Optical Pumping, Masers and Lasers

Project Presentation - Phy 312

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An Overview

Fundamentals of Lasers

- Energy levels in atoms are discrete
- When electrons move from a higher energy level to a lower energy level, photons of a particular frequency are emitted
- If we have many electrons at a higher energy level (population inversion), many photons of the same frequency can be emitted, i.e. lasers

To establish how a population inversion can be achieved, we first have to delve into two-level systems

Two level systems

- The probability of finding the electrons in these two levels changes with time under external influences

- This change of probabilities results in transitions

- These transitions include absorption, stimulated emission or spontaneous emission. For laser action, we want stimulated emission to exceed the rest.

- We can determine the rates of spontaneous and stimulated emission to maximize the rate of stimulated emission

We can find the changing probabilities using the time-dependent perturbation theory.
Wavefunction $\psi(t)$ can be expressed as a superposition of eigenstates ψ_a and ψ_b of unperturbed Hamiltonian :

$$\psi(t) = c_a(t)\psi_a e^{-iE_a t/\hbar} + c_b(t)\psi_b e^{-iE_b t/\hbar}.$$

Assuming $E_b > E_a$, and defining $\omega_o = \dfrac{E_b - E_a}{\hbar}$:

$$\frac{d}{dt}c_a = -\frac{i}{\hbar}c_b \hat{H}'_{ab} e^{-i\omega_o t}, \qquad \frac{d}{dt}c_b = -\frac{i}{\hbar}c_a \hat{H}'_{ba} e^{i\omega_o t}. \tag{1}$$

For small $\hat{H}'$, we can find the coefficients by successive approximation using these equations.

First order approximation

Suppose a particle starts out in state ψ_a, at t = 0, the coefficients are :

$$c_a(0) = 1, \qquad c_b(0) = 0.$$

The first-order approximation is

$$c_a^{(1)} = 1$$

$$c_b^{(1)} = -\frac{i}{\hbar} \int_0^t \hat{H}'_{ba}(t') e^{i\omega_o t'} dt'. \tag{2}$$

Sinusoidal Perturbations

Since we are dealing with electromagnetic waves, the perturbations will be sinusoidal. Hence,

$$\hat{H}'_{ab} = V_{ab} \cos \omega t,$$

Using this in (2), we can find the probability of the particle being in the ψ_b to be

$$P_{a \to b} = |c_b|^2 = \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2 (\omega_0 - \omega) t/2}{(\omega_0 - \omega)^2}.$$

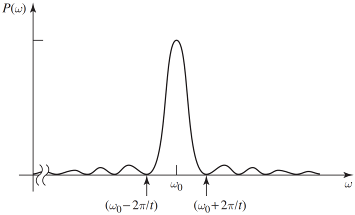


Figure – Transition probability as a function of ω_o [1].

Emission and Absorption of Radiation

Since the perturbation is sinusoidal,

$$V_{ba} = -\wp E_0. \tag{3}$$

where $\wp$ represents the off-diagonal elements of the z-component of the dipole moment operator. Inserting (3) in our expression for probability, we get

$$P_{a\to b} = \left(\frac{|\wp| E_0}{\hbar}\right)^2 \frac{\sin^2\left[(\omega_0 - \omega)t/2\right]}{(\omega_0 - \omega)^2}.$$

When an excited particle is exposed to photons, it makes a transition to the lower state. This is called **stimulated emission**. A photon with energy $\hbar\omega_0$ is emitted. If you shine one photon, two photons come out which leads to amplification.

If many electrons are excited, hitting them with a photon results in a chain reaction. Many photons are hence emitted at virtually the same instant and same frequency.

To achieve this, having more electrons in the upper state is essential. This is achieved through a process called **population inversion**. Otherwise, shining a photon on an atom may result in emission or absorption with the same probability. There will be no amplification.

Incoherent Perturbations

In a system with radiation coming from all directions of varying frequencies, energy density is not constant :

- Integration of energy density accounts for varying frequencies
- Averaging polarizations from all directions
- Probabilities are no longer oscillating
- Transition rate becomes time-independent

Transition rate

$$R_{b\to a} = \frac{\pi}{3\hbar^2\epsilon_0}|\vec{\wp}|^2\rho(\omega_0) = B_{ba}\rho(\omega_0). \tag{4}$$

Where B_{ba} is the rate of stimulated transitions per unit energy density.

Rate of Spontaneous Emission

Using B and studying a system in dynamic equilibrium,

$$\frac{dN_b}{dt} = B_{ab} N_a \rho(\omega_0) - A N_b - B_{ba} N_b \rho(\omega_0) = 0.$$

This gives us the rate of spontaneous emission per unit energy density,

$$A = \frac{\omega_0^3 |\vec{\wp}|^2}{3\epsilon_0 \hbar \pi c^3}.$$

Einstein's A and B coefficients

$$\frac{A}{B} = \frac{\hbar \omega_0^3}{\pi^2 c^3} \tag{5}$$

The stimulated emission rate must exceed the stimulated simultaneous rate for laser action.

As frequency increases, spontaneous emission increases, making it harder to make a laser and vice versa.

Lifetime of an Excited state

A determines the rate at which the population of the excited state decreases.

So, lifetime τ of an excited state becomes,

$$\tau = \frac{1}{A} \tag{6}$$

Selection Rules

The matrix elements for the perturbation are given by

$$\langle n^{'} l^{'} m^{'} | \vec{r} | nlm \rangle$$

The matrix elements are non-zero only if $\Delta l / \Delta m = 0 \text{ or } \pm 1$.
Matrix elements are also zero when $\Delta l = 0$ as $l + l'$ is even.

$$\Delta l \equiv l^{'} - l = \pm 1.$$

$$\Delta m \equiv m^{'} - m = 0 \text{ or } \pm 1.$$

Transition is only possible when these selection rules are satisfied

Multi Level Systems

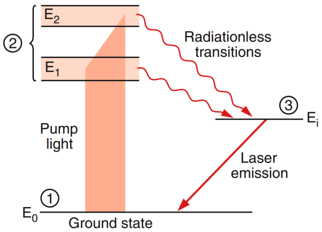
We cannot get population inversion in Two-Level Systems. The system will eventually be saturated.

We can alternatively use Three (or Four) Level Systems.

Three Level Systems

We can achieve population inversion as follows :

- Suppose we have three states, 1, 3 and 2, with increasing energy levels. At equilibrium $N_1 > N_3 > N_2$
- Pump electrons from 1 to 2. State 2 (small lifetime) rapidly decays into 3.
- 3 is a metastable state (long lifetime).
- Hence electrons can remain in 3 for a longer period
- A population inversion is achieved between 1 and 3.



Four Level Systems

- Four states 1, 2, 3 and 4. 2 and 4 have very short lifetimes.
- Electrons pumped from 1 to 2 (short lifetime) which rapidly transition to state 3 (longer lifetime).
- State 4 is empty while state 3 is populated : population inversion is achieved easily.
- When an electron from state 3 undergoes spontaneous emission, a cascade of photons is generated.
- τ_4 is also small, so electrons rapidly move to state 1. They are again pumped to state 2, resulting in a continuous laser.
- Four-level systems utilise lower pump energy (4 is empty), making it easier to achieve population inversion between states 3 and 4.

An example of such a four-level system is a He-Ne laser.

1. Optical Resonator
2. Active Medium
3. Energy Pump

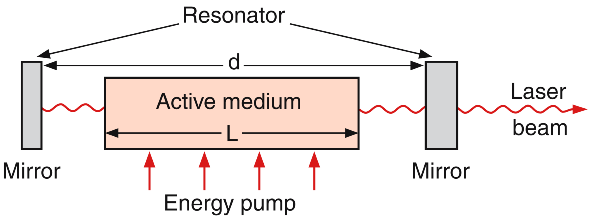


Figure – Basic illustration of a laser [3]

Gain Coefficient

$$I(z) = I_0 e^{G(\omega) z} \qquad G(\omega) = \frac{n \Delta N \pi^2 c^2}{\hbar \omega^3 \tau}$$

- $I(z) > I_0$ for light amplification
- For that $G(\omega) > 0$ i.e $N_b > N_a$
- Accounts for losses due to absorption by the active medium
- $G(\omega)$ becomes constant after ΔN_{thr} is reached
- Light is then emitted

Accounting for Losses

Defining a loss factor γ to include all losses such that occur after one round-trip

$$I(z) = I_0 e^{-(\gamma + G(\omega) z)}$$

Now for amplification,

$$\gamma + z \frac{n \Delta N \pi^2 c^2}{\hbar \omega^3 \tau} < 0$$

$$\Delta N > \gamma \frac{\hbar \omega^3 \tau}{z n \pi^2 c^2} = \Delta N_{thr},$$

where ΔN_{thr} is the minimum difference between N_b and N_a required for amplification to occur.

Frequency Spectrum in Optical Resonators

Only precise frequencies can be excited in the cavity of a resonator of length ‘L’ as the nodes must exist at the ends of a closed resonator :

$$2L = m\lambda \tag{7}$$

there is finite distance between allowed frequencies of $\Delta\omega = \frac{\pi c}{nL}$.

The energy difference between the excited and ground states fixes the emission frequency.

Resonance occurs when both conditions are fulfilled.

Still, a frequency spectrum is obtained due to the uncertainty principle, collisional and Doppler broadening.

Optical Resonators contd.

For shorter wavelengths, frequency spacing increases.

This allows for more modes to exist within a given spectral range.

Results in fewer emitted photons per mode.

Laser emission is in multiple directions, and directionality is lost.

More pumping energy is also required.

Hence, closed cavities are not suitable resonators.

Open Optical Resonators

Concentrates induced emission onto just a few modes.
Any suitable arrangement of optical mirrors can fulfil this condition.
One such open resonator contains two mirrors of reflectivity R1 and R2.
Due to reflection losses, the intensity per round trip decreases per round-trip as :

$$I(2d) = I_0 R_1 R_2 = I_0 e^{-\gamma_r}$$
$$\gamma_r = \ln(R_1 R_2)$$

Now, the mean lifetime of a photon τ is given by

$$\tau = \frac{2d}{c \cdot ln(R_1 R_2)}$$

A resonator with higher reflectivities or a shorter length will have a longer mean lifetime for photons.

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Thank You !

Any Questions ?