CHAPTER 1

Sets

All of mathematics can be described with sets. This becomes more and more apparent the deeper into mathematics you go. It will be apparent in most of your upper level courses, and certainly in this course. The theory of sets is a language that is perfectly suited to describing and explaining all types of mathematical structures.

1.1 Introduction to Sets

A **set** is a collection of things. The things are called **elements** of the set. We are mainly concerned with sets whose elements are mathematical entities, such as numbers, points, functions, etc.

A set is often expressed by listing its elements between commas, enclosed by braces. For example, the collection $\{2,4,6,8\}$ is a set which has four elements, the numbers $2,4,6$ and 8. Some sets have infinitely many elements. For example, consider the collection of all integers,

$$\{\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots\}.$$

Here the dots indicate a pattern of numbers that continues forever in both the positive and negative directions. A set is called an **infinite** set if it has infinitely many elements; otherwise it is called a **finite** set.

Two sets are **equal** if they contain exactly the same elements. Thus $\{2,4,6,8\} = \{4,2,8,6\}$ because even though they are listed in a different order, the elements are identical; but $\{2,4,6,8\} \neq \{2,4,6,7\}$. Also

$$\{\ldots-4,-3,-2,-1,0,1,2,3,4\ldots\} = \{0,-1,1,-2,2,-3,3,-4,4,\ldots\}.$$

We often let uppercase letters stand for sets. In discussing the set $\{2,4,6,8\}$ we might declare $A = \{2,4,6,8\}$ and then use A to stand for $\{2,4,6,8\}$. To express that 2 is an element of the set A, we write $2 \in A$, and read this as "2 *is an element of* A," or "2 *is in* A," or just "2 *in* A." We also have $4 \in A$, $6 \in A$ and $8 \in A$, but $5 \notin A$. We read this last expression as "5 *is not an element of* A," or "5 *not in* A." Expressions like $6,2 \in A$ or $2,4,8 \in A$ are used to indicate that several things are in a set.