MATH 420 Advanced Calculus: Homework 2.

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1. Prove that if $x_n > 0$ for all n and $x_n \to 0$ as $n \to \infty$, then $\frac{1}{x_n} \to \infty$.

   Proof. Suppose $\epsilon > 0$. Let $|x_n| < \epsilon$ where $n \geq N \in \mathbb{R}$. Let $M \in \mathbb{R}$ and set $\frac{1}{M} = \epsilon$. Then $|x_n| < \epsilon \implies |\frac{1}{x_n}| > \frac{1}{\epsilon} \implies |\frac{1}{x_n}| > \frac{1}{\frac{1}{M}} \implies |\frac{1}{x}| > M$. □

2. Suppose $x_n \to x$.

   (a) Prove that if $x_n \leq M$ for all n, then $x \leq M$ also.

   Proof. Suppose that $x > M$. Then $x_n \leq M < x$, so $0 < |x - x_n|$ for all n. But then $x_n \not\to x$, contradicting our supposition, so $x \to M$. □

   (b) Is it true that, if $x_n < M$ for all n, then we must have $x < M$ also? Prove it or give a counter-example. Let $M = 0$ and $x_n = \frac{1}{n}$. Then all $x_n < 0$, but the limit x is obviously M, so the statement is not true.