Übung 00

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1 Matrix equations

1. Let X, YA be arbitrary matrices, A invertible. Solve for X:

   $XA + A^\top = \mathbf{I}$

2. Let X, A, B be arbitrary matrices, $(C - 2A^\top)$ invertible. Solve for X:

   $X^\top C = [2A(X + B)]^\top$

3. Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^d$, $A \in \mathbb{R}^{d\times n}$, $A^\top A$ invertible. Solve for x:

   $(Ax - y)^\top A = \mathbf{0}_n^\top$

4. As above, additionally $B \in \mathbb{R}^{n\times n}$, B positive-definite. Solve for x:

   $(Ax - y)^\top A + x^\top B = \mathbf{0}_n^\top$

1. $XA + A^T = I$

$X = (I - A^T)A^{-1}$

$X = A^{-1} - A^T A^{-1}$

2. $X^T C = (X+B)^T 2A^T = (X^T + B^T) 2A^T$

$X^T(C - 2A^T) = B^T \cdot 2A^T$

$X^T = 2(AB)^T \cdot (C - 2A^T)^{-1}$

$X = [2(AB)^T \cdot (C - 2A^T)^{-1}]^T$

$X = 2[(C - 2A^T)^{-1}]^T \cdot AB = 2(C^T - 2A)^{-1} \cdot AB$

3. $(AX - y)^T A = 0_n^T$

$(X^T A^T - y^T) A = 0_n^T$

$X^T A^T A = y^T A$

$X^T = y^T A (A^T A)^{-1}$

$X = (A^T A)^{-1} A^T y$

4. $(AX - y)^T A + X^T B = 0_n^T$

$X^T A^T A - y^T A + X^T B = 0_n^T$

$X^T (A^T A + B) = y^T A$

$(A^T A + B^T) X = A^T y$

$\because$ B positive definite, $A^T A$ invertible

$x^T (A^T A) x = (Ax)^T Ax \geq 0$

$x^T B x > 0$

$\therefore x^T (A^T A + B) x > 0$

$\therefore$ $A^T A + B$ is positive difinite

$\therefore$ $A^T A + B$ invertible

$X = (A^T A + B^T)^{-1} A^T y$

2 Vector derivatives

Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^d$, $A \in \mathbb{R}^{d\times n}$.

1. What is $\frac{\partial}{\partial x} x$? (Of what type/dimension is this thing?)
2. What is $\frac{\partial}{\partial x}[x^\top x]$?
3. Let B be symmetric and positive definite. What is the minimum of $(Ax-y)^\top(Ax-y) + x^\top Bx$ w.r.t. x?

1. identity matrix with n dimension

2. $\frac{\partial}{\partial x}[x^T x] = 2X$

3. $F(x) = (Ax-y)^T(Ax-y) + x^T Bx$

let $\frac{\partial}{\partial x} F(x) = 2A^T(Ax-y) + 2Bx$

$= 2(A^TAx - A^Ty + Bx)$

$= 0$

$A^TAx - A^Ty + Bx = 0$

$(A^TA+B)x = A^Ty$

$A^TA + B$ is positive difinite

$\therefore \quad x = (A^TA+B)^{-1}A^Ty$

3 Error Measures

Let $y, \hat{y} \in \mathbb{R}^n$ be n true and predicted values of a regression problem.

1. Formally define the error measures *Mean Squared Error* (MSE) and *Mean Absolute Error* between y and $\hat{y}$.
2. How would choosing MAE or MSE as objective function for a regression problem impact the resulting prediction model? How do MSE and MAE differ?
3. Let $y, \hat{y} \in \{0,1\}^n$ be n true and predicted labels of a binary classification problem. What would the MSE and MAE calculate in this case?

1. MSE: $= \frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})^2$

MAE: $= \frac{1}{n}\sum_{i=1}^{n}|y-\hat{y}|$

2. MSE will square the error and thus having a larger result if there is a large error, MSE is more sensitive to the errors so it has higher accuracy but it is not as robust as MAE.

3. the probability of the prediction being false

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