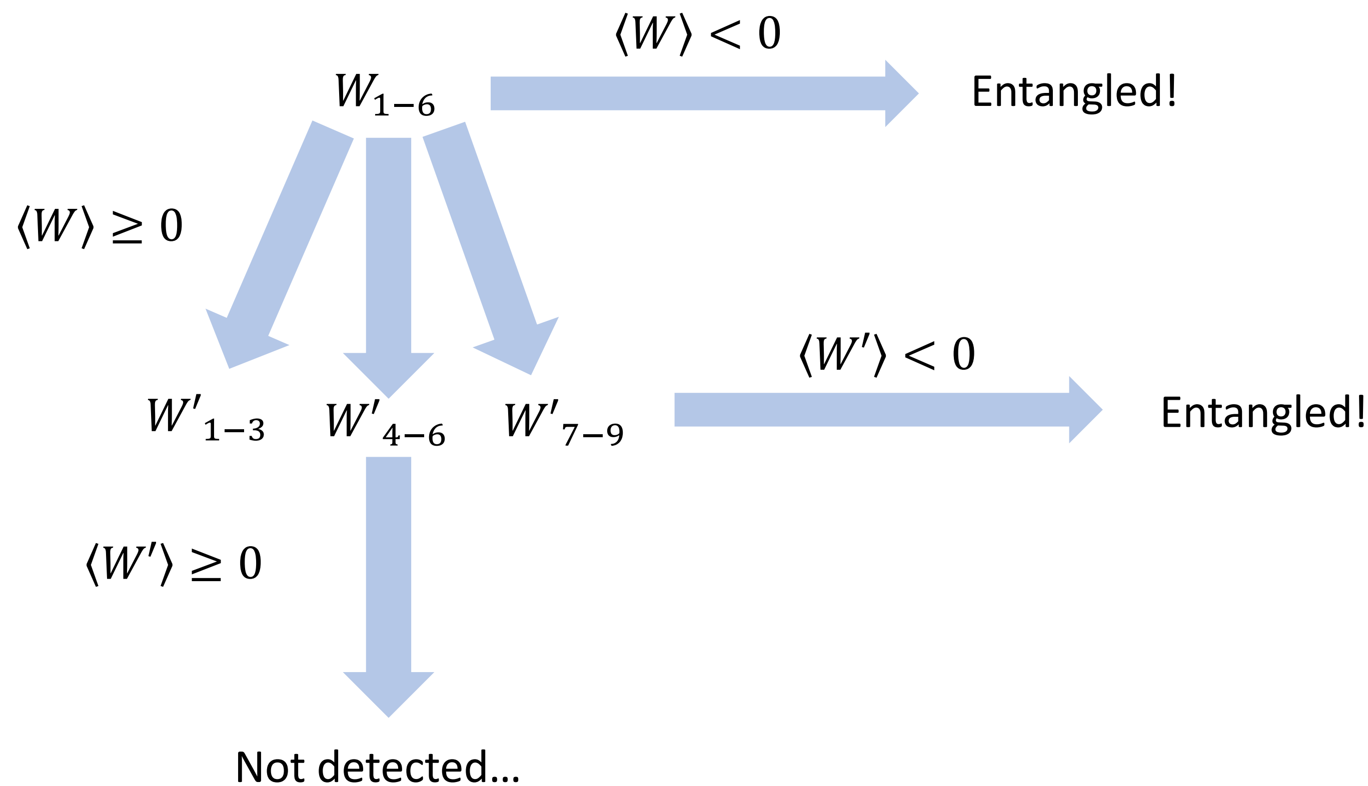
The Two-Step Process

$\rho_{ent}?$



The Two-Step Process

ρ_{ent}?



The Two-Step Process

ρ_{ent}?

W_{1-6} $\xrightarrow{\langle W \rangle < 0}$ Entangled!

$\langle W \rangle \geq 0$

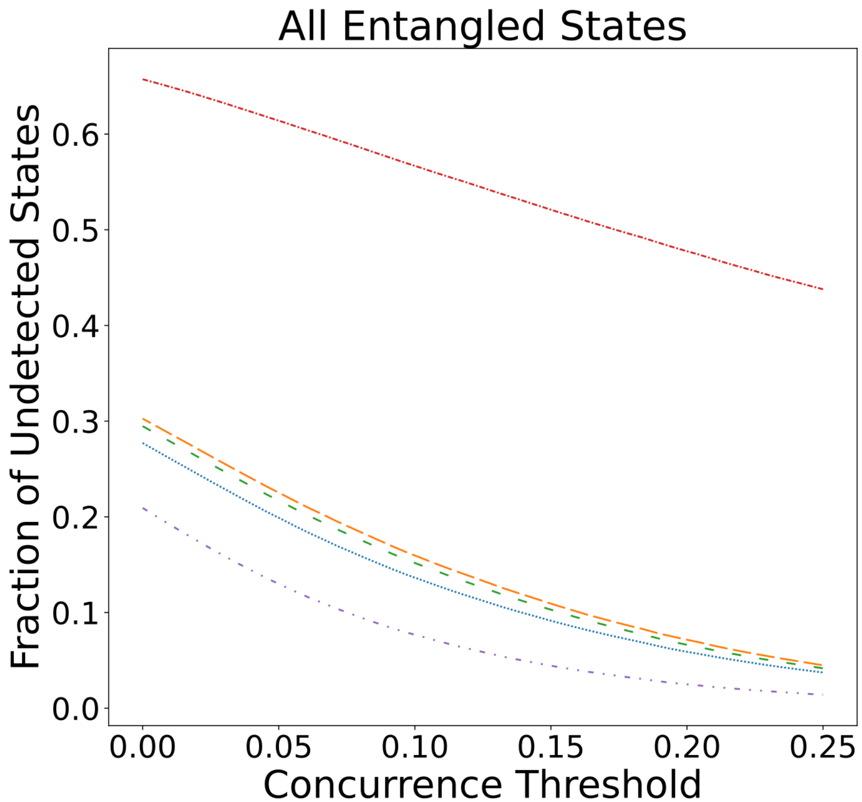
(neural network) W'_{1-3} W'_{4-6} W'_{7-9} $\xrightarrow{\langle W' \rangle < 0}$ Entangled!

$\langle W' \rangle \geq 0$

Not detected…

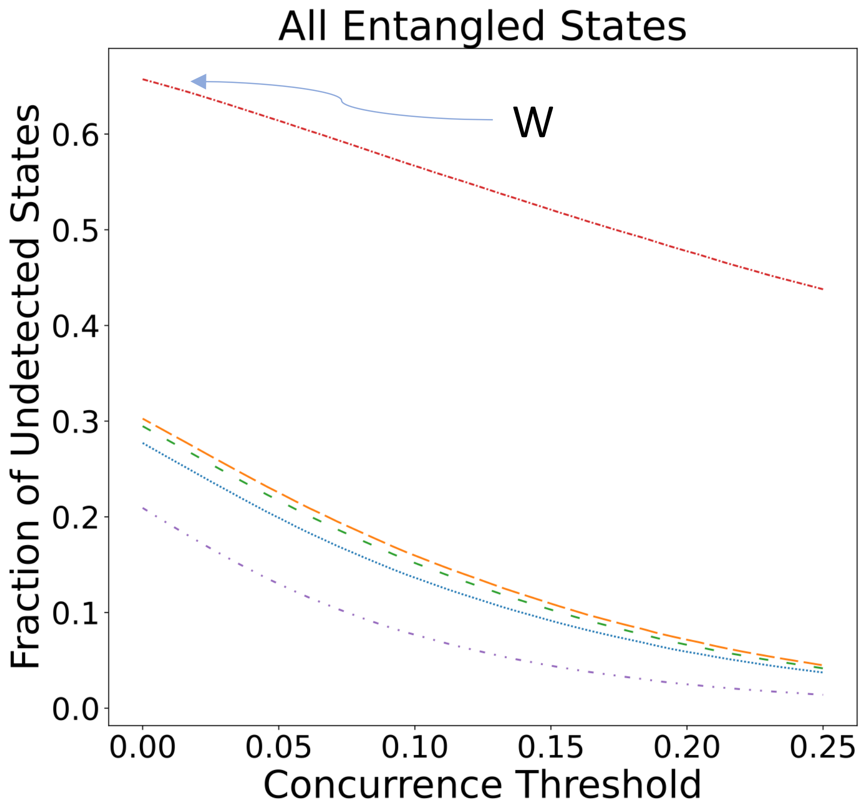
Yang, Verghese, Scholin, Unpublished

Two-Step Process Performance



Scholin, Zheng, Roberson, Lynn, Unpublished

Two-Step Process Performance



Scholin, Zheng, Roberson, Lynn, Unpublished

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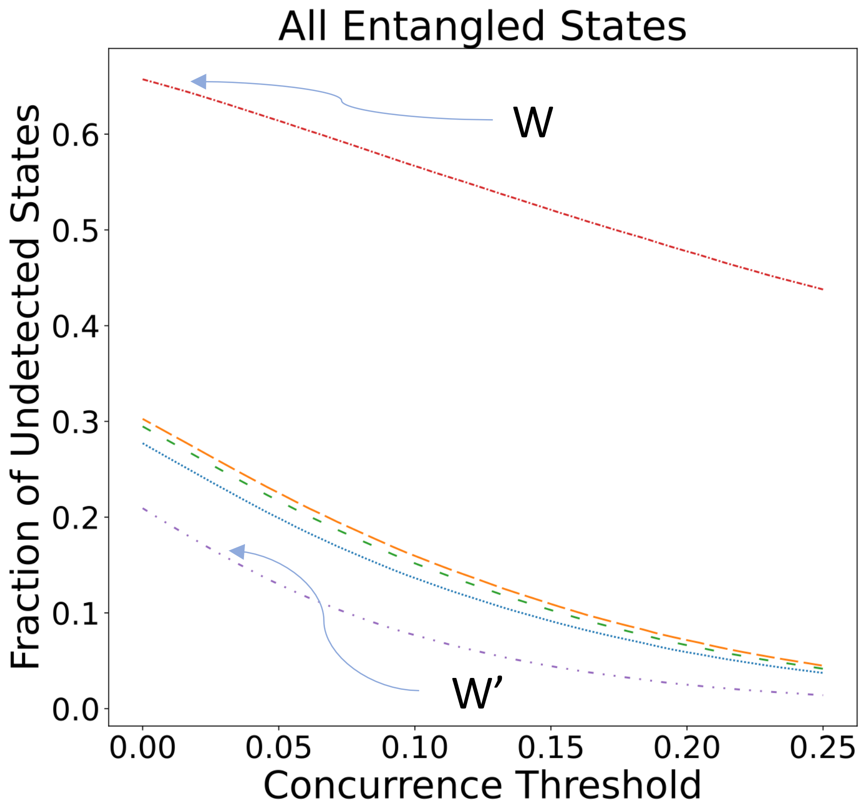

Scholin, Zheng, Roberson, Lynn, Unpublished

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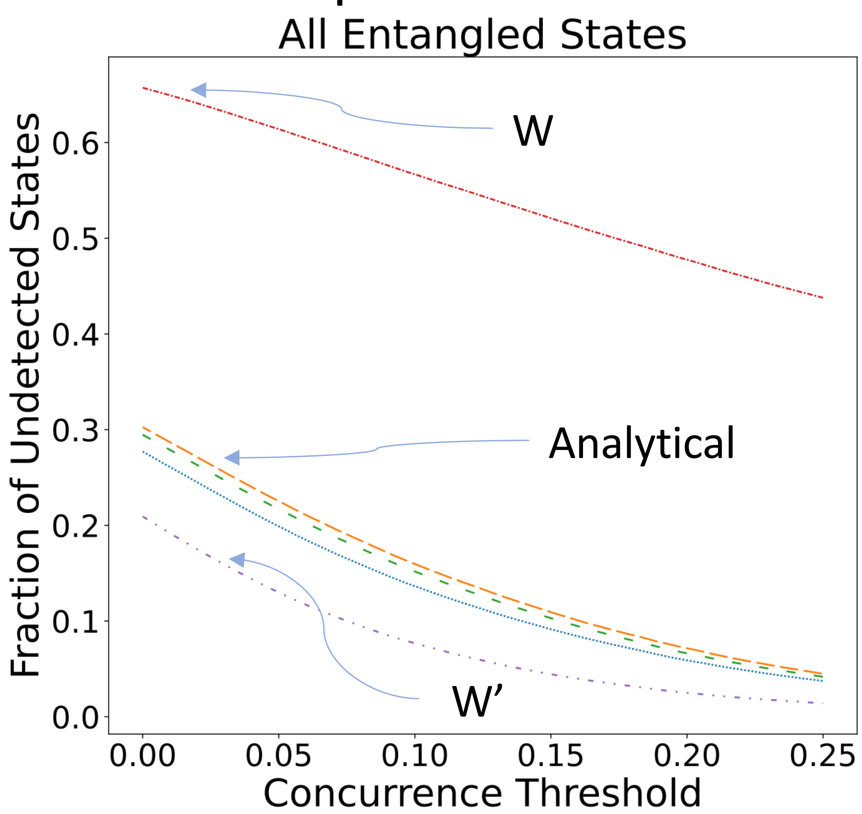

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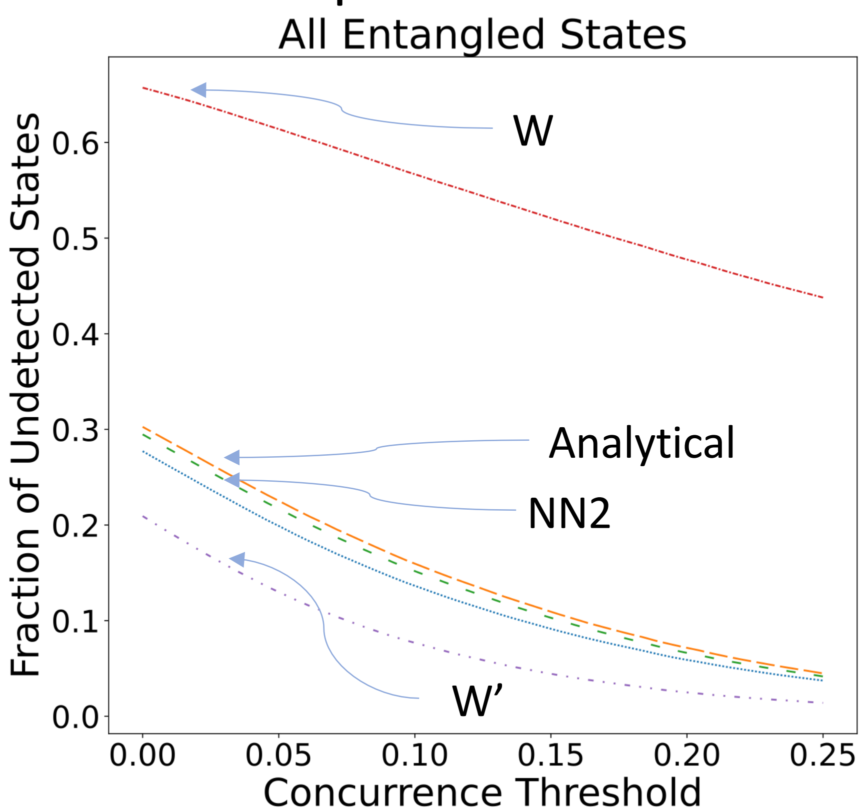
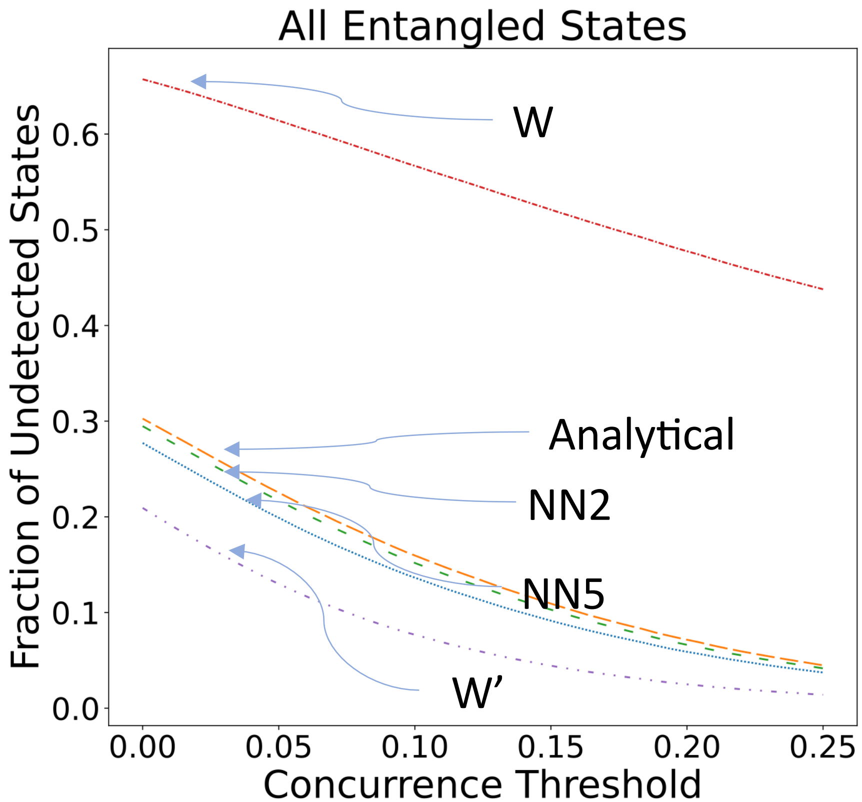


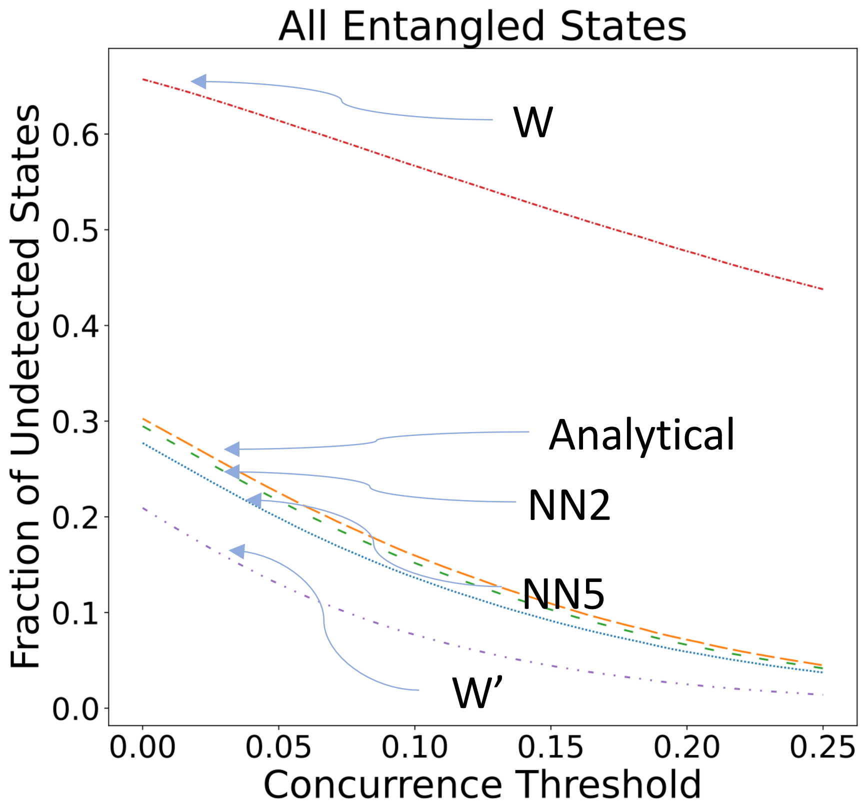
Scholin, Zheng, Roberson, Lynn, Unpublished

Two-Step Process Performance



Scholin, Zheng, Roberson, Lynn, Unpublished

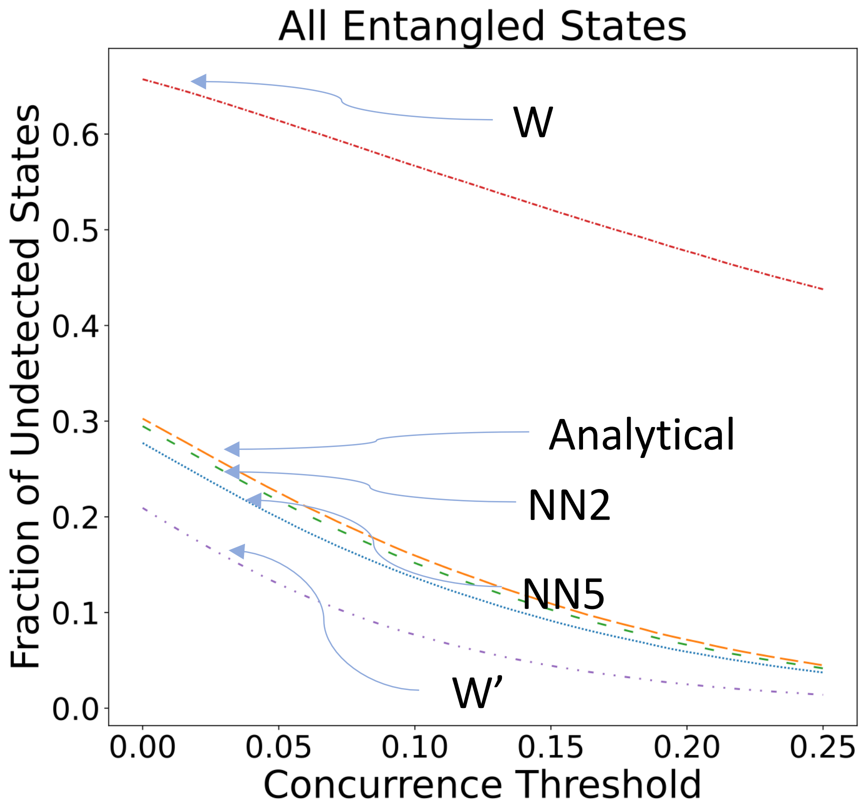
Two-Step Process Performance



5/9 of the measurements witness 2/3 of the states!

Scholin, Zheng, Roberson, Lynn, Unpublished

Two-Step Process Performance



5/9 of the measurements witness 2/3 of the states!

More work to be done…

Scholin, Zheng, Roberson, Lynn, Unpublished

Solving the Problem: Expand the W’ Options? (In Progress)

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

- Another subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
  - W'_{10-12}?

Expanding the W' Options... Impossible!

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\cancel{\boldsymbol{\sigma_x \otimes \sigma_z}}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\cancel{\boldsymbol{\sigma_y \otimes \sigma_z}}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

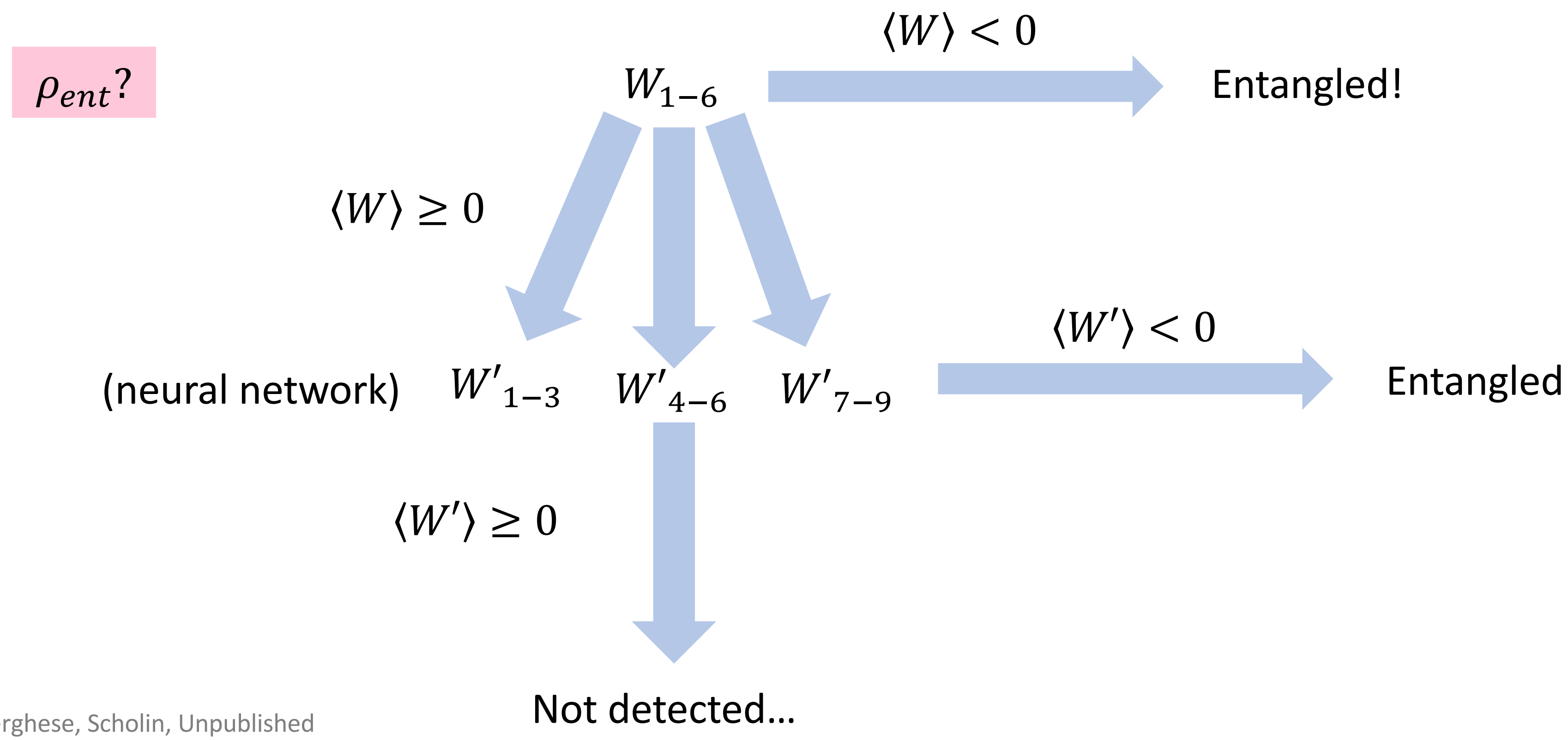
- Another subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
  - W'_{10-12}?
- The $\boldsymbol{\sigma_i \otimes \sigma_j}$ groups must come with a paired $\boldsymbol{\sigma_j \otimes \sigma_i}$
- $W = |\varphi_k\rangle\langle\varphi_k|^{\Gamma}$

The Three-Step Process

$\boldsymbol{\sigma_x \otimes \sigma_x}$	$\boldsymbol{\sigma_x \otimes \sigma_y}$	$\boldsymbol{\sigma_x \otimes \sigma_z}$
$\boldsymbol{\sigma_y \otimes \sigma_x}$	$\boldsymbol{\sigma_y \otimes \sigma_y}$	$\boldsymbol{\sigma_y \otimes \sigma_z}$
$\boldsymbol{\sigma_z \otimes \sigma_x}$	$\boldsymbol{\sigma_z \otimes \sigma_y}$	$\boldsymbol{\sigma_z \otimes \sigma_z}$

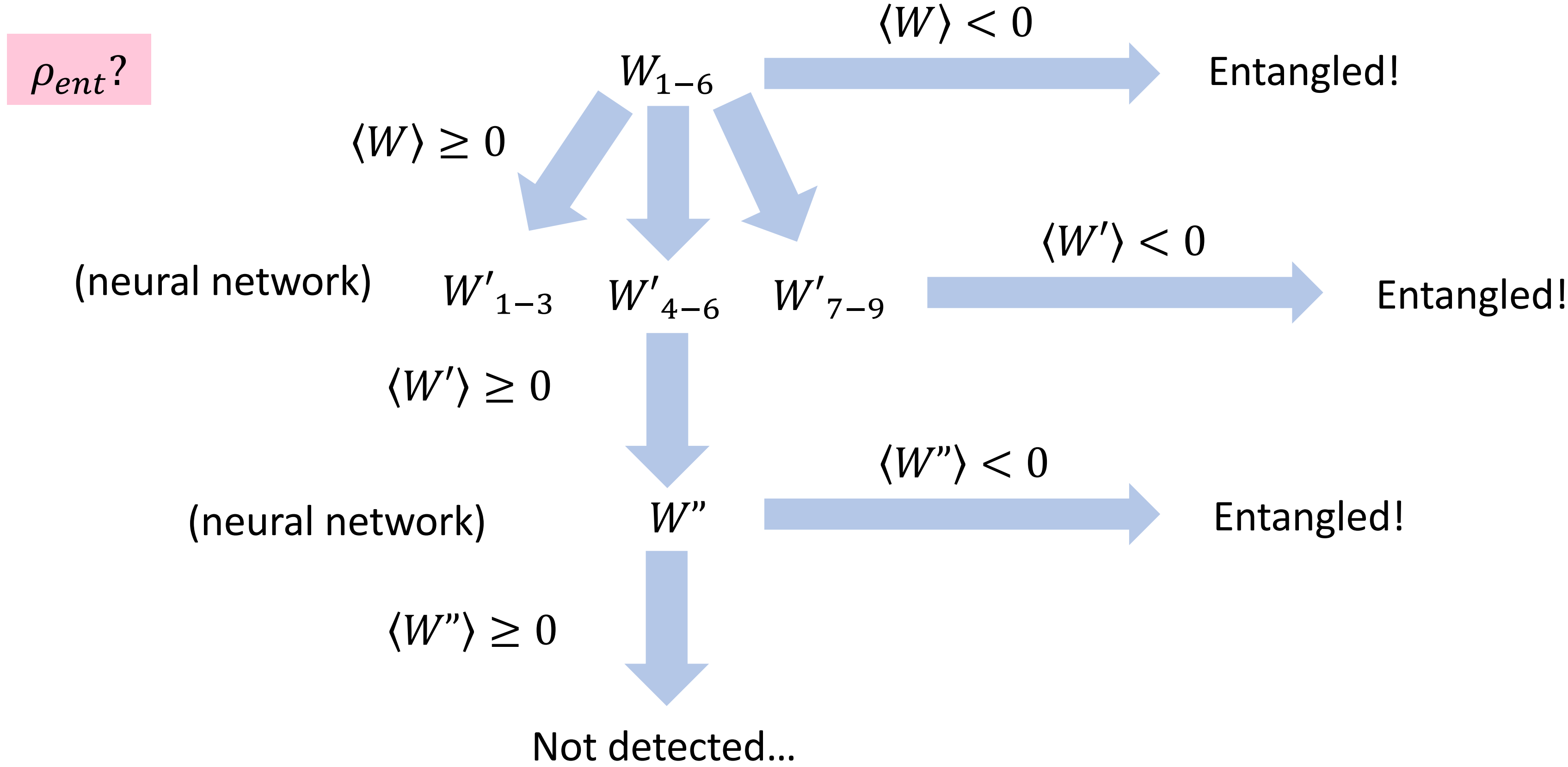
- One subgroup
  - W'_{1-3}
  - W'_{4-6}
  - W'_{7-9}
- Now W", just one additional measurement!
  - $W'_{4-6} + W''_{10-12}$
  - One extra measurement, or 3 measurements together, may be useful

The Two-Step Process



Yang, Verghese, Scholin, Unpublished

The Three-Step Process



Key Takeaways

Remember…

Goal: high chance of witnessing two-qubit entanglement if present from a small fraction of measurements required for full state tomography

Our work!

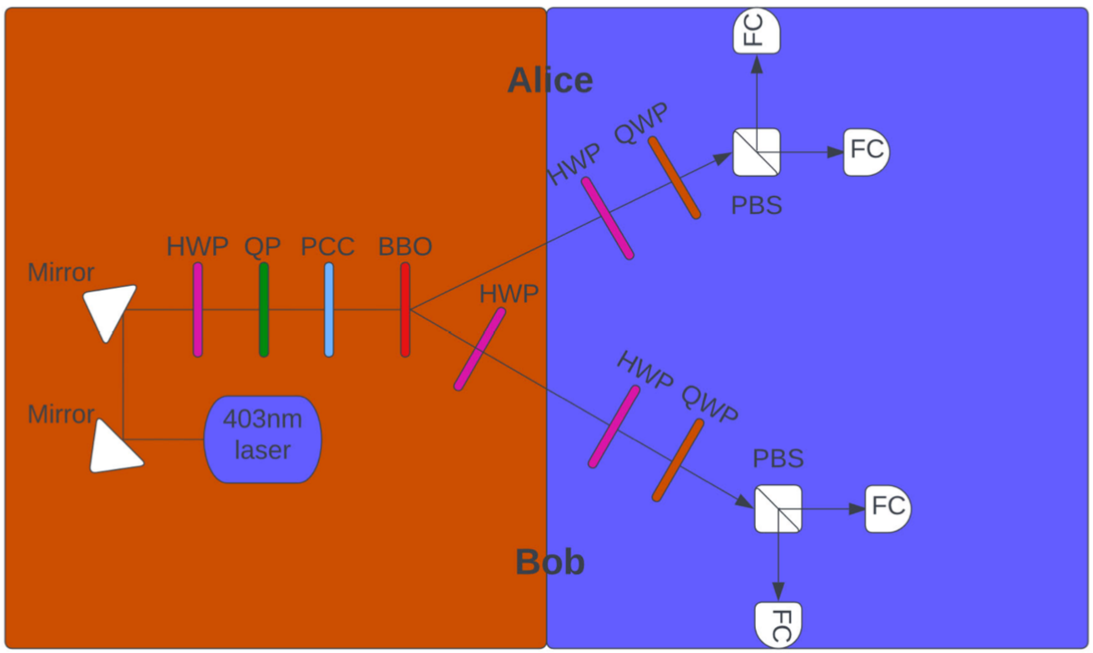
- W’ witness 2/3 states with 5/9 measurements
- W’ extension might improve this ratio

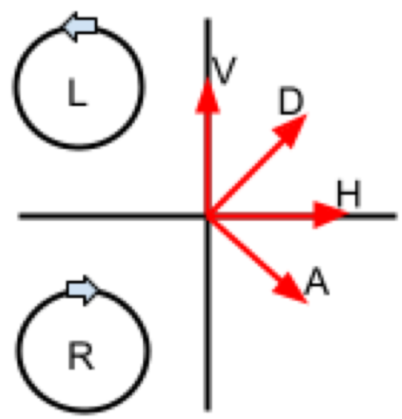
W" impossible (backup)

$$\begin{bmatrix} 1 & e^{-i\varphi} & e^{-i\alpha} & e^{-i\beta} \\ e^{i\varphi} & 1 & e^{i(\varphi-\alpha)} & e^{i(\varphi-\beta)} \\ e^{i\alpha} & e^{i(\alpha-\varphi)} & 1 & e^{i(\alpha-\beta)} \\ e^{i\beta} & e^{i(\beta-\varphi)} & e^{i(\beta-\alpha)} & 1 \end{bmatrix}$$

$$|HH\rangle + e^{i\varphi}|HV\rangle + e^{i\alpha}|VH\rangle + e^{i\beta}|VV\rangle$$

Experimental Apparatus (backup)





References

Alberto Riccardi, Dariusz Chruściński, and Chiara Macchiavello. “Optimal entanglement wit- nesses from limited local measurements”. en. In: Physical Review A 101.6 (June 2020), p. 062319. ISSN: 2469-9926, 2469-9934. DOI: 10.1103/PhysRevA.101.062319. URL: https://link.aps.org/doi/10.1103/PhysRevA.101.062319 (visited on 05/24/2023).

Eritas Yang, Becca Verghese, and Ben Hartley. “Entanglement Witness Writeup”. In: Unpublished (July 2022). URL: https://github.com/Lynn-Quantum-Optics/Summer-Spring-2022-3/blob/main/Summer2022/summer- 2022- QO_write_up.pdf.

Jan Roik et al. “Accuracy of Entanglement Detection via Artificial Neural Networks and Human- Designed Entanglement Witnesses”. In: Physical Review Applied 15 (May 2021). Publisher: American Physical Society, p. 054006. DOI: 10.1103/PhysRevApplied.15.054006. URL: https://link.aps.org/doi/10.1103/PhysRevApplied.15.054006 (visited on 05/30/2023).

Oscar Scholin, Richard Zheng, Alec Roberson, Theresa W. Lynn, “Entanglement Witnessing: Neural Network Optimization and Experimental Realization”. Presented at SQUINT 2023.