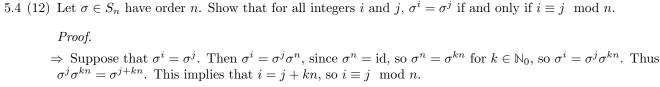
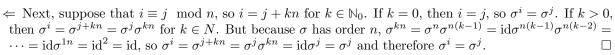
## Modern Alegbra Homework 6

Name: Parker Lockary





(13) Let  $\sigma = \sigma_1 \cdots \sigma_m \in S_n$  be the product of disjoint cycles. Prove that the order of  $\sigma$  is the least common multiple of the lengths of the cycles  $\sigma_1, \cdots, \sigma_m$ .

*Proof.* Let the order of  $\sigma$  be n. Since the cycles are disjoint, they commute and so  $(\sigma_1 \cdots \sigma_m)^n = \sigma_1^n \cdots \sigma_m^n$ . Let the length of each  $\sigma_i^n$  be  $k_i$ .  $\sigma_i$  is the identity permutation if and only if  $k_i|n$ . Therefore, the order of  $\sigma$  is the smallest integer n that is divisible be the lengths of all the cycles  $\sigma_1, \dots, \sigma_m$ . That is the definition of the least common multiple, and so the order of  $\sigma$  is the least common multiple.  $\square$