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Homework Assignment 1

H 1.1 (Edge Detection) 12 Points

Let the following 5 × 5 2-D signal be given:

0	0	0	0	0
0	0	0	0	0
5	5	0	0	0
5	5	0	0	0
5	5	0	0	0

x

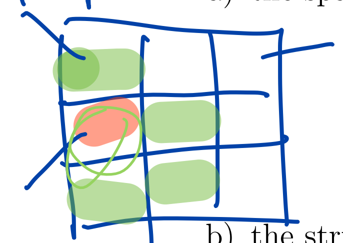
y

$$K_\rho * \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}$$

For the inner pixels (3 × 3 - square), compute the structure tensor (with $\rho = 0$, i.e. without convolution). To this end, you should compute

rank:1

Edge



flat rank=0

corner

rank ≥ 2

edge

a) the spatial derivatives in x- and y-direction:

$$[f_x]_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2}$$

$$[f_y]_{i,j} = \frac{f_{i,j+1} - f_{i,j-1}}{2}$$

$$\begin{matrix} - & 0 & 0 & 0 & - \\ - & 0 & 0 & 0 & - \\ - & -2.5 & -2.5 & 0 & - \\ - & -2.5 & -2.5 & 0 & - \\ - & -2.5 & -2.5 & 0 & - \end{matrix} \qquad \begin{matrix} - & - & - & - & - \\ 2.5 & 2.5 & 0 & 0 & 0 \\ 2.5 & 2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ - & - & - & - & - \end{matrix}$$

b) the structure tensor:

$$J_0 = \begin{pmatrix} [f_x]^2_{i,j} & [f_x]_{i,j} \cdot [f_y]_{i,j} \\ [f_x]_{i,j} \cdot [f_y]_{i,j} & [f_y]^2_{i,j} \end{pmatrix}$$

$$f_x^2 = \begin{matrix} 0 & 0 & 0 \\ 6.25 & 6.25 & 0 \\ 6.25 & 6.25 & 0 \end{matrix} \qquad f_y^2 = \begin{matrix} 6.25 & 0 & 0 \\ 6.25 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$$

$$f_x f_y = \begin{matrix} 0 & 0 & 0 \\ -6.25 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$$

We assumed grid sizes $h_x = h_y = 1$ here.

c) Decide for each of the nine pixels if it belongs to a flat area, an edge or a corner. Explain your decisions using the eigenvalues of the structure tensor.

d) For the central pixel, perform convolution (of the structure tensor) with the binomial kernel

$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

$$\frac{1}{8} \times 5 + \frac{1}{16} \times 5 = \frac{15}{16}$$

$$\frac{(4+2+2+1)}{16} = \frac{9}{16} \times \frac{25}{4}$$

and decide again for the central pixel. Explain your findings.

before: $\begin{pmatrix} 6.25 & 0 \\ 0 & 0 \end{pmatrix}$

after: $6.25 \times \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16}\right) =$

$= \sqrt{\sum |f_x|^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} =$

Let the following 2D-RGB-signal be given:

$$R = \boxed{\begin{matrix} 5 & 5 & 0 \\ 5 & 5 & 0 \\ 5 & 5 & 0 \end{matrix}}\,, \qquad G = \boxed{\begin{matrix} 0 & 0 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & 5 \end{matrix}}\,, \qquad B = \boxed{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}\,.$$

In order to detect the edge in the central pixel, compute

$\nabla f_R = \begin{pmatrix} -\frac{5}{2} \\ 0 \end{pmatrix}$ $\quad \nabla f_G = \begin{pmatrix} \frac{5}{2} \\ 0 \end{pmatrix}$ $\quad \nabla f_B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

e) the norm of the sum of gradients $|\nabla f_R + \nabla f_G + \nabla f_B|$, $= 0$

f) the joint colour gradient $\left| \Big(|\nabla f_R|,\ |\nabla f_G|,\ |\nabla f_B| \Big)^\top \right|$,

g) the joint colour structure tensor $\nabla f_R \nabla f_R^\top + \nabla f_G \nabla f_G^\top + \nabla f_B \nabla f_B^\top$.

Which of these expressions are useful for edge detection? Explain your answer.

H 1.2 (Cooccurrence Matrices) 8 Points

Compute the cooccurrence matrix of the 4×4 image

0	3	2	1
1	1	3	2
0	0	2	1
3	0	1	0

$$\frac{1}{9} \times \begin{array}{cc} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} & \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \\ \begin{matrix} 0 & \ 1 & \ 2 & \ 3 \end{matrix} & \\ i & \end{array} \quad j$$

with $\mathbf{d} = (-1, -1)^\top$. Assume that the x-axis points to the right and the y-axis downwards.

$(1,0)$ $\quad \frac{1}{9}\left(2\times 1 + 1\times 1 + 1\times 2^2 + 1\times 2^2 + 1\times 1\right) = \frac{4}{3}$

a) Determine the highest probability as well as the contrast of the cooccurrence matrix.

$2 + 1 + 4 + 4 + 1 = 12$

b) Assume $\mathbf{d} = (1, 1)^\top$. Would this yield the same highest probability? Would it yield the same contrast? You may either compute the solution or answer with logical arguments.

Same contrast, mode $= (0,1)$

P 1.1 (Hough Transform for Circles)

Please download the required file `cv20_ex01.tgz` from ILIAS. To unpack the data, use `tar xvfz cv20_ex01.tgz` .

(a) In the file `gradient_map.c`, supplement the routine `gradient_magnitude` with the missing code such that it computes the gradient magnitude via a finite difference approximation. Compile the program with

```
gcc -O3 -o gradient_map gradient_map.c -lm.
```

(b) Try to find appropriate parameters for the standard deviation σ of the Gaussian pre-smoothing kernel and the threshold of the gradient magnitude T_{Edge}. The program will write out the computed edge map.

(c) In the file `hough_transform.c`, supplement the routine `vote_hough` with the missing code such that it implements the voting step of the Hough transform for circles. To this end, you should use the provided routine

```
vote_circle (image, c_list, r_max, r_min, nx, ny, x, y, r) ,
```

that draws a circle with center (x, y) and radius r in the 2D-array "image". Compile the program with

```
gcc -O3 -o hough_transform hough_transform.c -lm.
```

(d) The program will read in the edge map from (b). Adjust the remaining parameters: $r_{\text{min}}, r_{\text{max}}$ for the minimum and maximum radius, respectively, and the thresholding parameter T_{Hough} in the hough space (given as percentage of points on the circle, $0 < T_{\text{Hough}} < 1$) such that all coins are detected.

Submission

Please remember that up to three people can work and submit their solutions together. The theoretical problem(s) have to be submitted in handwritten form before the deadline.

Deadline for Submission is: Friday, November 27th, 11:59 pm

Guidelines for the Tutorials

- The exercise sheets contain **three types of problems:**
- **Homework Assignments (H)** have to be submitted before the specified deadline. They will be corrected and you will receive feedback on ILIAS.
- **Programming Assignments (P)** are intended to help you to further familiarize with the contents of the lecture. They **don't** have to be submitted and will not be graded, but example solutions will be provided online later on.
- **Classroom Assignments (C)** are intended to be solved **within** the next tutorial. They **don't** have to be submitted and will not be graded, but your tutor will help you to complete them correctly.

- In order to gain **admission to the exam**, you have to achieve **50% of the total points** from the homework **(H)** assignments within the semester.

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Classroom Assignment 1

C 1.1 (Derivative Approximation)

A sufficiently often continuously differentiable function $f(x)$ is sampled with pixel distance h, resulting in a discrete signal (f_i). The goal is to approximate the second derivative $f''(x)$ in pixel i using the four values $f_{i-3}, f_{i-2}, f_{i-1}, f_i$.

1. Deduce the corresponding system of equations which determines the coefficients of the approximation.
2. Determine the order of consistency of the approximation

$$f_i'' = \frac{-f_{i-3} + 4f_{i-2} - 5f_{i-1} + 2f_i}{h^2}$$

for the second derivative $f''(x)$ in pixel i.

$$f_{i+1} = f_i + hf_i' + \frac{h^2}{2} f_i''$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!} \cdot f^{(n)}(a)$$

$$f_{i-1} = f_i - h \cdot f_i' + \frac{h^2}{2} f_i''$$

$$f_{i-2} = f_i - 2hf_i' + \frac{4h^2}{2} f_i''$$

$$f_i'' =$$

$$f_{i-3} = f_i - 2$$

$$f_i'' = af_i + bf_{i+1} + cf_{i-1}$$

$$= (a+b+c) f_i + h(b-c) f_i' + (b+c) \cdot \frac{h^2}{2} f_i''$$

$$b = c \quad \therefore \quad h^2 \cdot b = 1 \quad \therefore c = b = \frac{1}{h^2} \quad a = -(b+c) = -\frac{2}{h^2}$$