Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Homework Assignment 2

H 2.1 (Lucas and Kanade) 4 Points

Minimising the local energy that corresponds to the approach of Lucas and Kanade requires to solve the following linear system of equations:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} ,$$

with the abbreviations

$$\begin{aligned} a_{11} &= \int_{B_\rho} f_x^2 \, dxdy \, , & b_1 &= -\int_{B_\rho} f_x f_z \, dxdy \, , \\ a_{12} &= \int_{B_\rho} f_x f_y \, dxdy \, , & b_2 &= -\int_{B_\rho} f_y f_z \, dxdy \, , \\ a_{22} &= \int_{B_\rho} f_y^2 \, dxdy \, . \end{aligned}$$

Derive closed form solutions for the unknowns u and v, i.e. come up with formulae how to compute u and v from $a_{11}, a_{12}, a_{22}, b_1$ and b_2.

Hint: Use Cramer's rule: For a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$, the components of $\mathbf{x}$ are given by

$$x_i = \frac{\det\left(\mathbf{A}_{i \to \mathbf{b}}\right)}{\det\left(\mathbf{A}\right)} ,$$

where $\mathbf{A}_{i \to \mathbf{b}}$ is obtained by replacing column i in matrix $\mathbf{A}$ by vector $\mathbf{b}$.

H 2.2 (Variational Methods) **10 Points**

Instead of using the grey value constancy assumption, let us assume that the y-derivative of the image f remains constant over time, i.e.

$$f_y(x+u, y+v, t+1) = f_y(x, y, t) \tag{1}$$

(a) Linearise the constancy assumption (1) w.r.t the flow functions u and v.

(b) Write down an energy functional similar to Horn and Schunck based on the linearised constancy assumption from (a).

(c) Compute the Euler-Lagrange equations for this energy functional.

(d) Discretise the Euler-Lagrange equations from (c).

(e) Starting from the discrete equations computed in (d), derive an iterative scheme that computes the minimiser.

H 2.3 (Stereo) **6 Points**

Consider a camera with focal distance 2. Its image coordinate system is orthogonal with square pixels of size 1. The principal point in this coordinate system is located in $(2,3)^\top$. Furthermore, the position of the world coordinate system relative to the camera coordinate system is given by a rotation around the z-axis by an angle of 90° and a translation by the vector $(5, 0, -1)^\top$.

(a) Compute the intrinsic matrix A_{int} (including the focal length f).

(b) Compute the extrinsic matrix A_{ext}.

(c) Compute the full projection matrix P.

$$A_{int} = \left(\begin{array}{ccc|c} fk & \cdot & \cdot & 2 \\ 0 & k & \cdot & 3 \\ \hline 0 & 0 & & 1 \end{array}\right)$$

A_{ext}

P 2.1 (Lucas and Kanade)

Please download the required file `cv20_ex02.tgz` from ILIAS. To unpack the data, use `tar xvfz cv20_ex02.tgz` .

The programme `lkTemplate.c` should be extended to the method of Lucas and Kanade. Starting the program with two frames of a sequence yield two output images: the magnitude of the optic flow and a flow classification. This classification distinguishes three cases:

no information (black) – only normal flow (grey) – full flow (white)

(a) In the method `create_eq_systems`, supplement the missing code such that it computes the entries of the linear system of equations solved in the Lucas/Kanade approach.

(b) The aim of the method `lucas_kanade` is to reuse the entries calculated before and to solve the linear system of equations. The method should distinguishe the three cases given above. The normal flow or the full flow should be calculated if possible, otherwise u and v should be set to zero. Supplement the missing code. You can use the result from Problem 2 to compute the full flow, if possible.

(c) Compile the program using `gcc -O3 -o lkTemplate lkTemplate.c -lm` and test the program with the image pairs `pig1,2.pgm` and `sphere1,2.pgm`. What is the influence of the integration scale ρ? You can use a threshold $\varepsilon = 0.1$ for testing.

P 2.2 (Horn and Schunck)

In the routine `flow` in `hsTemplate.c`, supplement the missing code such that it computes one Jacobi iteration step. Make sure the image boundaries are treated correctly. To compile the program, type

```
gcc -O3 -o hsTemplate hsTemplate.c -lm
```

The filling-in effect that is characteristic for variational methods can be studied with the image pair `pig1.pgm`, `pig2.pgm`. To this end, investigate the result for different numbers of iterations. What is a good value for the regularisation parameter α in this case?

Submission:

The theoretical problem(s) have to be submitted in handwritten form before the deadline.

Deadline for Submission is: Monday, December 21st, 11:59 pm

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Classroom Assignment 2

C 2.1 (Affine Lucas and Kanade)

Derive the matrix $J_0 = \mathbf{r}\,\mathbf{r}^\top$ for the affine Lucas and Kanade model.

C 2.2 (Eigenvalue Analysis)

Let $\mathbf{J} \in \mathbb{R}^{n \times n}$ be a symmetric $(n \times n)$ matrix with real components. We consider its corresponding quadratic form given by

$$E : \mathbb{R}^n \longrightarrow \mathbb{R}, \qquad E(\mathbf{v}) = \mathbf{v}^\top \mathbf{J} \mathbf{v}$$

Show that among all vectors $\mathbf{v} \in \mathbb{R}^n$ with $|\mathbf{v}| = 1$, the function value $E(\mathbf{v})$ is minimal for the eigenvector of $\mathbf{J}$ corresponding to its smallest eigenvalue. What can we say about E if $\mathbf{J}$ is positive definite?