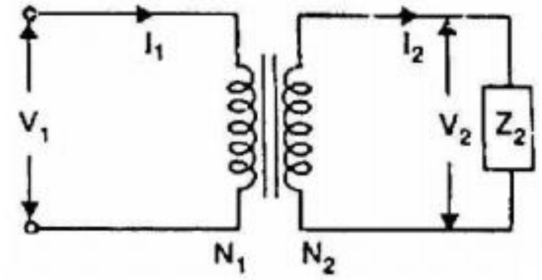
EET 3316
ELECTRICAL MACHINES

LECTURE 4 : Tx EQUIVALENT CIRCUITS

Impedance Ratio

- Consider a transformer having impedance Z_2 in the secondary as shown
- Hence impedance ratio (Z_2/Z_1) is equal to the square of voltage transformation ratio.
- In other words, an impedance Z_2 in secondary becomes Z_2/K^2 when transferred to the primary.
- Likewise, an impedance Z_1 in the primary becomes K^2Z_1 when transferred to the secondary.
- Similarly, $R_2/R_1 = K^2$ and $X_2/X_1 = K^2$



$$Z_2 = \frac{V_2}{I_2} \quad \text{and} \quad Z_1 = \frac{V_1}{I_1}$$

$$\frac{Z_2}{Z_1} = \left(\frac{V_2}{V_1}\right) \times \left(\frac{I_1}{I_2}\right)$$

$$\frac{Z_2}{Z_1} = K^2$$

Impedance Ratio

- The importance of above relations is that we can transfer the parameters from one winding to the other. Thus:

i. A resistance R_1 in the primary becomes K^2*R_1 when transferred to the secondary.

ii. A resistance R_2 in the secondary becomes R_2/K^2 when transferred to the primary.

iii. A reactance X_1 in the primary becomes K^2*X_1 when transferred to the secondary.

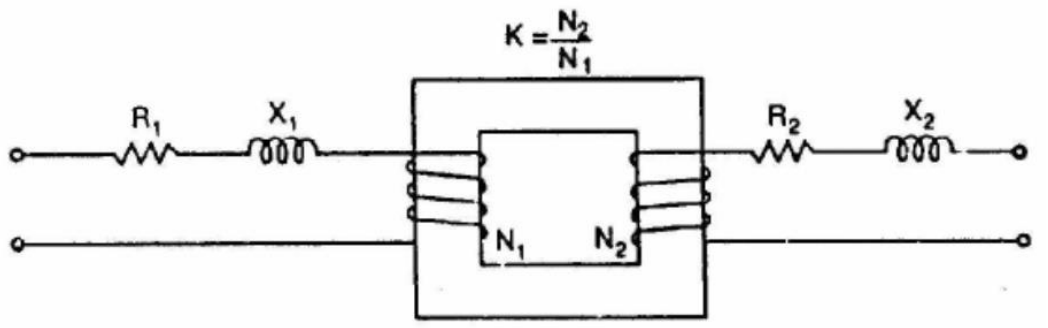
iv. A reactance X_2 in the secondary becomes X_2/K^2 when transferred to the primary.

Note: It is important to remember that:

- When transferring impedance (R or X) from primary to secondary, multiply it by K^2
- When transferring impedance (R or X) from secondary to primary, divide it by K^2
- When transferring voltage or current from one winding to the other, only K is used.

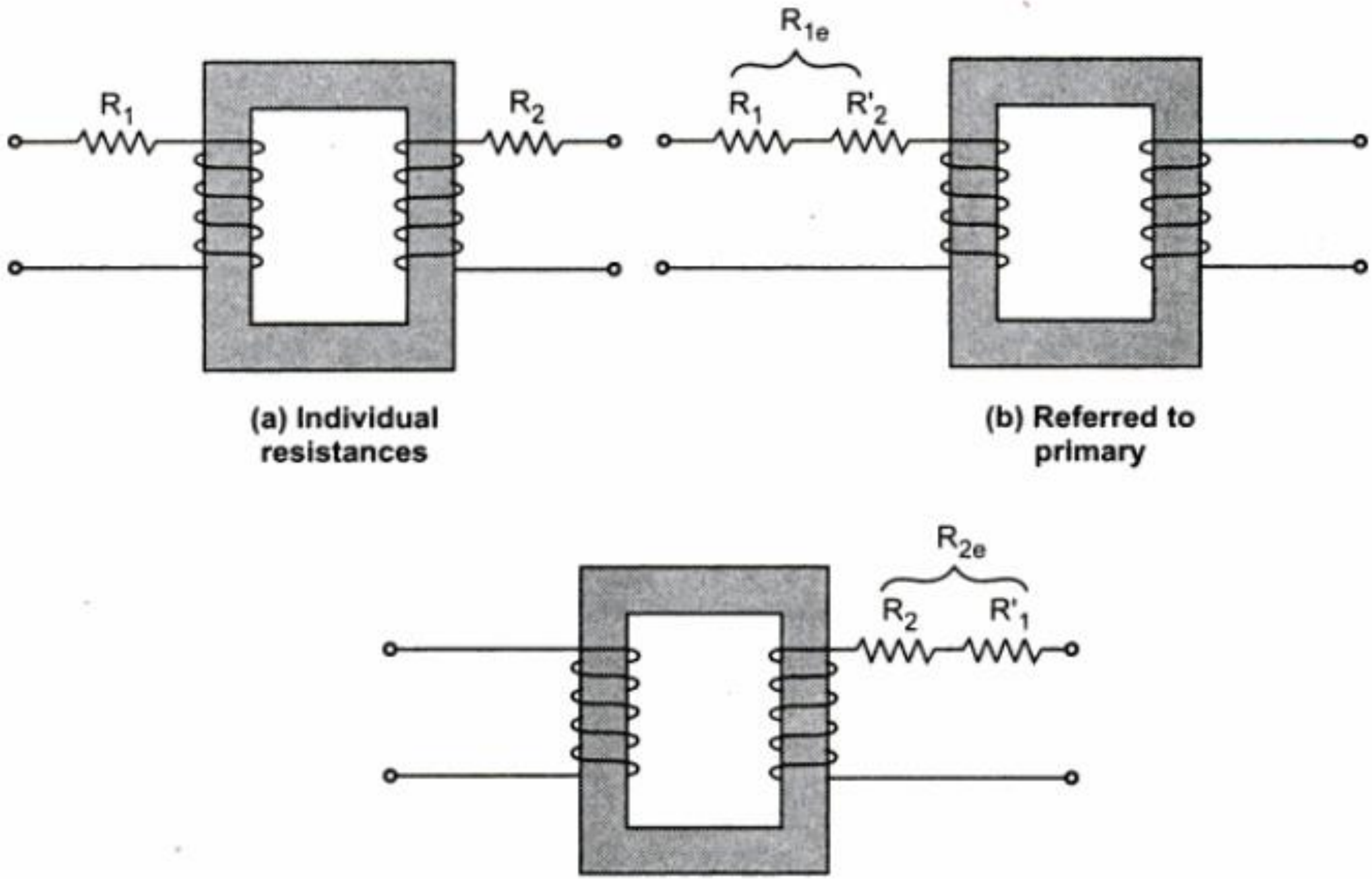
Shifting Impedances in A Transformer

- The figure below shows a transformer where resistances and reactance are shown external to the windings.



- The resistance and reactance of one winding can be transferred to the other by appropriately using the factor K^2.
- This makes the analysis of the transformer a simple affair because then we have to work in one winding only.
- Therefore, the transformer can be:
    1. Referred to primary
    2. Referred to secondary

Equivalent Resistances



(a) Individual resistances

(b) Referred to primary

(c) Referred to secondary

Equivalent Resistance

- The resistance of the two windings can be transferred to any side either primary or secondary without affecting the performance of the transformer.
- The transfer of the resistances on any one side is advantageous as it makes the calculations very easy.
- Consider the total loss due to both the resistances:

$$\text{total copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2\left[R_1 + \frac{I_2^2}{I_1^2} R_2\right] = I_1^2\left[R_1 + \frac{1}{K^2} R_2\right]$$

$$\frac{I_2}{I_1} = \frac{1}{K} \quad \text{neglecting no load current.}$$

This means $\frac{R_2}{K^2}$ is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with I_2.

This value of resistance R_2/K^2 which is the value of R_2 referred to primary is called **equivalent resistance of secondary referred to primary**.

It is denoted as R'_2.

$$\boxed{R'_2 = \frac{R_2}{K^2}}$$

Equivalent Resistance

$$\boxed{\text{Total copper loss} = I_1^2 R_{1e} = I_1^2 R_1 + I_2^2 R_2}$$

So equivalent resistance R_{1e} simplifies the calculations as we have to calculate parameters on one side only.

Similarly it is possible to refer the equivalent resistance to secondary winding.

$$\text{total copper loss} = I_2^2 \left[\frac{I_1^2}{I_2^2} R_1 + R_2 \right] = I_2^2 \, [\, K^2 R_1 + R_2]$$

Thus the resistance $K^2 R_1$ is primary resistance referred to secondary denoted as R'_1.

$$\boxed{R'_1 = K^2 R_1}$$

Transformer Referred to Primary

- When secondary impedance is transferred to the primary, it is divided by K^2.
- It is then called equivalent secondary resistance or reactance referred to primary and is denoted by R'_2 or X'_2.
- Equivalent resistance of transformer referred to primary

$$R_{01} = R_1 + R_2' = R_1 + R_2/K^2$$

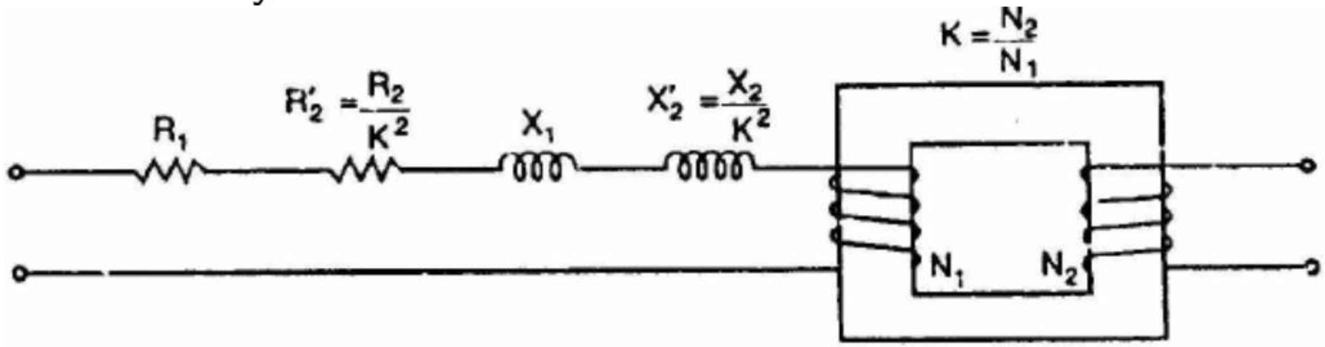
- Equivalent reactance of transformer referred to primary

$$X_{01} = X_1 + X_2' = X_1 + X_2/K^2$$

- Equivalent impedance of transformer referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

- Note that secondary now has no resistance or reactance.



Transformer Referred to Secondary

- When primary impedance is transferred to the secondary it is multiplied by K^2.
- It is then called equivalent secondary resistance or reactance referred to secondary and is denoted by R'_1 or X'_1.
- Equivalent resistance of transformer referred to secondary

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$

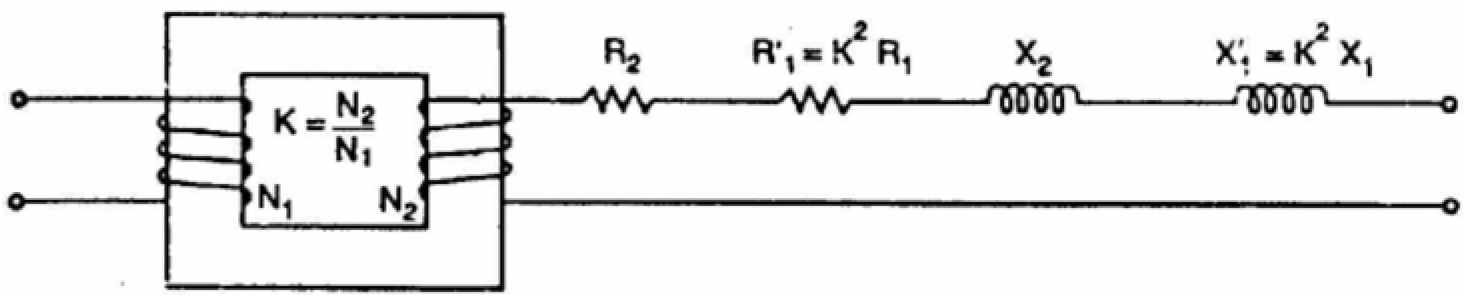
- Equivalent reactance of transformer referred to secondary

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1$$

- Equivalent impedance of transformer referred to secondary

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

- Note that primary now has no resistance or reactance.



Impedance Equivalent

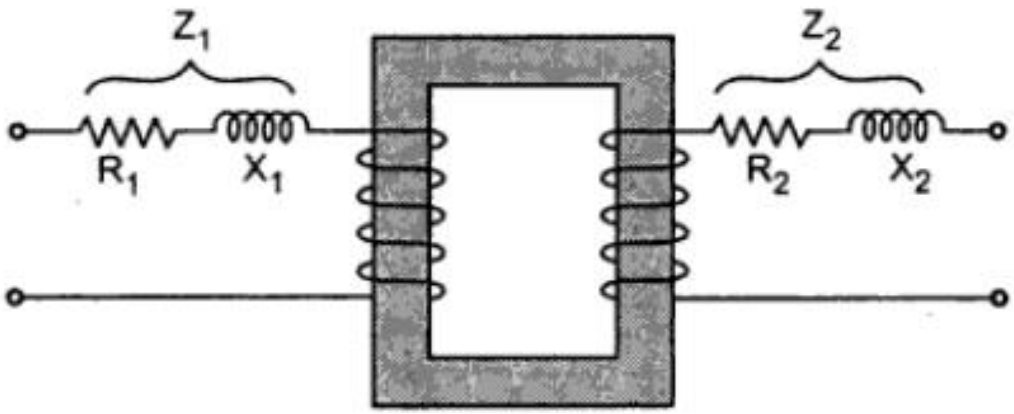
The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reactance X_2.

Thus we can say that the total impedance of primary winding is Z_1 which is,

$$Z_1 = R_1 + j X_1 \,\Omega$$

And the total impedance of the secondary winding is Z_2 which is ,

$$Z_2 = R_2 + j X_2 \,\Omega$$



$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \qquad Z_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2}$$

$$Z_{2e} = \sqrt{R_{2e}^2 + X_{2e}^2} \qquad Z_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1$$

Practice Question

- **Qn 1:** A 33 kVA, 2200/220V, 50Hz single phase transformer has the following parameters. Primary winding resistance $R_1 = 2.4\Omega$, Leakage reactance $X_1 = 6\ \Omega$, Secondary winding resistance $R_2 = 0.03\Omega$, Leakage reactance $X_2 = 0.07\ \Omega$. Then find Primary, Secondary and equivalent resistance and reactance.

- **Qn 2:** A single phase transformer having voltage ratio 2500/250V (primary to secondary) has a primary resistance and reactance 1.8 Ω and 4.2Ω, respectively. The corresponding secondary values are 0.02Ω and 0.045Ω. Determine the total resistance and reactance referred to secondary side. Also calculate the impedance of transformer referred to secondary side

Solution Q1

Here, Rating of transformer = 33 kVA; $V_1 = 2200$ V; $V_2 = 220$ V;

$$f = 50 \text{ Hz}; R_1 = 2.4\ \Omega; X_1 = 6\ \Omega; R_2 = 0.03\ \Omega; X_2 = 0.07\ \Omega$$

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{220}{2200} = 0.1$$

Transformer resistance referred to primary side;

$$R_{ep} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 2.4 + \frac{0.03}{(0.1)^2} = 2.4 + 3 = \mathbf{5.4}\ \Omega\ (Ans.)$$

Transformer reactance referred to primary side;

$$X_{ep} = X_1 + X_2' = X_1 + \frac{X_2}{K^2} = 6 + \frac{0.07}{(0.1)^2} = 6 + 7 = \mathbf{13}\ \Omega\ (Ans.)$$

Transformer resistance referred to secondary side;

$$R_{es} = R_2 + R'_1 = R_2 + R_1 \times K^2 = 0.03 + 2.4(0.1)^2 = \mathbf{0.054}\ \Omega\ (Ans.)$$

Transformer reactance referred to secondary side;

$$X_{es} = X_2 + X_1' = X_2 + X_1 \times K^2 = 0.07 + 6 \times (0.1)^2 = \mathbf{0.13}\ \Omega\ (Ans.)$$

Solution Q2

Here, $R_1 = 1{\cdot}8\ \Omega$; $X_1 = 4{\cdot}2\ \Omega$; $R_2 = 0{\cdot}02\ \Omega$; $X_2 = 0{\cdot}045\ \Omega$

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{250}{2500} = 0.1$$

Total resistance referred to secondary side,

$$R_{es} = R_2 + R_1' = R_2 + R_1 \times K^2$$

$$= 0{\cdot}02 + 1{\cdot}8 \times (0{\cdot}1)^2 = \mathbf{0{\cdot}038}\ \Omega\ (Ans.)$$

Total reactance referred to secondary side,

$$X_{es} = X_2 + X_1' = X_2 + X_1 \times K^2$$

$$= 0{\cdot}045 + 4{\cdot}2 \times (0{\cdot}1)^2 = \mathbf{0{\cdot}087}\ \Omega\ (Ans.)$$

Impedance of transformer referred to secondary side,

$$Z_{es} = \sqrt{(R_{es})^2 + (X_{es})^2} = \sqrt{(0 \cdot 038)^2 + (0 \cdot 087)^2}$$

$$= \mathbf{0{\cdot}095}\ \Omega\ (Ans.)$$

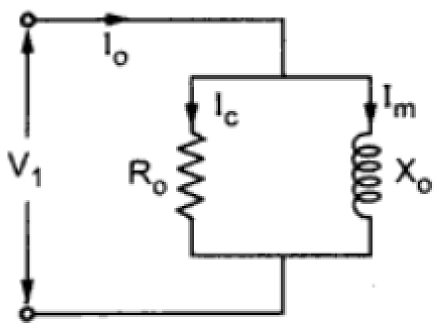
Practice Questions

1. A 10 kVA 2000/400 V single phase transformer has $R_1 = 5\ \Omega$; $X_1 = 12\ \Omega$; $R_2 = 0.2\ \Omega$ and $X_2 = 0.48\ \Omega$. Determine the equivalent impedance of the transformer reffered to (i) primary side and (ii) secondary side **[Z01=26 Ω, Z02=1.04 Ω]**.
2. A 100 kVA, 2200/440 V transformer has $R_1 = 0.3\ \Omega$; $X_1 = 1.1\ \Omega$; $R_2 = 0.01\ \Omega$ and $X_2 = 0.035\ \Omega$. Calculate:
   1. the equivalent impedance of the transformer referred to the primary
   2. total copper losses[**R01 = 0.55 X01=1.975 Z01=2.05 P=1136 W**]
3. A 50-kVA, 4,400/220-V transformer has $R_1 = 3.45\ \Omega$; $X_1 = 5.2\ \Omega$; $R_2 = 0.009\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate:
   1. equivalent resistance as referred to primary **[7.05]**
   2. equivalent resistance as referred to secondary **[0.0176]**
   3. equivalent reactance as referred to both primary and secondary **[X01=11.2; X01=0.028]**
   4. equivalent impedance as referred to both primary and secondary **[Z01=13.23; Z01=0.03311]**

Equivalent Circuit of a Transformer

The term equivalent circuit of a machine means the combination of fixed and variable resistances and reactances, which exactly simulates performance and working of the machine.

For a transformer, no load primary current I_o has two components,



No load equivalent circuit

$$I_m = I_o \sin \phi_0 = \text{magnetising component}$$

$$I_c = I_o \cos \phi_0 = \text{active component}$$

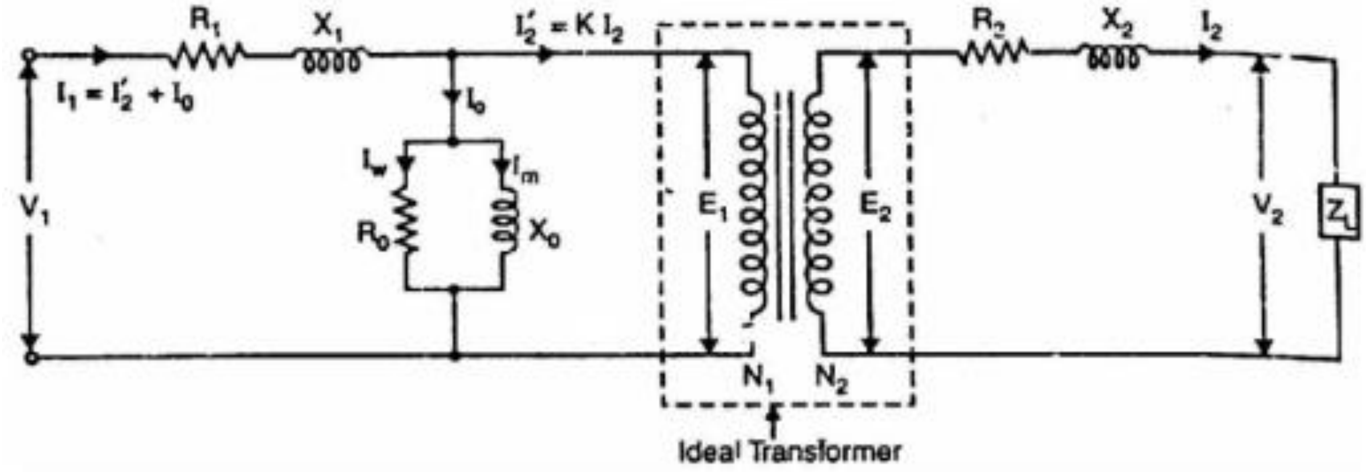
I_m produces the flux and is assumed to flow through reactance X_o called no load reactance while I_c is active component representing core losses hence is assumed to flow through the resistance R_o. Hence equivalent circuit on no load can be shown as in the Fig. This circuit consisting of R_o and X_o in parallel is called **exciting circuit**. From the equivalent circuit we can write,

$$R_o = \frac{V_1}{I_c} \qquad X_o = \frac{V_1}{I_m}$$

When the load is connected to the transformer then secondary current I_2 flows. This causes voltage drop across R_2 and X_2. Due to I_2, primary draws an additional current $I'_2 = I_2 / K$. Now I_1 is the phasor addition of I_o and I'_2. This I_1 causes the voltage drop across primary resistance R_1 and reactance X_1.

Equivalent Circuit of a Transformer

- It is important to describe the behavior of an electrical machine in terms of its equivalent circuit so that the analysis can be done using normal methods of circuit theory.
- The exact equivalent circuit of a transformer on load is as shown below:



- Here R_1 is the primary winding resistance and R_2 is the secondary winding resistance.
- Similarly, X_1 is the leakage reactance of primary winding and X_2 is the leakage reactance of the secondary winding.
- The parallel circuit $R_0 - X_0$ is the no-load equivalent circuit of the transformer

- The resistance R_0 represents the core losses (hysteresis and eddy current losses) so that current I_W which supplies the core losses is shown passing through R_0.
- The inductive reactance X_0 represents a loss-free coil which passes the magnetizing current I_m.
- The phasor sum of I_W and I_m is the no-load current I_0 of the transformer
- Note that in the equivalent circuit, the imperfections (losses) of the transformer have been taken into account by various circuit elements. Therefore, the transformer is now the ideal one.
- Note that equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_2'}{I_2}$$

Points to note

- The following points may be noted from the equivalent circuit:

i. When the transformer is on no-load (i.e., secondary terminals are open-circuited), there is no current in the secondary winding. However, the primary draws a small no-load current I_0. The no-load primary current I_0 is composed of (a) magnetizing current (I_m) to create magnetic flux in the core and (b) the current I_W required to supply the core losses.

ii. When the secondary circuit of a transformer is closed through some external load Z_L, the voltage E_2 induced in the secondary by mutual flux will produce a secondary current I_2. There will be I_2 R_2 and I_2 X_2 drops in the secondary winding so that load voltage V_2 will be less than E_2

$$V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2 Z_2$$

Points to Note

(iii) When the transformer is loaded to carry the secondary current I_2, the primary current consists of two components:

(a) The no-load current I_0 to provide magnetizing current and the current required to supply the core losses.

(b) The primary current I'_2 (= K I_2) required to supply the load connected to the secondary.

$\therefore$ Total primary current $I_1 = I_0 + (-KI_2)$

(iv) Since the transformer in Fig. is now ideal, the primary induced voltage E_1 can be calculated from the relation:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

If we add I_1R_1 and I_1X_1 drops to E_1, we get the primary input voltage V_1

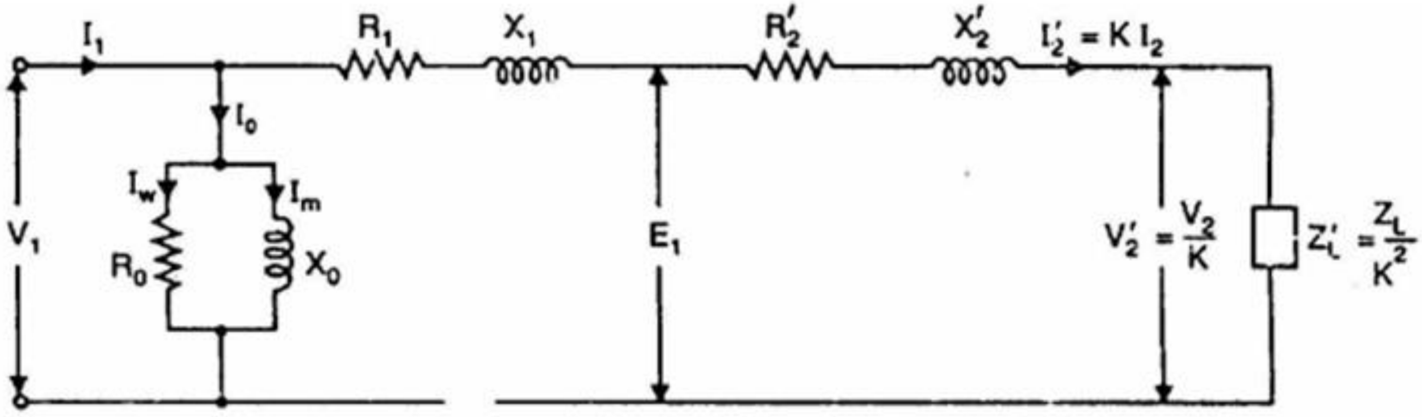
$$V_1 = -E_1 + I_1(R_1 + j\,X_1) = -E_1 + I_1Z_1$$

Equivalent circuit referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in Fig. (i)). This further reduces to Fig. (ii)). Note that when secondary quantities are referred to primary, resistances/reactances/impedances are divided by K^2, voltages are divided by K and currents are multiplied by K.

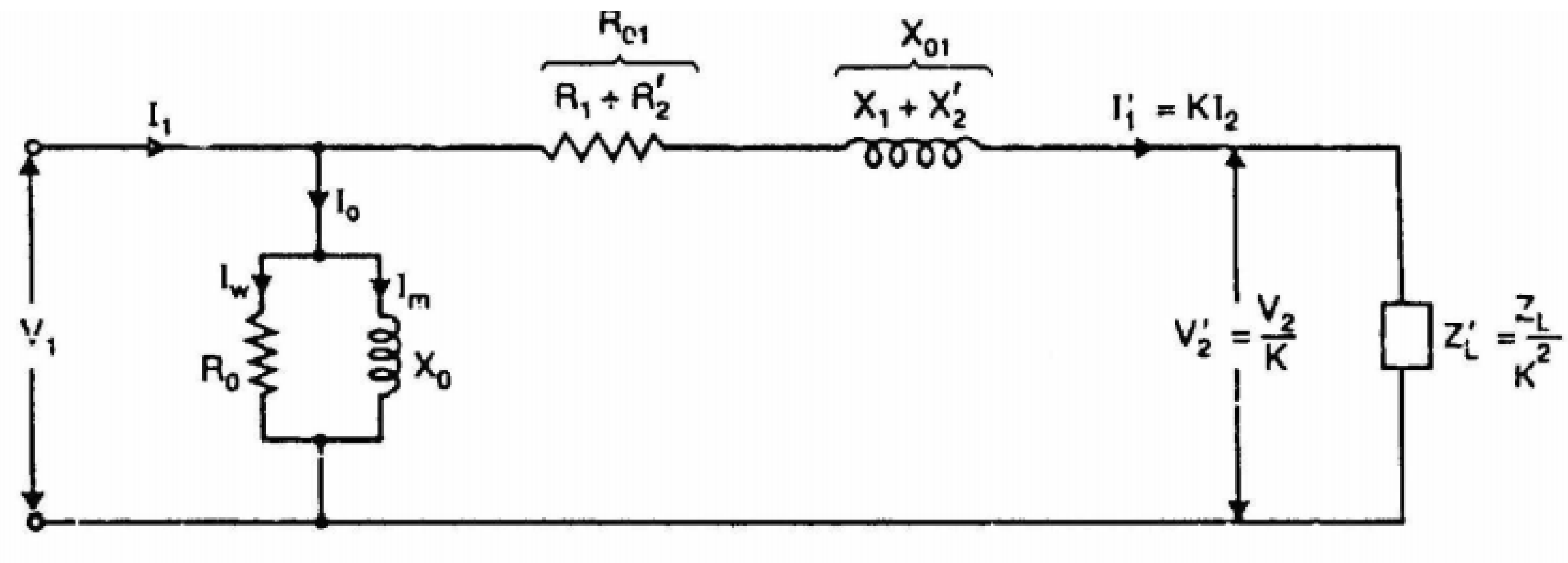
$$\therefore \quad R'_2 = \frac{R_2}{K^2}; \quad X'_2 = \frac{X_2}{K^2}; \quad Z'_L = \frac{Z_L}{K^2}; \quad V'_2 = \frac{V_2}{K}; \quad I'_2 = K\, I_2$$

$$Z_{01} = R_{01} + j\, X_{01} \quad \text{where} \quad R_{01} = R_1 + R'_2; \quad X_{01} = X_1 + X'_2$$



(i)

Equivalent circuit referred to primary

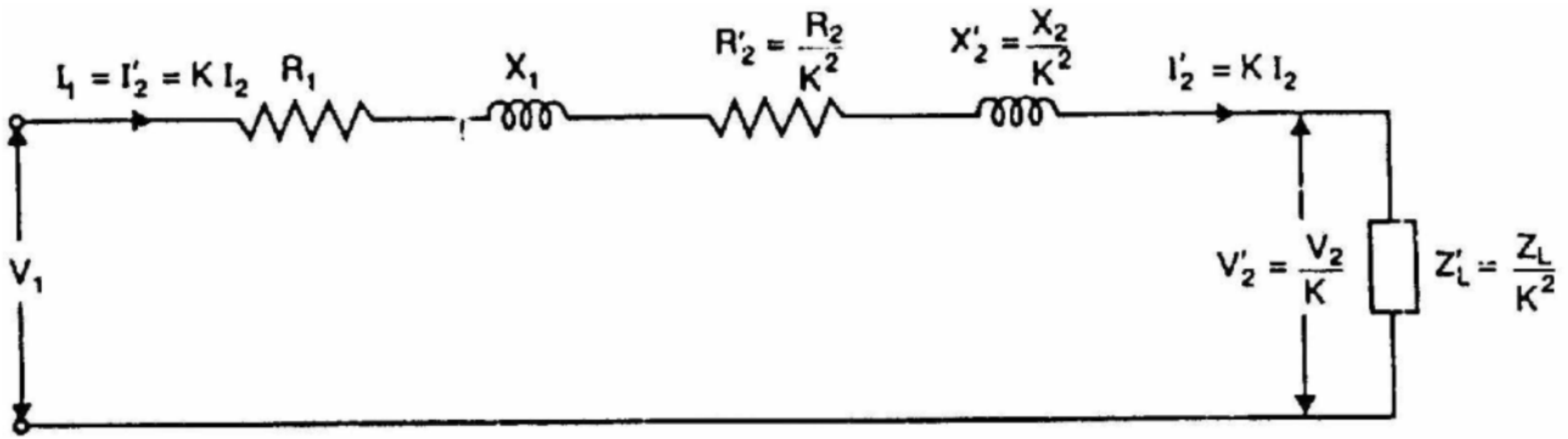


(ii)

- *The no-load current I_0 in a transformer is only 1-3% of the rated primary current and may be neglected without any serious error.*

Equivalent circuit of transformer referred to primary

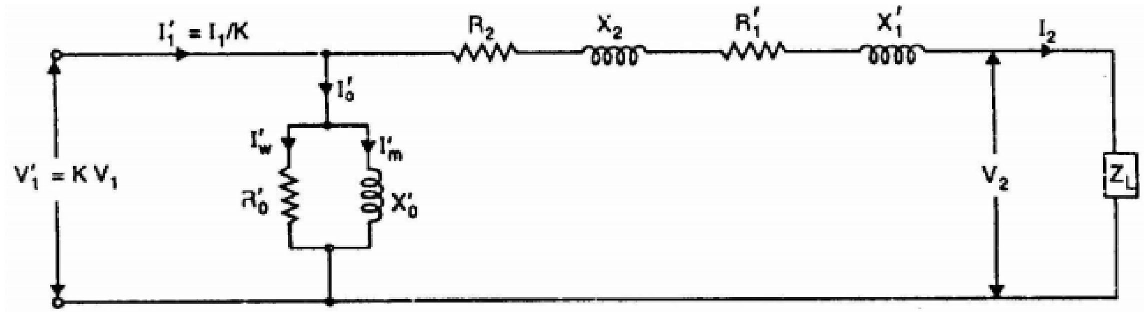
- If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to primary as shown.



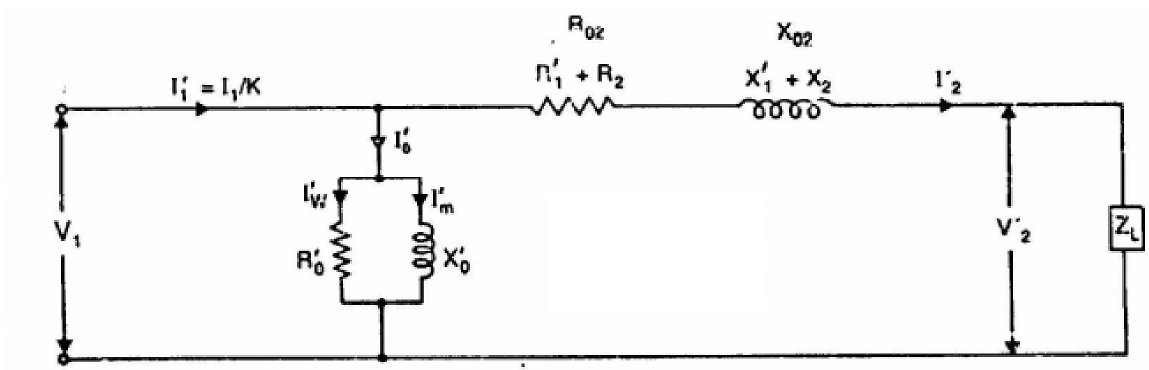
- When secondary quantities are referred to primary, resistances/reactances are divided by K^2 , voltages are divided by K and currents are multiplied by K.
- The equivalent circuit shown is an electrical circuit and can be solved for various currents and voltages
- Thus if V_2' and I_2' are obtained , then:
  - Actual secondary voltage, $V_2 = KV_2'$
  - Actual secondary current, $I_2 = I_2'/K$

Equivalent circuit of transformer referred to secondary

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (i)). This further reduces to Fig. (ii)). Note that when primary quantities are referred to secondary resistances/reactances/impedances are multiplied by K^2, voltages are multiplied by K. and currents are divided by K.



(i)



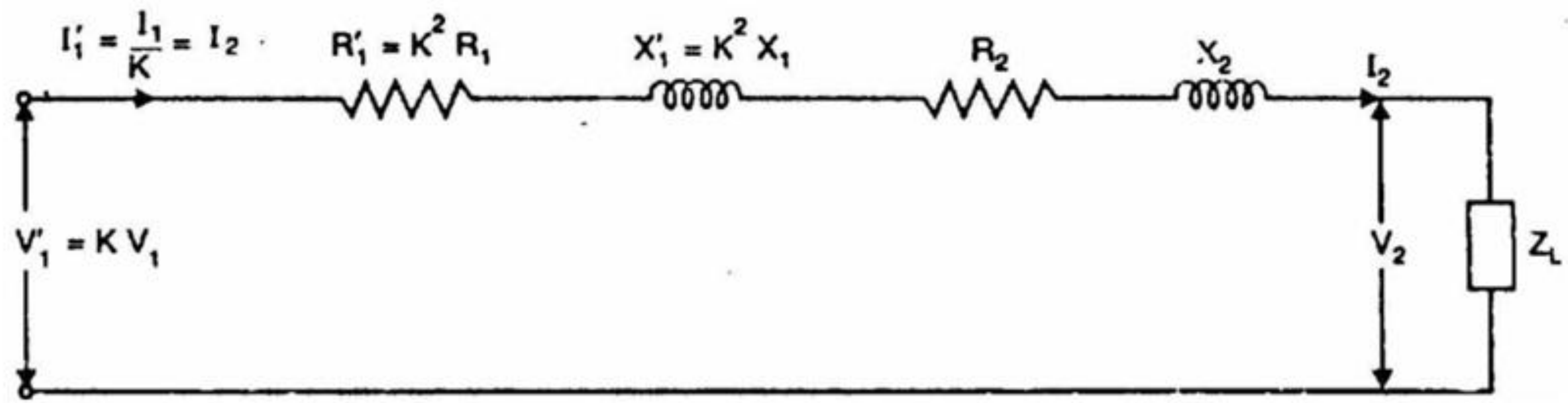
(ii)

$$\therefore \quad R'_1 = K^2 R_1; \qquad X'_1 = K^2 X_1; \qquad V'_2 = K\ V_1; \quad I'_1 = \frac{I_1}{K}$$

$$Z_{02} = R_{02} + j\,X_{02} \qquad \text{where } R_{02} = R_2 + R'_1; \quad X_{02} = X_2 + X'_1$$

Equivalent circuit of transformer referred to secondary

- If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown.

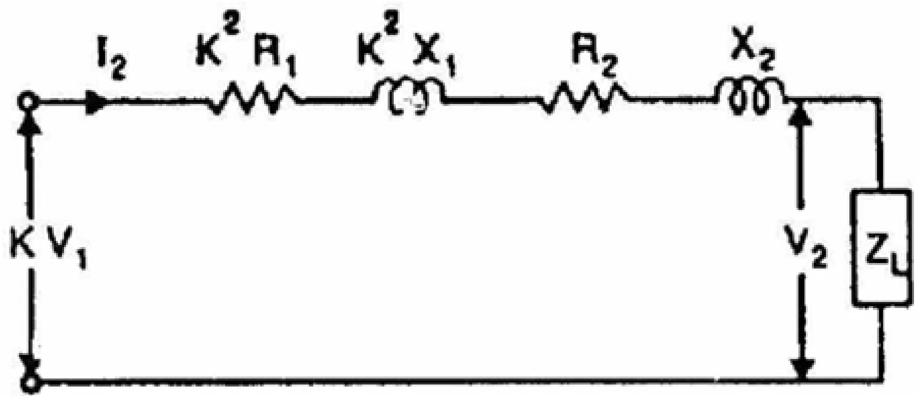


- When secondary quantities are referred to primary resistances/reactances are multiplied by K^2 , voltages are multiplied by K and currents are divided by K.
- After V_2' and I_2' are obtained , then:
  - Actual primary voltage, $V_1 = V_1'/K$
  - Actual primary current, $I_2 = KI_1'$

Note: *The same final answers will be obtained whether we use the equivalent circuit referred to primary or secondary. The use of a particular equivalent circuit would depend upon the conditions of the problem.*

Approximate Voltage Drop in a Transformer

- Consider the approximate equivalent circuit of transformer referred to secondary shown;



- At no-load, the secondary voltage is KV_1.
- When a load having a lagging p.f. $\cos\phi_2$ is applied, the secondary carries a current I_2 and voltage drops occur in $(R_2 + K^2R_1)$ and $(X_2 + K^2X_1)$.
- Consequently, the secondary voltage falls from KV_1 to V_2.
- Referring to the Figure, we have,

$$V_2 = KV_1 - I_2[(R_2 + K^2R_1) + j(X_2 + K^2X_1)]$$
$$= KV_1 - I_2(R_{02} + jX_{02}) = KV_1 - I_2Z_{02}$$

Approximate Voltage Drop in a Transformer

- Drop in secondary voltage $= I_2 Z_{02} = KV_1 - V_2$

$$= I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2$$

- For a load having a leading p.f. $\cos\phi_2$, we have,

$$\text{Approximate voltage drop} = I_2 R_{02} \cos\phi_2 - I_2 X_{02} \sin\phi_2$$

Note: If the circuit is referred to primary, then it can be easily established that:

$$\text{Approximate voltage drop} = I_1 R_{01} \cos\phi_1 \pm I_1 X_{01} \sin\phi_1$$

Voltage Regulation

- The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage (${}_0V_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage i.e.

$$\text{\%age voltage regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2}$$

where:

  - ${}_0V_2$ = No-load secondary voltage $= KV_1$.
  - V_2 = Secondary voltage on load

- As discussed above (*voltage drop*)

$$ {}_0V_2 - V_2 = I_2 R_{02} \cos\phi_2 \pm I_2 X_{02} \sin\phi_2 $$

The $+$ve sign is for lagging p.f. and -ve sign for leading p.f.

- The %age voltage regulation is the same whether primary or secondary side is considered.

Practice Question

- **Qn 1.:** A 250/125V, 5 kVA single phase transformer has primary resistance of 0.2 Ω and reactance of 0.75Ω. The secondary resistance is 0.05 Ω and reactance of 0.2 Ω. Determine:

i. Its regulation while supplying full load on 0.8 leading p.f

ii. The secondary terminal voltage on full load and 0.8 leading p.f.

- Qn 2.: A 40 kVA, 6600/250 V, 50 Hz transformer is having total reactance of 35 Ω when referred to primary side whereas its primary and secondary winding resistance is 10 Ω and 0.02 Ω, respectively. Find full load regulation of at a p.f. 0.8 lagging.

Solution Q1

$R_1 = 0.2\ \Omega,\ X_1 = 0.75\ \Omega,\ R_2 = 0.05\ \Omega,\ X_2 = 0.2\ \Omega,\ \cos\phi = 0.8$ leading

$$K = \frac{E_2}{E_1} = \frac{125}{250} = \frac{1}{2} = 0.5$$

$$(I_2)\ \text{F.L.} = \frac{\text{kVA}}{V_2} = \frac{5\times 10^3}{125} = 40\ \text{A} \quad \text{... full load}$$

$$R_{2e} = R_2 + K^2R_1 = 0.05+(0.5)^2\times 0.2 = 0.1\ \Omega$$

$$X_{2e} = X_2 + K^2X_1 = 0.2+(0.5)^2\times 0.75 = 0.3875\ \Omega$$

i) Regulation on full load, $\cos\phi = 0.8$ leading

$$\sin\phi = 0.6$$

$$\therefore \quad \%R = \frac{I_2 R_{2e}\cos\phi - I_2 X_{2e}\sin\phi}{V_2}\times 100,$$

$$I_2 = \text{Full load current}$$

$$= \frac{(40\times 0.1\times 0.8 - 40\times 0.3875\times 0.6)}{125}\times 100 = \mathbf{-4.88\%}$$

ii) For secondary terminal voltage, use basic expression of voltage drop.

On no load, $E_2 = 125$ V

$$E_2 - V_2 = I_2[R_{2e}\cos\phi - X_{2e}\sin\phi]$$

$$= 40\,[0.1\times 0.8 - 0.3875\times 0.6] = -6.1\ \text{V}$$

$$\therefore \quad V_2 = E_2 - [-6.1] = 125 + 6.1$$

$= \mathbf{131.1\ V}$ for leading p.f. $E_2 < V_2$.

Solution Q 1

Rating of transformer, $= 40 \text{ kVA} = 40 \times 10^3$ VA

Primary resistance, $R_1 = 10\ \Omega$

Transformation ratio, $K = \dfrac{250}{6600} = 0.03788$

Secondary resistance, $R_2 = 0.02\ \Omega$

Total resistance, referred to primary side,

$$R_{ep} = R_1 + R_2' = R_1 + \frac{R_2}{K^2} = 10 + \frac{0.02}{(0.03788)^2} = 23.94\ \Omega$$

Total reactance referred to primary side,

$$X_{ep} = 35\ \Omega$$

$$V_1 = \sqrt{(E_1 \cos\phi + I_1 R_{ep})^2 + (E_1 \sin\phi + I_1 X_{ep})^2}$$

where, $$I_1 = \frac{40 \times 10^3}{6600} = 6.06 \text{ A}$$

$$\cos\phi = 0.8;\ \sin\phi = \sin\cos^{-1} 0.8 = 0.6$$

$\therefore$ $$V_1 = \sqrt{(6600 \times 0.8 + 6.06 \times 23.94)^2 + (6600 \times 0.6 + 6.06 \times 35)^2}$$

$$= 6843.7 \text{ V}$$

$$\%\ Reg = \frac{V_1 - E_1}{V_1} \times 100 = \frac{6843.7 - 6600}{6843.7} \times 100 = 3.56\%\ (Ans.)$$

Practice Question

- **Qn 1.:** A single phase transformer with a ratio 5: 1 has primary resistance of 0.4 ohm and reactance of 1.2ohm and the secondary resistance of 0.01 and reactance of 0.04 ohm. Determine the percentage regulation when delivering 125 A at 600 V at

  (i) 0.8 p.f. lagging

  (ii) 0.8 p.f. leading.

- **Qn 2.:** If the ohmic loss of a transformer is 1% of the output and its reactance drop is 5% of the voltage, determine its regulation when the power factor is

  (i) 0.8 lagging

  (ii) 0.8 leading

  (iii) unity.