MOWNiT lab4

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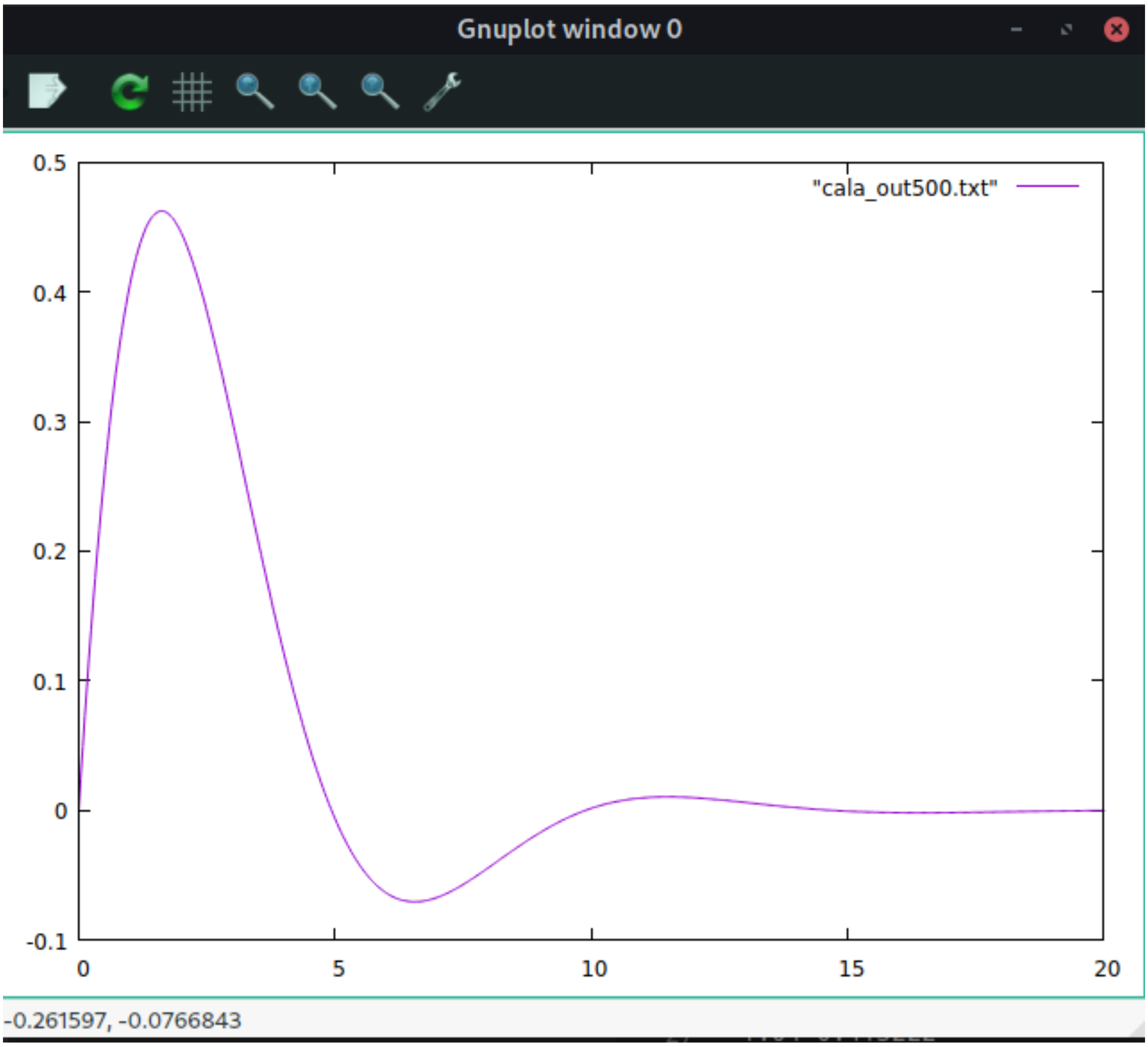
4/20/2020

zad2

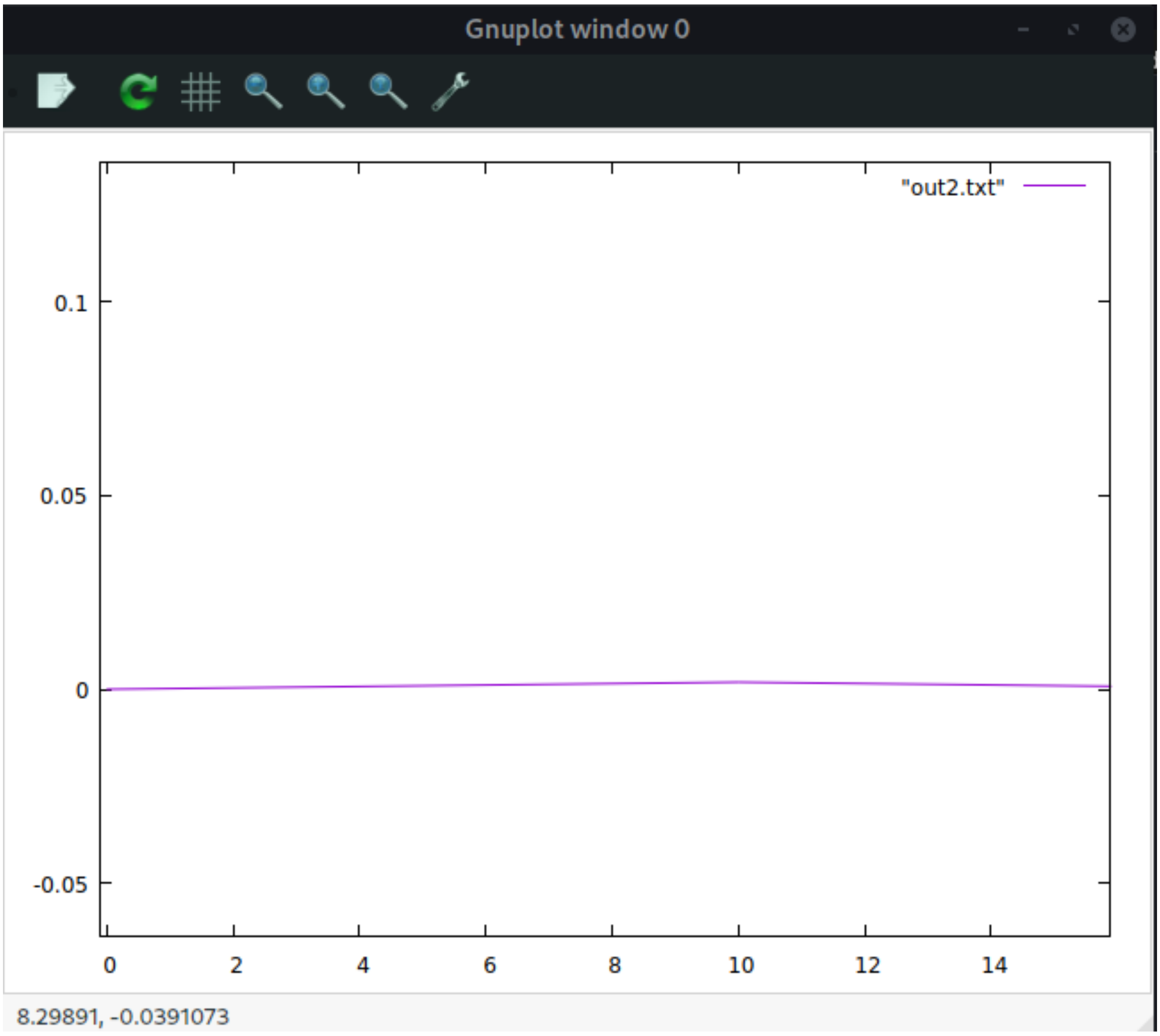
Funkcja testowa: $f(x) = \sin(\frac{kx}{\pi})e^{\frac{-mx}{\pi}}$

a. Przedstaw na wykresie jej dokładny przebieg.



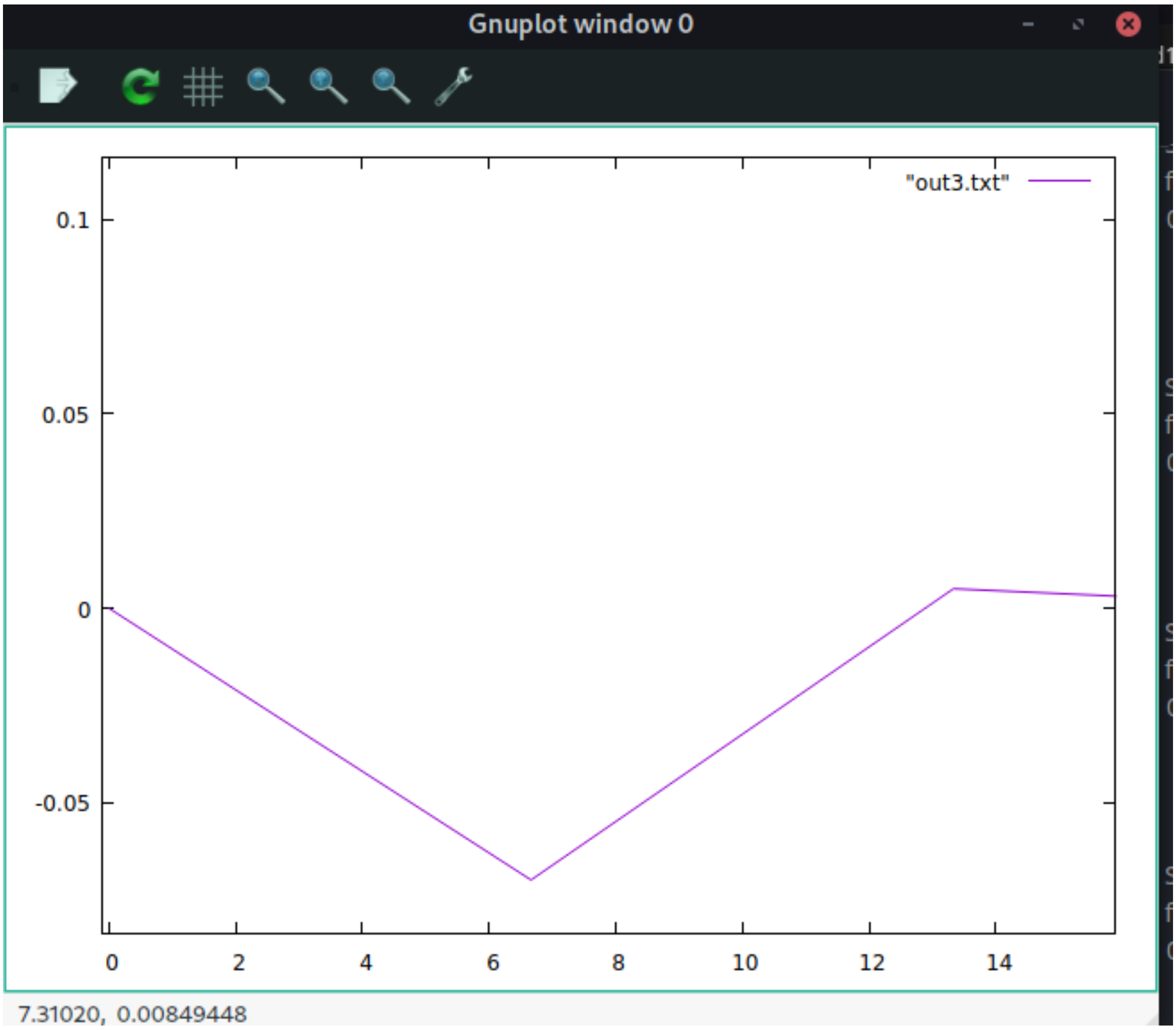
b) Przedstaw na wykresie jej przebieg dla różnej liczby węzłów

2



Gnuplot window 0
"out2.txt"
0.1
0.05
0
-0.05
0
2
4
6
8
10
12
14
8.29891, -0.0391073

3


Gnuplot window 0
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0
-0.05
0
2
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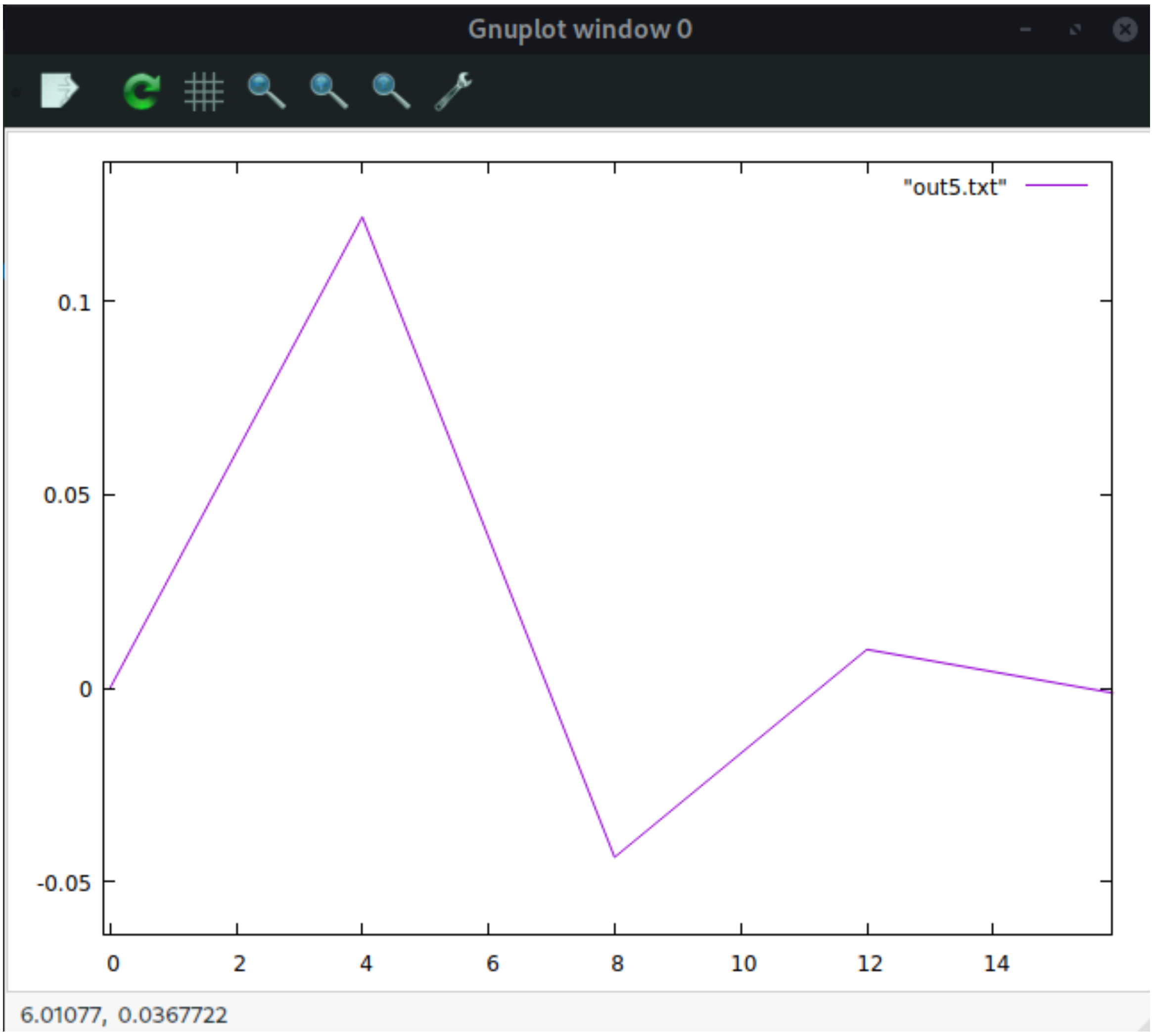
6
8
10
12
14
7.31020, 0.00849448

5


Gnuplot window 0
"out5.txt"
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0
2
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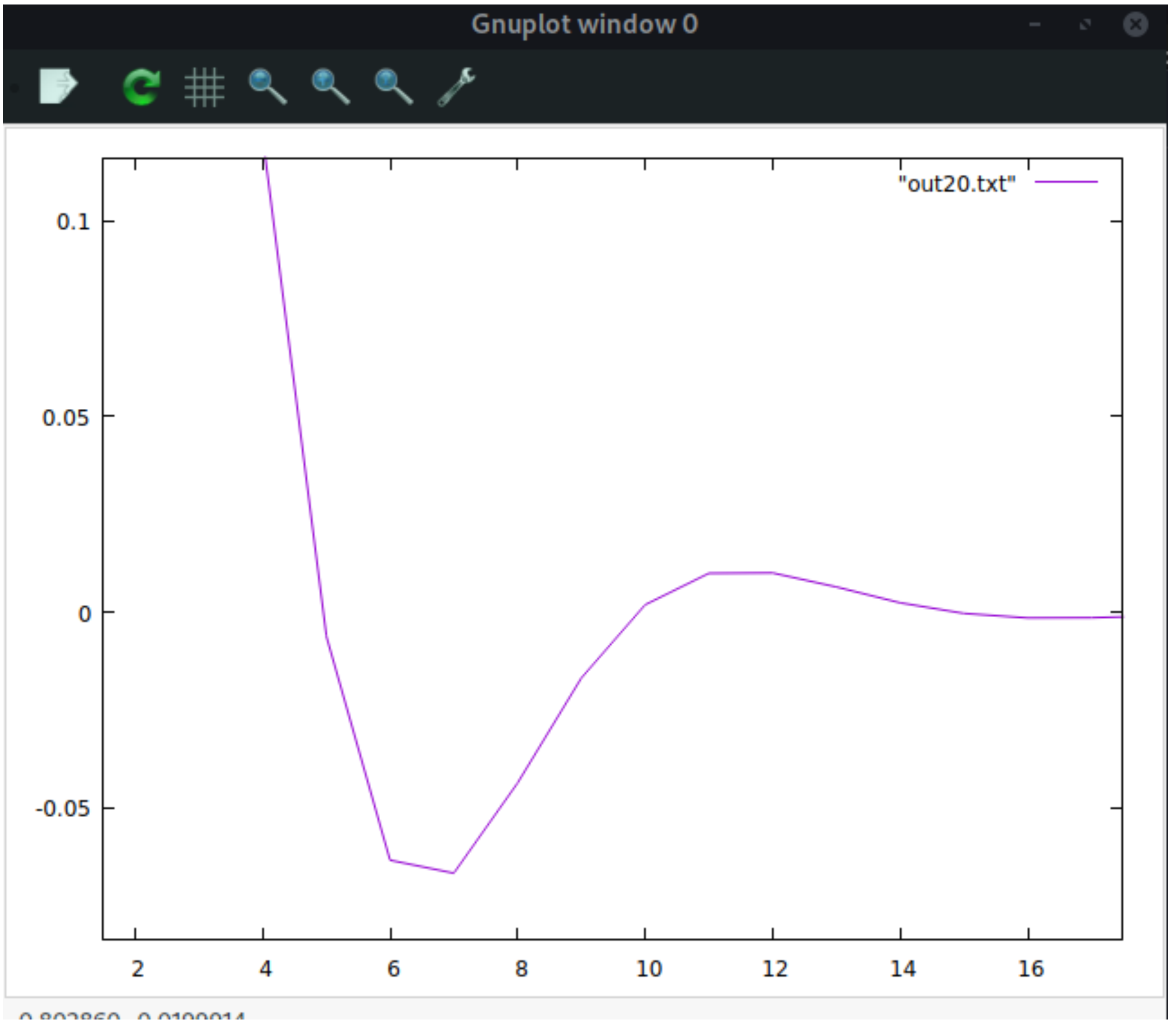
6
8
10
12
14
6.01077, 0.0367722

20



c)

Implementajca metody Newtona:

```
std::vector<struct point> calc_func_Newton(std::vector<struct point> points, double s
tep){

    std::vector<std::vector<double>> diffs = Newton_diff_calc(points);
    std::vector<struct point> result;
    for(double i = points[0].x; i < points[points.size()-1].x; i += step){
        struct point pt;
        pt.x = i;
        pt.y = Newton(diffs, points, i);
                    std::cout << "x: " << pt.x << " y: " << pt.y << std::endl;
        result.push_back(pt);
    }

    return result;

}
```

```cpp
double Newton(std::vector<std::vector<double>> diff_matrix, std::vector<struct point>
points, double x){

    double sum = diff_matrix[0][0];
    double u = (x - points[0].x) / (points[1].x - points[0].x);
    for (int i = 1; i < diff_matrix.size(); i++) {
        sum = sum + (uCoefNewton(u, i) * diff_matrix[0][i]) / fact(i);
    }

    return sum;
}
```

```cpp
std::vector<std::vector<double>> Newton_diff_calc(std::vector<struct point> points){

    std::vector<std::vector<double>> diff_matrix(points.size());
    for(int i = 0; i < points.size(); i++){
        diff_matrix[i].resize(points.size());
        diff_matrix[i][0] = points[i].y;
    }
    for (int i = 1; i < points.size(); i++) {
        for (int j = 0; j < points.size() - i; j++)
            diff_matrix[j][i] = diff_matrix[j + 1][i - 1] - diff_matrix[j][i - 1];
    }
    return diff_matrix;
}
```

```cpp
double uCoefNewton(double u, int n)
{
    double temp = u;
    for (int i = 1; i < n; i++)
        temp = temp * (u - i);
    return temp;
}

int fact(int n)
{
    int f = 1;
    for (int i = 2; i <= n; i++)
        f *= i;
    return f;
}
```

Implementacja metody Lagrange'a:

```
std::vector<struct point> calc_func_Lagrange(std::vector<struct point> points, double
step){

    std::vector<struct point> result;
    for(double i = points[0].x; i < points[points.size()-1].x; i += step){
        struct point pt;
        pt.x = i;
        pt.y = Lagrange(points, i);
        result.push_back(pt);
    }

    return result;

}
```

```
double Lagrange(std::vector<point> points, int x)
{
    double result = 0;

    for (int i=0; i<points.size(); i++)
    {

        double term = points[i].y;
        for (int j=0;j<points.size();j++)
        {
            if (j!=i)
                term = term*(x - points[j].x)/double(points[i].x - points[j].x);
        }

        result += term;
    }

    return result;
}
```

Implementacja metody Hermite’a

```
std::vector<struct point> calc_func_Hermite(std::vector<struct point> points, double step){
    std::vector<struct point> hermite_prep = Hermite_prep(points);
    std::vector<std::vector<double>> diffs = Hermite_diff_calc(hermite_prep);
    std::vector<struct point> result;
    for(double i = points[0].x; i < points[points.size()-1].x; i += step){
        struct point pt;
        pt.x = i;
        pt.y = Hermite(diffs, hermite_prep, i);
        std::cout << "x: " << pt.x << " y: " << pt.y << std::endl;
        result.push_back(pt);
    }

    return result;

}
```

```
double Hermite(std::vector<std::vector<double>> y, std::vector<struct point> x, double value)
{
    float sum = y[0][0];

    for (int i = 1; i < y.size(); i++) {
      sum = sum + (proterm(i, value, x) * y[0][i]);
    }
    return sum;
}
```

```
std::vector<struct point> Hermite_prep(std::vector<struct point> points){
    // std::vector<std::vector<double>> temp;
    std::vector<double> z;
    std::vector<struct point> pts;
    z.resize(points.size()*2);
    pts.resize(points.size()*2);
    for(int i = 0; i < points.size(); i++){
        z[i*2] = z[i*2+1] = points[i].x;
    }

    for(int i = 0; i < z.size(); i++){
        pts[i].x = z[i];
        pts[i].y = func1(z[i]);
    }

    return pts;
}
```

```
std::vector<std::vector<double>> Hermite_diff_calc(std::vector<struct point> points){

    std::vector<std::vector<double>> diff_matrix(points.size());
    for(int i = 0; i < points.size(); i++){
        diff_matrix[i].resize(points.size());
        diff_matrix[i][0] = points[i].y;
    }

    for (int i = 1; i <  points.size(); i++) {
        for (int j = 0; j <  points.size() - i; j++) {
            if(points[i+j].x == points[j].x){
                diff_matrix[j][i] = func1prim(points[j].x);
            }else{
            diff_matrix[j][i] = (diff_matrix[j][i - 1] - diff_matrix[j + 1][i - 1]) /
(points[j].x - points[i + j].x);
            }
        }
    }

    return diff_matrix;
}
```

```
double proterm(int i, double value, std::vector<struct point> x)
{
    double pro = 1;
    for (int j = 0; j < i; j++) {
        pro = pro * (value - x[j].x);
    }
    return pro;
}
```

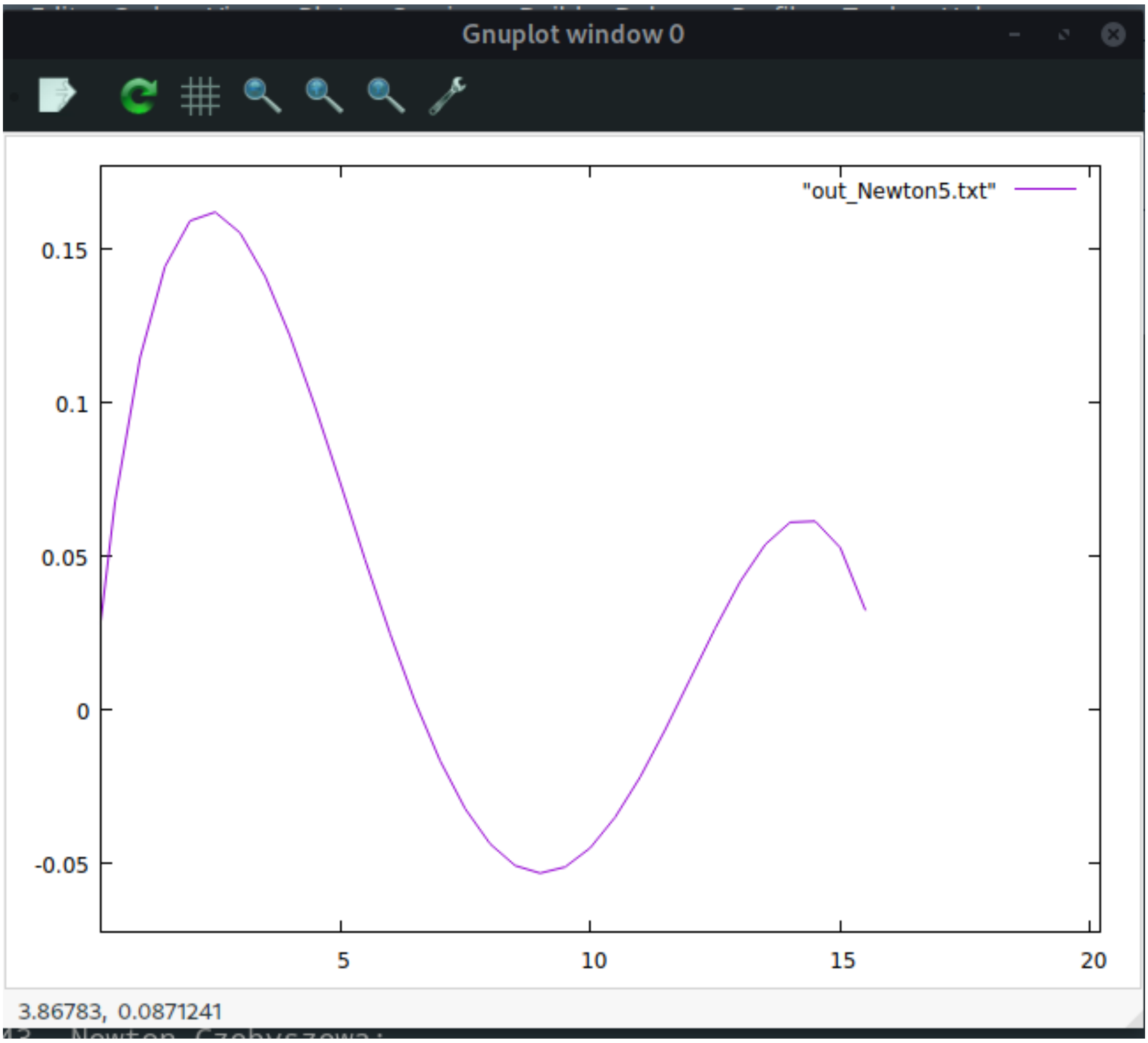
```
double func1prim(double x){
    double h = 0.001;
    return (func1(x) - func1(x+h)) / h;
}
```

Generowanie punktów metodą Czebyszewa:

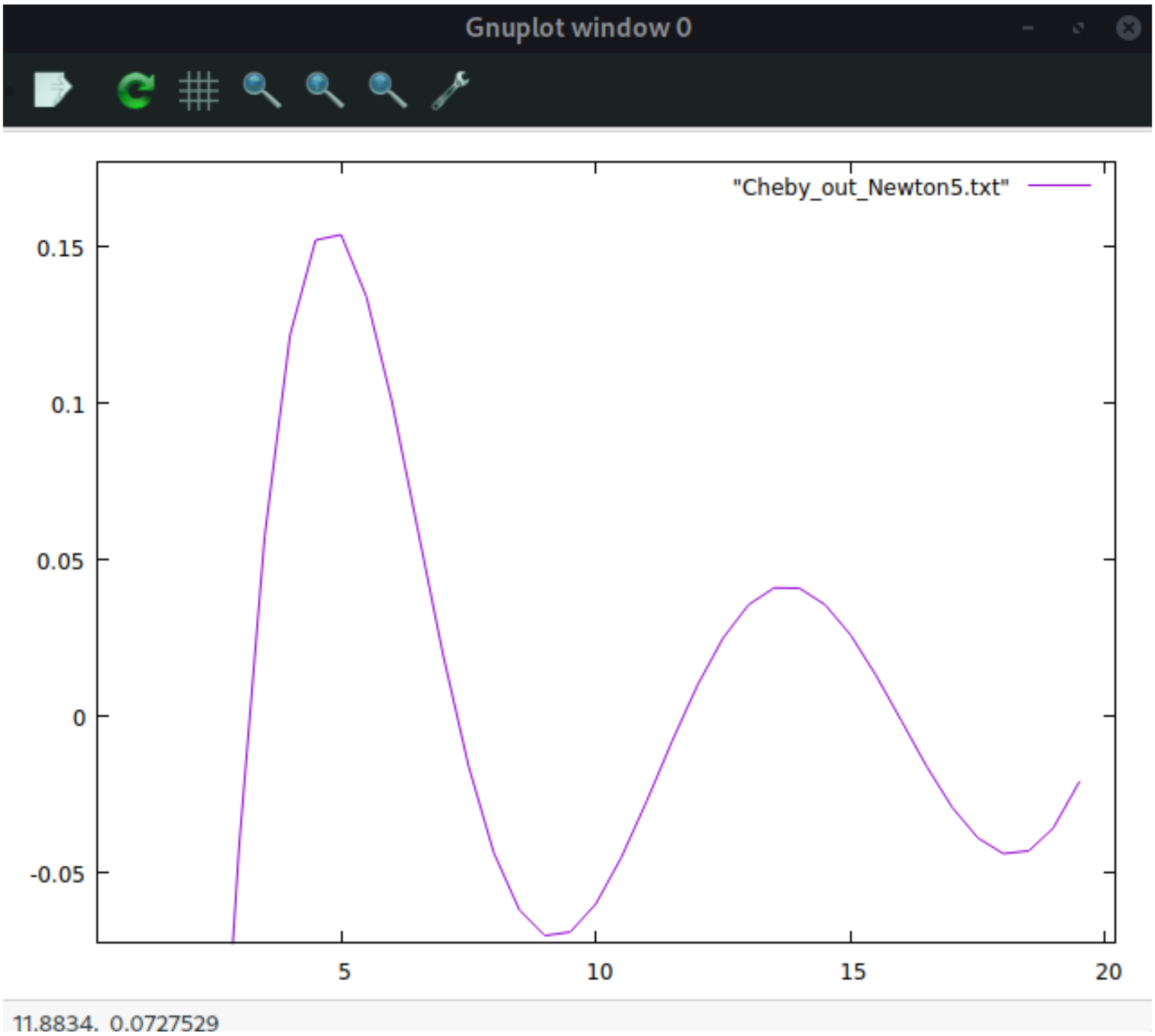
```
double cheby_point(double k, double n, double a, double b){
    k += 1;
    double val = 0.5*(a+b) + 0.5*(b-a)*cos(((2*k-1)*M_PI)/(2*n));
    return val;
}
```

Porównanie wyników interpolacji:

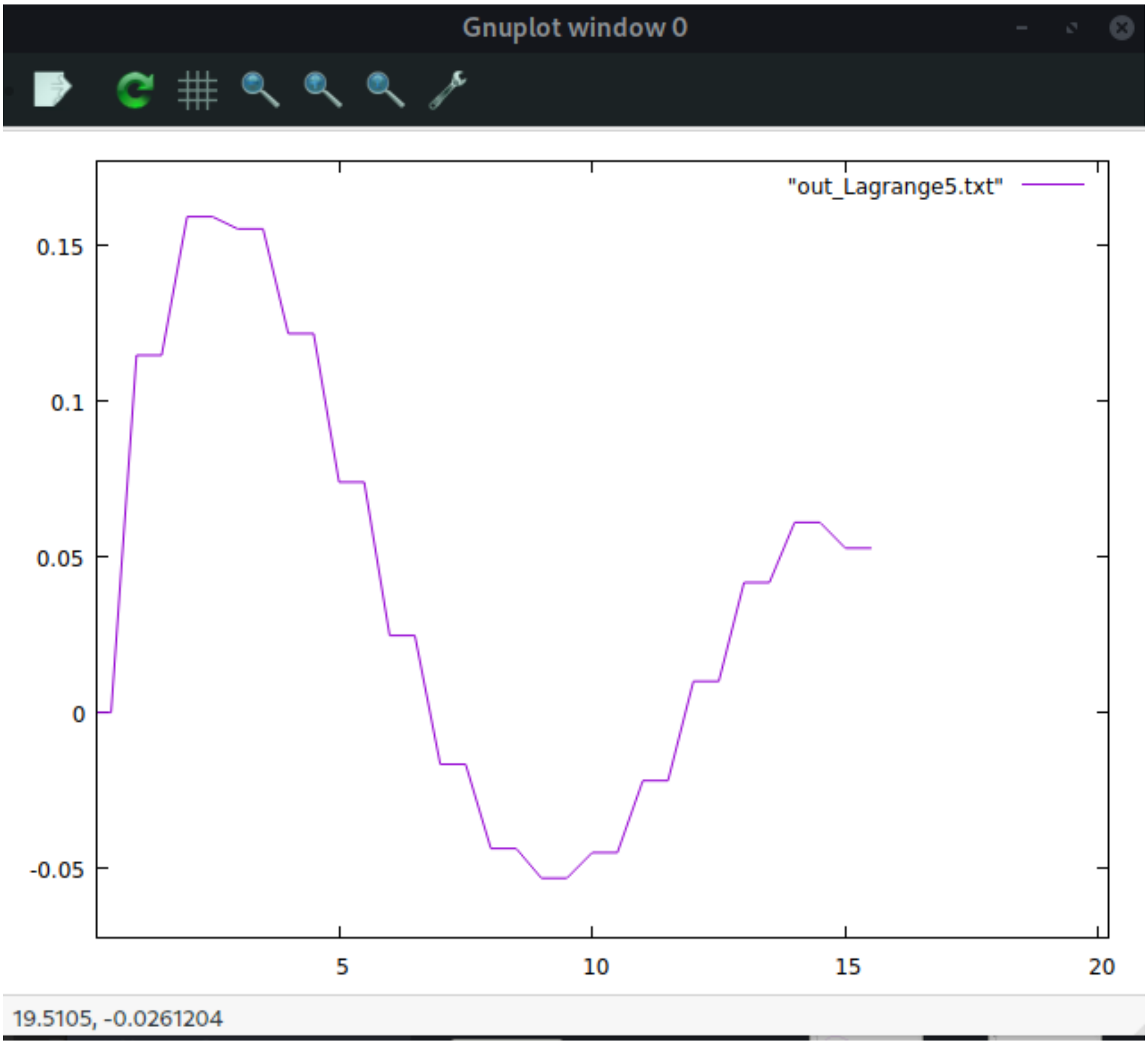
Metoda Newtona, punkty równodległe:



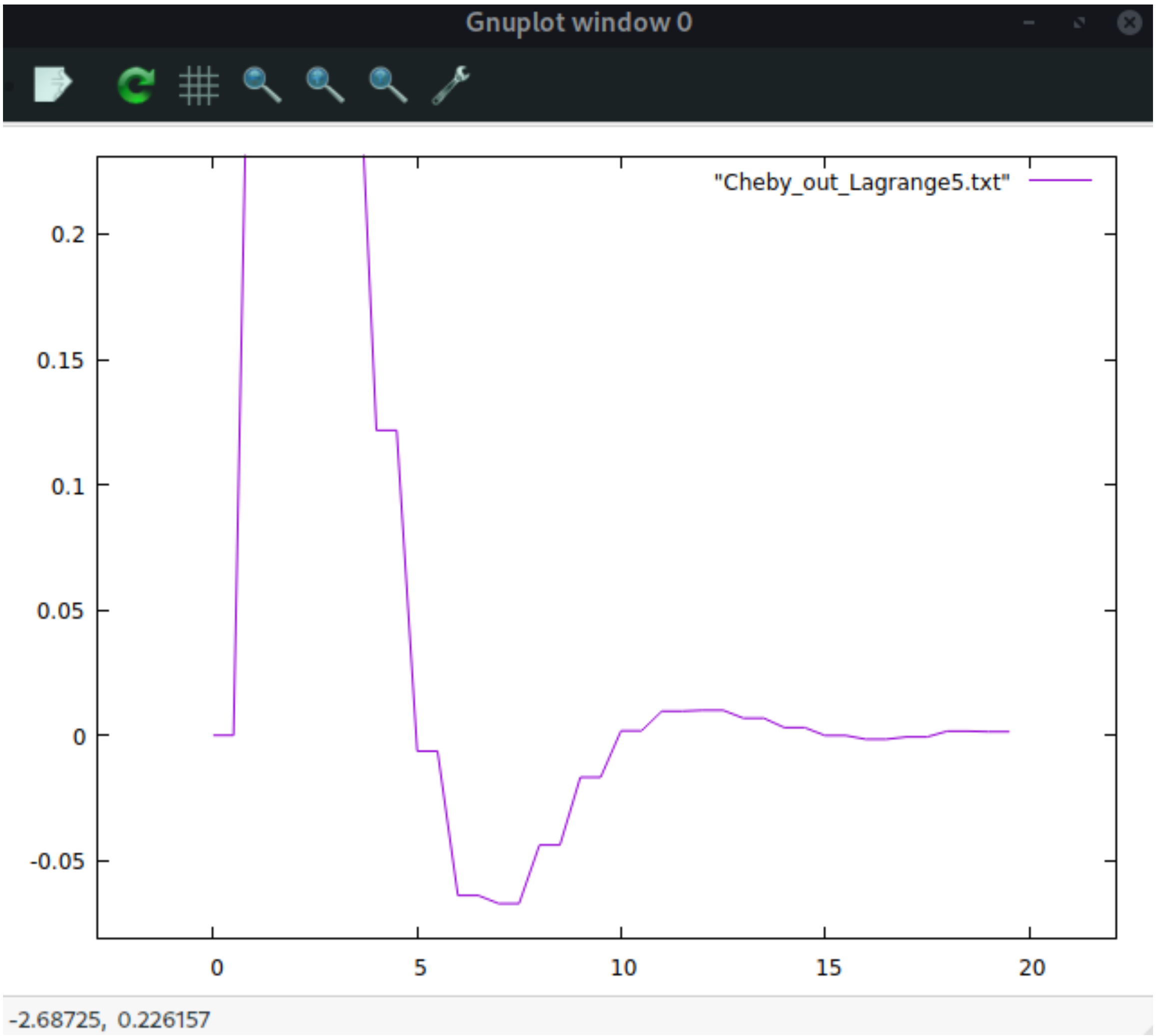
Metoda Newtona, punkty generowane metodą Czebyszewa:



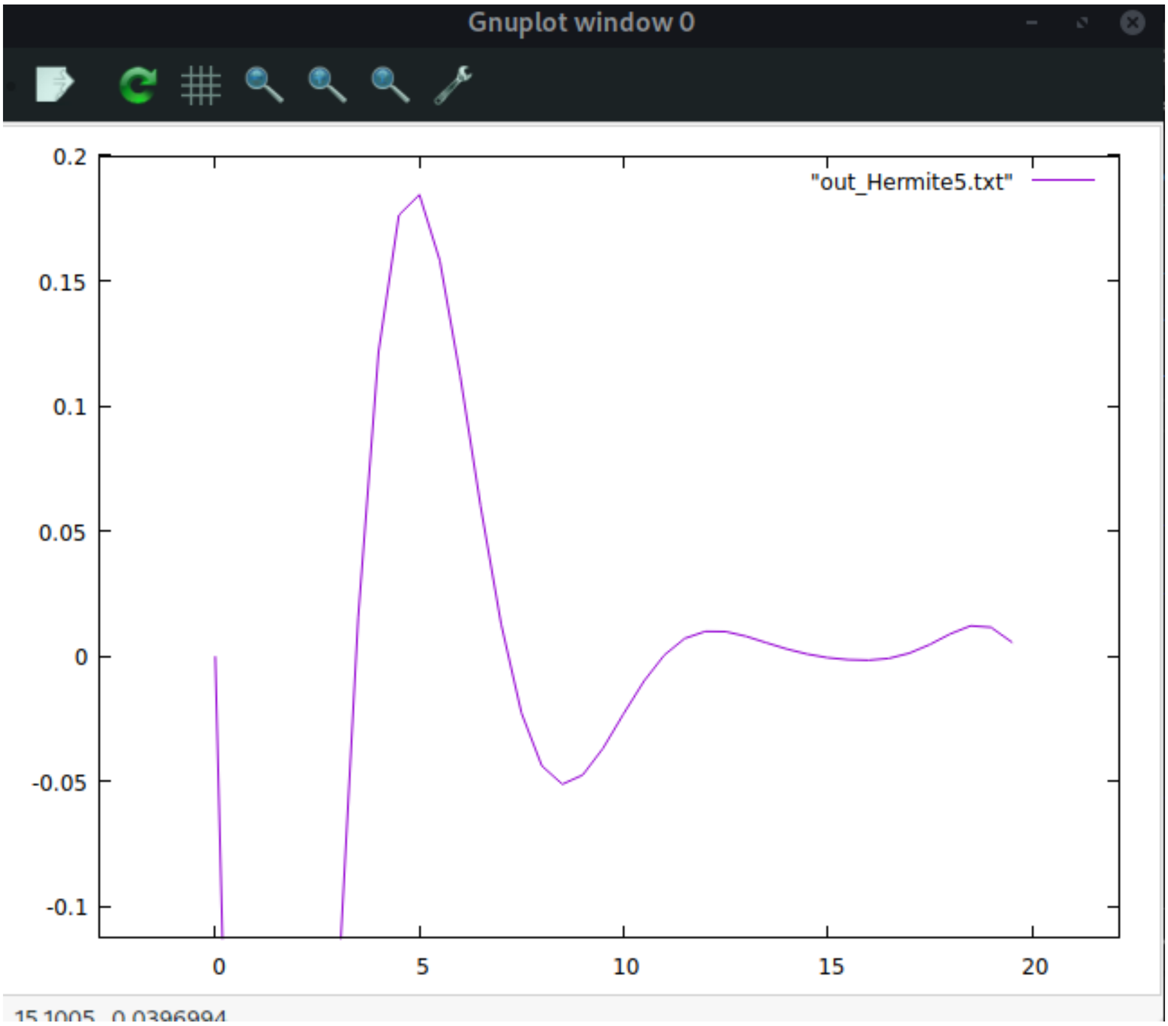
Metoda Lagrange'a, punkty równodległe:



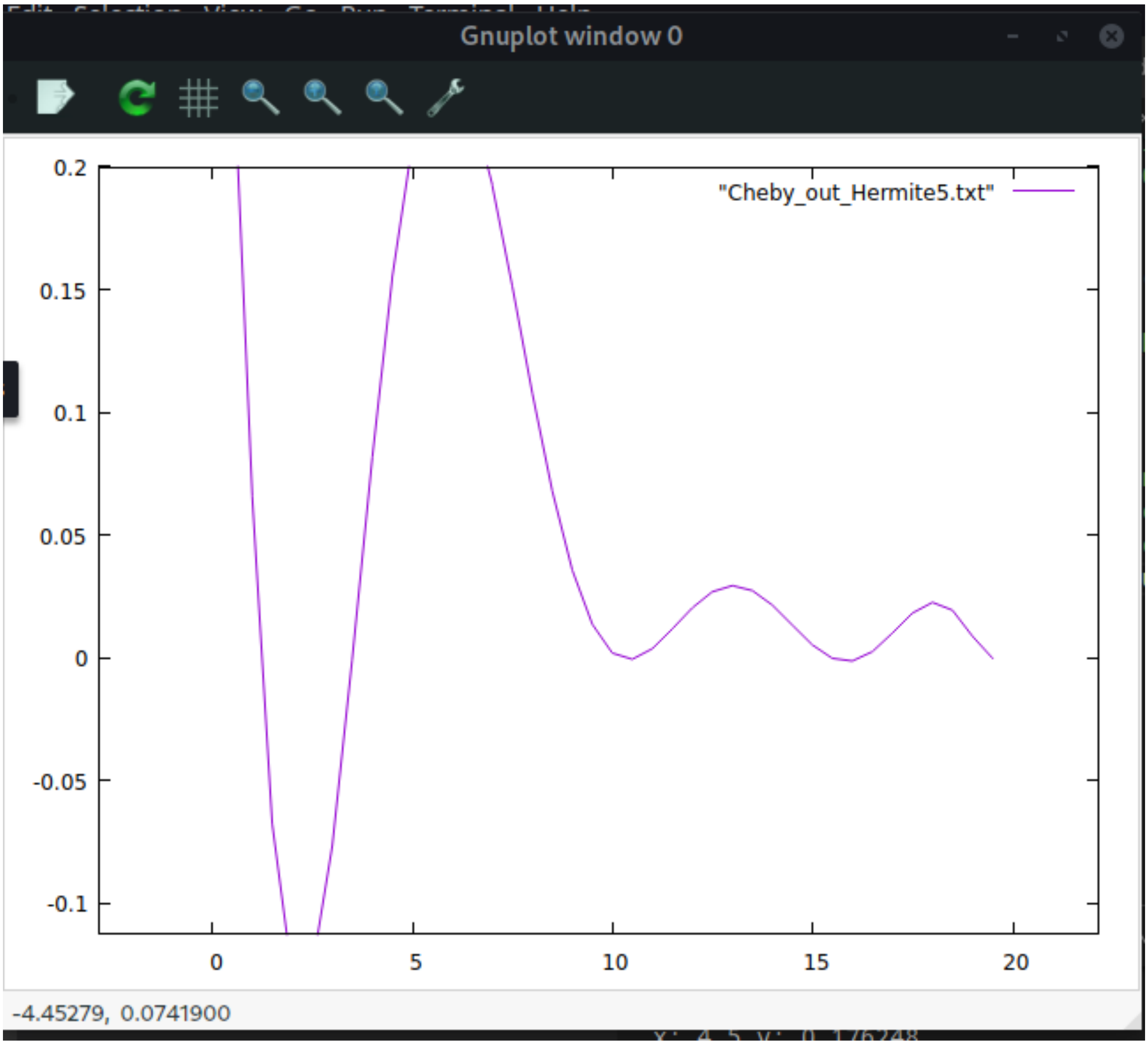
Metoda Lagrange'a, punkty generowane metodą Czebyszewa:



Metoda Hermite'a, punkty równoodległe:

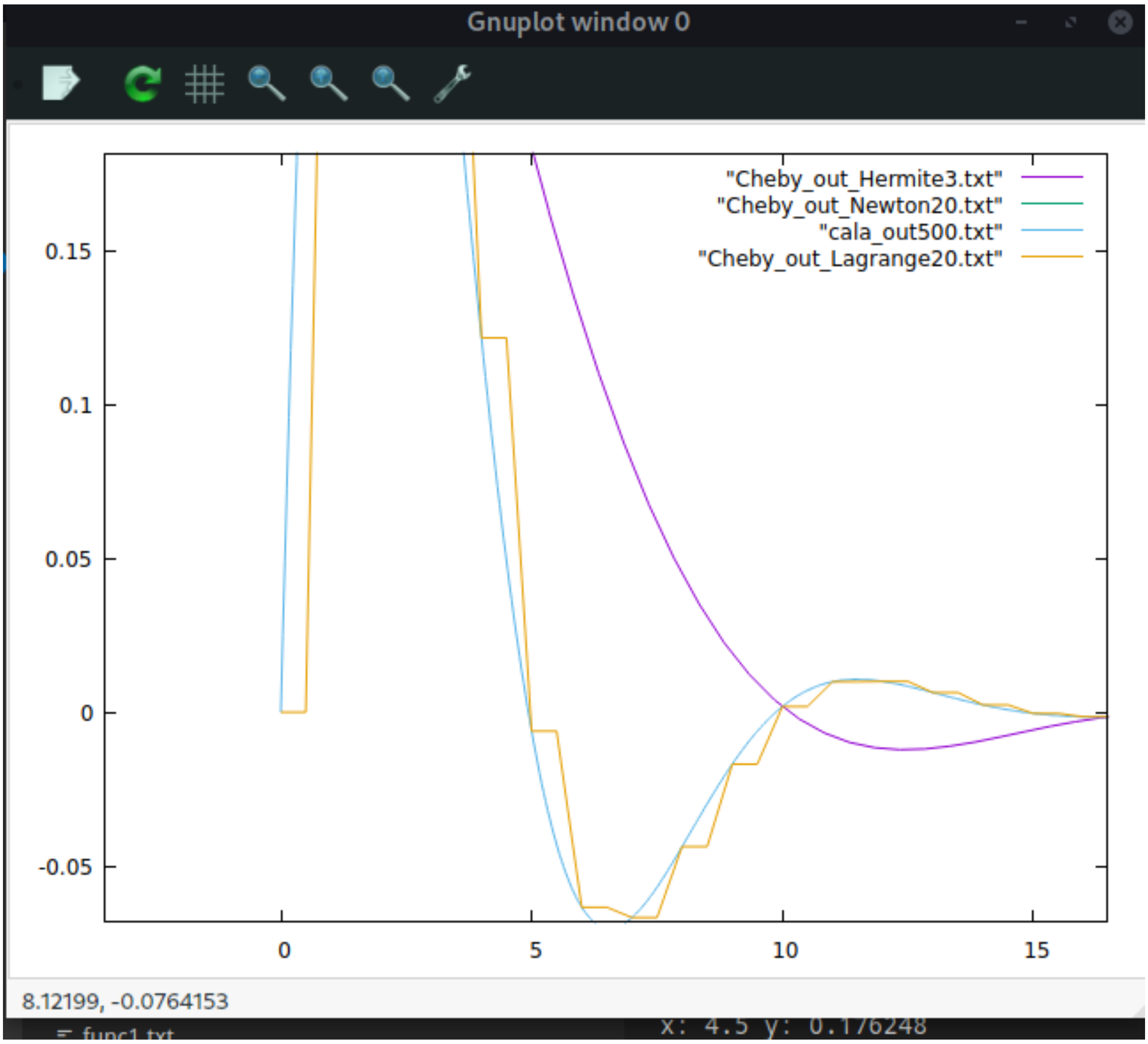


Metoda Hermite’a, punkty generowane metodą Czebyszewa:



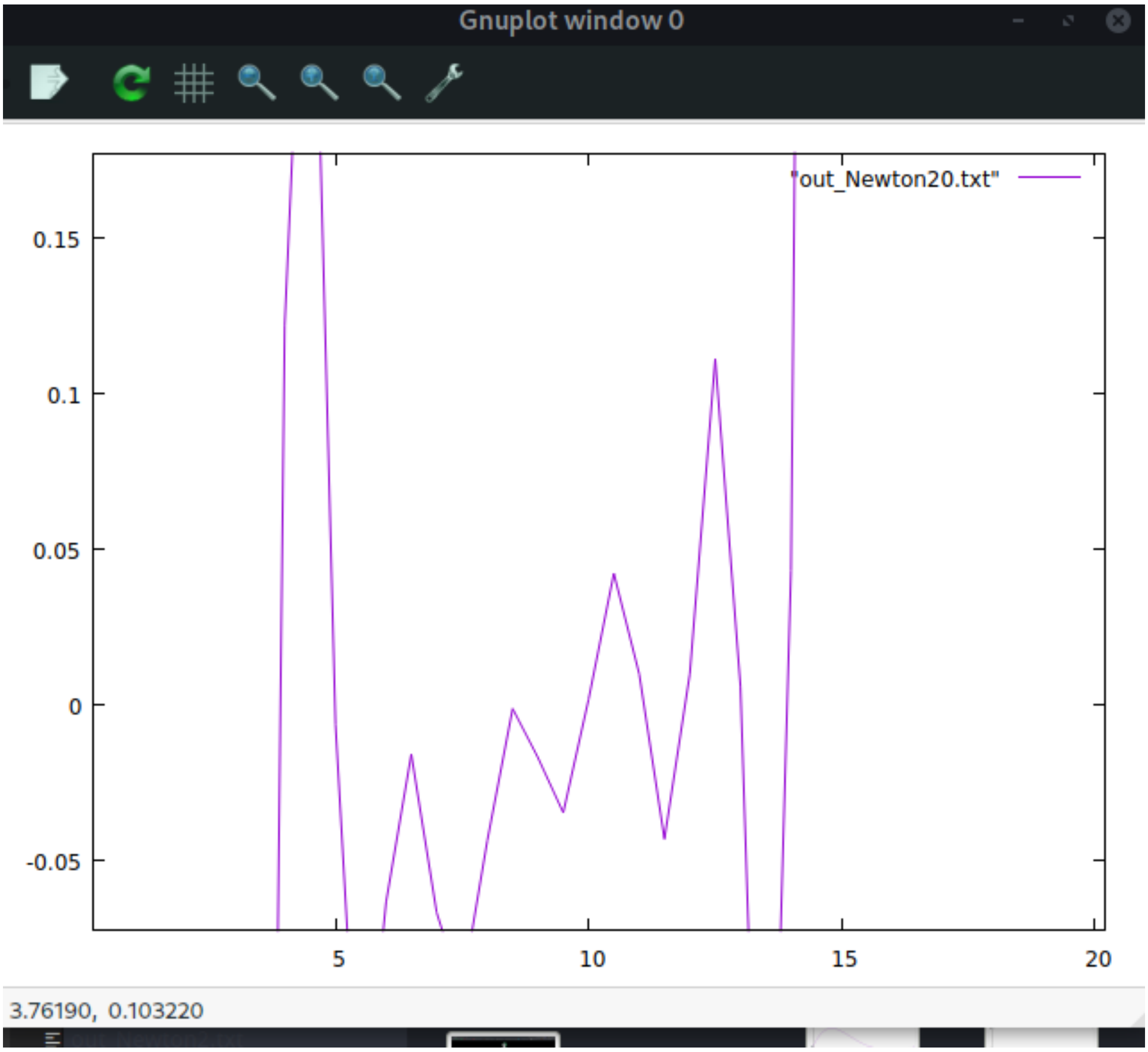
d)

- Dla metody Newtona wielomian ze stopniem 5 najlepiej przybliża funkcje.
- Dla metody Lagrange'a wielomian ze stopniem 20 najlepiej przybliża funkcje.
- Dla metody Hermite'a wielomian ze stopniem 3 najlepiej przybliża funkcje.



e)

Przy wielomianie ze stopniem 20 widać efekt Runge'go


Gnuplot window 0
"out_Newton20.txt"
0.15
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0.05
0
-0.05
5
10
15
20
3.76190, 0.103220

normal