Marks & final feedback on last page. In-text comments begin with the rubric criterion they are related to.

IB AA HL Math IA

A: IA should be double spaced so we can write comments between lines as examiners.

A: IA should have a front cover with the course name, title, and page count.

How can the quantity of garland needed to decorate my Christmas tree be determined, striking a balance between meeting personal aesthetic preferences and minimizing waste?

: Unlike the EE, the IA need not have an RQ. A mple title would suffice and might even be preferred.

Introduction

A: This is technically not a necessary part of the IA. In fact, some IA examiners discourage the use of TOCs or even section headers, as that has the potential to reduce the flow of a paper. It is fine to leave it, and the same goes for headers, but make sure that you do not lose flow!

During Christmas, my family bought a Christmas tree to place at home. Unfortunately, it lacked decorations, so we decided to go to the store to buy some garland for the tree. However, we were uncertain of how much we needed to buy, so we guessed the amount we needed. In our attempt to minimize waste, we tried to buy as little garland as we thought we needed, which ended up needing more garland. We promptly returned to the store to buy more garland, but this time we ended up with way too much garland. Faced with these troubles, I decided that for my investigation, I would find a method to calculate the amount of garland required to wrap around a tree based on certain parameters. Then, I will manipulate the spacing of the garland to obtain a balance between meeting personal aesthetic preferences and minimizing waste.

C: Personal connection to topic.

Aim and Methodology

The goal of this paper is to devise a general formula which accounts for certain parameters of the tree as well as the spacing between successive rotations of the garland to obtain the amount of the garland I would need to buy. To do so, I must first mathematically model the Christmas tree and the garland that wraps around the tree. This involves making some assumptions and approximations, to simplify the model:

A: Clear aim identified.

E: If you end up needing more rigour of math, it might be interesting to consider a different bounding curve than a straight line.

1. **The Christmas tree is "ideal"** – The Christmas tree is modelled as a *cone*, based on the assumption that an "ideal" tree would be radially symmetrical all around and that the slant of the tree with respect to its vertical axis is a straight line.

D: Good consideration of constraints and assumptions of modeling process.

2. **The garland wraps uniformly** – This means that it wraps in a perfect spiral around the tree, without sagging. The garland should also wrap around the tree with equal spacings between subsequent rotations. Since the tree is approximated to be a cone, the garland wrapping around the tree can be modelled as a *circular conical spiral*.

D: At some point, discuss the validity of these assumptions in real life.

The following parameters will be considered for the calculations:

A: This description should accompany the respective figure.

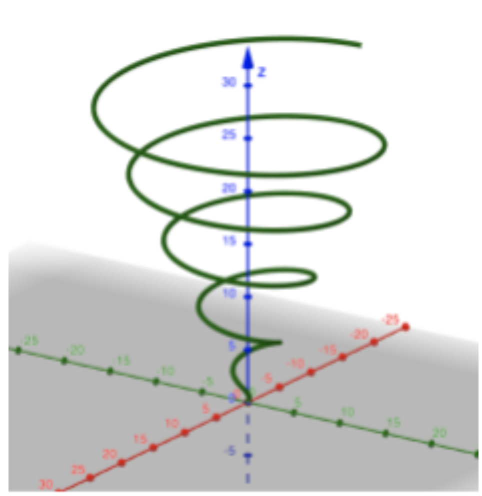


Figure 1: Circular Conical Spiral (modelled using Geogebra)

GeoGebra

A: Consider a higher DPI export to not be blurry

A: This looks copy-and-pasted. Consider retyping in LaTeX.

Parameter	***Description***
H	height of the tree
R	radius of the base of the tree
λ	slant distance (spacing) between successive rotations of the garland

B: Are there any units you will be using?

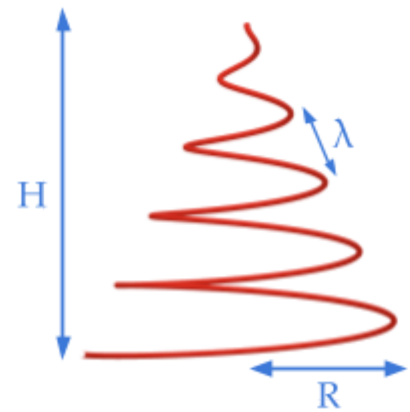


Figure 2: Parameters

Modelling the Garland

The garland can be modelled as a *circular conical spiral*, which has the following parametric function (Rejbrand, n.d.):

A: Reduce empty space.

E: To demonstrate understanding of this model, consider what we discussed in class or show different values of t plotted in GeoGebra.

$$C(t) = \begin{pmatrix} at\cos t \\ at\sin t \\ bt \end{pmatrix}, \qquad \forall t \in [0, H] \tag{1}$$

where the constants $a, b \in \mathbb{R}$. t is the independent variable, and the bounds of the function are restricted between 0 and H because that is the range which represents the height of the tree, and hence only that part of the curve is useful. For a given value t, the parametric function outputs a point on the curve, and as we generate an infinite amount of points between 0 and H, a locus of points are generated, forming the shape of the space curve. The z-axis of the spiral coincides with the tree's axis of symmetry and $r(0)$ represents the tip of the tree. B: Use math mode for all variables.

To ensure that the spiral sits on the surface of the cone (the Christmas tree), I want to find the appropriate values a and b by relating them to the parameters I have chosen. Based on methodology inspired by a blog post by Alan Stewart and Heighway, $\rho(t)$ is defined as the radial distance from the tree's axis of symmetry. Using Pythagorean's theorem:

$$\rho(t) = \sqrt{x(t)^2 + y(t)^2}$$

B: Make sure you are typing the formulae yourself.

where $x(t)$ and $y(t)$ represent the x and y components of $C(t)$ respectively.

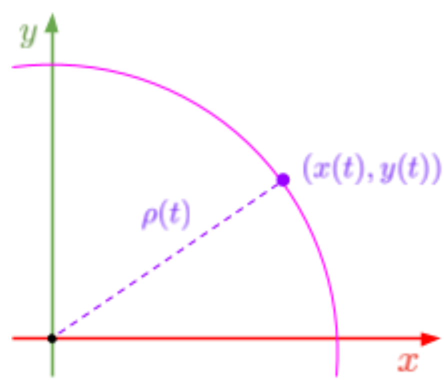


Figure 3: Radial distance from *z*-axis

➢ Substituting for $x(t)$ and $y(t)$

$$\begin{aligned} \rho(t) &= \sqrt{a^2t^2\cos^2 t + a^2t^2\sin^2 t} \\ &= at\sqrt{\cos^2 t + \sin^2 t} \\ &= at \end{aligned} \tag{2}$$

A: These look copy-and-pasted. The a is even cut.

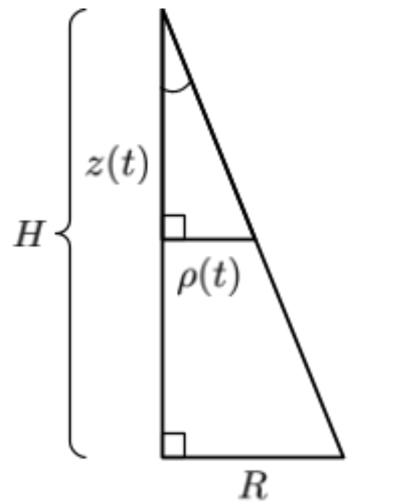


Figure 4: Proportional Relationship

From Figure 4, it can be seen that the triangle formed by $z(t)$ and $\rho(t)$ is similar to the triangle formed by the vertical cross-section of the cone, as a result of the shared interior angle. This allows us to establish the following proportional relationship:

E: State by Angle-Angle.

$$\begin{aligned} \frac{R}{H} &= \frac{\rho(t)}{z(t)} = \frac{at}{bt} \\ \Rightarrow \frac{R}{H} &= \frac{a}{b} \end{aligned} \tag{3}$$

E: Unclear why z(t) is bt.

In other words, for the spiral to lie on the surface of the cone, the ratio a over b must be proportional to R over H. In addition, this also tells us how modifying a and b changes the shape of the circular conical spiral. A larger value for a corresponds to a wider cone and vice versa, while a larger value for b corresponds to a larger vertical gap between consecutive spirals of the garland and vice versa.

D: Some reflection shown.

C: Consider showing example of different conical spirals with the same axis scale and differing a and b values.

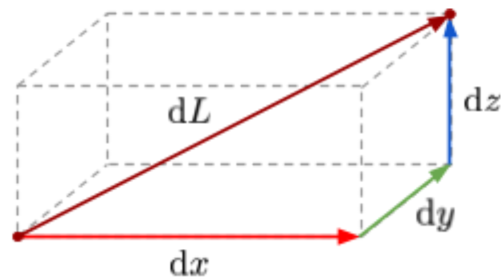
Deriving an Equation for the Length of the Garland

With a function which models the garland, we can now calculate the length of the garland by calculating the *arc length* of $C(t)$. The arc length, L, is defined as the distance travelled along the path of the curve from one point to another (“8.1: Arc Length” 2017), and it can be evaluated by decomposing the curve $C(t)$ as an infinite sum of infinitesimally small line segments, $\mathrm{d}L$. Thus, the length of the garland is represented by the definite integral:

$$L = \int_0^H \mathrm{d}L \tag{4}$$

The lower and upper bounds on the integral are a result of the restrictions imposed on $C(t)$.

However, what is $\mathrm{d}L$? To find what $\mathrm{d}L$ is, it is useful to think of it as a 3-dimensional vector. One important property of vectors is that they can be expressed as a sum of multiple vectors, and thus 3D vectors can be decomposed into their x, y, and z components. Using this line of thinking, it can similarly be said that $\mathrm{d}L$ can be decomposed into the infinitesimals $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$. The geometric intuition for this is visualized in Figure 5. Therefore, the relation between $\mathrm{d}L$ with $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$ can be found by applying the 3D Pythagorean theorem:



B: Sate if you made this.

Figure 5: dL in terms of dx, dy, and dz

$$\mathrm{d}L = \sqrt{(\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2}$$

However, this is not of much use, as $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$ are arbitrary. Thus, $\mathrm{d}t$ is introduced into the equation by multiplying the equation by $\mathrm{d}t/\mathrm{d}t$, and with some algebraic manipulation, $\mathrm{d}L$ can be expressed in terms of the derivatives of the x, y, and z components of the curves, which can be evaluated (Schlicker et al., n.d.):

$$\begin{aligned}\mathrm{d}L &= \sqrt{\frac{(\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2}{(\mathrm{d}t)^2}}\,\mathrm{d}t \\ &= \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2}\,\mathrm{d}t\end{aligned}$$

A: Keep on 1 line.

- Substituting for $x(t)$, $y(t)$, and $z(t)$:

$$\begin{aligned}\mathrm{d}L &= \sqrt{\left(\frac{\mathrm{d}}{\mathrm{d}t}(at\cos t)\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}t}(at\sin t)\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}t}(bt)\right)^2}\,\mathrm{d}t \\ &= \sqrt{(a\cos t - at\sin t)^2 + (a\sin t + at\cos t)^2 + b^2}\,\mathrm{d}t\end{aligned}$$

B: Consider putting a dot product symbol before the dt to separate it from the radical.

- Expanding and simplifying:

$$\begin{aligned}\mathrm{d}L &= \sqrt{(a^2\cos^2 t - \cancel{2a^2t\sin t\cos t} + a^2t^2\sin^2 t) + (a^2\sin^2 t + \cancel{2a^2t\sin t\cos t} + a^2t^2\cos^2 t) + b^2}\,\mathrm{d}t \\ &= \sqrt{a^2\sin^2 t + a^2\cos^2 t + a^2t^2\sin^2 t + a^2t^2\cos^2 t + b^2}\,\mathrm{d}t \\ &= \sqrt{a^2\cancel{(\sin^2 t + \cos^2 t)} + a^2t^2\cancel{(\sin^2 t + \cos^2 t)} + b^2}\,\mathrm{d}t \\ &= \sqrt{a^2 + a^2t^2 + b^2}\,\mathrm{d}t \\ &= \sqrt{a^2 + b^2}\sqrt{1 + \frac{a^2}{a^2 + b^2}t^2}\,\mathrm{d}t \qquad (5)\end{aligned}$$

E: Explain why the final 'simplification' was necessary.

As such, an equation for $\mathrm{d}L$ has been derived. Substituting it for $\mathrm{d}L$ in ***Eqn. 4***, the integral is now expressed in terms of a and b:

B: The font changed for the equation here.

$$L = \int_0^H \sqrt{a^2+b^2}\sqrt{1+\frac{a^2}{a^2+b^2}t^2}\,\mathrm{d}t \tag{6}$$

Although the constants a and b determine the shape of the curve, they do not represent any physical quantity which is of practical use to us. Therefore, they should be substituted with expressions based on the chosen input parameters H, R, and λ.

D: Evidence of some reflection.

Substituting for Expressions with a and b

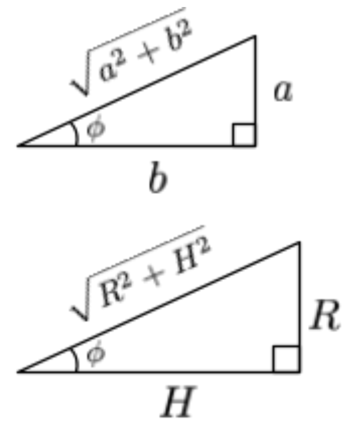


Figure 6: Similar Triangles

Let us first focus on the expression $\frac{a^2}{a^2+b^2}$. From ***Eqn. 3***, we know that a and b is proportional to H and R respectively. As such, similar triangles can be employed again to achieve a proportional relationship, as visualized in Figure 6.

➢ Let the interior angle of triangle be ϕ. Recall that the sine of an acute angle of a right triangle is equal to the side opposite to the angle over the hypotenuse. Thus:

$$\sin\phi = \frac{a}{\sqrt{a^2+b^2}} = \frac{R}{\sqrt{R^2+H^2}}$$

C: Evidence of personal engagement.

➢ Squaring both sides, we find an expression for $\frac{a^2}{a^2+b^2}$ in terms of R and H:

$$\frac{a^2}{a^2+b^2} = \frac{R^2}{R^2+H^2} \tag{7}$$

Now, onto the other expression containing a and b, which is $\sqrt{a^2+b^2}$. For this expression, trigonometric ratios are to our advantage, because it is not a fraction. However, another property of right triangles can be used, which is that scaling one side of a right triangle also scales the other sides by the same factor. This can be shown by multiplying Pythagoras' theorem by a scale factor:

$$a^2+b^2=c^2 \Leftrightarrow ka^2+kb^2=kc^2$$

We previously established that the triangle formed by a and b is similar to the triangle formed by the vertical cross-section of the cone, and since the parameter specifying the spacing of the garland, λ, lies on the slant of the tree, λ must therefore equal to $k\sqrt{a^2+b^2}$, where $k \in \mathbb{R}$ is some scale factor.

For every rotation of the garland, t increases by 2π since that is the period of sine and cosine. Thus, for a given t value, the radial distance after one rotation would be $\rho(t + 2\pi)$ and the height after one rotation $z(t + 2\pi)$. Therefore:

$$\begin{aligned} \rho(t+2\pi) &= a[t+2\pi] \\ &= at + 2\pi a \\ &= \rho(t) + 2\pi a \end{aligned} \qquad\qquad \begin{aligned} z(t+2\pi) &= b[t+2\pi] \\ &= bt + 2\pi b \\ &= z(t) + 2\pi b \end{aligned}$$

This tells us that for every rotation, the radial distance ρ of the spiral increases by $2\pi a$ and the vertical change increases by $2\pi b$. Since the slant distance between consecutive rotations of the garland is λ, the following equation is reached using Pythagoras' theorem:

B: Should there be a square here for lambda?

$$\begin{aligned} \lambda^2 &= \sqrt{(2\pi a)^2 + (2\pi b)^2} \\ &\Rightarrow \lambda = 2\pi\sqrt{a^2 + b^2} \end{aligned}$$

- Isolating for $\sqrt{a^2 + b^2}$:

$$\sqrt{a^2 + b^2} = \frac{\lambda}{2\pi} \tag{8}$$

Thus, we can now substitute $\frac{R^2}{R^2+H^2}$ for $\frac{a^2}{a^2+b^2}$ and $\frac{\lambda}{2\pi}$ for $\sqrt{a^2 + b^2}$ into ***Eqn. 6*** to finally obtain:

$$L = \int_0^H \frac{\lambda}{2\pi}\sqrt{1 + \frac{R^2}{R^2 + H^2}t^2}\,\mathrm{d}t \tag{9}$$

B: Deductive methods are clearly laid out.
E: Math is correct so far.

Evaluating the Integral

Now, we evaluate the integral so that we can finally obtain a general solution for the length of the garland based on our chosen parameters. D: Good link back to aims

- To begin, we can factor out $\frac{\lambda}{2\pi}$ from the integral since it is constant:

$$L = \frac{\lambda}{2\pi}\int_0^H \sqrt{1 + \frac{R^2}{R^2 + H^2}t^2}\,\mathrm{d}t$$

- Factor $R^2 + H^2$ from the square root:

A: Keep descriptions with their equations.

$$\Rightarrow L = \frac{\lambda}{2\pi\sqrt{R^2+H^2}} \int_0^H \sqrt{R^2+H^2+R^2t^2}\,\mathrm{d}t$$

➢ Recognizing that $S = \sqrt{R^2 + H^2}$ is the slant height of a cone, we substitute it in the equation,

$$\Rightarrow L = \frac{\lambda}{2\pi S} \int_0^H \sqrt{S^2+R^2t^2}\,\mathrm{d}t$$

Since the integral is of the form $\sqrt{c^2 + x^2}$, the integral can be evaluated using trigonometric substitution.

E: Good connection to mathematics commensurate with the course level.

➢ Let $t = \frac{S}{R}\tan\theta$. Thus $\mathrm{d}t = \frac{S}{R}\sec^2\theta\,\mathrm{d}\theta$. Substituting them into the integral,

$$\begin{aligned}
\Rightarrow L &= \frac{\lambda}{2\pi \cancel{S}} \int_0^{t=H} \sqrt{S^2+R^2\left(\frac{S}{R}\tan\theta\right)^2} \cdot \frac{\cancel{S}}{R}\sec^2\theta\,\mathrm{d}\theta \\
&= \frac{\lambda}{2\pi R} \int_0^{t=H} \sqrt{S^2+S^2\tan^2\theta} \cdot \sec^2\theta\,\mathrm{d}\theta \\
&= \frac{\lambda}{2\pi R} \int_0^{t=H} S\sec\theta \cdot \sec^2\theta\,\mathrm{d}\theta \\
&= \frac{\lambda S}{2\pi R} \int_0^{t=H} \sec^3\theta\,\mathrm{d}\theta \qquad (10)
\end{aligned}$$

Then, using integration by parts, the integral $sec^3\theta$ can be evaluated.

➢ Let $u = \sec\theta$ and $\mathrm{d}v = sec^2\theta\,\mathrm{d}\theta$. Therefore $du = \sec\theta\,\mathrm{d}\theta$ and $v = \tan\theta$.

$$\Rightarrow L = \frac{\lambda S}{2\pi R}\sec\theta\tan\theta\Big|_0^{t=H} - \frac{\lambda S}{2\pi R}\int_0^{t=H}\sec\theta\tan^2\theta\,\mathrm{d}\theta$$ E: Mathematics is correct so far

➢ Using the trigonometric identity $\tan^2\theta = \sec^2\theta - 1$, $\sec\theta\ \tan^2\theta = \sec^3\theta - \sec\theta$,

$$\begin{aligned}
\Rightarrow L &= \frac{\lambda S}{2\pi R}\sec\theta\tan\theta\Big|_0^{t=H} - \frac{\lambda S}{2\pi R}\int_0^{t=H}(\sec^3\theta - \sec\theta)\,\mathrm{d}\theta \\
&= \frac{\lambda S}{2\pi R}\sec\theta\tan\theta\Big|_0^{t=H} - \frac{\lambda S}{2\pi R}\int_0^{t=H}\sec^3\theta\,\mathrm{d}\theta + \frac{\lambda S}{2\pi R}\int_0^{t=H}\sec\theta\,\mathrm{d}\theta
\end{aligned}$$

➢ Since $\frac{\lambda S}{2\pi R}\int_0^{t=H} \sec^3\theta\, d\theta = L$ ***(Eqn. 10)***,

$$\Rightarrow L = \frac{\lambda S}{2\pi R}\sec\theta\tan\theta\Big|_0^{t=H} - L + \frac{\lambda S}{2\pi R}\int_0^{t=H}\sec\theta\, d\theta$$

$$\Rightarrow 2L = \frac{\lambda S}{2\pi R}\left[\sec\theta\tan\theta\,\Big|_0^{t=H} + \int_0^{t=H}\sec\theta\, d\theta\right]$$

$$\Rightarrow L = \frac{\lambda S}{4\pi R}\left[\sec\theta\tan\theta\,\Big|_0^{t=H} + \int_0^{t=H}\sec\theta\, d\theta\right]$$

B: This is slightly confusing here in terms of layout and conditional notation. It is correct, but just pointing it out that it is hard to see the L as not being par of the integrals.

➢ By $\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + c$,

$$\Rightarrow L = \frac{\lambda S}{4\pi R}\left[\,\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\,\right]_0^{t=H} \tag{11}$$

➢ Since we substituted $t = \frac{S}{R}\tan\theta$, then $\tan\theta = \frac{R}{S}t$. Then, using the trigonometric identity $\sec^2\theta = 1 + \tan^2\theta$, we can find an equation for $\sec\theta$:

$$\begin{aligned}\sec\theta &= \sqrt{1+\tan^2\theta}\\ &= \sqrt{1+\frac{R^2}{S^2}t^2}\\ &= \frac{\sqrt{S^2+R^2t}}{S}\end{aligned}$$

➢ Thus:

$$\begin{aligned}\Rightarrow L &= \frac{\lambda S}{4\pi R}\left[\frac{\sqrt{S^2+R^2t^2}}{S}\cdot\frac{R}{S}t + \ln\left|\frac{\sqrt{S^2+R^2t^2}}{S} + \frac{R}{S}t\right|\right]_0^H\\ &= \frac{\lambda S}{4\pi R}\left[\frac{Rt\sqrt{S^2+R^2t^2}}{S^2} + \ln\left|\sqrt{S^2+R^2t^2}+Rt\right| - \ln(S)\right]_0^H\end{aligned}$$

➢ Finally, we arrive at the final solution by using **FTC Part 2**:

$$L(\lambda, R, H) = \frac{\lambda S}{4\pi R}\left[\frac{RH}{S^2}\sqrt{S^2+R^2H^2} + \ln\left(\sqrt{S^2+R^2H^2}+RH\right) - \ln(S)\right] \tag{12}$$

E: Show this work to demonstrate how the ln(S) cancels when t=0

where $S = \sqrt{R^2+H^2}$

E: Could this look simpler if S is replaced with this value?

D: How can you assess the validity of your model?

Finding the Correct Spacing of Garland

Now that I have derived the general solution for the length of the garland based on my chosen input parameters, I can now input the approximate parameters height $H = 72\ in$ and $R = 15\ in$ of my Christmas tree to find the specific solutions for my particular Christmas tree. However, I have yet to decide on the spacing of the garland, because my next objective is to find values of λ which meet my personal aesthetic preferences as well as minimize waste.

A: Before getting to this section, it would be good to show a numerical solution with a given value of lambda.

Minimizing Waste

Let G be the length of one piece of garland. First, we want to first narrow down possible solutions which minimize waste. When buying garland, it always comes in standard lengths. For example, the one which my family bought is $6\ ft$ long ($72\ in$). Thus, we want solutions where the total length of garland equal to multiples of the length of one length of garland so that there is no excess garland. Hence:

$$kG = \frac{\lambda S}{4\pi R}\left[\frac{RH}{S^2}\sqrt{S^2+R^2H^2} + \ln\left(\sqrt{S^2+R^2H^2}+RH\right) - \ln(S)\right], \quad k \in \mathbb{Z}^+ \quad (13)$$

➢ Now, isolating for λ, we obtain:

$$\lambda = kG \cdot \frac{4\pi RS}{RH\sqrt{R^2H^2+S^2} + S^2\left(\ln\left(\sqrt{R^2H^2+S^2}+RH\right) - \ln S\right)} \quad (14)$$

➢ Replacing for $R = 15\ in$, $H = 72\ in$, $G = 72\ in$, and $S = \sqrt{15^2 + 72^2} = 74.5\ in$,

$$\lambda = k(72) \cdot \frac{4\pi(72)(73.5)}{(15)(72)\sqrt{(15)^2(72)^2+(73.5)^2} + (73.5)^2\left(\ln\left(\sqrt{(15)^2(72)^2+(73.5)^2}+(15)(72)\right) - \ln 73.5\right)}$$

E: I have not verified the accuracy of the computation.

➢ Simplifying, we finally obtain that the spacing between consecutive rotation of garland should equal:

D: Reflect on what this means exactly. Is this only a 0.84 inch difference between the strands of the garland if k=1?

$$\lambda = k(0.841), \quad k \in \mathbb{Z}^+ \quad (15)$$

E: Discuss level of accuracy.

This is a linear relationship, and this is visualized in Figure 7:

A: Keep descriptions along with their visuals.

A: Unclear what these graphs are doing for us in the conical spiral.

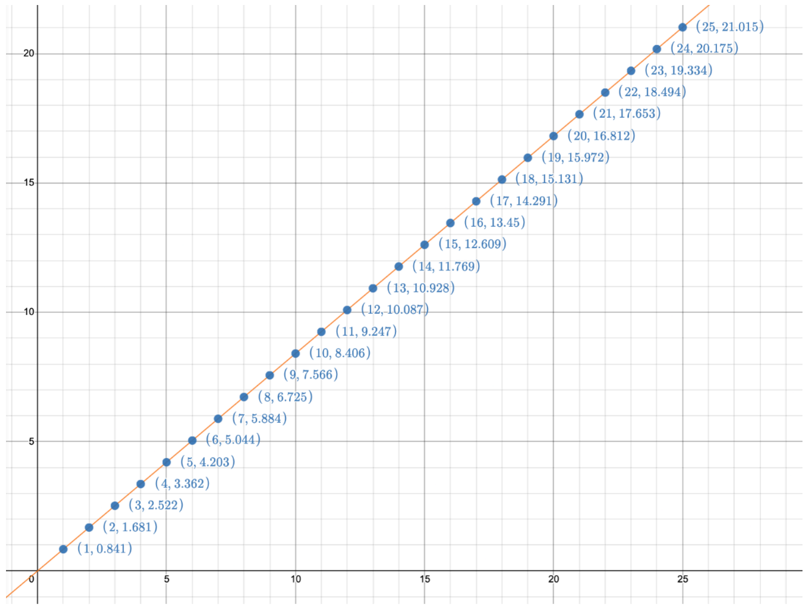
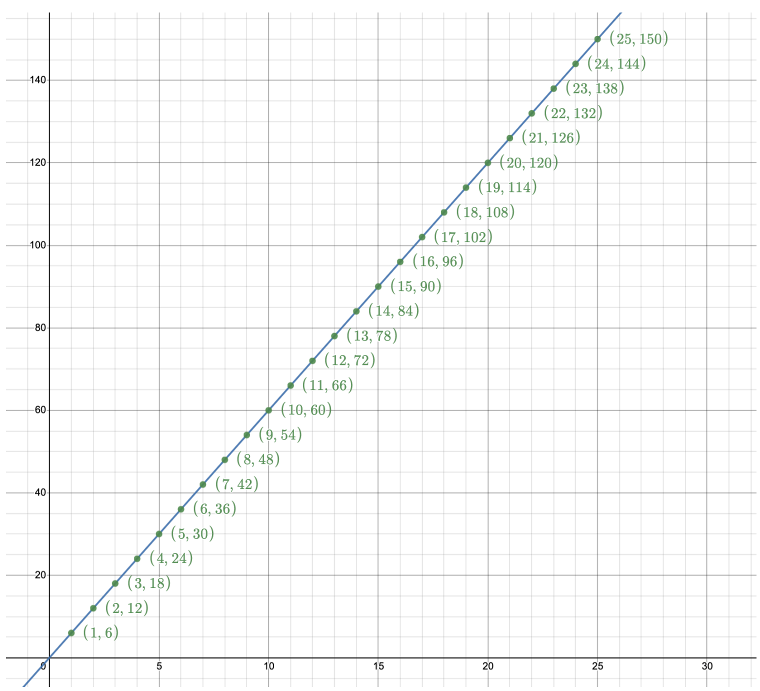


Figure 7: Graph showing the values for λ (in) for k from 1 to 25. (generated using *Desmos)*

Now that I have found different possible values for λ, I can finally calculate the total length of garland used, which is $L = kG,\ k \in \mathbb{Z}^{+}$ (***Eqn. 13***). The relationship between k and L is visualized in Figure 8.



A: Same here

Figure 8: Graph showing the values of L (ft) for given values of k from 1 to 25. (generated using *Desmos)*

D: Need to link back to aims.

A: Consider having this in an appendix instead of here unless it is necessary.

Here is the table of values for k values from 1 to 25.

k	$\lambda\ (in)$	$L\ (ft)$	k	$\lambda\ (in)$	$L\ (ft)$
1	0.84	6	14	11.8	84
2	1.68	12	15	12.6	90
3	2.52	18	16	13.4	96
4	3.36	24	17	14.3	102
5	4.20	30	18	15.1	108
6	5.04	36	19	16.0	114
7	5.88	42	20	16.8	120
8	6.72	48	21	17.7	126
9	7.57	54	22	18.5	132
10	8.41	60	23	19.3	138
11	9.25	66	24	20.0	144
12	10.1	72	25	21.0	150
13	10.9	78			

Factoring in Personal Aesthetics

optimal distance ~ 1 foot

will narrow down and find the best one

model around tree using geogebra

Dear Kenneth,

This is a decent first submission for the IA, and with some fine-tuning, has the potential to be a strong final IA. In general, the approach to the study was good, but there is a bit of a question about where it is headed considering it was incomplete. I do think you have many possible next steps available to you but make sure you adjust the aim of your study in the introduction if you end up deviating away from the original plan. At some points, the IA was getting very 'mathy' for a lack of better word. This is a good time to either consider testing predictions out or showing numerical examples to validate what you're doing. Still, you made a good attempt at explaining your reasoning, which helped. I would suggest staying consistent with one math typesetting tool, as the different fonts made it seem like there was some copy-and-pasting happening. One thing to consider is showing exemplars in 3d. You can even plot a cone in GeoGebra and make it translucent and then plot the conical spirals that you are testing out. Showing these screenshots would demonstrate more personal engagement and testing of predictions as well as an exploratory approach. It would also help make the math outcomes more clear.

In terms of the assessment criteria, and starting with the Presentation (marked out of 4) the exploration provided a clear rationale and aim. The IA was generally well organized and concise. Avoid redundant computations. Move big tables to appendices and only include summative results, even if the end result is a substantially shorter IA. Otherwise, the study was logically developed and generally easy to follow, even if the flow was not always there. Make sure you include visuals when you reference them. Also, make sure you have a cover page and are double spacing, and include a bibliography. For the Mathematical Communication (marked out of 4), you used a good variety of mathematical forms, and they were almost always relevant and appropriate. Deductive methods were clearly laid out, and good notation was used throughout. For the Personal Engagement (marked out of 3), it was clear that you had made the topic your own and applied it to an area of personal interest. There was evidence of significant personal engagement. To reach the outstanding echelon, however, you will need to go in deeper, show some creative thinking, and show more evidence of testing of predictions. It might also be helpful to discuss different perspectives to solving this problem or testing out different boundary curves (although I suspect this might take your IA and make it less accessible). For the Reflection criterion (marked out of 3), there was evidence of this throughout the IA, but at times this was only descriptive. I would suggest considering more extensions, commenting on the mathematics learning and making sure the reflections are both critical and of high quality. For the Use of Mathematics (marked out of 6), there is a lot of good math in here and I can see you reaching a high level in this criterion if you address the feedback included in the essay. Make sure you are justifying the level of accuracy of your results (and not just in the intro), and that there is clear rigour and sophistication in your investigation. Naturally, this feedback is based on the limited evidence you showed. More links to 3d plots and spirals are necessary.

Please do not mistake any of the feedback as overly harsh criticism. This is meant to give you as much information as possible to improve the IA, as this is the only time I can do so in writing. If you have any questions or concerns about this, please do not hesitate to reach out.

Good luck, and I look forward to your final submission!
MH

A mark range as it stands: 10-13 (IB 4 to 5-)
LCC moderated % mark range as it currently stands: 75-84%
Moderated mark on report card for this draft = 84% - 10% (late submission penalty) = 74%