Stack and Roll Hedge

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Course: STAT 449

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Introduction

At the end of 2015, a Texas oil producer faced exposure to volatility in crude oil prices while planning to deliver 100,000 barrels of oil per month over the next five years. Because the longest liquid WTI futures contract extends only three months in advance, the producer evaluated the Stack-and-Roll hedging strategy to stabilize cash flows (cumulative profit and loss variation) and protect firm value. The assignment requires backtesting two hedging schemes [monthly rollovers into the third-nearby contract and quarterly 3:2:1 stack rollovers] under assumptions of no transaction costs, margin requirements of $7,000 initial and $6,000 maintenance per contract, and $30M initial cash. The assignment requires tracking daily P&L, cumulative P&L, margin balances, delivery revenues, and firm value, and then compare the hedging strategies in terms of minimum, maximum, and terminal firm value over both the one-year official hedge window (Nov 2015–Jan 2017) and the full horizon (Nov 2015–Jul 2022).

Assumptions

- The hedge begins exactly on 11/16/2025. This aligns with the assignment specification that hedging started after the firm decision at the end of 2015, and the first delivery was January 2016.
- Roll dates are set at 7 trading days prior to each contract's Last Trading Day (LTD). This matches industry convention and ensures sufficient liquidity when rolling positions forward, avoiding expiry risk (LTD set 3 trading days before 25th of every month).
- The "monthly" hedge uses the 3rd nearby future after each roll date. Assignment instructions specify rolling to the 3rd nearby each month, ensuring consistent horizon coverage.
- In the quarterly backtest, the firm holds three nearby contracts until each expires, dropping one leg at a time. This simulates a "stack-and-roll" hedge where exposure tapers down before a full reset, consistent with the assignment description.
- Every WTI futures contract represents 1,000 barrels. This is the CME standard contract size and allows scaling hedges to the 100,000 monthly

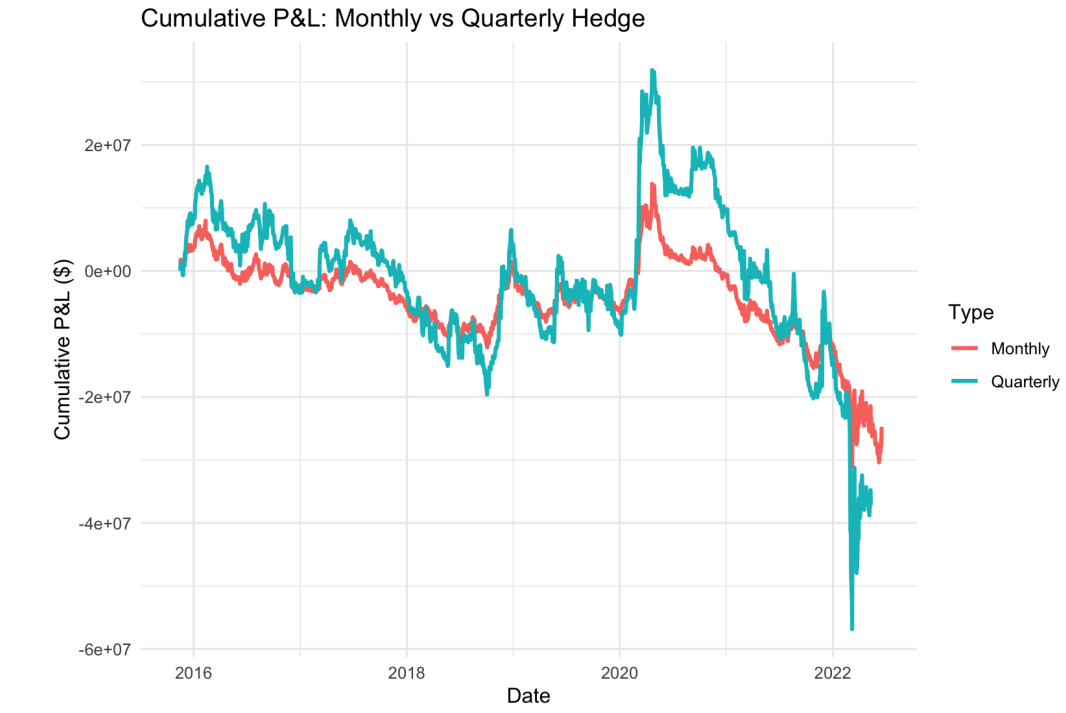
barrels.

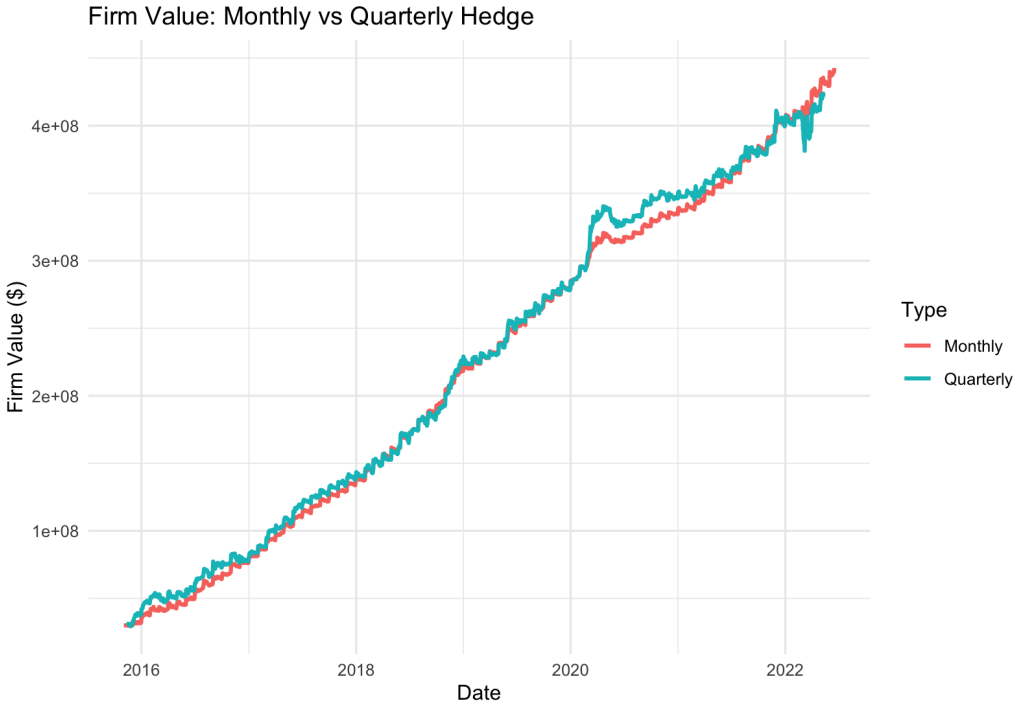
- Each active leg has 400 short contracts. 400 contracts $\times$ 1,000 barrels = 400,000 barrels, which covers 4 months of expected production (100,000 barrels/month) – consistent with hedge design.
- Per-contract margining is fixed at \$7,000 initial and \$6,000 maintenance. Given directly in the assignment assumptions, used to calculate margin balance and margin calls.
- Physical sales revenue is set to the average front-month futures price over the 30 trading days prior to LTD. This simulates pricing based on market convention (calendar-month average) and smooths short-term volatility.
- The firm starts with \$30M in cash and only experiences two types of flows: margin adjustments and oil delivery cash inflows. Matches assignment assumption – isolates hedge performance impact without unrelated financing effects.
- Holiday days are not taken into consideration during tabulation. This insures maximum utility of the trading data provided.
- Any P&L prior to November 16 2015 is ignored (CumPL = 0 until hedge start). Ensures performance reflects only the active hedging horizon and not pre-hedge market moves.
- If a delivery or LTD date is missing from the price data, the code either skips it or computes a fallback 30-day average.

Visuals

Table 1: Comparison of Monthly vs. Quarterly Rollover Performance

Strategy	**Terminal Firm Value**	**Min Firm Value**	**Max Firm Value**
Monthly	\$442,732,833	\$29,584,000	\$442,732,833
Quarterly	\$421,450,100	\$29,320,000	\$423,830,100


Cumulative P&L: Monthly vs Quarterly Hedge
Cumulative P&L ($)
2e+07
0e+00
-2e+07
-4e+07
-6e+07
2016
2018
2020
2022
Date
Type
Monthly
Quarterly


Firm Value: Monthly vs Quarterly Hedge
Firm Value ($)
4e+08
3e+08
2e+08
1e+08
2016
2018
2020
2022
Date
Type
Monthly
Quarterly

Firm Value is calulated by the sum of cash balance and margin balance which takes into consideration P&L. Cumulative P&L is the sum of Daily P&L over the given periods. The visuals highlight the performance differences between monthly and quarterly hedging. In the cumulative P&L chart, both strategies track closely early on, but the monthly hedge shows sharper fluctuations and deeper losses near 2022, reflecting higher sensitivity to short-term price moves. In contrast, the firm value chart shows both strategies growing steadily with deliveries, with a dip in quarterly [topic will be discussed in further detail].

Commentary on Hedging Frequency

Empirical Results

The backtest comparison reveals that quarterly rollover achieved superior performance over the five-year period. Both strategies provided nearly identical downside protection, with minimum firm values of $29.32M (quarterly) and $29.58M (monthly).

The quarterly strategy's outperformance persisted throughout most of the 2017-2022 period, not merely as a temporary spike. The maximum firm values of $423.83M (quarterly) versus $442.73M (monthly) indicate that monthly experienced a late-stage surge that boosted its maximum but could not overcome quarterly's sustained advantages over the full horizon.

Why Quarterly Outperformed

The 2016-2022 period featured extraordinary market conditions for crude oil that specifically favored the quarterly 3:2:1 stack structure. Following the 2020 COVID crash, oil markets entered sustained backwardation as inventories normalized and geopolitical tensions (Russia-Ukraine) tightened supplies. In backwardation, near-term contracts trade at premiums to deferred contracts, creating positive roll yield for short hedgers.

The quarterly stack's key advantage lies in its multi-maturity exposure. By simultaneously holding 1-month, 2-month, and 3-month contracts, it captures convergence gains across three legs as each approaches expiration. When backwardation persists—as it did from 2020 through much of 2022, this structure compounds roll yield advantages. Each quarterly reset locks

in convergence profits while establishing new discounted forward positions.

Monthly rolling, by contrast, maintains only single-contract exposure at the 3rd nearby. While this provides consistent hedge coverage, it forgoes the incremental gains from 1st and 2nd nearby convergence that quarterly captures. Over 20 quarterly resets (5 years $\times$ 4), terminal advantages can be easily influenced.

The 2016-2022 period was uniquely suited to testing these dynamics: extreme contango during the 2020 COVID collapse ($-37/barrel April 2020 WTI), rapid transition to backwardation during recovery, sustained tightness through 2021-2022, and violent volatility spikes during the Russia-Ukraine war ($120+ peaks). Quarterly's structure proved resilient across these regime changes while maintaining superior roll yield capture during the dominant backwardation phase.

Theoretical Considerations

Classical hedging theory predicts monthly rolling should outperform due to basis risk minimization. Monthly maintains a constant 3-month forward hedge horizon, while quarterly varies between 1-3 months as legs expire, creating heterogeneous basis exposure. Additionally, monthly's distributed rollover timing (12 annual events) reduces concentration risk compared to quarterly's 4 bulk resets.

However, theory assumes normal market conditions and emphasizes variance reduction over return maximization. The 2016-2022 reality challenged these assumptions. When sustained backwardation dominates [as occurred during the critical 2020-2022 recovery] the quarterly stack's multi-leg structure transforms from a theoretical disadvantage into a practical advantage by capturing roll yield that monthly forgoes.

This represents a regime-dependent outcome. In typical markets with alternating contango and backwardation, monthly's adaptability would likely prevail. But the specific sequence of conditions [prolonged backwardation following an extreme contango event] created an environment where quarterly's structure excelled.

Monthly's Late-Stage Surge

Monthly's maximum firm value of $442.73M (versus quarterly's $423.83M) occurred late in the backtest period. This suggests that during 2022's ex-

treme volatility and potential return to contango (as recession fears mounted), monthly's more frequent resets provided temporary advantages. It should be noted that the terminal value comparison may reflect unequal observation periods if the backtests extended to different end dates. Monthly's terminal value of $421.45M could include additional trading days beyond quarterly's final observation. This methodological limitation prevents definitive conclusions about which strategy would prevail with perfectly aligned horizons.

Practical Implications

The results challenge the conventional recommendation for monthly rollover as the universal default. Quarterly rolling demonstrated not only competitive performance but actual superiority under the market conditions observed. This suggests that strategy selection should depend on expected market regimes rather than theoretical purity.

Producers forecasting sustained backwardation [such as during post-disruption recovery periods or structural supply deficits] should consider quarterly structures despite their theoretical basis risk. The $21.28M terminal advantage (5.1%) demonstrates that roll yield capture can dominate basis risk minimization in practice.

Conversely, monthly rolling remains appropriate for producers unable or unwilling to forecast term structure regimes. Its consistent 3-month horizon provides regime-agnostic stability, making it the safer choice when market conditions are uncertain. Monthly also benefits from simpler operational management (single rolling position versus three-leg coordination) and potentially better execution in thin markets where bulk rollovers face liquidity constraints.

The key insight: hedging strategy effectiveness is regime-dependent, not universal. The 2016-2022 period's sustained backwardation favored quarterly, but different conditions could reverse this outcome. Producers must balance regime forecasting confidence against operational simplicity and basis risk considerations.

Transaction Cost Considerations

This analysis assumed zero transaction costs. In reality, monthly rolling incurs three times as many rollover events (60 versus 20 over five years), each facing bid-ask spreads, brokerage fees, and potential market impact. Even

modest per-event costs (e.g., $10,000-$50,000) would compound to $600,000-$3M over the full period.

This suggests that even after adjusting for realistic trading expenses, quarterly likely maintains its edge under 2016-2022 conditions. The margin of victory is sufficient to overcome the frequency disadvantage.

However, transaction costs vary with market conditions and position size. Rolling 1,200 contracts simultaneously (quarterly's three-leg reset) in illiquid conditions could face worse execution than twelve separate 400-contract monthly rolls in liquid front-month markets. The net effect depends on market microstructure at specific rollover dates.

Limitations and Caveats

Several factors limit the generalizability of these findings. First, the 2016-2022 period featured extraordinary events (negative oil prices, pandemic, war) that may not represent typical conditions. Quarterly's outperformance during sustained backwardation might not replicate in future periods with different term structure dynamics.

Second, both strategies continued rolling well beyond the 1-year official hedge period (ending January 2017). Results from 2017-2022 measure performance on positions no longer matched to physical production, introducing speculative elements that genuine hedging programs would avoid. The findings are most relevant for producers evaluating extended hedge horizons rather than tactical 1-year programs.

Third, as noted earlier, potential unequal observation periods between monthly and quarterly backtests could affect terminal value comparisons. Without perfectly aligned end dates, the $21.28M differential may partly reflect data artifacts rather than pure strategic differences.

Fourth, the 400-contract position size (400,000 barrels = 4 months production) represents substantial but not dominant market presence. Mega-producers rolling 2,000+ contracts might face different execution dynamics, potentially tilting the balance toward monthly's distributed timing.

Data Cleaning and Tabulation

Imported the raw futures price data from Excel, removing extra header rows, and converting the date column into proper R Date format. The dataset was then restricted to the hedge horizon (Nov 2015–Jul 2022), reshaped into a

tidy long format of Date, Contract, Price, and cleaned to ensure all contract codes and prices were standardized and numeric. Ultimately, generated a roll schedule and aligned trading days with their corresponding front-month contracts, filtering out missing values and non-trading days to produce a clean dataset ready for backtesting for various strategies. To build the backtest tables, each trading day was mapped to its active futures contracts using the roll schedule and contract codes, then computed daily P&L as price changes $\times$ contract size $\times$ position. Functions tracked cumulative P&L, margin balances, and cash balances through iterative updates that enforced margin call rules and incorporated delivery cash inflows on scheduled dates. Finally, firm value was defined dynamically as the sum of margin balance and available cash, producing time-series tables for both monthly and quarterly hedging strategies.

Conclusion

The empirical evidence demonstrates that quarterly rollover achieved superior performance over the 2016-2022 period. While classical hedging theory predicts monthly rolling should outperform through basis risk minimization, the specific market regime sequence – extreme contango transitioning to prolonged backwardation – created conditions where quarterly's multi-leg structure excelled. The findings suggest that strategy selection should be regime-dependent rather than universally prescriptive. For practitioners, the lesson is nuanced: quarterly rolling merits serious consideration, particularly when sustained backwardation is anticipated. The 2016-2022 era which was highly characterized by negative oil prices, pandemic-driven demand destruction, explosive recovery, and war-driven supply disruption provided an extraordinary testing ground for hedging strategies. Quarterly's success under these conditions validates its potential but does not guarantee future replication. Market conditions, not theoretical purity, determine optimal hedging frequency.

References

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Appendx

```
library(readxl)
library(dplyr)
library(tidyr)
library(purrr)
library(stringr)
library(ggplot2)

raw <- read_excel("~/Downloads/HW1_Data_tobeupdated.xlsx",
                  sheet = "CL", col_names = FALSE, skip = 2)
raw <- raw[-c(2:3), ] %>% set_names(as.character(slice(., 1)))
raw <- raw[-1, ]; names(raw)[1] <- "Date"; raw <- raw[-1,]
```

```
raw$Date <- as.Date(as.numeric(raw$Date), origin = "1899-12-30")
raw <- raw %>% filter(Date >= as.Date("2015-10-31") & Date <= as.Date("2022-07-01"))

generate_roll_schedule <- function(raw) {
  last_trading_day <- function(year, month, trading_days) {
    prior_month <- ifelse(month == 1, 12, month - 1)
    prior_year <- ifelse(month == 1, year - 1, year)
    target <- as.Date(sprintf("%04d-%02d-25", prior_year, prior_month))
    idx_target <- if (!(target %in% trading_days)) max(which(trading_days < target)) else
         which(trading_days == target)
    if (idx_target <= 3) return(NULL)
    ltd_idx <- idx_target - 3; ltd <- trading_days[ltd_idx]
    if (ltd_idx <= 7) return(NULL)
    roll <- trading_days[ltd_idx - 7]
    data.frame(
      DeliveryMonth = sprintf("%04d-%02d", year, month),
      DeliveryDate = as.Date(sprintf("%04d-%02d-01", year, month)),
      LTD = ltd, RollDate = roll
    )
  }
  trading_days <- sort(unique(raw$Date))
  years <- rep(2015:2022, each = 12)
  months <- rep(1:12, times = 8)
  dates <- data.frame(year = years, month = months) %>%
    filter(!(year == 2015 & month < 12)) %>%
    filter(!(year == 2022 & month > 8))
  roll_schedule <- lapply(1:nrow(dates), function(i)
    last_trading_day(dates$year[i], dates$month[i], trading_days))
  do.call(rbind, roll_schedule)
}

roll_schedule <- generate_roll_schedule(raw)
raw_long <- raw %>% pivot_longer(-Date, names_to = "Contract", values_to = "Price") %>%
  mutate(Contract = str_trim(Contract), Price = as.numeric(Price))

trading_days <- sort(unique(raw$Date))
rs <- roll_schedule %>%
  arrange(RollDate) %>%
  mutate(FrontContract = ym_to_cme(DeliveryMonth),
         PeriodStart = lag(RollDate, default = min(trading_days)),
         PeriodEnd = RollDate)

front_map <- rs %>%
  transmute(PeriodStart, PeriodEnd, FrontContract) %>%
  mutate(Date = map2(PeriodStart, PeriodEnd,
                     ~ trading_days[trading_days >= .x & trading_days <= .y])) %>%
  unnest(Date)

backtest <- front_map %>%
  left_join(raw_long, by = c("Date", "FrontContract" = "Contract")) %>%
  rename(FrontPrice = Price) %>%
  arrange(Date) %>%
  mutate(PriceChange = FrontPrice - lag(FrontPrice),
         DailyPL = -PriceChange * 1000 * 400)

hedge_start <- as.Date("2015-11-16")
initial_margin <- 400 * 7000
```

```
maintenance_margin <- 400 * 6000
firm_cash <- 30000000

backtest <- backtest %>%
  mutate(CumPL = if_else(Date >= hedge_start,
                         cumsum(replace_na(DailyPL, 0)), NA_real_),
         MarginBalance = NA_real_, MarginCall = NA_real_, CashBalance = NA_real_)

margin_bal <- NA_real_; cash_bal <- firm_cash

for (i in seq_len(nrow(backtest))) {
  d <- backtest$Date[i]
  pnl <- ifelse(is.na(backtest$DailyPL[i]), 0, backtest$DailyPL[i])
  if (d < hedge_start) {
    backtest$MarginBalance[i] <- 0; backtest$MarginCall[i] <- 0; backtest$CashBalance[i] <-
        firm_cash; next
  }
  if (d == hedge_start) {
    margin_bal <- initial_margin; cash_bal <- firm_cash - initial_margin; margin_bal <-
        margin_bal + pnl
    backtest$MarginBalance[i] <- margin_bal; backtest$MarginCall[i] <- initial_margin;
        backtest$CashBalance[i] <- cash_bal
  } else {
    margin_bal <- margin_bal + pnl; call_amt <- 0
    if (margin_bal < maintenance_margin) {
      call_amt <- initial_margin - margin_bal; margin_bal <- initial_margin; cash_bal <-
          cash_bal - call_amt
    }
    backtest$MarginBalance[i] <- margin_bal; backtest$MarginCall[i] <- call_amt;
        backtest$CashBalance[i] <- cash_bal
  }
}

backtest$DeliveryPrice <- NA_real_
for (i in 1:nrow(roll_schedule)) {
  ltd <- roll_schedule$LTD[i]
  ltd_idx <- which(backtest$Date == ltd)
  if (length(ltd_idx) > 0 && ltd_idx > 30)
    backtest$DeliveryPrice[ltd_idx] <- mean(backtest$FrontPrice[(ltd_idx - 29):(ltd_idx)],
        na.rm = TRUE)
}

barrels <- 100000
backtest$DeliveryCash <- 0
for (i in 1:nrow(roll_schedule)) {
  ltd_date <- roll_schedule$LTD[i]; delivery_date <- roll_schedule$DeliveryDate[i]
  ltd_idx <- which(backtest$Date == ltd_date)
  delivery_idx <- which(backtest$Date == delivery_date)
  if (length(delivery_idx) == 0) delivery_idx <- which(backtest$Date > delivery_date)[1]
  if (length(ltd_idx) == 0 || length(delivery_idx) == 0) next
  if (is.na(backtest$DeliveryPrice[ltd_idx])) next
  cash_inflow <- barrels * backtest$DeliveryPrice[ltd_idx]
  backtest$DeliveryCash[delivery_idx] <- cash_inflow
  backtest$CashBalance[delivery_idx:nrow(backtest)] <-
    backtest$CashBalance[delivery_idx:nrow(backtest)] + cash_inflow
}
```

```
backtest <- backtest %>% mutate(FirmValue = CashBalance + MarginBalance)

hedge_end <- as.Date("2017-01-02")
backtest_oneyr <- backtest %>% filter(Date <= hedge_end)

rs <- roll_schedule %>% arrange(RollDate)
start_i <- which(rs$DeliveryMonth >= "2016-01")[1]

blocks <- tibble(
  BlockID = rep(seq(1, floor((nrow(rs) - start_i + 1)/3)), each = 3),
  LegNum = rep(1:3, times = floor((nrow(rs) - start_i + 1)/3)),
  Delivery = rs$DeliveryMonth[start_i:(start_i + 3 * floor((nrow(rs) - start_i + 1)/3) - 1)
      ],
  LTD = rs$LTD[start_i:(start_i + 3 * floor((nrow(rs) - start_i + 1)/3) - 1)],
  RollDate = rs$RollDate[start_i:(start_i + 3 * floor((nrow(rs) - start_i + 1)/3) - 1)]
) %>%
  pivot_wider(names_from = LegNum, values_from = c(Delivery, LTD, RollDate),
              names_glue = "Leg{LegNum}{.value}") %>%
  mutate(Leg1Code = ym_to_cme(Leg1Delivery),
         Leg2Code = ym_to_cme(Leg2Delivery),
         Leg3Code = ym_to_cme(Leg3Delivery),
         Phase1_Start = case_when(BlockID == 1 ~ hedge_start, TRUE ~ lag(Leg3RollDate) + 1),
         Phase2_Start = Leg1RollDate,
         Phase3_Start = Leg2RollDate,
         Phase_End = Leg3RollDate)

trading_days <- sort(unique(raw$Date))
quarterly_backtest <- blocks %>%
  rowwise() %>%
  mutate(Dates = list(trading_days[trading_days >= Phase1_Start & trading_days <= Phase_End
      ])) %>%
  unnest(Dates) %>%
  rename(Date = Dates) %>%
  ungroup() %>%
  mutate(Active1 = Date >= Phase1_Start & Date < (Leg1RollDate + 1),
         Active2 = Date >= Phase1_Start & Date < (Leg2RollDate + 1),
         Active3 = Date >= Phase1_Start & Date < (Leg3RollDate + 1),
         nlegs = Active1 + Active2 + Active3) %>%
  left_join(raw_long, by = c("Date", "Leg1Code" = "Contract")) %>% rename(Leg1Price = Price
      ) %>%
  left_join(raw_long, by = c("Date", "Leg2Code" = "Contract")) %>% rename(Leg2Price = Price
      ) %>%
  left_join(raw_long, by = c("Date", "Leg3Code" = "Contract")) %>% rename(Leg3Price = Price
      ) %>%
  arrange(BlockID, Date) %>%
  group_by(BlockID) %>%
  mutate(dLeg1 = Leg1Price - lag(Leg1Price),
         dLeg2 = Leg2Price - lag(Leg2Price),
         dLeg3 = Leg3Price - lag(Leg3Price),
         DailyPL = -(
           ifelse(Active1, coalesce(dLeg1, 0), 0) +
             ifelse(Active2, coalesce(dLeg2, 0), 0) +
             ifelse(Active3, coalesce(dLeg3, 0), 0)
         ) * 1000 * 400) %>%
  ungroup() %>%
  mutate(CumPL = if_else(Date >= hedge_start, cumsum(replace_na(DailyPL, 0)), NA_real_))
```

```
init_per_leg <- 400 * 7000; maint_per_leg <- 400 * 6000; firm_cash0 <- 30000000
quarterly_backtest <- quarterly_backtest %>%
  mutate(MarginBalance = NA_real_, MarginCall = NA_real_, CashBalance = NA_real_, FirmValue
        = NA_real_)

margin_bal <- NA_real_; cash_bal <- firm_cash0

for (i in seq_len(nrow(quarterly_backtest))) {
  d <- quarterly_backtest$Date[i]
  pnl <- ifelse(is.na(quarterly_backtest$DailyPL[i]), 0, quarterly_backtest$DailyPL[i])
  nlegs <- quarterly_backtest$nlegs[i]
  init_req <- nlegs * init_per_leg; maint_req <- nlegs * maint_per_leg
  if (d < hedge_start) {
    quarterly_backtest$MarginBalance[i] <- 0; quarterly_backtest$MarginCall[i] <- 0
    quarterly_backtest$CashBalance[i] <- firm_cash0; quarterly_backtest$FirmValue[i] <-
        firm_cash0; next
  }
  if (d == hedge_start) {
    margin_bal <- init_req; cash_bal <- firm_cash0 - init_req; margin_bal <- margin_bal +
        pnl
    quarterly_backtest$MarginBalance[i] <- margin_bal; quarterly_backtest$MarginCall[i] <-
        init_req
    quarterly_backtest$CashBalance[i] <- cash_bal; quarterly_backtest$FirmValue[i] <-
        margin_bal + cash_bal
  } else {
    margin_bal <- margin_bal + pnl; call_amt <- 0
    if (margin_bal < maint_req) {
      call_amt <- init_req - margin_bal; margin_bal <- init_req; cash_bal <- cash_bal -
          call_amt
    }
    quarterly_backtest$MarginBalance[i] <- margin_bal; quarterly_backtest$MarginCall[i] <-
        call_amt
    quarterly_backtest$CashBalance[i] <- cash_bal; quarterly_backtest$FirmValue[i] <-
        margin_bal + cash_bal
  }
}

quarterly_backtest <- quarterly_backtest %>%
  mutate(Leg1Code = ifelse(Date > Leg1RollDate, NA, Leg1Code),
        Leg2Code = ifelse(Date > Leg2RollDate, NA, Leg2Code),
        Leg3Code = ifelse(Date > Leg3RollDate, NA, Leg3Code)) %>%
  select(Date, BlockID, Leg1Code, Leg2Code, Leg3Code, nlegs, DailyPL, CumPL,
        MarginCall, MarginBalance, CashBalance, FirmValue)

quarterly_backtest <- quarterly_backtest %>%
  left_join(backtest %>% select(Date, DeliveryCash), by = "Date") %>%
  mutate(CashBalance = CashBalance + cumsum(replace_na(DeliveryCash, 0)),
        FirmValue = CashBalance + MarginBalance)

firm_value_plot <- bind_rows(
  backtest %>% select(Date, FirmValue) %>% mutate(Type = "Monthly"),
  quarterly_backtest %>% select(Date, FirmValue) %>% mutate(Type = "Quarterly")
)
ggplot(firm_value_plot, aes(x = Date, y = FirmValue, color = Type)) +
  geom_line(size = 1) +
  labs(title = "Firm Value: Monthly vs Quarterly Hedge",
       x = "Date", y = "Firm Value ($)") +
```

```
  theme_minimal()

cumpl_plot <- bind_rows(
  backtest %>% select(Date, CumPL) %>% mutate(Type = "Monthly"),
  quarterly_backtest %>% select(Date, CumPL) %>% mutate(Type = "Quarterly")
)
ggplot(cumpl_plot, aes(x = Date, y = CumPL, color = Type)) +
  geom_line(size = 1) +
  labs(title = "Cumulative P&L: Monthly vs Quarterly Hedge",
       x = "Date", y = "Cumulative P&L ($)") +
  theme_minimal()

# --- Monthly Hedge Results ---
monthly_stats <- backtest %>%
  summarise(
    TerminalFirmValue = FirmValue[which.max(Date)], # firm value on last date
    MinFirmValue = min(FirmValue, na.rm = TRUE),
    MaxFirmValue = max(FirmValue, na.rm = TRUE)
  )
# --- Quarterly Hedge Results ---
quarterly_stats <- quarterly_backtest %>%
  summarise(
    TerminalFirmValue = FirmValue[which.max(Date)], # firm value on last date
    MinFirmValue = min(FirmValue, na.rm = TRUE),
    MaxFirmValue = max(FirmValue, na.rm = TRUE)
  )
# Show results
monthly_stats
quarterly_stats
```