









Applications of TBT BPM data

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Using TBT data for measurements and analysis

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Applications of TBT data

- Model Independent Analysis (MIA) techniques
 - → Increased BPM precision
 - → Lattice error diagnostic tools
- Phase and beta function approximations
- Mixed BPM tune measurements



MIA basics

Assume b is the transverse beam position of a pulse

$$b = b_0(\bar{x}_1, \bar{x}_1', \bar{\delta}, ...) + \sum_{\nu \in (x_1, x_1', ...)} \frac{\partial b}{\partial \nu} \Big|_{\nu = \bar{\nu}} \Delta \nu + ...$$

Zero order is sensitive to unknown BPM offset errors. Subtracting out the average < b > will remove this term

$$b - \langle b \rangle = \sum_{\nu \in (x_1, x_1', \dots)} \frac{\partial b}{\partial \nu} \Big|_{\nu = \bar{\nu}} (\Delta \nu - \langle \Delta \nu \rangle) + \dots$$



MIA basics

$$b - \langle b \rangle = B = QF^T + N$$

Where B is a BPM data matrix with P turns and M BPMs

$$B = \sum_{p=1}^{turns} \sum_{m=1}^{BPMs} (BPM)_{pm} = \begin{bmatrix} BPM_{11} & BPM_{12} & BPM_{1m} \\ BPM_{21} & BPM_{22} & BPM_{2m} \\ BPM_{p1} & BPM_{p2} & BPM_{pm} \end{bmatrix}$$

$$Q_{P\times d} = \begin{bmatrix} \vec{q}_1 & \vec{q}_2 & \dots & \vec{q}_d \end{bmatrix} \quad F_{M\times d} = \begin{bmatrix} \vec{f}_1 & \vec{f}_2 & \dots & \vec{f}_d \end{bmatrix}$$

N (P x M) contains noise associated with each BPM.



MIA basics

Perform a principal components analysis (PCA) on matrix B via single value decomposition (SVD)

$$B \to SVD(B) = USV^{T} = \sum_{i=1}^{a} \sigma_{i}\mu_{i}\nu_{i}^{T}$$

$$U_{P\times P} = \begin{bmatrix} \mu_{1} & \mu_{2} & \dots & \mu_{P} \end{bmatrix}$$

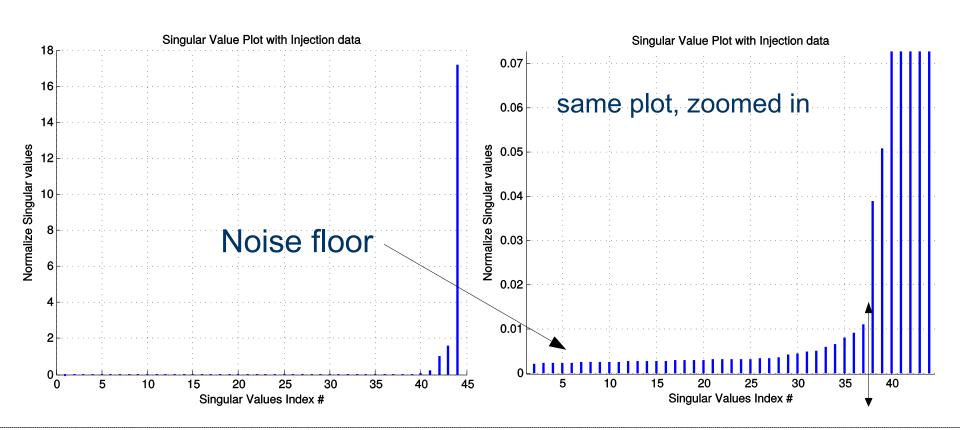
$$V_{M\times M} = \begin{bmatrix} \nu_{1} & \nu_{2} & \dots & \nu_{M} \end{bmatrix}$$

$$S_{P\times M} = \operatorname{diag} \begin{bmatrix} \sigma_{1} & \sigma_{2} & \dots & \sigma_{M} \end{bmatrix}$$

$$d = \operatorname{rank}(B)$$

MIA – singular values

Plotting the singular values in S (diagonal elements)





MIA – increased resolution technique

- 1) Create BPM matrix B via excited beam
- 2) Compute an SVD of B

$$B \to SVD(B) = USV^T + \mathcal{O}(\sigma_{BPM})$$

3) Set singular values due to the noise floor to zero

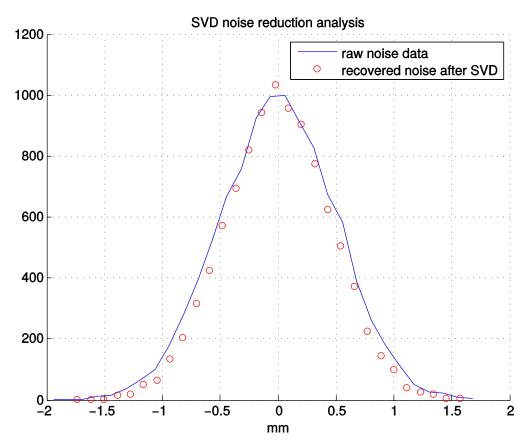
$$\bar{S}_{ii} = \begin{cases} 0 & i = \text{noise floor} \\ S_{ii} & i \neq \text{noise floor} \end{cases}$$

4) Recompute noise reduced BPM matrix B

$$\bar{B} = U\bar{S}V^T + \mathcal{O}(\sigma_{BPM}\sqrt{\frac{d}{M}})$$



MIA – simulation

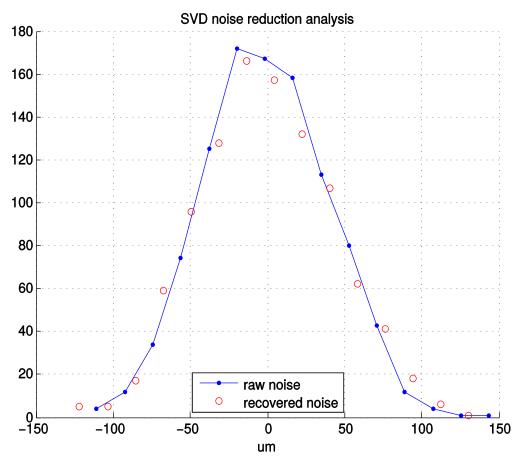


10,000 turn, 3mm kick in the horzontal and vertical with random Gaussian noise added to a single BPM.

95% CI between -0.014 – 0.012 mm



MIA - simulation

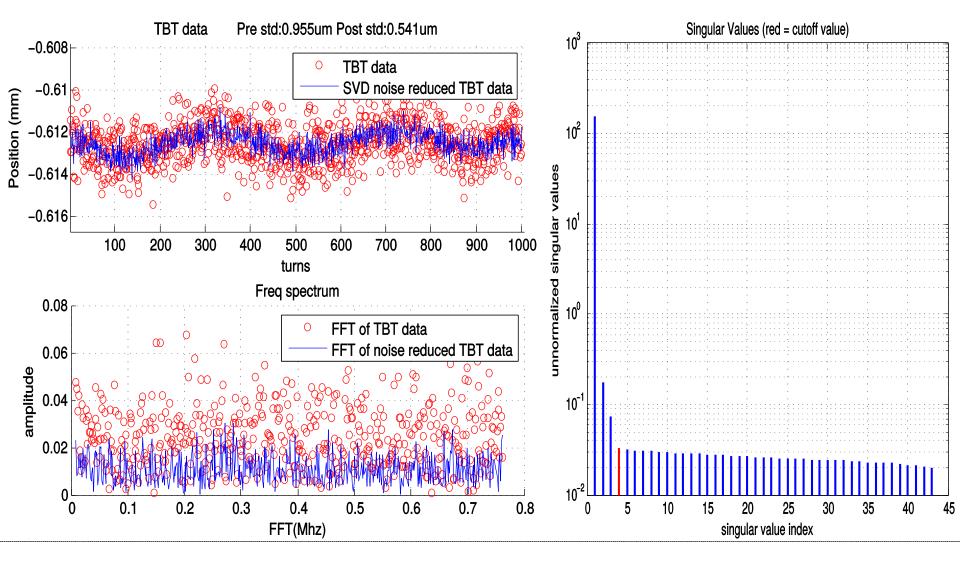


10,000 turn, 3mm kick in the horzontal and vertical with random Gaussian noise added to a single BPM.

95% CI between -2.9 – 2.4 um

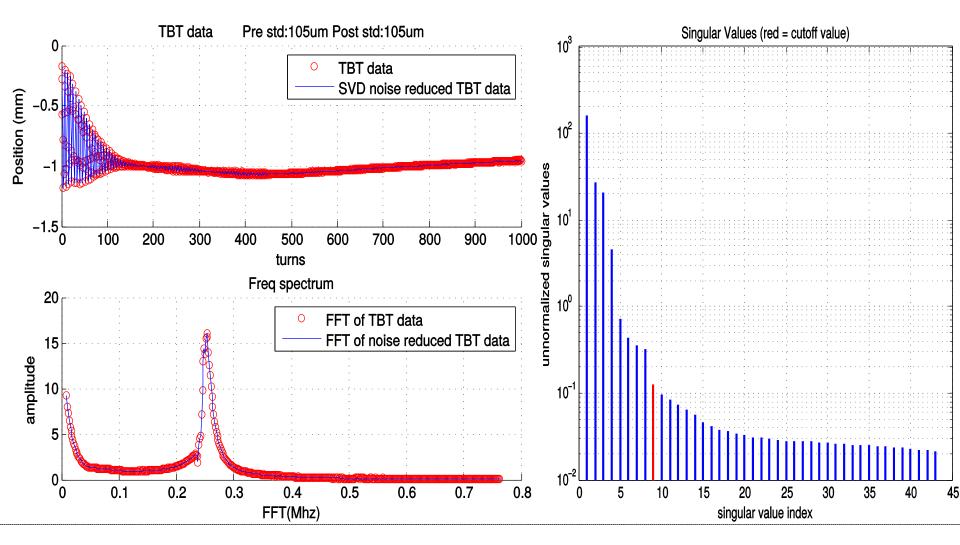


MIA - stable beam



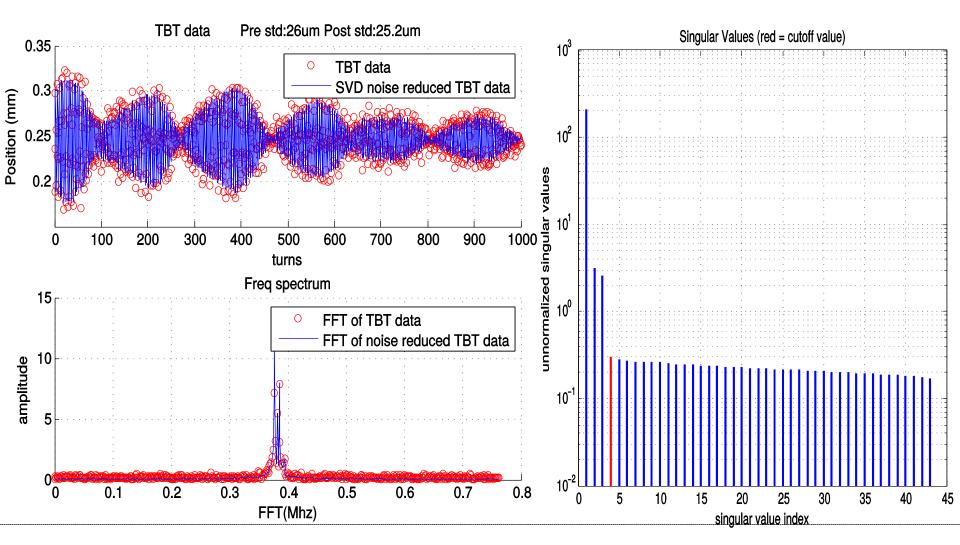


MIA - excited beam





MIA – pinger excited beam (Vertical)





MIA – spatial and temporal vectors

$$b - \langle b \rangle = B = QF^{T} + N$$

$$B \to SVD(B) = USV^{T} = \sum_{i=1}^{d} \sigma_{i}\mu_{i}\nu_{i}^{T}$$

$$U_{P \times P} = \begin{bmatrix} \mu_{1} & \mu_{2} & \dots & \mu_{P} \end{bmatrix}$$

$$V_{M \times M} = \begin{bmatrix} \nu_{1} & \nu_{2} & \dots & \nu_{M} \end{bmatrix}$$

$$S_{P \times M} = \operatorname{diag} \begin{bmatrix} \sigma_{1} & \sigma_{2} & \dots & \sigma_{M} \end{bmatrix}$$

$$d = \operatorname{rank}(B)$$



MIA – spatial and temporal vectors

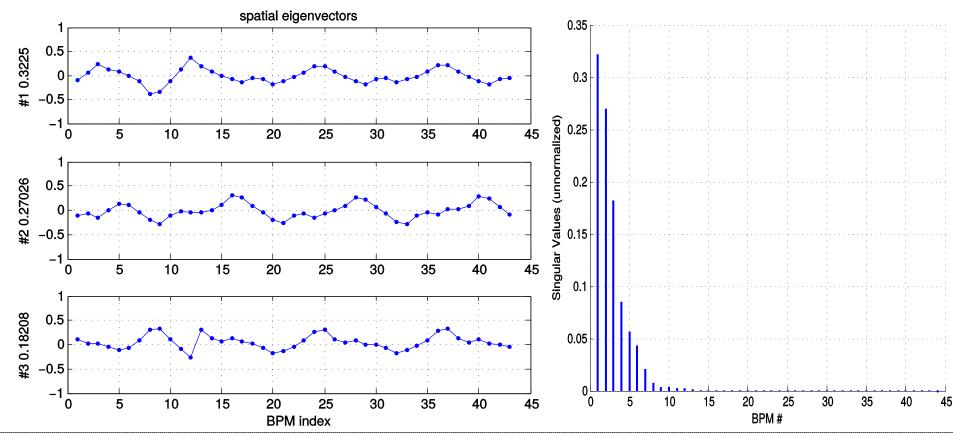
$$B \to SVD(B) = USV^T = \sum_{i=1}^{d} \sigma_i \mu_i \nu_i^T$$

- Singular values are eigenvalues and U & V are the corresponding eigenvectors
- ullet Column vectors of V & U are the eigenvectors of B^TB BB^T
- B^TB Is the variance-covariance matrix of B; this should always stay the same.
- Therefore V is a stable machine property while U changes from one group to the next.



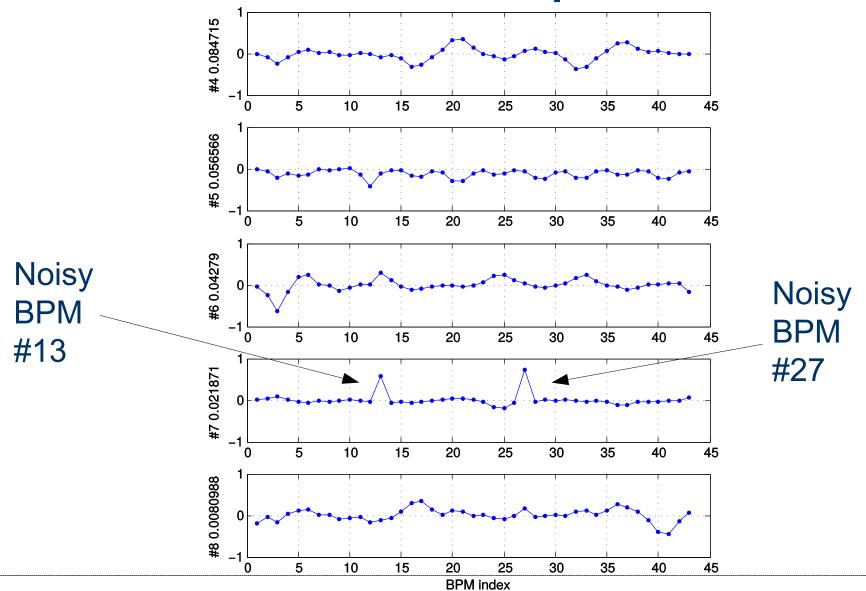
MIA – simulation of spatial vector

10,000 turn, 3mm kick in the horzontal and vertical with random Gaussian noise added to two BPMs.





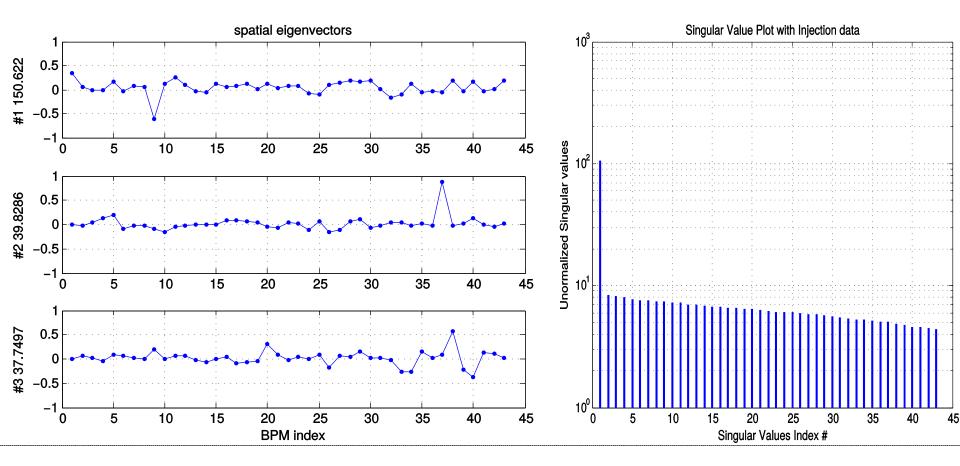
MIA – simulation of spatial vector





MIA – magnetic error – stable beam

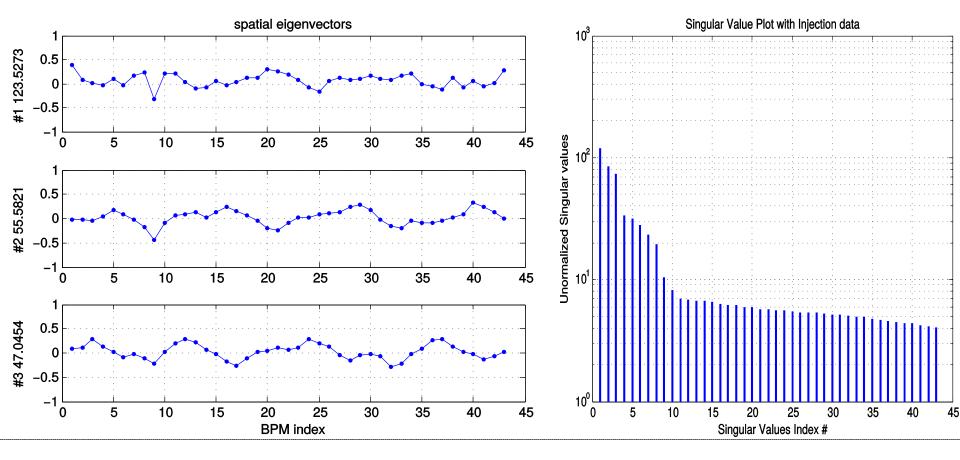
Increased QF (10,2) strength by 2 Amps (~2%)





MIA – magnetic error – excited beam

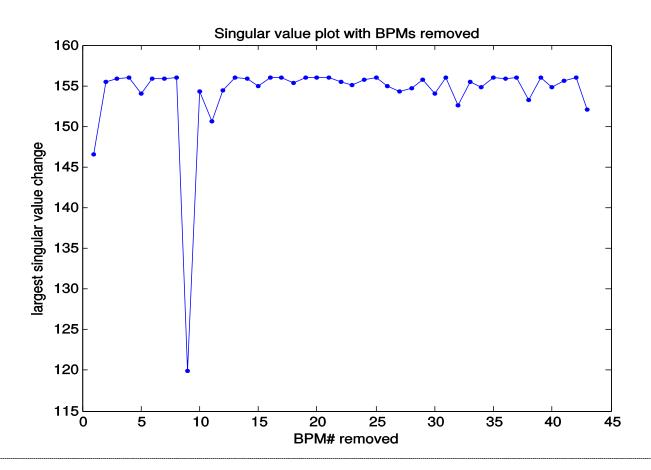
Increased QF (10,2) strength by 2 Amps (~2%)





MIA – Singular values w/ removed BPMs

Increased QF (10,2) strength by 2 Amps (~2%)





MIA – summary

• BPM precision increased by a factor of $\sqrt{\frac{d}{M}}$

- Works best with stable beam
- Deciding where the noise floor starts can be better optimized
- Works well in simulation up to ~ 15-20 micron noise
- Spatial vectors can be used to detect small irregularity



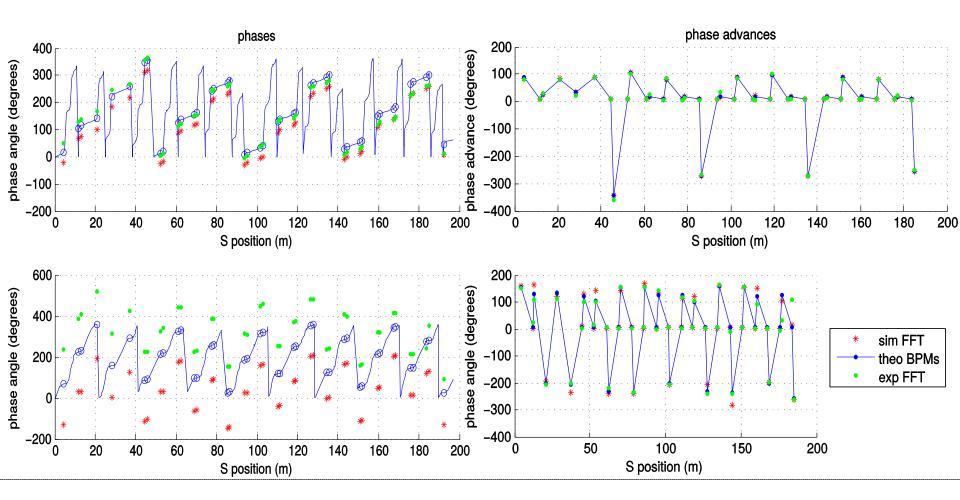
Phase & Beta measurements

- Phase measurements can be made at each BPM
 - \Rightarrow With excited beam $\phi = \tan \frac{Im(a_i)}{Re(a_i)}$ where a_i is the amplitude at the frequency corresponding to the tune.
- Taking the phase measurement differences will give the phase advances between BPMs
- Use the phase advances at three consecutive BPMs to measure an approximate Beta function.

$$B_{\rm exp} = B_{\rm theo} \left(\frac{\cot \phi_{\rm 12exp} - \cot \phi_{\rm 13exp}}{\cot \phi_{\rm 12theo} - \cot \phi_{\rm 13theo}} \right)$$

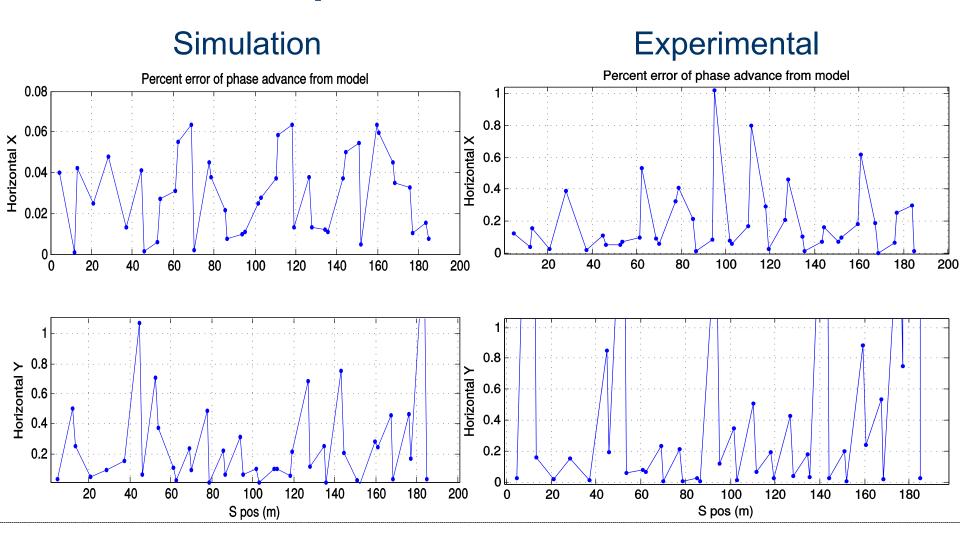


BPM phase advance



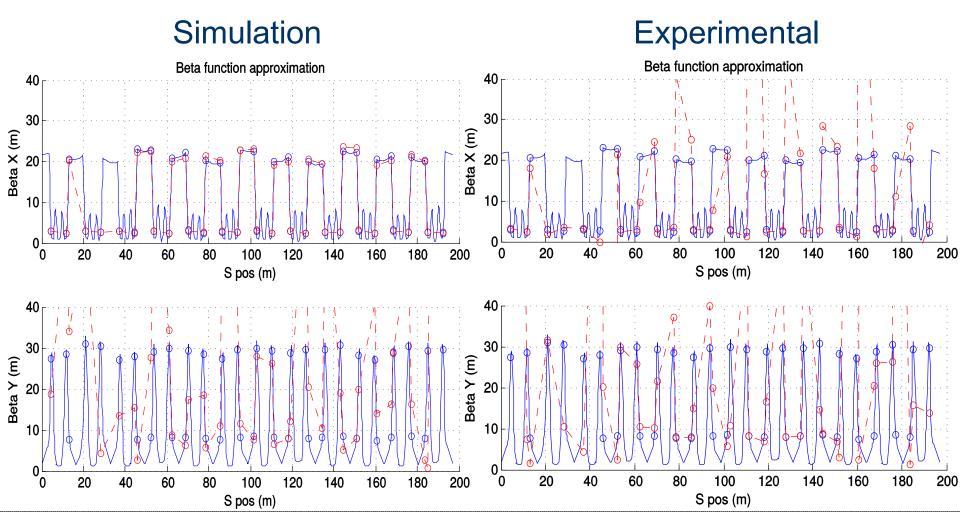


BPM phase advance errors



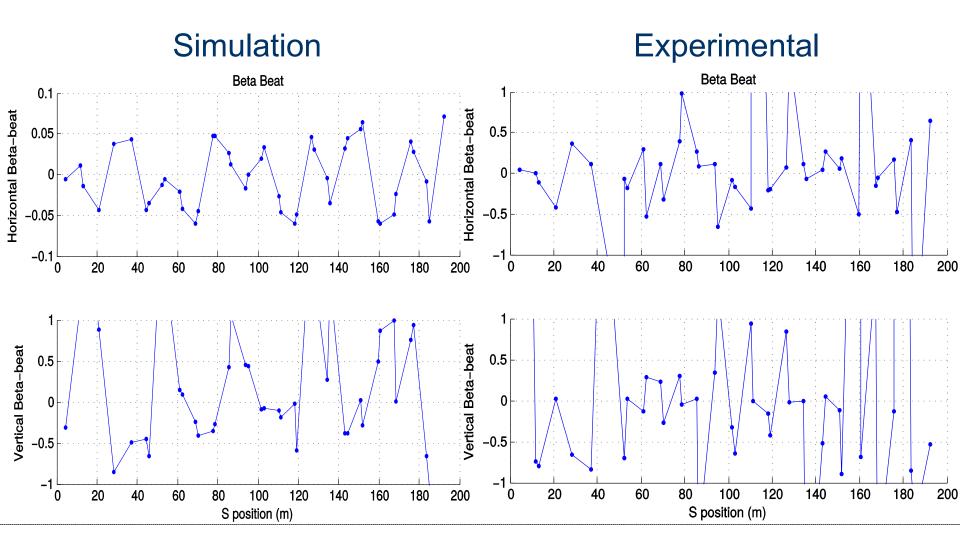


Beta Function approximations





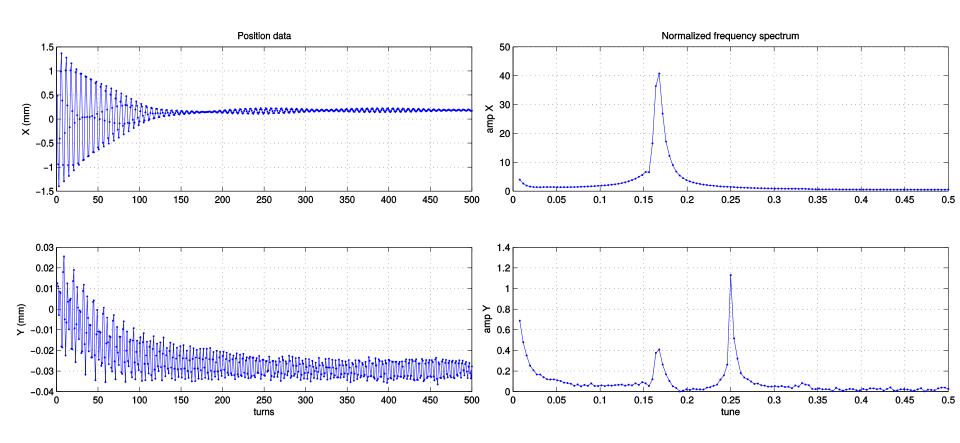
Beta-beat





Tune Measurements

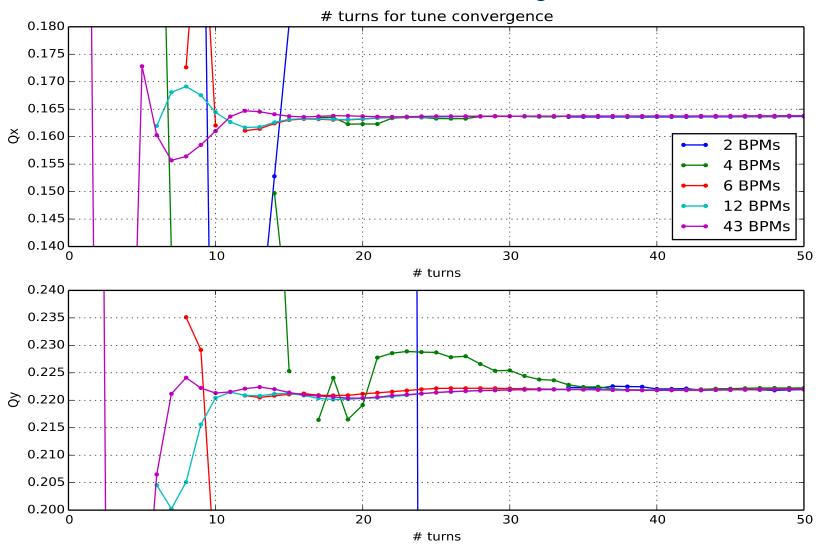
Single BPM Position & Frequency Data





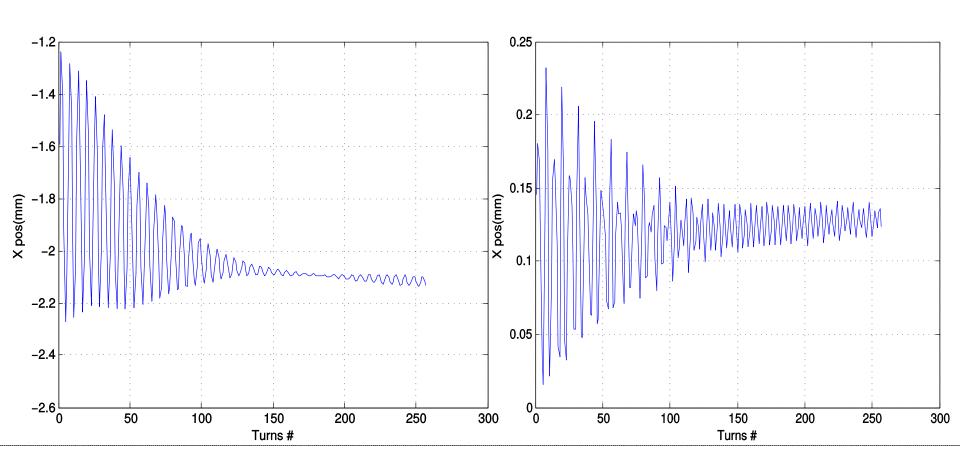
Tune Measurements

Mixed BPM tune convergence



Tune Measurements

TBT data (decoherence)





Tune, Phase, and Beta functions

- Multi BPM tune measurements look promising
- BPM gains might be affecting mixed BPM tune measurements
- Phase & phase advance measurements worked well
- Beta function approximations did not work out well
- Try measurements again when more BPMs are installed



For the future

- Degrees of freedom plot analysis with singular values
- Coupling correction with TBT data
- Linear optics with TBT data (alternative to LOCO)
- Try to further analyze mixed BPM tune measurements
- Phase & Beta function measurements with MIA techniques

