

Advanced 3D Monte Carlo Algorithms for Biophotonic and Medical Applications

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This thesis is submitted in partial fulfilment for the degree of
PhD
at the
University of St Andrews

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Declaration

I, Lewis McMillan, hereby certify that this thesis, which is approximately ***** words in length, has been written by me, that it is the record of work carried out by me, or principally by myself in collaboration with others as acknowledged, and that it has not been submitted in any previous application for a higher degree.

I was admitted as a research student in September 2015 and as a candidate for the degree of PhD in September 2015; the higher study for which this is a record was carried out in the University of St Andrews between 2015 and 2019.

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I hereby certify that the candidate has fulfilled the conditions of the Resolution and Regulations appropriate for the degree of PhD in the University of St Andrews and that the candidate is qualified to submit this thesis in application for that degree.

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Abstract

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Contents

Declaration	iii
Abstract	v
Acknowledgements	vii
Abbreviations	xi
List of Figures	xv
1 Monte Carlo Radiation Transport Technique	1
1.1 Introduction and Background	1
1.2 Monte Carlo Radiation Transport Algorithm	4
1.2.1 Introduction & background	4
1.2.2 Optical Properties	7
1.2.2.1 Derived Parameters	10
1.2.3 MCRT Algorithm	12
1.2.4 Code Details	17
1.3 Validation of MCRT Code	20
1.4 Conclusion	21
2 Computational Modelling of Tissue Ablation	23
2.1 Introduction and Background	23
2.2 Methods	24
2.2.1 Monte Carlo radiation transport (MCRT)	24
2.2.2 Heat Transport	25
2.2.3 Tissue Damage	32
2.2.4 Validation	35
2.3 <i>In silico</i> results	37
2.3.1 Introduction	37
2.3.2 Results	38
2.3.2.1 Investigating initial temperature	45
2.3.2.2 Investigating voxel temperature after ablation	47
2.4 Conclusion	47
3 3D Phase Tracking Monte Carlo Algorithm	51
3.1 Introduction	51
3.2 Theory	52
3.2.1 Huygens-Fresnel Principle	53

3.2.2	Validation of Phase Tracking Algorithm	54
3.3	Gaussian Beams	58
3.4	Bessel Beams	61
3.4.1	Validation	62
3.5	Higher order Bessel beams	67
3.6	Comparison	69
3.7	Discussion	70
3.8	Conclusion	70
4	AF	71
4.1	Problem	71
4.2	Validation	71
4.3	Practical application	71
4.4	Conclusion	71
5	Modelling fluorescent images	73
5.1	Problem	73
5.2	Validation	73
5.3	Practical application	73
5.4	Conclusion	73

Abbreviations

T_a ablation temperature.

AMR adaptive mesh refinement.

BPM beam propagation method.

FDM finite difference method.

FDTD finite difference time domain.

K-M theory Kubelka-Munk Theory.

MCRT Monte Carlo radiation transfer.

MPI Message-passing interface.

OCT optical coherence tomography.

PDF probability density function.

PDT photo-dynamic therapy.

PSTD pseudo-spectral time-domain.

RTE radiative transfer equation.

List of Figures

- 1.1 Sample Buffon needle experiment. 100 needles are dropped on a 10×10 cm area with lines spaced 1.5 cm apart. If a needle lands on a line it is recorded and coloured blue, else it is red. This simulation gave a value of $\pi \approx 3.10$.
- 1.2 Illustration of the rejection method for determining π from the area of a circle inscribed within a square. The ratio of the area of the circle to the square is $\frac{\pi}{4}$. Thus the ratio of darts landing in the circle to those that land outside the circle is $\pi \approx \frac{4N_{inner}}{N_{total}}$, where N_{total} is the total number of darts, and N_{inner} is the total number of darts that land in the circle. Using 200 darts gave a value of $\pi \approx 3.12$.
- 1.3 Computer generated imagery using ray tracing. The Monte Carlo method is used to “compute radiance along ray paths between lights and the camera”, in order to generate CGI images [15].
- 1.4 Energy flow through area dA within solid angle $d\Omega$ in a direction \hat{s} . Adapted from [23, 24]
- 1.5 Cylindrical volume element, $ds dA$, with solid angle $d\Omega$ in direction \hat{s} and solid angle $d\Omega'$ in direction \hat{s}' . Energy flowing through this element is used to derive the radiation transfer equation. Adapted from [23, 24].
- 1.6 Figure show the g factor for the Henyey-Greenstein phase function, for various configurations of back, forward or isotropic scattering.
- 1.7 Examples of wavelength dependent absorption coefficients for some common tissue chromophores [35, 39–47].
- 1.8 Flowchart of the Monte Carlo radiation transport algorithm as described in this section.
- 1.9 Example of a possible voxel model, with three different layers, various holes due to ablative pixel beam lasers (see Chapter 2). Each voxel can represent a different optical/thermal property of the tissue medium.
- 1.10 Illustration of photon propagation through a 2D grid. d_{x1} , and d_{y1} are the distances to the voxel walls in the x and y directions in the μ_1 voxel. In this case $S_1 = d_{x1}$ as d_{x1} is smaller than d_{y1} , thus the photon hits the voxel wall in the x direction. For the μ_5 voxel, d_y is smaller, thus the photon hits the voxel wall in the y^{th} direction.
- 1.11 Illustration of rotating the centre of mass frame to the lab frame. \mathbf{n} is the direction vector of the photon before scattering, and \mathbf{n}_s is the scattered direction vector. θ and φ are the scattering angles. z_s is in the same direction as \mathbf{n} .
- 1.12 Source code hierarchy, showing the relationship between different modules. Green is the entry point for the simulation. Red are the data modules, light blue are the routine modules, and grey are the external dependencies.
- 1.13 Performance of the parallelisation of the MCRT code using MPI.
- 1.14 Figure shows the fluence as a function of depth. Figure also shows comparison to the Jacques MCRT simulation and the MCRT as described in this chapter.

- 2.1 Flowchart of the tissue ablation algorithm.
- 2.2 Red lines are packet paths within a voxel. Black lines packet paths out with the voxel. Red packet paths, weighted by μ_a , are summed up to calculate the absorbed energy within each voxel.
- 2.3 Finite difference method stencil for simple explicit scheme
- 2.4 Computational domain decomposition. Total computational domain (red outline) is evenly divided between cores in the CPU. This is done via layers of the domain in the z direction. Information is passed to/from cores via the “halo swap” process (see Fig. 2.5).
- 2.5 Halo swapping. Process A updates the area in red and blue on the left. It updates the blue area which is sent to process B as B’s “halo”. Process B cannot update its own halo, but rather updates the halo for process A.
- 2.6 Figure show the speed up gained by parallelisation of the heat simulation using the “halo” swapping technique, for various sizes of computational domain (voxels). Data taken from a Intel Xeon E3-1245 v5, 8 cores @ 3.5GHz machine.
- 2.7 Ablation of a dog aorta, as viewed under a microscope. Steam vacuoles are clearly visible either side of the ablation area. Carbonisation is also evident at the edges of the ablation fronts. Adapted from [73].
- 2.8 Temperature of the cube for various times, comparing between analytical solution and numerical method.
- 2.9 Simulation of 81 pixel beams. Figure a) shows a slice through the optical properties at the end of the simulation in the z-y plane. Figure b) shows the optical properties in the x-y plane at the top surface. Yellow is unchanged tissue, and purple is completely ablated tissue. Figure shows that the ablation craters do not overlap one another.
- 2.10 Simulation of 70 W CO₂ ablative laser, with a circular beam profile. Crater depths as a function of pixel beam energy for various ablation temperature (T_a)’s.
- 2.11 Simulation of 70 W CO₂ ablative laser, with a Gaussian beam profile. Crater depths as a function of pixel beam energy for various T_a ’s.
- 2.12 Temperature bore hole though centre of medium as a function of time, for $T_a=500$ °C.
- 2.13 Tissue thermal damage around the ablation crater (white). Thermal tissue damage values of 3 refer to 3rd degree burns, 2 to 2nd, and 1 to 1st degree burns respectively. P is the power in Watts, T_a is the ablation temperature in Kelvin, and E_p is the energy per pixel beam in mJ.
- 2.14 Figure shows the maximum horizontal extent of thermal damage as a function of energy per pixel beam, for different T_a ’s.
- 2.15 Figure show the time taken for 1st, 2nd, and 3rd to occur as a function of depth, for a range of T_a ’s at 400 mJ.
- 2.16 Figure show the time taken for 1st, 2nd, and 3rd to occur as a function of depth, for a range of T_a ’s at 50 mJ.
- 2.17 Comparison of the different pulse profiles trialled for a pulselength of 0.2 s.
- 2.18 Comparison of various pulse shapes for the pixel beams.
- 2.19 Comparison of ablation depths for different initial temperatures in the porcine skin.
- 2.20 Comparison of maximum horizontal damage distance for different initial porcine skin temperatures.

- 2.21 Comparison of different voxel temperatures after ablation. Half refers to setting the temperature of a voxel to half that of the ablation temperature. Room refers to room temperature, and ablation leaves the temperature at the ablation temperature.
- 3.1 Example of phase calculation when a photon has travelled a distance l . Figure also show an example of interference between two photons via addition of the complex amplitudes at the point P_0 .
- 3.2 Illustration of the Huygens-Fresnel principle. At t_0 a wave is incident on an aperture. Times t_1 , t_2 , and t_3 show the evolution of the wavefront using the Huygens-Fresnel principle.
- 3.3 Comparison of theory and simulation for the double slit experiment. λ is xnm , b is, and d
- 3.4 Fresnel diffraction at a square aperture.
- 3.5 Comparison of theory and simulation for diffraction through a square aperture in the Fresnel and Fraunhofer regimes.
- 3.6 Illustration of a Gaussian beam focusing to its waist then diverging away. Image shows the various defined properties of a Gaussian beam along side the radius of curvature changing direction at the waist.
- 3.7 Simulation setup of focusing a Gaussian beam through a lens. Lens is convex-plano and is modelled on ThorLabs LA4249 UV fused silica lens [119].
- 3.8 Results of *in-silico* experiment of focusing a Gaussian beam though a convex-plano lens. L_t is the lens thickness, D is the $\frac{1}{e^2}$ input beam diameter, W_d is the working distance or back focal length, $2z_{max}$ is the depth of the medium, and w_0 is the beam waist.
- 3.9 Slice through the real part of the complex electric field of the *in-silico* experiment as in Fig. 3.7. Figure shows the radius of curvature changing direction at the waist as predicted by theory.
- 3.10 Illustration of φMC 's ability to model spherical aberrations. Image generated using same setup as in Fig. 3.7, but with a $D = 2\text{ mm}$.
- 3.11 Geometry of a Bessel beam, generated by an axicon lens. β is the angle with the optical axis, and the angle of the conical waves. α is the axicon angle.
- 3.12 Comparison of theoretical and MCRT simulation of a Bessel beams, with intensity normalised. The results from φMC show good agreement with the theory.
- 3.13 Bessel beam in the far field.
- 3.14 Experimental setup for propagating a Bessel beam through a cuvette filled with varying concentrations of Intralipid 20%. Bessel beam is imaged by an $20\times$ objective lens and a Grasshopper 3 camera.
- 3.15 Scattering properties of 20% Intralipid [124].
- 3.16 Comparison of experimental and simulation data for propagation of a Bessel beam produced by an axicon, through mediums of various turbidity. Images a) to g) is the data from φMC , and h) to n) are the experimental data. Volumes along the top is the volume of Intralipid in each solution as in Table 3.1. All images cropped so they are the same size.
- 3.17 Comparison of a larger medium, 2 mm^3 versus that of a smaller medium, $0.5\text{ mm}\times 0.5\text{ mm}\times 0.0\text{ mm}$.
- 3.18 Helical delay element attached to an axicon. Axicon introduces an additional radial delay in addition to that of the helical element. Input beam is a Gaussian, output beam is a higher order Bessel beam, $l > 0$.

- 3.19 Higher order Bessel beams. a) to d) show the phase shift due to the helical element. e) to h) show line plots of the simulation data compared to the theory. i) to l) and m) to p) show the higher order Bessel beam images for theory and simulation data respectively.
- 3.20 First comparison of Bessel and Gaussian beams, with equal power used to generate both beams.
- 3.21 Second comparison of Bessel and Gaussian beams for the case where the maxima of both Gaussian and Bessel beams match in the non scattering case.

Chapter 1

Monte Carlo Radiation Transport Technique

1.1 Introduction and Background

This chapter will provide an overview of the Monte Carlo method and how it is used within the context of Monte Carlo radiation transfer (MCRT). The chapter will then present the details of the MCRT code used as the basis of the subsequent chapters. Validation of this code and details of computational speed up are also presented. Subsequent chapters will expand upon the code modifications and extensions for each individual projects needs.

Monte Carlo Method

The Monte Carlo method is a numerical analysis technique based upon random numbers, which are used to calculate unknown variables in problems [1, 2].

The earliest use of the method is in Buffon's needle experiment of the 18th century [3–5]. Buffon asked the question;

“Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?”

The solution to this question is: for a needle length l , strip separation s , where x is the distance from the needle to the closest line, and θ is the angle of the needle with respect to the wood strips. Then using a simple geometrical argument, a needle crosses a strip if $x \leq \frac{l}{2} \sin\theta$.

x is distributed uniformly in $[0, \frac{s}{2}]$, and θ in $[0, \frac{\pi}{2}]$. Therefore the probability density function for x is $p(x) = \frac{2}{s}$, and θ is $p(\theta) = \frac{2}{\pi}$. The probability density function (PDF), is a function of a variable that gives probability for a variable to take a given value. The PDF is normalised over the whole range of the variable, in this case x , and θ . Thus, as x and θ are independent variables, giving a joint probability of $p(x, \theta) = \frac{4}{s\pi}$. So the probability of a needle of length l ($l < s$) is:

$$P = \int_0^{\frac{\pi}{2}} \int_0^{\frac{l}{2} \sin\theta} \frac{4}{s\pi} dx d\theta = \frac{2l}{s\pi} \quad (1.1)$$

Equation (1.1) can be used to carry out a Monte Carlo estimation of π . A simple rearrangement yields: $\pi = \frac{2l}{sP}$ where P is the ratio of needles crossing the line to the total number dropped.

Laplace was the first to suggest that Buffon's needle experiment could be used to estimate π [4]. Figure 1.1 shows an example of a simulation of Buffon's needle experiment.

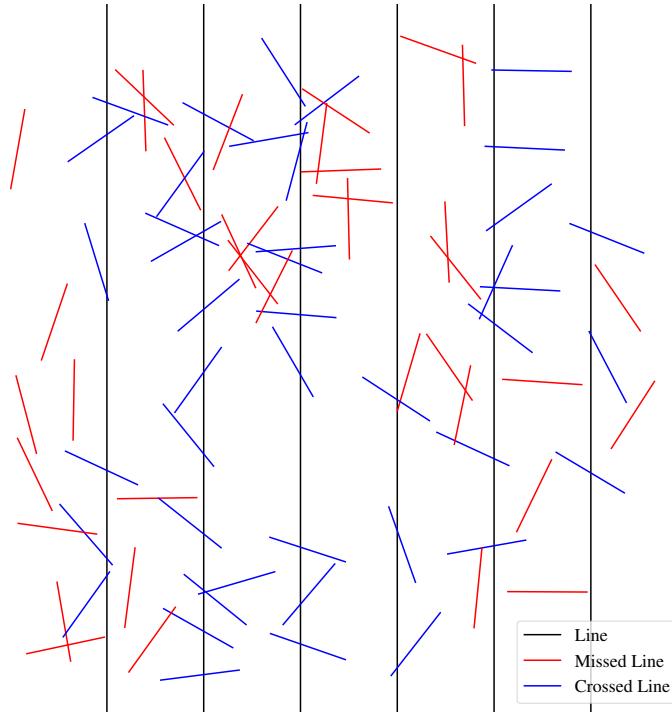


Figure 1.1: Sample Buffon needle experiment. 100 needles are dropped on a 10×10 cm area with lines spaced 1.5 cm apart. If a needle lands on a line it is recorded and coloured blue, else it is red. This simulation gave a value of $\pi \approx 3.10$.

There are various different approaches to using the Monte Carlo method to obtain randomly sampled variables. One analytical way of achieving this is the inverted sampling method. The inverted sampling method can be summarised by the following steps for drawing a sample X_i from an arbitrary PDF $p(x)$:

1. Compute the CDF* $P(x) = \int_0^x p(x')dx'$
2. Compute the inverse $P^{-1}(x)$
3. Obtain a uniformly distributed random number ξ
4. Finally, compute $X_i = P^{-1}(\xi)$

If a given problem cannot use the inverted sampling method, as it may not be possible to get a PDF or analytically invert the CDF, then the rejection method can be used. The rejection method is essentially a dart throwing method. This means that points are drawn and compared to the function. If the point lies under the function then the point is accepted, if it lies above the function then it is rejected. For example, if a function, $f(x)$ that does not have an analytical PDF, we can use a PDF $p(x)$ such that $f(x) < cp(x)$ where c is a constant. Therefore sampling from $p(x)$, and if the sampled point lies under $f(x)$ it is accepted, else it is rejected. Figure 1.2 shows an example of this process.

*The CDF is the cumulative distribution function.

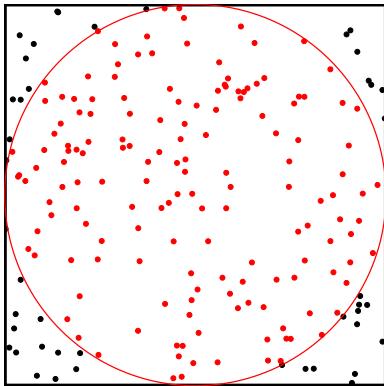


Figure 1.2: Illustration of the rejection method for determining π from the area of a circle inscribed within a square. The ratio of the area of the circle to the square is $\frac{\pi}{4}$. Thus the ratio of darts landing in the circle to those that land outside the circle is $\pi \approx \frac{4N_{inner}}{N_{total}}$, where N_{total} is the total number of darts, and N_{inner} is the total number of darts that land in the circle. Using 200 darts gave a value of $\pi \approx 3.12$

The Monte Carlo method is used in various different disciplines. Ranging from use in the financial sector to analyse investments and stocks by simulating the sources of uncertainty which affect their values [6, 7], use in statistical analysis [8], and in modern computer generated images (see Fig. 1.3) [9, 10]. It is also widely used in astronomy [11, 12] and medicine [13, 14], in order to simulate the propagation of radiation through scattering (turbid) media. This technique, MCRT, is what makes up the bulk of this thesis and is described in depth in the following sections.

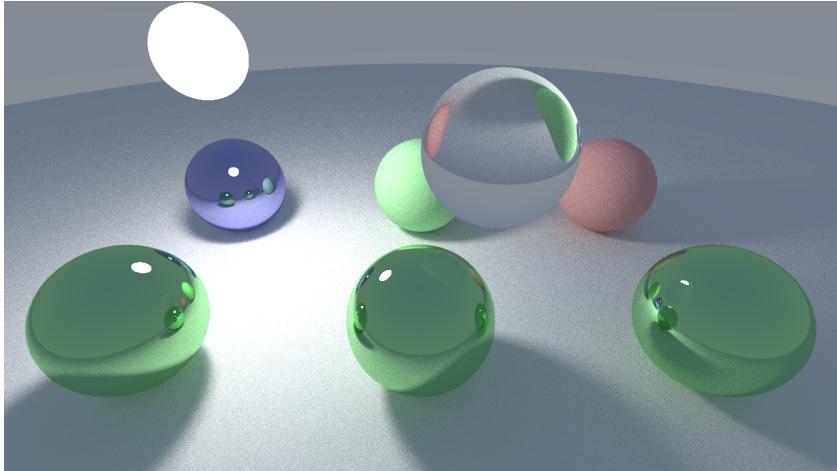


Figure 1.3: Computer generated imagery using ray tracing. The Monte Carlo method is used to “compute radiance along ray paths between lights and the camera”, in order to generate CGI images [15].

1.2 Monte Carlo Radiation Transport Algorithm

1.2.1 Introduction & background

The technique that makes up the bulk of this thesis, is the MCRT technique. This method was developed at the end of the Second World War at the Los Alamos National Laboratory, for the purpose of calculating neutron diffusion through shielding material [16–19]. It has since found a myriad of applications from light transport through dusty galactic clouds [20], calculating doses for radiotherapy [21] to light transport through tissue [22].

The theory that governs the transport of radiation through a medium is the radiative transfer equation. Before describing MCRT which is a numerical simulation of the radiative transfer equation (RTE), the theory of radiation transport must be examined.

Radiative Transfer

Transport of radiant energy through turbid media, can be modelled analytically using the RTE. The RTE models the radiative losses, and gains by a beam of radiation as it travels through a medium, including: loss of energy due to absorption, loss/gain of energy due to scattering, and energy gain due to emission. Before deriving the RTE, definitions of some terms and physical quantities is required.

The first term is spectral irradiance, L_ν . Spectral irradiance is defined as the energy flow in a direction $\hat{\mathbf{n}}$, for a solid angle $d\Omega$, per unit time per unit temporal frequency bandwidth. Irradiance is defined as the spectral irradiance over a small frequency range $[\nu, \nu + \Delta\nu]$:

$$L(\vec{r}, \hat{s}, t) = L_\nu(\vec{r}, \hat{s}, t)\Delta\nu \quad (1.2)$$

Where:

- \vec{r} is the position;
- \hat{s} is the unit normal vector;
- t is the time;
- and $L(\vec{r}, \hat{s}, t)$ is the irradiance [$W m^{-2} sr^{-1}$].

The irradiance can be used to determine the energy, dE , transported across an area dA , in a solid angle $d\Omega$ in a time dt (see Fig. 1.4) is:

$$dE = L(\vec{r}, \hat{s}, t) \cdot \cos(\theta) dA d\Omega dt \quad (1.3)$$

Where:

- \hat{n} is the unit normal to dA ;
- and $\cos(\theta)$ is the angle between \hat{n} and \hat{s} .

Irradiance can also be used to determine the fluence rate, ϕ , which is defined as the energy flow per unit time, independent of the flow direction.

$$\phi(\vec{r}, t) = \int_{4\pi} L(\vec{r}, \hat{s}, t) d\Omega \quad (1.4)$$

Where:

- ϕ is the fluence rate [Wm^{-2}].

Solving the RTE yields the irradiance which gives the distribution of light in the medium, and gives information on the state of the system and all the physical properties of it.

With the irradiance defined, as well as the other quantities that follow, the RTE can be derived [23, 24]. First considering the conservation of energy, as shown in Eq. (1.5).

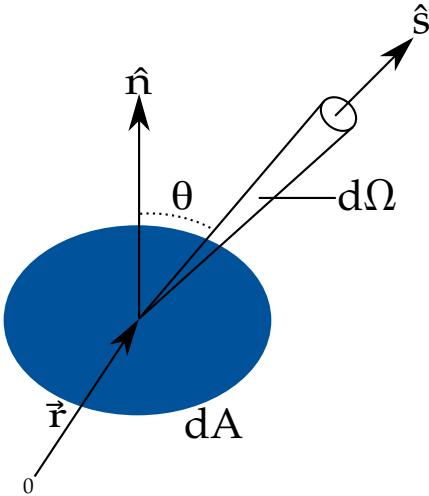


Figure 1.4: Energy flow through area dA within solid angle $d\Omega$ in a direction \hat{s} . Adapted from [23, 24]

$$dP = -dp_{div} - dp_{ext} + dP_{scatt} + dP_{src} \quad (1.5)$$

Where:

dP is the total change in energy in the volume $dA ds$ within the solid angle, $d\Omega$, per unit time (see Fig. 1.5);

dP_{div} is the energy loss due to the divergence of the radiation beam per unit time;

dP_{ext} is the energy loss due to absorption and scattering within the volume $dA ds$ within the solid angle, $d\Omega$;

dP_{scatt} is the energy gain due to scattering from \hat{s}' into $d\Omega$ per unit time;

and dP_{src} is the energy gain due to emission within the medium, per unit time.

The total change in energy, dP , in the volume element within the solid angle $d\Omega$ is equal to:

$$dP = \frac{1}{c} \frac{\partial L(\vec{r}, \hat{s}, t)}{\partial t} dA ds d\Omega \quad (1.6)$$

Where c is the speed of light.

The first loss term, dP_{div} , is the energy loss due to divergence of the radiation beam. This is modelled as:

$$dP_{div} = \frac{\partial L}{\partial s} d\Omega dV \quad (1.7)$$

$$= \hat{s} \cdot \nabla L(\vec{r}, \hat{s}, t) d\Omega dV \quad (1.8)$$

dP_{ext} is the second loss term, and accounts for energy loss due to scattering and absorption in the volume element within the solid angle $d\Omega$. This is modelled as:

$$dP_{ext} = \mu_t ds L(\vec{r}, \hat{s}, t) dA d\Omega \quad (1.9)$$

Where μ_t is the extinction coefficient [m^{-1}], see [Optical Properties](#) for further details.

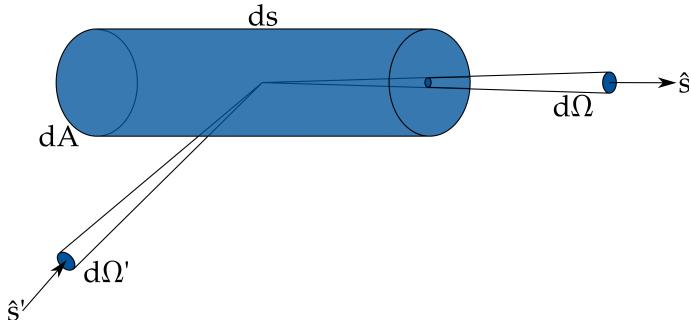


Figure 1.5: Cylindrical volume element, $ds dA$, with solid angle $d\Omega$ in direction \hat{s} and solid angle $d\Omega'$ in direction \hat{s}' . Energy flowing through this element is used to derive the radiation transfer equation. Adapted from [23, 24].

The first energy gain term, dP_{src} , is due to emission in the volume element within the solid angle $d\Omega$.

$$dP_{src} = S(\vec{r}, \hat{s}, t) dV d\Omega \quad (1.10)$$

The second energy gain term, and final term, is due to the incident energy on the volume element within the solid angle $d\Omega$ in direction \hat{s} due to scattering from any direction \hat{s}' .

$$dP_{scatt} = N_s dV \left(\int_{4\pi} L(\vec{r}, \hat{s}', t) P(\hat{s}', \hat{s}) \sigma_s d\Omega' \right) d\Omega \quad (1.11)$$

$$= \mu_s dV \left(\int_{4\pi} L(\vec{r}, \hat{s}', t) P(\hat{s}', \hat{s}) d\Omega' \right) d\Omega \quad (1.12)$$

Where:

N_s is the number density of scatters [$\# m^{-3}$];
 $P(\hat{s}', \hat{s})$ is the scattering phase function (see Section 1.2.2 for further discussion);
and σ_s is the cross section of the scatters [m^2], thus $\mu_s = N_s \sigma_s$, where μ_s is the scattering coefficient [m^{-1}].

Finally substituting Eqs. (1.6), (1.8) to (1.10) and (1.12) into Eq. (1.5) yields the RTE:

$$\frac{1}{c} \frac{\partial L(\vec{r}, \hat{s}, t)}{\partial t} + \mathbf{s} \cdot \nabla L(\vec{r}, \hat{s}, t) = -\mu_t L(\vec{r}, \hat{s}, t) + \mu_s \int_{4\pi} p(\hat{s}, \hat{s}') L(\vec{r}, \hat{s}', t) d\Omega' + S(\vec{r}, \hat{s}, t) \quad (1.13)$$

In general, the RTE is hard to solve in arbitrary 3D geometries, however there are a number of approximations, and numerical methods available. Diffusion approximation, Kubelka-Munk Theory (K-M theory), and MCRT are the common methods used to approximate or solve the RTE.

Kubelka-Munk Theory

K-M theory was originally developed in order to calculate the light distribution in thin layered materials, such as paint or paper [25]. The theory is rather simple and makes many assumptions about the medium and the incident light. The main assumptions of K-M theory are: only

scattering and absorption take place in the medium, the incident light is already diffuse, the medium is uniform with only isotropic scattering, no external or internal reflections, and the medium is planar and infinitely wide [26–28].

These assumptions make K-M theory poor for modelling light-tissue interactions. This is because in tissue, scattering is not isotropic but rather forward biased (see Section 1.2.2). Tissue is rarely, planar and infinitely wide. Tissue also has some reflections at its external and internal boundaries, due to changes in refractive indices. Many medical and biophotonic treatments/methods use laser light which is not diffuse. Finally tissue can also exhibit fluorescence, which K-M theory is not able to model, along with polarization. K-M theory does have some positive aspects. It is good at calculating the diffuse reflectance of simple media, and can be used to roughly estimate calculations. Though it is not well suited for modelling light-tissue applications [29].

Diffusion Approximation

The diffusion approximation for the RTE, is where the irradiance is separated into two components:

$$L(\vec{r}, \hat{s}) = L_c(\vec{r}, \hat{s}) + L_d(\vec{r}, \hat{s}) \quad (1.14)$$

Where L_c is the unscattered contribution, which satisfies Beer's law[†], and L_d is the diffuse contribution. The L_d component is expanded using Legendre polynomials and truncated. The diffusion approximation also has a number of assumptions and restrictions. The main assumption is that scattering dominates over absorption, and that the scattering is nearly isotropic. This restricts the types of scattering the Diffusion approximation can model, though using similarity relations can partially model scattering in tissue [30, 31].

Diffusion theory is computationally fast, and simple to implement. However it is poor at modelling light-tissue interactions due to its assumptions and restrictions, mainly the inaccurate modelling near the boundaries of the medium and its lack of modelling fluorescence and other microphysics. However it can be used to speed up MCRT in optically thick regions [32, 33].

MCRT

The final method, MCRT, is a method that is numerically equivalent to the RTE [23]. MCRT is a flexible method, it can model arbitrary 3D geometries, various microphysics including fluorescence, and polarisation. It can also model various different light sources, from collimated laser beams, to diffuse light sources. The only downside that is noted in the literature is that the MCRT can be expensive computationally. However with computational power growing faster with each year, this is less of a problem going forward. The next several sections give an in depth description of the MCRT method and its flexibility, along with a description of the code used in this thesis to solve various medical and biophotonic problems.

1.2.2 Optical Properties

Before an in depth description of the MCRT method is outlined, a discussion of the optical properties of materials is presented, which the MCRT method needs in order to simulate the transport of photons in a material.

[†]Beer's law (or Beer-Lambert law) states that the transmission, T , is equal to $e^{-\mu L}$, where L is the distance and μ is the attenuation coefficient.

Optical properties of a medium are the properties that describe how light is transported through that medium. Usually the optical properties of a medium are defined by three main parameters: the scattering and absorption coefficients (μ_s and μ_a), and the anisotropy coefficient (g). There are several other optical properties the medium can be defined with, however these in general are only used for specific applications, such as Raman cross-sections for Raman scattering.

Scattering

The scattering coefficient, along with the anisotropy value (see [Anisotropy](#)), define how light is scattered in a medium. Scattering occurs in skin due to a number of different scatterers, and inhomogeneities found within the skin. The main scatters in the skin are filamentous proteins such as collagen and elastin. These proteins are generally found within the dermis and epidermis [34]. In the upper layers of the skin, the main scatters are keratins and various chromophores such as melanin. The size of the scatters affect how light is scattered and into which direction that light is scattered into.

The scattering of light within tissue is usually defined as μ_s or μ'_s ; the scattering coefficient and the reduced scattering coefficient, where $\mu'_s = \mu_s(1-g)$. The scattering coefficient is defined such that the probability of transmission without scattering and neglecting absorption in a path length L is:

$$T = e^{-\mu_s L} \quad (1.15)$$

This gives units of inverse length for the scattering coefficient (usually measured in cm^{-1}). The reduced scattering coefficient is quite often given in place of the scattering coefficient, as the reduced coefficient is more easily measured than the “normal” coefficient [35].

Anisotropy

Anisotropy is the degree of deviation that light undergoes at each scattering event. The anisotropy value is taken from the phase function for the medium. The phase function is defined as the angular distribution of light intensity scattered by a particle. The phase function, $\Phi(\theta, \phi)$, is usually normalised over all angles:

$$\int_{\Omega} \Phi(\theta, \phi) d\Omega = 1 \quad (1.16)$$

Where θ , and ϕ are the usual polar and azimuthal spherical angles, and $d\Omega = \sin\theta d\theta d\phi$. Thus for Rayleigh and isotropic scattering, their phase function's are:

$$\Phi_{isotropic}(\theta, \phi) = \frac{1}{4\pi} \quad (1.17)$$

$$\Phi_{Rayleigh}(\theta, \phi) = \frac{3}{8\pi} (1 + \cos^2(\theta)) \quad (1.18)$$

For simplicity, the phase function is usually cast as the anisotropy value g , which is defined as the average angle of deflection:

$$g = \langle \cos(\theta) \rangle = \int_{\Omega} \cos(\theta) \Phi(\theta, \phi) d\Omega \quad (1.19)$$

The anisotropy factor, g , can take on any value from -1 to 1 . Where a value of -1 is totally back scattering, 0 is isotropic scattering, and 1 is totally forward scattering (see Fig. 1.6).

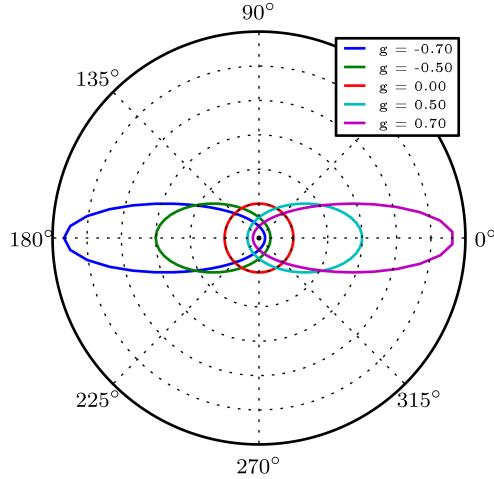


Figure 1.6: Figure show the g factor for the Henyey-Greenstein phase function, for various configurations of back, forward or isotropic scattering.

There are many phase functions that can be used to model the anisotropy factor in a medium. The standard phase function in biological tissue is the Henyey-Greenstein phase function. The Henyey-Greenstein phase function, was originally created to model scattering of diffuse radiation in the galaxy [36, 37]. It has since become the *de-facto* phase function for biological tissue. This is due to the phase functions relative simplicity and due to it being regarded as a “good” phase function for approximating scattering in biological tissue [38]. The Henyey-Greenstein phase function is shown in Eq. (1.20):

$$\Phi_{H.G}(\theta, \phi) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos(\theta))^{\frac{3}{2}}} \quad (1.20)$$

Absorption

Absorption of light by a medium is defined by the absorption coefficient μ_a . The absorption coefficient is defined in a similar fashion to the scattering coefficient, by considering the probability of transmission without absorbing and neglecting scattering in a path length L:

$$T = e^{-\mu_a L} \quad (1.21)$$

This, again like the scattering coefficient, gives inverse distance for the unit of the absorption coefficient (and its is also usually measured in units of cm^{-1}).

There are various sources of absorbers in tissue: blood, water, fat, melanin, β -carotene, bilirubin are among the more absorbing chromophores. These chromophores can all contribute, depending on the wavelength, with some more absorbing than others, see Fig. 1.7. The absorbed photons can then be remitted as fluorescence or stored as absorbed energy/heat.

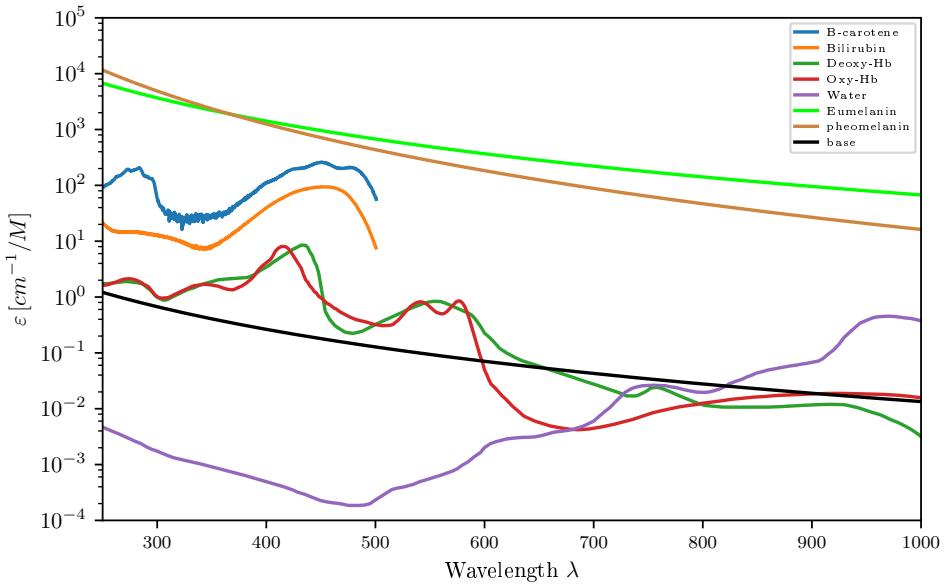


Figure 1.7: Examples of wavelength dependent absorption coefficients for some common tissue chromophores [35, 39–47].

1.2.2.1 Derived Parameters

There are also some derived parameters that are useful to use. These are the albedo and the total attenuation coefficient.

The total attenuation coefficient is defined as the sum of the scattering coefficient and the absorption coefficient:

$$\mu_t = \mu_s + \mu_a \quad (1.22)$$

The albedo, or scattering probability, is defined as the ratio of the scattering coefficient to the total attenuation coefficient:

$$a = \frac{\mu_s}{\mu_a + \mu_s} = \frac{\mu_s}{\mu_t} \quad (1.23)$$

Other Parameters

The preceding subsection, described the optical properties that this thesis will use in every chapter. However there are other optical properties that can be used to define a medium. These other parameters generally are used to model microphysics such as Raman scattering, polarization, fluorescence or reflection/refraction. This section will give a brief overview of these other optical properties.

Refractive Index

The refractive index of a medium, defines how fast light propagates through that medium. Generally, for tissue, the refractive index is given as a bulk refractive index. Meaning that the medium is divided into sections, with each section given a refractive index. For example, skin's refractive indices are divided up by the different layers of skin. Details on how refraction is implemented with the code can be found in Chapter 4.

Raman Scattering

Raman scattering is where a photon is scattered inelastically, which excites the molecule the photon scattered off, thus decreasing the energy of the photon and increasing the photons wavelength. The optical property needed to model Raman scattering is the Raman scattering cross section. The cross section, like the absorption or scattering coefficient, is the likelihood of a photon undergoing a Raman scattering event. Raman scattering has been modelled in MCRT in order to simulate spatially offset Raman spectroscopy for breast tumour analysis [48].

Fluorescence

Fluorescence occurs when a photon is absorbed by a fluorescent molecule and re-emitted with a new wavelength. Fluorescence is a reactively common phenomena, and is heavily utilised in biophotonics and medicine, in order to image, or monitor molecules in tissue. Again the optical property that models fluorescence is a coefficient that gives the probability of absorption and re-emission of a photon by a certain molecule. Usually this is in the form of an absorption coefficient or extinction coefficient. The extinction coefficient is a measurement of absorption in terms of the concentration of that absorber. Thus if a medium has many fluorophores, then the total absorption coefficient is the bulk absorption of the medium plus the contribution from the fluorophores as in Eq. (1.24):

$$\mu_a = \ln(10) \sum_i C_i \varepsilon_i \quad (1.24)$$

Where C_i is the concentration of the i^{th} fluorophore, and ε_i is the extinction coefficient of the i^{th} fluorophore.

Fluorescence will be described in more depth in Chapters 4 and 5.

1.2.3 MCRT Algorithm

A MCRT algorithm can range in complexity from a simple 20 line program, to tens of thousands of lines, depending on the application. This section will provide an in depth description of the MCRT algorithm for the propagating photons thorough a spherical medium with optical properties μ_s , and μ_a . The section following this one, will provide details of how the MCRT algorithm is implemented in the Fortran programming language, along with the various code details, such as the parallelisation of the code.

Figure 1.8 shows a flow chart of the MCRT algorithm described in this chapter.

Medium and Grid Set-up

The first step of any MCRT algorithm, is to set-up the medium the photons will propagate through. There are a variety of ways that the medium can be set-up, for this section, it is assumed the medium is an isotropic sphere, radius R , and centred at the origin. For simplicity one wavelength is considered, λ . As the MCRT algorithm presented here is run on a 3D Cartesian grid, the grid is setup before creating the spherical medium. The grid is composed of $n_x \times n_y \times n_z$ voxels[‡], where each voxel can have its own optical properties. The grid is setup by first setting an array that stores the locations of the voxel boundary walls in the x , y , and z directions. The next step is to setup the actual medium. This is achieved by discretising the medium onto a grid. For this example a sphere is inscribed into a cubic volume, by setting the optical properties of a voxel to that of the medium if the sphere encloses that voxel. The voxels out with sphere are set to that of the ambient medium. An example of a voxelised medium can be seen in Fig. 1.9.

Photon Launch and Initialisation

The second step in the MCRT algorithm, is to initialise the photon. Initialisation of the photon involves setting its initial position and direction. Again how this is done depends on the experiments being simulated. Here the photon is initialised to the centre of the sphere. The initial direction is sampled isotropically, and set accordingly:

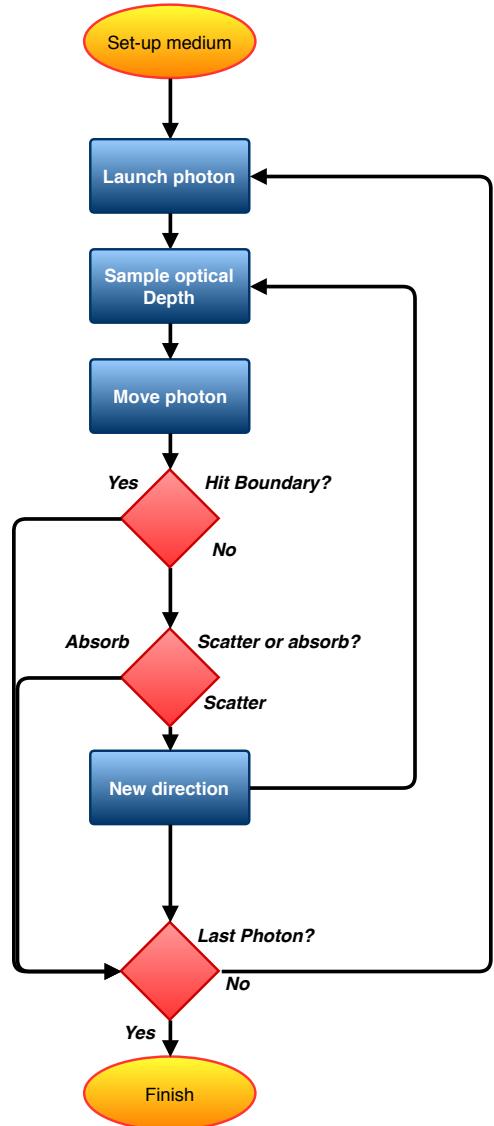


Figure 1.8: Flowchart of the Monte Carlo radiation transport algorithm as described in this section.

[‡]A voxel is a 3D pixel

$$n_{xp} = \sin(\theta) \cdot \cos(\phi) \quad (1.25)$$

$$n_{yp} = \sin(\theta) \cdot \sin(\phi) \quad (1.26)$$

$$n_{zp} = \cos(\theta) \quad (1.27)$$

With θ and ϕ sampled uniformly between $[0, \cos^{-1}(2\xi - 1)]$ and $[0, 2\pi\xi]$ respectively, where ξ is a random number in the range $[0,1]$.

The next step is to launch a photon packet. Depending on the source of photon packets for a given simulation, this step varies from simulation to simulation. The general idea of launching a photon packet is that the packet is given an initial direction vector and position (which consists of a physical position and a voxel position)[§]:

$$\text{direction} = \begin{bmatrix} n_{xp} \\ n_{yp} \\ n_{zp} \end{bmatrix} \quad (1.28)$$

$$\text{position} = [x_p, y_p, z_p] \quad (1.29)$$

$$\text{voxel} = [x_{cell}, y_{cell}, z_{cell}] \quad (1.30)$$

To set the direction vectors, the components of the direction vectors must be first set. The packets position is tracked using a Cartesian coordinate system, however for ease of computation for calculating scattering angles (see [Photon Interaction Event](#)), the direction vectors are computed in a spherical system thus the direction vectors are in Eqs. (1.25) to (1.27).

θ and ϕ are generated dependent on the photon source used. The individual sine and cosine terms are saved for use in the scattering routines (see [Photon Interaction Event](#)). The position is then set according to the light source used. For this example the photons are released from the origin of the sphere. Using this position the voxel the packet is in is calculated.

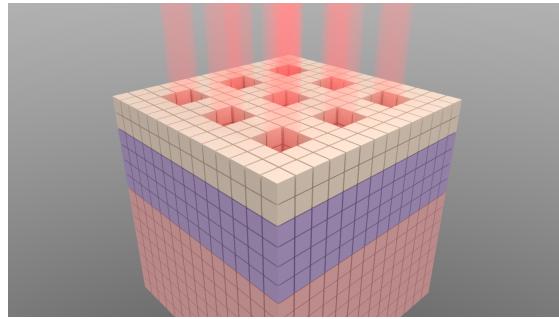


Figure 1.9: Example of a possible voxel model, with three different layers, various holes due to ablative pixel beam lasers (see Chapter 2). Each voxel can represent a different optical/thermal property of the tissue medium.

Photon Move

The next step in the algorithm is moving a packet to the next interaction point. The probability a packet will interact over a distance dL is $\mu_t dL$, where μ_t is the total extinction coefficient (see [Optical Properties](#)). Thus, the probability of travelling dL without any interaction is $1 -$

[§]all variables given in this section are the same as they are in the code.

$\mu_t dL$. Therefore over a distance L , with N segments of length L/N the probability of travelling L before any interaction:

$$P(L) = (1 - \mu_t \frac{L}{N}) \cdot (1 - \mu_t \frac{L}{N}) \dots (1 - \mu_t \frac{L}{N}) = (1 - \mu_t \frac{L}{N})^N \quad (1.31)$$

$$P(L) = \lim_{N \rightarrow \infty} (1 - \mu_t \frac{L}{N})^N = e^{-\mu_t L} = e^{-\tau} \quad (1.32)$$

Where τ is the number of mean free paths in a distance L . Eq. (1.32) is now a PDF for the distance a packet will travel before an interaction occurs. To be able to get a random optical depth, the PDF has to be able to be sampled from either analytically or via the rejection method. Using the Monte Carlo method described in Section 1.1, with ξ as our random number, gives:

$$\xi = \int_0^\tau e^{-\tau'} = 1 - e^{-\tau} \rightarrow \tau = -\ln(1 - \xi) \quad (1.33)$$

As ξ is symmetric about 0.5, $1 - \xi$ can be substituted for ξ yielding:

$$\tau = -\ln(\xi) \quad (1.34)$$

τ is now the optical distance, however this needs to be converted into a physical distance so that the photon packet can be moved. From our definition of τ we know that $\tau = \int_0^L \mu_t dS$, and if the medium is smooth and homogeneous (i.e not a gridded medium):

$$L = \frac{\tau}{\mu_t} \quad (1.35)$$

Therefore in order to update the packets position it is simply:

$$x_p = x_p + L \cdot n_{xp} \quad (1.36)$$

$$y_p = y_p + L \cdot n_{yp} \quad (1.37)$$

$$z_p = z_p + L \cdot n_{zp} \quad (1.38)$$

However as the code in this thesis is a 3D gridded Cartesian code, the method of updating and moving the packets position is slightly adjusted. As stated in [Medium and Grid Set-up](#), the medium has been discretised onto a grid, so that each voxel can have a different μ_t , thus Eq. (1.35) becomes:

$$L = \frac{\tau}{\mu_{t,\zeta}} \quad \zeta = (x, y, z) \quad (1.39)$$

with $\mu_{t,\zeta}$ the μ_t for the ζ^{th} voxel.

Moving the photon through a voxelised medium is more involved than propagating a photon through a non voxelised medium. This is because the voxel the photon is in needs to be updated as the photon moves from voxel to voxel. The first step of moving the photon through a voxelised medium is drawing a random optical depth. This optical depth will be the full optical depth the photon travels before an interaction event. The generation of a random optical depth is as outlined above, $\tau = -\log(\xi)$. As the photon travels through the voxel grid, a running total of the current optical distance travelled is kept. This is then compared to the randomly generated optical depth. When the running total optical depth equals the randomly generated optical depth the photon propagation is stopped, and the photon undergoes an interaction.

The next step is to calculate the distance to the nearest wall in the x , y , and z directions. The distance is calculated for each direction. Equation (1.40) shows for the x direction:

$$d_x = \frac{x_{face} - x_{cur}}{n_{xp}} \quad (1.40)$$

Where d_x is the distance to the nearest wall in the x direction. x_{face} is the voxel wall position in the x direction, and n_{xp} is the x direction vector. With three distances calculated, $[d_x, d_y, d_z]$, the minimum of these is thus the distance to the nearest voxel wall.

The next step is to calculate the optical depth for this distance. The optical depth is calculated by rearranging Eq. (1.39) for τ , with L now the distance to the nearest wall. With the optical distance to the nearest wall calculated, the next step is to determine if there is “enough” optical distance left to travel the full distance to the nearest wall. Therefore the running total optical distance is compared to the randomly generated optical distance. If the running total + the new optical distance to the nearest wall, is less than the randomly generated optical depth, then the photon travels to the nearest wall. The photon is then placed in the next voxel by a distance δ , where δ is just larger than machine precision. If the running total + the new optical distance to the nearest wall is greater than the generated optical distance then an interaction event occurs in the current voxel. The distance to the interaction event is calculated and the photon moved to this location.

Figure 1.10 illustrates this whole process for a 2D example.

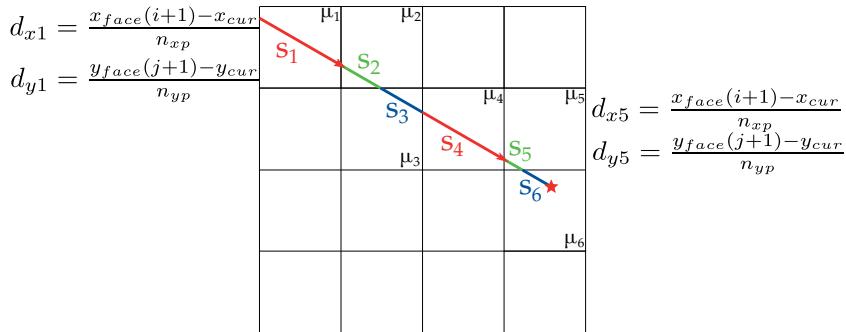


Figure 1.10: Illustration of photon propagation through a 2D grid. d_{x1} , and d_{y1} are the distances to the voxel walls in the x and y directions in the μ_1 voxel. In this case $S_1 = d_{x1}$ as d_{x1} is smaller than d_{y1} , thus the photon hits the voxel wall in the x direction. For the μ_5 voxel, d_y is smaller, thus the photon hits the voxel wall in the y^{th} direction.

This whole process is repeated until the photon undergoes an interaction event or leaves the voxel medium. The next step in the algorithm is the interaction event, which can consist of either: scattering, absorbing or another microphysics phenomena.

Photon Interaction Event

The next section of the algorithm is to decide how the photon interacts with the medium, either via scattering or absorption. There are other interaction events that can occur, however descriptions of these are left for the chapters that detail these behaviours.

To decide whether a packet scatters or absorbs involves generating a random number, ξ , and comparing it against the albedo, a . If $\xi < a$ then the packet scatters, otherwise it is absorbed.

Packet Absorption

If the interaction event is a photon packet absorption, then the algorithm terminates the photon packets and starts the next photon packet, see [Termination](#).

Packet Scattering

If the interaction event is a packet scattering, then the packet is scattered into a new direction and the above processes are carried out until a termination clause is met, see [Termination](#).

Depending on the medium being simulated, it can either be isotropic or anisotropic scattering. For the isotropic case, new $\cos(\theta)$ and ϕ angles are sampled uniformly, and the direction vectors set as in section [Photon Launch and Initialisation](#). For the case where the scattering is anisotropic the calculation of the scattering angles, θ and ϕ , is more complicated. The random sampling of the scattering angles, θ and ϕ , are valid in the “centre of mass” frame containing the scatter, incident and scattered ray. The photons position is updated in the lab frame, thus the direction vectors also have to be updated in the lab frame. This means that the scattering angles need to be rotated into the lab frame. For the isotropic case assume that the scattering is also isotropic in the lab frame, thus the new direction vector is easily calculated. However this is not the case for anisotropic scattering, as the centre of mass frame has to be rotated into the lab frame.

Figure 1.11 and Eq. (1.41) show how this process is achieved. Where $\mathbf{n} = (n_x, n_y, n_z)$, $\mathbf{n}_s = (n_x^{new}, n_y^{new}, n_z^{new})$, θ_s is chosen from the phase function Eq. (1.42), and $\varphi_s = 2\pi\xi$ with ξ being a random number in the range 0 to 1.

$$\begin{aligned} n_x^{new} &= \frac{\sin\theta_s}{\sin\theta} (n_x n_y \cos\varphi_s - n_y \sin\phi_s) + n_x \cos\theta_s \\ n_y^{new} &= \frac{\sin\theta_s}{\sin\theta} (n_y n_z \cos\varphi_s + n_x \sin\phi_s) + n_y \cos\theta_s \\ n_z^{new} &= -\sin\theta_s \cos\varphi_s + n_z \cos\theta_s \end{aligned} \quad (1.41)$$

$$\cos\theta_s = \frac{1 + g^2 - \left(\frac{1-g^2}{(1-g+2g\xi)^{3/2}} \right)^2}{2g} \quad (1.42)$$

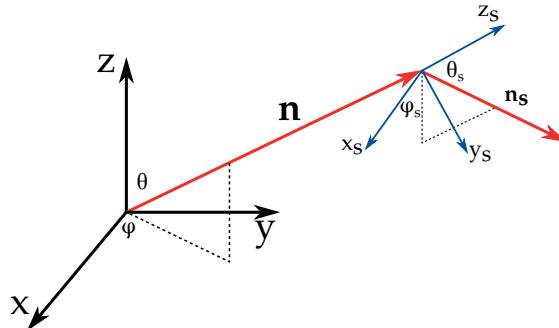


Figure 1.11: Illustration of rotating the centre of mass frame to the lab frame. \mathbf{n} is the direction vector of the photon before scattering, and \mathbf{n}_s is the scattered direction vector. θ and φ are the scattering angles. z_s is in the same direction as \mathbf{n} .

Termination

The final section of the MCRT algorithm is to check if it should be terminated. This is a simple check to see if there are any more photons to run. If there are more photons to run then the algorithm goes back to the [Photon Launch and Initialisation](#) section and continues from there. If there are no more photons the algorithm terminates and any results are written out.

Scored Quantities

As MCRT is a computational method, a wealth of information is able to be recorded during the simulation. From the paths of individual photons, to average scattering angles and more. However it is not practical to record all this information for every simulation, as this would lead to inefficient simulations, and expensive data storage solutions. Thus for a given problem only the pertinent information is stored.

One important recorded variable is fluence. Fluence is the number of photons entering a sphere per unit cross section area [2]. In practise the average fluence per area is used, Eq. (1.43), as this is easier to calculate in an MCRT code. Lucy showed that the average fluence per area is proportional to the sum of the path length through a volume [49]:

$$J_i = \frac{L}{NV_\zeta} \sum l \quad (1.43)$$

Where:

J_i is the mean intensity such that the fluence is $\Phi = 4\pi J [W m^{-2}]$;

L is the luminosity or power of the light source [W];

N is the total number of photon packets [-];

V_ζ is the volume of the ζ^{th} voxel [m^3];

and l is the path length of a photon packet through the ζ^{th} voxel [m].

The majority of the chapters in the thesis make use of Eq. (1.43) or modified versions of it as the main scored quantity, e.g. to determine absorbed energy.

Other common scored quantities are the exit location of a photon, the wavelength of an exiting photon or the distribution of photon packet absorption.

1.2.4 Code Details

The preceding section gave an overview of the algorithm and how it works. This section describes the various implementation details such as how the code is parallelised.

Code

All code in this thesis is written in modern Fortran[¶]. All subroutines and functions are contained in modules (with the exception of the main program—main.f90). This is done in order to be able to “hide” data from subroutines and functions, and to arrange the code that relates to other parts of the code in the same file. Having the code in modules also allows the use of runtime allocation of memory for arrays. This enables the user to specify the size of arrays depending on the need of the user for the problem at hand.

Modules are classified into three different types: data, routines and dependencies. Data modules are modules that contain no function or routines, but store variables that can be accessed anywhere in the program when required. Routine modules contain the subroutines and functions

[¶]modern Fortran is considered anything past Fortran 95 [50].

used in the code. Finally dependency modules are the modules that have not been written by me, and thus the code depends upon them in order to run.

Figure 1.12 show the relationship between the various modules, for a basic version of the MCRT as described in [MCRT Algorithm](#).

Using Fig. 1.12 as a reference each module contains:

`mcpolar.f90` is the entry point of the code. It calls all other subroutines and functions, as well as setting up various variables and printing progress.

`ch_opt` is the module where the optical properties are set or changed.

`gridset_mod` is where the optical properties grid and voxel walls are set.

`subs` contains general purpose routines that are used in various different parts of the code.

`writer_mod` contains routines that write out the results of the simulation.

`inttau2` is the module that contains the routines that propagate the photon through the voxel grid.

`sourceph_mod` contains the routines that initialise the photon position and direction.

`stokes_mod` contains the routine that calculates the scattering direction after a scattering event.

`iarray` is a data module that contains all the arrays in the code.

`constants` is a data module that contains all the constants and filepaths needed in the code.

`ieee_arithmetic` is an external dependency that gives various arithmetic checking routines such as `is_nan()`.

`vector_class` is a module that contains the vector type, and all its associated operations such as cross and dot products of vectors.

`photon_vars` is a data module that contains the data pertaining to each photon, such as wave length or energy.

Finally, `opt_prop` contains the data about the current optical properties such as the albedo, and absorption coefficient.

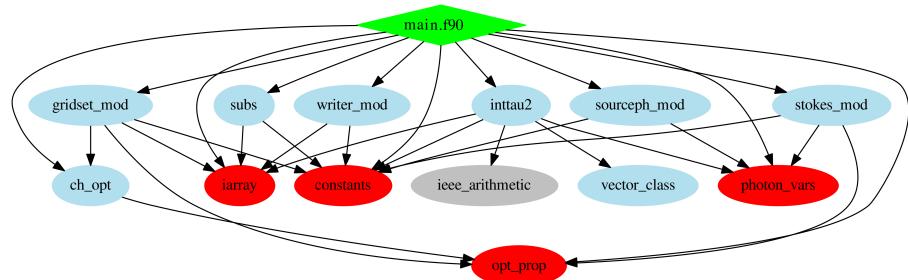


Figure 1.12: Source code hierarchy, showing the relationship between different modules. Green is the entry point for the simulation. Red are the data modules, light blue are the routine modules, and grey are the external dependencies.

Parallelisation of the MCRT Algorithm

As mentioned in the previous sections, MCRT can be computationally intensive, especially when dealing with highly scattering mediums. Fluorescence can also cause simulations times to dras-

tically increase as photons are no longer “killed” off, but rather re-emitted at a new wavelength. Other optical processes such as Raman scattering are highly unlikely events, which again can lead to a dramatic increase in simulation times, as many photons are required to be simulated in order to get “good” statistics.

Fortunately MCRT is classed as an “embarrassingly parallel” problem^{||}. This means that it is trivial to parallelise in comparison to other algorithms. The reason that MCRT is classed as “embarrassingly parallel”, is that the algorithm can be split up onto separate processors, with little need for communication between them. In reality this means that n copies of the algorithm can run on n cores in a processor, with communication taking place at the start and end of each simulation run.

All the code in this thesis is parallelised using Message-passing interface (MPI) [52, 53], with the only communication taking place at the end, where the results are collated on to all processes. The one exception to this is in Chapter 2, where the heat diffusion calculation needs communication between the processes during the calculation.

The parallel efficiency of a code depends on the problem, and the number of photon packets run. To determine the speedup of a given problem Amdahl’s law is used [54]:

$$speedup = \frac{1}{(1 - P) + P/N} \quad (1.44)$$

Where P is the fraction of the code that is parallel, and N is the number of cores the code is run on. The consequence of Amdahl’s law is as N tends to infinity the speedup tends to a maximum:

$$speedup_{max} = \frac{1}{1 - P} \quad (1.45)$$

The value of P varies from problem to problem, and the number of photon packets run. Figure 1.13 shows the results of the profiling of the code, for various numbers of cores. This test consisted of running the same number of photons, in a highly scattering medium of size 2 cm^3 . This yielded a P of 0.999010 ± 0.000045 , and a maximum speedup of 1010.1.

There are other ways the code could be parallelised, including task farms and domain decomposition [55]. However these methods are more involved to set up and validate, so the simplest approach was taken.

^{||}However this is not true for all MCRT applications. For example, using the Bjorkman & Wood [51] immediate temperature corrections method, turns MCRT into a different class of parallel problem [11].

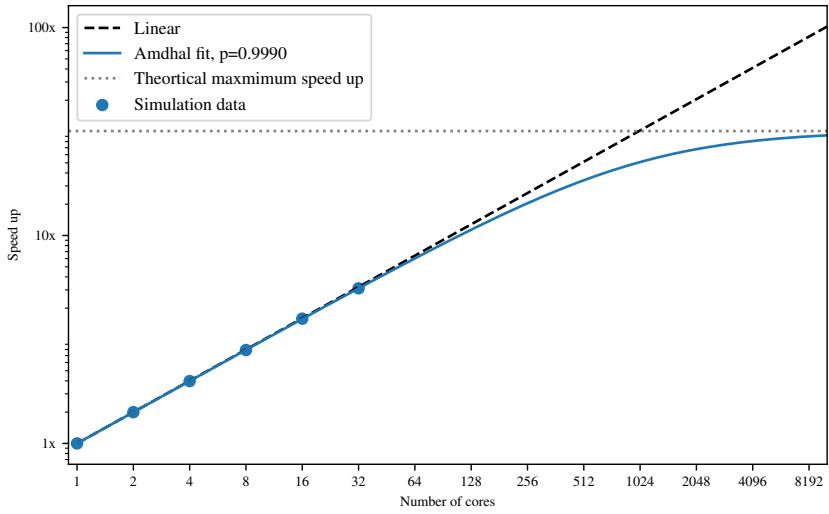


Figure 1.13: Performance of the parallelisation of the MCRT code using MPI.

1.3 Validation of MCRT Code

As the Monte Carlo method is an algorithm that depends upon random numbers, it is sometimes hard to ensure the correct result is obtained. Or to put it another way:

“Monte Carlo is easy to do wrong!” G.W. Collins III [56]

Thus the code has to be validated against various theoretical/experimental and other simulations, to determine whether the results are correct.

The main benchmark of the MCRT code, is a comparison against an expression for fluence as a function of depth [57]. This expression has also been fitted to by other MCRT simulations [58].

$$\Psi(z) = \Psi_0(C_1 e^{-k_1 z/\delta} - C_2 e^{-k_2 z/\delta}) \quad (1.46)$$

Where:

$\Psi(z)$ is the penetration of the excitation light, or equivalently the fluence rate [$W\text{ cm}^{-2}$];

Ψ_0 is a normalisation constant [$W\text{ cm}^{-2}$];

C_n and k_n are fitted coefficients [-];

and δ is the optical penetration depth, defined as $\delta = 1/\sqrt{3\mu_a(\mu_a + \mu_s(1-g))}$, [cm].

Jacques *et al.*, in their simulation used two different wavelengths, 420 nm and 630 nm. The medium in the simulation is a infinitely wide slab with a depth of 1 cm, with uniform optical properties. The medium has a refractive index of 1.38. The g value is in the range 0.7 – 0.9, and the optical properties are as in Table 1.1.

Wavelength/nm	Absorption		Scattering		Penetration			δ/cm
	μ_a/cm^{-1}	$\mu_s(1-g)/\text{cm}^{-1}$	C1	k1	C2	k2		
420	1.8	82	5.76	1.00	1.31	10.2	0.047	
630	0.23	21	6.27	1.00	1.18	14.4	0.261	

Table 1.1: Table of optical properties and determined coefficients from Jacques *et al.* [58].

Using these values Jacques *et al.* calculated values for C_1 , C_2 , k_1 and k_2 using their MCRT code. The above optical properties and medium dimensions^{**} are recreated in the code and a value of 0.9 was chosen for g . 8 million photons were run for the simulation. This yielded the result as in Fig. 1.14.

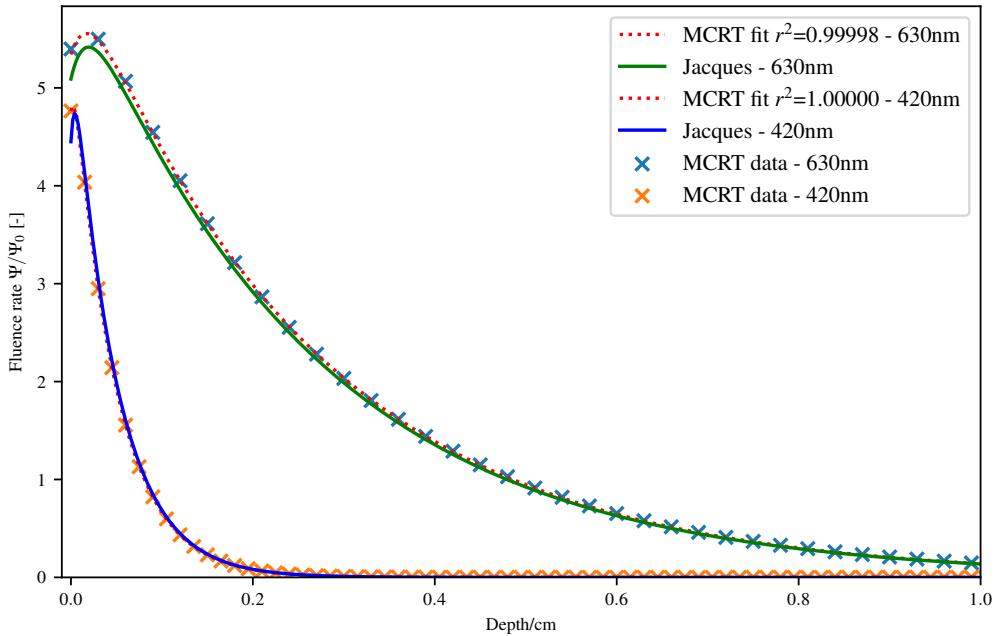


Figure 1.14: Figure shows the fluence as a function of depth. Figure also shows comparison to the Jacques MCRT simulation and the MCRT as described in this chapter.

Fitting Eq. (1.46) to the data calculated by our MCRT code for 630 nm, gave: $C_1 = 6.425$, $C_2 = 1.083$, $k_1 = 1.0$, and $k_2 = 12.966$. For 420 nm gave: $C_1 = 5.600$, $C_2 = 0.838$, $k_1 = 1.003$, and $k_2 = 9.846$. These are in good agreement (with in code differences) with Jacques *et al.* results.

1.4 Conclusion

The Monte Carlo method relies on generating random numbers in order to calculate unknown variables in problems. The Monte Carlo method can be used to calculate radiation transport in order to numerically solve the RTE. MCRT is the most flexible of the methods available to solve the RTE, allowing arbitrary geometries, light sources and turbid media. MCRT also allows the inclusion of various microphysics such as polarisation, Raman scattering and fluorescence. However the MCRT method does have the downside, that for some problems, many photon packets must be run in order to achieve a good signal to noise ratio. Though this is becoming less of an issue due to increased computational available capacity and MCRT being classed as an “embarrassingly parallel” problem, allowing it to be easily parallelised.

^{**}The infinitely wide slab is implemented so that when a photon leaves the one of the sides of the voxel grid, it is moved to the other side of the grid, retaining its original direction vectors.

The MCRT algorithm as described in this chapter forms the basis for this thesis. Each subsequent chapter builds upon the algorithm in order to solve new problems.

Chapter 2

Computational Modelling of Tissue Ablation

2.1 Introduction and Background

This chapter uses MCRT techniques coupled to a heat transfer simulation, to study the thermal damage to tissue due to a laser, with its power spread over many beams to leave viable tissue around zones of damaged/necrotic tissue [59]. This class of laser is called a fractionated ablative laser. This chapter presents experimental work carried out on porcine tissue by our collaborators at the University of Dundee and the photobiology department at Ninewells hospital, along side my computational model of tissue ablation.

Ablative lasers are used in a wide variety of medical procedures including: coagulating scalpels, port wine stain removal, tattoo removal, hair removal, and skin rejuvenation [60–64]. One class of laser used in these procedures are ablative lasers. Ablative lasers are usually high powered lasers ($>30\text{ W}$) targeted at a specific chromophore in the skin, to partially or fully remove layers of skin. These types of lasers are commonly used for aesthetic procedures such as: skin rejuvenation [64], and removal of various diseases such as Rhinophyma [65] or lesions/nodules [66]. Ablative lasers have also been recently investigated as a means of better drug penetration into the skin for various therapies such as photo-dynamic therapy (PDT). The ablative laser “drills” holes in the skin, which allows topical treatments to better diffuse into the skin [67].

One downside to using lasers to remove tissue, is that unlike a scalpel where the surgeon has full control of the depth of the incision, ablative lasers are not as predictable. Lasers can cause thermal damage to the surrounding areas, leading to potentially unwanted effects, though some applications of ablative lasers utilise the thermal damage, particularly aesthetic procedures [68].

Currently the only reliable method to measure the depth of the ablative holes, is via a biopsy, which is an invasive procedure. In this work an optical coherence tomography (OCT) system is used to measure the ablative crater non-invasively *in-vivo*. The OCT measurements are then compared to a computational model developed as part of this project. It is hoped this computational model could be used to predict the depth of the ablative crater when using a certain laser power for various different applications such as: laser assisted drug delivery, and various cosmetic applications.

2.2 Methods

To replicate the experimental work *in silico*, the numerical model has three main portions. The first is the MCRT code that models light transport through tissue so that we can calculate the laser energy deposited as a function of time and space. The second, a finite difference method (FDM) which is used to calculate the heat diffusion within the tissue due to the absorbed laser energy. Finally, a tissue damage model to track the tissue damage caused by the laser. All these individual functions are connected together to create a full numerical model.

2.2.1 Monte Carlo radiation transport (MCRT)

MCRT is used here to calculate the energy deposited by the laser. This is then passed to the heat transport simulation, which calculates the heat diffusion in the medium. The algorithm for the three coupled simulations is presented in Fig. 2.1.

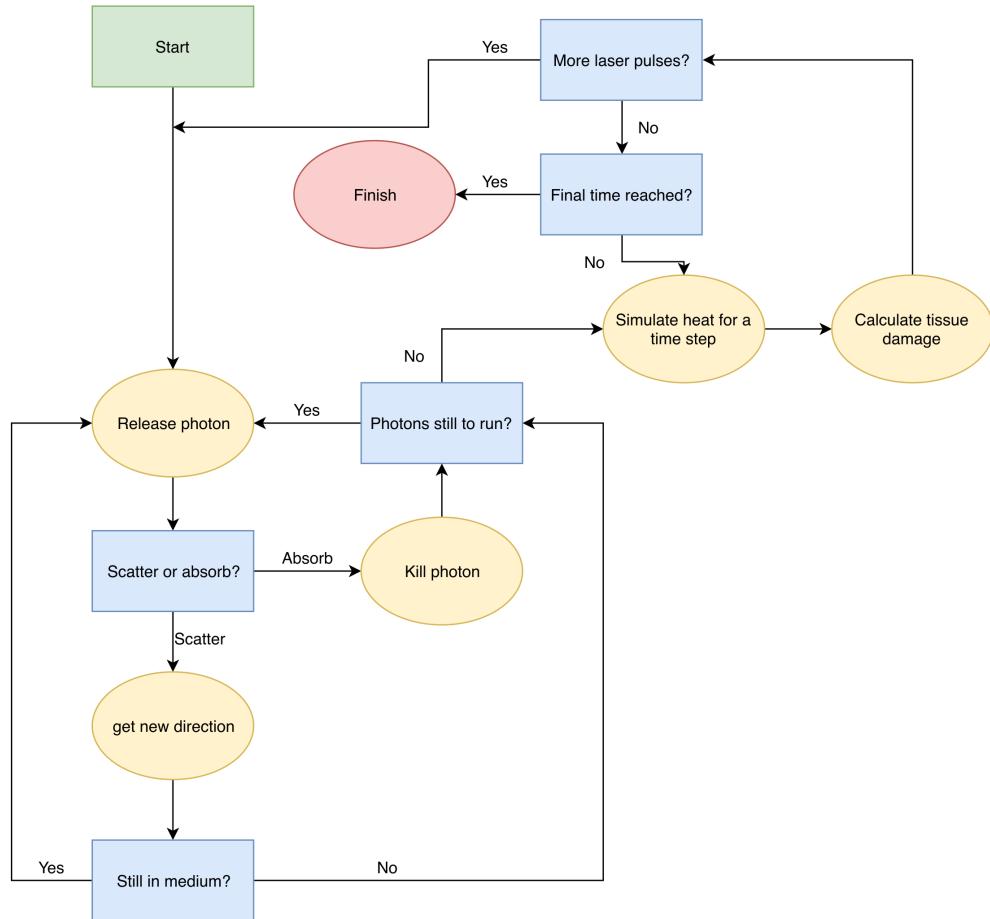


Figure 2.1: Flowchart of the tissue ablation algorithm.

The MCRT algorithm is largely the same as described in Chapter 1, with some important adjustments.

The first adjustment is that the path length counter for fluence is changed to track absorbed energy. This is achieved by multiplying the pathlength in a voxel by the absorption coefficient of that voxel. Figure 2.2 show this process graphically, and Equation (2.34) shows the mathematical expression:

$$E_i^{abs} = \frac{P}{NV_i} \sum \mu_{a,i} s \quad (2.1)$$

Where:

- E_i^{abs} is the energy absorbed in the i^{th} voxel [$J s^{-1} m^{-3}$];
- P is power [W];
- N is the number of packets, representing a power, P;
- V_i is the volume of the i^{th} voxel [m^{-3}];
- $\mu_{a,i}$ is the absorption coefficient of the i^{th} voxel [cm^{-1}];
- and s is the pathlength of a packet through the i^{th} voxel [cm].

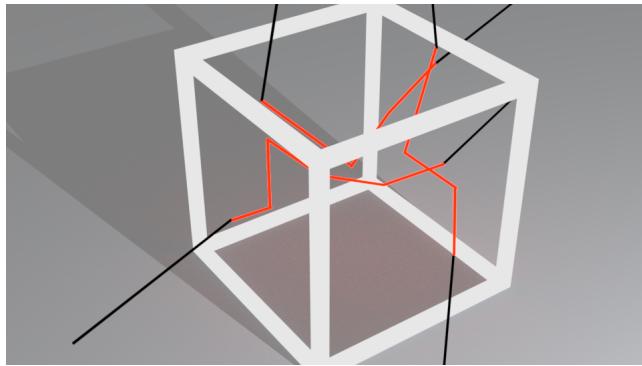


Figure 2.2: Red lines are packet paths within a voxel. Black lines packet paths out with the voxel. Red packet paths, weighted by μ_a , are summed up to calculate the absorbed energy within each voxel.

This grid of absorbed energy is then passed to the heat transport portion of the simulation, so that the heat diffusion in the porcine tissue can be calculated.

The next adjustment to the MCRT algorithm, is that the MCRT algorithm is run for every heat simulation time step, as the medium could change at every time step due to the optical, and thermal properties changing as a function of tissue damage.

Finally, to match the experiment undertaken the medium and laser for the *in-silico* experiments must match the practical experiments. As the laser used in the experiments emits an infra-red wavelength ($10.6 \mu m$), the optical properties are dominated by the water content of the tissue. Due to this it is assumed that there is just absorption in the medium, with no scattering. Further discussion can be found in Section 2.3.1. The laser in some of the *in silico* modelling, has multiple beams and the source photon packet routine is adjusted to accommodate this when needed.

2.2.2 Heat Transport

The diffusion of heat can be modelled using the heat equation (Eq. (2.2)), which is derived from Fourier's law and the principle of conservation of energy [69]. The standard heat equation is a partial differential equation of the parabolic form. Solutions and analytical methods are readily

available for lower dimensions (i.e. 1D heat diffusion), but for higher dimensions, numerical models must be used for all except the simplest problems. The simplest form of the heat equation is shown below:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) + \dot{q} \quad (2.2)$$

Where:

- $T(x, y, z, t)$ is the temperature as a function of time and space [K];
- κ is the thermal conductivity [$Wm^{-1}K^{-1}$];
- ρ is the density [Kgm^{-3}];
- c_p the specific heat capacity [JK^{-1}];
- $\dot{q}(x, y, z, t)$ is the source/sink term as a function of time and space [Wm^{-3}].

Equation (2.2) is for a homogeneous system where the thermal properties do not change as a function of time, space and temperature. However in order to model a moving ablation front the nonlinear heat equation must be used, where the thermal properties can be a function of time, space and temperature (Eq. (2.3)).

$$\frac{\partial T}{\partial t} = \frac{1}{(\rho c_p)_\xi} (\nabla k_\xi T + k_\xi \nabla^2 T) + \dot{q}, \quad \text{where } \xi = (i, j, k) \quad (2.3)$$

Included in Eq. (2.3) is a source and sink term, \dot{q} to allow the modelling of heat loss/gain from external sources/sinks. The heat source in this simulation is due to the laser, and it is assumed that the only loss of heat to the surrounding medium is via conduction.

The medium is considered to be at a constant temperature of 5°C, as the porcine skin was kept cooled prior to experimental work and the simulation volume is smaller than the porcine tissue samples.

Where:

- h is the heat transfer coefficient [$Wm^{-2}K$];
- A is the area of the grid element, that is radiating/convicting heat away [m^{-2}];
- and T , and T_∞ are the temperature in a voxel and the surrounding medium temperature respectively [K].

As Eq. (2.3) is generally hard to solve in arbitrary geometries with complex boundary conditions a numerical method is employed to solve it. The numerical method employed is a FDM, derived from the Taylor series, see Eq. (2.4).

A function $f(x)$ is discretised onto a grid with N nodes a distance Δx apart. Equation (2.4) is then truncated and rearranged and it is assumed that the remainder term R_1 is sufficiently small enough, to yield an approximation for the first derivative of a function $f(x)$ at a point $x_0 + \Delta x$, see Eq. (2.5). Equation (2.5) is the so called forward difference, due to it using a point in the ‘forward’ direction. The “backward” and central difference terms can be calculated by using a node at $x_0 - \Delta x$ for the backward difference Eq. (2.6b). The central difference (Eq. (2.6c)) is an average of the forward and backwards differences. Expressions can also be given for the 2nd derivatives for backward, forward and central (forward and backward 2nd order equations omitted for brevity) Eq. (2.6d).

$$f(x_0 + \Delta x) = f(x_0) + \frac{f'(x_0)}{1!} \Delta x + \frac{f''(x_0)}{2!} \Delta x^2 + \dots + \frac{f^{(n)}(x_0)}{n!} \Delta x^n + R_n(x) \quad (2.4)$$

$$f'(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (2.5)$$

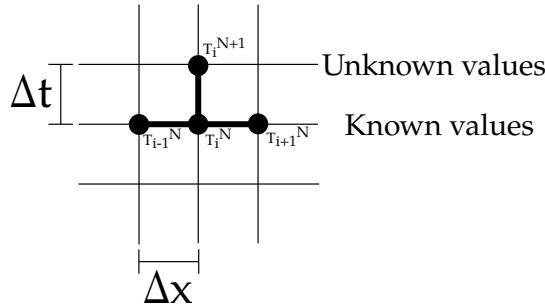


Figure 2.3: Finite difference method stencil for simple explicit scheme

$$\frac{df}{dx} = \frac{f_{i+1} - f_i}{\Delta x} \quad (\text{forward}) \quad (2.6a)$$

$$\frac{df}{dx} = \frac{f_i - f_{i-1}}{\Delta x} \quad (\text{backward}) \quad (2.6b)$$

$$\frac{df}{dx} = \frac{f_{i+1} - f_{i-1}}{2\Delta x} \quad (\text{central}) \quad (2.6c)$$

$$\frac{d^2f}{dx^2} = \frac{f_{i-1} - 2f_i + f_{i+1}}{\Delta x^2} \quad (\text{central}) \quad (2.6d)$$

Thus the linear heat equation Eq. (2.2), in 1D, taking a 1st order forward time derivative, and a 2nd order central spatial derivative gives:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i-1}^n - T_i^n + T_{i+1}^n}{\Delta x^2} + \frac{\dot{q}}{\rho c_p} \quad (2.7a)$$

$$T_i^{n+1} = \alpha \Delta t \frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} + \frac{\Delta t \dot{q}}{\rho c_p} \quad (2.7b)$$

Where $\alpha = \frac{\kappa}{\rho c}$.

Equation (2.7b) is called the “simple explicit form of finite-difference approximation” [70]. Figure 2.3 shows the “stencil” of this scheme, where there are three known points at time N , and just one unknown at time $N+1$. There are various other schemes that can be used to calculate the temperature at the next time step. However a simple explicit scheme is used here, due to its ease of implementation despite there being a constraint on the stability in comparison to an implicit method. This method is also easily scaled up to 3D with little difficulty.

For the more complicated nonlinear heat equation there is a possibility that the medium is not continuously smooth between nodes, in terms of optical and thermal properties. The two easiest methods [70] of achieving this are: (1), lag the value behind by one step, i.e $c_p^{n+1} = c_p^n$. (2), average κ , ρ , and c_p using a half difference scheme where the thermal property used in the calculation is the thermal property half way between two nodes, i.e the average of the two nodes:

*For brevity $f(x_0 + \Delta x)$ is defined as f_{i+1} , and $f(x_0 - \Delta x)$ as f_{i-1} , etc.

$$\kappa^\pm = \frac{\kappa_i + \kappa_{i\pm 1}}{2} \quad (2.8)$$

$$\rho^\pm = \frac{\rho_i + \rho_{i\pm 1}}{2} \quad (2.9)$$

$$c_p^\pm = \frac{c_{p,i} + c_{p,i\pm 1}}{2} \quad (2.10)$$

Thus for the simple 1D case as in Eq. (2.7b), the thermal properties are averaged between nodes when computing the coefficients of the temperature nodes, and lag the thermal properties when adding the heat from the laser:

$$T^{N+1} = \Delta t (AT_{i-1}^N - 2BT_i^N + DT_{i+1}^N) + T_i^N + \frac{\Delta t}{\rho c_p} q_L \quad (2.11)$$

Where (in the x direction):

$$\begin{aligned} A &= \frac{\kappa^-}{\rho^- c_p^- 2\Delta x^2} \\ B &= \frac{\kappa^+}{\rho^+ c_p^+ 2\Delta x^2} \\ D &= \frac{(A + B)}{2} \end{aligned} \quad (2.12)$$

Equation (2.11) is straightforward to generalise to higher dimensions. The 3D case gives:

$$U_{xx} = (AT_{i-1,j,k}^N - 2BT_{i,j,k}^N + DT_{i+1,j,k}^N) \quad (2.13)$$

$$U_{yy} = (AT_{i,j-1,k}^N - 2BT_{i,j,k}^N + DT_{i,j+1,k}^N) \quad (2.14)$$

$$U_{zz} = (AT_{i,j,k-1}^N - 2BT_{i,j,k}^N + DT_{i,j,k+1}^N) \quad (2.15)$$

$$T_{i,j,k}^{N+1} = \Delta t (U_{xx} + U_{yy} + U_{zz}) + T_{i,j,k}^N + \frac{\Delta t}{\rho c_p} q_L \quad (2.16)$$

Where:

$T_{i,j,k}^{N+1}$ is the new temperature at node i, j, k [K];

$T_{i,j,k}^N$ is the temperature at node i, j, k at the current time step [K];

α is the thermal diffusivity [$m^2 s^{-1}$];

κ is the thermal conductivity [W/mK];

Δx etc. is the size of the grid element in the p^{th} direction [m];

and A, B, D are the coefficients in their respective dimension (Eq. (2.12)).

Equation (2.16) gives the full numerical solution to the nonlinear heat equation with a laser heat source. This will allow the calculation of the heat diffusion in the porcine tissue due to laser heating.

As the laser used in the experimental work, operates in a pulsed mode, this is accounted for this in the simulation. The laser pulse shape is a triangular pulse, with the peak power, P_{peak} , and pulse length, τ [71]. In the heat simulation there has to be an additional variable in the term $laserOn(t) \cdot \frac{\alpha \Delta t}{\kappa} q_L$ in Eq. (2.16). This additional variable, $laserOn(t)$, is a boolean value and a function of time, which is defined as:

$$laserOn = \begin{cases} 1, & \text{Laser on} \\ 0, & \text{Laser off.} \end{cases}$$

In the instance where there is a train of laser pulses, the laser is turned on and off based upon the pulse frequency.

Due to a simple explicit FDM being used, the time step is constrained in order to make the solution stable. For a cubic 3D FDM without prescribed flux boundary conditions, this yields the constraint: $\Delta t \leq \frac{1}{\delta_\alpha}$ where $\delta = \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}$. Along with this time constraint, the pulse length of the laser also has to be considered. If the time step of the heat simulation is too large it will not account for the heat deposited by the laser. Thus, the timestep has to be at least an order of magnitude smaller than the shortest laser pulse.

As the time step is small, and the grid resolution large, the resultant simulation is slow. Thus the code has been fully parallelised to improve performance. Both the MCRT and heat simulation are independently parallelised.

Parallelisation of the heat simulation is more involved than the “embarrassingly parallel” class of problems where MCRT belongs. This is due to the heat simulation being dependent on neighbouring nodes to update the temperature at the current node. Thus if the medium were to be split up on to separate cores, there would have to be communication between the cores, in order for the simulation to be completed successfully. Therefore it is not possible to take the ‘easy’ route of running the simulation concurrently N times and collating the result at the end of all the simulations.

The heat simulation is parallelised using a technique called “halo swapping”. This involves splitting up the computational domain (see Fig. 2.4), in this case the tissue medium, and doing the calculations on each domain on a separate core. The “halo swapping” comes in when cores need to communicate with each other about updating their boundary temperature nodes (see Fig. 2.5).

Figure 2.6 shows the speed up gained from using the technique. The “halo swapping” technique is efficient for situations where the computational domain can be split up with large ‘chunks’ being calculated on each core. However if the computational domain is small, and the number of cores large then bottlenecks occur due to too much communication between cores taking place. Thus to efficiently use ‘halo swapping’ careful thought has to be given to the size of the computational domain, and the number of cores running the simulation. Evidence of this bottlenecking can be seen in Fig. 2.6 for problems where the size of the grid, in voxels, is 40^3 and 24^3 . These problems also show superlinear speed up, for certain number of cores. This is not unfeasible, due to a number of reasons, in particular the underlying computer architecture [72].

After one time step of the heat simulation has been completed, the temperature grid is passed to the tissue damage portion of the simulation to calculate the tissue damage that may have accrued during the heat simulation time step.

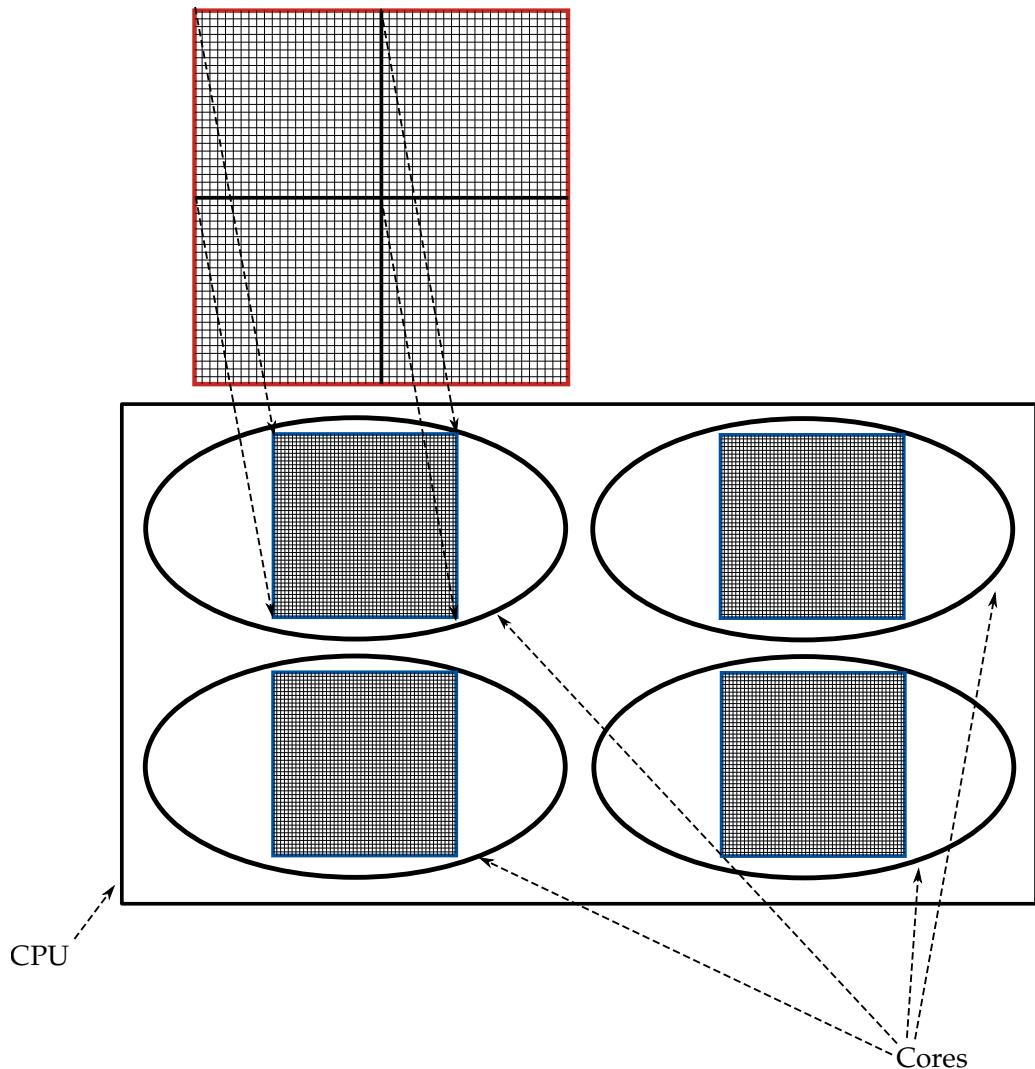


Figure 2.4: Computational domain decomposition. Total computational domain (red outline) is evenly divided between cores in the CPU. This is done via layers of the domain in the z direction. Information is passed to/from cores via the “halo swap” process (see Fig. 2.5).

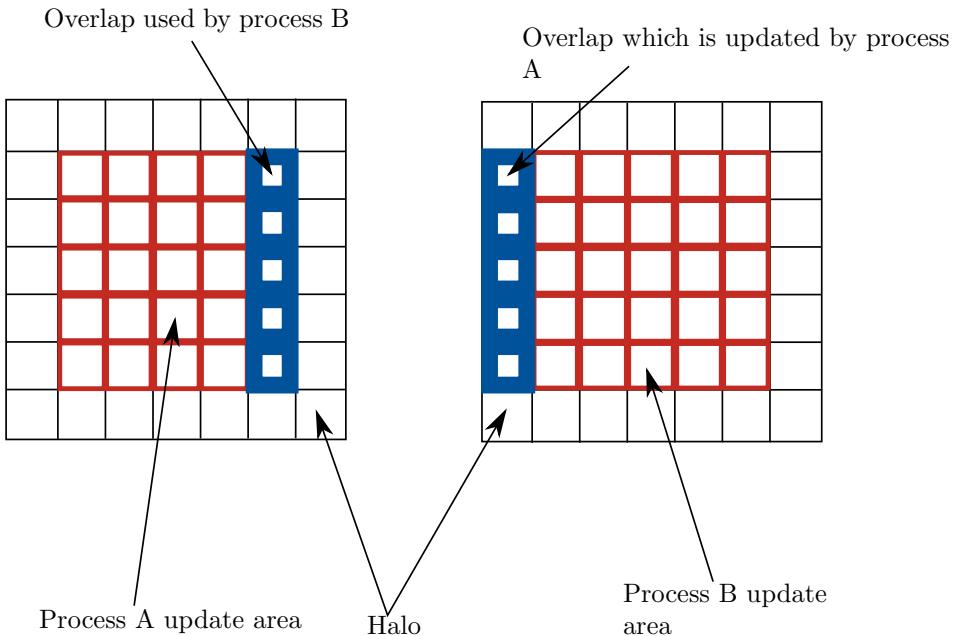


Figure 2.5: Halo swapping. Process A updates the area in red and blue on the left. It updates the blue area which is sent to process B as B's "halo". Process B cannot update its own halo, but rather updates the halo for process A.

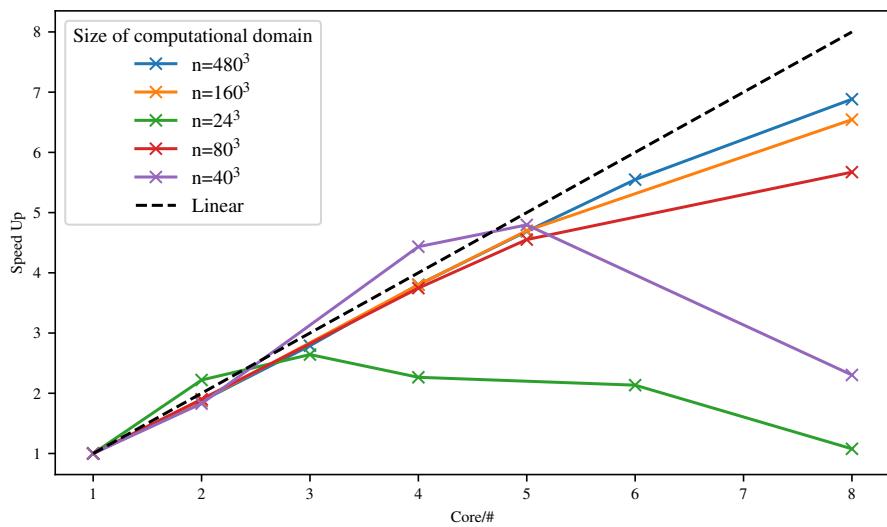


Figure 2.6: Figure show the speed up gained by parallelisation of the heat simulation using the "halo" swapping technique, for various sizes of computational domain (voxels). Data taken from a Intel Xeon E3-1245 v5, 8 cores @ 3.5GHz machine.

2.2.3 Tissue Damage

Introduction

The final portion of the simulation is the tissue damage model. To be able to model damage to the tissue, the process tissue undergoes upon heating due to the laser needs to be able to be described.

When the laser is turned on, the temperature starts to rise within the tissue due to the absorption of photons by the tissue. The temperature rise causes damage to the tissue when above a threshold temperature, T_d , approximately 43°C [73]. From the temperature, T_d , four main areas of tissue damage are defined:

$$T = \begin{cases} \text{coagulation}, & T_d \leq T \leq 100^\circ\text{C} \\ \text{water boils}, & T = 100^\circ\text{C} \\ \text{carbonisation}, & 100^\circ\text{C} \leq T \leq T_a \\ \text{ablation}, & T = T_a. \end{cases} \quad (2.17)$$

The area of tissue damage termed “coagulation” is a multifaceted process. At 43°C - 50°C , bonds break within cell membranes, causing ruptures, and some cell death [73, 74]. This process is usually termed *hyperthermia*. Around 50°C , enzyme activity decreases, cells become immobile, and various cell repair mechanisms are disabled, leading to increased cell death. When temperatures exceed 60°C , proteins become denatured. Thermal denaturation is a structural and functional change in a protein due to the heating it undergoes. This means they change from a highly organised structure with specific purposes, to disorganised structures with little to no function at all [75].

The next stage in the tissue damage process is the vaporisation of water. As the temperature of the tissue starts to approach 100°C (at 1 atm), water starts to vaporise. If the vaporised water cannot escape the tissue it forms steam vacuoles, small pockets of steam. These vacuoles can easily been seen when viewing tissue samples after tissue has been treated with a high powered laser (see Fig. 2.7). In certain conditions these steam pockets can explode [76].

The third stage of tissue damage is carbonisation of the tissue. This occurs when most of the water has boiled off, leaving the remaining tissue to heat up and reduce to its elemental carbon form. This carbonisation of tissue, when it occurs, is generally only a thin layer of 5-20 μm [73, 77].

The final stage of tissue damage is the removal of the remaining tissue, i.e tissue ablation. There is no agreement in the literature how tissue undergoes ablation with a number of methods proposed. The three main methods are: photochemical, thermal, and explosive [78–80]. Photochemical ablation is when the energy of a photon from the irradiating laser, is sufficient enough that it excites the electronic state of the tissues molecules into an anti-boding state, leading to broken bonds and conversion from electronic energy into kinetic energy, and thus ablation. Thermal ablation is where tissue is heated sufficiently so that tissue vaporisation takes place. Finally, explosive ablation is an extreme version of thermal ablation. Explosive ablation occurs when large amounts of energy is deposited in a small time scale, so that none of the energy can thermally diffuse away, resulting in explosive ablation. Photochemical ablation, is usually applied to UV laser ablation, whereas thermal and explosive ablation regimes are the more likely candidates for IR ablation which is considered here.

The theoretical models behind explosive and thermal ablation models are also not well understood, with many models proposed in order to try to and explain experimental results. These models range from heuristic models to sophisticated models that relate the underlying physical mechanisms to ablation damage. The two main heuristic models are: the blow off model, and

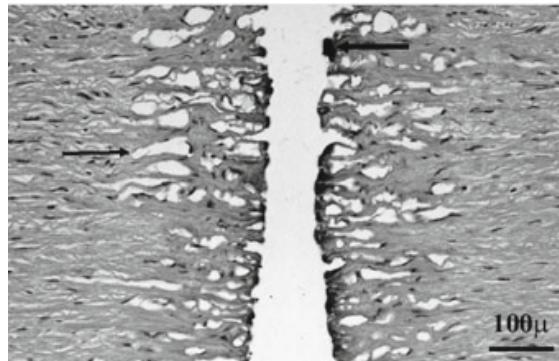


Figure 2.7: Ablation of a dog aorta, as viewed under a microscope. Steam vacuoles are clearly visible either side of the ablation area. Carbonisation is also evident at the edges of the ablation fronts. Adapted from [73].

the steady state model. The blow off model, assumes that there is thermal confinement (i.e no propagation of heat in time t), that material is removed after the laser irradiation. There is a radiant threshold that has to be met in order to ablate material, and that Beer-Lambert's law describes the spatial distribution of light. For laser pulses of < 10 ns, these conditions are normally met. However for lasers with pulse length larger than this, these conditions are not usually met [81–83].

The steady state heuristic model, assumes that the pulse length is of the order of ms or larger, that material starts to be removed shortly after laser irradiation begins, and that some radiant threshold exists in order for ablation to begin. The steady state model also assumes that a fixed energy is required to remove a unit of tissue [81]. However this does not always hold, as there are many circumstances where there is no one fixed energy, but rather many energies (due to various phase changes) that must be met in order for ablation to occur. There are also many other sophisticated models, that try to describe what happens physically when ablation occurs [84–86].

Due to the above mentioned reasons, there is no defined ablation temperature. The literature however, does suggest that it takes place when the tissue temperature is between 177 °C and 500 °C [85, 87, 88].

To model all these tissue damage processes the tissue damage model is split into two sections: “physical” damage and coagulation damage. “Physical” damage changes the tissue optical and thermal properties. Coagulation damage has no effect on the tissue’s bulk optical or thermal properties.

Modelling coagulation damage

With the description of the various process that tissue undergoes during ablation, a numerical model of these processes can be created. First, in order to model the full extent of the damage done under 100°C, i.e in the coagulation regime, the Arrhenius damage model is used. The Arrhenius damage model was originally used as a kinetic model of reaction products in chemistry [89]. It has since been adapted by various authors for modelling tissue damage, and is the *de facto* standard [90, 91]. These authors and various others, adapted this model by fitting Eq. (2.18) to experimental data for burn damage. The two parameters fitted are A , the frequency factor, and ΔE , the activation energy.

$$\Omega(t) = \int_{t_i}^{t_f} A e^{(-\frac{\Delta E}{RT})} d\tau \quad (2.18)$$

Where:

- Ω is the damage value [-];
- A is “frequency factor” [s^{-1}];
- ΔE is activation energy [$Jmol^{-1}$];
- R is the universal gas constant [$Jmol^{-1}K^{-1}$];
- T is the temperature [K];
- and t_i and t_f are the initial time and final time at t_{crit} .

It is reported that a value of Ω of 0.53, 1.0, and 10^4 relate to first, second, and third degree burns respectively [92]. The Arrhenius damage model is used to better understand the amount of damage caused by the laser in the non-ablated areas of tissue. Values of $A = 3.1 \times 10^{98}$ and $\Delta E = 6.3 \times 10^5$ are adopted [88, 90, 93].

Modelling physical tissue damage

As tissue mostly consists of water [94] when the temperature of the tissue approaches $100^\circ C$ (at 1 atm), water in the tissue begins to boil off. This acts as a large heat sink for the absorbed laser energy, slowing down the rate of ablation. The energy required to boil the water is $Q_{vapor} = m_v \cdot L_v$, where m_v is the mass of a voxel, and L_v is the latent heat of vaporisation. The energy to boil off the water is provided via the laser and heat diffusing into the voxel:

$$Q_{vapor} = \underbrace{laserOn(t) \cdot \dot{q} \cdot \Delta t \cdot V_{i,j,k}}_{\text{laser heating}} + \underbrace{c \cdot M_{i,j,k} \cdot \Delta T}_{\text{heat diffusion}} \quad (2.19)$$

Where:

- Q_{vapor} is the current energy in Joules that has been used to boil off the water in the voxel [J];
- $laserOn$ is a boolean variable that determine if the laser is on or off [-];
- \dot{q} is the energy absorbed by the voxel due to the laser [Wm^{-3}];
- Δt is the timestep [s];
- $V_{i,j,k}$ is the volume of the voxel labelled i, j, k [m^3];
- c is the heat capacity of the voxel [JK^{-1}];
- $M_{i,j,k}$ is the mass of the voxel labelled i, j, k [kg];
- and ΔT is the change in temperature the voxel would undergo, if the water was not boiling off.

As water boils off, the water content of each voxel changes. This affects the absorption coefficient, density, thermal conductivity, and heat capacity. Each of these vary with water content per voxel [95];

$$W = W_{init} - \left(W_{init} \cdot \left(\frac{Q_{current}}{Q_{vaporisation}} \right) \right) \quad (2.20)$$

$$\rho = \frac{1000}{W + 0.649 \cdot P} \quad (2.21)$$

$$c_p = 4.2 \cdot 10^3 \cdot W + 1.09 \cdot 10^3 \cdot P \quad (2.22)$$

$$\kappa = \rho \cdot (6.28 \cdot 10^{-4} \cdot W + 1.17 \cdot 10^{-4} \cdot P) \quad (2.23)$$

$$\mu_a = W \cdot \mu_{water} + \mu_{protein} \quad (2.24)$$

$$(2.25)$$

Where:

W is the water content (i.e $W = 0.7$ equates to 70% water content);

W_{init} is the initial water content;

$Q_{current}$ is the total energy absorbed by the i^{th} voxel since the temperature reached 100°C [J];

P is the protein content (i.e $P = 1.0 - W$);

κ is the Thermal conductivity [$Wm^{-1}K^{-1}$];

c_p is the heat capacity [$Jkg^{-1}K^{-1}$];

and μ_a is the total absorption coefficient, and μ_{water} and $\mu_{protein}$ are the absorption coefficients of water and protein respectively.

T_a is defined as occurring between 177 and 500 °C [85, 87, 88]. At T_a the tissue is removed and the thermal, optical, and physical properties set to that of air.

The updated damaged tissue structure is then fed back to the MCRT model and the whole process repeats until the predefined time limit is reached. This whole process of photon propagation, heat diffusion and tissue damage is outlined in Fig. 2.1.

2.2.4 Validation

Heat transport validation

To thoroughly validate the numerical method employed to solve the heat equation, the numerical method is compared against an easily solvable analytical case. The heat equation is solved on a cube, side L , in a surrounding medium of 0°C. The cube is initially at temperature 20°C and the temperature is calculated at various times. Thus the boundary conditions are:

$$T(0, y, z, t) = T(x, 0, z, t) = T(x, y, 0, t) = 0^\circ\text{C} \quad (2.26)$$

$$T(L, y, z, t) = T(x, L, z, t) = T(x, y, L, t) = 0^\circ\text{C} \quad (2.27)$$

The thermal diffusivity (α), density (ρ), and heat capacity (c_p) are all set to 1. This corresponds to a material which has the thermal diffusivity between copper and aluminium [96, 97]. Assuming a separable solution in Cartesian coordinates yields:

$$\begin{aligned} T(x, y, z, t) = & (A_1 \cos(\alpha x) + A_1 \sin(\alpha x)) \cdot \\ & (B_1 \cos(\beta y) + B_1 \sin(\beta y)) \cdot \\ & (C_1 \cos(\gamma z) + C_1 \sin(\gamma z)) \cdot e^{-\alpha \mu^2 t} \end{aligned} \quad (2.28)$$

$$\mu^2 = \alpha^2 + \beta^2 + \gamma^2 \quad (2.29)$$

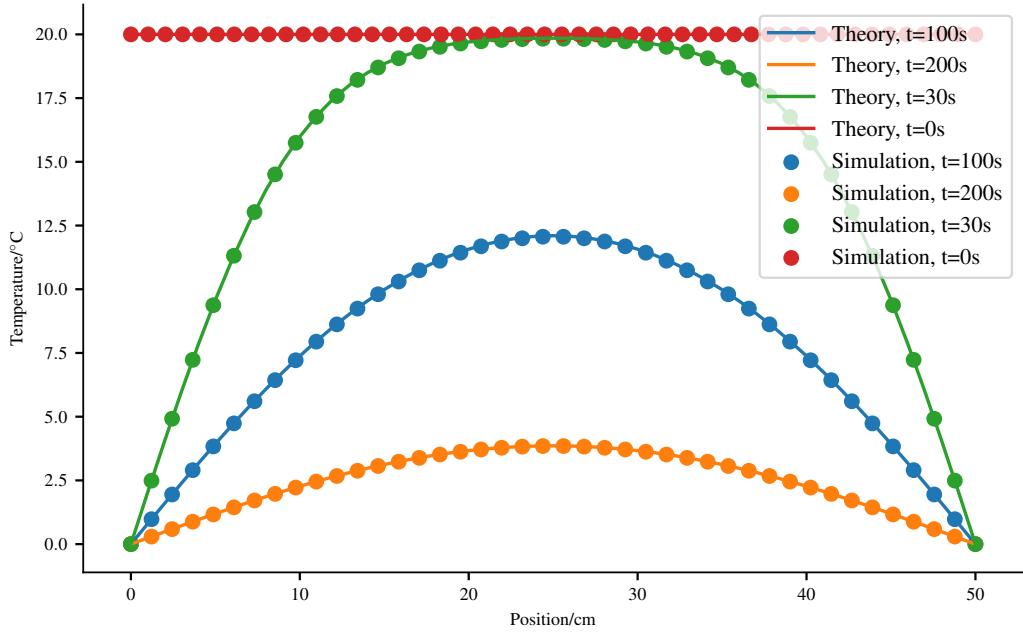


Figure 2.8: Temperature of the cube for various times, comparing between analytical solution and numerical method.

Applying the boundary conditions (Eqs. (2.26) and (2.27)) gives:

$$A_1 = B_1 = C_1 = 0 \text{ and } \alpha = \frac{\pi n}{L}, \beta = \frac{\pi m}{L}, \gamma = \frac{\pi p}{L} \quad (2.30)$$

$$\therefore T_{nmp}(x, y, z, t) = A_{nmp} \cdot \sin\left(\frac{\pi nx}{L}\right) \cdot \sin\left(\frac{\pi my}{L}\right) \cdot \sin\left(\frac{\pi pz}{L}\right) \quad (2.31)$$

This yields the following solution for the heat equation using the principle of superposition, and solving Eq. (2.32) with $f(x, y, z)$ as the initial temperature profile of the cube:

$$A_{nmp} = \frac{8}{L^3} \int_0^L \int_0^L \int_0^L f(x, y, z) \cdot \sin\left(\frac{\pi nx}{L}\right) \cdot \sin\left(\frac{\pi my}{L}\right) \cdot \sin\left(\frac{\pi pz}{L}\right) dx \cdot dy \cdot dz \quad (2.32)$$

$$T(x, y, z, t) = \sum_{n=1,3,\dots}^{\infty} \sum_{m=1,3,\dots}^{\infty} \sum_{p=1,3,\dots}^{\infty} \frac{2368}{\pi^3 nmp} \cdot \sin\left(\frac{\pi nx}{L}\right) \cdot \sin\left(\frac{\pi my}{L}\right) \cdot \sin\left(\frac{\pi pz}{L}\right) \cdot e^{(-\lambda^2 t)} \quad (2.33)$$

Where:

$$\lambda^2 = \alpha\pi^2\left(\frac{n^2}{L^2} + \frac{m^2}{L^2} + \frac{p^2}{L^2}\right);$$

n, m, p are odd integers;

and L is the length of the cube.

A slice through the middle of the cube, $L = 50 \text{ cm}$, yields Fig. 2.8, which shows that the numerical method matches the analytical solution closely.

MCRT & heat transport validation

As a first test of the code, both MCRT and heat simulation, is compared to a simple analytical model of ablation. The simple model of ablation is as: The ablation energy (E_a), is defined as the minimum energy required to raise the temperature of the medium to 100 °C, and then boil off the water in a volume dV , mass M . Thus in one dimension Eq. (2.34), where the symbols have their usual meanings. If the energy for ablation is delivered in a time dt by a laser of intensity ($W\text{cm}^{-2}$), P , this gives Eq. (2.35). Equation (2.35) can be rearranged in order to give an ablation front velocity, Eq. (2.36).

$$E_a = c_p \rho dx \Delta T + L_v \rho dx \quad (2.34)$$

$$P \cdot dt = \rho dx (c_p \Delta T + L_v) \quad (2.35)$$

$$u = \frac{P}{\rho(c_p \Delta T + L_v)} \quad (2.36)$$

Assuming the ablation front moves with constant velocity during the ablation, and using $L_v = 2.53 \cdot 10^6 \text{ Jkg}^{-1}$, $c_p = 4181 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ and the medium is a cube side 2 mm, with a starting temperature is 37 °C with a water content of 70% giving a density of $700 \text{ kg} \cdot \text{m}^{-3}$. For these parameters this gives an ablation velocity, $u \simeq 0.77 \text{ cm} \cdot \text{s}^{-1}$, and a time to ablate through 2 mm of tissue of $\simeq 0.26 \text{ s}$. As the code developed in this chapter simulates the diffusion of heat in a medium due to an incident laser, the expected time to ablate through the same medium should be slightly larger as heat diffuses away from the voxel while it is being heated. When the full heat + MCRT code is used to simulate this experiment, it gives a time, $t \simeq 0.33 \text{ s}$.

2.3 *In silico* results

2.3.1 Introduction

To match the experimental results, an accurate model of the experimental setup *in silico* must be created. However due to computational constraints, such as memory and time available, some approximations to the experimental set-up have to be made. The porcine skin was a large thin slice of the top most layers of the skin. However as the area of interest is where the ablation occurs, initially the porcine skin is modelled as a cuboid, dimensions: $1.1 \times 1.1 \times 0.5 \text{ cm}$. The initial temperature of the porcine skin is assumed to be around 5°, as the tissue was kept on ice or was kept cooled. As mentioned in the previous sections, there are several unknowns in the model: T_a , water content, temperature of air after ablation, and the exact thermal and optical properties of the porcine tissue. Therefore several models are run so that the full parameter space of these unknowns can be explored. Results from these *in silico* experiments are presented in this section along with a comparison of the model to the experimental work carried out in collaboration with the University of Dundee and the Photobiology department at Ninewells hospital.

Optical & thermal properties

The thermal and optical properties of porcine tissue are not known exactly for any given tissue sample. As such the thermal and optical properties used in this section are taken from various literature sources.

The laser used in the experimental work is a CO₂ laser operating at 10.6 μm . This means that the optical properties of the tissue are dominated by water absorption. The laser used in the experiment is the Pixel CO₂ [98]. The Pixel CO₂ laser has a wavelength 10.6 μm which

	Thermal conductivity, κ	Density, ρ	Heat capacity, c
Tissue	$\rho \cdot (6.28 \cdot 10^{-4} \cdot W + 1.17 \cdot 10^{-4} \cdot P)$	$\frac{1000}{W+0.649 \cdot P}$	$4.2 \cdot 10^3 \cdot W + 1.09 \cdot 10^3 \cdot P$
Air	$a e^{-b(T-273.15)} + c$	$\frac{p_{atm}}{R_{spec} T}$	1006

Table 2.1: Optical and thermal properties for porcine tissue and air. W and P are the percentage of water and protein respectively. ρ is the density of the skin, p_{atm} is the pressure of air at 1 atmosphere, and R_{spec} is the gas constant. a , b , and c are constants.

corresponds to an absorption of coefficient in water of $\sim 850 \text{ cm}^{-1}$. As the absorption coefficient is large, it is assumed that scattering is negligible at these wavelengths. Table 2.1 summarises the thermal properties for tissue and air used in the simulations.

The laser was used in “Pixel beam” mode. This means that the laser beam is split into an array of smaller beams. The laser used an array 9×9 of 81 pixel beams, each with a diameter of $250 \mu\text{m}$. The Pixel CO₂ rated laser power is $\sim 70 \text{ W}$.

The laser delivered one single pulse of varying total energy delivered over the range 50 mJ to 400 mJ , in so called “super pulsed mode”. The experiment consisted of ablating the porcine tissue, as a function of energy per “pixel” beam. This was achieved by adjusting the pulse length of the laser, τ , so that the energy per pulse was varied over a range 50 mJ to 400 mJ . The energy range for the laser was kept the same pre and post-upgrade, with the pulse length differing.

Computational speed up:

As discussed in the Section 2.1, the volume of interest is the area around the ablation craters. The volume is $1.1 \text{ cm} \times 1.1 \text{ cm} \times 0.5 \text{ cm}$. However, in order for the simulation to have good resolution of the ablation craters, this volume would require a large number of voxels for the tissue model. This is unfeasible due to: the memory required to store the various counters, grids, and variables, and the time that would be required in order to carry out the computation. Thus the volume of interest is reduced to focus on just one of the ablation craters that is created by the laser (a volume of $0.06 \text{ cm} \times 0.06 \text{ cm} \times 0.18 \text{ cm}$) As a check to ensure that no physical phenomena are omitted by focusing on just one ablation crater, an initial simulation that models the full volume of interest was carried out to investigate the possibility of overlapping craters or other related phenomena. The simulation, as shown in Fig. 2.9, gives reassurance that the shrinking of the volume of interest is a valid approximation to make, as there is no overlap between the separate ablation crater.

2.3.2 Results

Investigating ablation temperature, T_a

Various literature sources report the ablation temperature ranging widely from 177° to 500° [85, 87, 88]. Thus, several models are run over this range in order to establish the T_a which fits the experimental results. Figures 2.10 and 2.11 show how T_a , and beam profile affect the crater depth as a function of pixel beam energy for the CO₂ laser. The data suggests that, a T_a around $T_a = 500^\circ \text{C}$ is appropriate for the studies carried out, the upper limit of T_a from the literature.

Increasing the ablation temperature, has the obvious effect of requiring more energy to be deposited by the laser before ablation takes place. This also allows more heat to diffuse away from the ablation crater increasing the thermal damage done to the surrounding tissue. Decreasing the ablation temperature has the converse affect, and allows the ablation crater to become deeper.

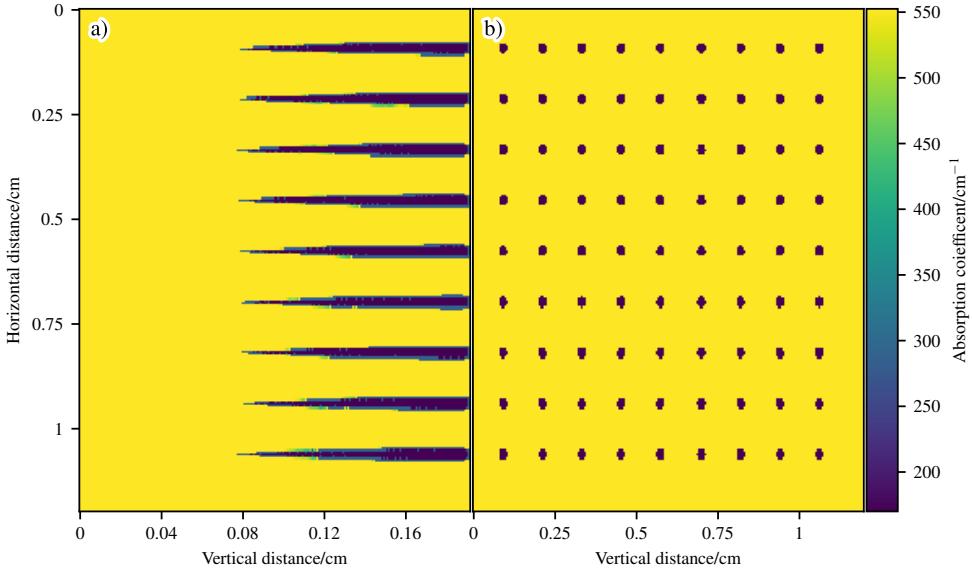


Figure 2.9: Simulation of 81 pixel beams. Figure a) shows a slice through the optical properties at the end of the simulation in the z-y plane. Figure b) shows the optical properties in the x-y plane at the top surface. Yellow is unchanged tissue, and purple is completely ablated tissue. Figure shows that the ablation craters do not overlap one another.

Over the full range of T_a , as the energy per pixel beam increases, there is a trend that at higher energies the crater depth begins to taper off. This is potentially due to a number of reasons. As the ablation craters grows the volume of tissue that is ablated is replaced with air, allowing more heat loss from the tissue to the environment. As well as heat loss to the environment, more heat is diffused away into the surrounding tissue as the crater grow, due to the availability of more tissue for the heat to diffuse into.

Temperature during ablation

Investigating thermal damage

As stated in Section 2.2.3, the Arrhenius damage integral is used to estimate the thermal damage due to the laser. To calculate the tissue damage around the ablation craters, Eq. (2.18) is first transformed into a summation:

$$\Omega(t) = \int_{t_p}^{t_f} A e^{(-\frac{\Delta E}{R T})} d\tau \quad (2.37)$$

$$\Omega(t) = \sum_{m=m_p}^{m_f} A e^{(-\frac{\Delta E}{R T_\xi^m})} \Delta t \quad (2.38)$$

Where:

ΔE , R , T , and A have the same meanings as before;

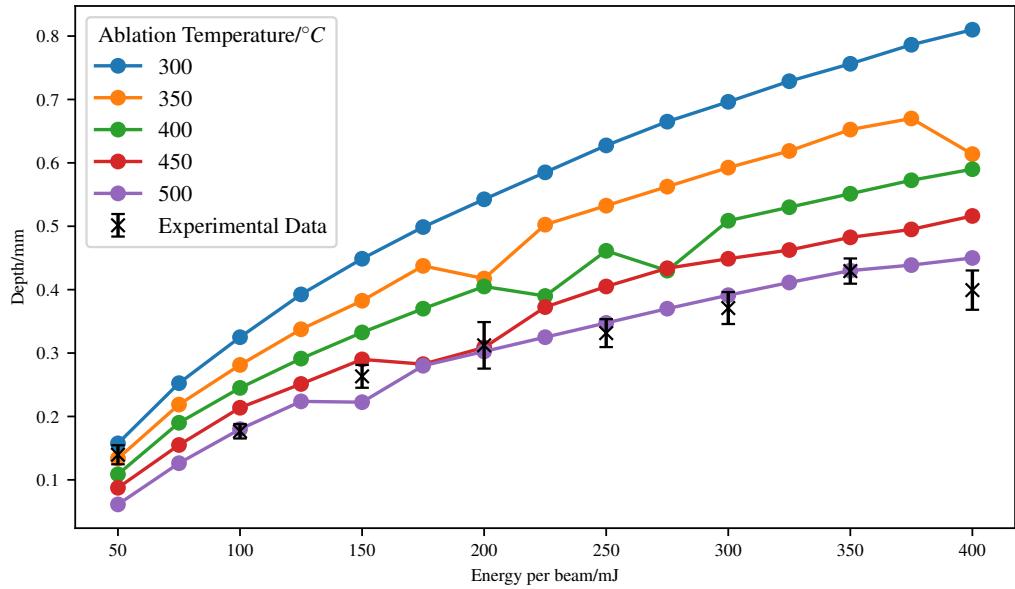


Figure 2.10: Simulation of 70 W CO₂ ablative laser, with a circular beam profile. Crater depths as a function of pixel beam energy for various T_a's.

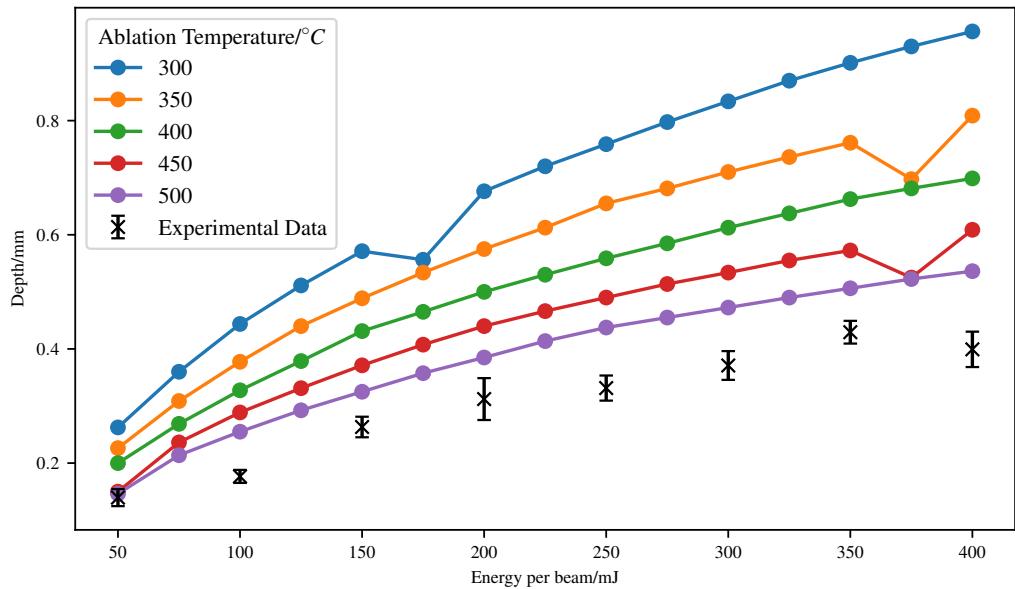


Figure 2.11: Simulation of 70 W CO₂ ablative laser, with a Gaussian beam profile. Crater depths as a function of pixel beam energy for various T_a's.

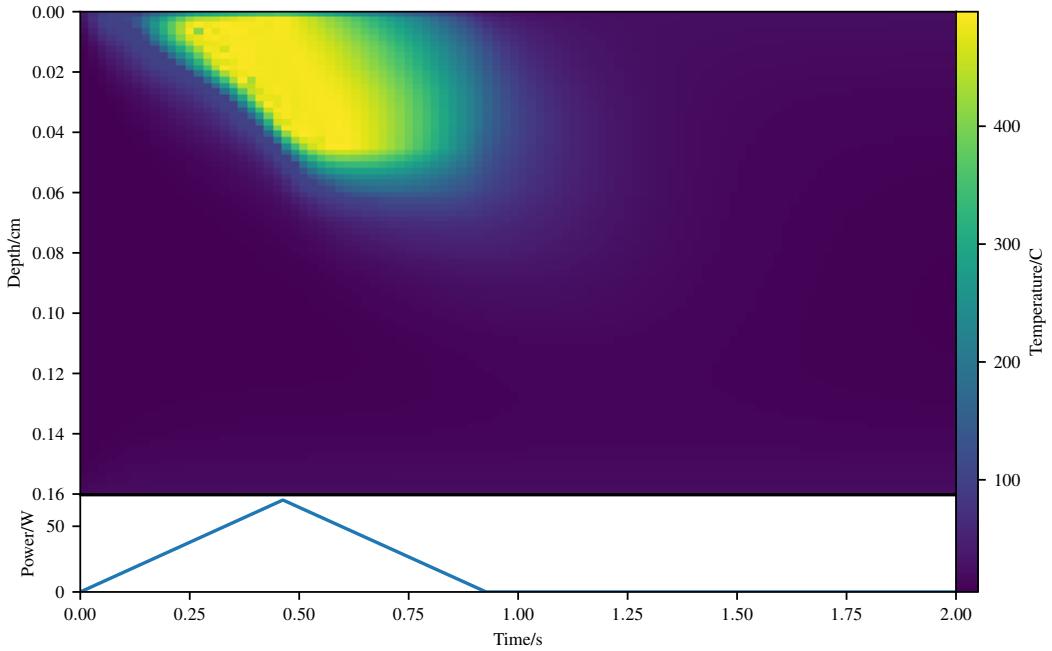


Figure 2.12: Temperature bore hole though centre of medium as a function of time, for $T_a=500\text{ }^{\circ}\text{C}$.

ξ is the i^{th}, j^{th}, k^{th} node;

and m_p is the p^{th} timestep when the ξ^{th} node is above the threshold temperature.

Using Eq. (2.38) it can thus be estimated that the damage to the tissue on a voxel by voxel basis. Figure 2.13 show how far the thermal damage extends around the ablation crater. For ease of visualisation 1-3 is mapped to their respective burns via the following scheme, with η as burn severity:

$$\eta = \begin{cases} 3, & \Omega \geq 10000 \\ 2, & 1 \leq \Omega < 10000 \\ 1, & 0.53 \leq \Omega < 1 \\ 0, & 0.0 \leq \Omega < 0.53. \end{cases} \quad (2.39)$$

As shown in Fig. 2.13, the thermal damage zone extends for a small distance around the ablation crater, due to the diffusion of heat into these areas. Figure 2.14 shows the maximum horizontal thermal damage distance as a function of T_a , and pixel beam energy. For values of T_a less than $\sim 425\text{ }^{\circ}\text{C}$, it appears that the maximum horizontal extent of the thermal damage tapers off. This is most likely because for lower values of T_a , there is a larger ablation crater, meaning that the energy form the laser is deposited deeper in the tissue in comparison to higher values of T_a . The higher values of T_a allow greater diffusion of the heat, thus yielding larger zones of damage. Overall there is little difference in the maximum horizontal extent of thermal injury, when using different energies (of the order of $\sim 0.01\text{ mm}$).

Investigations for the time it takes for different areas of the tissue to become thermally damaged, were also carried out. This can be easily achieved by saving the time each voxel passes

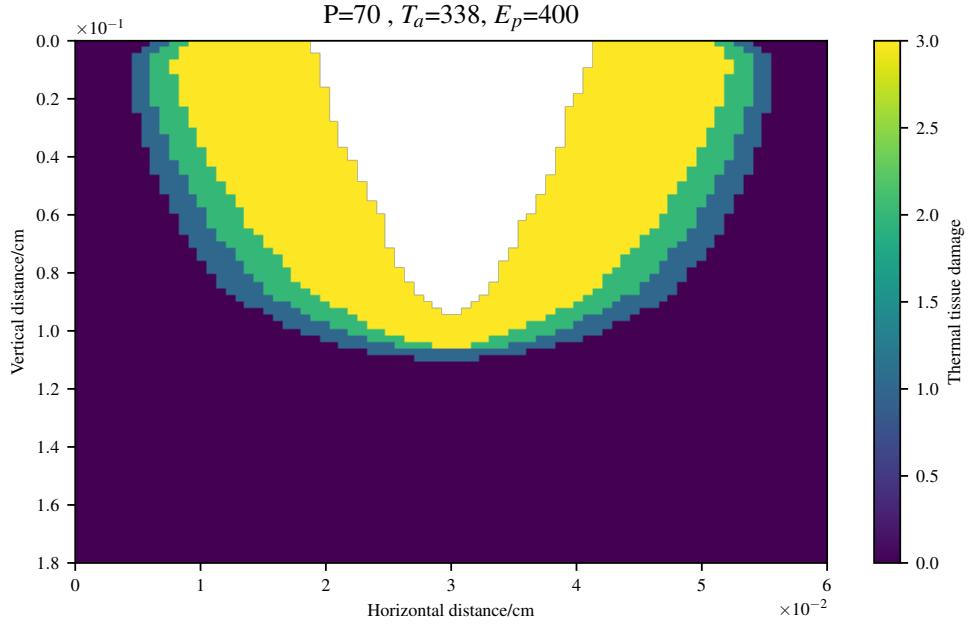


Figure 2.13: Tissue thermal damage around the ablation crater (white). Thermal tissue damage values of 3 refer to 3rd degree burns, 2 to 2nd, and 1 to 1st degree burns respectively. P is the power in Watts, T_a is the ablation temperature in Kelvin, and E_p is the energy per pixel beam in mJ.

one of the damage boundaries in Eq. (2.39). Figures 2.15 and 2.16 show the minimum time taken for 1st, 2nd, and 3rd degree burns to occur as a function of depth. Figure 2.15 shows that there is little to no time (upon the order of 0.5 ms) between 1st and 2nd, and 3rd degree burns. Figure 2.16 shows there is a slightly greater time difference between 1st and 2nd, and 3rd degree burns, however this is almost as negligible as the 400 mJ case.

The reason that there is almost no time between 1st and 2nd, and 3rd degree burns, is most likely because there is little time for heat to diffuse, whilst the laser is still illuminating the medium. The laser pulses are on the order of seconds, and tissue is not thermally conductive. This leads to the results presented here.

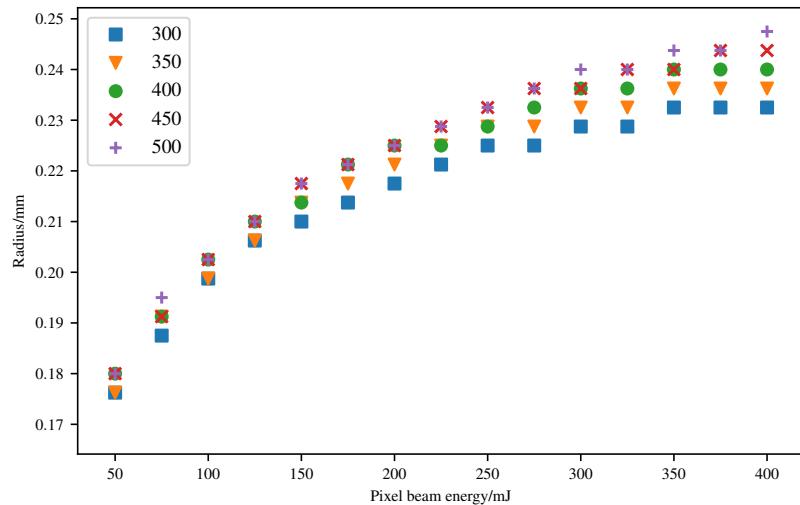


Figure 2.14: Figure shows the maximum horizontal extent of thermal damage as a function of energy per pixel beam, for different T_a 's.

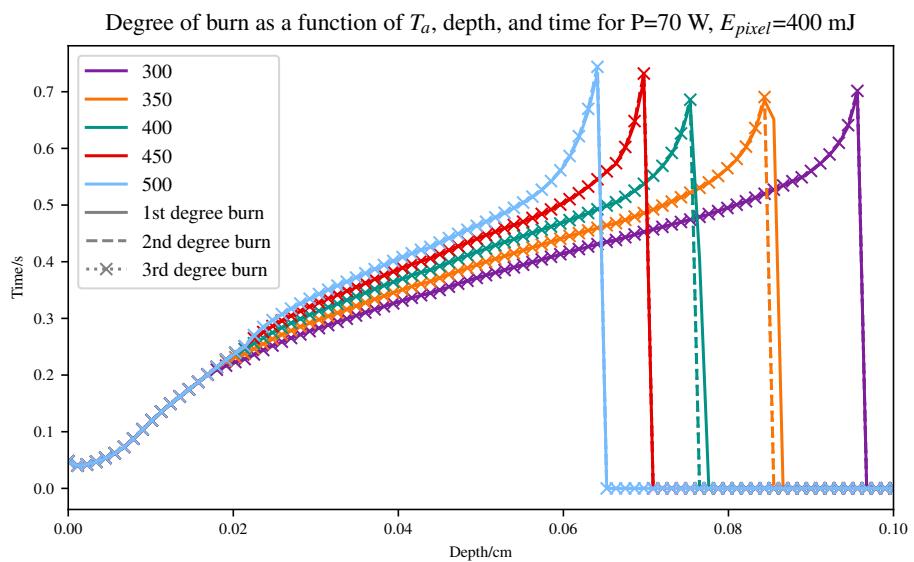


Figure 2.15: Figure show the time taken for 1st, 2nd, and 3rd to occur as a function of depth, for a range of T_a 's at 400 mJ.

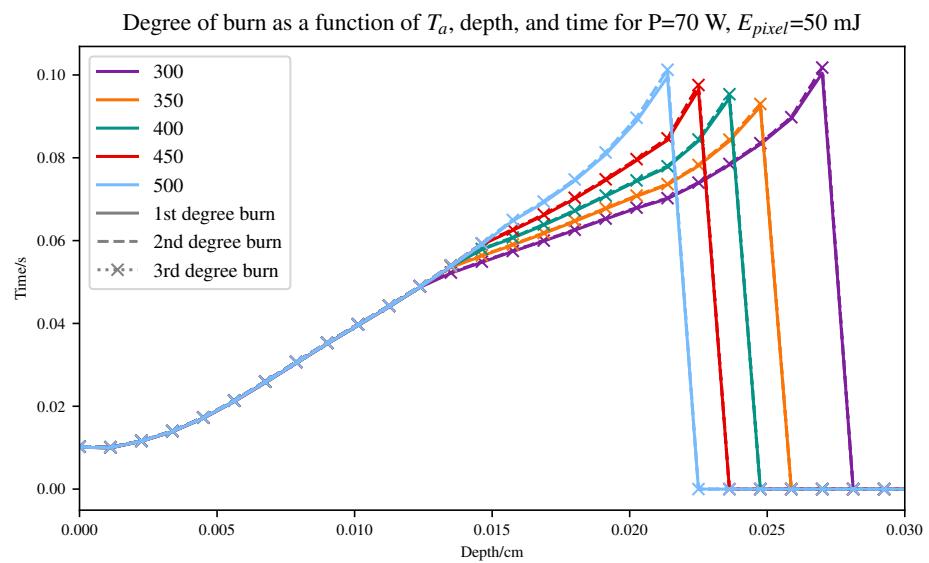


Figure 2.16: Figure show the time taken for 1st, 2nd, and 3rd to occur as a function of depth, for a range of T_a 's at 50 mJ.

Investigating beam type

As the manufacturer does not provide information on the beam profile of the pixel beams and the lack of equipment available to measure the beam profile, the shape of the beam profile has to be assumed. Two different beam types are tried: Gaussian, and circular (top-hat). Figures 2.10 and 2.11 show the result of these *in-silico* experiments. The Gaussian beam ablates deeper holes than the circular beam type, which is to be expected due to the distribution of power in the Gaussian beam. The beam that best fits the data, is the circular beam. For the Gaussian beam to fit the data ablation would have to take place at temperatures above 500 °C which does not fit with the literature. Without knowing the exact profile of the beam, it is assumed for the rest of the *in-silico* experiments that the beam profile is circular.

Investigating laser pulse profile

Pulsed laser have a variety of pulse profiles. The pulse profiles are usually modelled as triangular, tophat, or Gaussian. However the pulse profiles in reality are normally less well defined, and rather the pulse profile is something in between these perfect models.

The laser used in the above experiments, the Pixel CO₂, uses a triangular pulse profile for the laser pulses. Thus in the section the effect of the laser pulse profile has on ablation and surrounding thermal injury is investigated.

Four different laser pulses profiles are investigated: tophat, triangular and two different Gaussian profiles. The two different Gaussian pulses trialled are, a Gaussian profile with σ equal to the pulselength, and a pulse profile where the same energy is expended.

Figure 2.17 show the pulse profiles for a pulselength of 0.2 s.

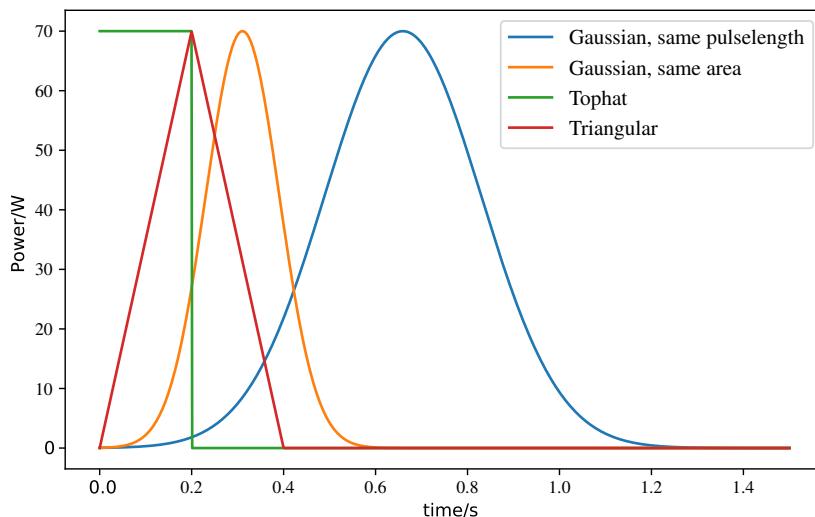


Figure 2.17: Comparison of the different pulse profiles trialled for a pulselength of 0.2 s.

2.3.2.1 Investigating initial temperature

As the experiment was carried out on porcine tissue that was kept on ice before the experiment was conducted, we assumed that the initial temperature of the porcine tissue was around 5 °C. This section investigates whether this is an accurate assumption.

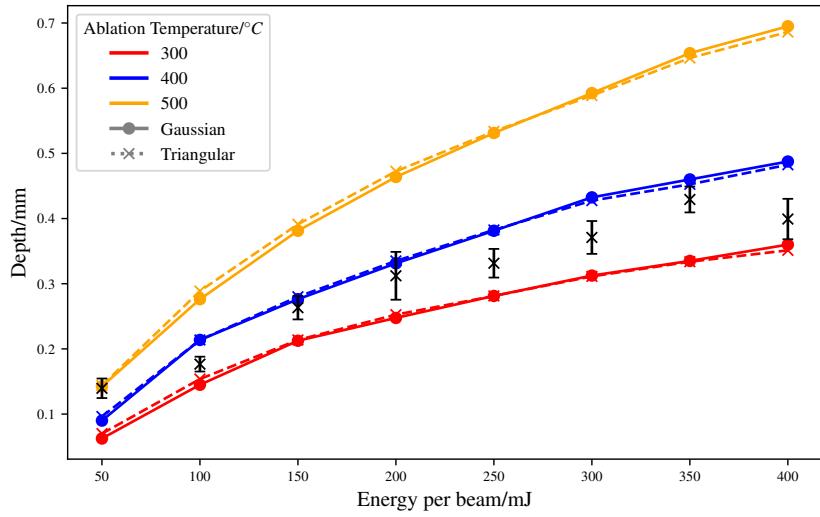


Figure 2.18: Comparison of various pulse shapes for the pixel beams.

To investigate this, three different temperatures were trialled: 25 °C, 0 °C, and 5 °C. These temperature correspond to room temperature, the temperature of ice and the original temperature we assumed. Figure 2.19 shows the results of this *in-silico* investigation.

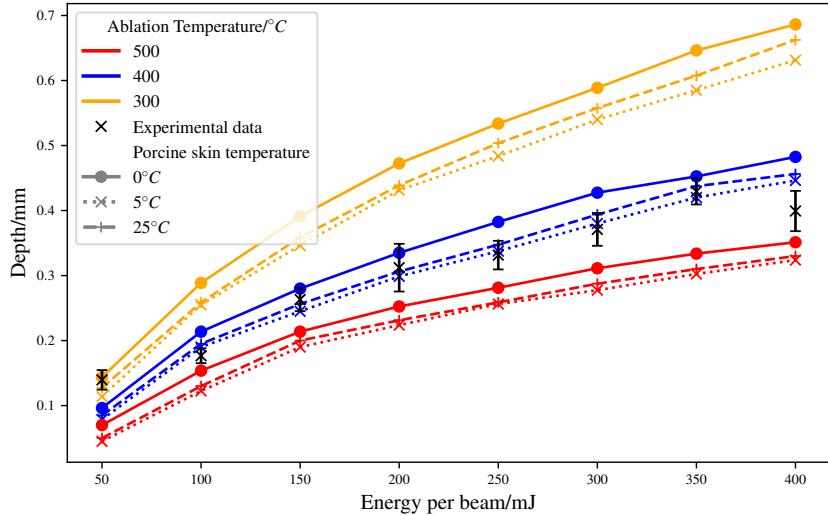


Figure 2.19: Comparison of ablation depths for different initial temperatures in the porcine skin.

As expected the hotter the porcine skin is initially the larger the ablation depth. This occurs as less energy is required to bring the porcine skin to its ablation temperature. In the previous subsections it was assumed that the temperature of the porcine skin was around 5 °C. This assumption was based upon the fact that the porcine skin was kept on ice before the experiment, thus the temperature of the skin must be between 0 and room temperature. This investigation shows that over small variations of temperature ($\lesssim 5$ °C), the ablation depth does not vary too

much (on the order of $\approx 0.01 \text{ mm}$).

However, there is a greater difference in the maximum extent of thermal damage to the skin for different initial temperatures in the porcine skin. Figure 2.20 shows this difference.

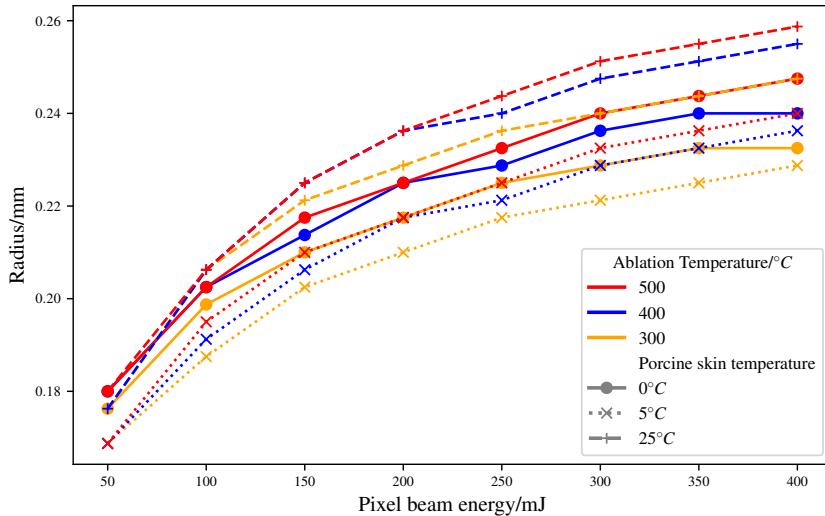


Figure 2.20: Comparison of maximum horizontal damage distance for different initial porcine skin temperatures.

2.3.2.2 Investigating voxel temperature after ablation

In the previous section it is assumed that the temperature of a voxel remains unchanged after the tissue is removed from that voxel via ablation. However this assumption may not be accurate. To test if the temperature of the voxel after ablation effects the depth of ablation or the thermal injury to the surrounding tissue various voxel temperature were tried. Two different temperatures were tried: half the ablation temperature and room temperature ($\approx 25 \text{ }^{\circ}\text{C}$). Figure 2.21 shows the effect of the voxel temperature after ablation has on the ablation depth.

Setting the voxel temperature to either half the ablation temperature or to room temperature has a large effect on the ablation depth, with a difference of $\approx 0.1 \text{ mm}$. However there is a small difference between setting the voxel temperature to room temperature and to half of the ablation temperature though.

2.4 Conclusion

Using MCRT and a finite difference method, a fully 3D model of photon and heat transport within tissue has been created. This model can be used to simulate the heat deposited by laser, the ablation craters formed via high powered lasers and the resultant thermal damage surrounding the ablation crater.

The model has been fully compared with both analytical solutions and experimental results. The model was found to match with experimental results that a tissue ablation temperature T_a of around $500 \text{ }^{\circ}\text{C}$ has to be adopted, towards the higher end of the range previously observed in the literature.

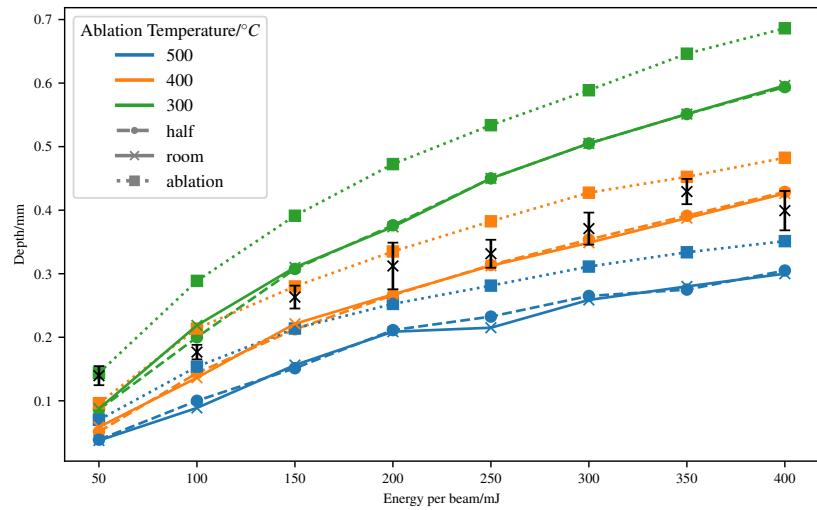


Figure 2.21: Comparison of different voxel temperatures after ablation. Half refers to setting the temperature of a voxel to half that of the ablation temperature. Room refers to room temperature, and ablation leaves the temperature at the ablation temperature.

The simulations allow us to predict for a given laser power and pulse length, how much thermal damage is caused in the tissue, and how deep an ablation crater that will form. The computational model could be used in future to help develop treatment regimes for both aesthetic and medical procedures. For example, currently there is considerable amount of “down time” after skin rejuvenation, in which the patient displays inflammation, erythema, edema, pain, and crusting [99–101]. Simulations of thermal damage due to fractional ablation could help design treatment regimes that minimise these effects, whilst still delivering skin rejuvenation. The model can also be applied to help optimise laser assisted drug delivery. Laser assisted drug delivery consists of using a laser to “drill” holes into the skin in order to help topical medicines diffuse into the skin better, than just applying the medicines to skin with no holes. Our model can help predict the laser parameters needed to reach a certain hole depth, thus minimising thermal damage and pain to patients.

There are many avenues available with regards to future work on this model. The model presented here in this chapter was on a initially homogeneous skin model. In reality skin is compromised of several distinctive layers, with each layer containing varying amounts of different chromophores. Our model can easily incorporate an multi-layered skin model complete with various fractions of chromophores. However as the laser used in these studies is an infra-red laser, water is the highest absorbing chromophore, meaning that a physically accurate model, with various chromophores is not need for this application. The current model is a voxel based model, where all the voxels are the same size. This allows the model presented in this chapter to be easily set-up, with regards to parallelisation, optical/thermal properties and ease of programming. However voxel models, where all the voxels are the same size, are not computationally efficient. Particularly in order to achieve good resolution, many voxels are needed, which requires large amounts of RAM, due to a $\sim n^3$ scaling of voxels to memory in 3D. A more efficient way, would be to allow different sizes of voxels, depending on parts of the model which need high resolution, and parts that do not need high resolution. Such a voxel model is called an adaptive mesh refinement (AMR). There are downsides to AMR: complex implementation for parallelisation and set-up of

optical/thermal properties, slower optical depth integration routines due to neighbour lookups.

Chapter 3

3D Phase Tracking Monte Carlo Algorithm

3.1 Introduction

Complex shaped light beams have been used for a variety of tasks in biophotonics. From using Airy beams to move particles and cells [102], Bessel beam ‘tractor beams’ [103], using Airy and Bessel beams for better field of view in light-sheet microscopy [104], and using Laguerre-Gaussian beams to optical trap optically reflective particles [105].

However simulation techniques for modelling complex shaped beams in biological tissue is lacking. Currently there are a number of techniques that can model these beams in biological tissue, however they all have downsides. These methods include diffusion approximation to the RTE, finite difference time domain (FDTD), pseudo-spectral time-domain (PSTD), beam propagation method (BPM), and MCRT. As discussed in [Diffusion Approximation](#), this method has many downside when it comes to modelling light propagation in biological tissue. [FDTD](#) involves using a finite difference method in order to solve Maxwell’s equations. However this turns out to be computationally intensive and requires a grid resolution of $\lambda/20$ and thus most models are restricted to 2D [106, 107].

This chapter modifies the MCRT method, which is ‘ballistic’ photon method into a quasi ‘ballistic/wave’ photon method so that the wave behaviour of photons can be modelled. This allows the modelling of complex shaped beams such as Bessel beams and Gaussian beams.

Bessel beams have been the subject of intense research since their discovery in 1987 [108, 109]. Durnin noticed that the blah blah.

This chapter examines how Bessel beams compare to other beam in a scattering medium. We investigate if the Bessel beams self-healing property has any effect in a turbid medium. We examine Bessel beams and the other beams by creating a novel MCRT algorithm that allows the tracking of a photon as it propagates through a medium. The main focus of this chapter, is validation of our new novel technique, followed by using the new algorithm (φMC) to compare Gaussian and Bessel beams, to see which one preforms better in a turbid medium. This chapter also extends out novel algorithm to other complex, diffraction less beams

3.2 Theory

The MCRT algorithm as described in Chapter 1, must be adjusted so that wave phenomena such as interference and diffraction can be modelled. Modelling these wave behaviours allows us to model complex beams, where these phenomena are required to form the beam, e.g Bessel beams. As MCRT is a ballistic simulation of photon packets, meaning that the MCRT simulation presented thus far in this thesis only modelled the ballistic behaviour of photons. However for the work presented in this chapter, wave like behaviour is crucial to modelling the various experiments and phenomena.

To convert a ballistic simulation of photon packets into a ballistic/wave-like simulation, the complex phase of each photon packet is tracked. This is achieved, by simply tracking the complex phase of the photon as it propagates through a medium. The packet is also given an initial complex electric field of the form:

$$E_0 = \frac{1}{N} \sqrt{\frac{P}{A}} \quad (3.1)$$

Where N is the number of photons run in a simulation, P is the power of the incident beam, and A is the area of the beam.

Equation (3.2) shows how the phase is calculated.

$$\varphi = \cos\left(\frac{2\pi l}{\lambda}\right) + i \sin\left(\frac{2\pi l}{\lambda}\right) \quad (3.2)$$

Where φ [–] is the phase of a photon packet, l [m] is the distance the photons has travelled, λ [m] is the wavelength of the photon, and i is the solution to $x^2 = -1$. Now we can calculate the phase of a photon at a position P_0 , if we know the distance it has travelled, and its original phase, Fig. 3.1.

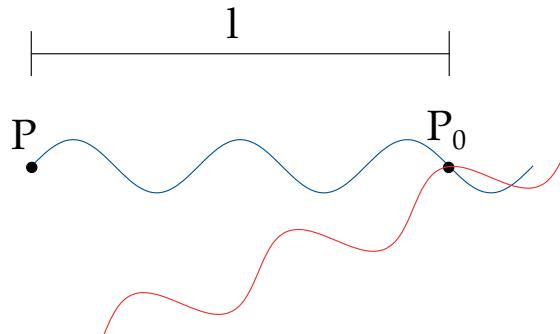


Figure 3.1: Example of phase calculation when a photon has travelled a distance l . Figure also show an example of interference between two photons via addition of the complex amplitudes at the point P_0 .

To be able model the wave-like behaviour of photons, we let the photons packets interfere with one another in a volume or area element. We do not model the interference at a point in space where photons packets cross one another as due to the ballistic nature of the MCRT simulation, this does not occur with enough frequency in order to give a good signal to noise ratio. Thus, interference takes place in a volume, dV , or area element, dA , instead. To calculate the interference from the phase, the phase is summed in each volume or area element and the absolute value taken, and then squared. Equation (3.3) shows the equation for interference for

a volume element dV . A similar relation for calculating the interference on an area element dA also exists.

$$I(\xi) = \left| \sum_{\xi} E_0 \cos\left(\frac{2\pi l}{\lambda}\right) + i \sum_{\xi} E_0 \sin\left(\frac{2\pi l}{\lambda}\right) \right|^2, \quad \xi = (x, y, z) \quad (3.3)$$

Where:

- l is the total distance travelled by a photon [m];
- λ is the wavelength of the photon [m];
- I is the intensity at the ξ^{th} cell [Wm^{-2}];
- E_0 is the initial electric field of the packets as in ?? [Vm^{-1}]
- and ξ is the x^{th}, y^{th}, z^{th} cell, volume dV .

As the MCRT simulation is now a quasi ballistic/wave simulation of photon behaviour, comparisons between the simulations and, theoretical and experimental data to prove this model is accurate. However before validation of the model takes place, one further principle needs to be introduced that is required for our model to work.

3.2.1 Huygens-Fresnel Principle

The Huygens-Fresnel principle is a method that is used to help model the propagation of waves in the far field limit and the near field limit.

The Huygens principle states [110–112]:

“Every point on a propagating wavefront serves as the source of spherical secondary wavelets, such as the source at some time later is the envelope of these wavelets.”

The principle is illustrated in Fig. 3.2. Christiaan Huygens postulated this principle in 1678. The principle allowed Huygens to derive laws of refraction and reflection, but it failed to describe diffraction effects. This led to Augustin-Jean Fresnel in 1818, combining the Huygens principle with his own theory of interference [112,113]. This principle, the Huygens-Fresnel principle, gave an accurate description of the propagation of light and diffraction effects. This was achieved by allowing the secondary wavelets to self interfere with one another, giving rise to an accurate description of the physical phenomena. Later, Gustav Kirchhoff gave a rigorous mathematical description of the Huygens-Fresnel principle, which is the basis of diffraction theory [114,115].

The Huygens-Fresnel principle allows the modelling of diffraction in both the near and far field. As the principle states that every point on the wavefront is a source of secondary spherical waves, this implies that there are “backward” waves. These “backward” waves are unphysical, and there is no evidence of their existence. Thus Fresnel introduced an inclination factor to eliminate these “backward” waves. This inclination factor was later put on rigorous mathematical standing by Kirchoff, as it naturally fell out of his theory [114,115]. Equation (3.4) shows the equation for the complex field at a point on a plane as derived by Kirchoff with the inclination factor.

$$u(\mathbf{r}_1) = \frac{1}{i\lambda} \int \int u(\mathbf{r}_0) \frac{\hat{s}_0 \cdot (\mathbf{r}_1 - \mathbf{r}_0)}{|\mathbf{r}_1 - \mathbf{r}_0|^2} e^{ik|\mathbf{r}_1 - \mathbf{r}_0|} dS_0 \quad (3.4)$$

Where:

- u is the complex electric field [Vm^{-1}];
- λ is the wavelength [m];
- S_0 is a plane with surface normal \hat{s}_0 [-];

k is the wavenumber [m^{-1}];
and r_n are spatial coordinates [•].

Our algorithm uses the Huygens-Fresnel principle to simulate diffraction effects, that would otherwise be absent from the simulation. The principle allows the algorithm to calculate the complex amplitude at a point, and thus the intensity at that point. The Huygens-Fresnel principle is implemented by sampling the light source on the surface of any lens or in a slit. In practise this means when for example, a plane wave is incident on a slit width a , and length b , the slit area is uniformly sampled for the initial position of the photon packets. The packets are then given a random direction, sampled towards the detector thus avoiding the non-existent “backward” waves. For the case of modelling propagation through a lens, the usual geometric optics approach is taken to propagate the packets through the lens. When the packet lies on the surface of the lens, the Huygens-Fresnel principle is invoked, and the packet is given a random direction (in the direction of the medium) and propagated as usual. Essentially the Huygens-Fresnel method calculates the complex amplitude of the electric field at a point. Thus running a Huygens-Fresnel simulation gives the solution to Eq. (3.4).

This is the principle that underpins the algorithm that allows various complex beams, and wave phenomena to be simulated within a ballistic method. The following sections validate the method against the theory and experimental data for various complex beam propagation.

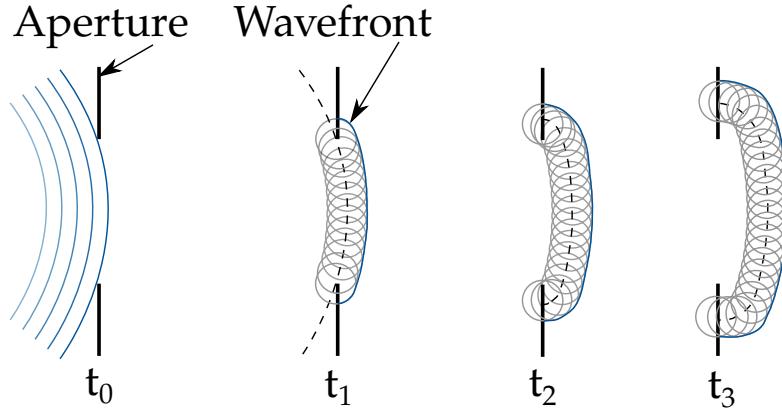


Figure 3.2: Illustration of the Huygens-Fresnel principle. At t_0 a wave is incident on an aperture. Times t_1 , t_2 , and t_3 show the evolution of the wavefront using the Huygens-Fresnel principle.

3.2.2 Validation of Phase Tracking Algorithm

Double Slit Experiment

The first test of our phase tracking algorithm, is to compare our simulation to a double slit experiment. The double slit experiment, is a simple experiment where monochromatic plane wave of light is incidence on two slits distance apart a , and width b , and the interference pattern is observed on a screen a distance d away from the slits.

This experiment, first carried out by Thomas Young and thus sometimes called Young’s slits experiment, is usually carried out with the detector screen in the far field. The so called Fraunhofer regime. The intensity pattern on the detector screen is as in Eq. (3.5):

$$I(\theta) \propto \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \text{sinc}^2\left(\frac{\pi b \sin\theta}{\lambda}\right) \quad (3.5)$$

Where the *sinc* function is defined as $\frac{\sin(x)}{x}$, for $x \neq 0$, d is the slit separation and θ is the angular spacing of the fringes.

The simulation was carried out for a wavelength of λ , a slit width of b , and a slit separation of d . Using the Huygens-Fresnel principle, each slit is a source of Huygens wavelets. The initial position of the photon packets is sampled uniformly from the slit area, after randomly choosing one of the slits. A random direction is then chosen to ensure that the packets will hit the detector screen. The simulation was run with photon packets, which took to run on an 8 core Intel Xeon machine. This gave an accurate match to the theoretical expression, as seen in Fig. 3.3.

Figure 3.3: Comparison of theory and simulation for the double slit experiment. λ is xnm , b is, and d

Diffraction by a Square Slit

φMC is also validated by simulating diffraction from a square aperture in the far and near field, the so call Fresnel and Fraunhofer regimes. Fresnel diffraction occurs in the near field when the *Fresnel number*, Eq. (3.6), is greater than 1.0. Fraunhofer diffraction occurs when the *Fresnel number* is less than 1.0.

$$F = l \sqrt{\frac{2}{\lambda r_0}} \quad (3.6)$$

Equation (3.6) is the Fresnel number, a measure of whether diffraction is in the Fresnel regime or the Fraunhofer regime. l is the slit width, λ is the wavelength of the incident radiation, and r_0 is the distance from the aperture to the detector screen, as shown in Fig. 3.4.

In order to compare φMC to the theory, the theory must first be examined. Consider the setup as shown in Fig. 3.4, in order to calculate the intensity at a point P , the contribution by an area element dS at the point a , to the optical disturbance at a point P is considered. Accounting for the the unobstructed optical disturbance from S as well, yields:

$$U(P) = \frac{1}{i\lambda} \iint_{\Sigma} \frac{A e^{i(k\rho - \omega t)}}{\rho} \frac{e^{ikr}}{r} \cos(\theta) dS \quad (3.7)$$

In the case where ρ_0 and r_0 are large compared to the size of the aperture, then $\cos(\theta) = 1$ and $\frac{1}{\rho r} = \frac{1}{\rho_0 r_0}$. The lengths of r_0 and ρ_0 are:

$$r = \sqrt{r_0^2 + y^2 + z^2} \quad (3.8)$$

$$\rho = \sqrt{\rho_0^2 + y^2 + z^2} \quad (3.9)$$

Using the binomial theorem to expand Eqs. (3.8) and (3.9) yields:

$$\rho + r \approx \rho_0 + r_0 + (y^2 + z^2) \frac{\rho_0 r_0}{2\rho_0 r_0} \quad (3.10)$$

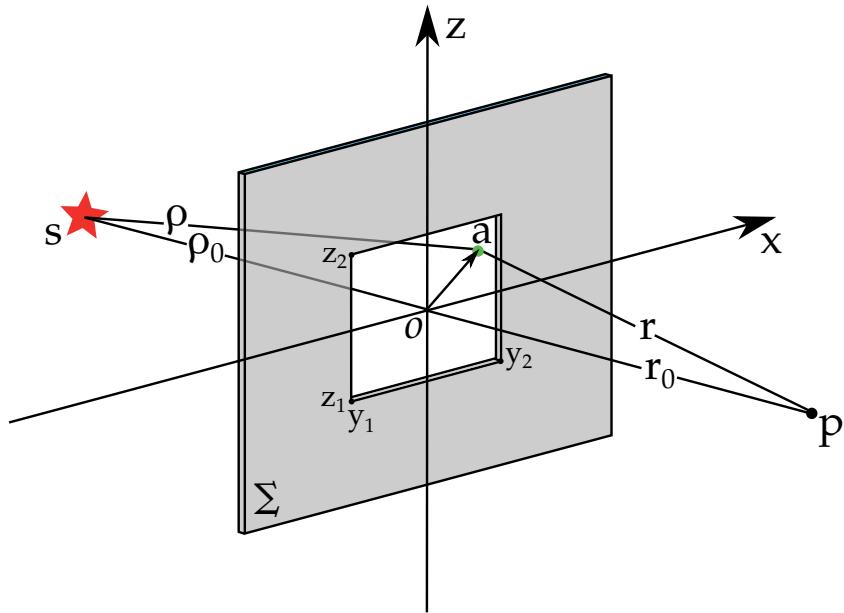


Figure 3.4: Fresnel diffraction at a square aperture.

Substituting Eq. (3.10) into Eq. (3.7) with $k = 2\pi/\lambda$

$$U(P) = \frac{Ae^{-i[k(\rho_0 + r_0)\omega t]}}{i\lambda\rho_0r_0} \iint_{\Sigma} e^{i2\pi y^2 \frac{(\rho_0+r_0)}{2\lambda\rho_0r_0} + i2\pi z^2 e^{\frac{i\pi u^2}{2}} \frac{(\rho_0+r_0)}{2\lambda\rho_0r_0}} dS \quad (3.11)$$

Introducing the dimensionless variables u and v

$$u = y\sqrt{\frac{2(\rho_0 + r_0)}{\lambda\rho_0r_0}} \quad (3.12)$$

$$v = z\sqrt{\frac{2(\rho_0 + r_0)}{\lambda\rho_0r_0}} \quad (3.13)$$

and substituting them into Eq. (3.11).

$$U(P) = \frac{\tilde{E}_u}{2} \int_{u_1}^{u_2} e^{\frac{i\pi u^2}{2}} du \int_{v_1}^{v_2} e^{\frac{i\pi v^2}{2}} dv \quad (3.14)$$

Equation (3.14) describes the optical disturbance at the point P , with \tilde{E}_u the unobstructed disturbance at P . Equation (3.14) can be evaluated using the Fresnel integrals, $C(w)$ and $S(w)$:

$$\int_0^w e^{i\pi w'^2/2} dw' = C(w) + iS(w) \quad (3.15)$$

$$S(w) = \int_0^w \sin\left(\frac{\pi w'^2}{2}\right) dw' \quad (3.16)$$

$$C(w) = \int_0^w \cos\left(\frac{\pi w'^2}{2}\right) dw' \quad (3.17)$$

Using Eq. (3.15), where $C(w)$ and $S(w)$ are the Fresnel integrals as in Eqs. (3.16) and (3.17), Eq. (3.14) can then be transformed into an intensity, by taking the absolute value and squaring, yielding Eq. (3.18):

$$I_p = \frac{I_u}{4} \{ [C(u_2) - C(u_1)]^2 + [S(u_2) - S(u_1)]^2 \} \times \{ [C(v_2) - C(v_1)]^2 + [S(v_2) - S(v_1)]^2 \} \quad (3.18)$$

Equation (3.18) gives the intensity of the field at the point P on axis for a square aperture where I_u is the unobstructed intensity at the point P .

As the mathematics of calculating the optical disturbances at all points on a plane at point P is difficult, instead the aperture is moved by small displacements, with \overrightarrow{SOP} fixed. This effectively achieves the translation of the origin, O , with respect to the fixed aperture. Thus, for each displacement new aperture coordinates y_1, y_2, z_1 , and z_2 are generated and therefore new u_1, u_2, v_1 , and v_2 . Therefore the intensity at a point $P + \delta d$, where δd is the displacement, can be calculated. This approximation holds for displacements that are small compared to the ρ_0 [111, 115, 116]. Using this method and Eq. (3.18) gives the theoretical curves curves in Fig. 3.5.

In φMC , the above experiment is simulated. A square slit is uniformly sampled in the y , and z direction in order to get the packets initial position. A random direction is then sampled, ensuring that the direction points towards the detector screen. As we assume a plane wave is incident of the aperture each photon is given the same initial complex electric field.

The detector screen's distance from the aperture is then varied and the intensity on the screen is measured for $\sim 10^{10}$ photons released from the aperture, as Huygens wavelets. For *Fresnel numbers* greater than 1.0, the number of bins is 300, covering a distance of 600 μm . For the case of Fraunhofer diffraction, the number of bins is 100 covering a distance of 6000 μm . The simulations take ~ 3 minutes for 10^{10} packets to be run on an Intel Xeon E3-1245 v5, 8 cores @ 3.5GHz machine. The number of bins, and photons packets simulated had to be increased for the cases where the Fresnel number was large (i.e the detector screen was near the aperture) Fig. 3.5 shows the comparison between the theory and the φMC simulations.

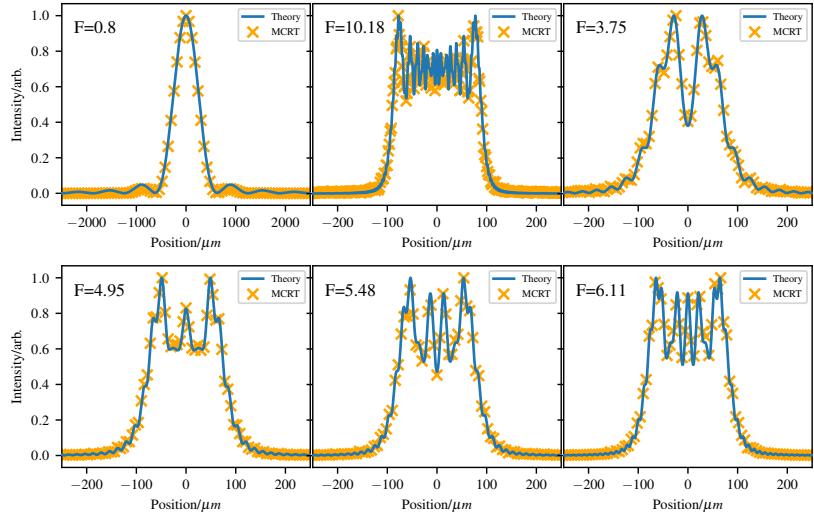


Figure 3.5: Comparison of theory and simulation for diffraction through a square aperture in the Fresnel and Fraunhofer regimes.

3.3 Gaussian Beams

Now that the method of tracking the phase of photon packets has been verified against theoretical results, we can now turn our attention to modelling the propagation of beams that require the wave behaviour of light in order to either form or propagate.

The first beam type we will examine is the Gaussian beam. Traditional MCRT cannot model Gaussian beams, as Gaussian beams have a finite beam waist at their focus (see Fig. 3.6). MCRT (along with geometric optics) predicts that Gaussian beams have an infinitely small waist. Various authors have tried to model Gaussian beams in a MCRT or geometric optics regime. Some of the techniques used by these authors include: artificial beam steering [117], generating skew rays [118] ... ***blah However all these techniques fail in one way or another to accurately model all the physical phenomena presented by a Gaussian beams. This section will show that φMC can accurately model all the physical phenomena of Gaussian beams, within the MCRT regime.

Before modelling a Gaussian beam, the theory and various physical parameters must be described. The electric field of a Gaussian beam can be defined as in Eq. (3.19):

$$E(r, z) = E_0 \frac{w_0}{w(z)} e^{\frac{-r^2}{w(z)^2}} e^{-i(kz + k \frac{r^2}{2R(z)} - \varphi(z))} \quad (3.19)$$

Where:

- r is the radial distance from the optical axis [m];
- z is the axial distance from the beam's waist [m];
- k is the wavenumber, $k = \frac{2\pi}{\lambda}$ [m^{-1}];
- E_0 is the electric field amplitude at the origin [$V m^{-1}$];
- $w(z)$ is the radius of the beam at which the amplitude has fallen to $\frac{1}{e}$, at the distance z along the beam, Eq. (3.20) [m];
- w_0 is the waist radius [m];
- $R(z)$ is the radius of curvature of the beam's wavefronts at z , Eq. (3.21) [m];

and finally, $\varphi(z)$ is the Gouy phase at z , Eq. (3.22) [-].

Equations (3.20) to (3.24) give the definitions of key physical properties as outlined above or as shown in Fig. 3.6. z_r is the Rayleigh length, Eq. (3.23), and defines the point in which the beam's waist grows to $\sqrt{2}$ times the size of the beam at its waist. The waist of the beam at the focal point is defined as Eq. (3.24), where f is the focal length and D is the $\frac{1}{e^2}$ diameter of the beam at the lens.

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_r}\right)^2} \quad (3.20)$$

$$R(z) = z \left[1 + \left(\frac{z_r}{z}\right)^2 \right] \quad (3.21)$$

$$\varphi(z) = \arctan\left(\frac{z}{z_r}\right) \quad (3.22)$$

$$z_r = \frac{\pi w_0^2}{\lambda} \quad (3.23)$$

$$w_0 = \frac{2\lambda f}{\pi D} \quad (3.24)$$

$$(3.25)$$

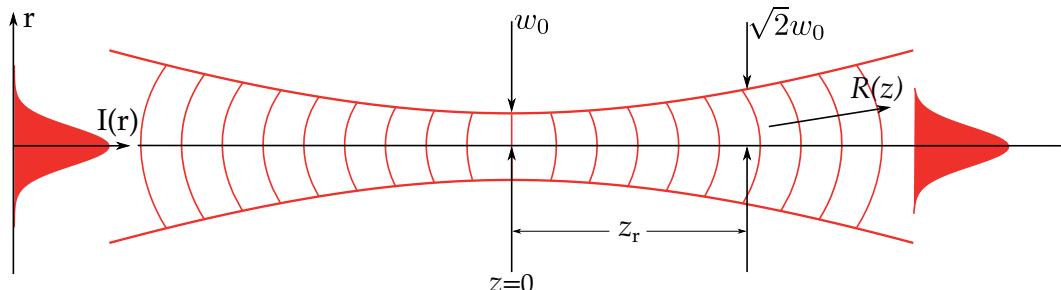


Figure 3.6: Illustration of a Gaussian beam focusing to its waist then diverging away. Image shows the various defined properties of a Gaussian beam along side the radius of curvature changing direction at the waist.

With the physical properties of the Gaussian beam outlined, a Gaussian beam can now be modelled using our algorithm. In order to simulate the Gaussian beam, we set up the simulation as shown in Fig. 3.7. The lens used is convex-plano lens, with radius of curvature, 4.6 mm , thickness, L_t , of 2.2 mm , and working distance, W_d , 8.5 mm . A Gaussian beam wavelength 488 nm and $\frac{1}{e^2}$ waist diameter 0.5 mm , is incident on the lens. Using Eq. (3.24) yields the size of the focal spot as $3\mu\text{m}$.

To model the lens in φ the photons initial z position is set just in front of the lens. The x and y are randomly sampled from a Gaussian distribution with a waist of $\sqrt{2}w_0$. The factor of $\sqrt{2}$ accounts for the conversion from intensity to electric field beam waist. This is due to the fact that the electric field is $\propto e^{-r^2/4\sigma'^2}$, and the intensity is $\propto e^{-r^2/2\sigma'^2}$. Thus for the input electric field waist to be equal to the intensity waist, i.e $\sigma' = \sigma$, $\sigma' = \sqrt{2}\sigma$. The packet is given an electric field of the form ??, with $P = 1\text{ mW}$, and $A = \frac{1}{2}\pi w_0^2$.

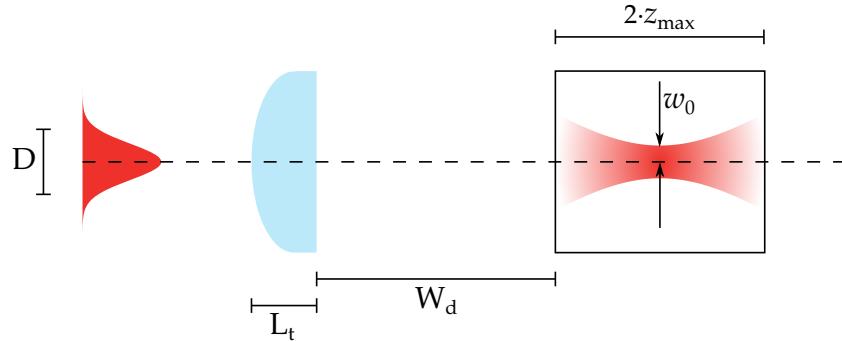


Figure 3.7: Simulation setup of focusing a Gaussian beam through a lens. Lens is convex-plano and is modelled on ThorLabs LA4249 UV fused silica lens [119].

The photon is then propagated to the surface of the convex side of the lens. This is achieved by finding the intersection of a sphere which represents the convex side of the lens, and the packets path. With the packet on the surface of the lens, Fresnel coefficients are calculated to determine if the packet is reflected or refracted. If the packet is reflected the packet is killed and the process starts again. If the packet is refracted, and moved in the new direction to the planar surface of the lens. The new direction vector is calculated using a vector version of Snell's law, as shown in ???. The packets is then uniformly sampled onto the surface of the voxel medium and the usual MCRT method is used to propagate the packet whilst tracking the phase.

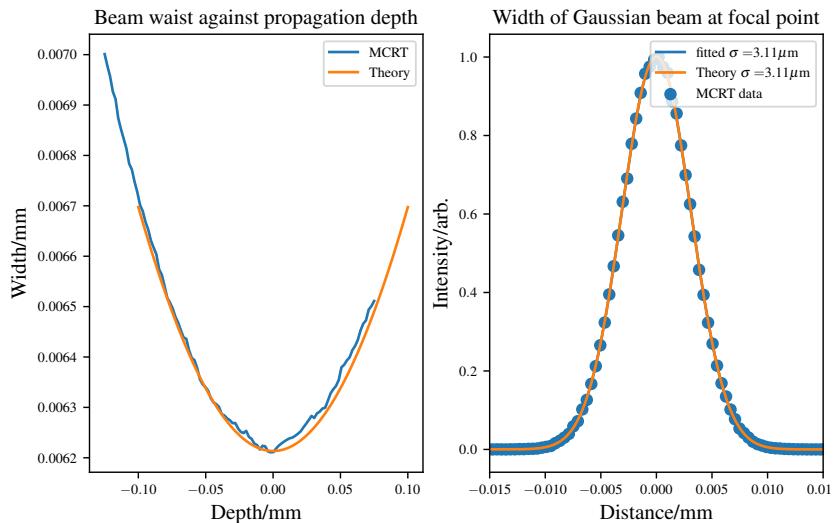


Figure 3.8: Results of *in-silico* experiment of focusing a Gaussian beam though a convex-plano lens. L_t is the lens thickness, D is the $\frac{1}{e^2}$ input beam diameter, W_d is the working distance or back focal length, $2z_{max}$ is the depth of the medium, and w_0 is the beam waist.

Figure 3.8 shows the comparison of theory and *in-silico* experiment, with excellent agreement between the two.

φMC also correctly models the change of direction of the radius of curvature, $R(z)$, as is predicted by theory. This can be seen in Fig. 3.9

Figure 3.9: Slice through the real part of the complex electric field of the *in-silico* experiment as in Fig. 3.7. Figure shows the radius of curvature changing direction at the waist as predicted by theory.

φMC can also model spherical aberrations caused by lenses. Fig. 3.10 shows aberrations caused by a plano-convex lens.

Figure 3.10: Illustration of φMC ’s ability to model spherical aberrations. Image generated using same setup as in Fig. 3.7, but with a $D = 2$ mm.

This section has shown that Gaussian beams, and their physical phenomena can be accurately modelled using φMC . A convex-plano lens was used to focus a Gaussian beam, but it is simple to implement other other lenses given a triangulated mesh of the lens or an equation that describes the shape of the lens.

3.4 Bessel Beams

Bessel beams, as described in the introduction to this chapter, are “non-diffracting” beams, that can “self heal”. This means, in reality, that the central core of the beam does not spread out, and that if a blockage is placed in front of the beam, the beam reforms further down the optical axis. There is some debate amongst physicists as to weather these phenomena are justly labelled, or if they glib terms used to make Bessel beams seem like the solution to everything.

The first “complex” beam simulated using φMC is a Bessel beam. Bessel beams are non-diffractive solutions to the wave equation. Bessel beams were first shown to blah

From the scalar description of the electric component of the beam, we get:

$$E(r, z) = E_0 \sqrt{\frac{2\pi k z w_0 \sin(\beta)}{z_{max}}} \exp\left(-\frac{z^2}{z_{max}^2} - \frac{i\pi}{4}\right) J_0(kr \sin(\beta)) \exp(ikz \cos(\beta)) \quad (3.26)$$

Where:

k is the wavevector, $k = \frac{2\pi}{\lambda}$ [m];

z is the distance from the axicon tip [m];

β is the angle the wavefront propagates at (see Fig. 3.11) [rad];

w_0 is the $\frac{1}{e^2}$ width of the input Gaussian beam [m];

J_0 is the Bessel function of the first order;

r is radial distance from the optical axis [m].

Equation (3.26) gives the electric field for a Bessel beam. The intensity can be calculated using:

$$I(r, z) = \frac{c\epsilon_0 |E|^2}{2} \quad (3.27)$$

Using the definition total power transmitted by a beam as:

$$P = \frac{\pi I_0 w_0^2}{2} \quad (3.28)$$

Where I_0 is defined as on axis intensity of the incident Gaussian beam.

$$I_0 = \frac{c\epsilon_0 E_0^2}{2} \quad (3.29)$$

Substituting Eqs. (3.26), (3.28) and (3.29) into Eq. (3.27) yields:

$$I(r, z) = \frac{4k_r P}{w_0} \frac{z}{z_{max}} J_0^2(k_r r) \exp\left(-\frac{2z^2}{z_{max}^2}\right) \quad (3.30)$$

Where:

k_r is the radial wavevector, $k_r = k \sin(\beta)$;
 P is the power of the incident Gaussian beam.

Bessel beam can be formed by an axicon lens or by diffraction through a ring. All the simulations of Bessel beams in this thesis use the axicon method of generating a Bessel beam, thus only axicons will be discussed. Figure 3.11 shows the geometry of a Bessel beam formed by an axicon. Using simple geometry and Snell's law the following equation can be derived to describe various properties of a Bessel beam formed by an axicon [?, 120].

The propagation depth of a Bessel beam is defined as the distance from the tip of the axicon to the end of the "Bessel region". However in reality the Bessel beam will continue to propagate slightly past this depth. Equation (3.31) shows the propagation depth of a Bessel beam where \cot is the cotangent function.

$$z_{max} = R(\cot(\beta) - \tan(\alpha)) \quad (3.31)$$

The propagation angle of the conical waves, β can be calculated using Snell's law and α the angle of the axicon:

$$\beta = \arcsin(n \sin(\alpha)) - \alpha \quad (3.32)$$

The central core of a Bessel beam is defined as the distance to the first zero of the Bessel beam. Equation (3.33) shows the radius of the core, where 2.405 is derived from position of the first zero of the Bessel function.

$$r_o = \frac{2.405}{k \sin(\beta)} \quad (3.33)$$

Finally, the spacing between Bessel beam rings is:

$$\Delta\rho = \frac{\lambda}{2 \sin(\beta)} \quad (3.34)$$

3.4.1 Validation

To ensure that the method described in Theory works as intended for Bessel beams several tests are run against theoretical expressions and experimental data.

Comparison to theoretical Bessel beam

To compare against a theoretical Bessel beam, a Bessel beam is modelled in the MCRT phase simulation, and propagated through air into the "Bessel region" and then propagated into the far field to ensure the beam follows the theory in both these regions.

Figure 3.11 shows the setup for the *in-silico* experiments. The Bessel beam is created with an axicon (conical) lens with an opening angle (α) of 5° , and a radius of 12.7 mm . The input beam is Gaussian in profile with a $\frac{1}{e^2}$ diameter of 1 mm , and a wavelength of 488 nm . The Bessel beam is then propagated to a detector screen 10 mm away from the tip of the axicon, which is in the middle of the “Bessel region” for the first test. For the second test the Bessel beam is propagated past the “Bessel region” into the far field. The detector screen has a size of $40\text{ }\mu\text{m} \times 40\text{ }\mu\text{m}$ with a bin resolution of $1\text{ }\mu\text{m}$. 8^{10} photon packets were simulated taking ~ 1 hour on an 8 core Intel Xeon 3.5Ghz machine.

Equation (3.30) gives the profile of a theoretical Bessel beam at a depth z_{max} , this is plotted against the simulation when $\frac{4k_r P_z}{w_0 z_{max}} e^{-2\left(\frac{z}{z_{max}}\right)^2} = 1$, with the simulation similarly normalised, by normalising to the maximum intensity of the image generated. Figure 3.12 shows this comparison.

Figure 3.13 shows the profile of the Bessel beam in the far field, where the theory predicts that it becomes a circular beam.

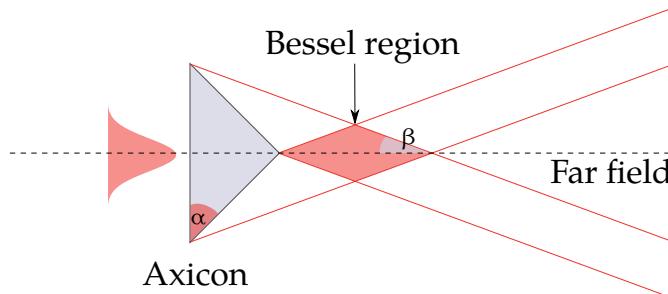


Figure 3.11: Geometry of a Bessel beam, generated by an axicon lens. β is the angle with the optical axis, and the angle of the conical waves. α is the axicon angle.

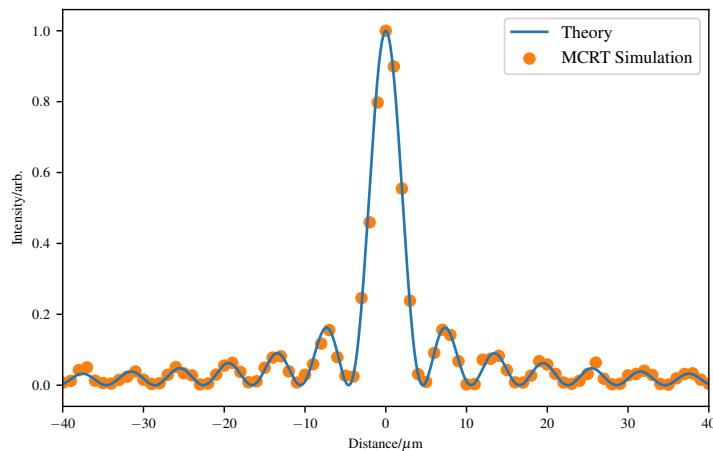


Figure 3.12: Comparison of theoretical and MCRT simulation of a Bessel beam, with intensity normalised. The results from φ MC show good agreement with the theory.

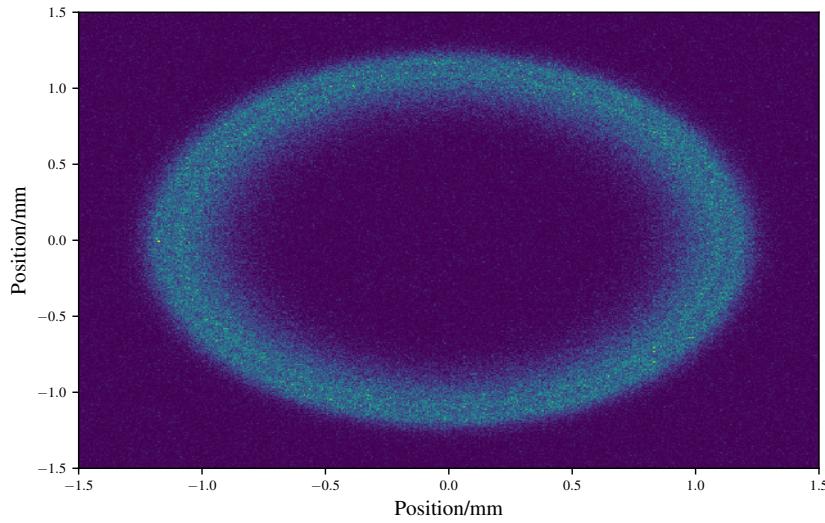


Figure 3.13: Bessel beam in the far field.

Comparison to experimental data

To ensure our algorithm works in turbid media, we carried out an experiment where a Bessel beam was propagated through a medium of varying turbidity. A laser, wavelength 488 nm , with a Gaussian profile is shone on an axicon lens, with angle 5° . The laser beam had a $\frac{1}{e^2}$ diameter of 2 mm . The Bessel beam was allowed to propagate through the air for 10 cm before entering a cuvette of side 2 mm . The cuvette was filled with $500\text{ }\mu\text{L}$ of water, and various volumes of a scattering agent added. The scattering agent used is intralipid 20 % (Sigma-Aldrich), which is diluted as shown in Table 3.1. Figure 3.15 shows the optical properties of Intralipid 20 %. Dilutions of Intralipid are kept below 2% scattering particle concentration, so that the scattering exhibited by the intralipid is in the independent scattering regime. This allows the linear scaling of the optical properties by concentrations [121–123]. Images of the Bessel beam as it emerges from the cuvette are taken for comparison with our algorithm. Figure 3.14 shows the experimental setup.

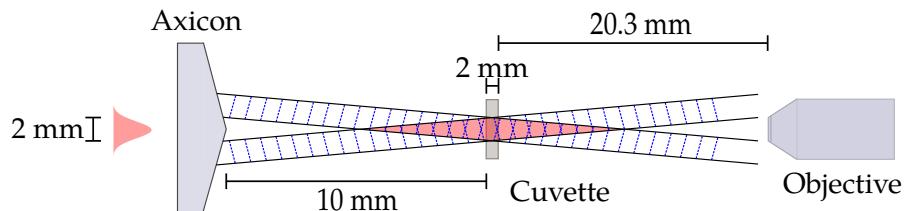


Figure 3.14: Experimental setup for propagating a Bessel beam through a cuvette filled with varying concentrations of Intralipid 20%. Bessel beam is imaged by a $20\times$ objective lens and a Grasshopper 3 camera.

To model within φMC , the experimental setup we simplify the setup considerably. The simulation models the propagation of a photon packet through the axicon to its conical surface. On the conical surface the Huygens-Fresnel principle is invoked, and the packet is sampled onto the surface of the medium (cuvette). The sampling of the photon onto the surface of the

Volume/ μL Intralipid H_2O	Intralipid concentration		Optical properties Scattering coefficient/ m^{-1}
	Volume/%	Scattering particle/%	
0	500	0.00	0.00
2	500	0.39841	557.14
4	500	0.79365	1114.28
6	500	1.18577	1671.42
8	500	1.57480	2228.56
10	500	1.96078	2785.71
12	500	2.34375	3342.84

Table 3.1: Intralipid solutions used for experiment.

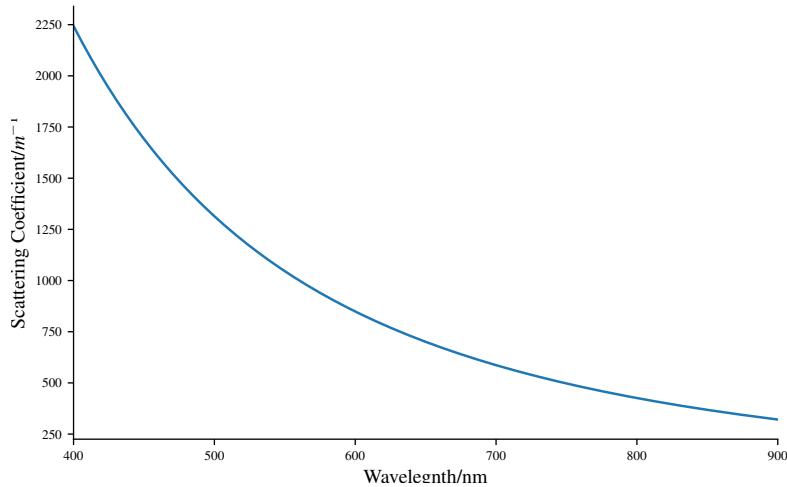


Figure 3.15: Scattering properties of 20% Intralipid [124].

medium, speeds the algorithm up, as it does not need to simulate the photons that would “miss” the medium. From there the usual MCRT method propagates the packet through the medium while tracking its phase, and scattering the packet until it leaves the medium. If the packet leaves the medium to any side other than the far side of the cuvette (e.g any side of the cuvette not facing the objective lens), then it is discarded. If the packet leaves the medium on the objective lens facing side, then the packet is recorded by its phase onto an area element. For each intralipid concentration 6.4×10^{10} photons are run over 64 cores, taking ~ 3 hours for the 0.5448% intralipid concentration. Once all the photon packets have been run, the phase is converted into intensity, as in Eq. (3.3), but in 2D.

Figure 3.16 shows the results from the experiment and simulation. The simulation shows good agreement with experimental data.

Discussion

Originally the medium was modelled as in the experiment, a 2 mm^3 volume. The image created was thus a 2001×2001 with a resolution of $1\text{ }\mu\text{m}$. In order to achieve a good signal to noise ratio for this setup 6.4×10^{12} packets needed to be run, taking ~ 70 hours on a computer cluster using 64 cores. This was enough packets to get a good signal to noise ration on all the simulations up to $6\text{ }\mu\text{L}$. However the amount of packets needed to get a good signal to noise ratio for $8\text{ }\mu\text{L}$ and above was prohibitively computationally costly. Therefore the modelled medium was shrunk in

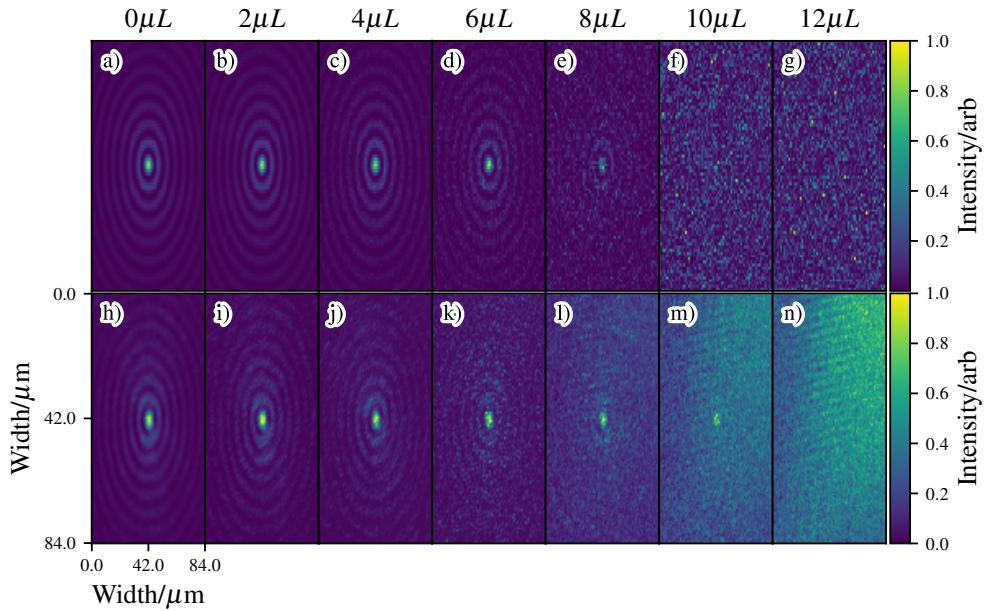


Figure 3.16: Comparison of experimental and simulation data for propagation of a Bessel beam produced by an axicon, through media of various turbidity. Images a) to g) is the data from φ MC, and h) to n) are the experimental data. Volumes along the top is the volume of Intralipid in each solution as in Table 3.1. All images cropped so they are the same size.

the x and y directions giving: $0.5 \text{ mm} \times 0.5 \text{ mm} \times 2.0 \text{ mm}$. This allowed a smaller image (501×501), whilst keeping the same resolution. Shrinking the medium also has the benefit that the photons are confined closer to the image plane, thus ensuring more photons are hit the plane in comparison to the larger medium.

Shrinking the medium size does have some draw backs. Firstly Bessel beams propagation depth rely on the input beams width see Eq. (3.31). The input beams width was kept constant between the shrinking of the volumes size. However shrink the medium size in the x and y directions gives the same effect as using a smaller input beam. Therefore the x and y dimension were carefully chosen such that the Bessel beam would still form a Bessel beam at the image plane. The second issue with shrinking the medium is that some packets may be lost. What this means is that, in the larger medium a packet may scatter towards an x or y medium wall and then scatter back into the centre of the medium and then is recorded. However this same packet in the smaller medium would be lost as the packet would exit the medium and ceased to be tracked. It is not expected that this will cause much of an issue as any scattering event already degraded the quality of the beam, as that packet is no longer coherent with the rest of the packets, thus it will not contribute positively to the Bessel beam. To ensure this is not an issue, results from a larger medium are compared to that of the smaller medium in Fig. 3.17. The larger and smaller medium yield the same results (within Monte Carlo noise) for Intralipid volumes less than $8 \mu\text{L}$. At $8 \mu\text{L}$ the smaller medium has a Bessel beams central core, whilst the larger medium is noisy, and forms no Bessel beam. This test has shown that shrinking the medium allows accurate modelling of the propagation of a Bessel beam through a turbid medium while using less computational resources.

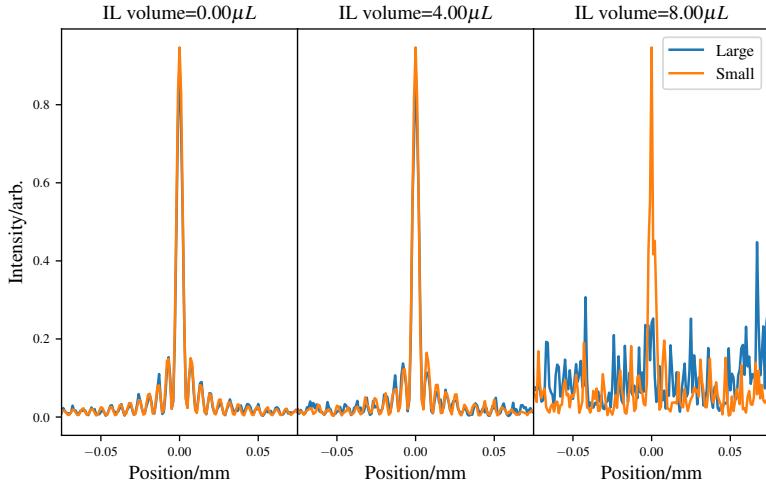


Figure 3.17: Comparison of a larger medium, 2 mm^3 versus that of a smaller medium, $0.5 \text{ mm} \times 0.5 \text{ mm} \times 0.00 \text{ mm}$.

3.5 Higher order Bessel beams

Our technique outlined in the preceding sections, can also be applied to arbitrary higher order Bessel beams.

The electric field of a Bessel beam is:

$$E(r, \varphi, z) = E_0 J_l(k_r r) e^{-ik_z z} e^{-il\varphi} \quad (3.35)$$

Where:

l is the order of the beam [-];

$k_z^2 + k_r^2 = k^2$, where k^2 is the wavevector [m^{-1}];

r, φ , and z are the cylindrical coordinates [m, rad, m];

and J_l is the l -order Bessel function of the first kind [-].

To generate higher order Bessel beam, a helicon is used. A helicon (shown in Fig. 3.18) is an axicon attached to a helix phase delay element. The helical element imparts a helical phase delay to photon packets as they pass through the element.

The distance travelled though the helicon is shown in Eqs. (3.36), (3.37) and (3.39) [125]. h_1 is the path length travelled by a photon through the helical element. h_2 is the path through an axicon.

$$h_1 = \frac{l\varphi\lambda}{(n-1)2\pi} \quad (3.36)$$

$$h_2 = r \tan(\alpha) \quad (3.37)$$

$$h_3 = h_1 + h_2 \quad (3.38)$$

$$\Delta h = \frac{l\lambda}{n-1} \quad (3.39)$$

Where φ is the azimuthal angle, r is the radial position, l is the order, and α is the axicon angle.

The path length in the above equations can be converted into a phase delay by considering the transmission functions of the individual elements [126–129]:

$$T_1(\varphi) = e^{-ik(n-1)h_1} = e^{-il\varphi} \quad (3.40)$$

$$T_2(r) = e^{-ik(n-1)h_2} = e^{-ik_r r} \quad (3.41)$$

$$T_3(r, \varphi) = T_1 * T_2 = e^{-ik_r r - il\varphi} \quad (3.42)$$

$$(3.43)$$

Where T_1 is the transmission function for the helical element, T_2 is the transmission function for the axicon, and T_3 is the total transmission function. Using the small angle approximation for β and Eq. (3.32), and knowing $k_r = \sin(\beta)$ yields the phase delay as a function of angle and radial position:

$$\phi(\varphi, r) = k(n - 1)r\alpha + l\varphi \quad (3.44)$$

To implement a helicon in the φMC algorithm, an additional helical phase delay is added. The additional delay is implemented by adding $l\varphi$ where $0 < \varphi < \frac{2\pi}{l}$. An actual helix element is not modelled explicitly in the code, but rather just the phase delay. This method is similar to using a spatial light modulator in an experiment to impart a phase delay on a beam.

Figure 3.19 shows the comparison between theoretical higher order Bessel beam and the higher order beam beam simulated by φMC .

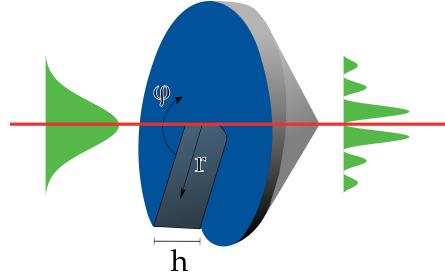


Figure 3.18: Helical delay element attached to an axicon. Axicon introduces an additional radial delay in addition to that of the helical element. Input beam is a Gaussian, output beam is a higher order Bessel beam, $l > 0$.

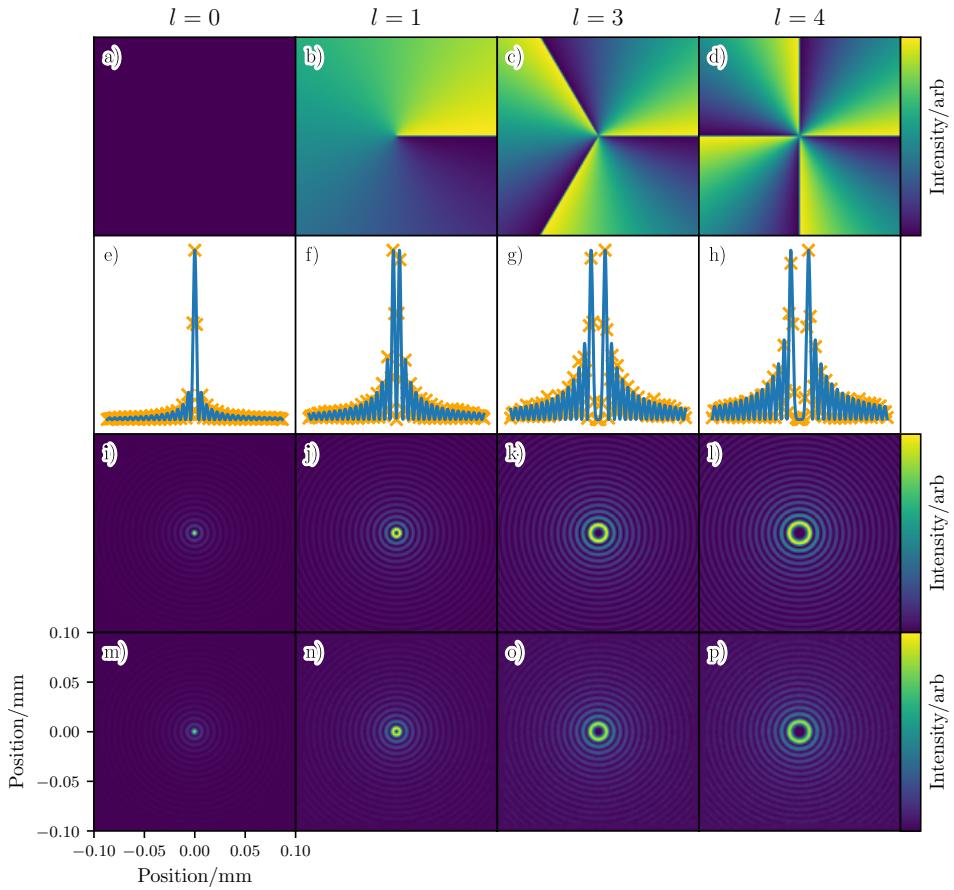


Figure 3.19: Higher order Bessel beams. a) to d) show the phase shift due to the helical element. e) to h) show line plots of the simulation data compared to the theory. i) to l) and m) to p) show the higher order Bessel beam images for theory and simulation data respectively.

3.6 Comparison

As Bessel and Gaussian beams are radically different from one another it is hard to directly compare the two beams. Gaussian beams carry all their power in the ‘central core’ of the beam, whereas in a Bessel beam, it carries the same amount of power in each ring. Bessel beams also have a much larger depth of focus than Gaussian beams. This section attempts to compare the two beams, to see which beam preform better in a heavily scattering medium.

To ensure a fairer comparison the Bessel beams central core width is set to that of the Gaussian beam’s waist.

$$r_0 = \frac{\kappa}{k \sin\beta} \quad (3.45)$$

Where κ is a constant that determines the metric used to measure the Bessel beam’s core, and the other symbols have the same meanings as before. For $\kappa = 2.408$ the radius is measured

from the maximum of the core to the first zero of the Bessel beam. $\kappa = 1.75$ measures the Bessel beam's core from the maximum to $\frac{1}{e^2}$ of the maximum. For both beams central cores to be equal, the axicon used to generate the Bessel beam is adjusted. This is achieved by calculating the 'correct' α based upon the optical setup used to focus the Gaussian beam. Using the small angle approximation* and $\kappa = 1.75$ we can compare the Bessel beam's core radius to a Gaussian beam's waist:

$$\frac{1.75\lambda}{2\pi\sin\beta} = \frac{2\lambda f}{\pi D} \quad (3.46)$$

$$\alpha = \frac{1}{n-1} \arcsin \left(\frac{1.75D}{4f} \right) \quad (3.47)$$

Where α axicon angle as before, n is the refractive index of the axicon, D is the $\frac{1}{e^2}$ diameter of the incident Gaussian beam on the lens, and f is the focal length of the lens. Both D and f are properties of the optical system used to focus the Gaussian beam.

The first comparison carried out between the Bessel and Gaussian beams is to use the same power to generate both beams. The beams are propagated through mediums of varying degrees of Intralipid solution. Volumes of 0, 20, 40, 60 μL are used of Intralipid in 500 μL of water. The medium has a volume of 0.04 mm \times 0.04 mm \times 0.2 mm. For both beams a wavelength of 488 nm and a power of 1 mW is used. One hundred million packets are simulated for each simulation. The results of this are shown in Fig. 3.20

Figure 3.20: First comparison of Bessel and Gaussian beams, with equal power used to generate both beams.

The results show that for the same power, Gaussian beams propagate deeper into the medium compared to Bessel beams. This is to be expected as in a Gaussian beam all the power is in its 'central core', whilst the power is evenly distributed between all the Bessel beam's rings. Therefore for a second comparison the power given to the Bessel beam is such that the central core maximum matches that of the Gaussian beam's for the case where there is no scattering. To achieve this the Bessel beam was given 1000mW. The results of this comparison are illustrated in Fig. 3.21.

Figure 3.21: Second comparison of Bessel and Gaussian beams for the case where the maxima of both Gaussian and Bessel beams match in the non scattering case.

3.7 Discussion

a [130]

3.8 Conclusion

*for small α and β : $\beta = (n - 1)\alpha$.

Chapter 4

AF

4.1 Problem

4.2 Validation

4.3 Practical application

4.4 Conclusion

Chapter 5

Modelling fluorescent images

5.1 Problem

Code ‘works’. Need to validate code against toy model to confirm (may need to do this on kennedy cluster). Then scale up to supercomputer level and run tests so we can get request time on Archer(Edinburgh).

Paper has a couple of sections complete, just theory so far. Most of paper to be written. Contact with Madrid again once code is on Archer??

5.2 Validation

5.3 Practical application

5.4 Conclusion

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