

# 1 Riemann integral is continuous

This small theorem supports the claim that “integrating functions makes them nicer”, since  $f$  is required only to be Riemann-integrable and not necessarily continuous on its domain.

**Theorem 1.** Suppose we have some interval  $I$ , some  $t_0 \in I$  and some Riemann-integrable function  $f : I \rightarrow \mathbb{R}$ . Define  $g : I \rightarrow \mathbb{R}$  by

$$g(x) = \int_{t_0}^x f(t) \, dt.$$

Then  $g$  is continuous.

*Proof.* Since  $f$  is Riemann-integrable, it is bounded by some  $M > 0$ . Let  $y \in I$  be given and fix some  $\varepsilon > 0$ . Then provided we take  $x \in I$  such that  $|x - y| < \frac{\varepsilon}{M}$ , we will have

$$\begin{aligned} |g(x) - g(y)| &= \left| \int_{t_0}^x f(t) \, dt - \int_{t_0}^y f(t) \, dt \right| \\ &= \left| \int_y^x f(t) \, dt \right| \\ &\leq \int_y^x |f(t)| \, dt \\ &\leq M |x - y| \\ &< \varepsilon, \end{aligned}$$

as desired. □