R_{50} Estimation and Comparison: Final Report

Choosing a Model for Estimating R_{50}

When first looking at this problem and the data I was given I knew that a logistic regression could be used to estimate R_{50}. I decided that a good estimate of R_{50} is the range where there is a 50% probability a target will be detected since R_{50} is the range where 50% of the targets will be detected. However, a prediction using a logistic model can't estimate R_{50} directly. Logistics output a probability given a continuous variable, whereas what I needed was the opposite. I researched to see if there was a way to create a model that would be able to do this "out-of-the-box" but I found nothing. I decided to use the logistic model and find a way to solve for R_{50} using that model. I will explain this method later.

I chose to create a logistic model using the R package `parsnip` since prediction output is in a dataframe and I could obtain confidence intervals easily from the model as well. At first while creating my model, I included the interaction term `range:turbines`. This would account for a difference in the rate the detection probability changes as the range of the target increases. Below is the how I created the model and the estimates of the model's terms.

```
logistic_reg() %>%
  set_engine("glm") %>%
  fit(as.factor(detection) ~ range * turbines, data = sample)
```

R_{50} Model With Interaction Term

term	estimate	std.error	statistic	p.value
(Intercept)	6.97	2.13	3.27	0.0011
range	-0.35	0.11	-3.32	0.0009
turbines	-3.65	2.47	-1.48	0.1392
range:turbines	0.12	0.13	0.94	0.3496

However, the interaction term was not significant so I simplified the model as shown below. All the terms of this model were significant and this was my final choice for a model.

```
R50_mod <- logistic_reg() %>%
  set_engine("glm") %>%
  fit(as.factor(detection) ~ range + turbines, data = sample)
```

R_{50} Model

term	estimate	std.error	statistic	p.value
(Intercept)	5.64	1.29	4.38	0.000012
range	-0.28	0.06	-4.70	0.000003
turbines	-1.47	0.69	-2.12	0.033855

Solving for R_{50} and Confidence Intervals

Since my model couldn't estimate R_{50} directly, I created a function `solve_R50` that could use the model to solve for R_{50} and the confidence intervals under each condition. `solve_R50` first creates a function `R50_root` that will have a root at where R_{50}, the upper bound, or the lower bound is (depending on `type`). It then solves for the root using `uniroot` for each turbine condition. If you are interested, I wrote a detailed blog post here on the bulk of the logic behind this.

In the function below, the argument `R50_mod` is the `parsnip` logistic model and `turbines` is a binary vector indicating the condition to predict under. The argument `type` is one of `"pred"`, `"lower"` or `"upper"`. `"pred"` gives the predicted R_{50} value. `"lower"` or `"upper"` give the corresponding side of the confidence interval. To see the full function see here.

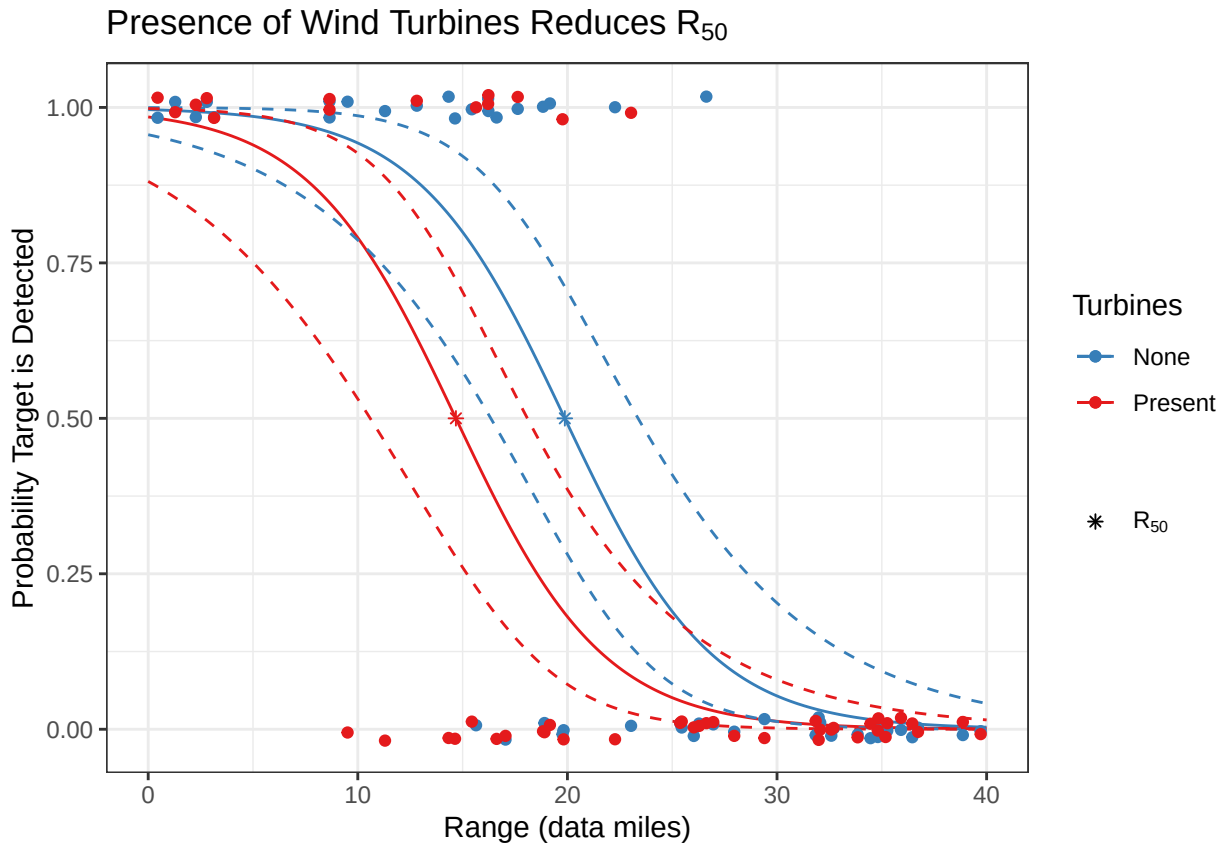
```
solve_R50 <- function(R50_mod, turbines, type) {

  # ... skipped logic for `type` argument

  # create a function that we will find the root of using `uniroot`
  R50_root <- function(range, turbines) {
    predict.model_fit(R50_mod,
                      tibble(range = range, turbines = turbines),
                      # pred_type variable determines if we are solving
                      # for R50 or a confidence interval
                      type = pred_type
    ) %>%
      # pick the column we need from predict.model_fit
      # subtracting .5 creates a root where we want it
      pull(pred_col) - .5
  }

  # solve for each `turbines` condition
  map_dbl(turbines,
          ~ uniroot(R50_root,
                    interval = range(R50_mod$fit$data$range),
                    turbines = .x)$root)
}
```

This is how I used the solver to obtain R_{50} and the confidence intervals for each turbine condition. Below is the information presented in table form and graphically.

```
R50_dat <-
  tibble(turbines = 0:1) %>%
  mutate(R50 = solve_R50(R50_mod, turbines, "pred"),
         lower = solve_R50(R50_mod, turbines, "lower"),
         upper = solve_R50(R50_mod, turbines, "upper"))
```

R_{50} Estimation and Confidence Intervals

Turbines	R_{50}	Lower	Upper
None	19.88	16.35	23.37
Present	14.68	10.59	18.09



Impact of Wind Turbines on R_{50}

We can determine whether the presence of wind turbines affects R_{50} by looking at the impact the `turbines` term has on the model. Below are the estimates of the model's terms. Since the `turbines` term is significant and it represents a horizontal shift in the two curves as shown above, the estimated R_{50} will be significantly different under the two conditions. In this sample, the presence of wind turbines reduced R_{50} by 5.2 data miles.

R_{50} Model Terms

term	estimate	std.error	statistic	p.value
(Intercept)	5.64	1.29	4.38	0.000012
range	-0.28	0.06	-4.70	0.000003
turbines	-1.47	0.69	-2.12	0.033855

Estimation of Power

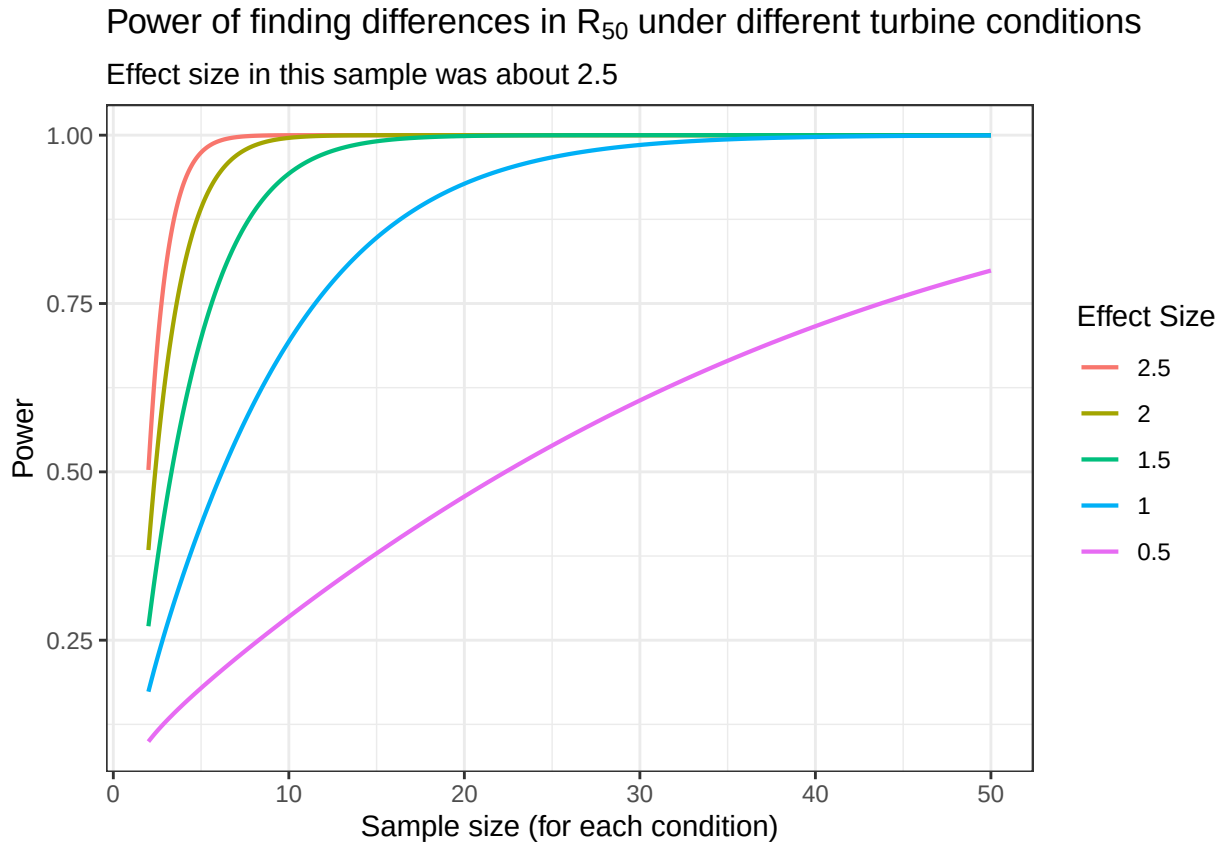
To calculate power I needed to estimate the standard deviation of R_{50}. I used the confidence intervals to make this estimation. Unfortunately, the confidence intervals were not symmetric after translating them to be in terms of range. More specifically, the distance from R_{50} to the upper bound was not equal to the distance from R_{50} to the upper bound. To be conservative in the power calculations I choose the largest margin to determine the standard deviation.

R_{50} Confidence Margins

Turbines	R_{50}	Lower	Upper	Lower Margin	Upper Margin
None	19.88	16.35	23.37	-3.53	3.50
Present	14.68	10.59	18.09	-4.09	3.41

Assuming there are two standard deviations from R_{50} to the interval bounds, my over-estimate for the standard deviation is about 2 (4.09 / 2). Since the difference in the R_{50} estimates is about 5 data miles when there are wind turbines versus no turbines, then the estimates are about 2.5 standard deviations from each other. In other words, the effect size is about 2.5 in this sample.

To calculate power across different sample sizes and effect sizes I used the function `pwr.t.test` from the R package `pwr`. Using a function normally used for t-test power calculations made sense in this situation because if R_{50} was estimated from several different samples of data under both turbine conditions, then a t-test would be used to compare the two sets of R_{50} estimates to prove that R_{50} is different under those conditions. Below is a power curve plot made using `pwr.t.test` for effect sizes and sample sizes smaller than the one in this sample.



Goals Accomplished

By the end of this project, I developed a model and a method for estimating R_{50} and I estimated the confidence intervals of R_{50}. I also identified the impact of the presence of wind turbines on R_{50} and I estimated the power of finding this impact for different sample sizes and effect sizes. If desired the code for this project can be found here.