Appendix 7: Supplementary Information for Chapter 4

IPM Equations

IPMs describe how the abundance and distribution of a continuously distributed trait changes in a population through discrete time. Vital rates are combined in projection kernels that describe state-dependent per-capita contributions of existing individuals to the population trait distribution in the following time step via survival and development (denoted ($P(z',z)$)) and sexual and asexual reproduction (denoted $F(z',z)$ and $C(z',z)$ respectively).

$$n(z',t+1) = \int_L^U [G(z'|z,\sigma,\theta) * s_a(z,\theta)]n(z,t)dz + s_s * r_a(z')sdl(t), \tag{4.1.1}$$

$$mf(t+1) = \int_L^U [p_f(z,\theta) * s_a(z,\theta) * r_f(z,\theta) * p_m * p_{fv}]n(z,t)dz, \tag{4.1.2}$$

$$sdl(t+1) = p_e * g_i * v_s * mf(t) + s_{sb} * g_{sb} * p_e * sb(t), \tag{4.1.3}$$

and

$$sb(t+1) = s_{sb} * (1 - g_{sb}) * sb(t) + (1 - g_i) * v_s * mf(t). \tag{4.1.4}$$

The survival probability of non-seedlings function, $s_a(z,\theta)$, is given by:

$$\begin{gathered} Logit(s_a(z,\theta)) = \beta_{0,s,i} + \beta_{s,z} * z + \\ \beta_{s,\theta_t,dry} * \theta_{t,dry,i} + \beta_{s,\theta_t,wet} * \theta_{t,wet,i} + \\ \beta_{s,\theta_p,dry} * \theta_{p,dry,i} + \beta_{s,\theta_p,wet} * \theta_{p,wet,i} + \\ \beta_{s,\theta_{s3},dry} * \theta_{s3,dry,i} + \beta_{s,\theta_{s3},wet} * \theta_{s3,wet,i} + \\ \beta_{s,\theta_t \times z,dry} * \theta_{t,dry,i} * z + \beta_{s,\theta_t \times z,wet} * \theta_{t,wet,i} * z + \\ \beta_{s,\theta_p \times z,dry} * \theta_{p,dry,i} * z + \beta_{s,\theta_p \times z,wet} * \theta_{p,wet,i} * z + \\ \beta_{s,\theta_{s3} \times z,dry} * \theta_{s3,dry,i} * z + \beta_{s,\theta_{s3} \times z,wet} * \theta_{s3,wet,i} * z, \end{gathered} \tag{4.1.5}$$

where *wet* and *dry* denote wet season and dry seasons covariation values, i indexs each site in Table 4.1, and the function $g()$ takes a site i and returns 0 for sites in the invaded range and 1 for sites in the native range. The development function, $G(z'|z,\sigma,\theta)$ is given by:

$$G(z'|z,\sigma,\theta) = f_G(z'|\mu_G(z,\theta), \sigma_G(z,i)), \tag{4.1.6}$$

where f_G denotes a normal probability density function, $\mu_G(z,\theta)$ is given by:

$$\mu_G(z,\theta) = \beta_{0,G,i} \tag{4.1.7}$$

where f_{t2} and f_s denote a set of spline basis functions created by `t2` and `s()` from the `mgcv` *R* package (Wood 2017). *mean* and *seas* denote the mean annual value and the seasonality of the monthly values, respectively. $\sigma_G(z,i)$ is given by:

$$\sigma_G(z,i) = \beta_{0,\sigma_G,i} + \beta_{z,\sigma_G} * z + ... \tag{4.1.8}$$

The probability of flowering function, $p_f(z,\theta)$, is given by:

$$\begin{gathered}
Logit(p_f(z,\theta)) = \beta_{0,p_f,i} + \beta_{z,p_f} * z + \\
f_s(\theta_{t,mean,i}) + f_s(\theta_{p,total,i}) + \\
\beta_{p_f,\theta_t,seas} * \theta_{t,seas,i} + \beta_{p_f,\theta_p,seas} * \theta_{p,seas,i} + \\
\beta_{p_f,\theta_{s2},mean} * \theta_{s2,mean,i} + \beta_{p_f,\theta_{s2},seas} * \theta_{s2,seas,i} + \\
\beta_{p_f,\theta_t \times z,seas} * \theta_{t,seas,i} * z + \beta_{p_f,\theta_p \times z,seas} * \theta_{p,seas,i} * z + \\
\beta_{p_f,\theta_{s2} \times z,mean} * \theta_{s2,mean,i} * z + \beta_{p_f,\theta_{s2} \times z,seas} * \theta_{s2,seas,i} * z.
\end{gathered} \tag{4.1.9}$$

The number of flowers produced conditional on flowering function, $r_f(z,\theta)$, is given by:

$$\begin{gathered}
Log(r_f(z,\theta)) = \beta_{0,r_f,i} + \beta_{z,r_f} * z + \\
\beta_{r_f,\theta_t,mean} * \theta_{t,mean,i} + \beta_{r_f,\theta_t,seas} * \theta_{t,seas,i} + \\
\beta_{r_f,\theta_p,total} * \theta_{p,total,i} + \beta_{r_f,\theta_p,seas} * \theta_{p,seas,i} + \\
\beta_{r_f,\theta_{s2},mean} * \theta_{s2,mean,i} + \beta_{r_f,\theta_{s2},seas} * \theta_{s2,seas,i} + \\
\beta_{r_f,\theta_t \times z,mean} + \theta_{t,mean,i} * z + \beta_{r_f,theta_t \times z,seas} * \theta_{t,seas,i} * z + \\
\beta_{r_f,\theta_p \times z,total} + \theta_{p,total,i} * z + \beta_{r_f,theta_p \times z,seas} * \theta_{p,seas,i} * z + \\
\beta_{r_f,\theta_{s2} \times z,mean} + \theta_{s2,mean,i} * z + \beta_{r_f,theta_s 2 \times z,seas} * \theta_{s2,seas,i} * z + \\
\beta_{r_f,native} * g(i) + \beta_{r_f,native \times z} * g(i) * z.
\end{gathered} \tag{4.1.10}$$

$g(i)$ is a function that returns 1 if site i in the native range (South Africa) and 0 when site i is located elsewhere. Finally, the size distribution of newly observed non-seedling plants, $r_a(z')$, is given by:

$$r_a(z') = f_{r_a}(z'|\mu_{r_a}, \sigma_{r_a}), \tag{4.1.11}$$

where f_{r_a} is a Gaussian probability density function.