a library of exchange and correlation functionals

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Octopus Workshop







- The xc functional is at the heart of DFT
- There are many approximations for the xc (probably of the order of 250–300)
- Most computer codes only include a very limited quantity of functionals, typically around 10–15
- Chemist and Physicists do not use the same functionals!
- Difficult to reproduce older calculations with older functionals
- Difficult to reproduce calculations performed with other codes
- Difficult to perform calculations with the newest functionals







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Kohn-Sham equations

The main equations of DFT are the Kohn-Sham equations:

$$\left[-\frac{1}{2}\nabla^2 + v_{\rm ext}(r) + v_{\rm H}(r) + v_{\rm xc}(r)\right]\varphi_i(r) = \epsilon_i\varphi_i(r)$$

where the exchange-correlation potential is defined as

$$v_{\rm xc}(r) = \frac{\delta E_{\rm xc}}{\delta n(r)}$$

In any practical application of the theory, we have to use an approximation to E_{xc} , or $v_{xc}(r)$.







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Local density approximation:

$$E_{\mathrm{xc}}^{\mathrm{LDA}}(r) = E_{\mathrm{xc}}^{\mathrm{LDA}}[n]\big|_{n=n(r)}$$

Generalized gradient approximation:

$$E_{\mathrm{xc}}^{\mathrm{GGA}}(r) = \left. E_{\mathrm{xc}}^{\mathrm{GGA}}[n, \nabla n] \right|_{n=n(r)}$$

Meta-generalized gradient approximation:

$$E_{\mathrm{xc}}^{\mathrm{mGGA}}(r) = \left. E_{\mathrm{xc}}^{\mathrm{mGGA}}[n, \nabla n, \tau] \right|_{n=n(r), \tau=\tau(r)}$$







Jacob's ladder

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What do we need? - I

The energy is usually written as:

$$E_{\mathrm{xc}} = \int\!\mathrm{d}^3 r\, e_{\mathrm{xc}}(r) = \int\!\mathrm{d}^3 r\, n(r) \epsilon_{\mathrm{xc}}(r)$$

The potential in the LDA is:

$$v_{xc}^{LDA}(r) = \left. \frac{d}{dn} e_{xc}^{LDA}(n) \right|_{n=n(r)}$$

In the GGA:

$$v_{\mathrm{xc}}^{\mathrm{GGA}}(r) = \left. \frac{\partial}{\partial n} e_{\mathrm{xc}}^{\mathrm{LDA}}(n, \nabla n) \right|_{n=n(r)} - \left. \nabla \left. \frac{\partial}{\partial (\nabla n)} e_{\mathrm{xc}}^{\mathrm{LDA}}(n, \nabla n) \right|_{n=n(r)}$$







What do we need? - II

For response properties we also need higher derivatives of exc

 1st-order response (polarizabilities, phonon frequencies, etc.):

$$f_{\mathrm{xc}}^{\mathrm{LDA}}(r) = \left. \frac{d^2}{d^2 n} e_{\mathrm{xc}}^{\mathrm{LDA}}(n) \right|_{n=n(r)}$$

2st-order response (hyperpolarizabilities, etc.):

$$k_{\mathrm{xc}}^{\mathrm{LDA}}(r) = \left. \frac{d^3}{d^3 n} e_{\mathrm{xc}}^{\mathrm{LDA}}(n) \right|_{n=n(r)}$$

And let's not forget spin...







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An example: Perdew & Wang 91 (an LDA)

Perdew and Wang parametrized the correlation energy per unit particle:

$$e_c(r_s,\zeta) = e_c(r_s,0) + \alpha_c(r_s) \frac{f(\zeta)}{f''(0)} (1-\zeta^4) + [e_c(r_s,1) - e_c(r_s,0)]f(\zeta)\zeta^4$$

The function $f(\zeta)$ is

$$f(\zeta) = \frac{[1+\zeta]^{4/3} + [1-\zeta]^{4/3} - 2}{2^{4/3} - 2},$$

while its second derivative f''(0) = 1.709921. The functions $e_c(r_s, 0)$, $e_c(r_s, 1)$, and $-\alpha_c(r_s)$ are all parametrized by the function

$$g = -2A(1 + \alpha_1 r_s) \log \left\{ 1 + \frac{1}{2A(\beta_1 r_s^{1/2} + \beta_2 r_s + \beta_3 r_s^{3/2} + \beta_4 r_s^2)} \right\}$$



Written in C from scratch

- Bindings both in C and in Fortrar
- Lesser GNU general public license (v. 3.0)
- Automatic testing of the functionals
- Contains at the moment 33 LDA functionals, 142 GGA functionals, 36 hybrids, and 14 mGGAs
- Contains functionals for exchange, correlation, and kinetic energy
- Contains functionals for 1D, 2D, and 3D calculations
- Returns ε_{xc} , v_{xc} , f_{xc} , and k_{xc}
- Quite mature: included in 16 codes (including abinit, espresso, cp2k, etc.)







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What is working!

	$\varepsilon_{ m xc}$	$V_{\rm xc}$	$f_{\rm xc}$	k _{xc}
LDA	OK	OK	OK	OK
GGA	OK	OK	OK	NO
HYB_GGA	OK	OK	OK	NO
mGGA	TEST	TEST	TEST	NO







An example in C

```
switch (xc_family_from_id(xc.functional))
case XC FAMILY LDA:
  if (xc.functional == XC_LDA_X)
    xc_lda_x_init(&lda_func, xc.nspin, 3, 0);
  else
    xc_lda_init(&lda_func, xc.functional, xc.nspin);
  xc_lda_vxc(&lda_func, xc.rho, &xc.zk, xc.vrho);
  xc_lda_end(&lda_func);
  break:
case XC FAMILY GGA:
  xc_gga_init(&gga_func, xc.functional, xc.nspin);
  xc_gga_vxc(&gga_func, xc.rho, xc.sigma, &xc.zk, xc.vrho, xc.vsigma
  xc_gga_end(&gga_func);
  break:
default:
  fprintf(stderr, "Functional '%d' not found\n", xc.functional);
  exit(1);
```







Another example in Fortran

```
program Ixctest
 use libxc
  implicit none
  real(8) :: rho, e_c, v_c
 TYPE(xc_func) :: xc_c_func
 TYPE(xc_info) :: xc_c_info
 CALL xc_f90_lda_init(xc_c_func, xc_c_info, &
    XC_LDA_C_VWN, XC_UNPOLARIZED)
 CALL xc_f90_lda_vxc(xc_c_func, rho, e_c, v_c)
 CALL xc_f90_lda_end(xc_c_func)
end program Ixctest
```







The info structure

```
typedef struct{
  int
       number; /* indentifier number */
  int kind: /* XC_EXCHANGE or XC_CORRELATION */
  char *name; /* name of the functional, e.g. "PBE" */
  int family; /* type of the functional, e.g. XC_FAMILY_GGA */
  char *refs: /* references
      provides; /* e.g. XC_PROVIDES_EXC | XC_PROVIDES_VXC */
  int
} xc_func_info_type;
This is an example on how you can use it:
  xc_gga_type b88;
  xc_gga_init(&b88, XC_GGA_X_B88, XC_UNPOLARIZED);
  printf("The functional '%s' is defined in the reference(s):\n%s",
    b88.info->name, b88.info->refs);
  xc_gga_end(&b88);
```







The future

- More functionals!
- More derivatives
- More codes using it!







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Where to find us!



Libxc: a library of exchange and correlation functionals for DFT M.A.L. Marques, M.J.T. Oliveira, and T. Burnus

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