Bara Wiktor #2

Numerical Methods (ENUME 2019) – Project
Assignment C: Solving ordinary differential equations

1. Develop a program for solving the following differential equation:

$$4y'' + 4y' + 5y = 0 \text{ for } t \in [0,10],\ y(0) = 4 \text{ and } y'(0) = -2$$

by means of the implicit Gauss-Legendre order 6 method defined by the following Butcher table:

$$\begin{array}{c|ccc}
\frac{1}{2} - \frac{\sqrt{15}}{10} & \frac{5}{36} & \frac{2}{9} - \frac{\sqrt{15}}{15} & \frac{5}{36} - \frac{\sqrt{15}}{30} \\
\frac{1}{2} & \frac{5}{36} + \frac{\sqrt{15}}{24} & \frac{2}{9} & \frac{5}{36} - \frac{\sqrt{15}}{24} \\
\frac{1}{2} + \frac{\sqrt{15}}{10} & \frac{5}{36} + \frac{\sqrt{15}}{30} & \frac{2}{9} + \frac{\sqrt{15}}{15} & \frac{5}{36} \\
\hline
 & \frac{5}{18} & \frac{4}{9} & \frac{5}{18}
\end{array}$$

Compare the solution, obtained by means of this program for the constant integration step $h = 0.01$, with the solution, obtained by means of the MATLAB operator ***ode113***. Optimise the parameters ***RelTol*** and ***AbsTol*** of the latter in such a way as to get the most accurate solution $\dot{\mathbf{y}}(t;h)$ (to be used as a reference in the following sections).

2. Carry out a systematic investigation of the dependence of the accuracy of the solution $\hat{\mathbf{y}}(t;h)$ on the integration step h. Use the following accuracy indicators for this purpose:

$$\delta_2(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_2}{\left\|\dot{\mathbf{y}}(t,h)\right\|_2} \quad \text{(the root-mean-square error)}$$

$$\delta_\infty(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_\infty}{\left\|\dot{\mathbf{y}}(t,h)\right\|_\infty} \quad \text{(the maximum error)}$$

Make the graphs $\delta_2(h)$ and $\delta_\infty(h)$.

3. Repeat the systematic investigation, defined in Section 3, for the implicit Euler method. Add the curves representative of $\delta_2(h)$ and $\delta_\infty(h)$ to the graph obtained in Section 2.