

HW 9

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Problem 12.10. If the polymer evaluated in Problem 12.9 is polypropylene, what would be the **(a)** coiled length and **(b)** extended length of the average molecule?

n Range	n_i (Mid Value)	Population fraction
1-100	50	—
101-200	150	—
201-300	250	0.01
301-400	350	0.10
401-500	450	0.21
501-600	550	0.22
601-700	650	0.18
701-800	750	0.12
801-900	850	0.07
901-1000	950	0.05
1001-1100	1050	0.02
1101-1200	1150	0.01
1201-1300	1250	0.01

Figure 1: table from problem 12.9

(a) In order to calculate $L = l\sqrt{m}$ where $m = 2n$ and l is the length of a single bond, we need the degree of polymerization from table 1 above. We can get this by multiplying the percent fraction (column 3) by the Mid values (column 2) and sum them up.

average n value (a.u)	2.5	35	94.5	121	117	90	59.5	47.5	21	11.5	12.5
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The sum of these values is $n = 612$. Now to get the length of a single bond, we can simply find the radius of a carbon atom and multiply by 2. This is assuming that each mer is touching the mer adjacent to it. The radius of carbon here is 0.077 nm [1].

$$L = l\sqrt{m} = l\sqrt{2n} = 2 * 0.077 \text{ nm} \sqrt{2(612)} = 5.39 \text{ nm}$$

(b) We know that carbon tends to bond in a tetrahedral form with angles of 109.5° . In order to get an uncoiled length, we use equation 12.5 [1] in the textbook.

$$L_{ext} = ml \sin(109.5^\circ/2) = 2 * 0.077 * 2 * 612 \sin(109.5/2) = 154 \text{ nm}$$

Problem 12.21. In Problem 11.5, the mass reduction in an automobile design was calculated based on trends in metal alloy selection. A more complete picture is obtained by noting that, in the same 1975 to 1985 period, the volume of polymers used increased from 0.064 m^3 to 0.100 m^3 . Estimate the mass reduction (compared to 1975) including this additional polymer data. (Approximate the polymer density as ThO_2)

Problem ref (11.5). Between 1975 and 1985, the volume of all iron and steel in a given automobile model decreased from 0.162 m^3 to 0.116 m^3). In the same time frame, the volume of all aluminum alloys increased from 0.012 m^3 to 0.023 m^3). Using the densities of pure Fe and Al, estimate the mass reduction resulting from this trend in materials substitution.

From appendix 1 [1], the density of Fe is 7.87 g/cm^3 and for Al 2.70 g/cm^3 .

We use this information to find the mass resulting in the difference of volume.

$$\begin{aligned} m_{\text{Fe}} &= \rho V = 7.87 \text{ g/cm}^3 * 10^6 \text{ cm}^3 * (0.162 - 0.116) * \frac{1}{10^3 \text{ kg}} = 362.02 \text{ kg} \\ m_{\text{Al}} &= \rho V = 2.70 \text{ g/cm}^3 * 10^6 \text{ cm}^3 * (0.023 - 0.012) * \frac{1}{10^3 \text{ kg}} = 29.7 \text{ kg} \\ m_{\text{Poly}} &= \rho V = 1 \text{ Mg/cm}^3 * 10^6 \text{ cm}^3 * (0.100 - 0.064) * 10^3 \text{ kg} = 36 \text{ kg} \end{aligned}$$

Now for the mass reduction we have $362.02 - 29.7 - 36 = 296.32 \text{ kg}$ mass reduction.

Problem 12.31. Calculate the average separation distance between the centers of adjacent glass spheres in the nylon described in Problem 12.30. (Assume a simple cubic array of dispersed particles as in Problem 11.20).

Problem ref 12.30. Some injection moldable nylon 66 contains 40 wt% glass spheres as a filler. Improved mechanical properties are the result. If the average glass sphere diameter is $100 \mu\text{m}$, estimate the density of such particles per cubic millimeter.

According to Ex 2.5 [1], the density of nylon 66 is $\rho_n = 1140 \text{ kg/m}^3$ and for glass $\rho_g = 2540 \text{ kg/m}^3$. We assume a total mass of 1 kg which means we have 0.4 kg glass and 0.6 kg nylon.

$$V_{tot} = V_n + V_g = \frac{m_n}{\rho_n} + \frac{m_g}{\rho_g} = \frac{0.6 \text{ kg}}{1140 \text{ kg/m}^3} + \frac{0.4 \text{ kg}}{2540 \text{ kg/m}^3} \approx 6.838 \times 10^{-4} \text{ m}^3$$

Next we want the # of glass spheres in a Volume of glass, (Note radius of glass sphere is 50 microns)

$$\frac{V_g}{V_{g-spheres}} = \frac{1.575 \times 10^{-4}}{(4\pi/3)(50 \times 10^{-6})^3} \approx 3 \times 10^8 \text{ glass spheres in a volume of glass}$$

Now we want the # of glass spheres per a unit of total volume,

$$\frac{\# \text{ of glass spheres}}{V_{tot}} = \frac{3 \times 10^8 \text{ spheres}}{6.838 \times 10^{-4} \text{ m}^3} \approx 4.39 \times 10^{11} \# \text{ spheres / total volume}$$

Now in order to make sure we have units of meters and not inverse meters cubed we take the cube root and inverse of the above number,

$$(\sqrt[3]{4.39 \times 10^{11}})^{-1} = 1.316 \times 10^{-4} \approx 0.132 \text{ mm}$$

Problem 12.47. On a plot similar to that displayed in Figure 12.17, show the composite modulus for (a) 60 vol% fibers (the result of Example 12.8) and (b) 50 vol% fibers (the result of Practice Problem 12.8). Include the individual glass and polymer plots.

Refer to the plot below.

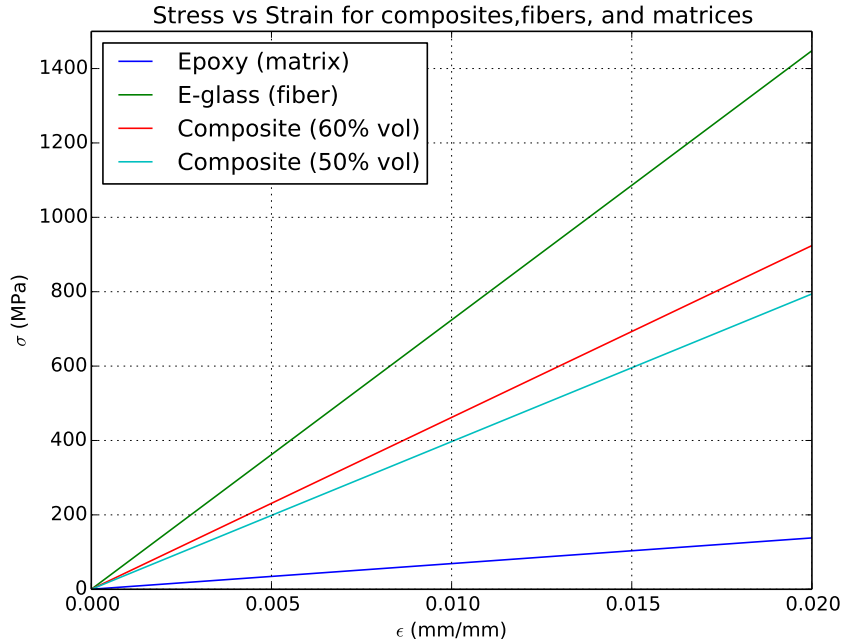


Figure 2: The slopes used are: $72.4 \times 10^3 \text{ MPa}$ for E-glass, $6.9 \times 10^3 \text{ MPa}$ for Epoxy, $46.2 \times 10^3 \text{ MPa}$ for 60% Composite, and $39.7 \times 10^3 \text{ MPa}$ for 50% Composite.

Problem 12.62. In Problem 6.9, a competition among various metallic pressure-vessel materials was illustrated. We can expand the selection process by including some composites, as listed in the following table:

Materials	ρ (Mg/m ³)	Cost (\$/kg)	Y.S. (MPa)	ρ /Y.S. (Mg*MPa/m ³)	ρ *Cost/Y.S. ((\$Mg*MPa/m ³))
1040 carbon Steel	7.80	0.63	–	–	–
304 stainless steel	7.80	3.70	–	–	–
3003-H14 aluminum	2.73	3.00	–	–	–
Ti-5Al-2.5Sn	4.46	15.00	–	–	–
Reinforced concrete	2.50	0.40	200	.0125	.005
Fiberglass	1.80	3.30	200	.009	.0297
C fiber-reinforced polymer	1.50	70.00	600	.0025	.175

Figure 3: Material cost, yield strength, and density

(a) From this expanded list, select the material that will produce the lightest vessel. (b) Select the materials that will produce the minimum-cost vessel.

(a) Looking at problem 6.9 [1], we see that the mass of the lightest vessel is $m = C \frac{\rho}{Y.S.}$ where C in this case is some arbitrary constant. So in order to minimize mass, we look for the smallest value of the ratio $\frac{\rho}{Y.S.}$. Looking at figure 3, column 5, we see that **carbon fiber-reinforced polymer** has the smallest value.

(b) Now in order to also minimize cost, we can simply multiply the cost by our ratio we calculated in part a. The idea is that if you want to minimize two variables, you know that their product must also be minimized. So we can multiply the costs by the ratio we calculated in part a. The last column in figure 3 contains this product. The smallest value in this case is 0.005 which corresponds to **reinforced concrete**.

1 References

1. James F. Shackelford, Introduction to Materials Science for Engineers, Seventh Edition, Pearson Higher Education, Inc., Upper Saddle River, New Jersey (2009).