A quadrupole passmethod with fringe field effect

X. Huang

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Outline

- A brief review of theory
  - Linear effects (soft edge)
  - Nonlinear effects (hard edge)
- The new AT passmethod
  - Implementation
  - Benchmarking with realistic quadrupole field pass
- The quadrupole fringe field effect for SPEAR3
  - Tune changes
  - Tune dependence on amplitude changes

The general Hamiltonian for a quadrupole

A general Hamiltonian (including longitudinal field variation) can be derived using a proper magnetic field expansion (El-Kareh; Forest; Bassetti & Biscari).

$$H(s) = \frac{1}{2}(P_x^2 + P_y^2) + \frac{1}{2}k(s)(x^2 - y^2) - \frac{1}{4}k'(s)(x^2 - y^2)(xP_x + yP_y) - \frac{1}{12}k''(s)(x^4 - y^4) + O(X^6)$$

J. Irwin, C.X. Wang

Usually a quadrupole is modeled as hard-edge. In the hard-edge model, the leading correction is from the last two terms, which are nonlinear. This effect has been derived by many authors: Lee-Whiting, Forest & Milutinovic, Irwin & Wang, Zimmermann.

The leading term for a soft fringe model is linear. This is studied by Irwin & Wang.

The linear correction to quadrupole map

$$H(s) = H_0(s) + \tilde{H}(s)$$

A perturbation approach

$$H_0(s) = \begin{cases} \frac{1}{2}(P_x^2 + P_y^2) + \frac{1}{2}k_0(x^2 - y^2) & s \le s_0 \\ \frac{1}{2}(P_x^2 + P_y^2) & s > s_0 \end{cases}$$

Hard-edge model, for exit edge

$$\tilde{H}(s) = \frac{1}{2}\tilde{k}(s)(x^2 - y^2) \qquad \tilde{k}(s) = \begin{cases} k(s) - k_0 & s \le s_0 \\ k(s) & s > s_0 \end{cases}$$

Perturbation term

The map

Irwin, C.X. Wang, PAC95

$$\mathcal{M}(s_1 \to s_2) = \mathcal{M}_Q(s_1 \to s_0)\ \tilde{\mathcal{Q}}_f\ \mathcal{M}_{drift}(s_0 \to s_2)$$

$$\tilde{\mathcal{Q}}_f = e^{:-\int_{s_1}^{s_2} ds\,\bar{H}(s) + \frac{1}{2}\int_{s_1}^{s_2} ds \int_{s}^{s_2} d\tilde{s}[\bar{H}(s),\bar{H}(\tilde{s})]:}$$

The generating function for the correction map (only leading contribution is shown)

$$f_2 = \frac{I_1}{2}(xP_x - yP_y) \qquad I_1 = \int_{-\infty}^{\infty} \tilde{k}(s)(s - s_0)\,ds$$

$\longrightarrow$ matrix $\mathrm{diag}(e^{I_1}, e^{-I_1}, e^{-I_1}, e^{I_1})$

For a symmetric quadrupole, the entrance edge has a reversed sign for I_1

I derived the tune change and found

$$\Delta\upsilon_x = -\frac{k_0 L \beta_x I_1}{\pi}, \qquad \Delta\upsilon_y = -\frac{k_0 L \beta_y I_1}{\pi}$$

The nonlinear correction

The generating function for the map (exit edge)

$$f_4 = \frac{1}{12(1+\delta)} k_0 (x^3 P_x + 3xy^2 P_x - y^3 P_y - 3x^2 y P_y) - \frac{1}{6(1+\delta)} k_{skew} (x^3 P_y + y^3 P_x)$$

The function for the entrance edge has an opposite sign.

Forest & Milutinovic point out the skew quadrupole part corresponds to a 'kick map'! A normal quadrupole can thus be modeled by a pair of pi/4 rotation and a kick map. This is the basis for the nonlinear part of the new AT passmethod.

F. Zimmermann derived the average Hamiltonian that include both edges.

$$\hat{H}_{1+2} \approx \frac{1}{12}(K_Q l_Q) K_Q \left[x^4 + 6x^2 y^2 + y^4\right]$$

Hard edge

$$\hat{H}_3 \approx \frac{5}{12} \Delta^2 K_Q^2 (K_Q l_Q) \left[x^4 - y^4\right]$$

Additional soft edge contribution.
2Δ is fringe length.

And tune dependence on amplitude (only showing hard edge contribution below)

$$\Delta Q_x = \frac{1}{8\pi} \sum_Q (K_Q l_Q) K_Q \left[\beta_{x,Q}^2 I_x + 2\beta_{x,Q}\beta_{y,Q} I_y\right]$$

$$\Delta Q_y = \frac{1}{8\pi} \sum_Q (K_Q l_Q) K_Q \left[\beta_{y,Q}^2 I_y + 2\beta_{x,Q}\beta_{y,Q} I_x\right]$$

$$\Delta = \sqrt{\frac{6I_1}{k_0}}$$

The new AT quad pass: QuadLinearFPass

Additional fields: I1a (entrance edge) and I1b (exit edge)

$$I_{1a} = \left| \int \left(\frac{k(s)}{k_0} - 1\right)(s - s_0)\,ds \right|, \qquad I_{1b} = \left| \int \left(\frac{k(s)}{k_0} - 1\right)(s - s_0)\,ds \right|$$

The I1 parameter is scaled by the focusing gradient and only magnitude is supplied to the passmethod. Normally one should have I1a=I1b, unless the quadrupole is split in the model.

The nonlinear part is independent of I1. However, if either I1a or I1b is zero, then the nonlinear effect for that edge is not evaluated (because most likely we have a magnet split).

Kick map for the fringe field of a skew quadrupole (entrance edge).

$$\mathcal{M} = \exp(:f:) = \exp\left(:\alpha \frac{x^3 p_y}{1+\delta}:\right) \exp\left(:\alpha \frac{y^3 p_x}{1+\delta}:\right) \qquad \alpha = \frac{q\hat{B}a_1}{6p_0}$$

Reverse sign at exit edge for α

$$\exp\left(:\alpha \frac{y^3 p_x}{1+\delta}:\right) x = x - \frac{\alpha y^3}{1+\delta},$$

$$\exp\left(:\alpha \frac{y^3 p_x}{1+\delta}:\right) p_y = p_y + \frac{3\alpha}{1+\delta} y^2 p_x,$$

Forest & Milutinovic, NIMA 269 (1988)

Switch x, y, etc for the other map.

Path length effect is ignored (higher order).

When I1a and I1b are not supplied, then the passmethod is identical to QuadLinearPass.

Realistic behavior of a SPEAR3 quad

The analytical quadrupole field map for SPEAR3 magnet.

Enge function

$$\Theta(z) = \frac{1}{1+\exp\left(\sum_{i=0} c_i \left(\frac{|z|-s_0}{g}\right)^i\right)}$$

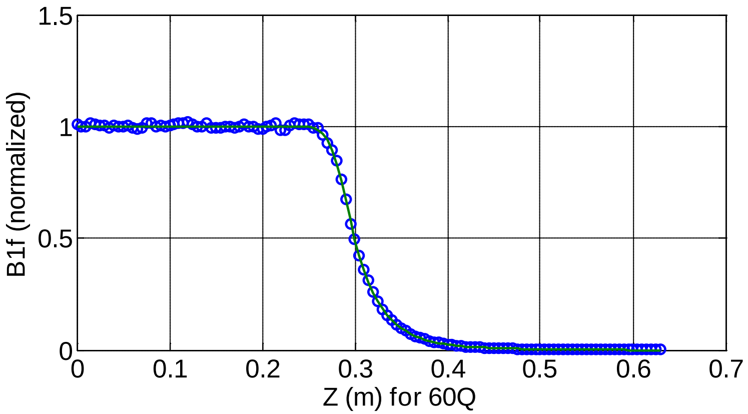
Magnetic field

$$B_x = B_1[y\Theta(z) - \frac{1}{12}\Theta''(z)(3x^2y + y^3)]$$

$$B_y = B_1[x\Theta(z) - \frac{1}{12}\Theta''(z)(x^3 + 3xy^2)]$$

$$B_z = \mathrm{sgn}(z)B_1\Theta'(z)xy$$

$$g = 0.05\,\mathrm{m},$$
$$c_{0..5} = -0.2394, 3.4633, -1.9048, 1.0301, -0.2941, 0.0315$$
$$s_0 = 0.166010\,\mathrm{m} \text{ for 34Q}$$



All SPEAR3 quads have identical fringe profile.

$$I_{1a} = \frac{I_1}{k_0} = 0.61\times 10^{-3}\,\mathrm{m}^2, \quad \Delta = \sqrt{\frac{6I_1}{k_0}} = 0.060\,\mathrm{m}$$

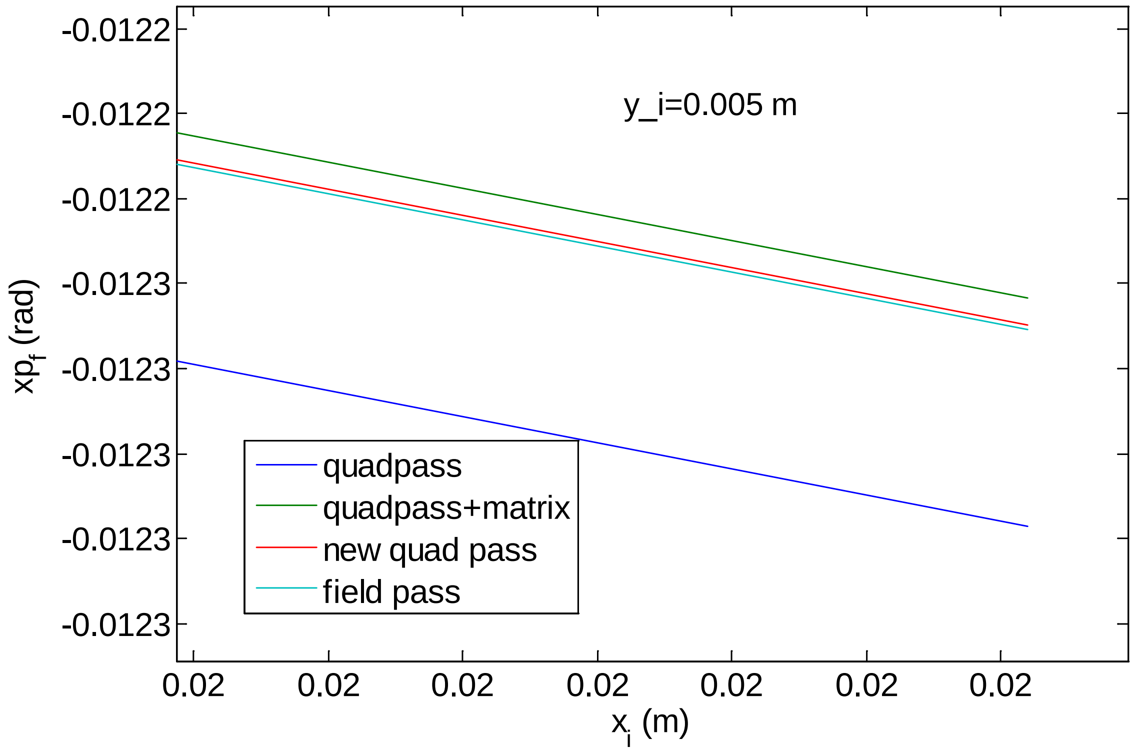
The field passmethod (SSRL AP-note-024) maps the phase space coordinates from entrance edge to exit edge by numerically integrating the Lorentz equation. With the realistic field, this passmethod reproduces the reality. Only that it is slow and not symplectic.

Verification of the passmethod

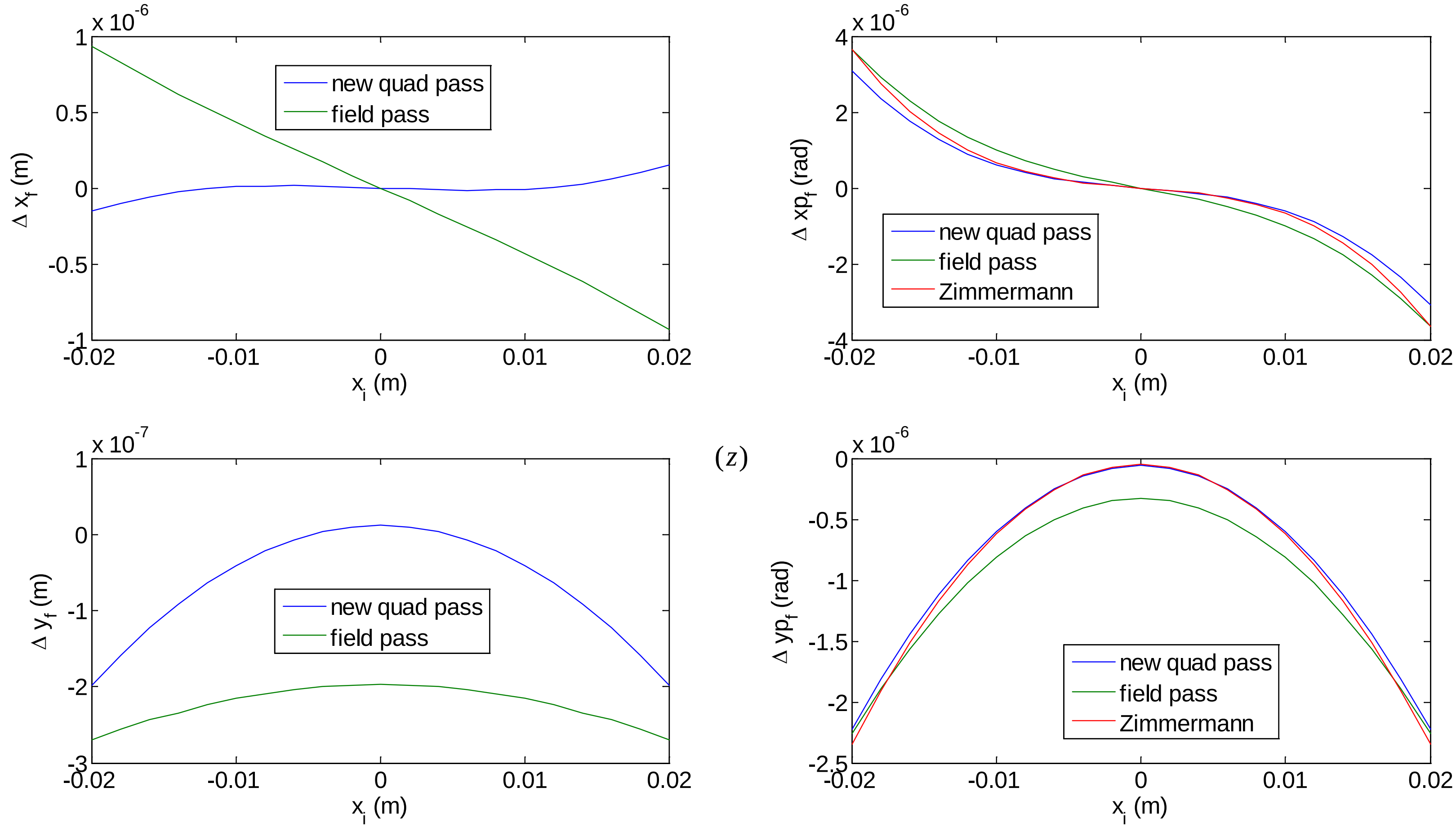
To compare 3 cases:

Case	Element
QuadLinearPass	Quad
QuadLinearFPass	Quad
Field Pass + Quad Field	[Drift QuadField Drift]

Negative drift

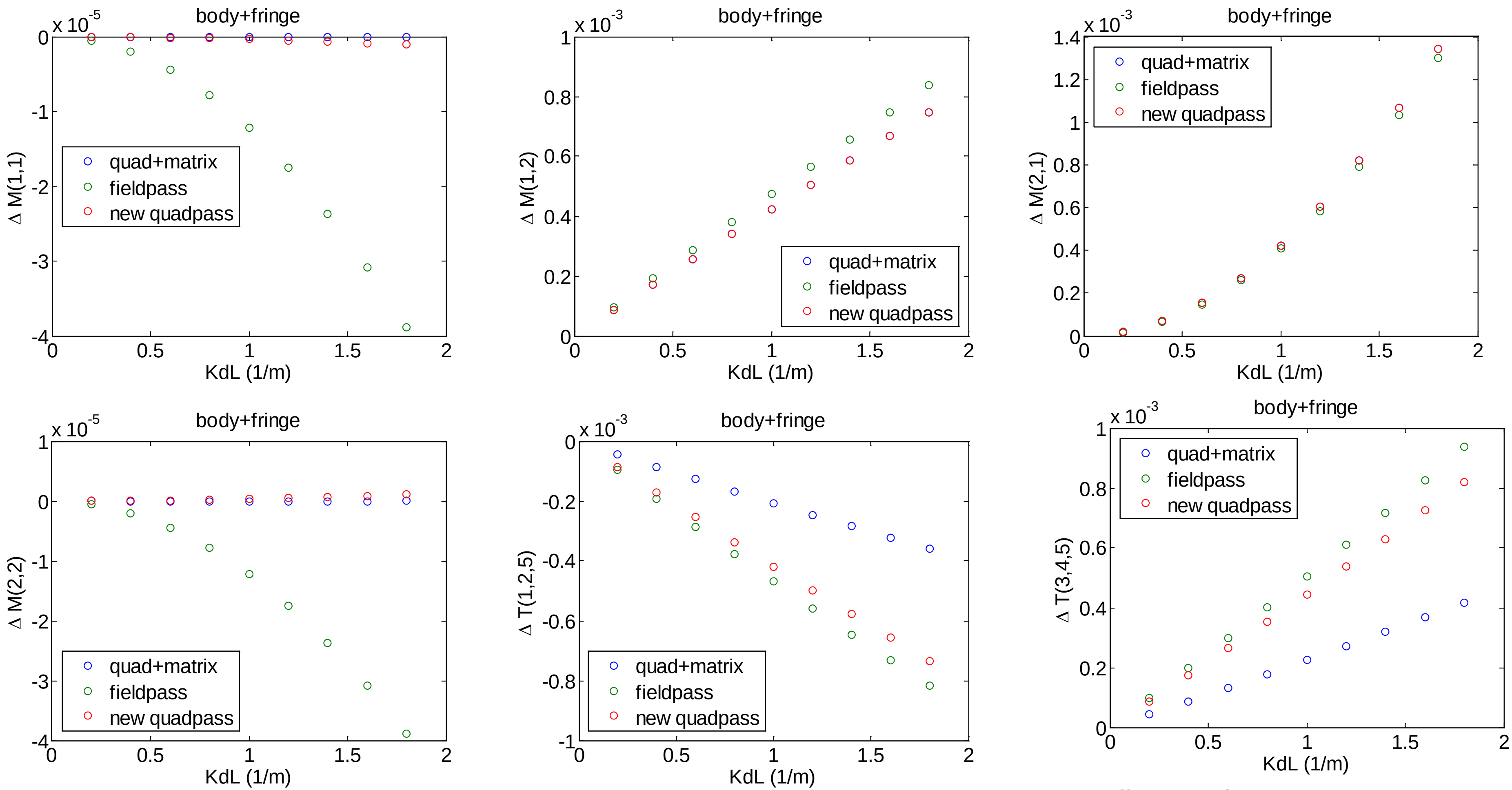


Compare the nonlinear effects



Plotted are the final coordinates with the result of (quad+matrix) tracking subtracted. Zimmerman result is from the average Hamiltonian H_{1+2} on slide 5.

Matrix Elements



Difference of Delta M11,M22 are higher order.

Note: coordinate 5 is momentum deviation for AT

Good agreement between the fieldpass and the new quad pass.

Quad Fringe Effects for SPEAR3

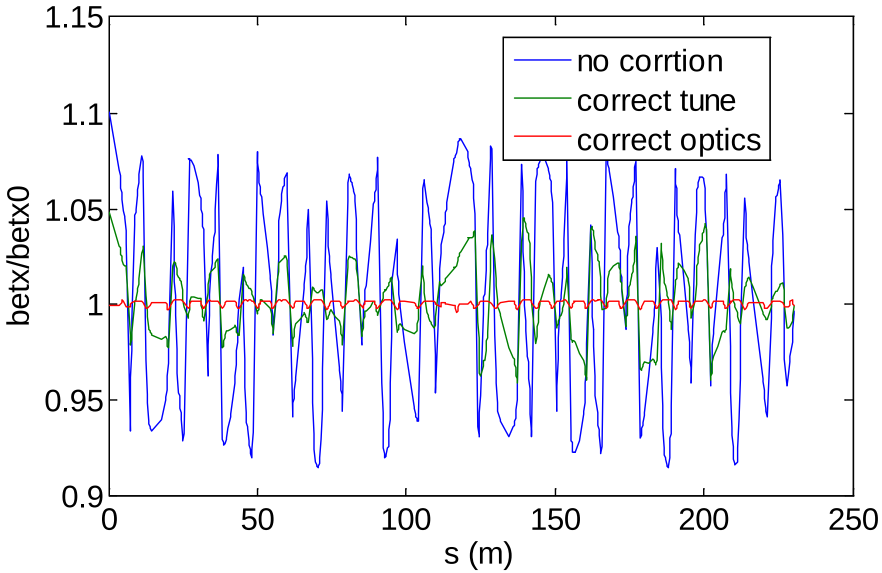
Tune and chromaticity changes

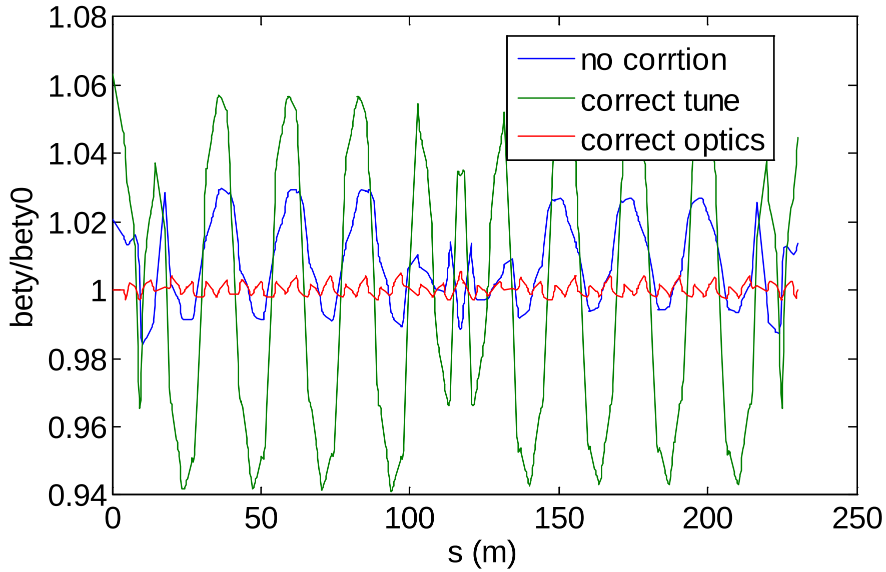
Parameter	No fringe	With fringe	Field pass*	Field pass, wrong scale
Nux	14.1060	14.0424	14.0384	14.1409
Nuy	6.1770	6.1210	6.1193	6.1399
Chromx	2.0002	1.8713	1.8633	2.1967
Chromy	2.0000	2.2782	2.2599	2.1655

1. “No fringe” is the baseline lattice (sp3v82_lelt).
2. “With fringe” applies the new quad pass to all quads.
3. “Field pass” uses the field pass and quad field model, keeping the integrated gradient fixed, for all quads except the Q9S triplet.
4. Caculated dtune (using eq. on slide 4) find [dnux, dnuy]=[-0.1575, -0.0452].
5. “Field pass, wrong scale” uses gradient values without considering the difference of effective length between new and old field profile (see note below).

Note: The quadrupole gradient in the field pass calculation for IPAC10 was slightly overestimated because the difference of the effective length in the AT model and its new value from the new field profile analysis was not accounted for. For example, in AT model, 34Q is 0.3534 m; the new field profile gives 0.3548 m. Since the field model is equivalent to the latter, the K values should be scaled down for 34Q magnets. Same for other types. Taking this into account, the IPAC10 paper should report tunes of [14.147, 6.123].

Beta beating





"No correction": just change to the new passmethod.
"correct tune": use QF and QD only to correct the tunes to [14.106, 6.177].
"correct optics": use 72 quad variables to correct optics.

Nonlinear effects

	Baseline	Correct tune*	Delta*	Correct optics	Delta	Delta (Zimmermann)**
dnuxdemx	1900	2093	193	2019	119	102
dnuxdemy	2133	2283	150	2300	185	156
dnuydemx	2114	2284	170	2314	181	156
dnuydemy	1756	1755	-1	1834	78	88

It is checked that the coefficients were derived from good linear fits.

* The discrepancy here is due to the large beta beating (after tune correction).

** Note the Zimermann formulae are given for dnux/dIx and the emittance is twice of action variable.

Conclusion

- A quadrupole passmethod with fringe field is developed.
- Comparison of the passmethod to direct field integration and theoretic calculation shows good agreement.
- The quadrupole fringe field has an appreciable effect on both the linear optics and the nonlinear optics.

Thanks to Boaz Nash for providing F. Zimmermann's paper.