HW 1

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E45

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Problem 6.1. The following three $\sigma - \epsilon$ data points are provided for a titanium alloy for aerospace. Calculate E for this alloy.

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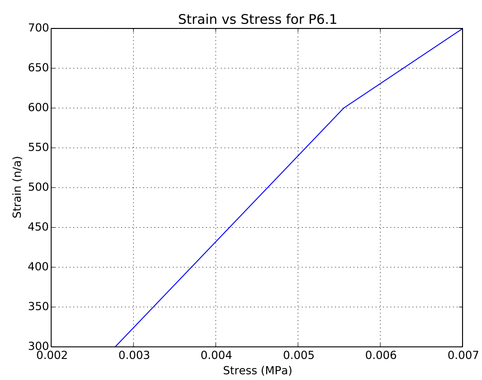
$$\epsilon = 0.002778 \,(\sigma = 300MPa)$$
$$\epsilon = 0.005556 \,(\sigma = 600MPa)$$
$$\epsilon = 0.009897 \,(\sigma = 900MPa)$$

Plotting the data points given in a strain vs stress plot, we see where the slope changes, and where the actual modulus of elasticity is.



Using the graphical version of Hooke's law, $\sigma = E\epsilon$, we can solve for the modulus E.

$$E = \frac{\sigma}{\epsilon} = \frac{600 \times 10^6 Pa}{0.005556} = 1.0799 \times 10^{11}\, Pa = 108\, GPa$$

Problem 6.2. If the Poisson's ration for the alloy in P6.1 is 0.35, calculate **(a)** the shear modulus G, and **(b)** the shear stress τ necessary to produce an angular displacement α of $0.2865°$.

(a) Use the following equation for small strains:

$$E = 2G(1 + \upsilon)$$
$$G = \frac{E}{2(1+\upsilon)} = \frac{1.0799 \times 10^{11} Pa}{2(1+0.35)} = 4.00 \times 10^{10} Pa = 40\, GPa$$

(b) Use the following equations for the shear modulus G and the shear strain γ,

$$\gamma = \tan\alpha, \quad G = \frac{\tau}{\gamma}$$

$$\tau = G\gamma = G\tan\alpha = 4.00 \times 10^{10} Pa * 0.2865^\circ * \frac{\pi}{180} = 2.0 \times 10^{8} = 0.2\, GPa$$

Problem 6.4. Consider the 1040 carbon steel listed in Table 6.1. **(a)** A 20-mm-diameter bar of this alloy is used as a structural member in an engineering design. The unstressed length of the bar is precisely 1m. The structural load on the bar is $9 \times 10^4 N$ in tension. What will be the length of the bar under this structural load? **(b)** A design engineer is considering a structural change that will increase the tensile load on this member. What is the maximum tensile load that can be permitted without producing extensive plastic deformation of the bar? Give your answer in both newtons (N) and pounds force (lb_f).

(a) To solve for the length of the bar under the structural load, l, we can use the equation $\epsilon = \frac{l-l_0}{l_0}$ where ϵ is the engineering strain and l_0 is the length of the unstretched bar. To solve for ϵ we can use Hook' law, $\sigma = E\epsilon$, and the relationship $\sigma = \frac{P}{A_0}$ where σ is the engineering stress, P is the load on the sample, and A_0 is the cross sectional area of the 1040 carbon steel.

$$\frac{l-l_0}{l_0} = \epsilon = \frac{\sigma}{E} = \frac{P}{A_0 E} = \frac{4P}{\pi d^2 E}$$

Rewriting and solving for l where $P = 9 \times 10^4 N, l_0 = 1m, d = 20 \times 10^{-3} m$, and $E = 200 \times 10^9 Pa$

$$l = \frac{4Pl_0}{\pi d^2 E} + l_0 = \frac{4 * 9 \times 10^4 N * 1m}{\pi * 400 \times 10^{-6} m^2 * 200 \times 10^9 Pa} + 1m = 1.00143m$$

(b) Anything above the yield strength starts to deform the bar. Using the equation $\sigma = \frac{P}{A_0}$ and the value of the yield strength for carbon 1040, we can solve for the maximum load.

$$P = \sigma A_0 = 600 \times 10^6 Pa * \pi 100 \times 10^{-6} m^2 = 1.88 \times 10^5 N$$

Using the conversion of $1 lb_f = 4.448N$, we see that $P = 1.88 \times 10^5 N * \frac{0.2248\, lb_f}{N} = 4.24 \times 10^4\, lb_f$.

Problem 6.16. A single crystal Al_2O_3 rod (precisely 6mm diameter x 50 mm long) is used to apply loads to small samples in a high-precision dilatometer (a length-measuring device). Calculate the resultign rod dimension if the crystal is subjected to a 25-kN axial compression load.

To solve for l_x using the engineering strain equation, $l_x = l_0\,(1+\epsilon)$. Also since this is axial compression, the force P must be negative.

$$\begin{aligned} l_x &= l_0(1+\epsilon) = l_0(1+\frac{\sigma}{E}) = l_0(1+\frac{-P}{A_0E}) = l_0(1+\frac{-P}{\pi r^2 E}) \\ &= 50\times 10^{-3}m(1+\frac{-25\times 10^3 N}{\pi * 9\times 10^{-6}m^2 * 380\times 10^9 Pa}) \\ &= 49.88\,mm \end{aligned}$$

To solve for the diameter expansion, l_y we use the Poisson's ratio to solve for ϵ_y

$$l_y = l_0(1+\epsilon_y) = l_0(1-\epsilon_x\upsilon)$$

Note that $\upsilon = 0.26$ and that $\epsilon_x = \frac{-P}{\pi r^2 E} = -2.33\times 10^{-3}$ from above.

$$l_y = 6\times 10^{-3}(1-(-2.33\times 10^{-3})0.26) = 6.0036\,mm$$

Problem 6.45. You are asked to measure nondestructively the yield strength and tensile strength of an annealed 65-45-12 cast iron structural member. Fortunately, a small hardness indentation in this structural design will not impair its future usefulness, which is a working definition of nodestructive. A 10-mm-diameter tungsten carbide sphere creates a 4.26-mm-diameter impression under a 3,000-kg load. What are the yield and tensile strengths?

Use the sphere hardness equation in table 6.9, where P = 3000kg, D = 10 mm, d = 4.26 mm

$$\begin{aligned} BHN &= \frac{2P}{\pi D(D-\sqrt{D^2-d^2})} = \frac{2(3000kg)}{\pi 10mm(10mm-\sqrt{10^2mm-4.26^2mm})} \\ &\approx 200 \end{aligned}$$

Using the graph in figure 6.29b, a BHN of 200 produces a yield strength of about 400 and a tensile strenth of about 550.