Modern Algebra Homework 1

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2.4

4

For $n = 1$, $(3(1) - 2)n = \frac{(1)(3(1)-1)x}{2}$, so the base case holds. Now suppose that $\sum_1^n (3n-2)x = \frac{n(3n-1)x}{2}$. We shall prove that for $k = n + 1$, it still holds.

$$\sum_1^k (3k-2)x = x + 4x + 7x + \cdots + (3n-2)x + (3k-2)x = \frac{k(3k-1)x}{2}$$
$$x + 4x + 7x + \cdots + (3n-2)x + (3(n+1)-2)x = \frac{(n+1)(3(n+1)-1)x}{2}$$
$$x + 4x + 7x + \cdots + (3n-2)x + (3n+1)x = \frac{(3n^2+5n+2)x}{2}$$

15.d

$$\begin{aligned}
471 &= 0 \cdot 562 + 471 \\
562 &= 1 \cdot 471 + 91 \\
471 &= 5 \cdot 91 + 16 \\
91 &= 5 \cdot 16 + 11 \\
16 &= 1 \cdot 11 + 5 \\
11 &= 2 \cdot 5 + 1 \leftarrow \text{So gcd(471,562)=1.} \\
5 &= 5 \cdot 1 + 0
\end{aligned}$$

So gcd(471,562)=1 with $r, s = (88, -105)$.

16

By theorem 2.9, $\gcd(a, b) = 1$ means that two integers are coprime. By theorem 2.10, $\gcd(a, b) = ar + bs$. Therefore, $1 = ar + bs \implies \gcd(a, b) = 1 \implies a$ and b are coprime. $\square$