Robotics
Exercise 8

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1 Probability Basics (5 Points)

Probability theory is very useful in Robotics. Probabilities allow us to express ==stochasticity== and to model ==uncertainty== (e.g. uncertainty about the robot state given sensor information). Make yourself familiar with probability theory: take a look at lecture slides **09-probabilities**.

a) Box 1 contains 8 apples and 4 oranges. Box 2 contains 10 apples and 2 oranges. Boxes are chosen with equal probability. What is the probability of choosing an apple? If an apple is chosen, what is the probability that it came from box 1? (1P)

$$P(Apple) = 0.5 \times \frac{8}{12} + 0.5 \times \frac{10}{12} = \frac{9}{12} = \frac{3}{4}$$

A = from box 1   B = apple

$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)} = \frac{8}{12} \times \frac{0.5}{0.75} = \frac{4}{9}$$

b) The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was: 30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan. Afterward it was: 24% Blue , 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown.

1994: Yellow: 20%   green : 10%   1996: yellow : 14%, green 20%

A friend of mine has two bags of M&Ms, and he tells me that one is from 1994 and one from 1996. He won't tell me which is which, but he gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag? (1P)

$$P(1994Y, 1996G \mid Y, G) = \frac{P((1994Y, 1996G) \cap (Y, G))}{P(Y, G)} = \frac{0.2 \times 0.2}{0.054} \approx 74.1\%$$

$$P(Y, G) = 0.2 \times 0.2 + 0.7 \times 0.14 = 0.04 + 0.014 = 0.054$$

c) The Monty Hall Problem: I have three boxes. In one I put a prize, and two are empty. I then mix up the boxes. You want to pick the box with the prize in it. You choose one box. I then randomly select *another* one of the two remaining boxes and show that it is empty (this is not supposed to be another random experiment; just assume that an empty box is picked). I then give you the chance to change your choice of boxes—should you do so? (1P)

let A = (Pick == Prize)   B = (Rest == Prize) $P(A) = \frac{1}{3}$ $P(B) = 1 - P(A) = \frac{2}{3}$ (since one box is shown empty) ∴ should change

d) Given a joint probability $P(X,Y)$ over 2 binary random variables as the table

for $\forall x, y$, $P(X, Y) = P(X) \cdot P(Y)$

	Y=0	Y=1
X=0	.06	.24
X=1	.14	.56

What are $P(X)$ and $P(Y)$? Are X and Y independent? (1P)

$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) = 0.3$

$P(X=1) = 0.7$

$P(Y=0) = P(X=0, Y=0) + P(X=1, Y=0) = 0.2$

$P(Y=1) = 0.8$

$P(X=1, Y=1) = 0.56 = P(X=1) \cdot P(Y=1) = 0.56$ ∴ independent.

e) What is the equation of a n-dim Gaussian distribution with mean μ and covariance matrix Σ? What does the mean and the covariance matrix tell us? (1P)

$$P(x) = \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$

μ is the bias, the mean of distribution, d is the dimensions.
Σ is the covariance-matrix, the uncertainty of distribution.

2 Particle Filtering the location of a car (7 points)

$\Sigma^{-1} \approx H_{log(P)}$, $\mathbb{E}_x(H_{log(P)}) = I$ ← Fisher information matrix

You are going to implement a particle filter. Access the code as usual:

1. For python run: *'jupyter-notebook course1-Lectures/07-particle_filter/particle_filter.ipynb'*

2. For C++ run: *'cd course1-Lectures/07-particle_filter'*, *'make'*, *'./x.exe'*
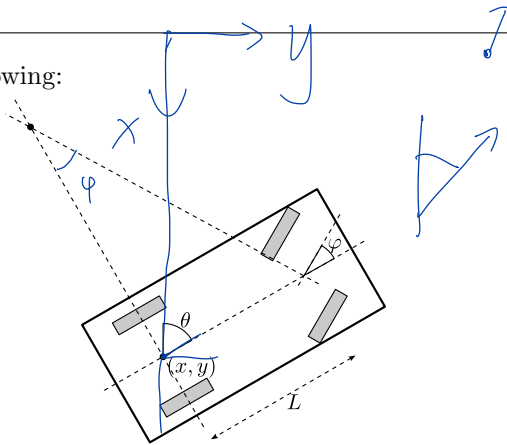
The motion of the car is described by the following:

State $q = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$ **Controls** $u = \begin{pmatrix} v \\ \varphi \end{pmatrix}$

Sytem equation $\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v \cos\theta \\ v \sin\theta \\ (v/L) \tan\varphi \end{pmatrix}$

$|\varphi| < \Phi$

Rotation matrix
SO2.
$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

The `CarSimulator` simulates the described car (using <u>Euler integration</u> with step size 1sec). At each time step a control signal $u = (v, \phi)$ moves the car a bit and Gaussian noise with standard deviation $\sigma_{\text{dynamics}} = .03$ is added to x, y and θ. Then, in each step, the car measures the relative positions of m landmark points (green cylinders), resulting in an observation $y_t \in \mathbb{R}^{m \times 2}$; these observations are Gaussian-noisy with standard deviation $\sigma_{\text{observation}} = .5$. In the current implementation the control signal $u_t = (.1, .2)$ is fixed (roughly driving circles).

a) Odometry (dead reckoning): First write a particle filter (with $N = 100$ particles) that ignores the observations. For this you need to use the cars system dynamics (described above) to propagate each particle, and add some noise σ_{dynamics} to each particle (step 3 on slide 10:22). Draw the particles (their x, y component) into the display. Expected is that the particle cloud becomes larger and larger. (3 P)

b) Next implement the likelihood weights $w_i \propto P(y_t | x_t^i) = \mathcal{N}(y_t | y(x_t^i), \sigma) \propto e^{-\frac{1}{2}(y_t - y(x_t^i))^2 / \sigma^2}$ where $y(x_t^i)$ is the (ideal) observation the car would have if it were in the particle position x_t^i. Since $\sum_i w_i = 1$, normalize the weights after this computation. (2 P)

c) Test the full particle filter including the likelihood weights and resampling. Test using a larger ($10\sigma_{\text{observation}}$) and smaller ($\sigma_{\text{observation}}/10$) variance in the computation of the likelihood. (2 P)

please see the code implementation.

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derivation of Bayes Filter

$P(A|B, C) = P(B|A, C) \cdot \frac{P(A|C)}{P(B|C)}$
$\quad X_t \quad Y_t \quad Y_{0:t-1}, U_{0:t-1}$

states : X_t
observations : Y_t
control : U_t

observation model:
$P(Y_t | X_t)$

transition model:
$P(X_t | X_{t-1}, U_{t-1})$
$\downarrow$
First-order Markov.

$P_t(X_t) = P(X_t | y_{0:t}, U_{0:t-1})$ likelihood.

$= P(Y_t | X_t, y_{0:t-1}, U_{0:t-1}) \cdot \frac{P(X_t | y_{0:t-1}, U_{0:t-1})}{P(Y_t | y_{0:t-1}, U_{0:t-1})}$ ←prior

    not needed.   ← evidence as constant C_t.
if X_t is given (obser. model)

$= C_t P(Y_t | X_t) \cdot P(X_t | y_{0:t-1}, U_{0:t-1})$

$= C_t \cdot P(Y_t | X_t) \cdot \int P(X_t, X_{t-1} | y_{0:t-1}, U_{0:t-1}) \, dX_{t-1}$ ← marginal probability.

$= C_t \cdot P(Y_t | X_t) \cdot \int P(X_t | X_{t-1}, y_{0:t-1}, U_{0:t-1}) \cdot P(X_{t-1} | y_{0:t-1}, U_{0:t-1}) \, dX_{t-1}$ ← Bayes rule.
    transition model.   recursive form.

$= C_t \cdot P(Y_t | X_t) \cdot \int P(X_t | X_{t-1}, U_{t-1}) \cdot P(X_{t-1} | y_{0:t-1}, U_{0:t-2}) \, dX_{t-1}$

$= C_t \cdot P(Y_t | X_t) \cdot \int P(X_t | X_{t-1}, U_{t-1}) \cdot P_{t-1}(X_{t-1}) \, dX_{t-1}$

    observation   transition   old
    model   model.   estimation

    new info.   predictive estimation.