Landau Theory of Fermi Liquids

Course Project for PHY 633 Magnetism : Theory and Experiment

Name: Tayyaba Noureen

Student Number: 25100223

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1 Introduction

Landau theory of Fermi liquids describes the effect of electron-electron interaction. While a Fermi gas consists of noninteracting electrons, a Fermi liquid includes these interactions, revealing new phenomena and physical properties. Landau theory of the Fermi liquid describes these interactions via a few parameters. The central assumption of the Landau theory of Fermi liquids is that as we increase the strength of the electron-electron interaction from zero to its physical value, the ground state and the excitations evolve smoothly as long as we do not cross a phase transition. Quasi-particles describe the single-particle excitations. These are very similar to excitations in an electron gas. A quasi-particle can be considered a single particle accompanied by a distortion cloud in the electron gas, which modifies its effective mass and lifetime.

In this report, we explore both the qualitative aspects of quasiparticles—how they emerge and their physical interpretation—and the quantitative aspects of Landau-Fermi theory, including calculations of measurable quantities. In the first section (§ 2), we focus on the quasi-particle properties and calculate their lifetime, whose growth near the Fermi sphere allows for describing low-energy excitations. The stability of quasiparticles provides grounds for building up the theory based on their description. Then in section (§ 3), we explore the relation between bare electrons and quasiparticles, focusing on the role of the spectral function in describing quasiparticle properties. Using Landau parameters in section (§ 4), we parameterize the excitation energies and connect these parameters to measurable quantities in section (§ 5). Specifically, we calculate the heat capacity, compressibility, and magnetic susceptibility, showing how Landau parameters encapsulate interaction effects. Ultimately, we derive the relation between Landau parameters and effective masses using Galilean invariance in section (§ 6). These parameters and how interactions result in an effective mass are the hallmarks of Fermi liquid theory, as they allow a smooth description of interactions.

In this report, we closely follow the treatment done in Kruger's notes (1) and a compact overview of Fermi liquids provided by Michael Kinza in (2). To motivate the Fermi liquid theory we used (4) and important physical implications of the calculations were discussed in (5) and (3).

2 Quasiparticles and their Lifetime

In an non interacting Fermi gas, excitations are long lived (infinite lifetime in the absence of an external perturbation) because there are no scattering processes that result due to interactions. Once an excitation happens, there is no particle-particle interaction that would result in a decay. However, in a Fermi liquid, electron-electron interactions result in scattering processes that lead to a finite lifetime of the quasiparticles. If the lifetime of these quasiparticles is long lived, that would mean that they are stable particles with well-defined momentum, energy, and mass.

We assume that similar to Fermi gas, in Fermi liquid theory there exists a well defined Fermi Surface. In a Fermi gas, available electron states are filled upto a Fermi momentum k_F, with Fermi energy:

$$\epsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

We consider a

In order to calculate the lifetime of a quasi-particle, let us consider the interactions. They have the following form in position space:

$$\hat{H}_{int} = \frac{1}{2}\int d^3r \int d^3r' V(|\mathbf{r}-\mathbf{r'}|\hat{n}(\mathbf{r})\hat{n}(\mathbf{r'},$$

where $\hat{n}(\mathbf{r}) = \sum_{\sigma=\uparrow,\downarrow} \hat{n}_\sigma = \sum_\sigma c_\sigma^\dagger c_\sigma$ and $V(r)$ is some potential. In momentum space, we use the transformations:

$$
\begin{aligned}
c_\sigma(\mathbf{r}) &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{k}} \hat{c}_{\mathbf{k},\sigma} e^{i\mathbf{k}.\mathbf{r}} \\
c^\dagger_\sigma(r) &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{k}'} \hat{c}^\dagger_{\mathbf{k}',\sigma} e^{-i\mathbf{k}'.\mathbf{r}} \\
c_\sigma(\mathbf{r}') &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{k}''} \hat{c}_{\mathbf{k}'',\sigma} e^{i\mathbf{k}''.\mathbf{r}'} \\
c^\dagger_\sigma(\mathbf{r}') &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{k}'''} \hat{c}^\dagger_{\mathbf{k}''',\sigma} e^{-i\mathbf{k}'''.\mathbf{r}'} \\
V(|\mathbf{r}-\mathbf{r}'|) &= \frac{\mathcal{V}}{(2\pi)^3}\int d^3q\, \tilde{V}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')},
\end{aligned}
$$

where $\sqrt{\mathcal{V}}$ is the volume and $\tilde{V}(\mathbf{q})$ is the Fourier transform of $V(r)$:

$$\tilde{V}(\mathbf{q}) = \int d^3r\, V(r) e^{-i\mathbf{q}\cdot\mathbf{r}}.$$

Substituting these, we get:

$$
\begin{aligned}
\hat{H}_{int} &= \frac{1}{2}\frac{V}{(2\pi)^3\mathcal{V}^2}\int d^3r \int d^3r' \int d^3q\, \tilde{V}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \sum_{\mathbf{k}'} \hat{c}^\dagger_{\mathbf{k}',\sigma} e^{-i\mathbf{k}'.\mathbf{r}} \sum_{\mathbf{k}} \hat{c}_{\mathbf{k},\sigma} e^{i\mathbf{k}.\mathbf{r}} \sum_{\mathbf{k}'''} \hat{c}^\dagger_{\mathbf{k}''',\sigma} e^{-i\mathbf{k}'''.\mathbf{r}'} \sum_{\mathbf{k}''} \hat{c}_{\mathbf{k}'',\sigma} e^{i\mathbf{k}''.\mathbf{r}'} \\
&= \frac{1}{2}\frac{V}{(2\pi)^3\mathcal{V}^2}\int d^3r \int d^3r' \int d^3q\, \tilde{V}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} \sum_{\mathbf{k},\mathbf{k}',\mathbf{k}'',\mathbf{k}'''} \sum_{\sigma,\sigma'} \hat{c}^\dagger_{\mathbf{k}',\sigma} \hat{c}_{\mathbf{k},\sigma} \hat{c}^\dagger_{\mathbf{k}''',\sigma'} \hat{c}_{\mathbf{k}'',\sigma'} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} e^{i(\mathbf{k}''-\mathbf{k}''')\cdot\mathbf{r}'}
\end{aligned}
$$

When integrating over $\mathbf{r}$ and $\mathbf{r}'$ we get delta functions:

$$\int d^3r\, e^{i(\mathbf{k}-\mathbf{k}'+\mathbf{q})\cdot\mathbf{r}} = \mathcal{V}\delta(\mathbf{k}-\mathbf{k}'+\mathbf{q}),$$

$$\int d^3r'\, e^{i(\mathbf{k}''-\mathbf{k}'''-\mathbf{q})\cdot\mathbf{r}'} = \mathcal{V}\delta(\mathbf{k}''-\mathbf{k}'''-\mathbf{q}).$$

Using these delta functions and changing the volume integral to a summation the Hamiltonian becomes:

$$\hat{H}_{\text{int}} = \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \tilde{V}(\mathbf{q}) \hat{c}^\dagger_{\mathbf{k}-\mathbf{q},\sigma} \hat{c}^\dagger_{\mathbf{k}'+\mathbf{q},\sigma'} \hat{c}_{\mathbf{k}',\sigma'} \hat{c}_{\mathbf{k},\sigma}. \tag{2.1}$$

In this interaction Hamiltonian acts on an initial quasiparticle state with momentum $|\mathbf{k}| > k_F$, it loses energy due to scattering such that the new momentum is $\mathbf{k}-\mathbf{q}$ and creates another quasiparticle with momentum $\mathbf{k}'+\mathbf{q}$, leaving behind a quasihole as shown in figure 1.

The lifetime of a quasiparticle is inversely related to the scattering rates. The inverse lifetime $\tau^{-1}_{\mathbf{k}}$ of the quasiparticle with momentum k and initial state $|i\rangle$ is given by summing the scattering rates $w_{i\to f}$ over all final states $|f\rangle$. Using Fermi's Golden rule:

$$\tau^{-1}_{\mathbf{k}} = \sum_f w_{i\to f} = \frac{2\pi}{\hbar}\sum_f \left|\langle f|\hat{H}_{int}|i\rangle\right|^2 \delta(E_f - E_i) \tag{2.2}$$

The delta function ensures energy conservation. The initial and final energies are:

$$
\begin{aligned}
E_i &= E_{FS} + \epsilon_{\mathbf{k}} \\
E_f &= E_{FS} + \epsilon_{\mathbf{k}-\mathbf{q}} + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k'}},
\end{aligned}
$$

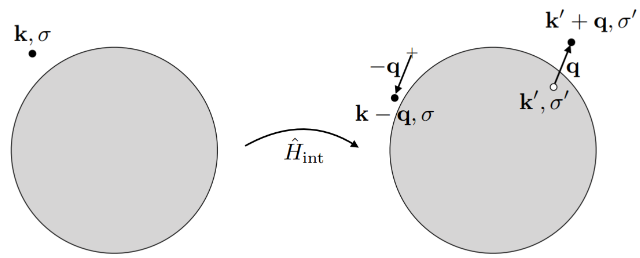

Figure 1: Quasi particle interaction on Fermi sphere (1)

where E_{FS} is the Fermi energy of the Fermi sea that is fully occupied. Inserting $\hat{H}_{int}$ from eq. 2.1 into eq. 2.2:

$$
\begin{aligned}
\tau_{\mathbf{k}}^{-1} &= \frac{\pi}{\hbar} \sum_{k,k',q,\sigma,\sigma'} |\tilde{V}(\mathbf{q})|^2 \sum_f \left| \langle f | \hat{c}^{\dagger}_{\mathbf{k}-\mathbf{q},\sigma} \hat{c}^{\dagger}_{\mathbf{k}'+\mathbf{q},\sigma'} \hat{c}_{\mathbf{k}',\sigma'} \hat{c}_{\mathbf{k},\sigma} | i \rangle \right|^2 \delta(E_f - F_i) \\
&= \frac{\pi}{\hbar} \sum_{k,k',q,\sigma,\sigma'} |\tilde{V}(\mathbf{q})|^2 n_F(\epsilon_k) n_F(\epsilon_{k'}) [(1 - n_F(\epsilon_{k-q})][1 - n_F(\epsilon_{k'+q})] \delta(E_f - F_i) \\
&= \frac{2\pi}{\hbar} \sum_{k',q,\sigma'} |\tilde{V}(\mathbf{q})|^2 n_F(\epsilon_k) n_F(\epsilon_{k'}) [(1 - n_F(\epsilon_{k-q})][1 - n_F(\epsilon_{k'+q})] \delta(E_f - F_i) \\
&= \frac{2\pi}{\hbar} \sum_{k',q,\sigma'} |\tilde{V}(\mathbf{q})|^2 n_F(\epsilon_k) n_F(\epsilon_{k'}) [(1 - n_F(\epsilon_{k-q})][1 - n_F(\epsilon_{k'+q})] \delta(\epsilon_{\mathbf{k}-\mathbf{q}} + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k'}} - \epsilon_{\mathbf{k}})
\end{aligned}
$$

The matrix element in the first step contributes factors of n_F and $1 - n_F$. $n_F(k)$ and $n_F k'$ ensures that the states k and k' are occupied. $[(1 - n_F(\epsilon_{k-q})][1 - n_F(\epsilon_{k'+q})]$ ensure that the final states are unoccupied and evaluating the sum over the final state gets us to the second line. In the third line, we evaluated the sum over k and σ, $\sum_k n_F(\epsilon_k) = 1$ and in the last step, substituted the values for E_f and E_i. Now, to solve this expression first note that:

$$
\sum_{k'} = \int d^3k' \frac{V}{(2\pi)^3}
$$

and the sum over σ' gives a factor of 2.

$$
\begin{aligned}
\tau_{\mathbf{k}}^{-1} &= \frac{2\pi}{\hbar} \int d^3k' \frac{V}{(2\pi)^3} \int d^3q |\tilde{V}(\mathbf{q})|^2 \delta(\epsilon_{\mathbf{k}-\mathbf{q}} + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k'}} - \epsilon_{\mathbf{k}}) \\
&= \frac{2\pi.4\pi}{\hbar} \frac{V}{(2\pi)^3} \int dk' k^{2'} \int d^3q |\tilde{V}(\mathbf{q})|^2 \delta(\epsilon_{\mathbf{k}-\mathbf{q}} + \epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k'}} - \epsilon_{\mathbf{k}}) \\
&= \frac{2\pi}{\hbar} \frac{V}{(2\pi)^3} \int d^3k' \int d^3q |\tilde{V}(\mathbf{q})|^2 \delta(\epsilon_{\mathbf{k}'+\mathbf{q}} - \epsilon_{\mathbf{k'}} - \hbar\omega_{\mathbf{q},\mathbf{k}}),
\end{aligned}
$$

$\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}-\mathbf{q}} = \frac{\hbar^2}{2m}[2\mathbf{k}\mathbf{q} - \mathbf{q}^2] \equiv \hbar\omega_{\mathbf{q},\mathbf{k}}$. The distributio functions that we had in the sum initially define the limits of integration. In order to evaluate this integral, we move to cylindrical coordinates, such that the axis of cylinder lies in the same direction as $\mathbf{q}$. $\mathbf{k}'$ gets decomposed into two components, the component parallel to cylinder's axis is $k'_\parallel$ and perpendicular to coordinate $k'_\perp$. Integration over the angular coordinates gives a factor of 2π.

$$\tau_{\mathbf{k}}^{-1} = \frac{1}{\hbar}\frac{V}{(2\pi)}\int d^3q|\tilde{V}(\mathbf{q})|^2\int_{k_2}^{k_1} dk'_\perp k'_\perp \int_0^{k_F} dk'_\parallel \delta(\frac{\hbar^2 q^2}{2m} + \frac{\hbar^2 q k'_\parallel}{2m} - \hbar\omega_{\mathbf{q},\mathbf{k}})$$

Now we do the integration over $k_\parallel$ using the delta function which sets $k'_\parallel = \frac{2m\omega_{\mathbf{q},\mathbf{k}} - \hbar\mathbf{q}^2}{2\hbar q}$. $k_1^2 = k_F^2 - (k'_{\parallel,0})^2$ and the other limit $k_2^2 = k_F^2 - (k'_{\parallel,0} + q)^2$. Using the delta function and then carrying out the integration over $k'_\perp$ and replacing constant terms with the density function $g(\epsilon_F)$ and with Fermi velocity $v_F = \frac{\hbar k_F}{m}$, we get

$$\tau_{\mathbf{k}}^{-1} = \frac{2\pi}{\hbar}\int d^3q|\tilde{V}(\mathbf{q})|^2 \frac{g(\epsilon_F)\omega_{\mathbf{q},k}}{2qv_F}$$

The integration limits are now determined by $(1 - n_F(\epsilon_{k-q})$. We integrate over values of $\mathbf{q}$ which lie outside k_F with momentum $\mathbf{k}$. So we get two conditions $k^2 \geq (\mathbf{k} - \mathbf{q})^2$ and $(\mathbf{k} - \mathbf{q})^2 \geq k_F^2$. If we integrate in spherical coordinates and call the angle between $\mathbf{k}$ and $\mathbf{q}$ θ, then opening the squares and simplifying the limits translate to $\cos\theta_1 \equiv \frac{q}{2k}$ and $\cos\theta_2 \equiv \frac{k^2+k_F^2+q^2}{2kq}$

$$\begin{aligned}
\tau_{\mathbf{k}}^{-1} &= \frac{g(\epsilon_F)V}{2\pi\hbar v_F}\int dq|\tilde{V}(\mathbf{q})|^2 q^2 \int_{\cos\theta_1}^{\cos\theta_2} d\cos\theta(2k\cos\theta - q) \\
&= \frac{g(\epsilon_F)V}{2\pi\hbar v_F}\int dq|\tilde{V}(\mathbf{q})|^2 \frac{1}{4k}(k^2 - k_F^2)^2
\end{aligned}$$

Now expanding the expression near k_F, and simplifying, we obtain

$$\tau_{\mathbf{k}}^{-1} = \frac{g(\epsilon_F)V}{2\pi\hbar v_F^2}\frac{m}{\hbar^4}(\epsilon_k - \epsilon_F)^2 \int dq|\tilde{V}(\mathbf{q})|^2$$

The integral over q goes from 0 to ∞ but $q \to \infty$ corresponds to small length scales which are unrealistic for a real material. Therefore, on physical grounds we set the upward limit for the cutoff. And as $q \to 0$ which corresponds to very large scales. We only consider short length interaction so this limit is not a problem for us. The potential usually has a coloumbic form, which as $q \to \inf$ goes to zero, resulting in lifetime of a quasiparticle approaching zero. However, that limit is not in question right now. Ignoring the constants and keeping realistic cutoffs for q, we obtain lifetime of the quasiparticle as

$$\tau_{\mathbf{k}}^{-1} = (\epsilon_k - \epsilon_F)^2$$

This expression shows that quasi particle life time becomes larger as $\epsilon_k \to \epsilon_F$. Therefore, quasiparticles are long lived near the Fermi surface and our description of Fermi liquids in terms of quasiparticles is valid.

3 Relation between Quasiparticles and Electrons

Lets say $c_{\mathbf{k}\sigma}$ is an annihilation operator of a bare electron with wave-vector $\mathbf{k}$ and spin σ. It acts on a ground state $|GS\rangle$ of the interacting system. Lets denote the amplitude of this action by $\sqrt{Z}$. Result will be a state $|\mathbf{k}\sigma\rangle$, that contains a quasihole if $k \leq k_F$ (because the ground system contained interacting quasiparticles and electron's action results in annihilation of a quasiparticle). Since the system is interacting, there will also be an additional amplitude that generates superpositions of many excitations. Lets denote these superpositions $|incoherent\rangle$. The assumption of Fermi liquid theory is that these excitations vary smoothly with $\mathbf{k}$. This process is summarized as:

$$c_{k\sigma} \approx \begin{cases} \sqrt{Z} + |\text{incoherent}\rangle & \text{if } k \leq k_F \\ \sqrt{Z} + |\text{incoherent}\rangle & \text{if } k \leq k_F \end{cases}$$

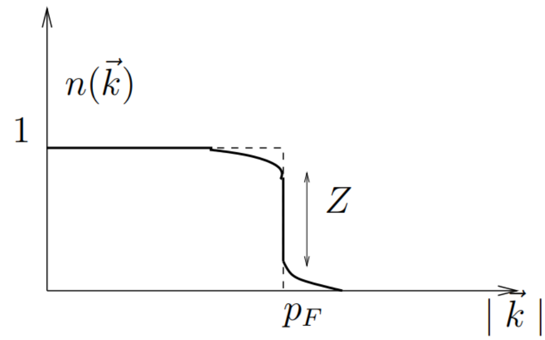

Figure 2: Discontinuity in the occupation number at Fermi surface (2)

At $T = 0$, Fermi occupation number $n_{\mathbf{k}\sigma}$ has a step size of Z at $k = k_F$. For Fermi gases this jump-size is 1 and so is the amplitude of annihilation operators acting on a non interacting ground state. This step size indicates the existence of a sharp Fermi surface in Fermi liquids that is even more defined in Fermi gases due to higher value of Z, as expected. This discontinuity is also called the Midgal discontinuity. It is shown in figure 2. Another important point to be noted here is that the interactions decrease Z, which in turn lead to higher effective masses of quasi particles as compared to non interacting fermions. Now, we see how relationship between momentum and energy of quasiparticles differs from that of bare fermions. We make use of the spectral function $A(\mathbf{k}, \omega)$ that describes the probability that a particle with momentum $\mathbf{k}$ has an energy ω. in the spectral function, the existence of quasi particles and the incoherent background can be observed. If $\omega > \epsilon_F$, $A(\mathbf{k}, \omega)$ is the probability that we can find the system in the state with momentum k and energy ω after adding an electron. If $\omega < \epsilon_F$, that means that we have extracted an electron and $A(\mathbf{k}, \omega)$ describes this probability for the particle being removed having an energy ω and momentum k. For Fermi gas, since there exists a well defined relation between energy ω and momentum k, the spectral function is a Dirac delta function:

$$A(\mathbf{k}, \omega) \approx \delta(\omega - \epsilon_k)$$

In the presence of interactions, the quasiparticle has a finite overlap with the fermion described by $\sqrt{Z} < 1$, so the spectral function is given by this interaction and the rest of the amplitude of the function is transferred to the incoherent excitations:

$$A(\mathbf{k}, \omega) = A_{\text{qp}}(\mathbf{k}, \omega) + A_{\text{inc}}(\mathbf{k}, \omega) \tag{3.1}$$

At $T = 0$ and at Fermi surface $\omega = \epsilon_F$, lifetime of the quasiparticle is infinite and its spectral function will become proportional to the Dirac delta function as $A(\mathbf{k}, \omega = \epsilon_F) \approx \sqrt{Z}\delta(\epsilon_F - \epsilon_k)$. As we move away from the Fermi surface, the lifetime of the quasi particle becomes finite and the peak of spectral functions broadens as more and more spectral weight is carried by the incoherent background.

4 Parametrising Excitation Energies

Fermi liquid theory assumes that excitations are described by quasiparticles that are very similar to electrons except that they include interactions and have different effective masses. This fact has also been observed experimentally. Kinetic energy of such quasi particles have a similar dispersion relation

$$\epsilon_0 \mathbf{k} = \frac{\hbar^2 k^2}{2m^*},$$

where $m*$ is the effective mass of quasiparticles which has been observed to be much larger than electron mass in some compounds. When we calculated life time of the quasiparticles we found that they are very well defined close to the fermi surface. Therefore, we can do an expansion of the disperson relation near the fermi surface with radius k_F

$$\epsilon_0(\mathbf{k}) \approx \epsilon_F + \frac{\hbar^2}{m^*} k_F (k - k_F).$$

We are assuming that the Fermi liquid of a fermi sphere is well defined. It is important to note here that Fermi surfaces have actually complicated geometry for strongly interacting real surfaces. Now, we have to see how do these energies change due to the interactions between the quasiparticles. Let's say quasiparticles are excited and their distribution changes as

$$n^{(0)}_{k\sigma} \to n_{k\sigma} = n^{(0)}_{k\sigma} + \delta n_{k\sigma}.$$

$n^{(0)}_{k\sigma}$ is the distribution at ground state before excitations and $n_{k\sigma}$ is the distribution resulting due to some perturbation. The resulting change in total energy up to a quadratic order is described by

$$\delta E = \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m*} \delta n_{k\sigma} + \frac{1}{2} \sum_{\mathbf{k}\sigma\mathbf{k}'\sigma'} f(\mathbf{k}\sigma, \mathbf{k}'\sigma') \delta n_{k\sigma} \delta n_{k'\sigma'} \tag{4.1}$$

We have introduced a function $\frac{\hbar^2 k^2}{2m*}$ is called the **Landau interaction function**. The first term shows the change in energy due to single particle contributions and the second term shows the change in energy due to interaction described by the Landau interaction function. As quasiparticles interact, the energy of each of them is dependent on distribution of other quasiparticles. Total energy is $E = sum_{\mathbf{k}\sigma} \epsilon_\sigma(\mathbf{k}) \implies \delta E = \sum_{\mathbf{k}\sigma} \epsilon_\sigma(\mathbf{k}) \delta n_{k\sigma}$. Using this and Eq. 4, we get

$$\begin{aligned} \epsilon_\sigma(\mathbf{k}) &= \frac{\delta(\delta E)}{\delta(\delta n_{\mathbf{k}\sigma})} \\ &= \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2} \sum_{\mathbf{k}'\sigma'} f(\mathbf{k}\sigma, \mathbf{k}'\sigma') \delta n_{k'\sigma'} \end{aligned} \tag{4.2}$$

The expression does not look useful because our expansion includes an undetermined function $f(\mathbf{k}\sigma, \mathbf{k}'\sigma')$. Since, we are close to the fermi surface $k' \approx k \approx k_F$, therefore $f(\mathbf{k}\sigma, \mathbf{k}'\sigma')$ and $\delta n_{k\sigma}$ primarily depend on the angle θ between $\mathbf{k}$ and $\mathbf{k}'$. Therefore, we can make use of spherical harmonics. Moreover, we have assumed that the Fermi surface is spherical, therefore, only zeroth and first harmonics are important. First, we decompose $f(\mathbf{k}\sigma, \mathbf{k}'\sigma')$ into two terms $f^{(s)}$ and $f^{(a)}$

$$\begin{aligned} f(\mathbf{k}\uparrow, \mathbf{k}'\uparrow) = f(\mathbf{k}\downarrow, \mathbf{k}'\downarrow) = f^{(s)}(\mathbf{k}, \mathbf{k}') + f^{(a)}(\mathbf{k}, \mathbf{k}'), \\ f(\mathbf{k}\uparrow, \mathbf{k}'\downarrow) = f(\mathbf{k}\downarrow, \mathbf{k}'\uparrow) = f^{(s)}(\mathbf{k}, \mathbf{k}') - f^{(a)}(\mathbf{k}, \mathbf{k}'). \end{aligned}$$

Since spins can be positive or negative, we can combine these equations as

$$f(\mathbf{k}\sigma, \mathbf{k}'\sigma') = f^{(s)}(\mathbf{k}, \mathbf{k}') + \sigma\sigma' f^{(a)}(\mathbf{k},\ \mathbf{k}')$$

Now, we can make use of spherical harmonics that reduces the expressions for $f^{(a,s)}$ into an expansion in terms of Legendre polynomials $P_\ell(x)$, $x = \cos\theta$,

$$f^{(a,s)}(\mathbf{k}, \mathbf{k}') = \sum_{\ell=0}^{\infty} f^{(a,s)}_\ell P_\ell(\cos\theta). \tag{4.3}$$

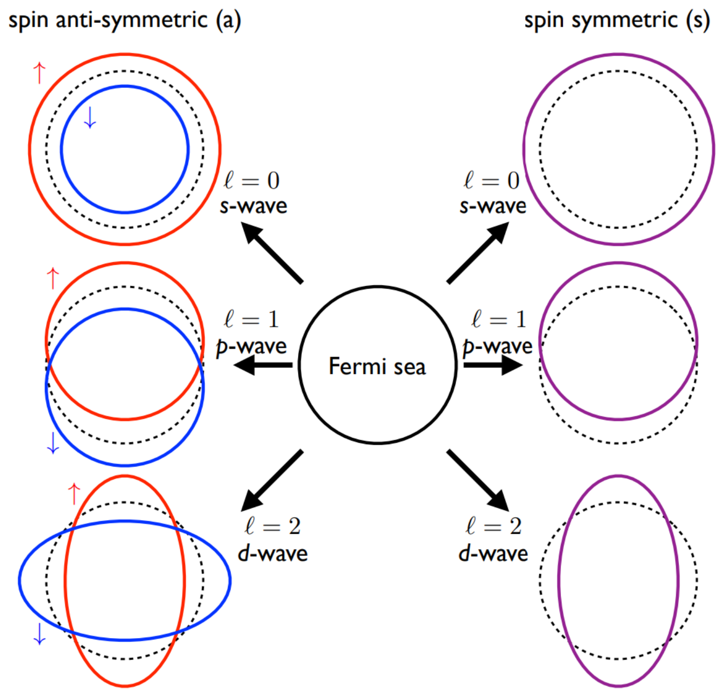

Figure 3: Deformations in Fermi surface

The first Legendre polynomials are given by $P_0(x) = 1$, $P_1(x) = x$ and $P_2(x) = \frac{1}{2}(3x^2 - 1)$. This distinction into symmetric and antisymmetric part based on the spins leads to deformations in the Fermi surface as shown in figure 3. $\ell = 0$ corresponds to an isotropic s-wave, uniform expansion or compression, $\ell = 1$ corresponds to displacement along a single direction and $\ell = 2$ corresponds to elongation along different direction depending on spin symmetry. $f^{(s)}$ leads to identical Fermi surface deformations for both spin-up and spin-down particles. $f^{(a)}$ results in opposite deformations for spin-up and spin-down particles. This reflects the different behavior of quasiparticles with opposite spins due to the antisymmetric part of the interaction. From Eq. , we can see that $f^{(a,s)}$ and and therefore $f_\ell^{(a,s)}$ carry units of energy. We introduce dimensionless Landau parameters

$$F_\ell^{(a,s)} = g^*(\epsilon_F) f_\ell^{(a,a)}$$

where density of states at the Fermi level is given by

$$g^*(\epsilon_F) = \frac{V}{2\pi^2} \left(\frac{2m^*}{\hbar^2} \right)^{3/2} \sqrt{\epsilon_F} = \frac{V m^* k_F}{\pi^2 \hbar^2}$$

Expanding Eq. 4.3 to the first two harmonics and expressing it in terms of the dimensionless parameters

$$f^{(a,s)}(\mathbf{k}, \mathbf{k}') \approx \frac{1}{g^*(\epsilon_F)} (F_0^{(a,s)} + F_1^{(a,s)} \cos\theta).$$

This way, using this expansion we have reduced $f(\mathbf{k}\mathbf{k}')$ to knowing two dimensionless parameters. In the next section, we see how to determine these.

5 Measuring the Landau parameters

The isotropic Fermi liquid is parametrised by a few dimensionless parameters,

$$\frac{m^*}{m}, F_0^{(s)}, F_0^{(a)}, F_1^{(s)}.$$

These parameters give us physical quantities such as specific heat, and magnetic susceptibility.

5.1 Heat Capacity

A finite temperature, distribution is uniform $n_{\mathbf{k}\sigma} = n^{(0)}_{\mathbf{k}\sigma} + \delta n_{\mathbf{k}\sigma}$ and the number of quasiparticles and quasiholes are the same so when we sum them up we get

$$\sum_{\mathbf{k}} \delta n_{\mathbf{k}\sigma} = 0.$$

The distribution is radially symmetric for which only $\ell = 0$ component contributes so we need to consider the parameters $F_0^{(s)}$ and $F_0^{(a)}$. In general, this follows from the orthogonality of the Legendre polynomials

$$\int_{-1}^{1} dx\, P_\ell(x) P_{\ell'}(x) = \int_0^\pi d\theta\, \sin\theta\, P_\ell(\cos\theta) P_{\ell'}(\cos\theta) = \frac{2}{2\ell+1}\delta_{\ell,\ell'},$$

implying that the ℓ - wave component can only couple to $F_\ell^{(s)}$ and $F_\ell^{(a)}$. Using Eq. 4.2 and decomposing into spin symmetric and antisymmetric parts

$$\begin{aligned}
\epsilon_\sigma(\mathbf{k}) &= \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2}\sum_{\mathbf{k}'\sigma'} f(\mathbf{k}\sigma, \mathbf{k}'\sigma')\delta n_{k'\sigma'} \\
&= \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2}\sum_{\mathbf{k}'\sigma'} \left[f^{(s)}(\mathbf{k}, \mathbf{k}') + \sigma\sigma' f^{(a)}(\mathbf{k}, \mathbf{k}') \right] \delta n_{k'\sigma'} \\
&= \frac{\hbar^2 k^2}{2m^*} + \frac{1}{2g^*(\epsilon_F)}\sum_{\mathbf{k}'\sigma'} \left[F_0^{(s)} + \sigma\sigma' F_0^{(a)} \right] \delta n_{k'\sigma'} \\
&= \frac{\hbar^2 k^2}{2m^*}.
\end{aligned}$$

In the third line we replaced the parameters with dimensionless Landau parameters and set $\ell = 0$ as only those Landau paramters contribute for an isotropic distribution. In the last line we used $\sum_{\mathbf{k}'} \delta n_{\mathbf{k}'\sigma} = 0$. Therefore, the interaction parameters do not contribute in this case. Interaction parameters only affect C_v through effective mass. Therefore, heat capacity for a Fermi liquid has the same expression as Fermi gases except that mass is replaced by effective mass

$$C_v = \frac{\pi^2}{3} k_B^2 g^*(\epsilon_F) T,$$

where $g^*(\epsilon_F)$ is

$$g^*(\epsilon_F) = \frac{m^* k_F}{\hbar^2 \pi^2}.$$

Comparing this expression with the heat capacity of a Fermi gas and considering the coefficient called the Sommerfeld coefficient that is given by

$$\gamma = \frac{C_v}{T},$$

we get the ratio of the Sommerfeld coefficients of Fermi liquid and Fermi gas (γ_0) as

$$\frac{\gamma}{\gamma_0} = \frac{g^*(\epsilon_F)}{g(\epsilon_F)} = \frac{m^*}{m}.$$

5.2 Magnetic Pauli Susceptibility and Compressibility

Consider a magnetic Zeeman field B that generates a spherically symmetric, spin antisymmetric change in quasiparticle distribution. Due to the applied field distribution changes as $\delta n_\uparrow = -\delta n_\downarrow$. Fermi energy is shifted as $\epsilon_F \to \epsilon_F + \delta n_\uparrow$ and for spin down electrons it is shifted as $\epsilon_F \to \epsilon_F - \delta n_\uparrow$ as shown in figure 4

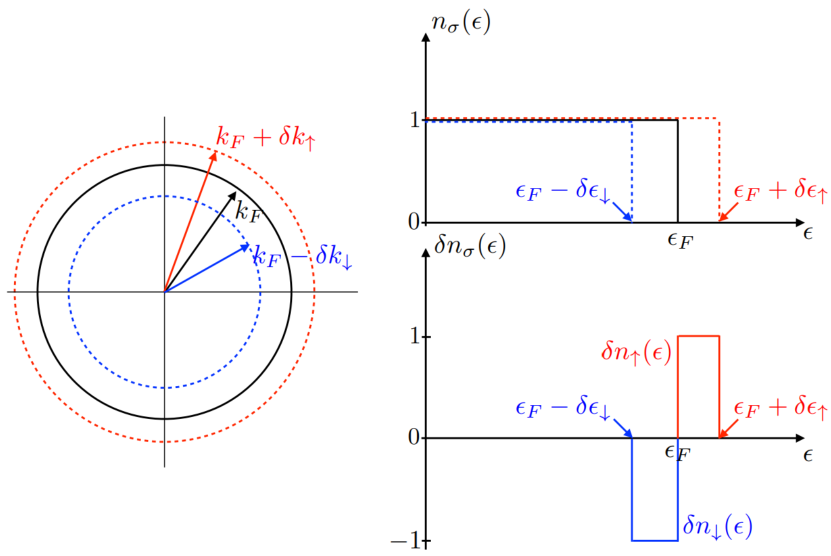

Figure 4: Shift in distribution due to applied magnetic field (1)

The change in energy results in change in density of states. Magnetization of the system due to the change in density is:

$$M = \frac{1}{2} g\mu_B(\delta n_\uparrow - \delta n_\downarrow) = g\mu_B \delta n$$

where g is the g-factor for the quasiparticle and μ_B is the Bohr magneton. Now we calculate the change in energy

$$
\begin{aligned}
\delta E &= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} \delta n_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k}'\sigma', \mathbf{k}\sigma} f(\mathbf{k}\sigma, \mathbf{k}'\sigma') \delta n_{k'\sigma'} \delta n_{\mathbf{k}\sigma} \\
&= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} n_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k}'\sigma', \mathbf{k}\sigma} \left[f^{(s)}(\mathbf{k}, \mathbf{k}') + \sigma\sigma' f^{(a)}(\mathbf{k}, \mathbf{k}') \right] \delta n_{k'\sigma'} \delta n_{\mathbf{k}\sigma} \\
&= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} n_{\mathbf{k}\sigma} + \frac{1}{2g^*(\epsilon_F)} \sum_{\mathbf{k}'\sigma', \mathbf{k}\sigma} \left[F_0^{(s)} + \sigma\sigma' F_0^{(a)} \right] \delta n_{k'\sigma'} \delta n_{\mathbf{k}\sigma} \\
&= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} n_{\mathbf{k}\sigma} + \frac{1}{2g^*(\epsilon_F)} \sum_{\mathbf{k}'\sigma'} \left[F_0^{(s)} (\delta n_\uparrow + \delta n_\downarrow) \sigma\sigma' + F_0^{(a)} (\delta n_\uparrow - \delta n_\downarrow) \right] \delta n_{k'\sigma'} \\
&= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} n_{\mathbf{k}\sigma} + \frac{1}{2g^*(\epsilon_F)} \sum_{\mathbf{k}'\sigma'} \left[F_0^{(s)} (\delta n_\uparrow + \delta n_\downarrow) + \sigma' F_0^{(a)} (\delta n_\uparrow - \delta n_\downarrow) \right] \delta n_{k'\sigma'}
\end{aligned}
$$

Using the relation $\delta n_\uparrow = -\delta n_\downarrow$, we see that the symmetric parameter does not contribute and for the antisymmetric part, the change in two density functions is added up, so we get

$$
\begin{aligned}
\delta E &= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} \delta n_{\mathbf{k}\sigma} + \frac{1}{2g^*(\epsilon_F)} \sum_{\mathbf{k}'\sigma'} \left[\sigma' F_0^{(a)} (2\delta n_\uparrow) \right] \delta n_{k'\sigma'} \\
&= \sum_{\mathbf{k}\sigma} \frac{\hbar^2 k^2}{2m^*} \delta n_{\mathbf{k}\sigma} + \frac{2}{g^*(\epsilon_F)} \left[F_0^{(a)} (\delta n_\uparrow)^2 \right]
\end{aligned}
$$

We can solve this by expanding first term to get energy change and then get Pauli Susceptibility. Instead, we consider the full expression for energy and consider the symmetric and antisymmetric changes to the Fermi surface and consider the case where Fermi surface expands to get how compressibility comes about and consider the case for the antisymmetric change which results due to the applied field.
We now consider the combined effects of symmetric and antisymmetric changes to the Fermi surface energy, where spin-up ($\uparrow$) and spin-down ($\downarrow$) densities are modified as follows:

$$
\delta n_\uparrow = \int_{\epsilon_F}^{\epsilon_F + \delta\epsilon_F + \mu_B B} g(\epsilon)\, d\epsilon, \quad \delta n_\downarrow = \int_{\epsilon_F}^{\epsilon_F + \delta\epsilon_F - \mu_B B} g(\epsilon)\, d\epsilon.
$$

We expand $g(\epsilon)$ around ϵ_F using a Taylor series:

$$
g(\epsilon) \approx g(\epsilon_F) + (\epsilon - \epsilon_F) \left. \frac{dg(\epsilon)}{d\epsilon} \right|_{\epsilon_F}.
$$

Substituting into the expressions for $\delta n_\uparrow$ and $\delta n_\downarrow$, we get:

$$
\delta n_\uparrow = \int_{\epsilon_F}^{\epsilon_F + \delta\epsilon_F + \mu_B B} \left[g(\epsilon_F) + (\epsilon - \epsilon_F) \left. \frac{dg(\epsilon)}{d\epsilon} \right|_{\epsilon_F} \right] d\epsilon,
$$

$$
\delta n_\downarrow = \int_{\epsilon_F}^{\epsilon_F + \delta\epsilon_F - \mu_B B} \left[g(\epsilon_F) + (\epsilon - \epsilon_F) \left. \frac{dg(\epsilon)}{d\epsilon} \right|_{\epsilon_F} \right] d\epsilon.
$$

Performing the integrals, we get:

$$
\delta n_\uparrow = g(\epsilon_F) (\delta\epsilon_F + \mu_B B) + \frac{1}{2} \left. \frac{dg(\epsilon)}{d\epsilon} \right|_{\epsilon_F} (\delta\epsilon_F + \mu_B B)^2,
$$

$$\delta n_\downarrow = g(\epsilon_F)\left(\delta\epsilon_F - \mu_B B\right) + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left(\delta\epsilon_F - \mu_B B\right)^2.$$

The total change in density, $\delta n = \delta n_\uparrow + \delta n_\downarrow$, and the magnetization-related change, $\delta m = \delta n_\uparrow - \delta n_\downarrow$, are given by:

$$\delta n = 2g(\epsilon_F)\delta\epsilon_F + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left[(\delta\epsilon_F + \mu_B B)^2 + (\delta\epsilon_F - \mu_B B)^2\right],$$

$$\delta m = 2g(\epsilon_F)\mu_B B + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left[(\delta\epsilon_F + \mu_B B)^2 - (\delta\epsilon_F - \mu_B B)^2\right].$$

Expanding these expressions

$$(\delta\epsilon_F + \mu_B B)^2 = (\delta\epsilon_F)^2 + 2\delta\epsilon_F\mu_B B + (\mu_B B)^2,$$

$$(\delta\epsilon_F - \mu_B B)^2 = (\delta\epsilon_F)^2 - 2\delta\epsilon_F\mu_B B + (\mu_B B)^2.$$

Substituting into δn and δm we get

$$\delta n = 2g(\epsilon_F)\delta\epsilon_F + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left[2(\delta\epsilon_F)^2 + 2(\mu_B B)^2\right],$$

$$\delta m = 2g(\epsilon_F)\mu_B B + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left[4\delta\epsilon_F\mu_B B\right].$$

We can use these expressions to get δE_F in terms of δn From the symmetric change in density:

$$\delta n = 2g(\epsilon_F)\delta\epsilon_F + \frac{1}{2}\frac{dg(\epsilon)}{d\epsilon}\bigg|_{\epsilon_F}\left[2(\delta\epsilon_F)^2 + 2(\mu_B B)^2\right].$$

Solving for $\delta\epsilon_F$ and approximating

$$\delta\epsilon_F \approx \frac{\delta n}{2g(\epsilon_F)} - \frac{(\delta n)^2}{4\epsilon_F g(\epsilon_F)^2}.$$

Substitute $\delta\epsilon_F$ into the energy change expression that contains both symmetric and asymmetric contributions:

$$\delta E = \epsilon_F\delta n + \frac{1 + F_0^{(s)}}{2g(\epsilon_F)}(\delta n)^2 - g\mu_B B\delta n_\uparrow + \frac{2}{g(\epsilon_F)}\left(1 + F_0^{(a)}\right)(\delta n_\uparrow)^2.$$

With $\delta n_\uparrow = \frac{\delta m}{2}$, we get

$$\delta E = (\epsilon_F)\,\delta n + \frac{1}{2g^*(\epsilon_F)}\left[(1 + F_0^{(s)})\right](\delta n)^2 + \left(-\frac{g\mu_B B}{2}\delta m\right) + \frac{1}{2g^*(\epsilon_F)}(1 + F_0^{(a)})(\delta m)^2.$$

This is the a combined expression accounting for both symmetric and antisymmetric Fermi surface changes. If we restore the sum over spins and now consider the case where there is no expansion of the Fermi surface and the distribution is given by $\delta n_\uparrow = -\delta n_\downarrow$, and $\delta n = \delta n_\uparrow + \delta n_\downarrow$ and we get

$$\delta E = -g\mu_B B\delta n_\uparrow + \frac{2}{g^*(\epsilon_F)}\left(1 + F_0^{(a)}\right)(\delta n_\uparrow)^2. \tag{5.1}$$

If we consider the case where no magnetic field was applied and hence $\delta m = 0$, we retain

$$\delta E = (\epsilon_F)\,\delta n + \frac{1}{2g^*(\epsilon_F)}\left[(1 + F_0^{(s)})\right](\delta n)^2 \tag{5.2}$$

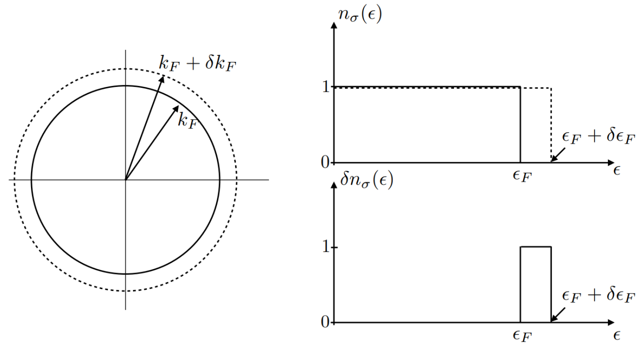

Figure 5: Change in occupation number for isothermal expansion

where δn isotropic and spin independent and throughout these calculations we were at at $T = 0$, which accounts for isothermal compressibility. This part of change in energy only accounts for the case where $\delta\epsilon_F \to \epsilon_F + \delta\epsilon_F$ as shown in the Figure 5. Now, the expression for inverse compressibility κ^{-1} is

$$\kappa^{-1} = -V\frac{\partial p}{\partial V} = V\frac{\partial^2 E}{\partial V^2} \tag{5.3}$$

and the expression is evaluated at constant T and N. Approximating $\delta n \approx \delta V$ and using Eq. 5.2, we get compressibility of a Fermi liquid as

$$\kappa \approx \frac{g^*(\epsilon_F)}{1 + F_0^{(s)}}.$$

Now lets consider Eq. 5.1 to get magnetic susceptibility. From the expression we can quickly note that the energy is minimum when $\delta n_\uparrow = \frac{1}{4}g\mu_B B\frac{g^*(\epsilon_F)}{1+F_0^{(a)}}$ and the magnetization is then given by

$$M = g\mu_B\delta n_\uparrow = \frac{1}{4}g^2\mu_B^2 B\frac{g^*(\epsilon_F)}{1 + F_0^{(a)}}.$$

Using this, we find out susceptibility as

$$\chi = \frac{\partial M}{\partial B} = \frac{\mu_B^2 g^*(\epsilon_F)}{1 + F_0^{(a)}},$$

where g factor for quasiparticles is approximated as. Its ratio with the spin susceptibility of Fermi gas is given by

$$\frac{\chi}{\chi_0} = \frac{m^*/m}{1 + F_0^{(a)}}$$

From the expression for magnetic susceptibility we see that susceptibility of a Fermi liquid is larger as the Landau parameter becomes negative and as $F_0^{(a)} \to -1$, χ diverges which leads to ferromagnetism and is called Stoner's instability. Now that we have heat capacity and susceptibility, we can calculate Wilson ratio R_W of the fermi liquid.

$$R_w = \frac{\pi^2 k_B^2 \chi}{3\mu_B^2\gamma} = \frac{\pi^2}{3\mu_B^2}\frac{\frac{m/m^*}{1+F_0^a}}{m/m^*\gamma_0} = \frac{1}{1 + F_0^{(a)}}$$

where all the constants give Wilson ratio of Fermi gas which is 1. Therefore, from this expression we can see that Wilson ratio much larger than one results in a ferromagnetic instability.

6 Effective Mass and Galilean Invariance

In this section we see how effective mass m^* is related to bare mass and Fermi liquid parameters. We can get a simple expression relating the three by considering a Galilean invariant system. Consider a Galilean transformation to a frame that moves at speed $\vec{v}$, its Hamiltonian, energy and momentum transform as

$$\begin{aligned} H' &= H - \vec{P}.\vec{v} + \frac{1}{2}M\vec{v}^2 \\ E' &= E - \vec{P}.\vec{v} + \frac{1}{2}M\vec{v}^2 \\ \vec{P}' &= \vec{P} - M\vec{V}. \end{aligned}$$

where $\vec{P}$ is the momentum in lav frame. Total mass of the system is Nm. Addition of a quasiparticle with mass m changes it as $M \to M + m$ and in lab frame momentum increases by $\vec{p} = m\vec{v}$ and energy by $\epsilon_\sigma(\vec{p})$. While in the moving frame momentum changes as

$$\delta\vec{P}' = \vec{p} - m\vec{v}$$

and energy changes by

$$\epsilon(\vec{p}) = \vec{p}.\vec{v} + \frac{1}{2}m\vec{v}^2$$

quasiparticle energy in the moving frame is then

$$\begin{aligned} \epsilon'(\vec{p} - m\vec{v}) &= \epsilon(\vec{p}) - \vec{p}.\vec{v} + \frac{1}{2}m\vec{v}^2 \\ \epsilon'(\vec{p}) &= \epsilon(\vec{p} + m\vec{v}) - \vec{p}.\vec{v} - + \frac{1}{2}m\vec{v}^2 \end{aligned}$$

We expand the energy $\epsilon(\vec{p} + m\vec{v})$ to first order in $\vec{v}$

$$\epsilon(\vec{p} + m\vec{v}) \approx \epsilon(\vec{p}) + \left(\frac{\partial\epsilon}{\partial\vec{p}}\right) \cdot (m\vec{v}),$$

Near the Fermi surface, the quasiparticle velocity is defined as

$$\frac{\partial\epsilon}{\partial\vec{p}} = \frac{\vec{p}}{m^*}.$$

Substituting this into the expansion

$$\epsilon(\vec{p} + m\vec{v}) \approx \epsilon(\vec{p}) + \frac{\vec{p}}{m^*} \cdot (m\vec{v}).$$

Substituting $\epsilon(\vec{p} + m\vec{v})$ into the original expression gives

$$\epsilon'(\vec{p}) \approx \epsilon(\vec{p}) + \frac{\vec{p}}{m^*} \cdot (m\vec{v}) - \vec{p} \cdot \vec{v} + \frac{1}{2}mv^2.$$

$$\epsilon'(\vec{p}) \approx \epsilon(\vec{p}) + \left(\frac{m}{m^*} - 1\right)\vec{p} \cdot \vec{v}. \tag{6.1}$$

Now, let's consider the particle distribution. From the moving frame, the ground state appears as a Fermi surface shifted by $-m\vec{v}$ resulting in the transformation of occupation number as

$$n'_{\vec{p}} = n^0_{\vec{p}+m\vec{v}} \approx n^0_{\vec{p}} + m\vec{v} \cdot \frac{\partial n^0_{\vec{p}}}{\partial\vec{p}},$$

where $n^0_{\vec{p}}$ is the equilibrium distribution in the lab frame. As a function of occupation number the quasiparticle energy in the moving frame is given by:

$$\epsilon'_{\vec{p}} = \epsilon_{\vec{p}}[n'_{\vec{p}}] = \epsilon_{\vec{p}}[n^0_{\vec{p}+m\vec{v}}].$$

Substituting the occupation number expansion to get the energy

$$\epsilon'_{\vec{p}} = \epsilon_{\vec{p}} + \sum_{\vec{p}'} \frac{\partial \epsilon_{\vec{p}}}{\partial n_{\vec{p}'}} \cdot m\vec{v} \cdot \frac{\partial n^0_{\vec{p}'}}{\partial \vec{p}'}.$$

Using Landau's interaction parameter

$$\frac{\partial \epsilon_{\vec{p}}}{\partial n_{\vec{p}'}} = \frac{f_{\vec{p}\vec{p}'}}{V},$$

we get

$$\epsilon'_{\vec{p}} = \epsilon_{\vec{p}} + \frac{1}{V} \sum_{\vec{p}'} f_{\vec{p}\vec{p}'} \cdot m\vec{v} \cdot \frac{\partial n^0_{\vec{p}'}}{\partial \vec{p}'}.$$

Near the fermi surface we have

$$\frac{\partial n^0_{\vec{p}'}}{\partial \vec{p}'} \approx -\delta(\epsilon_{\vec{p}'} - \epsilon_F) \cdot \frac{\vec{p}'}{m^*}.$$

Substituting this into the energy expansion

$$\epsilon'_{\vec{p}} = \epsilon_{\vec{p}} - \frac{1}{V} \sum_{\vec{p}'} f_{\vec{p}\vec{p}'} \cdot \frac{\vec{p}'}{m^*} \cdot m\vec{v} \cdot \delta(\epsilon_{\vec{p}'} - \epsilon_F).$$

From the figure, we can see that the displaced Fermi surface and the one that retains spherical symmetry is characterized by the $\ell = 1$ symmetric Landau parameter.

$$\frac{1}{V} \sum_{\vec{p}'} f_{\vec{p}\vec{p}'} \cdot \frac{\vec{p}'}{m^*} \cdot \delta(\epsilon_{\vec{p}'} - \epsilon_F) = -\frac{F_1^S}{3} \frac{\vec{p}}{m^*}.$$

Thus, the energy becomes:

$$\epsilon'_{\vec{p}} = \epsilon_{\vec{p}} - \frac{F_1^S}{3} \frac{m}{m^*} \vec{p} \cdot \vec{v}.$$

Comparing this energy with the energy we got from expansion near the Fermi surface in Eq. 6.1, we get

$$\frac{m}{m^*} - 1 = -\frac{m}{m^*} \frac{F_1^{(s)}}{3}$$
$$\frac{m^*}{m} = 1 + \frac{F_1^{(s)}}{3}$$

This expression shows that the deviation of effective mass from bare mass is determined $F_1^{(s)}$.

7 Conclusions

By considering the quasiparticles in a Fermi liquid as elementary excitations above the Fermi surface, we have discussed a framework that effectively describes many of the thermodynamic and transport properties of metals and other Fermi systems. The key concept in Landau's theory is the idea of the effective mass of quasiparticles, which determines their response to external fields and interactions. In Fermi liquids, despite strong interactions between particles, the system behaves in many ways as though the particles are non-interacting, with modified properties. The theory is successful in describing a wide range of phenomena, including specific heat capacity, spin susceptibility, and electrical conductivity in metals. We also introduced the notion of Landau parameters, which characterize the interactions between quasiparticles. These parameters provide a detailed description of how the interactions modify the properties of the system. In conclusion, the Landau Fermi liquid theory remains a fundamental and powerful framework for describing the low-temperature behavior of interacting Fermionic systems. It has broad applications in understanding weakly interacting systems and continues to influence the study of strongly correlated systems.

8 Bibliography

References

[1] Krüger, F. *PHAS0059 Theoretical Condensed Matter Lecture Notes*, University College London, Department of Physics and Astronomy.

[2] Kinza, M. *Theory of Fermi-liquids in Metals: A Compact Overview as an Introduction to Theoretical Solid-State Physics*, Springer.

[3] University of Geneva Designer Quantum Materials Lab. *Lecture Notes on Fermi Liquids*, Retrieved from http://eduardo.physics.illinois.edu/phys561/FL.pdf.

[4] Kittel, C. *Introduction to Solid State Physics*, John Wiley & Sons.

[5] White, R. M. *Quantum Theory of Magnetism*, Springer Series in Solid-State Sciences, Springer Berlin.