Gravitational Collapse

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SBASSE LUMS

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- Interior of star is a perfect pressureless fluid called dust

Exterior Geometry

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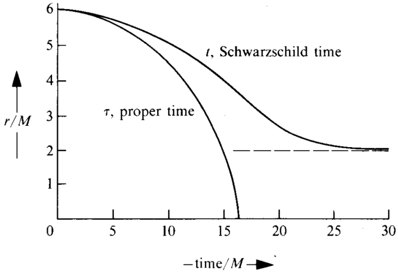
- Metric:

$$ds^2 = -\left(1 - \frac{2M}{R}\right) dt^2 + \frac{1}{-\left(1 - \frac{2M}{R}\right)} dr^2 + r^2 d\Omega^2$$

- Surface of star freely falls radially

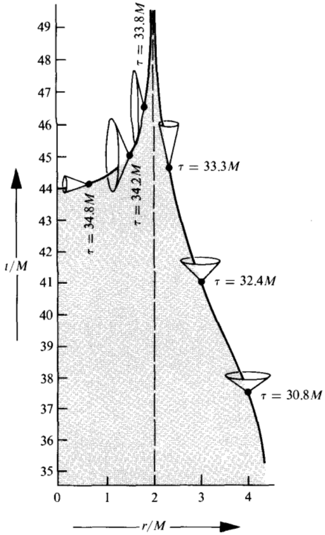
$$r = \frac{R_i}{2}(1+\cos\eta) \quad \tau = \frac{R_i}{2}\left(\frac{R}{2M}\right)^{1/2}(\eta+\sin\eta)$$

$$t = 2M\ln\left[\frac{(R_i/2M-1)^{1/2}+\tan\eta/2}{(R_i/2M-1)^{1/2}-\tan\eta/2}\right] + 2M(R_i/2M-1)^{1/2}(\eta+(R_i/4M)(\eta+\sin\eta))$$



- Time elapsed for comoving observer:

$$\Delta\tau = \pi\sqrt{\frac{R_i^3}{8M}}.$$



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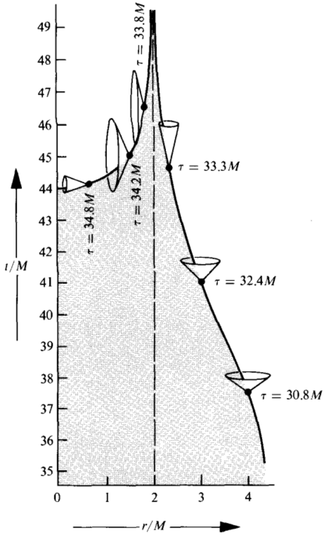
- Conclusion:
  - Event Horizon (R = 2M)
  - No communication beyond R = 2M
  - External observer never sees the star crossing horizon



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$$ds^2 = -d\tau^2 + a^2(\tau)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)].$$

$$a = \frac{a_m}{2}(1 + \cos\eta)$$
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- Energy Density

$$\rho = \frac{3a_m}{8\pi a^3} = \frac{3}{8\pi a_m^2}\left(\frac{1}{2}(1 + \cos\eta)\right)^{-3}$$

Surface of Star
First Junction Condition

- Induced metric on both side must be the same

$$h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b} = g_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial y^{a}} \frac{\partial x^{\beta}}{\partial y^{b}}$$

$$ds^2_{\Sigma} = -d\tau^2 + a^2(\tau) \sin^2 \chi_0 d\Omega^2.$$

$$ds^2_{\Sigma} = -(F\dot{T}^2 - F^{-1}\dot{R}^2)d\tau^2 + R^2(\tau)d\Omega^2$$

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- Comparing the induced metrics we get:

$$R(\tau) = a(\tau) \sin \chi_0$$

$$F\dot{T}^2 - F^{-1}\dot{R}^2 = 1 \implies F\dot{T} = \sqrt{\dot{R}^2 + F} = \beta(R, \dot{R}).$$

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- Implies energy conservation
- For smooth transition, hypersurface must be generated by geodesics of interior and exterior geometries.

Conclusions

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- Horizon and Singularity theorems