TA - check 7 23. 10. 2021

1 a) $A = \{x \in Z \mid \exists y \in Z\ (x = 2y)\}$

$B = \{x \in Z \mid \exists y \in Z\ (x = 2y+1)\}$

i $f: A \to B$, $f(x) = x + 1$

ii

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

$x_1 = x_2 \Rightarrow x_1 = x_2$ True $\Rightarrow f$. injective

$x \in Z \Rightarrow x+1 \in Z \Rightarrow f$ surjective

f is a bijective function from $A \to B \Rightarrow$
$|A| = |B|$

iii $g: A \to Z$ $g(x) = \frac{x}{2}$

$g(x_1) = g(x_2) \Rightarrow x_1 = x_2$

$\frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$ true $\Rightarrow g$ injective

$\cancel{x \in A}$

$x = 2y$, $y \in Z \Rightarrow \frac{x}{2} \in Z \Rightarrow g$ surjective

g - bijective $\Rightarrow |A| = |Z|$

2 $f: TREE \to N$ $D \cdot N$

$$f(\cancel{t}) = \begin{cases} \cancel{0, \quad t = (x,} \\ \cancel{t} \end{cases}$$

a) $$f(t) = \begin{cases} 0 & , \quad t = \emptyset \\ 0 & , \quad t = (x, \emptyset) \wedge x \% 2 = 0 \\ 1 & , \quad t = (x, \emptyset) \wedge x \% 2 = 1 \\ 0 + \sum_{i=1}^{k} f(i) & , \quad t = (x, (T_1, \dots, T_k)) \wedge x \% 2 = 0 \\ 1 + \sum_{i=1}^{n} f(i) & , \quad t = (x, (T_1 \dots T_k)) \wedge x \% 2 = 1 \end{cases}$$

b) $f: TREE \times N \to TREE$

$$f(t, n) = \begin{cases} \emptyset & , \quad t = \emptyset \\ (x+n, \emptyset) & , \quad t = (x, \emptyset) \\ (x+n, (f(T_1, n), \dots f(T_k, n))) & , \quad t = (x, (T_1 \dots T_k)) \end{cases}$$

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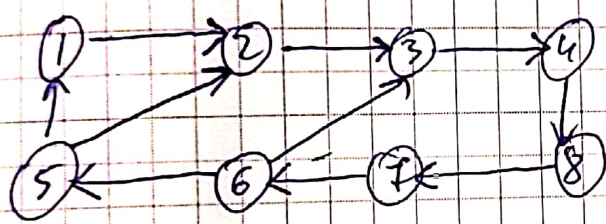
c) $f: TREE \to R$ $\qquad D = R \cup \{+,-,/,*\}$

$$f(t) = \begin{cases} 0 , & t = \emptyset \\ x , & t = (x, \emptyset) \cancel{\land x \in D} \\ f(T_1) \times \dots f t. \times f(T_k) , & t = (x, (T_1 \dots T_k)) \\ & x \in \{+, -, /, *\} \end{cases}$$

d) $G(V, E)$ $\qquad |V| = 8$ $\qquad 10 \leq |E| \leq 15$



e) $f: V \times V \to \{0, 1\}$

$f = \{(1, 2), (1, 3) \dots \}$

$\cancel{f(x, y)} = \{$

3) a) i. $A(x)$ x is ~~cool~~ amazing

$C(x)$ x is cool

$P(x)$ x is a puzzle

$\forall x \quad A(x) \land C(x) \to P(x)$

ii $C(x)$ - x is a conte
$L(x, y)$ - x loves y

$$\forall x y z \; C(x) \wedge C(y) \wedge C(z) \wedge x \neq y \wedge x \neq z \wedge y \neq z \wedge L(x, y) \rightarrow \neg L(x, z)$$

iii $p \xrightarrow{q} r$

iv CAP - 2 games 1W 1L, 5 goals
SAS - 1 game 1W 2 goals, got 0
ONF 0W 1 0 1

CAP must have played 1 game if owls lied
SAS 2 games
ONF 1 games, because is the only configuration possible

	4 0	4 0	
CAP	1	1	SAS
SAS	0	1	ONF
CAP			ONF not played

~~We know that CAP played 2 games, but didn't~~

We know that CAP played 1 game and that they didn't win and didn't lose a game, which means they made a draw. CAP played with SAS, since SAS played 2 games.

ONF played 1 game and won. Also owls lied that they got 1 goal against them, which means that the only possible score is when they got 0 goals against them.

SAS scored a total of 2 goals is a lie, which means that in the match with CAP they scored 0 or 1 goal.