2.e. for example from $(0,0) \to (1,-1)$ with const. acc.
if we use $d = |a| = |-\frac{1}{2}| = \frac{1}{2}$ but we actually can never reach it.

2.d. "grow" using RRT*, rewiring the tree to provide optimal solution.

pseudo code: $X_{near} \leftarrow Near(G, x_{new}, |V|)$

for all $x \in X_{near} \setminus \{x_{min}\}$ do // x_{min} is $x_{nearest}$

if ObstacleFree(x, x_{new}) and

Cost$(x) >$ Cost(x_{new}) + Line(x, x_{new})

then $x_{parent} \leftarrow$ Parent(x)

$E' \leftarrow E' \setminus \{(x_{parent}, x)\}$ // delete

$E' \leftarrow E' \cup \{x_{new}, x\}$ // add

Robotics
Exercise 7

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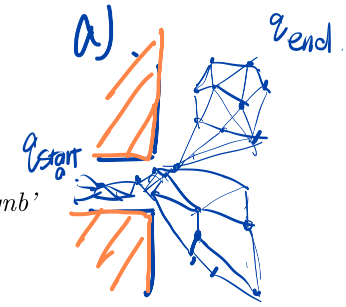
a)



1 RRTs for path finding (7 points)

1. For python you can run: *'jupyter-notebook course1-Lectures/06-rrt/06-rrt.ipynb'*
2. For C++ run: *'cd ourse1-Lectures/06-rrt/'*, *'make'*, *'./x.exe'*

The code demonstrates an RRT exploration that randomly samples from Q and displays the explored endeffector positions.

a) First grow an RRT *backward* from the target configuration $q^* = (0.945499, 0.431195, -1.97155, 0.623969, 2.22355, -0.665206, -1.48356)$. Grow the RRT directly towards $q = 0$ with a probability of $\beta = 0.5$. Stop when there exists a node close (<`stepSize`) to the $q = 0$ configuration. Read out the collision free path from the tree and display it. Why would it be more difficult to grow the tree *forward* from $q = 0$ to q^*? (2 points)

b) Find a collision free path using bi-directional RRTs (that is, 2 RRTs growing together). Use q^* to root the backward tree and $q = 0$ to root the forward tree. Stop when a newly added node is close to the other tree. Read out the collision free path from the tree and display it. (3 points)

c) Propose a simple method to make the found path smoother (while keeping it collision free). Implement the method and display the smooth trajectory. (2 points)

Please see the code implementation.

2 A distance measure in phase space (5 points)

Consider the 1D point mass with mass $m = 1$ state $x = (q, \dot{q})$. The 2D space of $(q, \dot{q})$ combining position and velocity is also called phase space. In RRT's (in line 4 on slide 07:29) we need to find the nearest configuration q_{near} to a q_{target}. But what does "near" mean in phase space? This exercise explores this question.

a) Explain why the Euclidean distance in phase space is not a good meassurement of "near". (1P)

b) Consider a current state $x_0 = (q_0, \dot{q}_0)$. A simple motion model is to use constant acceleration a that takes time T to reach a target state $x_1 = (q_1, \dot{q}_1)$. Come up with equations to calculate a and T. (1P)

c) Given the solution from task b), how would you quantify/meassure the distance? Give an analytic expression. (1P)

d) Given a set (tree) of states $x_{1:n}$ and you pick the closest to x_{target}, how would you "grow" the tree from this closest point towards x_{target}? (1P)

e) We can not go to every point in phase space with constant acceleration motion. Come up with example configurations x_0 and x_1 where this is not possible. What would happen to your distance measure when plugging in those configurations? Can we still use the distance measure from c) in the context of RRTs? (1P)

(a) Euclidian distance: $d = \sqrt{(q_1 - q_2)^2 + (\dot{q}_1 - \dot{q}_2)^2}$ Since it is squared, (1, 1) and (1, -1) same distance to (0, 0) but (1, -1) is actually closer, because it moves towards the (0, 0)

(b) $\dot{q}_1 = \dot{q}_0 + aT$, $q_1 = q_0 + \dot{q}_0 T + \frac{1}{2} a T^2$

$a = \frac{1}{T}(\dot{q}_1 - \dot{q}_0)$

$q_1 = q_0 + \dot{q}_0 T + \frac{1}{2}(\dot{q}_1 - \dot{q}_0) T = q_0 + \frac{1}{2}(\dot{q}_1 + \dot{q}_0) T$

$T = 2 \frac{q_1 - q_0}{\dot{q}_1 + \dot{q}_0}$, $a = \frac{1}{2} \frac{\dot{q}_1^2 - \dot{q}_0^2}{q_1 - q_0}$

(c) Possi.

① $d = |a|$ for constant acceleration model

because if q_0 and q_1 is close, no need to have big acc.

② Work: $W = \frac{1}{2} m \dot{q}_1^2 - \frac{1}{2} m \dot{q}_0^2 = \frac{1}{2} m (\dot{q}_1^2 - \dot{q}_0^2) = \frac{1}{2}(\dot{q}_1^2 - \dot{q}_0^2)$

$\therefore d = c_1 |q_0^2 - q_1^2| + c_2 |\dot{q}_0^2 - \dot{q}_1^2| + c_3 (q_0 - q_1)^2 + c_4 (\dot{q}_0 - \dot{q}_1)^2$ ← absolute