2. a) $q^* = q_0 + J^\# \cdot (y^* - y_0) = q_0 + W^{-1}J^T(JW^{-1}J^T + C^{-1})^{-1}(y^*-y_0)$ for $C \to \infty$, $q^* = q_0 + W^{-1}J^T(JW^{-1}J^T)^{-1}(y^*-y_0)$

need to prove $\phi(q^*) = y^*$, with linearity $\delta y = J\delta q \Rightarrow y = y_0 + J\cdot(q-q_0)$

$\therefore J\cdot(q^*-q_0) = J(q_0 + W^{-1}J^T(JW^{-1}J^T)^{-1}(y^*-y_0) - q_0) = (JW^{-1}J^T)(JW^{-1}J^T)^{-1}(y^*-y_0)$

$\therefore J(q^*-q_0)+y_0 = y^*-y_0+y_0 = y^*$

Robotics

Exercise 3

Lecturer: Jim Mainprice

TAs: Philipp Kratzer, Janik Hager, Yoojin Oh

Machine Learning & Robotics lab, U Stuttgart

Universitätsstraße 38, 70569 Stuttgart, Germany

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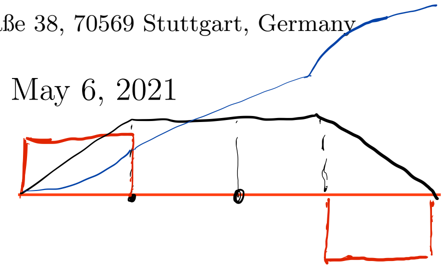

$$MP(s) = \begin{cases} 4s^2 & s \in [0, \frac{1}{4}] \\ s & s \in (\frac{1}{4}, \frac{3}{4}) \\ -4(s-1)^2+1 & s \in [\frac{3}{4}, 1] \end{cases}$$

1 Motion profiles (2 points)

Construct a motion profile that accelerates constantly in the first quarter of the trajectory, then moves with constant velocity, then decelerates constantly in the last quarter. Write down the equation $\text{MP}(s) : [0, 1] \mapsto [0, 1]$.

2 Verify some things from the lecture (6 points)

a) On slides 12 + 13 we derived the basic inverse kinematics law. Verify that (assuming linearity of ϕ, i.e., $J\delta q = \delta y$) for $C \to \infty$ the desired task is fulfilled exactly, i.e., $\phi(q^*) = y^*$. By writing $C \to \infty$ we mean that C is a matrix of the form $C = \varrho C_0$, $\varrho \in \mathbb{R}$, and we take the limit $\varrho \to \infty$. (2P)

b) On slide 20 there is a term $(\mathbf{I} - J^\# J)$ called nullspace projection. Verify that for $\varrho \to \infty$ (and $C = \varrho\mathbf{I}$) and any choice of $\delta a \in \mathbb{R}^n$

$$\delta q = (\mathbf{I} - J^\# J)\delta a \;\Rightarrow\; \delta y = 0$$

(assuming linearity of ϕ, i.e., $J\delta q = \delta y$). Note: this means that any choice of δa, the motion $(\mathbf{I} - J^\# J)\delta a$ will not change the "endeffector position" y. (2P)

(b) $y = \phi(q)$ with linearity $\delta y = J\delta q$

$\therefore \delta y = J(I - J^\# J)\delta a = (J - JW^{-1}J^T(JW^{-1}J^T + C^{-1})^{-1}J)\delta a$

for $C = \rho I \to \infty$, $\delta y = (J - I\cdot J)\cdot\delta a = 0$.

c) On slides 31 + 32 it says that

$$\underset{q}{\operatorname{argmin}} \|q - q_0\|^2_W + \|\Phi(q)\|^2$$
$$\approx q_0 - (J^\top J + W)^{-1} J^\top\, \Phi(q_0) = q_0 - J^\# \Phi(q_0)$$

where the approximation $\approx$ means that we use the local linearization $\Phi(q) = \Phi(q_0) + J(q - q_0)$. Verify this. (2P)

pls. see below.

3 IK in the simulator (6 points)

Installation instructions:

1. On github https://github.com/humans-to-robots-motion/robotics-course you can find the course repository and an instruction on how to install it.
2. To make sure you have an updated version of the repository, run '*git pull*' and '*git submodule update*'
3. You can find the exercises in the directory *course1-Lectures*. From there, you can run:
   - For python: *'jupyter-notebook 01-kinematics/kinematics.ipynb'*
   - For C++: *'cd 01-kinematics'*, *'make'*, *'./x.exe -mode 2'*

The goal of this task is to reach the coordinates $y^* = (-0.2, -0.4, 1.1)$ with the right hand of the robot. Assume $W = \mathbf{I}$ and $\sigma = .01$.

a) The provided code already generates a motion using inverse kinematics $\Delta q = J^\sharp\ \Delta y$ with $J^\sharp = (J^\top J + \sigma^2 W)^{-1} J^\top$. Record the task error, that is, the deviation of hand position from y^* after each step. You can plot the error using *'plt.plot(err)'* and *'plt.show()'* in python or *'gnuplot(err)'* in C++ (*err* is the array of errors). Why is it initially large? (1P)

b) Try to do 100 smaller steps $\delta q = \alpha J^\sharp\ \delta y$ with $\alpha = .1$ (each step starting with the outcome of the previous step). How does the task error evolve over time? (1P)

c) Generate a nice trajectory composed of $T = 100$ time steps. Interpolate the target linearly $\hat{y} \leftarrow y_0 + (t/T)(y^* - y_0)$ in each time step. How does the task error evolve over time? (2P)

d) Generate a trajectory that moves the right hand in a circle centered at $(-0.2, -0.4, 1.1)$, aligned with the xz-plane, with radius 0.2. (2P)

2. c). let $\phi(q) = \phi(q_0) + J\cdot(q-q_0)$

$$f(q) = \|q-q_0\|_W^2 + \|\phi(q)\|^2 = (q-q_0)^T W (q-q_0) + \phi^T(q_0)\phi(q_0) + (q-q_0)^T J^T J (q-q_0) + 2\phi^T J (q-q_0)$$

$$\therefore \frac{\partial f(q)}{\partial q} = 2(q-q_0)^T W + 2(q-q_0)^T J^T J + 2\phi^T J \overset{!}{=} 0$$

$$\therefore (q-q_0)^T (W + J^T J) = -\phi^T J$$

$$q - q_0 = -(J^T J + W)^{-1} J^T \phi(q_0)$$

$$q = q_0 - (J^T J + W)^{-1} J^T \phi(q_0)$$