Robotics
Exercise 8

Lecturer: Jim Mainprice
TAs: Philipp Kratzer, Janik Hager, Yoojin Oh
Machine Learning & Robotics lab, U Stuttgart
Universitätsstraße 38, 70569 Stuttgart, Germany

July 1, 2021

1 Probability Basics (5 Points)

Probability theory is very useful in Robotics. Probabilities allow us to express stochasticity and to model uncertainty (e.g. uncertainty about the robot state given sensor information). Make yourself familiar with probability theory: take a look at lecture slides **09-probabilities**.

a) Box 1 contains 8 apples and 4 oranges. Box 2 contains 10 apples and 2 oranges. Boxes are chosen with equal probability. What is the probability of choosing an apple? If an apple is chosen, what is the probability that it came from box 1? (1P)

$P(\text{Apple}) = 0.5 \times \frac{8}{12} + 0.5 \times \frac{10}{12} = \frac{9}{12} = \frac{3}{4}$ A = from box 1, B = apple $P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)} = \frac{8}{12} \times \frac{0.5}{0.75} = \frac{4}{9}$

b) The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was: 30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan. Afterward it was: 24% Blue , 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown.

1994: yellow: 20% green: 10% 1996: yellow: 14%, green 20% $P(Y,G) =$

A friend of mine has two bags of M&Ms, and he tells me that one is from 1994 and one from 1996. He won't tell me which is which, but he gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag? (1P)

$P(1994Y, 1996G \mid Y, G) = \frac{P((1994Y, 1996G) \cap (Y,G))}{P(Y,G)} = \frac{0.2 \times 0.2}{0.054} \approx 74.1\%$

$P(Y,G) = 0.2 \times 0.2 + 0.1 \times 0.14 = 0.04 + 0.014 = 0.054$

c) The Monty Hall Problem: I have three boxes. In one I put a prize, and two are empty. I then mix up the boxes. You want to pick the box with the prize in it. You choose one box. I then randomly select *another* one of the two remaining boxes and show that it is empty (this is not supposed to be another random experiment; just assume that an empty box is picked). I then give you the chance to change your choice of boxes—should you do so? (1P)

let A = (Pick == Prize) B = (Rest == Prize) $P(A) = \frac{1}{3}$ $P(B) = 1 - P(A) = \frac{2}{3}$ (Since one box is shown empty) ∴ should change

d) Given a joint probability $P(X, Y)$ over 2 binary random variables as the table

	Y=0	Y=1
X=0	.06	.24
X=1	.14	.56

for $\forall x,y$, $P(x,y) = P(x) \cdot P(y)$

$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) = 0.3$
$P(X=1) = 0.7$
$P(Y=0) = P(X=0, Y=0) + P(X=1, Y=0) = 0.2$
$P(Y=1) = 0.8$

What are $P(X)$ and $P(Y)$? Are X and Y independent? (1P)

$P(X=1, Y=1) = 0.56 = P(X=1) \cdot P(Y=1) = 0.56$ ∴ independent.

e) What is the equation of a n-dim Gaussian distribution with mean μ and covariance matrix Σ? What does the mean and the covariance matrix tell us? (1P)

$P(x) = \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}} \exp(-\frac{1}{2}(x-\mu)^T \cdot \Sigma^{-1}(x-\mu))$

μ is the biase, the mean of distribution, d is the dimensions.
Σ is the covariance-matrix, the uncertainty of distribution.

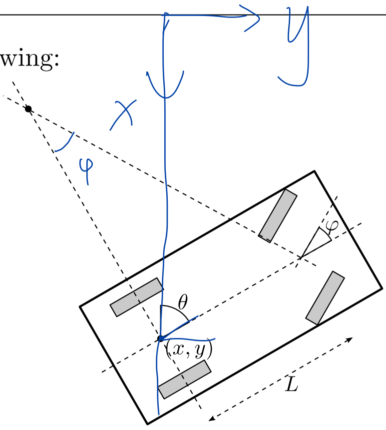
$\Sigma^{-1} \approx H_{\log(P)}$, $E_x(H_{\log(P)}) = I$
↑ Fisher information matrix

2 Particle Filtering the location of a car (7 points)

You are going to implement a particle filter. Access the code as usual:

1. For python run: *'jupyter-notebook course1-Lectures/07-particle_filter/particle_filter.ipynb'*
2. For C++ run: *'cd course1-Lectures/07-particle_filter'*, *'make'*, *'./x.exe'*

The motion of the car is described by the following:



Rotation matrix SO2.

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

State $q = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$ **Controls** $u = \begin{pmatrix} v \\ \varphi \end{pmatrix}$

Sytem equation $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\ \cos\theta \\ v\ \sin\theta \\ (v/L)\ \tan\varphi \end{pmatrix}$

$|\varphi| < \Phi$

The `CarSimulator` simulates the described car (using Euler integration with step size 1sec). At each time step a control signal $u = (v, \phi)$ moves the car a bit and Gaussian noise with standard deviation $\sigma_{\text{dynamics}} = .03$ is added to x, y and θ. Then, in each step, the car measures the relative positions of m landmark points (green cylinders), resulting in an observation $y_t \in \mathbb{R}^{m\times 2}$; these observations are Gaussian-noisy with standard deviation $\sigma_{\text{observation}} = .5$. In the current implementation the control signal $u_t = (.1, .2)$ is fixed (roughly driving circles).

a) Odometry (dead reckoning): First write a particle filter (with $N = 100$ particles) that ignores the observations. For this you need to use the cars system dynamics (described above) to propagate each particle, and add some noise σ_{dynamics} to each particle (step 3 on slide 10:22). Draw the particles (their x, y component) into the display. Expected is that the particle cloud becomes larger and larger. (3 P)

b) Next implement the likelihood weights $w_i \propto P(y_t|x_t^i) = \mathcal{N}(y_t|y(x_t^i), \sigma) \propto e^{-\frac{1}{2}(y_t - y(x_t^i))^2/\sigma^2}$ where $y(x_t^i)$ is the (ideal) observation the car would have if it were in the particle position x_t^i. Since $\sum_i w_i = 1$, normalize the weights after this computation. (2 P)

c) Test the full particle filter including the likelihood weights and resampling. Test using a larger ($10\sigma_{\text{observation}}$) and smaller ($\sigma_{\text{observation}}/10$) variance in the computation of the likelihood. (2 P)

please see the code implementation.

derivation of Bayes Filter

$$P(\underset{x_t}{A} \mid \underset{y_t}{B}, \underset{y_{0:t-1},\, u_{0:t-1}}{C}) = \frac{P(B|A,C)\cdot P(A|C)}{P(B|C)}$$

states : x_t
observations : y_t
control : u_t.

observation model: $P(y_t|x_t)$

transition model: $P(x_t|x_{t-1}, u_{t-1})$ → First-order Markov.

$P_t(x_t) = P(x_t|y_{0:t}, u_{0:t-1})$

$$= \frac{\underbrace{P(y_t|x_t, y_{0:t-1}, u_{0:t-1})}_{\text{likelihood}} \cdot \underbrace{P(x_t|y_{0:t-1}, u_{0:t-1})}_{\text{prior}}}{P(y_t|y_{0:t-1}, u_{0:t-1})}$$

← evidence as constant C_t.

not needed. If x_t is given (obser. model)

$$= C_t \cdot P(y_t|x_t)\cdot P(x_t|y_{0:t-1}, u_{0:t-1})$$

$$= C_t\cdot P(y_t|x_t)\int P(x_t, x_{t-1}|y_{0:t-1}, u_{0:t-1})\, dx_{t-1}$$ ← marginal probability.

$$= C_t\cdot P(y_t|x_t)\int \underbrace{P(x_t|x_{t-1}, \cancel{y_{0:t-1}}, u_{t-1})}_{\text{transition model}}\cdot \underbrace{P(x_{t-1}|y_{0:t-1}, u_{0:t-1})}_{\text{recursive form}}\, dx_{t-1}$$ ← conditional prob.

x_{t-1} independent of u_{t-1}

$$= C_t\cdot P(y_t|x_t)\int P(x_t|x_{t-1}, u_{t-1})\cdot P(x_{t-1}|y_{0:t-1}, u_{0:t-2})\, dx_{t-1}$$

$$= C_t\cdot \underbrace{\underbrace{P(y_t|x_t)}_{\substack{\text{observation}\\ \text{model}}}}_{\text{new info.}} \cdot \underbrace{\int \underbrace{P(x_t|x_{t-1}, u_{t-1})}_{\substack{\text{transition}\\ \text{model.}}}\cdot \underbrace{P_{t-1}(x_{t-1})}_{\substack{\text{old}\\ \text{estimation}}}\, dx_{t-1}}_{\text{predictive estimation.}}$$