Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Homework Assignment 4

H 4.1 (Classification Problem) 6 Points

Assume the following conditional class densities to be given:

$$P(x \mid \omega_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$
$$P(x \mid \omega_2) = \frac{1}{\sqrt{4\pi}} e^{-\frac{1}{4}x^2}$$

(a) Assuming equal priors and symmetric losses, derive a decision rule based on the likelihood ratio test (LRT).

(b) Now, assume equal priors but $\lambda_{1,1} = \lambda_{2,2} = 0$, $\lambda_{1,2} = 1$ and $\lambda_{2,1} = 2$. How does the decision rule change?

(c) What happens if we assume $\lambda_{1,1} = \lambda_{2,2} = 0$, $\lambda_{1,2} = 1$ but $\lambda_{2,1} = 0$? State the decision rule!

(d) **Bonus** **2 Points**
Let us now return to the assumptions of equal priors and symmetric losses, and introduce a third class to our classification problem, with the conditional class density

$$P(x \mid \omega_3) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-2)^2}$$

Based on the maximum likelihood criterion, derive the new decision rule for ω_1.
Hint: We already know in which cases ω_1 dominates ω_2, so you only have to find out in which cases ω_1 dominates ω_3.

(e) **Bonus** **2 Points**
Set up the complete decision rules for all three classes.

H 4.2 (Principal Component Analysis) **6 Points**

Assume the following set of 2D points to be given:

$$\{\mathbf{x}_1 : (18, 18)^\top, \mathbf{x}_2 : (2, 2)^\top, \mathbf{x}_3 : (7, 13)^\top, \mathbf{x}_4 : (13, 7)^\top,$$
$$\mathbf{x}_5 : (16, 16)^\top, \mathbf{x}_6 : (4, 4)^\top, \mathbf{x}_7 : (9, 19)^\top, \mathbf{x}_8 : (11, 1)^\top,$$
$$\mathbf{x}_9 : (1, 9)^\top, \mathbf{x}_{10} : (19, 11)^\top, \mathbf{x}_{11} : (13, 13)^\top, \mathbf{x}_{12} : (7, 7)^\top\}$$

(a) Compute the mean and the total scatter matrix for the given set of points.

(b) Perform a projection for $\mathbf{x}_5$ and $\mathbf{x}_9$ on the dominant direction (this performs dimensionality reduction, you obtain one coefficient for each point).

(c) **Bonus** **4 Points**
By doing back-projection, using the mean and the coefficient (from part (b)), compute the approximated 2D points for $\mathbf{x}_5$ and $\mathbf{x}_9$. What is your finding?

P 4.1 (Mean Curvature Motion)

Please download the required file `cv20_ex04.tgz` from ILIAS. To unpack the data, use `tar xvfz cv20_ex04.tgz` .

In the file `segmentation.c`, supplement the routine `mcm_update` such that it computes the update term for mean curvature motion. Compile the program with

```
gcc -O3 -o segmentation segmentation.c -lm
```

and start it by typing `./segmentation mcm`

and test its functionality with the image `head.pgm`. You can specify the presmoothing scale, the time step size (check script for stability codition), the stopping time and the “Number of iterations between writes“, i.e. after how many iterations an intermediate result is written out. The intermediate stopping time for these images is thus $\tau \times$ (Number of iterations). You can take a look at the evolution by typing `animate *_<s>`, where `<s>` is the output image name you specified. The final result is stored as `final-<s>`.

P 4.2 (Chan-Vese Segmentation)

Now, your task is to complete the program such that it implements Chan-Vese segmentation. To this end, let us slighlty reformulate the Euler-Lagrange equations:

$$
\begin{aligned}
& 0 &=& \left((f-u_{\text{in}})^2-(f-u_{\text{out}})^2\right)|\nabla v|-\lambda\,|\nabla v|\,\text{div}(\frac{\nabla v}{|\nabla v|}) \\
\Leftrightarrow\quad & 0 &=& \frac{1}{\lambda}\left((f-u_{\text{in}})^2-(f-u_{\text{out}})^2\right)|\nabla v|-|\nabla v|\,\text{div}(\frac{\nabla v}{|\nabla v|})
\end{aligned}
$$

Thus, the reformulated gradient descent scheme that has to be implemented reads

$$
\begin{aligned}
& \partial_t v &=& \frac{1}{\lambda}\left((f-u_{\text{out}})^2-(f-u_{\text{in}})^2\right)|\nabla v|+|\nabla v|\ \text{div}\left(\frac{\nabla v}{|\nabla v|}\right) \\
\Leftrightarrow\quad & \frac{v^{k+1}-v^k}{\tau} &=& \frac{1}{\lambda}\left(\left(f-u_{\text{out}}^{k+1}\right)^2-\left(f-u_{\text{in}}^{k+1}\right)^2\right)\left|\nabla v^k\right|+\left|\nabla v^k\right|\ \text{div}\left(\frac{\nabla v^k}{|\nabla v^k|}\right) \\
\Leftrightarrow\quad & v^{k+1} &=& v^k+\tau\left(\frac{1}{\lambda}\left(\left(f-u_{\text{out}}^{k+1}\right)^2-\left(f-u_{\text{in}}^{k+1}\right)^2\right)\left|\nabla v^k\right|+\left|\nabla v^k\right|\ \text{div}\left(\frac{\nabla v^k}{|\nabla v^k|}\right)\right)
\end{aligned}
$$

The minimum is found by alternating between the computation of $u_{\text{in}}^{k+1}, u_{\text{out}}^{k+1}$ and the actual iteration step for updating v^{k+1}. Since we already computed the contribution of the MCM-term, we only have to consider the IDM term here. To this end, supplement the routine `idm_updates` with the missing code such that it computes the mean values $u_{\text{in}}^{k+1}, u_{\text{out}}^{k+1}$ as well as the coefficient of the intensity-driven motion term. Moreover, in the routine `chan_vese_segmentation`, supplement the missing iteration step that computes v^{k+1}. Compile the program again with

```
gcc -O3 -o segmentation segmentation.c -lm
```

but start it this time by typing `./segmentation cv`

The evolution and the final result can be visualized using the same commands as for the previous problem, but an additional image `overlay-<s>` is written out that shows the original image together with the contour of the segmentation (zero level line).

Submission:

The theoretical problem(s) have to be submitted in handwritten form before the deadline.

Deadline for Submission is: Monday, February 15th, 11:59 pm

Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Classroom Assignment 4

C 4.1 (Likelihood Ratio Test) Check the correctness of the formula for the likelihood ratio test on page 20, lecture 20.

C 4.2 (Discriminant Functions) Check the correctness of the simplifications of the discriminant functions leading to the two minimum distance classifiers (Euclidean and Mahalanobis).