Introduction to Data Processing and representation

236201

**HW3**

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**Theory**

1. ***Circulant Matrices***
2. Considering the matrix , we want to compute ,

for :

Let’s check the behavior of for

As can be seen from the outcome above, we got matrix with one cyclic shift of the rows downward or cyclic shifts of the rows upward. Thus for we get , meaning the result is matrix with cyclic shifts of the rows downward. For we get , meaning the result is matrix with cyclic shifts of the rows downward or one cyclic shift of the rows upwards. The first row of would be the row of the outcome matrix now, after shifting all the rows we get

This means, for the result is the identity matrix of order .

is called permutation matrix which means a square [binary matrix](https://en.wikipedia.org/wiki/Binary_matrix) that has exactly one entry of 1 in each row and each column and 0s elsewhere.

1. We want to compute the eigenvalues of

Using the definition, if there is a vector such that

For some scalar , then is called the eigenvalues of with corresponding eigenvector .

This is equivalent to , and nontrivial solution is available iff the determinant vanishes, so the solutions of are given by

By applying the determinant of upper or lower triangular matrix is the product of all the diagonal elements of the matrix, we get

Using the polar form, we denote by , then we get

Since every matrix has exactly complex eigenvalues, we get that the eigenvalues of are given by:

1. To complete the eigendecomposition of , we need to compute the corresponding eigenvectors of the previously computed eigenvalues in (b):

We get

For (since eigenvectors are up to multiplication by a scalar), we get

Thus, we get

Therefore, is diagonalizable and can be decomposed to the form

Where is a matrix composed of the eigenvectors of , is the diagonal matrix constructed from the corresponding eigenvalues and is the matrix inverse of .

Thus, P is of the form

And is of the form

We can diagonalize in a unitary basis, since we can get by normalizing the matrix such that (eigenvectors are up to multiplication by a scalar), is symmetric and unitary since its rows and columns obey

and which makes the following statement true

1. Considering the general circulant matrix .

And recalling that is given by .

Using the results of (a) we get:

Thus, the polynomial on matrix is given by

Therefore,

1. We got that , by applying this we get that for all eigenvalue of matrix , , is an eigenvalue of with the same eigenvector.

To see that we first show that if is an eigenvalue of matrix then is the eigenvalue of the matrix , since is the eigenvalue of then as is the corresponding eigenvector. The eigenvalue of is computed by

This result indicates that is the eigenvalue of with the same eigenvector of the eigenvalue of .

Now, let’s compute the eigenvalues of

Using the results above, we get

Therefore, the eigenvalue of with as corresponding eigenvector is given by

is diagonalizable by the same unitary matrix with the eigenvalues computed above, meaning it is diagonalizable in a unitary basis.

1. We want to show that the diagonalization basis matrix can be chosen as the matrix:

We saw earlier that is diagonalizable in unitary basis using the basis matrix . Since is symmetric and each one of its elements is given by , therefore its conjugate is the conjugate of the Discrete Fourier Transform matrix, given that is diagonalizable using and since , we get

In short, both the and its complex conjugate can be chosen as the diagonalization basis matrix and it’s up to the choice of the eigenvalues (we can choose them to be or ).

1. Let be the diagonalization basis matrix and let’s denote , then we get

By applying matrix transpose operation on both sides and considering the symmetry of and and the fact that is diagonal matrix, we get

After multiplying both side by from the right, we get

Since both sides are equal, we get

Therefore, we get

1. We want to show that given the two circulant matrices , and commute, meaning :

Let be the diagonalization basis matrix and let’s denote , hence .

Let’s assume the following

Then we get

Since is unitary matrix, we get , therefore

Considering that and are diagonal matrices meaning they commute, we get

By using the fact again we get

Therefore, and commute.

is a circulant matrix, we show this using the results of section (d).

We saw that and are given by polynomial expression of the matrix , meaning

Hence,

As we saw earlier, is a circulant matrix, thus is a circulant matrix two, and since the outcome of adding two circulant matrices is also a circulant matrix we get that is a circulant matrix too.

1. We want to compute for :

Let’s first check the behavior of for , we get

Hence,

And we get

We notice that this is the vertically flipped form of , let’s denote it by .

Now for we get

To calculate we rely on the fact that for we get

Relying on the results we got above, we can conclude that for is given by:

1. We want to prove that a convolution of -dimensional signals can be computed by point-wise multiplication of the signals in the Fourier domain, up to a normalization.

The convolution of and can be computed using a circulant matrix built from (the same as by with the elements of ) then multiplying by as follows

We get

Considering we get

Considering that is a diagonal matrix and hence symmetric, we get

Relying on the results of (g), we get

Therefore, we get

1. **Fourier Transform**
2. and are two given functions, with convolution denoted by and given by

We want to find in terms of

We start by denoting and , then we get

By changing the integral variable by denoting and hence , we get

1. and are two given functions, we want to show that the following condition holds

Where and are the Fourier transform of and respectively.

We get

By changing the variables and denoting , hence and . We get

is the definition of convolution of the two functions and , meaning

Now we get

is the Fourier transform of , therefore we get

is the projection of to the Fourier family, as seen in the lecture, or the inverse Fourier transform, also

Thus,

We got as required.

**Question 3**

1. For :

So:

1. For :
2. For :

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**Implementation**

תמונה שמכילה טקסט, אדם, מקורה, עמידה

התיאור נוצר באופן אוטומטי

תמונה שמכילה טקסט, ישן, לדגמן

התיאור נוצר באופן אוטומטי





1. We use the index instead of , so as not to confuse with the imaginary number .

We will define: .

For :

If :

Else:

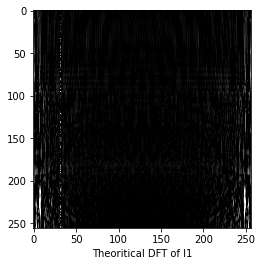
For :

If :

Else:

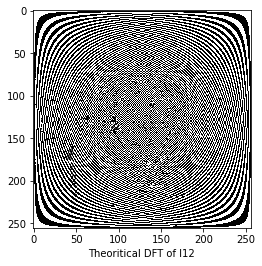
So:

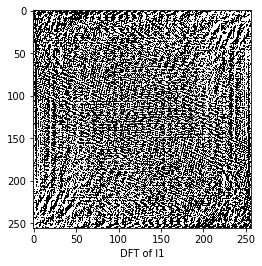
1. The weight of noise 1: ; The weight of noise 2:

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*תמונה שמכילה כהה

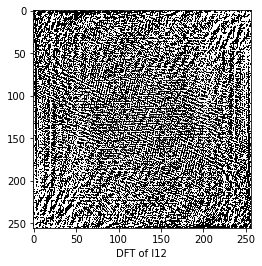
התיאור נוצר באופן אוטומטי*

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תמונה שמכילה טקסט, אבן

התיאור נוצר באופן אוטומטי



MSE of I1 is: 2.7109850297764546

MSE of I2 is: 2.7002239091778053

MSE of I12 is: 359.91252877302543

