The Methods of Numerical Relativity

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Outline

When *not* to try Numerical Relativity

We seek solutions to the gravitational field equations,

$$G_{ab} = R_{ab} - \frac{1}{2}\, g_{ab} R = 8\pi T_{ab}\,.$$

which describe how curvature of spacetime evolves and is related to matter distribution. Non-linear PDEs. How can we find solutions? Different physics:

- Is the desired solution time independent, or with a lot of symmetry? There might be an exact solution. Read a book.
- Is the desired solution close to an exact solution that we already know? Perturbation theory/asymptotic expansions. Second-order?

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When to try Numerical Relativity

On the other hand:

- Are the fields highly dynamical? Are non-linear terms expected to dominate the behavior? Is there no symmetry? Use Numerical Relativity; *numerical methods to find approximate solutions that converge to the continuum solution in the limit of infinite resolution.*

Ingredients

Mathematics:

- PDEs theory.
- Differential geometry.
- Numerical analysis.

Physics:

- GR.
- Matter models.

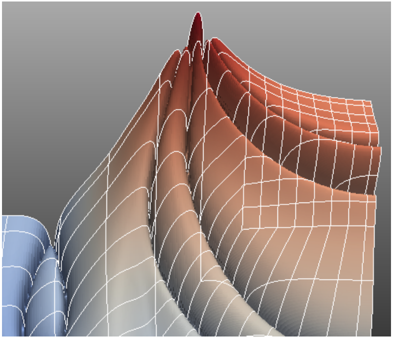
Computation:

- HPC.
- Numerical methods.
- Visualization.

Event horizon of merging blackholes. Thierfelder. 2009

Example problems

- Critical collapse. Can we make arbitrarily small blackholes?
- Computational astrophysics: supernova core collapse.
- Gravitational wave astronomy.



Curvature in GW collapse evolution. DH, Weyhausen, Brügmann. 2015

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LIGO Hanford

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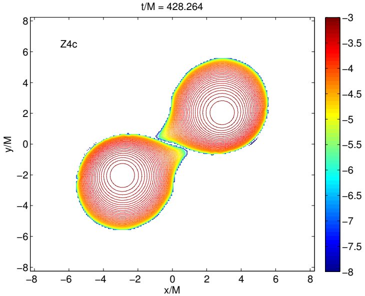


Contour plot of density of NS. DH+. 2013

A brief history of NR

From Wikipedia NR page and Smarr:

- Late 1950s: ADM decomposition.
- 1960s. First numerics, including Smarr [nice talk: http://online.itp.ucsb.edu/online/numrel00/smarr/].
- 1970s. Axisymmetric spacetimes. York 3+1 decomposition.
- 1980s. Numerical Relativity becomes a field.
- 1990s. Critical Phenomena. 3d work becomes feasible. Grand challenge.
- 2000s. Compact binaries.

Outline

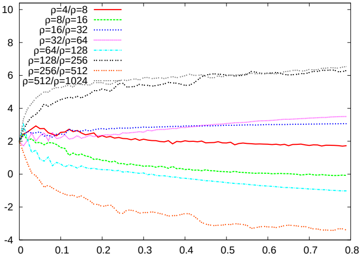
Evolution equations and constraints

Decomposition:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + 2D_{(i}\beta_{j)}\,,$$
$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha[R_{ij} - 2K^k{}_i K_{jk} + K K_{ij}] + \mathcal{L}_\beta K_{ij}\,,$$
$$H = R - K_{ij}K^{ij} + K^2 = 0\,,$$
$$M_i = D^j(K_{ij} - \gamma_{ij}K) = 0\,.$$

- Evolution and constraint equations.
- Constraint subsystem closed.
- Gauge freedom similar to E&M / YM.

Formulation crucial!



Cao and DH. 2011

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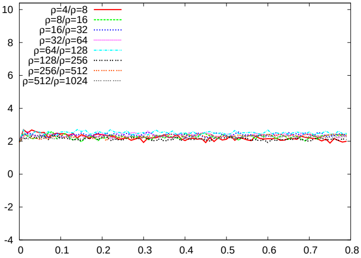

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Methods

Bare-bones summary:

- Grid-up (physical).
- Time integration: MoL.

Spatial derivatives:

- Finite differences.
- HRSC. (Fluids).
- Pseudospectral.

Boundaries:

- Elliptic for ID.
- CPBCs.

Grids:

- Moving-box/adaptive MR.
- Multipatch.
- Corotation.

Popular codes:

- The Einstein Toolkit
- Spectral Einstein Code.
- BAM and `bamps`. (Jena)

Various licences.

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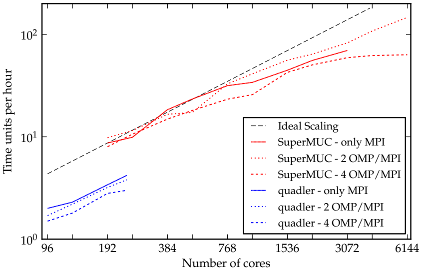
Parallelization

Implementation typically with:

- Automated generation.
- Mixture of C^{++}/C/Fortran.

Parallelization:

- MPI/OMP standard.
- GPU? Memory a problem. Some applications.
- MR & balancing an issue.



Strong scaling of BAM at LRZ.

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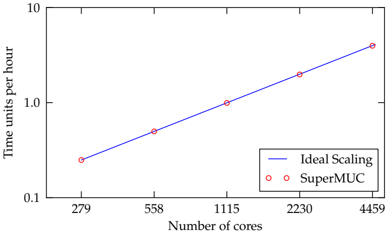

Strong scaling of `bamps` at LRZ.

Outline

Some parting thoughts...

Points for discussion:

- Solid mathematical foundation, although implementation of principle solutions not always clear.
- Smooth solutions often expected. Meaningful error analysis possible!
- Perfect parallelization hard. Libraries?

One expects a *great deal* of overlap with the rest of computational physics. Sadly much develoment in isolation.