

Efficient Adaptive Entanglement Witnessing

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Introduction

Quantum cryptography utilizes quantum entanglement to secure data against the threat of quantum computing. Detecting entanglement is necessary for secure and efficient long-distance quantum communication [2]. For example, in Figure 1 if Alice and Bob received particles that aren't entangled, then the information they exchange will be insecure and maybe unusable. Entanglement witnessing is an efficient method of detecting entanglement locally without requiring a full tomography.

Entanglement witnesses are operators that act on a quantum state and have a negative expectation value iff the state is entangled. Previous work [1] developed 6 witnesses, $\{W\}$, that use the measured probabilities of 12 polarization pairs. Our group previously expanded this set with three groups of witnesses, $\{W'\}$, doubling the number of detected entangled states with an additional 8 measurements. Our work experimentally verifies $\{W'\}$ and expands conceptual understanding.

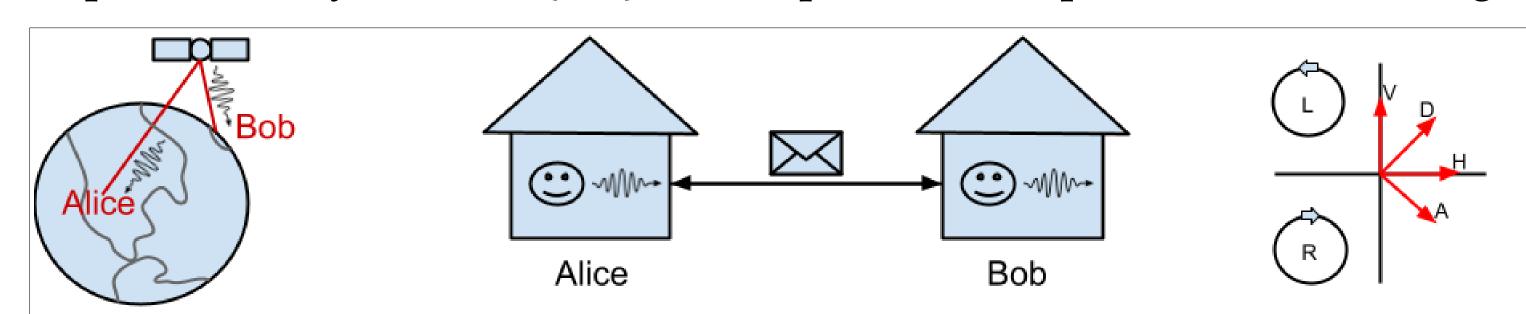


Figure 1: Alice and Bob each receive photons distributed from a source, in this case, a satellite. They separately measure their photons in polarizations of their choice: H,V,D,A,L,R. By classically communicating the measurement results on many copies of the two-photon state, Alice and Bob can verify that the source provides entangled photons.

Adaptive Witnessing Procedure

Figure 2A shows the flow of our adaptive witnessing procedure. Previous group members developed two models for adaptive choice if $\{W\}$ do not witness entanglement:

- The population model is an analytic model that uses 6 of the projective probabilities $p_{\alpha\alpha}$ already measured to determine $\{W\}$; we choose a $\{W'\}$ group based on the quantities $0.5 (p_{\alpha\alpha} + p_{\alpha_{\perp}\alpha_{\perp}})$ for $\alpha = H$, D, and R.
- The choice can also be made from the measurements done for $\{W\}$ using an artificial five-layer neural network trained on 4.4 million computationally generated entangled states. This method succeeds slightly more than the population model, as seen in Figure 2B.

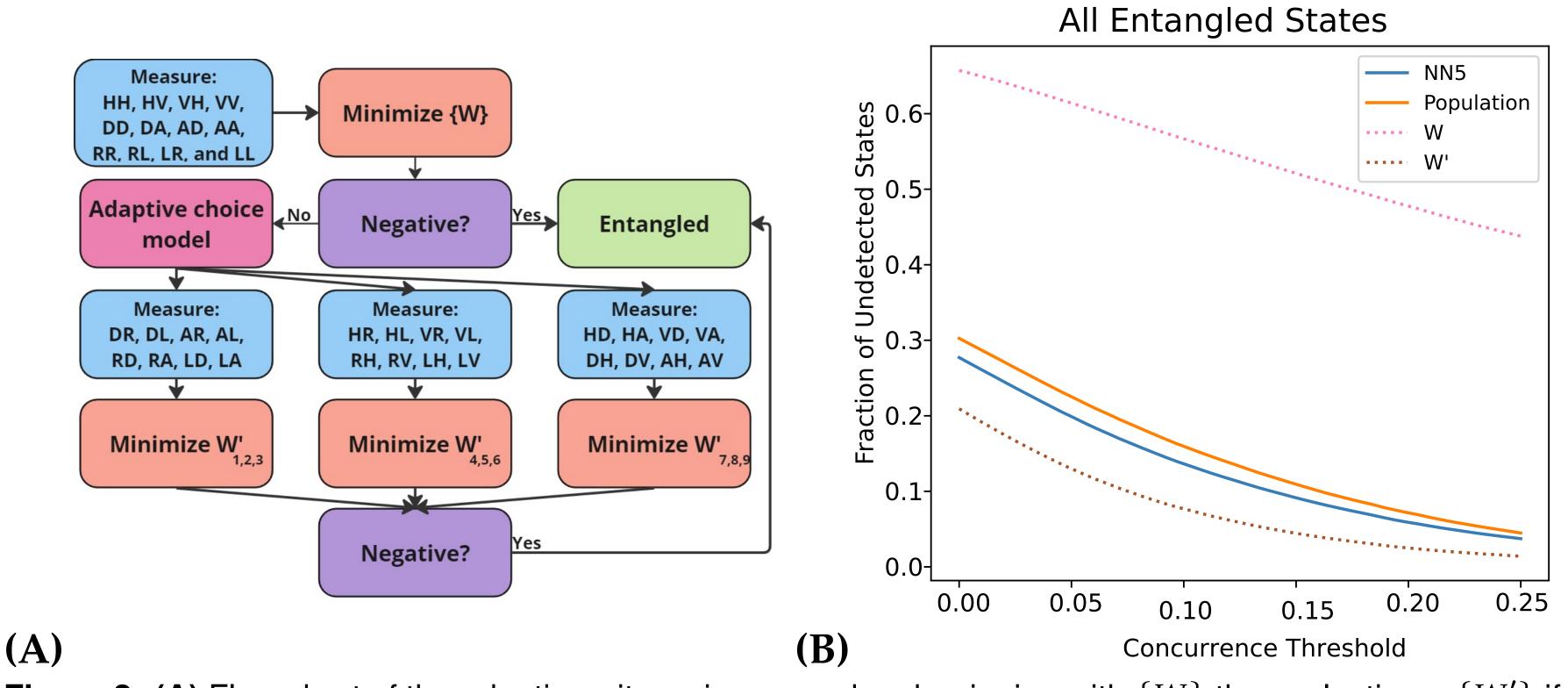


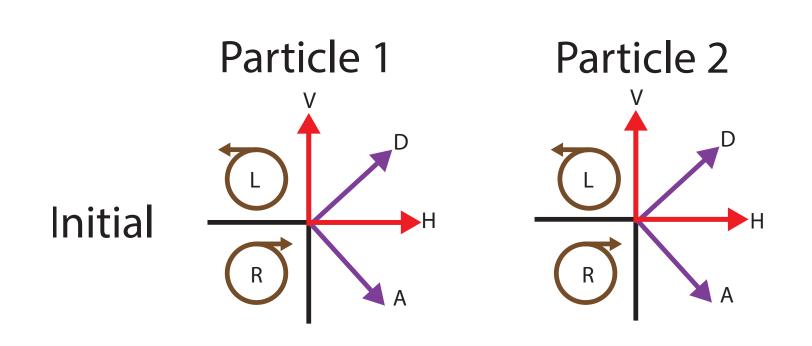
Figure 2: (A) Flow chart of the adaptive witnessing procedure beginning with $\{W\}$ then selecting a $\{W'\}$ if entanglement is not detected. **(B)** Comparison of fraction of computationally generated random entangled states witnessed by $\{W\}$, all W', and the W' triplet chosen by each adaptive method. $\{W\}$ witnesses 34.27% of all the entangled states and $\{W'\}$ detects 79.07%. The five-layer neural network (NN5) detected 72.28% of the states and the population model detected 69.74%. Detected fractions are shown as a function of minimum concurrence (degree of entanglement) in the states considered.

Geometric Witness Construction

The $\{W'\}$ witnesses are rotations of either or both of the photons about a given axis for the $\{W\}$ witnesses.

- $W'_{1\to 3}$ rotates about the H/V axis.
- $W'_{4\to 6}$ rotates about the D/A axis.
- $W'_{7\rightarrow 9}$ rotates about the R/L axis.

Using this framework, we can construct new sets of witnesses based on rotations of previous sets of witnesses. Additionally, this allows us to systematically introduce measurements by rotating the particles.



Rotate Particle 1 Around Y Axis

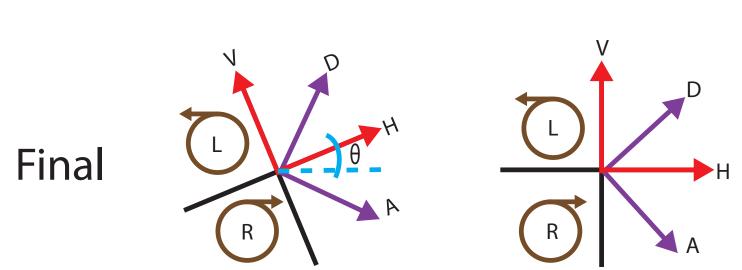


Figure 3: Rotating particle 1 about the y axis by and angle θ . With colors representing measurement pairs.

Experimental Implementation

Our experiment uses adjustable optical components to create and detect several entangled states. The setup is shown in Figure 4.

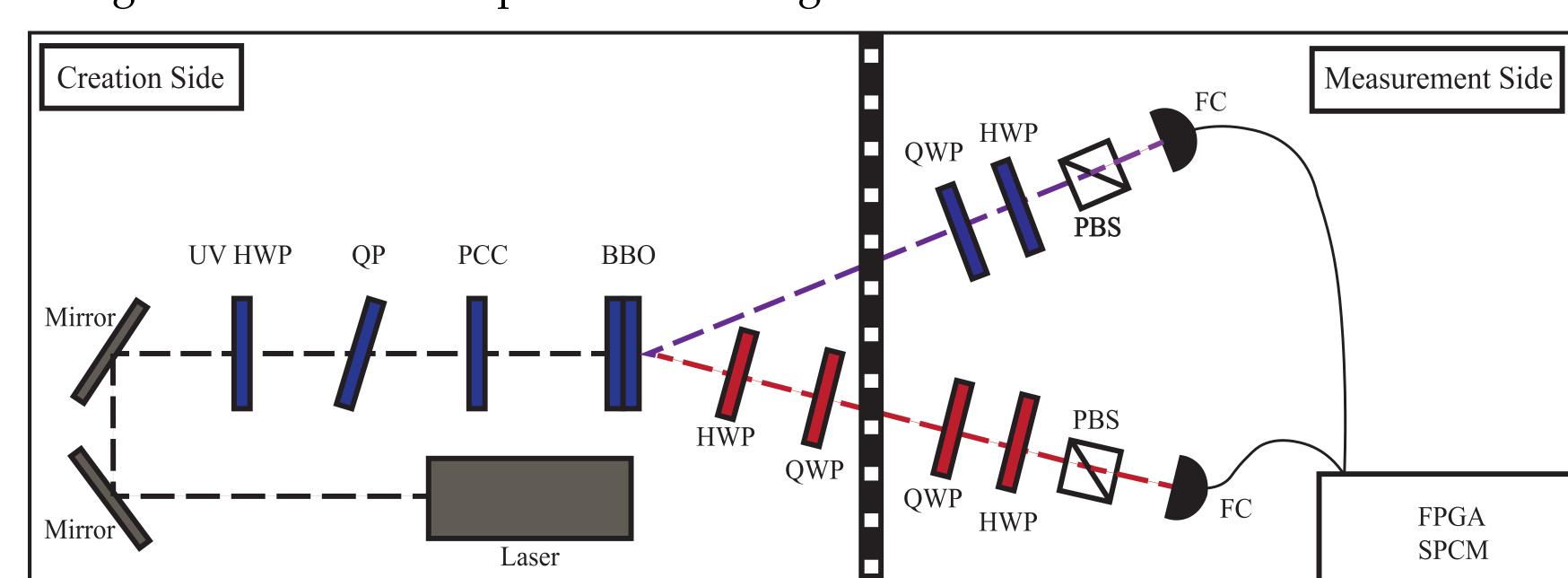
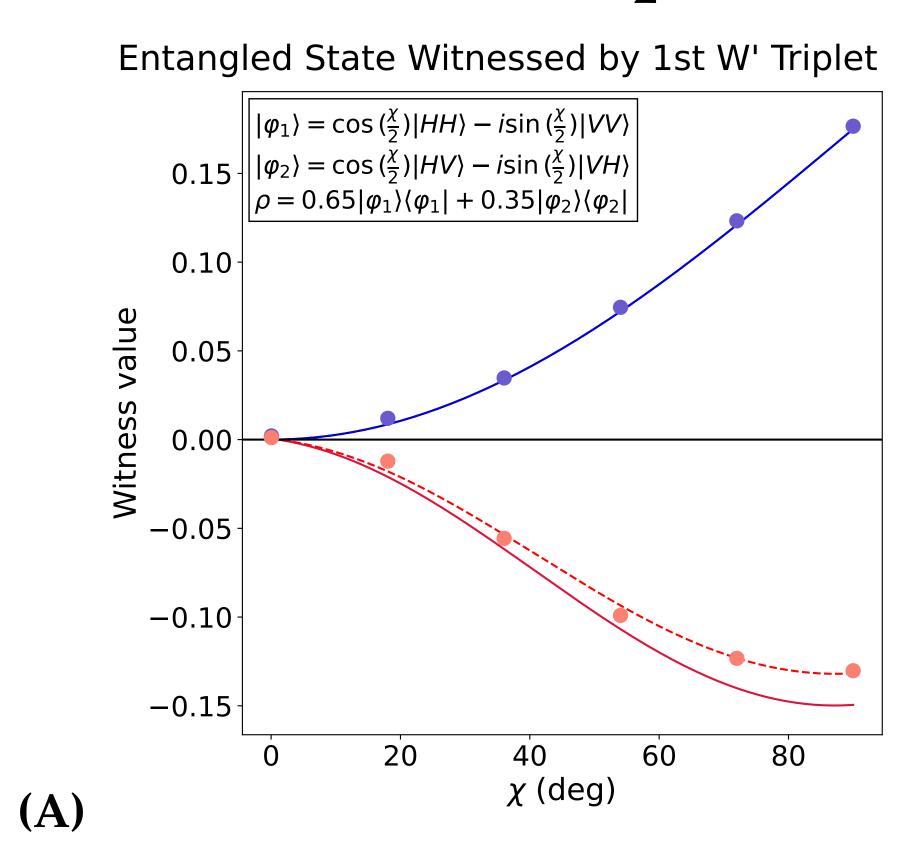
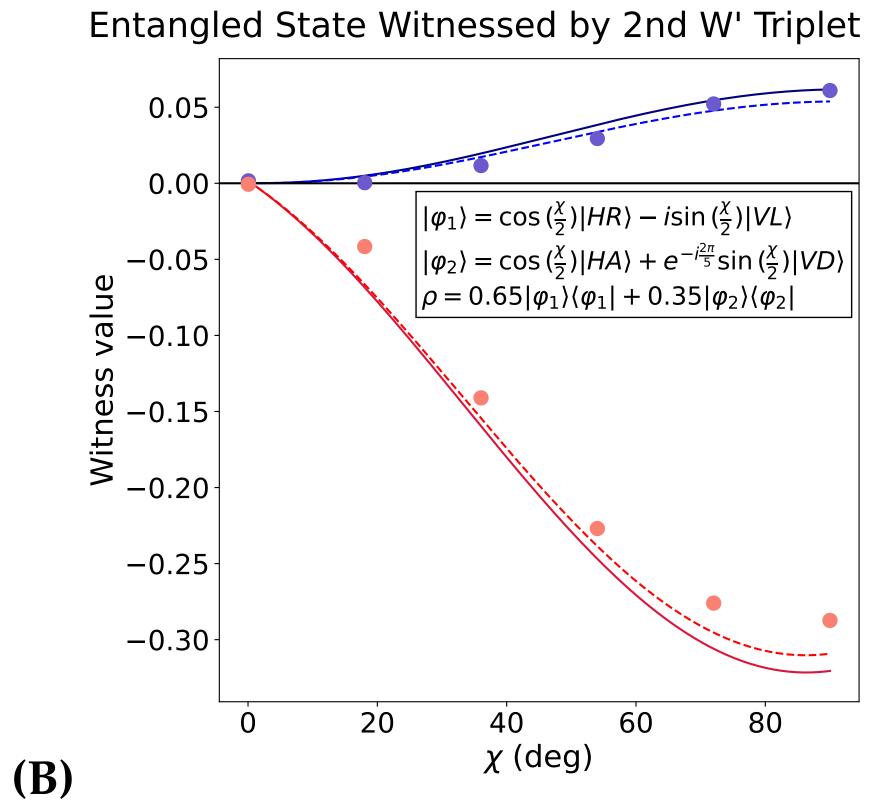


Figure 4: Entangled state creation occurs to the left of the vertical dashed line, while measurement occurs to the right. Measurement occurs in separate channels: Alice (top) and Bob (bottom). Creation components: $403\,\mathrm{nm}$ Diode Laser (laser), UVHWP (ultraviolet half wave plate), QP (quartz plate), PCC (precompensation crystal), BBO (β-barium borate crystal), QWP (quarter wave plate), HWP (half wave plate). Measurement components: QWP, HWP, PBS (polarizing beam splitter), FC (optical fiber coupler), SPCM + FPGA: (single-photon counting module).

Our experimental setup can create states of the form

$$|\varphi\rangle = \cos\left(\frac{\chi}{2}\right)|H\rangle_A|\alpha\rangle_B + e^{i\eta}\sin\left(\frac{\chi}{2}\right)|V\rangle_A|\alpha_\perp\rangle_B.$$
 (1)





We experimentally demonstrated three entangled states, each minimizing a $\{W'\}$ witness from a different triplet. Each state is a probabilistic mixture of two pure states as in equation 1 but with different $|\alpha\rangle$.

The experimental detection of these states by each $\{W'\}$ triplet when it was missed by $\{W\}$ illustrates that our adaptive procedure improves the success of entanglement detection.

The population model and neural net correctly predicted which $\{W'\}$ witness triplet to measure post- $\{W\}$ measurement in each experimental case, further demonstrating the usefulness of our adaptive protocol.

Entangled State Witnessed by 3rd W' Triplet $0.15 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HD\rangle - \sin(\frac{\chi}{2})|VA\rangle}{|\varphi_2\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle} = 0.10$ $0.05 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HD\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HD\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle - \sin(\frac{\chi}{2})|VL\rangle}{0.005} = 0.65|\varphi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|HR\rangle} - \frac{1}{|\varphi_1\rangle = 0.05}|\Psi_1\rangle\langle \varphi_1| + 0.35|\varphi_2\rangle\langle \varphi_2|$ $0.10 = \frac{1}{|\varphi_1\rangle = \cos(\frac{\chi}{2})|\Psi_1\rangle} + \frac{1}{|\varphi_1\rangle = 0.05}|\Psi_1\rangle\langle \varphi_1| + 0.35|\varphi_1\rangle\langle \varphi_2|$

Figure 5: Plots of states witnessed by $\{W'\}$ and not

by $\{W\}$. The solid blue lines represent the theoreti-

cal minimized value of $\{W\}$ for the target states, the

solid red lines represent the theoretical minimum of

References

[1] A. Riccardi, D. Chruściński, and C. Macchiavello, *Phys. Rev. A* **101**, 062319 (2020).

[2] Yin, Juan, et al. *Nature* 582.7813 (2020): 501-505.

$\{W'\}$, and the dotted line represents our adjusted theory. The adjusted theory accounts for our experimental setup being unable to create 100% pure states. (A), (B), (C), show experimental data of mixed states, each witnessed by a different $\{W'\}$ triplet.

Acknowledgments

We thank all previous members of Lynn Lab who contributed to the development of the $\{W'\}$ witnesses. We also thank Professor Theresa Lynn for her hours of support and vast knowledge. The HMC Physics Summer Research Fund and Campbell Summer Research funded this work.