Robotics
Exercise 2

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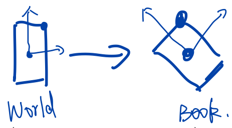
0 MLR robotics-course

We will start with coding exercises in the next exercise sheet. On github https://github.com/humans-to-robots-motion/robotics-course you can find our course repository and instructions on how to install it. Follow instructions for **Setup for Robotics Lecture Exercises** in https://github.com/humans-to-robots-motion/robotics-course#setup-for-robotics-lecture-exercises. Make yourself familiar with the code and run the tutorial (tutorials/1-basics.ipynb). For coding exercises we will provide both, c++ and python templates. You will be able to choose which one to use.

1 Geometry (3 Points) $p = \begin{pmatrix} x \\ y \end{pmatrix}$



$$T_{W\to B} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 1 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) You have a book (coordinate frame B) lying on the table (world frame W). Initially B and W are identical. Now you move the book 1 unit to the right, then rotate it by 45° counter-clock-wise around its origin. Given a dot p marked on the book at position $p^B = (1,1)^T$ in the book coordinate frame, what are the coordinates p^W of that dot with respect to the world frame? (1P)

$$p^W = T_{W\to B} \cdot p^B = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \quad \therefore p^W = (1, \sqrt{2})$$

b) Given a point x with coordinates $x^W = (0,1)^T$ in world frame, what are its coordinates x^B in the book frame? (1P)

c) What is the *coordinate* transformation from world frame to book frame, and from book frame to world frame? (1P)

Please use homogeneous coordinates to derive these answers. (See http://ipvs.informatik.uni-stuttgart.de/mlr/marc/notes/3d-geometry.pdf for more details on 3D geometry.)

$$b)\ x^B = T_{B\to W} \cdot x^W = \begin{pmatrix} R^{-1} & -R^{-1}t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \\ 1 \end{pmatrix} \quad \therefore x^B = (0, \sqrt{2})^T$$

2 Task spaces and Jacobians (7 Points)

c) coordinate transform $T_{B\to W} = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R^T & -R^T t \\ 0 & 1 \end{pmatrix}$, $T_{W\to B} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}$

In the lecture we introduced the basic kinematic maps $\phi^{\text{pos}}_{i,v}(q)$ and $\phi^{\text{vec}}_{i,v}(q)$, and their Jacobians, $J^{\text{pos}}_{i,v}(q)$ and $J^{\text{vec}}_{i,v}(q)$, where i may refer to any part of the robot and v is any point or direction on this part. In the following you may assume that we have routines to compute these for any q. The problem is to express other kinematic maps and their Jacobians in terms of these knowns. In the following you're asked to define more involved kinematic maps (a.k.a. task maps) $\phi(q)$ to solve certain motion problems. Please formulate all these maps such that the overall optimization problem is

$$f(q) = \|q - q_0\|^2_W + \|\phi(q)\|^2$$

that is, the motion aims to minimize $\phi(q)$ to zero (absorb the y^* in the definition of ϕ.)

a) You would like to control a standard endeffector position p_{eff} to be at y^*, as usual. Can you define a 1-dimensional task map $\phi : \mathbb{R}^n \to \mathbb{R}$ to achieve this? What is its Jacobian? (2P)

b) You would like the two hands of the robot to become parallel (e.g. for clapping). What task map can you define to achieve this? What is the Jacobian? (2P)

c) Assume you would like to control the pointing direction of the robot's head (e.g., its eyes) to point to a desired external world point x^W. What task map can you define to achieve this? What is the Jacobian? (3P)

2. a) $\phi^{pos}_{eff,p}(q) = T_{w\to eff}(q)\cdot p \in R^4$, $J^{pos}_{eff,p}(q) \in R^{4\times d}$, $q \in R^d$

$$\to \phi(q) = \|y^* - \phi^{pos}_{eff,p}\|_2 \in R = \sqrt{y^{*T}y^* - 2y^{*T}\cdot\phi^{pos}_{eff,p} + \phi^{pos\,T}_{eff,p}\cdot\phi^{pos}_{eff,p}}$$

$$J(q) = \frac{1}{2}\cdot\frac{1}{\|y^*-T\cdot p\|_2}\cdot\left(-2y^{*T} + 2\phi^{pos}_{eff,p}\right)J^{pos}_{eff,p}(q)$$

b) $\phi^{vec}_{eff,v}(q) = R_{w\to eff}(q)\cdot v \in R^3$, $J^{vec}_{eff,v}(q) \in R^{3\times d}$

minimizing $\phi(q)\to 0$, so that parellel.

$$\to \phi(q) = \left(\phi^{vec}_{eff,v}(q)\right)^T\cdot V_2 - 1 \in R$$

for orthogonal $\vec{a}\cdot\vec{b} = 0$

for parellel $\vec{a}\cdot\vec{b} = 1$

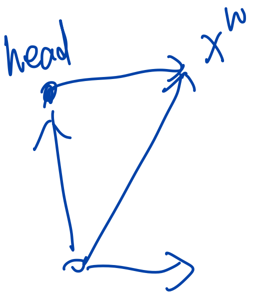
rotating V_1 so that it is parallel to V_2

$$J(q) = V_2^T\cdot J^{vec}_{eff,v}(q) \in R^{1\times d}$$ (use enumerator notation!)

c) need normalization!

direction vector attached to head (independent of q)

$$\phi(q) = \frac{\left(x^w - \phi^{pos}_{eff,p}\right)^T}{\|x^w - \phi^{pos}_{eff,p}\|}\cdot\phi^{vec}_{head,v} - 1$$



$$J(q) = \phi^{vec\,T}_{h,v}\cdot\frac{\partial}{\partial q}\frac{x^w - \phi^{pos}_{eff,p}}{\|x^w - \phi^{pos}_{eff,p}\|}$$

$$= \phi^{vec\,T}_{h,v}\left(\frac{1}{\|x^w - \phi^{pos}_{eff,p}\|}\left(-J^{pos}_{eff,p}\right) + \left(x^w - \phi^{pos}_{eff,p}\right)\frac{\partial}{\partial q}\frac{1}{\|x^w - \phi^{pos}_{eff,p}\|}\right)$$

$$= \phi^{vec\,T}_{h,v}\left(\frac{-J^{pos}_{eff,p}}{\|x^w - \phi^{pos}_{eff,p}\|} + \left(x^w - \phi^{pos}_{eff,p}\right)\frac{-1}{2\{(x^w-\phi)^T(x^w-\phi)\}^{\frac{3}{2}}}\left(2\phi^T J - 2x^{wT} J\right)\right)$$

$$= \phi^{vec\,T}_{h,v}\cdot\frac{1}{\|x^w - \phi^{pos}_{eff,p}\|}\left(-I + \frac{(x-\phi)(x-\phi)^T}{(x^w-\phi)^T(x^w-\phi)}\right)\cdot J^{pos}_{eff,p}$$