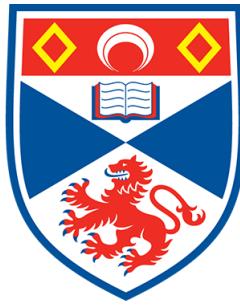


# Advanced 3D Monte Carlo Algorithms for Biophotonic and Medical Applications

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University of  
St Andrews

This thesis is submitted in partial fulfillment for the degree of  
PhD  
at the  
University of St Andrews

August 2019



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# Publications and Presentations

## Publications

- “*Development of a predictive Monte Carlo radiative transfer model for ablative fractional skin laser*”, **L. McMillan**, P. O’Mahoney, K. Feng, K. Zhou, IRM. Barnard, C. Li, S. Ibbotson, E. Eadie, CTA. Brown, and K. Wood, *Scientific Reports (in prep)*
- “*φMC, a phase tracking algorithm for Monte Carlo radiation transfer codes to model light propagation and interference in a turbid media*”, **L. McMillan**, S. Reidt, C. McNicol, IRM. Barnard, CTA. Brown, MP. MacDonald and K. Wood, *Biomedical Optics Express (in prep)*
- “*Quantifying direct DNA damage in the basal layer of skin exposed to UV radiation from sunbeds*” IRM. Barnard, P. Tierney, CL. Campbell, **L. McMillan**, H. Moseley, E. Eadie, CTA. Brown, K. Wood, *Photochemistry and photobiology* 94.5 (2018): 1017-1025.
- “*Could psoralen plus ultraviolet A1 (PUVA1) work? Depth penetration achieved by phototherapy lamps*”, IRM. Barnard, E. Eadie, **L. McMillan**, H. Moseley, T. Brown, K. Wood, R. Dawe, *British Journal of Dermatology* (2019)

## Presentations

- “*Investigating novel cardiovascular disease biomarkers using Monte Carlo simulations*”, British Medical and Lasers Association Conference (BMLA), Darlington, May 2016
- “*Recovering fluorescent depth information using Monte Carlo radiative transfer method and Genetic Algorithms*”, British Medical and Lasers Association Conference (BMLA), Manchester, May 2017
- “*MCRT in medical physics*”, Invited speaker, St Andrews Monte Carlo Summer School (SAMCSS), St Andrews, August 2017
- “*Parallel Computer Simulations of Light-Tissue Interactions for Applications in Medicine, Cosmetics Industry and Biophotonics Research*”, Invited speaker, St Andrews Computer Science School Seminar, St Andrews, April 2019
- “*Numerical model of laser tissue ablation and thermal injury*”, British Medical and Lasers Association Conference (BMLA), London, May 2019
- “*Medical applications of MCRT*”, Invited speaker, St Andrews Monte Carlo Summer School (SAMCSS), St Andrews, August 2019

## **Posters**

- “*Numerical model of laser tissue ablation and thermal injury*”, International Conference of Biophotonics (ICOB), St Andrews, May 2019

## **Awards**

- British Medical Laser Association (BMLA) Best talk, 2019

## **Summer Schools**

- Hands-on introduction to HPC, EPCC, Edinburgh 2016
- Message-passing programming with MPI, EPCC, Edinburgh, 2017

# Abstract

The Monte Carlo radiation transfer (MCRT) method can simulate the transport of light through turbid media. MCRT allows the modelling of multiple anisotropic scattering events, as well as a range of microphysics such as polarisation and fluorescence. This thesis concerns the development of several MCRT algorithms to solve various biophotonic and medically-related problems including modelling of tissue ablation and autofluorescent signals. An extension of the MCRT method through a theoretical quasi-wave/particle model is also demonstrated, allowing beam shapes with arbitrary phase profiles to be propagated.

Tissue ablation can be used to treat acne scarring, Rhinophyma, and it can also be used to help enhance topical drug delivery. Currently the depth of ablation is not easily elucidated from a given laser or laser power setting. Therefore, a numerical tissue ablation model is developed using a combination of MCRT, a heat diffusion model, and a numerical tissue damage model to assess ablation crater depth and thermal damage to the surrounding tissue.

Autofluorescence is the natural fluorescence of biological structures in tissue. Autofluorescence can be used as a biomarker of several diseases including: cardiovascular diseases, Alzheimers, and diabetes. However, the origin of the autofluorescence signal is not completely clear. The effect of tissue optics on the signal, which fluorophores contribute to the signal and by how much, and how different locations on the body can affect the signal are not well understood. This thesis presents a study of the effect of tissue optics on the autofluorescent signal. As part of this study, AmoebaMCRT was created to determine the relative concentrations of fluorophores for a given autofluorescent signal.

Finally, we developed an extension to the MCRT method which allows the simulation of quasi-wave/particles. This method relies on the Huygens-Fresnel principle and the tracking of the phase of each individual photon packet. The extension,  $\varphi$ MC, allows the modelling of complex beams that require the wave properties of light such as arbitrary order Bessel beams and Gaussian beams. We then use  $\varphi$ MC to predict which beam, Bessel or Gaussian, performs “better” in a highly turbid medium.



# Acknowledgements

First of all, thank you to my supervisors Dr. Kenny Wood and Prof. Tom Brown for allowing me the opportunity to undertake this PhD, and for all their support and encouragement over the course of my thesis. If it weren't for you both keeping me on the straight and narrow I'm sure I would still be coding instead of writing. Also, I swear my code is now bug free...

Thank you to Isla for answering all my stupid questions over the past three years, and for being a good source of moral support, and good company at conferences. Also thanks for co-founding, and mostly running, Code & Cake. I thoroughly enjoyed it, both the baking and the talks. Finally, thanks for reading over this thesis.

Thank you to Sascha, Salvo, Blanca, Faisal, Danni, and Mike for allowing me to collaborate with you, and for your kindness with the time you gave me.

Thank you to everyone at Ninewells for making me feel welcome every time I took up space in your research office. In particular thanks to Luke, Paul, and Ewan for letting me collaborate with you, and for your good company at the conferences we attended together.

Thank you to the whole McConnell family; Evelyn, Stevie, and Heather, for welcoming me to stay at your home whilst Kathleen and I were between homes, and for finding accommodation for us both. I would also like to thank you for everything that you have done for me over these past four years, it has made it so much easier to complete this thesis. Thank you also to the thesis fairy for the much needed Haribo as I was writing up.

A huge thank you to my Mum, Rachel, Eilidh, Allan, Caroline, and Archie, for all the support you have given to me over my last nine years at St Andrews. All your frequent messages on the family chat have been a source of entertainment and much needed distraction. I can't begin to list everything you have done for me, but I really appreciate knowing that you are always there for me.

Finally, last but certainly not least, to Kathleen. This thesis would never have been completed if it wasn't for your constant love and support. Thank you for understanding when I got stressed or chose to work weekends and nights on my PhD, and even on the occasions where I never noticed the time and forgot to meet you. Consider reading this as the favour repaid for when I read your **two** dissertations!

## **Funding**

This work was supported by the Engineering and Physical Sciences Research Council [grant number EP/K503162/1].

## **Research Data/Digital Outputs access statement**

Research data underpinning this thesis are available at TBC





For Lil and Marion.



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# Abbreviations

$T_a$  ablation temperature.

**AMR** adaptive mesh refinement.

**CDF** cumulative distribution function.

**FDM** finite difference method.

**K-M theory** Kubelka-Munk theory.

**MCRT** Monte Carlo radiation transfer.

**MPI** Message-passing interface.

**OCT** optical coherence tomography.

**PDF** probability density function.

**PDT** photo-dynamic therapy.

**RTE** radiative transfer equation.

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# Chapter 1

## Introduction

This thesis is concerned with the development of several Monte Carlo radiation transfer (MCRT) algorithms for biophotonic and medical applications. The MCRT method allows the simulation of light propagation through turbid media whilst undergoing multiple anisotropic scattering, absorption and a range of other micro-physics (see Chapter 2 for full discussion of the theory). MCRT was invented at the end of World War II for the purpose of calculating the paths of neutrons through various media [1–4]. As the codes developed around this period focused on modelling the transport of protons, gamma rays and other nuclear particles, it was a small leap to modelling the dose received by a patient undergoing radiation therapy [5]. However, the jump to modelling light transport in skin would take a further two decades [6], and another decade until it became popular and the *de-facto* gold standard in light tissue interaction modelling [7,8]. It has since been used to optimise photo-dynamic therapy, quantifying DNA damage from UV light sources, and modelling treatment of port wine stain removal amongst various other medical treatments and diagnosis methods [9–11]. The real power of the MCRT method is that it allows the model to be tailored to the patient. This means the medical treatment can be personalised to each individual patient rather than an average corpus of patients.

Personalised medicine entails having fine grained knowledge of the patient down to the genome level, to understand how various drugs or treatments will affect the patient. One particular area of research in personalised medicine is into the so called “digital twin”. A digital twin as defined by A. El Saddik as [12]:

“... is a digital replica of a living or non-living physical entity.”

Digital twins are currently heavily used in engineering to predict when various machinery will need to undergo maintenance. Digital twins operate by modelling their physical counterpart. This model is updated via information from their physical counterpart which allows the digital twin to predict its physical counterparts future behaviour. Companies like Phillips use this in their MRI machines to help schedule downtime, and predict which parts the engineer will need on site, both of which minimises the downtime of the machine which is important for the hospital or clinic [13].

At the heart of the digital twin method is the ability to accurately model the object or living thing being studied. This can be straightforward when the twin in question is a machine, as sensors can usually be attached to the various components to get feedback on the machine’s operation. Machines also have the bonus of (normally) being well understood so that modelling them is usually relatively straightforward. However, this is not as easy when dealing with living

systems. First, we still do not have a complete understanding of the biology within humans. Therefore, modelling a human accurately is not possible as various assumptions and approximations have to be made. Second, to get accurate information on what is happening inside a patient, generally either ionising radiation needs to be used or cameras inserted into the patient. Both of these cannot be done for indefinite periods without causing harm or discomfort. Therefore, continuous information on the inner functions of the body is not possible. One area where information is more readily available is the skin. Information on the skin function or dysfunction is normally diagnosed with light. Light is also used in various treatments such as photodynamic therapy and tissue ablation, over various internal and external sites on the body. Light's interaction over the whole spectrum, from the UV to the infrared, is readily modelled with techniques such as MCRT. MCRT allows a digital twin model of the individual patient's skin to be simulated. This can then be used to tailor treatment regimes for the patient, or to predict treatment outcomes for specific patients. The use of simulation techniques like MCRT allow testing *in-silico*, and can negate the need to test on humans or animals. MCRT is already heavily used to plan radiation therapy treatments [14, 15], though this has yet to make the leap to light-tissue interaction modelling.

## 1.1 Monte Carlo Method

The Monte Carlo method is a numerical analysis technique based upon random numbers, which are used to calculate unknown variables in problems [16, 17].

The earliest use of the method is in Buffon's needle experiment of the 18<sup>th</sup> century [18–20]. Buffon asked the question;

"Suppose we have a floor made of parallel strips of wood, each the same width, and we drop a needle onto the floor. What is the probability that the needle will lie across a line between two strips?"

The solution to this for a needle of length  $l$ , a strip separation  $s$ ,  $\theta$  is the angle of the needle with respect to the wood strips, and where  $x$  is the distance from the needle to the closest line,. Then, using a simple geometrical argument, a needle crosses a strip if  $x \leq \frac{l}{2} \sin\theta$ .

$x$  is distributed uniformly in  $[0, \frac{s}{2}]$ , and  $\theta$  in  $[0, \frac{\pi}{2}]$ . Therefore the probability density function for  $x$  is  $p(x) = \frac{2}{s}$ , and  $\theta$  is  $p(\theta) = \frac{2}{\pi}$ . The probability density function (PDF) is a function of a variable that gives the probability for a variable to take a given value. The PDF is normalised over the whole range of the variable, in this case  $x$ , and  $\theta$ . Thus, as  $x$  and  $\theta$  are independent variables, giving a joint probability of  $p(x, \theta) = \frac{4}{s\pi}$ . So the probability of a needle of length  $l$  crossing a line ( $l < s$ ) is:

$$P = \int_0^{\frac{\pi}{2}} \int_0^{\frac{l}{2} \sin\theta} \frac{4}{s\pi} dx d\theta = \frac{2l}{s\pi} \quad (1.1)$$

Equation (1.1) can be used to carry out a Monte Carlo estimation of  $\pi$ . A simple rearrangement yields:  $\pi = \frac{2l}{sP}$  where  $P$  is the ratio of needles crossing the line to the total number dropped. Laplace was the first to suggest that Buffon's needle experiment could be used to estimate  $\pi$  [19]. Figure 1.1 shows an example of a simulation of Buffon's needle experiment.

There are various different approaches to using the Monte Carlo method to obtain randomly sampled variables. One analytical way of achieving this is the inverted sampling method. The inversion method allows the mapping of one or more uniform random variables to the random variables from the desired distribution [21]. The inverted sampling method can be summarised by the following steps for drawing a sample  $X_i$  from an arbitrary PDF  $p(x)$ :

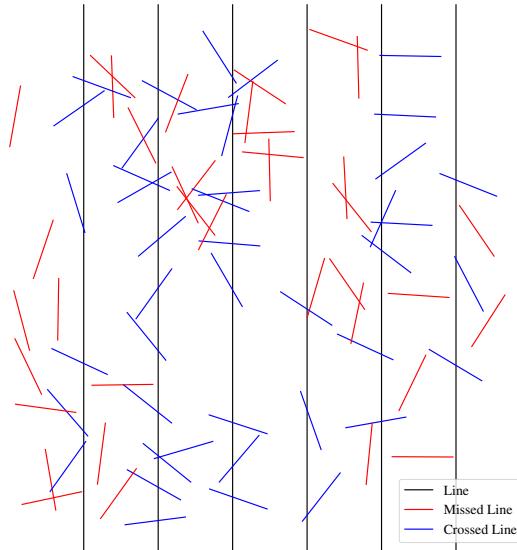
1. Compute the cumulative distribution function (CDF)  $P(x) = \int_0^x p(x')dx'$
2. Obtain a uniformly distributed random number  $\xi$  in the range  $[0.,1.)$
3. Compute the inverse  $P^{-1}(x)$
4. Finally, compute  $X_i = P^{-1}(\xi)$

For example, taking  $P(x) = \cos x$ , we want to map  $\xi$  to  $x$ :

$$\xi_i = \int_0^{x_i} P(x)dx = \sin \theta_i \quad (1.2)$$

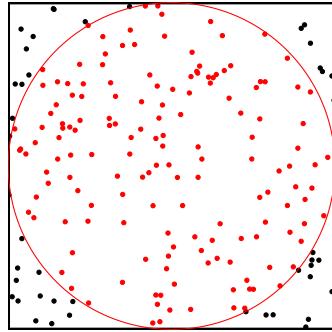
$$\therefore x_i = \sin^{-1} \xi_i \quad (1.3)$$

If a given problem cannot use the inverted sampling method, as it may not be possible to get a PDF or analytically invert the CDF, then the rejection method can be used. The rejection method is essentially a dart throwing method. This means that points are randomly chosen and compared to the function. If the point lies under the function then the point is accepted, if it lies above the function then it is rejected.



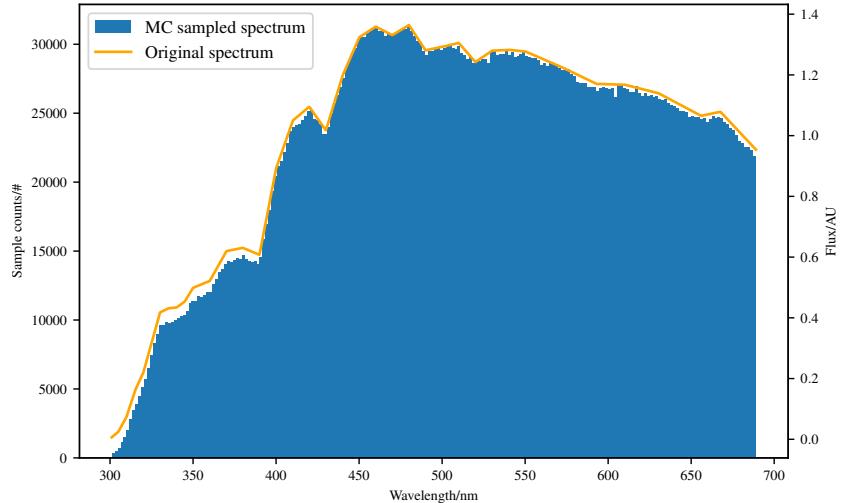
**Figure 1.1:** Sample Buffon needle experiment. 100 needles are dropped on a  $10 \times 10$  cm area with lines spaced 1.5 cm apart. If a needle lands on a line it is recorded and coloured blue, if it does not land on a line it is coloured red. This simulation gave a value of  $\pi \approx 3.10$ .

For example, if a function,  $f(x)$  does not have an analytical PDF, we can use a PDF  $p(x)$  such that  $f(x) < cp(x)$  where  $c$  is a constant. Therefore, if we sample from  $p(x)$  and if the point lies under  $f(x)$  then it is accepted, else it is rejected. Figure 1.2 shows an example of this process.



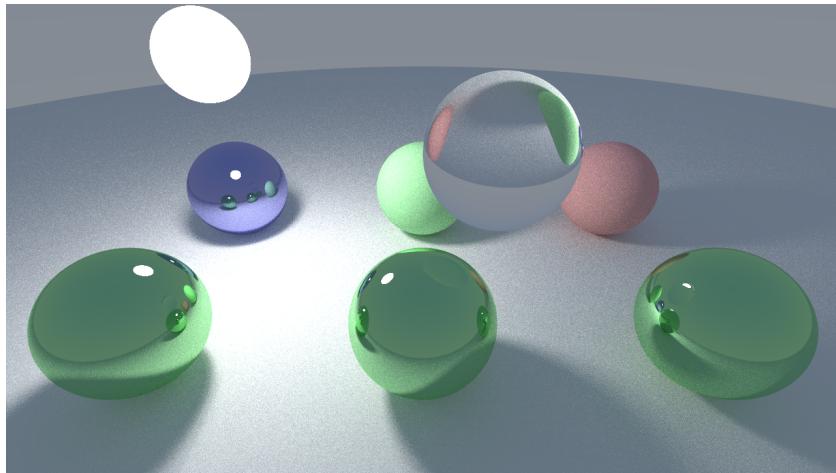
**Figure 1.2:** Illustration of the rejection method for determining  $\pi$  from the area of a circle inscribed within a square. The ratio of the area of the circle to the square is  $\frac{\pi}{4}$ . Thus, the ratio of darts landing in the circle to those that land outside the circle is  $\pi \approx \frac{4N_{inner}}{N_{total}}$ , where  $N_{total}$  is the total number of darts, and  $N_{inner}$  is the total number of darts that land in the circle. Using 200 darts gave a value of  $\pi \approx 3.12$

One common use of the Monte Carlo method is to randomly sample from a spectrum. To generate a random sample from a spectrum, first the CDF of the spectrum must be calculated. This is done by first normalising the PDF, where the PDF in this case is the spectrum itself. It is normalised such that the sum of the PDF is unity. The CDF is then just the cumulative sum of the PDF. Then using the above method as described above, a random number is drawn,  $\xi$ , and the bracketing values in the CDF are found. We then interpolate to get the x value corresponding to  $\xi$ . Figure 1.3 shows the result of this process for  $5 \times 10^6$  random samples.



**Figure 1.3:** Example of randomly sampling from a spectrum. Figure shows 100 random samples drawn to recreate a solar spectrum.

The Monte Carlo method is used in various different disciplines. Ranging from use in the financial sector to analyse investments and stocks by simulating the sources of uncertainty which affect their values [22, 23], use in statistical analysis [24], and in modern computer generated images (see Fig. 1.4) [25, 26]. It is also widely used in astronomy [27, 28] and medicine [11, 29], to simulate the propagation of radiation through scattering (turbid) media. This technique, Monte Carlo radiation transfer (MCRT), is what makes up the bulk of this thesis and is described in depth in the following sections.



**Figure 1.4:** Computer generated imagery using ray tracing. The Monte Carlo method is used to “compute radiance along ray paths between lights and the camera”, to generate CGI images [21].

## 1.2 Synopsis and Thesis Objectives

Chapter 2 details the MCRT method which is used for the bulk of this thesis. Presented in this chapter is the theory behind the MCRT method along with why it is considered the *de-facto* standard in light-tissue interaction modelling. This chapter also presents a discussion on how the MCRT method is translated into code. Details of speed up techniques such as parallelisation are also presented. Finally, to ensure the code is accurate, it is validated against another MCRT code.

Chapter 3 details the tissue ablation model developed as part of this thesis. The tissue ablation model was created to predict the depth of ablation craters in tissue for a given laser power. The model could also be used to help optimise treatment regimes in cosmetic and medical procedures. The numerical tissue ablation model consists of 3 numerical methods: MCRT to model the light transport in the tissue, numerical heat model to model the heat diffusion within the tissue, and a numerical tissue damage model to assess the damage to the tissue due to the laser. The predictive power of the method is demonstrated by the way of optimising spy disposal. Discussion of the model, alongside validation of the model against theoretical and experimental evidence is presented.

Chapter 4 presents an adaptation to the regular Monte Carlo model so it can model wave like properties of photons including diffraction and interference. The new algorithm is validated against several common experiments that exhibit the wave like behaviour of light, including: diffraction by slits and apertures. The algorithm is then used to model Bessel beams and shows their “self-healing” effect. Gaussian beams are also modelled using this new algorithm. The algorithm is also validated against real experimental data, taken by collaborators at the University of Dundee. Finally the algorithm is used to compare Bessel beams and Gaussian beams in highly turbid media, to determine which beam can image deepest.

Chapter 5 details the modelling of a novel biomarker for cardiovascular disease, autofluorescence. The theoretical groundwork for the biomarker is presented along with discussion of how MCRT can model fluorescence. Presented alongside this is ameombaMCRT, a Monte Carlo ra-

diation transfer simplex algorithm used to determine concentrations of fluorophores in different layers of tissue for a given spectrum. Finally, a study of how tissue optics affect the autofluorescent signal is presented. This study investigates the effect of blood, melanin, and skin thickness can have on the autofluorescent signal.

Finally, chapter 6 concludes this thesis and presents possible future avenues of research which could be undertaken.

# Chapter 2

# Monte Carlo Radiation Transport Technique

## 2.1 Introduction

This chapter will provide an overview of the Monte Carlo radiation transport method (MCRT) method and compares it to other light transport methods. Details of the MCRT code developed during this project and used as the basis of the results reported in subsequent chapters, validation of code, and details of computational speed ups, are also presented.

## 2.2 Monte Carlo Radiation Transport Algorithm

### 2.2.1 Introduction and Background

The technique that makes up the bulk of this thesis is the MCRT technique. This method was developed at the end of World War II at the Los Alamos National Laboratory, for the purpose of calculating neutron diffusion through shielding material [1–4]. It has since found a myriad of applications from light transport through dusty galactic clouds [30], calculating doses for radiotherapy [31] to light transport through tissue [32].

The theory that governs the transport of radiation through a medium is the radiative transfer equation (RTE). Before describing MCRT, which is a numerical simulation of the RTE, the theory of radiative transport must be examined.

### Radiative Transfer

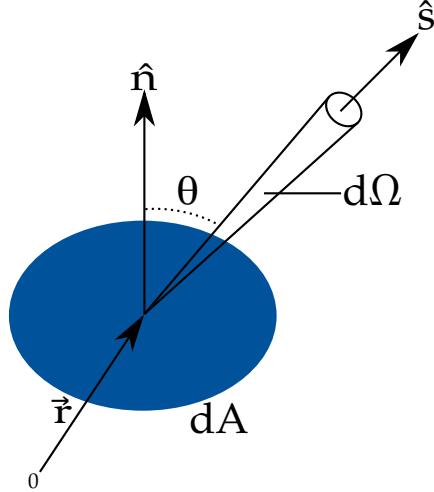
Transport of radiant energy through turbid media can be modelled analytically using the RTE. The RTE models the radiative losses and gains by a beam of radiation as it propagates, including: loss of energy due to absorption, loss/gain of energy due to scattering, and energy gain due to emission. Before deriving the RTE, definitions of some terms and physical quantities is required.

The first term is spectral irradiance,  $L_\nu$ . Spectral irradiance is defined as the energy flow in a direction  $\hat{s}$ , for a solid angle  $d\Omega$ , per unit time per unit temporal frequency bandwidth. Irradiance is defined as the spectral irradiance over a small frequency range  $[\nu, \nu + \Delta\nu]$ :

$$L(\vec{r}, \hat{s}, t) = L_\nu(\vec{r}, \hat{s}, t)\Delta\nu \quad (2.1)$$

Where:

- $\vec{r}$  is the position;
- $\hat{s}$  is a unit normal vector;
- $t$  is the time;
- and  $L(\vec{r}, \hat{s}, t)$  is the irradiance [ $W m^{-2} sr^{-1}$ ].



**Figure 2.1:** Energy flow through area  $dA$  within solid angle  $d\Omega$  in a direction  $\hat{s}$ . Adapted from [33, 34].

The irradiance can be used to determine the energy,  $dE$ , transported across an area  $dA$ , in a solid angle  $d\Omega$  in a time  $dt$  (see Fig. 2.1) is:

$$dE = L(\vec{r}, \hat{s}, t) \cdot \cos(\theta) \cdot dA \cdot d\Omega \cdot dt \quad (2.2)$$

Where:

- $\hat{n}$  is the unit normal to  $dA$ ;
- and  $\cos(\theta)$  is the angle between  $\hat{n}$  and  $\hat{s}$ .

Irradiance can also be used to determine the fluence rate,  $\phi$ , which is defined as the energy flow per unit time, independent of the flow direction.

$$\phi(\vec{r}, t) = \int_{4\pi} L(\vec{r}, \hat{s}, t) \cdot d\Omega \quad (2.3)$$

Where:

- $\phi$  is the fluence rate [ $W m^{-2}$ ].

Solving the RTE yields the irradiance which gives the distribution of light in the medium, and information on the state of the system.

With the irradiance defined, as well as the other quantities that follow, the RTE can be derived [33, 34]. First considering the conservation of energy, as shown in Eq. (2.4).

$$dP = -dP_{div} - dP_{ext} + dP_{scatt} + dP_{src} \quad (2.4)$$

Where:

- $dP$  is the total change in energy in the volume  $dA \cdot ds$  within the solid angle,  $d\Omega$ , per unit

time (see Fig. 2.2);

$dP_{div}$  is the energy loss due to the divergence of the radiation beam per unit time;

$dP_{ext}$  is the energy loss due to absorption and scattering within the volume  $dA ds$  within the solid angle,  $d\Omega$ ;

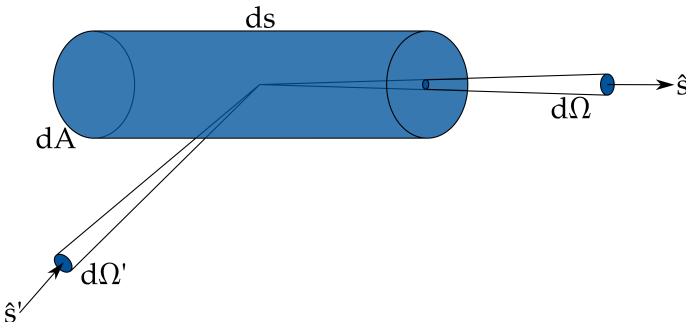
$dP_{scatt}$  is the energy gain due to scattering from  $\hat{s}'$  into  $d\Omega$  per unit time;

and  $dP_{src}$  is the energy gain due to emission within the medium, per unit time.

The total change in energy,  $dP$ , in the volume element within the solid angle  $d\Omega$  is equal to:

$$dP = \frac{1}{c} \frac{\partial L(\vec{r}, \hat{s}, t)}{\partial t} dV d\Omega \quad (2.5)$$

Where  $c$  is the speed of light.



**Figure 2.2:** Cylindrical volume element,  $ds dA$ , with solid angle  $d\Omega$  in direction  $\hat{s}$  and solid angle  $d\Omega'$  in direction  $\hat{s}'$ . Energy flowing through this element is used to derive the radiation transfer equation. Adapted from [33, 34].

The first loss term,  $dP_{div}$ , is the energy loss due to divergence of the radiation beam. This is modelled as:

$$dP_{div} = \frac{\partial L}{\partial s} d\Omega dV \quad (2.6)$$

$$= \hat{s} \cdot \nabla L(\vec{r}, \hat{s}, t) d\Omega dV \quad (2.7)$$

$dP_{ext}$  is the second loss term and accounts for energy loss due to scattering and absorption in the volume element within the solid angle  $d\Omega$ . This is modelled as:

$$dP_{ext} = \mu_t ds L(\vec{r}, \hat{s}, t) dA d\Omega \quad (2.8)$$

Where  $\mu_t$  is the extinction coefficient [ $m^{-1}$ ], see Section 2.2.2 for further details.

The first energy gain term,  $dP_{src}$ , is due to emission in the volume element within the solid angle  $d\Omega$  from a source,  $S(\vec{r}, \hat{s}, t)$  [ $Wm^{-3}sr^{-1}$ ].

$$dP_{src} = S(\vec{r}, \hat{s}, t) dV d\Omega \quad (2.9)$$

The second energy gain term, and final term, is due to the incident energy on the volume element within the solid angle  $d\Omega$  in direction  $\hat{s}$  due to scattering from any direction  $\hat{s}'$ .

$$dP_{scatt} = N_s dV \left( \int_{4\pi} L(\vec{r}, \hat{s}', t) P(\hat{s}', \hat{s}) \sigma_s d\Omega' \right) d\Omega \quad (2.10)$$

$$= \mu_s dV \left( \int_{4\pi} L(\vec{r}, \hat{s}', t) P(\hat{s}', \hat{s}) d\Omega' \right) d\Omega \quad (2.11)$$

Where:

$N_s$  is the number density of scatterers [Number of scatterers  $m^{-3}$ ];

$P(\hat{s}', \hat{s})$  is the scattering phase function (see Section 2.2.2 for further discussion);

and  $\sigma_s$  is the cross section of the scatterers [ $m^2$ ], thus  $\mu_s = N_s \sigma_s$ , where  $\mu_s$  is the scattering coefficient [ $m^{-1}$ ].

Finally substituting Eqs. (2.5), (2.7) to (2.9) and (2.11) into Eq. (2.4) yields the RTE:

$$\frac{1}{c} \frac{\partial L(\vec{r}, \hat{s}, t)}{\partial t} + s \cdot \nabla L(\vec{r}, \hat{s}, t) = -\mu_t L(\vec{r}, \hat{s}, t) + \mu_s \int_{4\pi} p(\hat{s}, \hat{s}') L(\vec{r}, \hat{s}', t) d\Omega' + S(\vec{r}, \hat{s}, t) \quad (2.12)$$

In general, the RTE is hard to solve in arbitrary 3D geometries, however there are several approximations, and numerical methods available. The diffusion approximation, Kubelka-Munk theory (K-M theory), and MCRT are the common methods used to approximate or solve the RTE. This section discusses in brief each of these methods and their respective pitfalls, and positive aspects.

### Kubelka-Munk Theory

The K-M theory was originally developed to calculate the light distribution in thin layered materials, such as paint or paper [35]. The theory is rather simple and makes many assumptions about the medium and the incident light. The main assumptions of K-M theory are: only scattering and absorption take place in the medium, the incident light is already diffuse, the medium is uniform with only isotropic scattering, no external or internal reflections, and the medium is planar and infinitely wide [36–38].

These assumptions make K-M theory poor for modelling light-tissue interactions. This is because in tissue, scattering is not isotropic but rather forward biased (see Section 2.2.2). Tissue is rarely planar and infinitely wide. Tissue also has some reflections at its external and internal boundaries, due to changes in refractive indices. Many medical and biophotonic treatments or methods use laser light which is not diffuse. Finally, tissue can also exhibit fluorescence, which the K-M theory is not able to model, along with polarisation. K-M theory does have some positive aspects. K-M theory is good at calculating the diffuse reflectance of simple media, and can be used to roughly estimate calculations. Though it is not well suited for modelling light-tissue applications [39].

### Diffusion Approximation

The diffusion approximation for the RTE is where the irradiance is separated into two components:

$$L(\vec{r}, \hat{s}) = L_c(\vec{r}, \hat{s}) + L_d(\vec{r}, \hat{s}) \quad (2.13)$$

Where  $L_c$  is the unscattered contribution, which satisfies Beer's law\*, and  $L_d$  is the diffuse contribution. The  $L_d$  component is expanded using Legendre polynomials and truncated. The diffusion approximation also has several assumptions and restrictions. The main assumption is that scattering dominates over absorption, and the scattering is nearly isotropic. This restricts the types of scattering the diffusion approximation can model, though using similarity relations can partially model scattering in tissue [40, 41].

The diffusion approximation is computationally fast and simple to implement. However, it is poor at modelling light-tissue interactions due to its assumptions and restrictions, mainly the inaccurate modelling near the boundaries of the medium and its lack of modelling fluorescence and other microphysics. Though it can be used to speed up MCRT in mediums where scattering dominates [42, 43].

## MCRT

The final method, MCRT, is numerically equivalent to the RTE [33]. MCRT is a flexible method. MCRT can model arbitrary 3D geometries, and various microphysics including fluorescence and polarisation. MCRT can also model various different light sources, from collimated laser beams to diffuse light sources. The only downside noted in the literature is that the MCRT can be expensive computationally. However, with computational power growing faster with time, this is less of a problem going forward.

The next several sections give an in-depth description of the MCRT method and its flexibility, along with a description of the code used in this thesis to solve various medical and biophotonic problems.

### 2.2.2 Optical Properties

Before an in-depth description of the MCRT method is outlined, a discussion of the optical properties of materials is presented, which the MCRT method requires to simulate the transport of photons in a medium.

Optical properties of a medium are the properties that describe how light is transported through that medium. Usually the optical properties of a medium are defined by three main parameters: the scattering ( $\mu_s$ ), anisotropy ( $g$ ), and absorption ( $\mu_a$ ) coefficients.

#### Scattering

The scattering coefficient, along with the anisotropy value (see [Anisotropy](#)), define how light is scattered in a medium. The main scatterers in the deeper layers of the skin are filamentous proteins such as collagen and elastin [44]. In the upper layers of the skin, the main scatterers are keratins and various chromophores such as melanin. The size of the scatterers affect how light is scattered and into which direction that light is scattered into.

The scattering of light within tissue is usually defined as  $\mu_s$  or  $\mu'_s$ : the scattering coefficient and the reduced scattering coefficient, where  $\mu'_s = \mu_s(1-g)$ . The scattering coefficient is defined such that the probability of transmission without scattering and neglecting absorption in a path length L is:

$$T = e^{-\mu_s L} \quad (2.14)$$

---

\*Beer's law (or Beer-Lambert law) states that the transmission,  $T$ , is equal to  $e^{-\mu L}$ , where  $L$  is the distance and  $\mu$  is the attenuation coefficient.

This gives units of inverse length for the scattering coefficient (usually measured in  $cm^{-1}$  in medical applications). The reduced scattering coefficient is often given in place of the scattering coefficient, as the reduced coefficient is more easily measured than the “normal” coefficient [45].

### Anisotropy

Anisotropy is the degree of deviation that light undergoes at each scattering event. The anisotropy value is taken from the phase function for the medium. The phase function is defined as the angular distribution of light intensity scattered by a particle. The phase function,  $\Phi(\theta, \phi)$ , is usually normalised over all angles:

$$\int_{\Omega} \Phi(\theta, \phi) d\Omega = 1 \quad (2.15)$$

Where  $\theta$  and  $\phi$  are the usual polar and azimuthal spherical angles, and  $d\Omega = \sin \theta d\theta d\phi$ . Thus, for Rayleigh and isotropic scattering, their phase functions are:

$$\Phi_{isotropic}(\theta, \phi) = \frac{1}{4\pi} \quad (2.16)$$

$$\Phi_{Rayleigh}(\theta, \phi) = \frac{3}{16\pi} (1 + \cos^2(\theta)) \quad (2.17)$$

For simplicity, the phase function is usually cast as the anisotropy value  $g$ , which is defined as the average angle of deflection:

$$g = \langle \cos(\theta) \rangle = \int_{\Omega} \cos \theta \Phi(\theta, \phi) d\Omega \quad (2.18)$$

The anisotropy factor,  $g$ , can take on any value from  $-1$  to  $1$ . Where a value of  $-1$  is totally back scattering,  $0$  is isotropic scattering, and  $1$  is totally forward scattering (see Fig. 2.3). For Rayleigh scattering the anisotropy factor is  $0$ , as the scattering is forward/backward symmetric.

There are many phase functions which can be used to model the anisotropy factor in a medium. The standard phase function in biological tissue is the Henyey-Greenstein phase function. The Henyey-Greenstein phase function was originally created to model scattering of diffuse radiation in the galaxy [46, 47]. It has since become the *de-facto* phase function for biological tissue. This is due to the phase functions relative simplicity and due to it being regarded as a “good” phase function for approximating scattering in biological tissue [48]. The Henyey-Greenstein phase function is shown in Eq. (2.19):

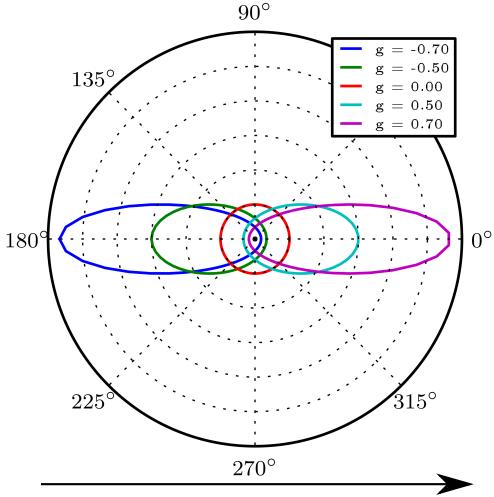
$$\Phi_{H.G}(\theta, \phi) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos(\theta))^{\frac{3}{2}}} \quad (2.19)$$

The Henyey-Greenstein phase function will be adopted as the phase function for the whole of this thesis.

### Absorption

Absorption of light by a medium is defined by the absorption coefficient  $\mu_a$ . The absorption coefficient is defined in a similar fashion to the scattering coefficient, by considering the probability of transmission without absorbing and neglecting scattering in a path length  $L$ :

$$T = e^{-\mu_a L} \quad (2.20)$$



**Figure 2.3:** Figure shows the  $g$  factor for the Henyey-Greenstein phase function, for various configurations of back, forward or isotropic scattering. Arrow indicates the photons initial direction before scattering.

This, again like the scattering coefficient, gives inverse distance for the unit of the absorption coefficient (and usually measured in units of  $\text{cm}^{-1}$ ).

There are various sources of absorbers in tissue including blood, water, fat, melanin,  $\beta$ -carotene, and bilirubin. These chromophores can all contribute, depending on the wavelength, with some more absorbing than others, as shown in Fig. 2.4. The absorbed photons can then be remitted as fluorescence or absorbed as heat.

### Derived Parameters

There are also some derived parameters that are useful to be defined. These are the albedo and the total attenuation coefficient.

The total attenuation coefficient is defined as the sum of the scattering coefficient and the absorption coefficient:

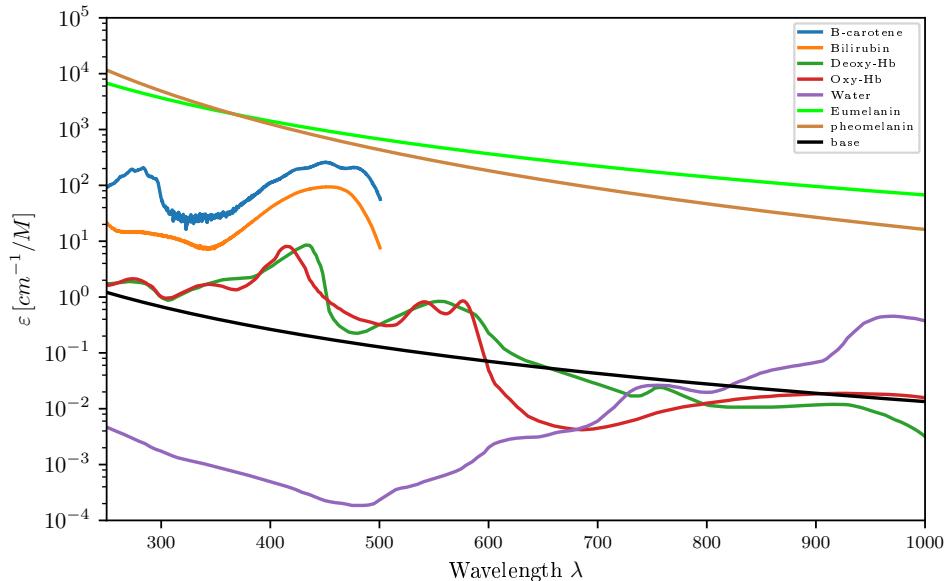
$$\mu_t = \mu_s + \mu_a \quad (2.21)$$

The albedo, or scattering probability, is defined as the ratio of the scattering coefficient to the total attenuation coefficient:

$$a = \frac{\mu_s}{\mu_a + \mu_s} = \frac{\mu_s}{\mu_t} \quad (2.22)$$

### Other Parameters

The preceding subsection described the optical properties which this thesis will use in every chapter. However, there are other optical properties that can be used to define a medium. These other parameters generally are used to model microphysics such as Raman scattering, polarization, fluorescence, or reflection and refraction. This section will give a brief overview of these other optical properties.



**Figure 2.4:** Examples of wavelength dependent absorption coefficients for some common tissue chromophores [45, 49–57].

### Fluorescence

Fluorescence occurs when a photon is absorbed by a fluorescent molecule and re-emitted with a new wavelength. Fluorescence is a reactively common phenomena and is heavily utilised in biophotonics and medicine, to image or monitor molecules in tissue. Again the optical property that models fluorescence is a coefficient that gives the probability of absorption and re-emission of a photon by a certain molecule. Usually this is in the form of an absorption coefficient or extinction coefficient. The extinction coefficient is a measurement of absorption in terms of the concentration of the absorber. Thus, if a medium has many fluorophores, then the total absorption coefficient is the bulk absorption of the medium plus the contribution from the fluorophores as in Eq. (2.23):

$$\mu_a = \ln(10) \sum_i C_i \varepsilon_i \quad (2.23)$$

Where  $C_i$  is the concentration of the  $i^{th}$  fluorophore, and  $\varepsilon_i$  is the extinction coefficient of the  $i^{th}$  fluorophore.

Fluorescence will be described in more depth in ??.

### 2.2.3 MCRT Algorithm

This section will provide an in depth description of the MCRT algorithm for the propagating photons through a spherical medium with optical properties  $\mu_s$ , and  $\mu_a$ . The subsequent section provides details of how the MCRT algorithm is implemented in the Fortran programming language, along with the various code details, such as the parallelisation of the code.

Figure 2.5 shows a flow chart of the MCRT algorithm described in this chapter.

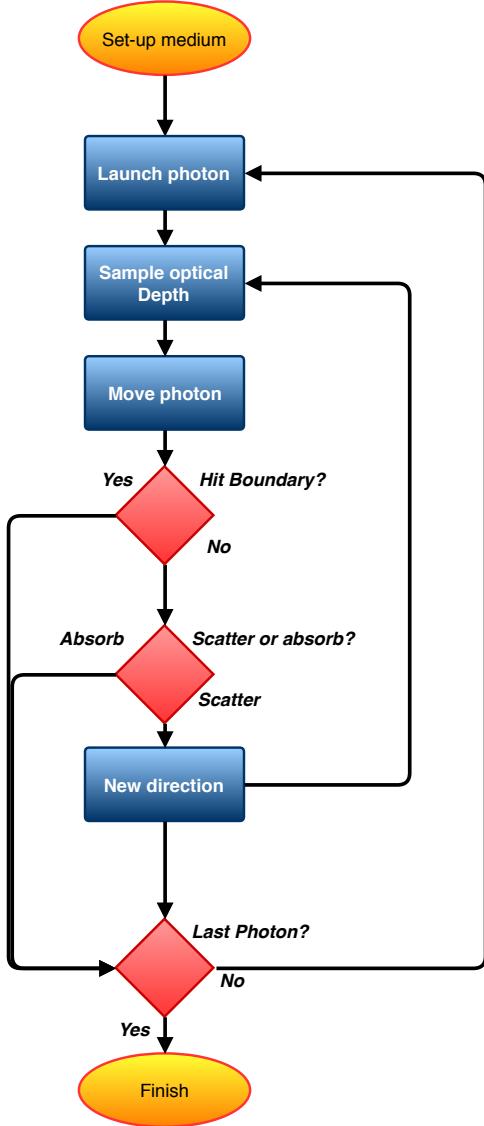
#### Medium and Grid Set-up

The first step of any MCRT algorithm is to set-up the medium the photons will propagate through. There are a variety of ways in which the medium can be set-up. For this section, it is assumed that the medium is an isotropic sphere, radius  $R$ , and centred at the origin. For simplicity, one wavelength is considered,  $\lambda$ . As the MCRT algorithm presented here is run on a 3D Cartesian grid, the grid is setup before creating the spherical medium. The grid is composed of  $n_x \times n_y \times n_z$  voxels<sup>†</sup>, where each voxel can have its own optical properties. The grid is setup by first setting an array that stores the locations of the voxel boundary walls in the  $x$ ,  $y$ , and  $z$  directions. The next step is to setup the actual medium. This is achieved by discretising the medium onto a grid. For this example a sphere is inscribed into a cubic volume, by setting the optical properties of a voxel to that of the medium if the sphere encloses that voxel. The voxels outside with sphere are set to that of the ambient medium. An example of a voxelised medium can be seen in Fig. 2.6.

#### Photon Launch and Initialisation

The second step in the MCRT algorithm, is to initialise the photon. Initialisation of the photon involves setting its initial position and direction.

Again, how this is done depends on the experiment being simulated. Here the photon is initialised to the centre of the sphere. The initial direction is sampled isotropically and set accordingly:



**Figure 2.5:** Flowchart of the Monte Carlo radiation transport algorithm as described in this section.

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<sup>†</sup>A voxel is a 3D pixel

$$n_{xp} = \sin \theta \cdot \cos \phi \quad (2.24)$$

$$n_{yp} = \sin \theta \cdot \sin \phi \quad (2.25)$$

$$n_{zp} = \cos \theta \quad (2.26)$$

With  $\theta$  and  $\phi$  sampled uniformly between  $[0, \cos^{-1}(2\xi - 1)]$  and  $[0, 2\pi\xi]$  respectively, where  $\xi$  is a random number in the range  $[0,1)$ .

The next step is to launch a photon packet. Depending on the source of photon packets for a given simulation, this step varies from simulation to simulation. The general idea of launching a photon packet is that the packet is given an initial direction vector and position (which consists of a physical position and a voxel position)<sup>‡</sup>:

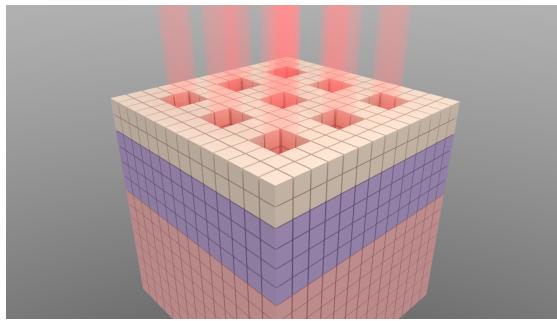
$$\text{direction} = \begin{bmatrix} n_{xp} \\ n_{yp} \\ n_{zp} \end{bmatrix} \quad (2.27)$$

$$\text{position} = [x_p, y_p, z_p] \quad (2.28)$$

$$\text{voxel} = [x_{cell}, y_{cell}, z_{cell}] \quad (2.29)$$

To set the direction vectors, the components of the direction vectors must be first set. The packets position is tracked using a Cartesian coordinate system. However for ease of computation for calculating scattering angles (see [Photon Interaction Event](#)), the direction vectors are computed in a spherical system thus the direction vectors are in Eqs. (2.24) to (2.26).

$\theta$  and  $\phi$  are generated dependent on the photon source used. The individual sine and cosine terms are saved for use in the scattering routines (see [Photon Interaction Event](#)). The position is then set according to the light source used. For this example the photons are released from the origin of the sphere. Using this position the voxel which the packet is in, is calculated.



**Figure 2.6:** Example of a possible voxel model, with three different layers, various holes due to ablative pixel beam lasers ( see Chapter 3). Each voxel can represent a different optical or thermal property of the tissue medium.

### Photon Propagation

The next step in the algorithm is moving a packet to the next interaction point. The probability that a packet will interact over a distance  $dL$  is  $\mu_t dL$ , where  $\mu_t$  is the total extinction coefficient (see [Optical Properties](#)). Thus, the probability of travelling  $dL$  without any interaction is  $1 -$

---

<sup>‡</sup>all variables given in this section are the same as they are in the code.

$\mu_t dL$ . Therefore, over a distance  $L$  with  $N$  segments of length  $L/N$  the probability of travelling  $L$  before any interaction:

$$P(L) = (1 - \mu_t \frac{L}{N}) \cdot (1 - \mu_t \frac{L}{N}) \dots (1 - \mu_t \frac{L}{N}) = (1 - \mu_t \frac{L}{N})^N \quad (2.30)$$

$$P(L) = \lim_{N \rightarrow \infty} (1 - \mu_t \frac{L}{N})^N = e^{-\mu_t L} = e^{-\tau} \quad (2.31)$$

Where  $\tau$  is the number of mean free paths in a distance  $L$ . Eq. (2.31) is now a PDF for the distance a packet will travel before an interaction occurs. To be able to get a random optical depth, the PDF has to be able to be sampled from either analytically or via the rejection method. Using the Monte Carlo method described in Section 1.1, with  $\xi$  as our random number, gives:

$$\xi = \int_0^\tau e^{-\tau'} = 1 - e^{-\tau} \rightarrow \tau = -\ln(1 - \xi) \quad (2.32)$$

As  $\xi$  is symmetric about 0.5,  $1 - \xi$  can be substituted for  $\xi$  yielding:

$$\tau = -\ln(\xi) \quad (2.33)$$

$\tau$  is now the optical distance, however this needs to be converted into a physical distance so the photon packet can be moved. From our definition of  $\tau$  we know that  $\tau = \int_0^L \mu_t dS$ , and if the medium is smooth and homogeneous (i.e not a gridded medium):

$$L = \frac{\tau}{\mu_t} \quad (2.34)$$

Therefore, to update the position of the packets it is simply:

$$x_p = x_p + L \cdot n_{xp} \quad (2.35)$$

$$y_p = y_p + L \cdot n_{yp} \quad (2.36)$$

$$z_p = z_p + L \cdot n_{zp} \quad (2.37)$$

However, as the code in this thesis is a 3D gridded Cartesian code, the method of updating and moving the packet's position is slightly adjusted. As stated in [Medium and Grid Set-up](#), the medium has been discretised onto a grid, so that each voxel can have a different  $\mu_t$ , thus Eq. (2.34) becomes:

$$L = \frac{\tau}{\mu_{t,\zeta}} \quad \zeta = (x, y, z) \quad (2.38)$$

with  $\mu_{t,\zeta}$  the attenuation coefficient for the  $\zeta^{th}$  voxel.

Moving the photon through a voxelised medium is more involved than propagating a photon through a non-voxelised medium. This is because the voxel the photon is in needs to be updated as the photon moves from voxel to voxel. The first step of moving the photon through a voxelised medium is drawing a random optical depth. This optical depth will be the full optical depth the photon travels before an interaction event. The generation of a random optical depth is as outlined above,  $\tau = -\ln(\xi)$ . As the photon travels through the voxel grid, a running total of the current optical distance travelled is kept. This is then compared to the randomly generated optical depth. When the running total optical depth equals the randomly generated optical depth, the photon propagation is stopped and the photon undergoes an interaction.

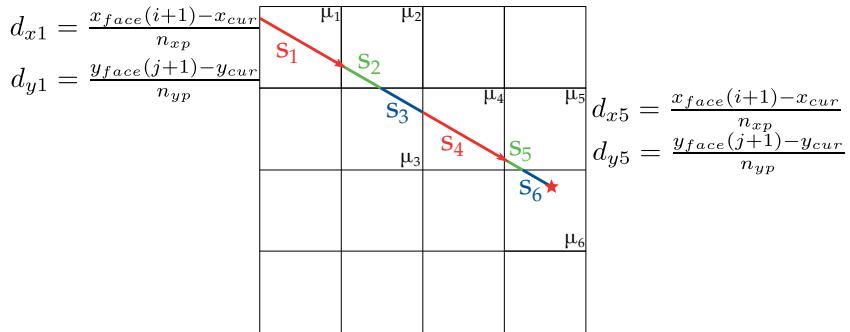
We then calculate the distance to the nearest voxel boundary in the  $x$ ,  $y$ , and  $z$  directions. The distance is calculated for each direction. Equation (2.39) shows for the  $x$  direction:

$$d_x = \frac{x_{face} - x_{cur}}{n_{xp}} \quad (2.39)$$

Where  $d_x$  is the distance to the nearest wall in the  $x$  direction.  $x_{face}$  is the voxel wall position in the  $x$  direction, and  $n_{xp}$  is the  $x$  direction vector. With three distances calculated,  $[d_x, d_y, d_z]$ , the minimum of these is thus the distance to the nearest voxel wall.

The next step is to calculate the optical depth for this distance. The optical depth is found by rearranging Eq. (2.38) for  $\tau$ , with  $L$  now the distance to the nearest wall. With the optical distance to the nearest wall calculated, the next step is to determine if there is “enough” optical distance left to travel the full distance to the nearest wall. Therefore, the running total optical distance is compared to the randomly generated optical distance. If the running total plus the new optical distance to the nearest wall is less than the randomly generated optical depth, then the photon travels to the nearest wall. The photon is then placed in the next voxel by a distance  $\delta$ , where  $\delta$  is just larger than machine precision. If the running total plus the new optical distance to the nearest wall is greater than the generated optical distance then an interaction event occurs in the current voxel. The distance to the interaction event is calculated and the photon moved to this location.

Figure 2.7 illustrates this whole process for a 2D example.



**Figure 2.7:** Illustration of photon propagation through a 2D grid.  $d_{x1}$ , and  $d_{y1}$  are the distances to the voxel walls in the  $x$  and  $y$  directions in the  $\mu_1$  voxel. In this case  $S_1 = d_{x1}$  as  $d_{x1}$  is smaller than  $d_{y1}$ , thus the photon hits the voxel wall in the  $x$  direction. For the  $\mu_5$  voxel,  $d_y$  is smaller, thus the photon hits the voxel wall in the  $y^{th}$  direction.

This whole process is repeated until the photon undergoes an interaction event or leaves the voxel medium. The next step in the algorithm is the interaction event, which can consist of either: scattering, absorbing or another microphysics phenomena.

### Photon Interaction Event

The next section of the algorithm is to decide how the photon interacts with the medium, either via scattering or absorption. There are other interaction events which can occur, however descriptions of these are left for the chapters that detail these behaviours.

To decide whether a packet scatters or absorbs a random number,  $\xi$ , is generated and compared against the albedo,  $a$ . If  $\xi < a$  then the packet scatters, otherwise it is absorbed.

## Packet Absorption

If the interaction event is a photon packet absorption, then the algorithm terminates the photon packets and starts the next photon packet, see [Termination](#).

## Packet Scattering

If the interaction event is a packet scattering, then the packet is scattered into a new direction and the above processes are carried out until a termination clause is met, see [Termination](#).

Depending on the medium being simulated, it can either be isotropic or anisotropic scattering. For the isotropic case, new  $\cos(\theta)$  and  $\phi$  angles are sampled uniformly and the direction vectors set as in section [Photon Launch and Initialisation](#). For the case where the scattering is anisotropic, the calculation of the scattering angles,  $\theta$  and  $\phi$ , is more complicated. The random sampling of the scattering angles,  $\theta$  and  $\phi$ , are valid in the “centre of mass” frame containing the scatter, incident and scattered ray. The photons position is updated in the lab frame, thus the direction vectors also have to be updated in the lab frame. This means the scattering angles need to be rotated into the lab frame. For the isotropic case we assume the scattering is also isotropic in the lab frame, thus the new direction vector is easily calculated. However, this is not the case for anisotropic scattering, as the centre of mass frame has to be rotated into the lab frame.

Figure 2.8 and Eq. (2.40) show how this process is achieved. Where  $\mathbf{n} = (n_x, n_y, n_z)$ ,  $\mathbf{n}_s = (n_x^{new}, n_y^{new}, n_z^{new})$ ,  $\theta_s$  is chosen from the phase function Eq. (2.41), and  $\varphi_s = 2\pi\xi$  with  $\xi$  being a random number in the range 0 to 1.

$$\begin{aligned} n_x^{new} &= \frac{\sin \theta_s}{T} (n_x n_z \cos \varphi_s - n_y \sin \varphi_s) + n_x \cos \theta_s \\ n_y^{new} &= \frac{\sin \theta_s}{T} (n_y n_z \cos \varphi_s + n_x \sin \varphi_s) + n_y \cos \theta_s \\ n_z^{new} &= -\sin \theta_s \cos \varphi_s T + n_z \cos \theta_s \\ T &= \sqrt{1 - n_z^2} \end{aligned} \quad (2.40)$$

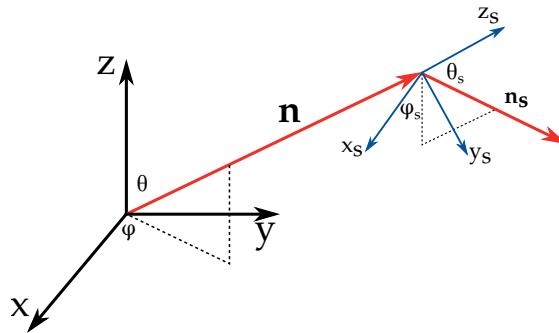
$$\cos \theta_s = \frac{1 + g^2 - \left( \frac{1-g^2}{(1-g+2g\xi)^{3/2}} \right)^2}{2g} \quad (2.41)$$

## Termination

The final section of the MCRT algorithm is to check if it should be terminated. This is a simple check to see if there are any more photons to run. If there are more photons to run then the algorithm goes back to the [Photon Launch and Initialisation](#) section and continues from there. If there are no more photons the algorithm terminates and any results are written out.

## Scored Quantities

As MCRT is a computational method, a wealth of information can be recorded during the simulation. From the paths of individual photons, to average scattering angles and more. However, it is not practical to record all this information for every simulation, as this would lead to inefficient simulations, and expensive data storage solutions. Thus, for a given problem only the pertinent information is stored.



**Figure 2.8:** Illustration of rotating the centre of mass frame to the lab frame.  $\mathbf{n}$  is the direction vector of the photon before scattering, and  $\mathbf{n}_s$  is the scattered direction vector.  $\theta$  and  $\varphi$  are the scattering angles.  $z_s$  is in the same direction as  $\mathbf{n}$ .

One important recorded variable is fluence. Fluence is the number of photons entering a sphere per unit cross section area [17]. In practice the average fluence per area is used, Eq. (2.42), as this is easier to calculate in an MCRT code. Lucy showed that the average fluence per area is proportional to the sum of the path length through a volume [58]:

$$J_i = \frac{L}{4\pi N V_\varsigma} \sum l \quad (2.42)$$

Where:

$J_i$  is the mean intensity such that the fluence is  $\Phi = 4\pi J$  [ $W m^{-2}$ ];

$L$  is the luminosity or power of the light source [ $W$ ];

$N$  is the total number of photon packets [-];

$V_\varsigma$  is the volume of the  $\varsigma^{th}$  voxel [ $m^3$ ];

and  $l$  is the path length of a photon packet through the  $\varsigma^{th}$  voxel [ $m$ ].

Most chapters in this thesis make use of Eq. (2.42) or modified versions of it as the main scored quantity, e.g. to determine absorbed energy.

Other common scored quantities are the exit location of a photon, the wavelength of an exiting photon or the distribution of photon packet absorption.

## 2.2.4 Code Details

This section describes the implementation of the MCRT and of the parallelisation of the code.

### Code

All code in this thesis is written in modern Fortran<sup>§</sup>. All subroutines and functions are contained in modules (with the exception of the main program—main.f90). This is done to be able to “hide” data from subroutines and functions and to arrange the code which relates to other parts of the code in the same file. Having the code in modules also allows the use of runtime allocation of memory for arrays. This enables the user to specify the size of arrays depending on the need of the user for the problem at hand.

Modules are classified into three different types: data, routines, and dependencies. Data modules are modules that contain no function or routines, but store variables that can be accessed

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<sup>§</sup>modern Fortran is considered anything past Fortran 95 [59].

anywhere in the program when required. Routine modules contain the subroutines and functions used in the code. Finally, dependency modules are the modules that have not been written by me, and thus the code depends upon them to run.

Figure 2.9 shows the relationship between the various modules, for a basic version of the MCRT as described in [MCRT Algorithm](#).

Using Fig. 2.9 as a reference each module contains:

`mcpolar.f90` is the entry point of the code. It calls all other subroutines and functions, as well as setting up various variables and printing progress.

`ch_opt` is the module where the optical properties are set or changed.

`gridset_mod` is where the optical properties grid and voxel walls are set.

`subs` contains general purpose routines which are used in various different parts of the code.

`writer_mod` contains routines which write out the results of the simulation.

`inttau2` is the module which contains the routines that propagate the photon through the voxel grid.

`sourceph_mod` contains the routines which initialise the photon position and direction.

`stokes_mod` contains the routine which calculates the scattering direction after a scattering event.

`iarray` is a data module which contains all the arrays in the code.

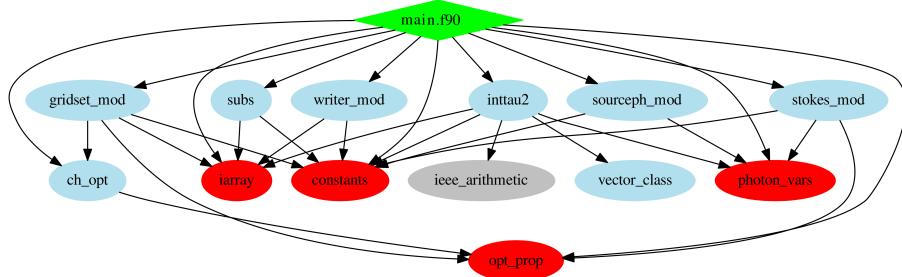
`constants` is a data module which contains all the constants and filepaths needed in the code.

`ieee_arithmetic` is an external dependency which gives various arithmetic checking routines such as `is_nan()`.

`vector_class` is a module which contains the vector type, and all its associated operations such as cross and dot products of vectors.

`photon_vars` is a data module which contains the data pertaining to each photon, such as wavelength or energy.

Finally, `opt_prop` contains the data about the current optical properties such as the albedo and absorption coefficient.



**Figure 2.9:** Source code hierarchy showing the relationship between different modules. Green is the entry point for the simulation. Red are the data modules, light blue are the routine modules, and grey are the external dependencies.

## Parallelisation of the MCRT Algorithm

As mentioned in the previous sections, MCRT can be computationally intensive, especially when dealing with highly scattering mediums. Fluorescence can also cause simulations times to drastically increase as photons are no longer “killed” off, but rather re-emitted at a new wavelength. Other optical processes such as Raman scattering are highly unlikely events, which again can lead to a dramatic increase in simulation times, as many photons are required to be simulated to get “good” statistics.

Fortunately MCRT is classed as an “embarrassingly parallel” problem<sup>¶</sup>. This means that it is trivial to parallelise in comparison to other algorithms. The reason that MCRT is classed as “embarrassingly parallel” is the algorithm can be split up onto separate processors, with little need for communication between them. In reality this means that  $n$  copies of the algorithm can run on  $n$  cores in a processor, with communication taking place at the start and end of each simulation run.

All the code in this thesis is parallelised using Message-passing interface (MPI) [61, 62], with the only communication taking place at the end, where the results are collated on to all processes. The one exception to this is in Chapter 3, where the heat diffusion calculation needs communication between the processes during the calculation.

The parallel efficiency of a code depends on the problem and the number of photon packets run. To determine the speedup of a given problem, Amdahl’s law is used [63]:

$$speedup = \frac{1}{(1 - P) + P/N} \quad (2.43)$$

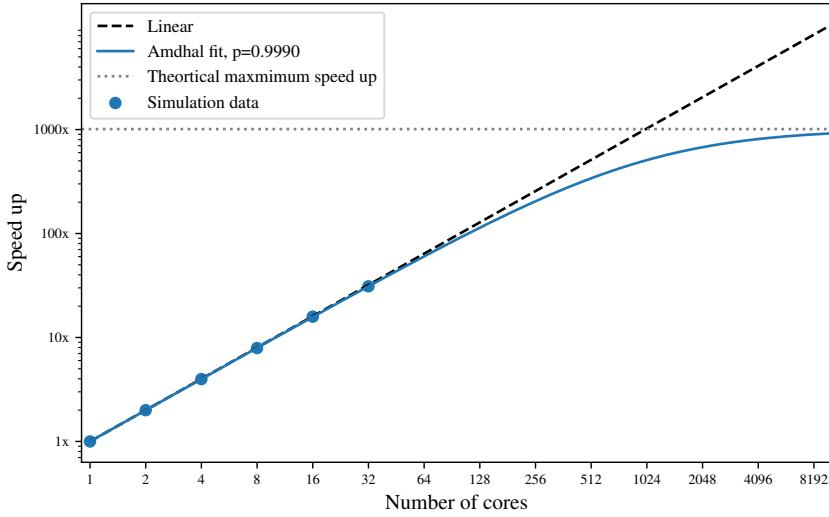
Where  $P$  is the fraction of the code that is parallel and  $N$  is the number of cores the code is run on. The consequence of Amdahl’s law is as  $N$  tends to infinity the speedup tends to a maximum:

$$speedup_{max} = \frac{1}{1 - P} \quad (2.44)$$

The value of  $P$  varies from problem to problem and the number of photon packets run. Figure 2.10 shows the results of the profiling of the code, for various numbers of cores. This test consisted of running the same number of photons, in a highly scattering medium of size  $2\text{ cm}^3$ . This yielded a  $P$  of  $0.999010 \pm 0.000045$  and a maximum speedup of 1010.1.

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<sup>¶</sup>However, this is not true for all MCRT applications. For example, using the Bjorkman & Wood [60] immediate temperature corrections method turns MCRT into a different class of parallel problem [27].



**Figure 2.10:** Performance of the parallelisation of the MCRT code using MPI.

## 2.3 Validation of MCRT Code

As the Monte Carlo method is an algorithm which depends upon random numbers, it is sometimes hard to ensure the correct result is obtained. Or to put it another way:

“Monte Carlo is easy to do wrong!” G.W. Collins III [64]

Thus, the code has to be validated against various theoretical and experimental and other simulations, to determine whether the results are correct.

The main benchmark of the MCRT code is a comparison against an expression for fluence as a function of depth [65]. This expression has also been fitted to by other MCRT simulations [66].

$$\Psi(z) = \Psi_0(C_1 e^{-k_1 z/\delta} - C_2 e^{-k_2 z/\delta}) \quad (2.45)$$

Where:

$\Psi(z)$  is the penetration of the incident light or equivalently the fluence rate [ $W\text{ cm}^{-2}$ ];

$\Psi_0$  is a normalisation constant [ $W\text{ cm}^{-2}$ ];

$C_n$  and  $k_n$  are fitted coefficients [-];

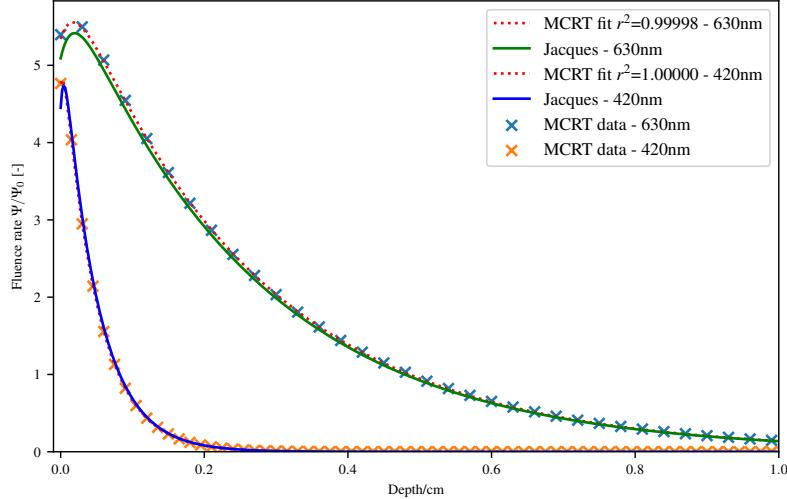
and  $\delta$  is the optical penetration depth, defined as  $\delta = 1/\sqrt{3\mu_a(\mu_a + \mu_s(1-g))}$  [cm].

Jacques *et al.* in their simulation used two different wavelengths, 420 nm and 630 nm. The medium in the simulation is an infinitely wide slab with a depth of 1 cm, with uniform optical properties. The medium has a refractive index of 1.38. The  $g$  value is in the range 0.7 — 0.9, and the optical properties are as in Table 2.1.

Wavelength/nm	Absorption	Scattering	Penetration				$\delta/cm$
	$\mu_a/cm^{-1}$	$\mu_s(1-g)/cm^{-1}$	C1	k1	C2	k2	
420	1.8	82	5.76	1.00	1.31	10.2	0.047
630	0.23	21	6.27	1.00	1.18	14.4	0.261

**Table 2.1:** Table of optical properties and determined coefficients from Jacques *et al.* [66].

Using these values Jacques *et al.* calculated values for  $C_1$ ,  $C_2$ ,  $k_1$  and  $k_2$  using their MCRT code. The above optical properties and medium dimensions<sup>||</sup> are recreated in the code and a value of 0.9 was chosen for  $g$ . 8 million photons were run for the simulation. This yielded the result as in Fig. 2.11.



**Figure 2.11:** Figure shows the fluence as a function of depth. Figure also shows comparison to the Jacques MCRT simulation and the MCRT as described in this chapter.

Fitting Eq. (2.45) to the data calculated by our MCRT code for 630 nm, gave:  $C_1 = 6.425$ ,  $C_2 = 1.083$ ,  $k_1 = 1.0$ , and  $k_2 = 12.966$ . For 420 nm gave:  $C_1 = 5.600$ ,  $C_2 = 0.838$ ,  $k_1 = 1.003$ , and  $k_2 = 9.846$ . These are in good agreement (within code differences) with Jacques *et al.* results.

## 2.4 Conclusion

There are various methods available to model the radiative transport equation. MCRT is the most flexible of the methods available, allowing arbitrary geometries, light sources, multiple anisotropic scattering, and various microphysics to be modelled. This chapter presented an overview of the MCRT algorithm that will be the basis of the results presented in the following chapters. The code described in this chapter is based upon K. Wood's MCRT code for light propagation in galactic dust clouds. It has been rewritten in modern Fortran and adapted so the code can model biological tissue and can be applied to various medical and biophotonic problems. The optical properties required in a MCRT simulation have been discussed and will be utilised to subsequent chapters. A description of how the code has been parallelised and validated against a standard literature code has also been presented.

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<sup>||</sup>The infinitely wide slab is implemented so that when a photon leaves one of the sides of the voxel grid, it is moved to the other side of the grid, retaining its original direction vectors.

# Chapter 3

## Computational Modelling of Tissue Ablation

### 3.1 Introduction and Background

This chapter uses MCRT techniques coupled to a heat transfer simulation, to study the thermal damage to tissue due to a laser, with its power spread over many beams to leave viable tissue around zones of damaged or necrotic tissue [67]. This class of laser is called a fractionated ablative laser. This chapter presents experimental work carried out on porcine tissue by our collaborators at the University of Dundee and the photobiology department at Ninewells Hospital, alongside my computational model of tissue ablation.

Ablative lasers are used in a wide variety of medical procedures including: coagulating scalpels, port wine stain removal, tattoo removal, hair removal, and skin rejuvenation [68–72]. One class of laser used in these procedures are ablative lasers. Ablative lasers are usually high powered lasers ( $>30\text{ W}$ ) targeted at a specific chromophore in the skin, to partially or fully remove layers of skin. These types of lasers are commonly used for aesthetic procedures such as: skin rejuvenation [72], and removal of various diseases such as Rhinophyma [73] or lesions/nodules [74]. Ablative lasers have also been recently investigated as a means of better drug penetration into the skin for various therapies such as photo-dynamic therapy (PDT). The ablative laser “drills” holes in the skin, which allows topical treatments to better diffuse into the skin [75].

One downside to using lasers to remove tissue is, that unlike a scalpel where the surgeon has full control of the depth of the incision, ablative lasers are not as predictable. Lasers can cause thermal damage to the surrounding areas, leading to potentially unwanted effects, though some applications of ablative lasers utilise the thermal damage, particularly aesthetic procedures [76].

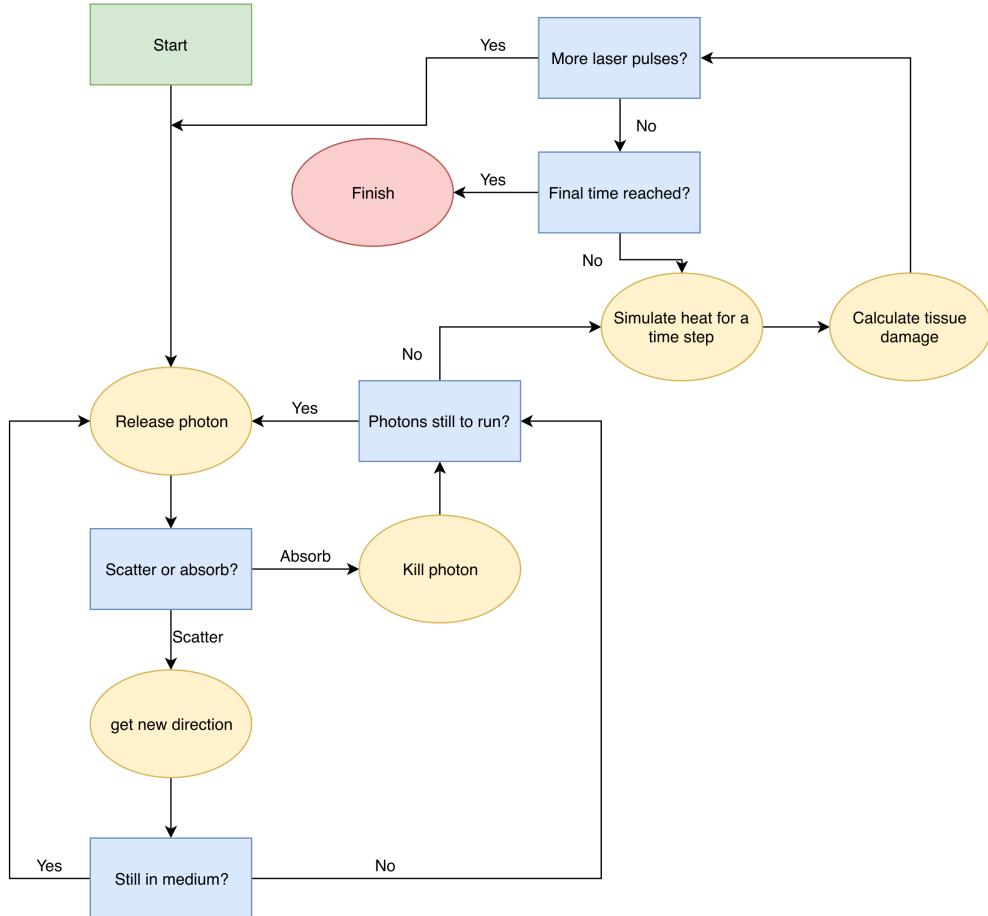
Currently, the standard method to measure the depth of the ablative holes is via a biopsy which is an invasive procedure. In this work, an optical coherence tomography (OCT) system is used to measure the ablative crater non-invasively *in-vivo*. The OCT measurements are then compared to a computational model developed as part of this project. It is hoped this computational model could be used to predict the depth of the ablative crater when using a certain laser power for various different applications such as: laser assisted drug delivery and various cosmetic applications.

## 3.2 Methods

To replicate the experimental work *in silico*, the numerical model has three main portions. The first is the MCRT code that models light transport through tissue so that we can calculate the laser energy deposited as a function of time and space. The second, a finite difference method (FDM) which is used to calculate the heat diffusion within the tissue due to the absorbed laser energy. Finally, a tissue damage model to track the tissue damage caused by the laser. All these individual functions are connected together to create a full numerical model. The full code from this chapter can be found at <https://github.com/lewisfish/Tissue-Ablation-MC>

### 3.2.1 Monte Carlo radiation transport (MCRT)

MCRT is used here to calculate the energy deposited by the laser. This is then passed to the heat transport simulation, which calculates the heat diffusion in the medium. The algorithm for the three coupled simulations is presented in Fig. 3.1.



**Figure 3.1:** Flowchart of the tissue ablation algorithm.

The MCRT algorithm is largely the same as described in Chapter 2, with some important

adjustments.

The first adjustment is that the path length counter for fluence is changed to track absorbed energy. This is achieved by multiplying the pathlength in a voxel by the absorption coefficient of that voxel. Figure 3.2 show this process graphically, and Equation (3.34) shows the mathematical expression:

$$E_i^{abs} = \frac{P}{NV_i} \sum \mu_{a,i} s \quad (3.1)$$

Where:

$E_i^{abs}$  is the energy absorbed in the  $i^{th}$  voxel [ $J s^{-1} m^{-3}$ ];

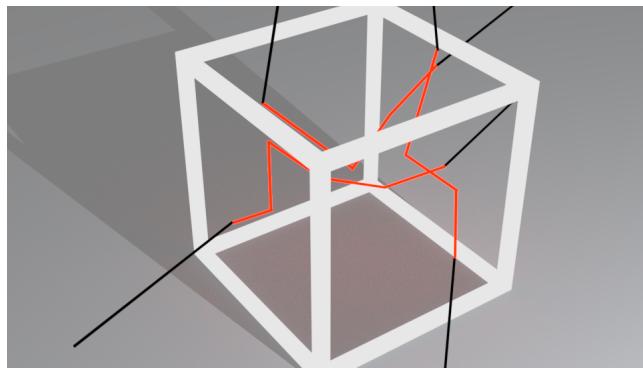
$P$  is power [W];

$N$  is the number of packets, representing a power,  $P_i$ ;

$V_i$  is the volume of the  $i^{th}$  voxel [ $m^{-3}$ ];

$\mu_{a,i}$  is the absorption coefficient of the  $i^{th}$  voxel [ $cm^{-1}$ ];

and  $s$  is the pathlength of a packet through the  $i^{th}$  voxel [cm].



**Figure 3.2:** Red lines are packet paths within a voxel. Black lines packet paths out with the voxel. Red packet paths, weighted by  $\mu_a$ , are summed up to calculate the absorbed energy within each voxel.

This grid of absorbed energy is then passed to the heat transport portion of the simulation, so that the heat diffusion in the porcine tissue can be calculated.

The next adjustment to the MCRT algorithm is that the MCRT algorithm is run for every heat simulation time step, as the medium could change at every time step due to the optical, and thermal properties changing as a function of tissue damage.

Finally, to match the experiment undertaken the medium and laser for the *in-silico* experiments must match the practical experiments. As the laser used in the experiments emits an infra-red wavelength ( $10.6\ \mu m$ ), the optical properties are dominated by the water content of the tissue. Due to this it is assumed that there is just absorption in the medium, with no scattering. Further discussion can be found in Section 3.3.1. The laser in some of the *in silico* modelling, has multiple beams and the source photon packet routine is adjusted to accommodate this when needed.

### 3.2.2 Heat Transport

The diffusion of heat can be modelled using the heat equation (Eq. (3.2)), which is derived from Fourier's law and the principle of conservation of energy [77]. The standard heat equation is a

partial differential equation of the parabolic form. Solutions and analytical methods are readily available for lower dimensions (i.e. 1D heat diffusion), but for higher dimensions, numerical models must be used for all except the simplest problems. The simplest form of the heat equation is shown below:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) + \dot{q} \quad (3.2)$$

Where:

- $T(x, y, z, t)$  is the temperature as a function of time and space [K];
- $\kappa$  is the thermal conductivity [ $Wm^{-1}K^{-1}$ ];
- $\rho$  is the density [ $kgm^{-3}$ ];
- $c_p$  the specific heat capacity [ $JK^{-1}$ ];
- $\dot{q}(x, y, z, t)$  is the source and sink term as a function of time and space [ $Wm^{-3}$ ].

Equation (3.2) is for a homogeneous system where the thermal properties do not change as a function of time, space and temperature. However, to model a moving ablation front the nonlinear heat equation must be used, where the thermal properties can be a function of time, space and temperature (Eq. (3.3)).

$$\frac{\partial T}{\partial t} = \frac{1}{(\rho c_p)_\xi} (\nabla k_\xi T + k_\xi \nabla^2 T) + \dot{q}, \quad \text{where } \xi = (i, j, k) \quad (3.3)$$

Included in Eq. (3.3) is a source and sink term,  $\dot{q}$ , to allow the modelling of heat loss and gain from external sources and sinks. The heat source in this simulation is due to the laser, and it is assumed that the only loss of heat to the surrounding medium is via conduction.

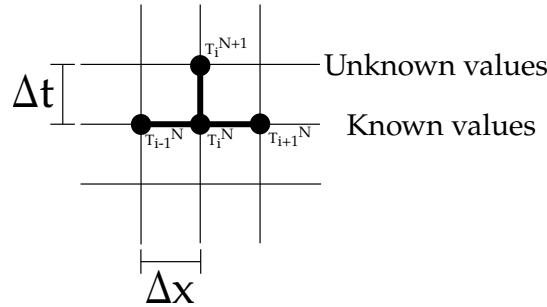
The medium is considered to be at a constant temperature of 5°C, as the porcine skin was kept cooled prior to experimental work and the simulation volume is smaller than the porcine tissue samples.

As Eq. (3.3) is generally hard to solve in arbitrary geometries with complex boundary conditions, a numerical method is employed to solve it. The numerical method employed is a finite difference method (FDM), derived from the Taylor series, see Eq. (3.4).

The FDM works by discretising a function,  $f(x)$ , onto a grid with  $N$  nodes a distance  $\Delta x$  apart. Equation (3.4) is then truncated and rearranged and it is assumed that the remainder term  $R_1$  is sufficiently small enough, to yield an approximation for the first derivative of a function  $f(x)$  at a point  $x_0 + \Delta x$ , see Eq. (3.5). Equation (3.5) is the so called “forward” difference, due to it using a point in the “forward” direction. The “backward” and “central” difference terms can be calculated by using a node at  $x_0 - \Delta x$  for the backward difference Eq. (3.6b). The central difference (Eq. (3.6c)) is an average of the forward and backward differences. Expressions can also be given for the 2<sup>nd</sup> derivatives for backward, forward and central (forward and backward 2<sup>nd</sup> order equations omitted for brevity) Eq. (3.6d).

$$f(x_0 + \Delta x) = f(x_0) + \frac{f'(x_0)}{1!} \Delta x + \frac{f''(x_0)}{2!} \Delta x^2 + \dots + \frac{f^{(n)}(x_0)}{n!} \Delta x^n + R_n(x) \quad (3.4)$$

$$f'(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (3.5)$$



**Figure 3.3:** Finite difference method stencil for simple explicit scheme.

$$\frac{df}{dx} = \frac{f_{i+1} - f_i}{\Delta x} * \quad (forward) \quad (3.6a)$$

$$\frac{df}{dx} = \frac{f_i - f_{i-1}}{\Delta x} \quad (backward) \quad (3.6b)$$

$$\frac{df}{dx} = \frac{f_{i+1} - f_{i-1}}{2\Delta x} \quad (central) \quad (3.6c)$$

$$\frac{d^2f}{dx^2} = \frac{f_{i-1} - 2f_i + f_{i+1}}{\Delta x^2} \quad (central) \quad (3.6d)$$

Thus, the linear heat equation Eq. (3.2), in 1D, taking a 1<sup>st</sup> order forward time derivative, and a 2<sup>nd</sup> order central spatial derivative gives:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} + \frac{\dot{q}}{\rho c_p} \quad (3.7a)$$

$$T_i^{n+1} = T_i^n + \alpha \Delta t \frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} + \frac{\Delta t \dot{q}}{\rho c_p} \quad (3.7b)$$

Where  $\alpha = \frac{\kappa}{\rho c}$ .

Equation (3.7b) is called the “simple explicit form of finite-difference approximation” [78]. Figure 3.3 shows the “stencil” of this scheme, where there are three known points at time  $N$ , and just one unknown at time  $N+1$ . There are various other schemes that can be used to calculate the temperature at the next time step. However, the simple explicit scheme is used here due to its ease of implementation despite there being a constraint on the time step used in comparison to an implicit method where there is none. This method is also easily scaled up to 3D with little difficulty.

For the more complicated nonlinear heat equation there is a possibility that the medium is not continuously smooth between nodes, in terms of optical and thermal properties. The two easiest methods [78] of achieving this are: (1), lag the value behind by one step, i.e  $c_p^{n+1} = c_p^n$ . (2), average  $\kappa$ ,  $\rho$ , and  $c_p$  using a half difference scheme where the thermal property used in the calculation is the thermal property halfway between two nodes, i.e the average of the two nodes:

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\*For brevity  $f(x_0 + \Delta x)$  is defined as  $f_{i+1}$ , and  $f(x_0 - \Delta x)$  as  $f_{i-1}$ , etc.

$$\kappa^\pm = \frac{\kappa_i + \kappa_{i\pm 1}}{2} \quad (3.8)$$

$$\rho^\pm = \frac{\rho_i + \rho_{i\pm 1}}{2} \quad (3.9)$$

$$c_p^\pm = \frac{c_{p,i} + c_{p,i\pm 1}}{2} \quad (3.10)$$

Thus, for the simple 1D case as in Eq. (3.7b), the thermal properties are averaged between nodes when computing the coefficients of the temperature nodes, and lag the thermal properties when adding the heat from the laser:

$$T^{N+1} = \Delta t (AT_{i-1}^N - 2BT_i^N + DT_{i+1}^N) + T_i^N + \frac{\Delta t}{\rho c_p} q_L \quad (3.11)$$

Where (in the  $x$  direction):

$$\begin{aligned} A &= \frac{\kappa^-}{\rho^- c_p^- 2\Delta x^2} \\ B &= \frac{\kappa^+}{\rho^+ c_p^+ 2\Delta x^2} \\ D &= \frac{(A + B)}{2} \end{aligned} \quad (3.12)$$

Equation (3.11) is straightforward to generalise to higher dimensions. The 3D case gives:

$$U_{xx} = (AT_{i-1,j,k}^N - 2BT_{i,j,k}^N + DT_{i+1,j,k}^N) \quad (3.13)$$

$$U_{yy} = (AT_{i,j-1,k}^N - 2BT_{i,j,k}^N + DT_{i,j+1,k}^N) \quad (3.14)$$

$$U_{zz} = (AT_{i,j,k-1}^N - 2BT_{i,j,k}^N + DT_{i,j,k+1}^N) \quad (3.15)$$

$$T_{i,j,k}^{N+1} = \Delta t (U_{xx} + U_{yy} + U_{zz}) + T_{i,j,k}^N + \frac{\Delta t}{\rho c_p} q_L \quad (3.16)$$

Where:

$T_{i,j,k}^{N+1}$  is the new temperature at node  $i, j, k$  [K];

$T_{i,j,k}^N$  is the temperature at node  $i, j, k$  at the current time step [K];

$\alpha$  is the thermal diffusivity [ $m^2 s^{-1}$ ];

$\kappa$  is the thermal conductivity [ $W/mK$ ];

$\Delta x$  etc. is the size of the grid element in the  $p^{th}$  direction [m];

and  $A, B, D$  are the coefficients in their respective dimension (Eq. (3.12)).

Equation (3.16) gives the full numerical solution to the nonlinear heat equation with a laser heat source. This will allow the calculation of the heat diffusion in the porcine tissue due to laser heating.

As the laser used in the experimental work operates in a pulsed mode, this is accounted for in the simulation. The laser pulse shape is a triangular pulse, with the peak power,  $P_{peak}$ , and pulse length,  $\tau$  [79]. In the heat simulation there has to be an additional variable in the term  $laserOn(t) \cdot \frac{\alpha \Delta t}{\kappa} q_L$  in Eq. (3.16). This additional variable,  $laserOn(t)$ , is a boolean value and a function of time, which is defined as:

$$laserOn = \begin{cases} 1, & \text{Laser on} \\ 0, & \text{Laser off.} \end{cases}$$

In the instance where there is a train of laser pulses, the laser is turned on and off based upon the pulse frequency.

Due to a simple explicit FDM being used, the time step is constrained to make the solution stable. For a cubic 3D FDM without prescribed flux boundary conditions, this yields the constraint:  $\Delta t \leq \frac{1}{\delta\alpha}$  where  $\delta = \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}$ . Along with this time constraint, the pulse length of the laser also has to be considered. If the time step of the heat simulation is too large it will not account for the heat deposited by the laser. Thus, the timestep has to be at least an order of magnitude smaller than the shortest laser pulse.

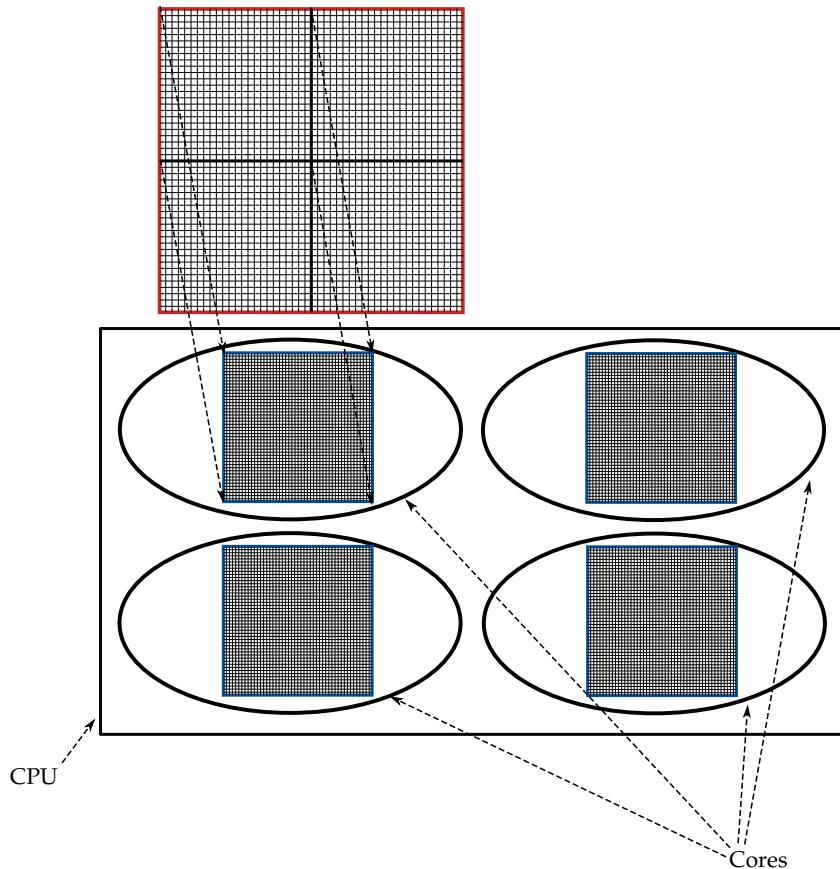
As the time step is small, and the grid resolution large the resultant simulation is slow. Thus, the code has been fully parallelised to improve performance. Both the MCRT and heat simulation are independently parallelised.

Parallelisation of the heat simulation is more involved than the “embarrassingly parallel” class of problems where MCRT belongs. This is due to the heat simulation being dependent on neighbouring nodes to update the temperature at the current node. Thus, if the medium were to be split up on to separate cores, there would have to be communication between the cores, in order for the simulation to be completed successfully. Therefore, it is not possible to take the “easy” route of running the simulation concurrently  $N$  times and collating the result at the end of all the simulations.

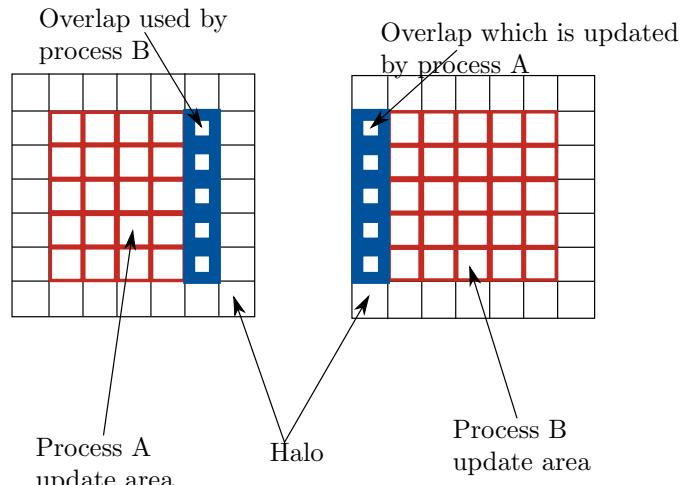
The heat simulation is parallelised using a technique called “halo swapping”. This involves splitting up the computational domain (see Fig. 3.4), in this case the tissue medium, and doing the calculations on each domain on a separate core. The “halo swapping” comes in when cores need to communicate with each other about updating their boundary temperature nodes (see Fig. 3.5).

Figure 3.6 shows the speed up gained from using the technique. The “halo swapping” technique is efficient for situations where the computational domain can be split up with large “chunks” being calculated on each core. However, if the computational domain is small, and the number of cores used is large then bottlenecks occur due to too much communication between cores taking place. Thus, to efficiently use “halo swapping” careful thought has to be given to the size of the computational domain, and the number of cores running the simulation. Evidence of this bottlenecking can be seen in Fig. 3.6 for problems where the size of the grid, in voxels, is  $40^3$  and  $24^3$ . These problems also show superlinear speed up, for a certain number of cores. This is not unfeasible, due to several reasons, including the start up time of MPI or the underlying computer architecture [80].

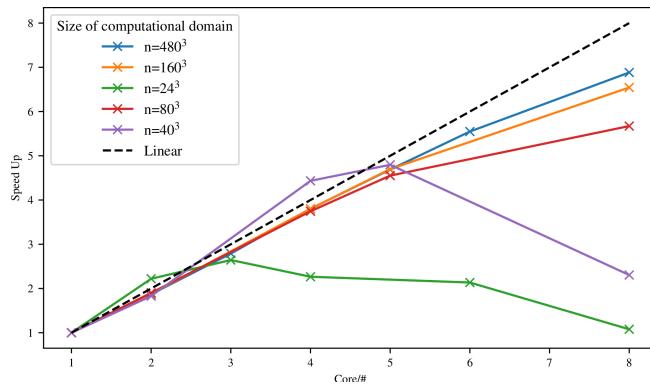
After one time step of the heat simulation has been completed, the temperature grid is passed to the tissue damage portion of the simulation to calculate the tissue damage that may have accrued during the heat simulation time step.



**Figure 3.4:** Computational domain decomposition. Total computational domain (red outline) is evenly divided between cores in the CPU. This is done via layers of the domain in the  $z$  direction. Information is passed to and from cores via the “halo swap” process (see Fig. 3.5).



**Figure 3.5:** Halo swapping. Process A updates the area in red and blue on the left. It updates the blue area which is sent to process B as B’s “halo”. Process B cannot update its own halo, but rather updates the halo for process A.



**Figure 3.6:** Figure shows the speed up gained by parallelisation of the heat simulation using the “halo swapping” technique, for various sizes of computational domain (voxels). Data taken from a Intel Xeon E3-1245 v5, 8 cores @ 3.5GHz machine.

### 3.2.3 Tissue Damage

#### Introduction

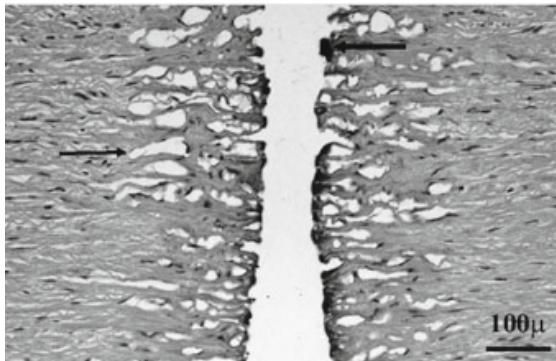
The final portion of the simulation is the tissue damage model. To be able to model the tissue damage process, the physical reality of this process must be understood. When the laser is turned on, the temperature starts to rise within the tissue due to the absorption of photons by the tissue. The temperature rise causes damage to the tissue when above a threshold temperature,  $T_d$ , approximately  $43^\circ\text{C}$  [81]. From the temperature,  $T_d$ , four main areas of tissue damage are defined:

$$T = \begin{cases} \text{coagulation,} & T_d \leq T \leq 100^\circ\text{C} \\ \text{water boils,} & T = 100^\circ\text{C} \\ \text{carbonisation,} & 100^\circ\text{C} \leq T \leq T_a \\ \text{ablation,} & T = T_a. \end{cases} \quad (3.17)$$

The area of tissue damage termed “coagulation” is a multifaceted process. At  $43^\circ\text{C}$  -  $50^\circ\text{C}$ , bonds break within cell membranes, causing ruptures and some cell death [81,82]. This process is usually termed *hyperthermia*. Around  $50^\circ\text{C}$ , enzyme activity decreases, cells become immobile, and various cell repair mechanisms are disabled, leading to increased cell death. When temperatures exceed  $60^\circ\text{C}$ , proteins become denatured. Thermal denaturation is a structural and functional change in a protein due to the heating it undergoes. This means they change from a highly organised structure with specific purposes, to disorganised structures with little to no function at all [83].

The next stage in the tissue damage process is the vaporisation of water. As the temperature of the tissue starts to approach  $100^\circ\text{C}$  (at 1 atm), water starts to vaporise. If the vaporised water cannot escape the tissue it forms steam vacuoles, small pockets of steam. These vacuoles can easily been seen when viewing tissue samples after tissue has been treated with a high powered laser (see Fig. 3.7). In certain conditions these steam pockets can explode [84].

The third stage of tissue damage is carbonisation of the tissue. This occurs when most of the water has boiled off, leaving the remaining tissue to heat up and reduce to its elemental carbon form. This carbonisation of tissue, when it occurs, is generally only a thin layer of 5-20  $\mu\text{m}$  [81,85].



**Figure 3.7:** Ablation of a dog aorta, as viewed under a microscope. Steam vacuoles are clearly visible on either side of the ablation area. Carbonisation is also evident at the edges of the ablation fronts. Adapted from [81].

The final stage of tissue damage is the removal of the remaining tissue, i.e tissue ablation. There is no agreement in the literature how tissue undergoes ablation with several methods proposed. The three main methods are: photochemical, thermal, and explosive [86–88]. Photochemical ablation is when the energy of a photon from the irradiating laser is sufficient enough that it excites the electronic state of the tissues molecules into an anti-boding state, leading to broken bonds and conversion from electronic energy into kinetic energy, and thus ablation. Thermal ablation is where tissue is heated sufficiently so that tissue vaporisation takes place. Finally, explosive ablation is an extreme version of thermal ablation. Explosive ablation occurs when large amounts of energy is deposited in a small time scale, so that none of the energy can thermally diffuse away, resulting in explosive ablation. Photochemical ablation, is usually applied to UV laser ablation, whereas thermal and explosive ablation regimes are the more likely candidates for IR ablation which is considered here.

The theoretical models behind explosive and thermal ablation models are also not well understood, with many models proposed to try and explain experimental results. These models range from heuristic models to sophisticated models that relate the underlying physical mechanisms to ablation damage. The two main heuristic models are: the blow off model and the steady state model. The blow off model assumes there is thermal confinement (i.e no propagation of heat in time  $t$ ), and that material is removed after the laser irradiation. There is a radiant threshold that has to be met to ablate material and that Beer-Lambert's law describes the spatial distribution of light. For laser pulses of  $< 10$  ns, these conditions are normally met. However, for lasers with pulse length larger than this, these conditions are not usually met [89–91].

The steady state heuristic model, assumes that the pulse length is of the order of  $ms$  or larger, that material starts to be removed shortly after laser irradiation begins, and that some radiant threshold exists in order for ablation to begin. The steady state model also assumes that a fixed energy is required to remove a unit of tissue [89]. However, this does not always hold, as there are many circumstances where there is no one fixed energy, but rather many energies (due to various phase changes) that must be met in order for ablation to occur. There are also many other sophisticated models that try to describe what happens physically when ablation occurs [92–94].

Due to the above mentioned reasons, there is no defined ablation temperature. The literature, however, does suggest that it takes place when the tissue temperature is between 177 °C and 500 °C [93, 95, 96].

To model all these tissue damage processes the tissue damage model is split into two sections: coagulation damage and “physical” damage. Coagulation damage has no effect on the tissue’s

bulk optical or thermal properties. “Physical” damage changes the tissue optical and thermal properties.

### Modelling coagulation damage

With the description of the various processes that tissue undergoes during ablation, a numerical model of these processes can be created. First to model the full extent of the damage done under 100°C, i.e in the coagulation regime, the Arrhenius damage model is used. The Arrhenius damage model was originally used as a kinetic model of reaction products in chemistry [97]. It has since been adapted by various authors for modelling tissue damage, and is the *de facto* standard [98, 99]. These authors and various others, adapted this model by fitting Eq. (3.18) to experimental data for burn damage. The two parameters fitted are A, the frequency factor, and  $\Delta E$ , the activation energy.

$$\Omega(t) = \int_{t_i}^{t_f} Ae^{(-\frac{\Delta E}{RT})} d\tau \quad (3.18)$$

Where:

- $\Omega$  is the damage value [-];
- A is “frequency factor” [ $s^{-1}$ ];
- $\Delta E$  is activation energy [ $Jmol^{-1}$ ];
- R is the universal gas constant [ $Jmol^{-1}K^{-1}$ ];
- T is the temperature [K];
- and  $t_i$  and  $t_f$  are the initial time and final time at  $t_{crit}$ .

It is reported that a value of  $\Omega$  of 0.53, 1.0, and 10<sup>4</sup> relate to first, second, and third degree burns respectively [100]. The Arrhenius damage model is used to better understand the amount of damage caused by the laser in the non-ablated areas of tissue. Values of  $A = 3.1 \times 10^{98}$  and  $\Delta E = 6.3 \times 10^5$  are adopted [96, 98, 101].

### Modelling physical tissue damage

As tissue mostly consists of water [102] when the temperature of the tissue approaches 100°C (at 1 atm), water in the tissue begins to boil off. This acts as a large heat sink for the absorbed laser energy, slowing down the rate of ablation. The energy required to boil the water is  $Q_{vapor} = m_v \cdot L_v$ , where  $m_v$  is the mass of a voxel, and  $L_v$  is the latent heat of vaporisation. The energy to boil off the water is provided via the laser and heat diffusing into the voxel:

$$Q_{vapor} = \underbrace{laserOn(t) \cdot \dot{q} \cdot \Delta t \cdot V_{i,j,k}}_{\text{laser heating}} + \underbrace{c \cdot M_{i,j,k} \cdot \Delta T}_{\text{heat diffusion}} \quad (3.19)$$

Where:

- $Q_{vapor}$  is the current energy in Joules that has been used to boil off the water in the voxel [J];
- $laserOn$  is a boolean variable that determine if the laser is on or off [-];
- $\dot{q}$  is the energy absorbed by the voxel due to the laser [ $Wm^{-3}$ ];
- $\Delta t$  is the timestep [s];
- $V_{i,j,k}$  is the volume of the voxel labelled  $i, j, k$  [ $m^3$ ];
- $c$  is the heat capacity of the voxel [ $JK^{-1}$ ];
- $M_{i,j,k}$  is the mass of the voxel labelled  $i, j, k$  [ $kg$ ];

and  $\Delta T$  is the change in temperature the voxel would undergo, if the water was not boiling off.

As water boils off, the water content of each voxel changes. This affects the absorption coefficient, density, thermal conductivity, and heat capacity. Each of these vary with water content per voxel [103];

$$W = W_{init} - \left( W_{init} \cdot \left( \frac{Q_{current}}{Q_{vaporisation}} \right) \right) \quad (3.20)$$

$$\rho = \frac{1000}{W + 0.649 \cdot P} \quad (3.21)$$

$$c_p = 4.2 \cdot 10^3 \cdot W + 1.09 \cdot 10^3 \cdot P \quad (3.22)$$

$$\kappa = \rho \cdot (6.28 \cdot 10^{-4} \cdot W + 1.17 \cdot 10^{-4} \cdot P) \quad (3.23)$$

$$\mu_a = W \cdot \mu_{water} + \mu_{protein} \quad (3.24)$$

$$(3.25)$$

Where:

$W$  is the water content (i.e  $W = 0.7$  equates to 70% water content);

$W_{init}$  is the initial water content;

$Q_{current}$  is the total energy absorbed by the  $i^{th}$  voxel since the temperature reached 100°C [J];

$P$  is the protein content (i.e  $P = 1.0 - W$ );

$\kappa$  is the Thermal conductivity [ $Wm^{-1}K^{-1}$ ];

$c_p$  is the heat capacity [ $Jkg^{-1}K^{-1}$ ];

and  $\mu_a$  is the total absorption coefficient, and  $\mu_{water}$  and  $\mu_{protein}$  are the absorption coefficients of water and protein respectively.

$T_a$  is defined as occurring between 177 and 500 °C [93, 95, 96]. At  $T_a$  the tissue is removed and the thermal, optical, and physical properties set to that of air.

The updated damaged tissue structure is then fed back to the MCRT model and the whole process repeats until the predefined time limit is reached. This whole process of photon propagation, heat diffusion and tissue damage is outlined in Fig. 3.1.

### 3.2.4 Validation

#### Heat transport validation

To thoroughly validate the numerical method employed to solve the heat equation, the numerical method is compared against an easily solvable analytical case. The heat equation is solved on a cube, side L, in a surrounding medium of 0°C. The cube is initially at temperature 20°C and the temperature is calculated at various times. Thus, the boundary conditions are:

$$T(0, y, z, t) = T(x, 0, z, t) = T(x, y, 0, t) = 0^\circ\text{C} \quad (3.26)$$

$$T(L, y, z, t) = T(x, L, z, t) = T(x, y, L, t) = 0^\circ\text{C} \quad (3.27)$$

The thermal diffusivity ( $\alpha$ ), density ( $\rho$ ), and heat capacity ( $c_p$ ) are all set to 1. This corresponds to a material which has the thermal diffusivity between copper and aluminium [104, 105]. Assuming a separable solution in Cartesian coordinates yields:

$$\begin{aligned} T(x, y, z, t) = & (A_1 \cos(\alpha x) + A_1 \sin(\alpha x)) \cdot \\ & (B_1 \cos(\beta y) + B_1 \sin(\beta y)) \cdot \\ & (C_1 \cos(\gamma z) + C_1 \sin(\gamma z)) \cdot e^{-\alpha \mu^2 t} \end{aligned} \quad (3.28)$$

$$\mu^2 = \alpha^2 + \beta^2 + \gamma^2 \quad (3.29)$$

Applying the boundary conditions (Eqs. (3.26) and (3.27)) gives:

$$A_1 = B_1 = C_1 = 0 \text{ and } \alpha = \frac{\pi n}{L}, \beta = \frac{\pi m}{L}, \gamma = \frac{\pi p}{L} \quad (3.30)$$

$$\therefore T_{nmp}(x, y, z, t) = A_{nmp} \cdot \sin\left(\frac{\pi n x}{L}\right) \cdot \sin\left(\frac{\pi m y}{L}\right) \cdot \sin\left(\frac{\pi p z}{L}\right) \quad (3.31)$$

This yields the following solution for the heat equation using the principle of superposition, and solving Eq. (3.32) with  $f(x, y, z)$  as the initial temperature profile of the cube:

$$A_{nmp} = \frac{8}{L^3} \int_0^L \int_0^L \int_0^L f(x, y, z) \cdot \sin\left(\frac{\pi n x}{L}\right) \cdot \sin\left(\frac{\pi m y}{L}\right) \cdot \sin\left(\frac{\pi p z}{L}\right) dx \cdot dy \cdot dz \quad (3.32)$$

$$T(x, y, z, t) = \sum_{n=1,3,\dots}^{\infty} \sum_{m=1,3,\dots}^{\infty} \sum_{p=1,3,\dots}^{\infty} \frac{2368}{\pi^3 nmp} \cdot \sin\left(\frac{\pi n x}{L}\right) \cdot \sin\left(\frac{\pi m y}{L}\right) \cdot \sin\left(\frac{\pi p z}{L}\right) \cdot e^{(-\lambda^2 t)} \quad (3.33)$$

Where:

$$\lambda^2 = \alpha \pi^2 \left( \frac{n^2}{L^2} + \frac{m^2}{L^2} + \frac{p^2}{L^2} \right);$$

$n, m, p$  are odd integers;

and  $L$  is the length of the cube.

A slice through the middle of the cube,  $L = 50 \text{ cm}$ , yields Fig. 3.8, which shows that the numerical method matches the analytical solution closely.

### MCRT and heat transport validation

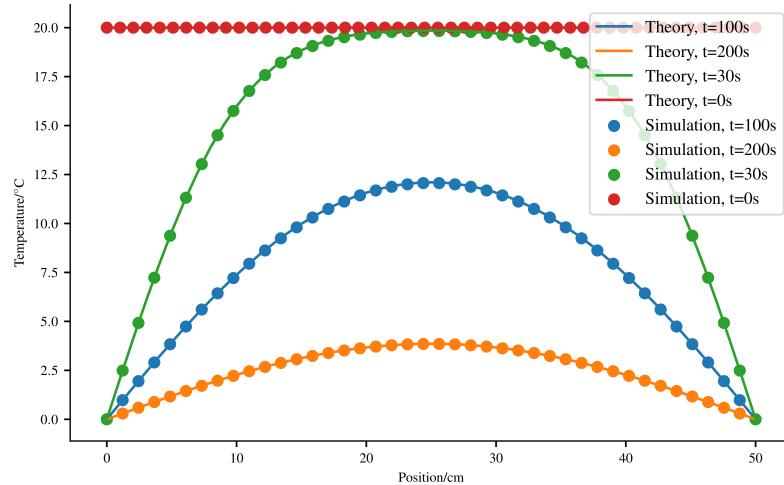
As a first test of the code, both the MCRT and the heat simulation are compared to a simple analytical model of ablation. The simple model of ablation is as: the ablation energy ( $E_a$ ) is defined as the minimum energy required to raise the temperature of the medium to  $100^\circ\text{C}$ , and then boil off the water in a volume  $dV$ , mass  $M$ . Thus, in one dimension Eq. (3.34), where the symbols have their usual meanings. If the energy for ablation is delivered in a time  $dt$  by a laser of intensity,  $P$  ( $\text{Wcm}^{-2}$ ), this gives Eq. (3.35). Equation (3.35) can be rearranged to give an ablation front velocity, Eq. (3.36).

$$E_a = c_p \rho dx \Delta T + L_v \rho dx \quad (3.34)$$

$$P \cdot dt = \rho dx (c_p \Delta T + L_v) \quad (3.35)$$

$$u = \frac{P}{\rho(c_p \Delta T + L_v)} \quad (3.36)$$

Assuming the ablation front moves with constant velocity during the ablation, and using  $L_v = 2.53 \cdot 10^6 \text{ Jkg}^{-1}$ ,  $c_p = 4181 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$  and the medium is a cube side  $2 \text{ mm}$ , with a



**Figure 3.8:** Temperature profiles of the cube for various times, comparing between analytical solution and numerical method.

starting temperature is  $37\text{ }^{\circ}\text{C}$  with a water content of 70% giving a density of  $700\text{ kg}\cdot\text{m}^{-3}$ . For these parameters this gives an ablation velocity,  $u \simeq 0.77\text{ cm}\cdot\text{s}^{-1}$ , and a time to ablate through 2 mm of tissue of  $\simeq 0.26\text{ s}$ . As the code developed in this chapter simulates the diffusion of heat in a medium due to an incident laser, the expected time to ablate through the same medium should be slightly larger as heat diffuses away from the voxel while it is being heated. When the full heat plus MCRT code is used to simulate this experiment, it gives a time,  $t \simeq 0.33\text{ s}$ .

### 3.3 *In silico* results

#### 3.3.1 Introduction

To match the experimental results, an accurate model of the experimental setup *in silico* must be created. However, due to computational constraints, such as memory and time available, some approximations to the experimental set-up have to be made. The porcine skin was a large thin slice of the top most layers of the skin. However, as the area of interest is where the ablation occurs, initially the porcine skin is modelled as a cuboid, dimensions:  $1.1 \times 1.1 \times 0.5\text{ cm}$ . The initial temperature of the porcine skin is assumed to be around  $5\text{ }^{\circ}\text{C}$ , as the tissue was kept on ice or was kept cooled. As mentioned in the previous sections, there are several unknowns in the model:  $T_a$ , water content, temperature of air after ablation, and the exact thermal and optical properties of the porcine tissue. Therefore, several models are run so that the full parameter space of these unknowns can be explored. Results from these *in silico* experiments are presented in this section along with a comparison of the model to the experimental work carried out in collaboration with the University of Dundee and the Photobiology department at Ninewells Hospital.

#### Optical & thermal properties

The thermal and optical properties of porcine tissue are not known exactly for any given tissue sample. As such the thermal and optical properties used in this section are taken from various literature sources.

	Thermal conductivity, $\kappa$	Density, $\rho$	Heat capacity, $c$
Tissue	$\rho \cdot (6.28 \cdot 10^{-4} \cdot W + 1.17 \cdot 10^{-4} \cdot P)$	$\frac{1000}{W+0.649 \cdot P}$	$4.2 \cdot 10^3 \cdot W + 1.09 \cdot 10^3 \cdot P$
Air	$a e^{-b(T-273.15)} + c$	$\frac{p_{atm}}{R_{spec}T}$	1006

**Table 3.1:** Thermal properties for porcine tissue and air.  $W$  and  $P$  are the percentage of water and protein respectively.  $\rho$  is the density of the skin,  $p_{atm}$  is the pressure of air at 1 atmosphere, and  $R_{spec}$  is the gas constant.  $a$ ,  $b$ , and  $c$  are constants.

The laser used in the experimental work is an CO<sub>2</sub> laser operating at 10.6  $\mu m$ . This means the optical properties of the tissue are dominated by water absorption. The laser used in the experiment is the Pixel CO<sub>2</sub> [106]. The Pixel CO<sub>2</sub> laser has a wavelength 10.6  $\mu m$  which corresponds to an absorption coefficient in water of  $\sim 850\text{ cm}^{-1}$ . As the absorption coefficient is large, it is assumed that scattering is negligible at these wavelengths. Table 3.1 summarises the thermal properties for tissue and air used in the simulations.

The laser was used in “Pixel beam” mode. This means that the laser beam is split into an array of smaller beams. The laser used an array  $9 \times 9$  of 81 pixel beams, each with a diameter of 250  $\mu m$ . The Pixel CO<sub>2</sub> rated laser power is  $\sim 70\text{ W}$ .

The laser delivered one single pulse of varying total energy delivered over the range 50  $mJ$  to 400  $mJ$ , in so called “super pulsed mode”. The experiment consisted of ablating the porcine tissue, as a function of energy per “pixel” beam. This was achieved by adjusting the pulse length of the laser,  $\tau$ , so that the energy per pulse was varied over a range 50  $mJ$  to 400  $mJ$ .

### Computational speed up:

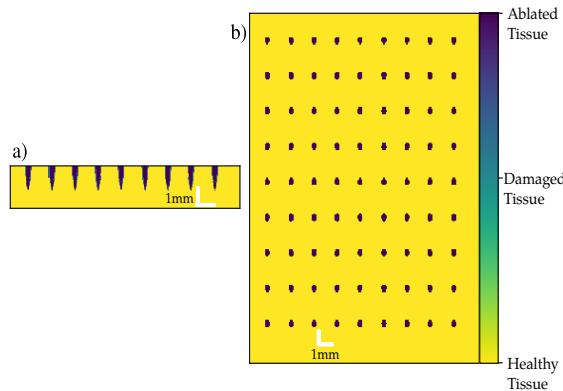
As discussed in the Section 3.1, the volume of interest is the area around the ablation craters. The volume is  $1.1\text{ cm} \times 1.1\text{ cm} \times 0.5\text{ cm}$ . However, for the simulation to have good resolution of the ablation craters this volume would require many voxels for the tissue model. This is unfeasible due to: the memory required to store the various counters, grids, and variables, and the time that would be required to carry out the computation. Thus, the volume of interest is reduced to focus on just one of the ablation craters that is created by the laser (a volume of  $0.06\text{ cm} \times 0.06\text{ cm} \times 0.18\text{ cm}$ ) As a check to ensure that no physical phenomena are omitted by focusing on just one ablation crater, an initial simulation that models the full volume of interest was carried out to investigate the possibility of overlapping craters or other related phenomena. The simulation, as shown in Fig. 3.9, gives reassurance that the shrinking of the volume of interest is a valid approximation to make as there is no overlap between the separate ablation crater.

### 3.3.2 Results

#### Investigating ablation temperature, $T_a$

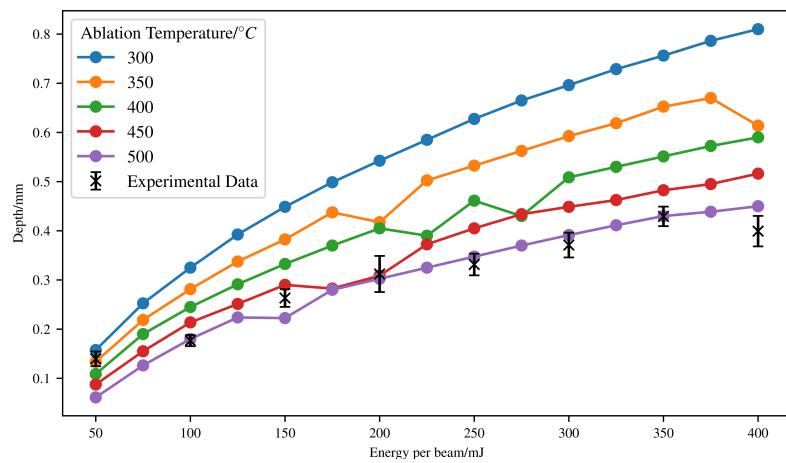
Various literature sources report the ablation temperature ranging widely from 177° to 500° [93, 95, 96]. Thus, several models are run over this range to establish the  $T_a$  which fits the experimental results. Figures 3.10 and 3.11 show how  $T_a$ , and beam profile affect the crater depth as a function of pixel beam energy for the CO<sub>2</sub> laser. The data suggests that a  $T_a$  around  $T_a = 500\text{ }^\circ C$  is appropriate for the studies carried out, the upper limit of  $T_a$  from the literature.

Increasing the ablation temperature has the obvious effect of requiring more energy to be deposited by the laser before ablation takes place. This also allows more heat to diffuse away from the ablation crater increasing the thermal damage to the surrounding tissue. Decreasing the ablation temperature has the converse effect and allows the ablation crater to become deeper.



**Figure 3.9:** Simulation of 81 pixel beams. Figure a) shows a slice through the optical properties at the end of the simulation in the  $z$ - $y$  plane. Figure b) shows the optical properties in the  $x$ - $y$  plane at the top surface. Yellow is unchanged tissue and purple is completely ablated tissue. Figure shows that the ablation craters do not overlap one another.

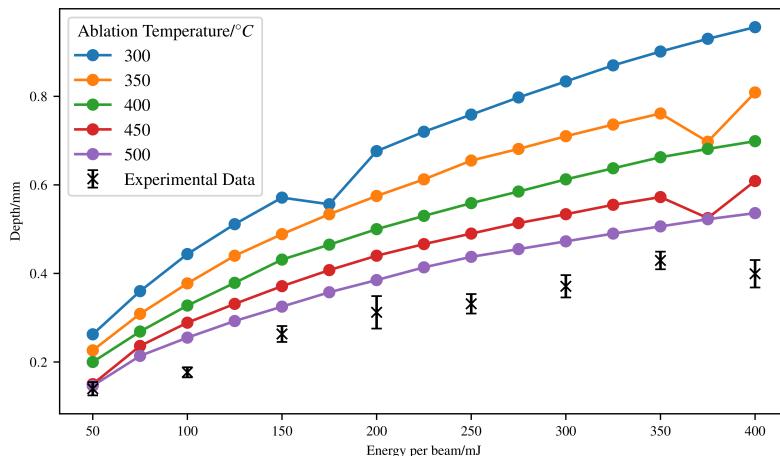
Over the full range of  $T_a$ , as the energy per pixel beam increases, there is a trend that at higher energies the crater depth begins to taper off. This is potentially due to several reasons. As the ablation craters grows the volume of tissue that is ablated is replaced with air, allowing more heat loss from the tissue to the environment. As well as heat loss to the environment, more heat is diffused away into the surrounding tissue as the crater grows, due to the availability of more tissue for the heat to diffuse into. Finally, there is the appearance of “kinks” in what should be smooth data. The cause of these “kinks” is due to a couple of voxels not fully ablating around the bottom of the crater. Why this occurs is unknown, but believed to be due to the numerical error in that particular run of the simulation. For example in Fig. 3.10  $T_a = 300$  the data is smooth, whilst in  $T_a = 350$  there are two unexplained “kinks”.



**Figure 3.10:** Simulation of 70 W  $\text{CO}_2$  ablative laser, with a circular beam profile. Crater depths as a function of pixel beam energy for various  $T_a$ 's.

## Investigating beam type

As the manufacturer does not provide information on the beam profile of the pixel beams and the lack of equipment available to measure the beam profile, the shape of the beam profile has to be assumed. Two different beam types are trialled: Gaussian and circular (top-hat). Figures 3.10 and 3.11 show the result of these *in-silico* experiments. The Gaussian beam ablates deeper holes than the circular beam type, which is to be expected due to the distribution of power in the Gaussian beam. The beam that best fits the data is the circular beam. For the Gaussian beam to fit the data ablation would have to take place at temperatures above 500 °C which does not fit with the literature. Without knowing the exact profile of the beam, it is assumed for the rest of the *in-silico* experiments that the beam profile is circular.

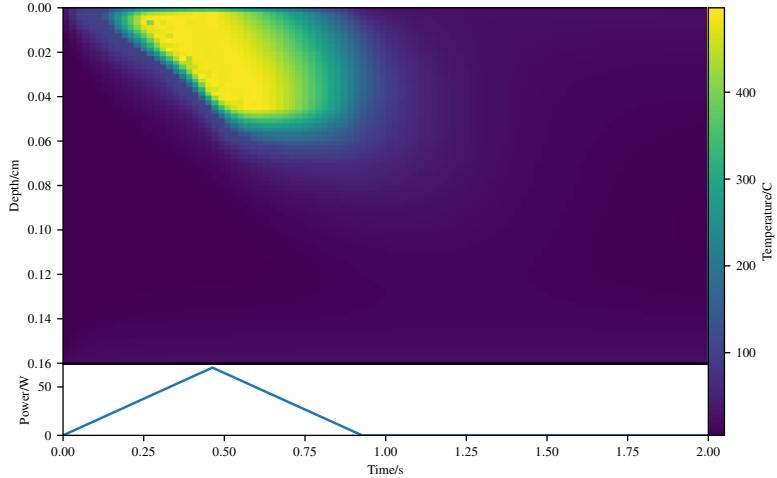


**Figure 3.11:** Simulation of 70 W CO<sub>2</sub> ablative laser, with a Gaussian beam profile. Crater depths as a function of pixel beam energy for various  $T_a$ 's.

## Temperature during ablation

Figure 3.12 shows slices of temperature as a function of time during a simulation for  $T_a = 500$  °C. This means that every column of pixels shows a bore hole through the medium (along the z axis) for a given time. Figure 3.12 also shows the laser pulse profile as a function of time as a reference so that knowledge of when the laser is on or off is easily elucidated. The figure shows that the temperature reaches a maximum temperature which is equal to  $T_a$ , regardless of ablation progress. This maximum temperature is researched roughly 0.25 s into the simulation and lasts until ~0.75 s. The maximum temperature reaches a depth of around 0.04 cm into the tissue. The region where water is boiling can be seen by the evidence of the “dark valley” before the abrupt jump in temperature to the maximum temperature.

The cause of this dark valley is due to the water in the tissue acting as a heat sink. The water needs a large amount of energy to boil off, thus it stops the temperature from increasing until it boils off, giving rise to this dark valley. This temperature maximum extends for a small distance into the tissue, before diffusion spreads it out. Once the tissue ablates the ablated volume cools over the period of around 0.5 s. Ablation continues until around 0.6 s, where the laser power has past its maximum value and no more ablation occurs.



**Figure 3.12:** Temperature bore hole through centre of medium as a function of time, for  $T_a=500\text{ }^{\circ}\text{C}$ . Laser power is also plotted for comparison.

### Investigating thermal damage

As stated in Section 3.2.3, the Arrhenius damage integral is used to estimate the thermal damage due to the laser. To calculate the tissue damage around the ablation craters, Eq. (3.18) is first transformed into a summation:

$$\Omega(t) = \int_{t_p}^{t_f} Ae^{-\frac{\Delta E}{RT}} d\tau \quad (3.37)$$

$$\Omega(t) = \sum_{m=m_p}^{m_f} Ae^{-\frac{\Delta E}{RT_\xi^m}} \Delta t \quad (3.38)$$

Where:

$\Delta E$ ,  $R$ ,  $T$ , and  $A$  have the same meanings as before;

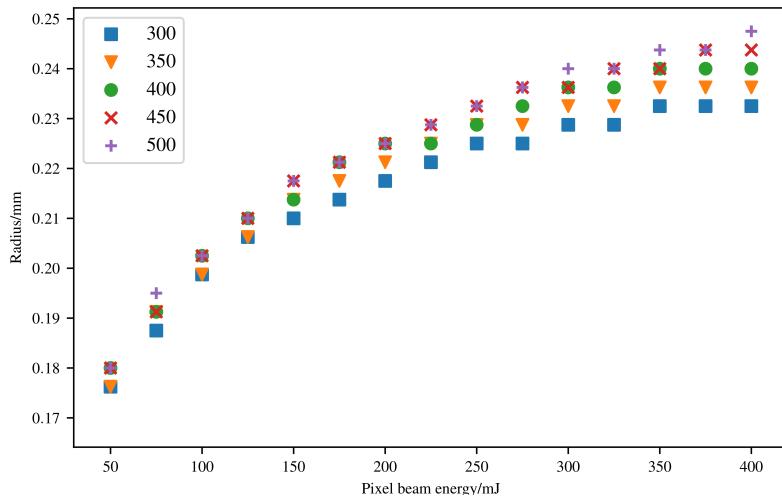
$\xi$  is the  $i^{th}$ ,  $j^{th}$ ,  $k^{th}$  node;

and  $m_p$  is the  $p^{th}$  timestep when the  $\xi^{th}$  node is above the threshold temperature.

Using Eq. (3.38) it can thus be estimated that the damage to the tissue on a voxel by voxel basis. Figure 3.14 shows how far the thermal damage extends around the ablation crater. For ease of visualisation 1-3 is mapped to their respective burns via the following scheme, with  $\eta$  as burn severity:

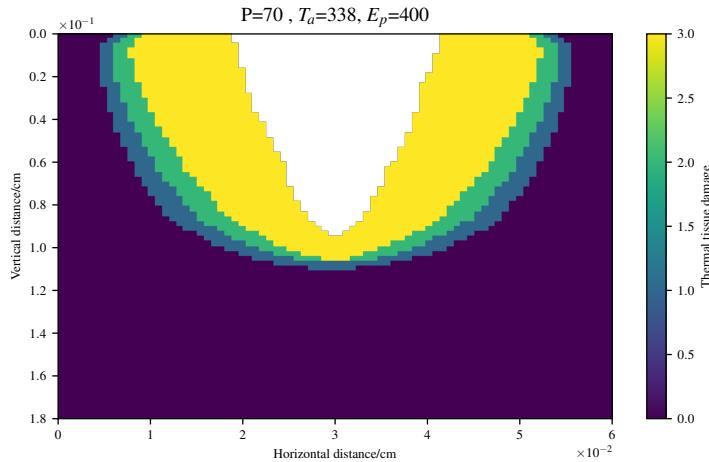
$$\eta = \begin{cases} 3, & \Omega \geq 10000 \\ 2, & 1 \leq \Omega < 10000 \\ 1, & 0.53 \leq \Omega < 1 \\ 0, & 0.0 \leq \Omega < 0.53. \end{cases} \quad (3.39)$$

As shown in Fig. 3.14, the thermal damage zone extends for a small distance around the ablation crater, due to the diffusion of heat into these areas. Figure 3.13 shows the maximum horizontal thermal damage distance as a function of  $T_a$ , and pixel beam energy. For values of



**Figure 3.13:** Figure shows the maximum horizontal extent of thermal damage as a function of energy per pixel beam, for different  $T_a$ 's.

$T_a$  less than  $\sim 425$  °C, it appears that the maximum horizontal extent of the thermal damage begins to taper off. This is most likely because for lower values of  $T_a$ , there is a larger ablation crater, meaning that the energy from the laser is deposited deeper in the tissue in comparison to higher values of  $T_a$ . The higher values of  $T_a$  allow greater diffusion of the heat, thus yielding larger zones of damage. Overall there is little difference in the maximum horizontal extent of thermal injury, when using different energies (of the order of  $\sim 0.01$  mm).

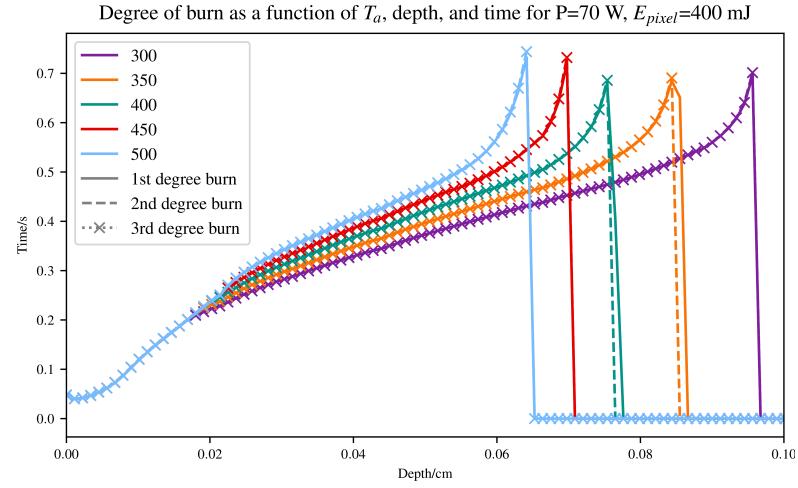


**Figure 3.14:** Tissue thermal damage around the ablation crater (white). Thermal tissue damage values of 3 refer to 3<sup>rd</sup> degree burns, 2 to 2<sup>nd</sup>, and 1 to 1<sup>st</sup> degree burns respectively.  $P$  is the power in Watts,  $T_a$  is the ablation temperature in Kelvin, and  $E_p$  is the energy per pixel beam in mJ.

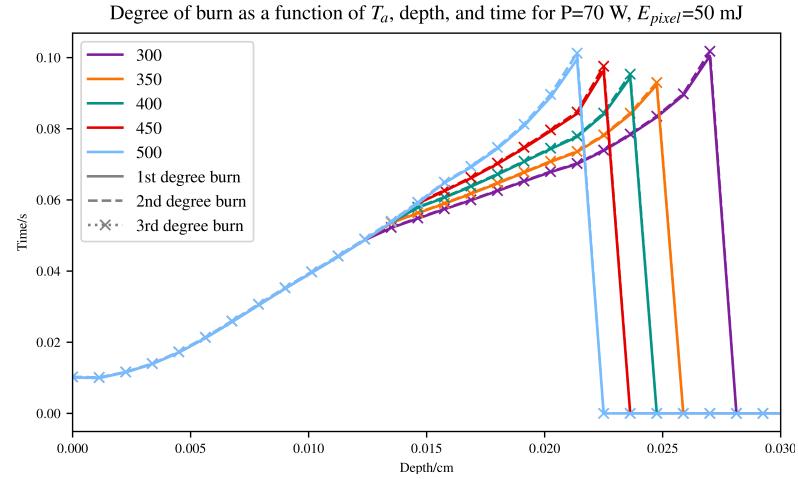
Investigations for the time it takes for different areas of the tissue to become thermally damaged, were also carried out. This can be easily achieved by saving the time each voxel passes

one of the damage boundaries in Eq. (3.39). Figures 3.15 and 3.16 show the minimum time taken for 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> degree burns to occur as a function of depth. Figure 3.15 shows that there is little to no time (upon the order of 0.5 ms) between 1<sup>st</sup> and 2<sup>nd</sup>, and 3<sup>rd</sup> degree burns. Figure 3.16 shows there is a slightly greater time difference between 1<sup>st</sup> and 2<sup>nd</sup>, and 3<sup>rd</sup> degree burns, however this is almost as negligible as the 400 mJ case.

The reason that there is almost no time between 1<sup>st</sup> and 2<sup>nd</sup>, and 3<sup>rd</sup> degree burns, is most likely because there is little time for heat to diffuse, whilst the laser is still illuminating the medium. The laser pulses are on the order of seconds, and tissue is not thermally conductive. This leads to the results presented here.



**Figure 3.15:** Figure shows the time taken for 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> to occur as a function of depth, for a range of  $T_a$ 's at 400 mJ.



**Figure 3.16:** Figure shows the time taken for 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> to occur as a function of depth, for a range of  $T_a$ 's at 50 mJ.

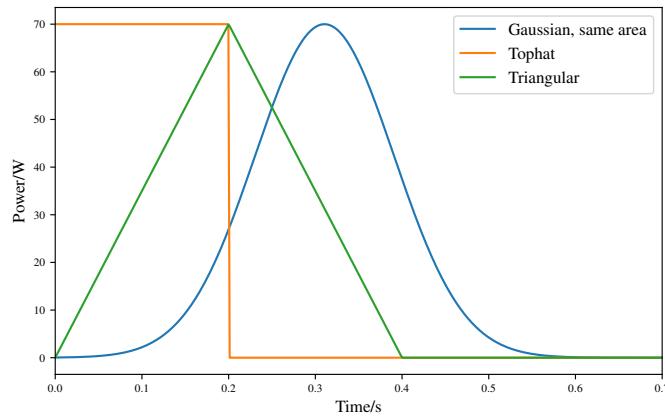
## Investigating laser pulse profile

Pulsed lasers have a variety of pulse profiles. The pulse profiles are usually modelled as triangular, top hat, or Gaussian. However, the pulse profiles in reality are normally less well defined, and rather the pulse profile is something in between these perfect models.

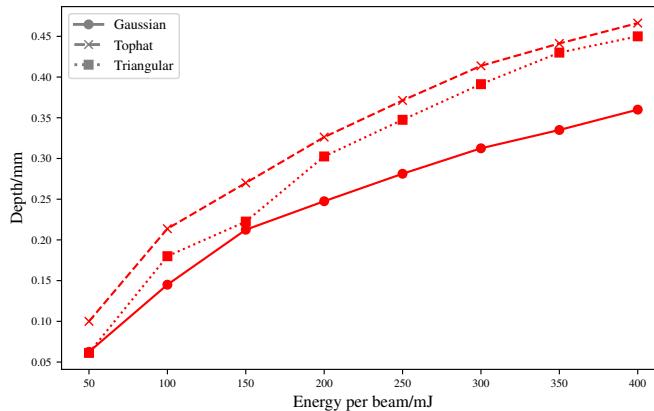
The laser used in the above experiments, the Pixel CO<sub>2</sub>, states that it has a triangular pulse profile for the laser pulses. Thus, in this section the effect of the laser pulse profile has on ablation and the surrounding thermal injury is investigated.

Three different laser pulses profiles are investigated: top hat, triangular, and a Gaussian profiles. The Gaussian profile used has approximately the same area as the two other pulses.

Figure 3.17 show the pulse profiles for a pulselength of 0.2 s. From Fig. 3.18 the top hat pulse profile causes the most ablation, where as the Gaussian pulse causes the least. This can be explained by the fact that the Gaussian pulse delivers energy over a longer time span, thus letting heat diffuse away before it can “build” up and cause damage, and thus ablation.



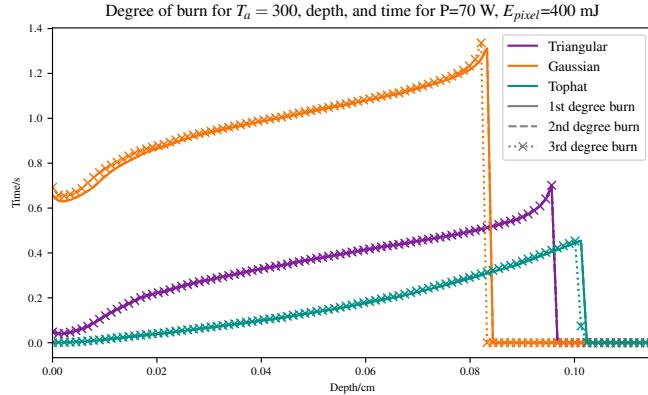
**Figure 3.17:** Comparison of the different pulse profiles trialled for a pulselength of 0.2 s.



**Figure 3.18:** Comparison of various pulse shapes for the pixel beams. Figure shows ablation depths for  $T_a=500\text{ }^{\circ}\text{C}$ .

Figure 3.19 shows the difference between the pulse types with respect to the time it takes to reach a type of burn. There is a large difference in profiles for each of the pulse types. The Gaussian pulse type takes approximately 0.7 s to inflict a burn of any type. Whereas the top hat pulse almost immediately inflict tissue damage, with the triangular pulse type takes

approximately 0.05 s. The profiles of the lines in Fig. 3.19 are also different when compared. Both the Gaussian and triangular pulses have broadly the same shape, whereas the top hat beam has a gradual increasing curve. This is due to the shape of the beams and their energy delivered per second. The top hat beam delivers constant energy at 70 W, where the other two beams peak output is 70 W.

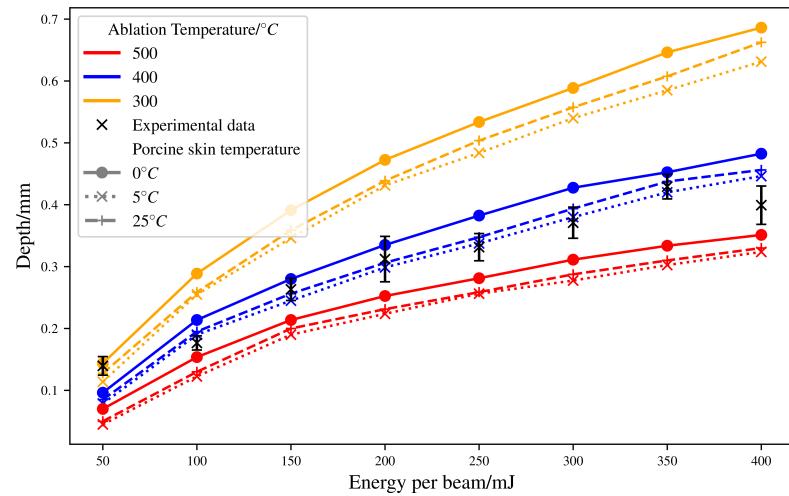


**Figure 3.19:** Figure shows a comparison of the time it takes to inflict a burn on tissue for laser with different pulse profiles.

### Investigating Initial Temperature

As the experiment was carried out on porcine tissue that was kept on ice before the experiment was conducted, we assumed that the initial temperature of the porcine tissue was around 5 °C. This section investigates whether this is an accurate assumption.

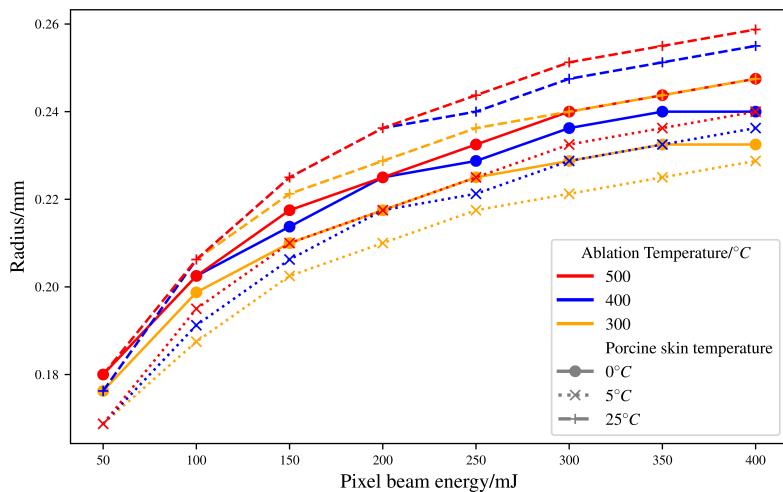
To investigate this, three different temperatures were trialled: 0 °C, 5 °C, and 25 °C. These temperatures correspond to room temperature, the temperature of ice and the original temperature we assumed. Figure 3.20 shows the results of this *in-silico* investigation.



**Figure 3.20:** Comparison of ablation depths for different initial temperatures in porcine skin.

As expected, the hotter the porcine skin is initially, the larger the ablation depth. This occurs as less energy is required to bring the porcine skin to its ablation temperature. In the previous subsections it was assumed that the temperature of the porcine skin was around  $5\text{ }^{\circ}\text{C}$ . This assumption was based upon the fact that the porcine skin was kept on ice before the experiment, thus the temperature of the skin must be between 0 and room temperature. This investigation shows that over small variations of temperature ( $\lesssim 5\text{ }^{\circ}\text{C}$ ), the ablation depth does not vary too much (on the order of  $\approx 0.01\text{ mm}$ ).

However, there is a greater difference in the maximum extent of thermal damage to the skin for different initial temperatures in porcine skin. Figure 3.21 shows this difference.

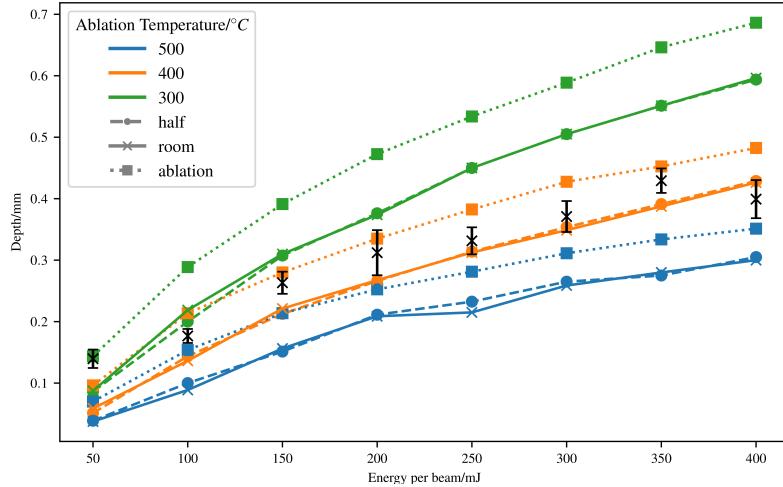


**Figure 3.21:** Comparison of maximum horizontal damage distance for different initial porcine skin temperatures.

### Investigating Voxel Temperature After Ablation

In the previous section it is assumed that the temperature of a voxel remains unchanged after the tissue is removed from that voxel via ablation. However, this assumption may not be accurate as there should be some energy expended in the ablation process which would affect the temperature of the system. To test how much this could affect the simulation, the temperature of the voxel after ablation was varied. Two different temperatures were trialled: half the ablation temperature and room temperature ( $\approx 25\text{ }^{\circ}\text{C}$ ). Figure 3.22 shows the effect of the voxel temperature after ablation has on the ablation depth.

Setting the voxel temperature to either half the ablation temperature or to room temperature has a large effect on the ablation depth, with a difference of  $\approx 0.1\text{ mm}$ . However, there is a small difference between setting the voxel temperature to room temperature and to half of the ablation temperature though. This suggests that if there is some energy expended in ablating tissue, it will only have an effect if the energy required to ablate the tissue is of the order of  $cm\Delta T$  with  $\Delta T$  being equal to at least half of the ablation temperature.



**Figure 3.22:** Comparison of different voxel temperatures after ablation. Half refers to setting the temperature of a voxel to half that of the ablation temperature. Room refers to room temperature, and ablation leaves the temperature at the ablation temperature.

### 3.4 Application of Model for Spy Disposal

In the 1964 James Bond film “Goldfinger”, James Bond is threatened with a laser by the titular antagonist. Would this laser actually cut Bond in half as the film implies, and could Goldfinger be more humane<sup>†</sup> in his choice of laser for the task?

As the first laser was demonstrated in 1960 was a ruby laser of  $694\text{ nm}$ , with the film being released in 1964 and the “laser”<sup>‡</sup> shown on screen being red, the likely laser portrayed is a ruby laser.

To assess whether Bond would die due to the laser, we used the model outlined in this chapter with the following parameters.

As Auric Goldfinger uses this laser to cut sheets of gold, we assumed the power of the laser was around 1 kW, as industrial lasers used to cut metal, are high powered continuous operation lasers. We assumed that the Bond is completely made of skin, with no organs or bones. We ran two simulations, one for the Ruby  $694\text{ nm}$  and one for the  $\text{CO}_2$   $10.6\text{ }\mu\text{m}$ . For the  $\text{CO}_2$  as before there is no scattering due to high absorption coefficient. The Ruby laser’s wavelength is highly scattering, so we model both scattering and absorption. The medium we model is a  $2\text{ cm}^3$  cube of homogeneous skin.

We found that the  $\text{CO}_2$  takes  $22\text{ ms}$ , and Ruby takes  $11\text{ s}$  to ablate through the  $2\text{ cm}$  medium. From timing the movie, the laser moves at a round  $1\text{ cms}^{-1}$ . Therefore the Ruby laser used by Goldfinger would only give Bond some serious burns, but would leave him in one piece. If Goldfinger used a  $\text{CO}_2$  laser then Bond would have been cleanly cut in two.

### 3.5 Conclusion

Using MCRT and a finite difference method, a fully 3D model of photon and heat transport within tissue has been created. This model can be used to simulate the heat deposited by

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<sup>†</sup>i.e could Goldfinger use a laser that would lessen Bond’s suffering.

<sup>‡</sup>A laser was not used on set, but rather was added in post-production.

laser, the ablation craters formed via high powered lasers and the resultant thermal damage surrounding the ablation crater.

The model has been fully compared with both analytical solutions and experimental results. The model was found to match with experimental results that a tissue ablation temperature  $T_a$  of around  $500\text{ }^{\circ}\text{C}$  has to be adopted, toward the higher end of the range previously observed in the literature.

The simulations allow us to predict for a given laser power and pulse length, how much thermal damage is caused in the tissue, and how deep an ablation crater that will form. The computational model could be used in future to help develop treatment regimes for both aesthetic and medical procedures. For example, currently there is considerable amount of “down time” after skin rejuvenation, in which the patient displays inflammation, erythema, edema, pain, and crusting [107–109]. Simulations of thermal damage due to fractional ablation could help design treatment regimes that minimise these effects, whilst still delivering skin rejuvenation. The model can also be applied to help optimise laser assisted drug delivery. Laser assisted drug delivery uses lasers to drill holes into the skin to help promote topical drug diffusion into the skin. Our model can help predict the laser parameters needed to reach a certain hole depth, thus minimising thermal damage and pain to patients.

There are many avenues available with regards to future work on this model. The model presented here in this chapter was on an initially homogeneous skin model. In reality skin is composed of several distinct layers, with each layer containing varying amounts of different chromophores. Our model can easily incorporate an multi-layered skin model complete with various fractions of chromophores. However, as the laser used in these studies is an infra-red laser, water is the highest absorbing chromophore, meaning that a physically accurate model, with various chromophores is not needed for this application. The current model is a voxel based model, where all the voxels are the same size. This allows the model presented in this chapter to be easily set-up, with regards to parallelisation, optical and thermal properties and ease of programming. However, voxel models, where all the voxels are the same size, are not computationally efficient. To achieve good resolution, many voxels are needed, which requires large amounts of RAM, due to a  $\sim n^3$  scaling of voxels to memory in 3D. A more efficient way, would be to allow different sizes of voxels, depending on parts of the model which need high resolution, and parts that do not need high resolution. Such a voxel model is called an adaptive mesh refinement (AMR). There are downsides to AMR: complex implementation for parallelisation and set-up of optical and thermal properties, and slower optical depth integration routines due to more computationally intensive voxel calculations.



# Bibliography

- [1] N. Metropolis. The beginning of the Monte Carlo method. *Los Alamos Science*, 15:125–130, 1987.
- [2] R. Eckhardt. Stan Ulam, John von Neumann, and the Monte Carlo method. *Los Alamos Science*, 15:131–136, 1987.
- [3] H.L Anderson. Metropolis, Monte Carlo, and the MANIAC. *Los Alamos Science*, 14:96–108, 1986.
- [4] S. Ulam, R.D Richtmyer, and J. Von Neumann. Statistical methods in neutron diffusion. *LAMS-551, Los Alamos National Laboratory*, pages 1–22, 1947.
- [5] W.H Ellett, A.B Callahan, and G.L Brownell. Gamma-ray dosimetry of internal emitters. Monte Carlo calculations of absorbed dose from point sources. *The British journal of radiology*, 37(433):45–52, 1964.
- [6] B.C Wilson and G. Adam. A Monte Carlo model for the absorption and flux distributions of light in tissue. *Medical physics*, 10(6):824–830, 1983.
- [7] L. Wang, S.L Jacques, and L. Zheng. MCML Monte Carlo modeling of light transport in multi-layered tissues. *Computer methods and programs in biomedicine*, 47(2):131–146, 1995.
- [8] H. Key, E.R Davies, P.C Jackson, and P.N.T Wells. Monte Carlo modelling of light propagation in breast tissue. *Physics in medicine & biology*, 36(5):591, 1991.
- [9] I.R.M Barnard, P. Tierney, C.L Campbell, L. McMillan, H. Moseley, E. Eadie, C.T.A Brown, and K. Wood. Quantifying direct DNA damage in the basal layer of skin exposed to UV radiation from sunbeds. *Photochemistry and photobiology*, 94(5):1017–1025, 2018.
- [10] D.J Smithies and P.H Butler. Modelling the distribution of laser light in port-wine stains with the Monte Carlo method. *Physics in Medicine & Biology*, 40(5):701, 1995.
- [11] C.L Campbell, K. Wood, R.M Valentine, C.T.A Brown, and H. Moseley. Monte Carlo modelling of daylight activated photodynamic therapy. *Physics in Medicine & Biology*, 60(10):4059, 2015.
- [12] A. El Saddik. Digital twins: The convergence of multimedia technologies. *IEEE MultiMedia*, 25(2):87–92, 2018.
- [13] H. van Houten. The rise of the digital twin: how healthcare can benefit. In <https://www.philips.com/a-w/about/news/archive/blogs/innovation-matters/20180830-the-rise-of-the-digital-twin-how-healthcare-can-benefit.html>. Philips, Aug 2018.

- [14] P. Andreo. Monte Carlo simulations in radiotherapy dosimetry. *Radiation Oncology*, 13(1):121, 2018.
- [15] P. Andreo. Monte Carlo techniques in medical radiation physics. *Physics in Medicine & Biology*, 36(7):861, 1991.
- [16] E.D Cashwell and C.J Everett. A practical manual on the Monte Carlo method for random walk problems. 1959.
- [17] D.W.O Rogers and A.F Bielajew. Monte Carlo techniques of electron and photon transport for radiation dosimetry. *The dosimetry of ionizing radiation*, 3:427–539, 1990.
- [18] L. Badger. Lazzarini’s lucky approximation of  $\pi$ . *Mathematics Magazine*, 67(2):83–91, 1994.
- [19] P. Beckmann. *A history of Pi*. St. Martin’s Griffin, 2015.
- [20] G.L Buffon. *Histoire naturelle générale et particulière*, volume 18. de l’Imprimerie de F. Dufart, 1785.
- [21] Matt Pharr, Wenzel Jakob, and Greg Humphreys. *Physically based rendering: From theory to implementation*. Morgan Kaufmann, 2016.
- [22] P. Jäckel. *Monte Carlo methods in finance*. J. Wiley, 2002.
- [23] D.B Hertz. Risk analysis in capital investment. *Harvard Business Review*, 42(1):95–106, 1964.
- [24] J.V Wall and C.R Jenkins. *Practical statistics for astronomers*. Cambridge University Press, 2012.
- [25] J.T Kajiya. The rendering equation. *SIGGRAPH Comput. Graph.*, 20(4):143–150, August 1986.
- [26] R.L Cook, T. Porter, and L. Carpenter. Distributed ray tracing. *SIGGRAPH Comput. Graph.*, 18(3):137–145, January 1984.
- [27] T.P Robitaille. HYPERION: an open-source parallelized three-dimensional dust continuum radiative transfer code. *Astronomy & Astrophysics*, 536:A79, 2011.
- [28] T. Harries. Torus: Radiation transport and hydrodynamics code. *Astrophysics Source Code Library*, 2014.
- [29] R.M Valentine, C.T.A Brown, K. Wood, H. Moseley, and S. Ibbotson. Monte Carlo modeling of in vivo protoporphyrin IX fluorescence and singlet oxygen production during photodynamic therapy for patients presenting with superficial basal cell carcinomas. *Journal of Biomedical Optics*, 16(4):048002, 2011.
- [30] K. Wood and R.J Reynolds. A model for the scattered light contribution and polarization of the diffuse  $h\alpha$  galactic background. *The Astrophysical Journal*, 525(2):799, 1999.
- [31] D.W.O Rogers, B.A Faddegon, G.X Ding, C-M. Ma, J. We, and T.R Mackie. BEAM: a Monte Carlo code to simulate radiotherapy treatment units. *Medical physics*, 22(5):503–524, 1995.

- [32] B.C Wilson and G. Adam. A Monte Carlo model for the absorption and flux distributions of light in tissue. *Medical Physics*, 10(6):824–830, 1983.
- [33] L.V Wang and H. Wu. *Biomedical optics: principles and imaging*. John Wiley & Sons, 2012.
- [34] S. Chandrasekhar. *Radiative transfer*. Courier Corporation, 2013.
- [35] V.DMZ Barbařić-Mikočević and K. Itrić. Kubelka-Munk theory in describing optical properties of paper (i). *Technical Gazette*, 18(1):117–124, 2011.
- [36] M. Jasiński. Modelling of light and human skin interaction using Kubelka-Munk theory. *Scientific Research of the Institute of Mathematics and Computer Science*, 10(1):71–81, 2011.
- [37] WF. Cheong, S.A Prahl, and A.J Welch. A review of the optical properties of biological tissues. *IEEE journal of quantum electronics*, 26(12):2166–2185, 1990.
- [38] M. Gabriela. Mathematical methods in biomedical optics. *ISRN Biomedical Engineering*, 2013, 2013.
- [39] S.A Prahl. Light transport in tissue. 1990.
- [40] R. Graaff, J.G Aarnoudse, F.F.M de Mul, and H.W Jentink. Similarity relations for anisotropic scattering in absorbing media. *Optical engineering*, 32(2):244–253, 1993.
- [41] G. Yoon, S.A Prahl, and A.J Welch. Accuracies of the diffusion approximation and its similarity relations for laser irradiated biological media. *Applied Optics*, 28(12):2250–2255, 1989.
- [42] T.P Robitaille. On the modified random walk algorithm for Monte-Carlo radiation transfer. *Astronomy & Astrophysics*, 520:A70, 2010.
- [43] M. Min, C.P Dullemond, C. Dominik, A. de Koter, and J.W Hovenier. Radiative transfer in very optically thick circumstellar disks. *Astronomy & Astrophysics*, 497(1):155–166, 2009.
- [44] S.L Jacques. Origins of tissue optical properties in the UVA, visible, and NIR regions. *OSA TOPS on advances in optical imaging and photon migration*, 2:364–369, 1996.
- [45] S.L Jacques. Optical properties of biological tissues: a review. *Physics in Medicine & Biology*, 58(11):R37, 2013.
- [46] T. Lister, P.A Wright, and P.H Chappell. Optical properties of human skin. *Journal of biomedical optics*, 17(9):090901, 2012.
- [47] L.G Henyey and J.L Greenstein. Diffuse radiation in the galaxy. *The Astrophysical Journal*, 93:70–83, 1941.
- [48] S.L Jacques, C.A Alter, and S.A Prahl. Angular dependence of hene laser light scattering by human dermis. *Lasers Life Sci*, 1(4):309–333, 1987.
- [49] J.M Dixon, M. Taniguchi, and J.S Lindsey. PhotochemCAD 2: A refined program with accompanying spectral databases for photochemical calculations. *Photochemistry and photobiology*, 81(1):212–213, 2005.

- [50] S. Prahl. PhotochemCAD spectra. <https://omlc.org/spectra/PhotochemCAD/index.html>, 2017. [Online; Last accessed 4-February-2019].
- [51] D.J Segelstein. *The complex refractive index of water*. PhD thesis, University of Missouri-Kansas City, 1981.
- [52] R.M Pope and E.S Fry. Absorption spectrum (380–700 nm) of pure water. II. integrating cavity measurements. *Applied optics*, 36(33):8710–8723, 1997.
- [53] R.L.P van Veen, H.J.C.M Sterenborg, A. Pifferi, A. Torricelli, and R. Cubeddu. Determination of VIS–NIR absorption coefficients of mammalian fat, with time-and spatially resolved diffuse reflectance and transmission spectroscopy. In *Biomedical Topical Meeting*, page SF4. Optical Society of America, 2004.
- [54] I.S Saidi. *Transcutaneous optical measurement of hyperbilirubinemia in neonates*. PhD thesis, Rice University, 1992.
- [55] J.A Iglesias-Guitian, C. Aliaga, A. Jarabo, and D. Gutierrez. A biophysically-based model of the optical properties of skin aging. In *Computer Graphics Forum*, volume 34, pages 45–55. Wiley Online Library, 2015.
- [56] A.N Bashkatov, E.A Genina, and V.V Tuchin. Optical properties of skin, subcutaneous, and muscle tissues: a review. *Journal of Innovative Optical Health Sciences*, 4(01):9–38, 2011.
- [57] T. Sarna and H.A Swartz. The physical properties of melanins. *The pigmentary system: physiology and pathophysiology*, pages 311–341, 2006.
- [58] L.B Lucy. Computing radiative equilibria with Monte Carlo techniques. *Astronomy and Astrophysics*, 344:282–288, 1999.
- [59] M. Metcalf, J. Reid, and M. Cohen. *Modern Fortran Explained*. Oxford University Press, 2011.
- [60] J.E Bjorkman and K. Wood. Radiative equilibrium and temperature correction in monte Carlo radiation transfer. *The Astrophysical Journal*, 554(1):615, 2001.
- [61] W. Gropp, E. Lusk, and A. Skjellum. *Using MPI: Portable Parallel Programming with the Message-Passing Interface*. Scientific and Engineering Computation. MIT Press, 2014.
- [62] W. Gropp, T. Hoefler, R. Thakur, and E. Lusk. *Using advanced MPI: Modern features of the message-passing interface*. MIT Press, 2014.
- [63] G.M Amdahl. Validity of the single processor approach to achieving large scale computing capabilities. In *Proceedings of the April 18-20, 1967, Spring Joint Computer Conference*, pages 483–485. ACM, 1967.
- [64] J. Bjorkman. Monte Carlo radiation transfer. Presented as SAMCSS 2013, 2013.
- [65] C.M Gardner, S.L Jacques, and A.J Welch. Fluorescence and reflectance spectra specify intrinsic fluorescence spectrum corrected for tissue optics distortion. In *Advances in Fluorescence Sensing Technology*, volume 1885, pages 122–129. International Society for Optics and Photonics, 1993.

- [66] S.L Jacques, R. Joseph, and G. Gofstein. How photobleaching affects dosimetry and fluorescence monitoring of PDT in turbid media. In *Optical Methods for Tumor Treatment and Detection: Mechanisms and Techniques in Photodynamic Therapy II*, volume 1881, pages 168–180. International Society for Optics and Photonics, 1993.
- [67] D. Manstein, G.S Herron, R.K Sink, H. Tanner, and R.R Anderson. Fractional photothermolysis: a new concept for cutaneous remodeling using microscopic patterns of thermal injury. *Lasers in Surgery and Medicine: The Official Journal of the American Society for Laser Medicine and Surgery*, 34(5):426–438, 2004.
- [68] S. Amini-Nik, D. Kraemer, M.L Cowan, K. Gunaratne, P. Nadesan, B.A Alman, and R.J Dwayne Miller. Ultrafast mid-IR laser scalpel: protein signals of the fundamental limits to minimally invasive surgery. *PLoS One*, 5(9):e13053, 2010.
- [69] O.T Tan, K. Sherwood, and B.A Gilchrest. Treatment of children with port-wine stains using the flashlamp-pulsed tunable dye laser. *New England Journal of Medicine*, 320(7):416–421, 1989.
- [70] M. Kuperman-Beade, V.J Levine, and R. Ashinoff. Laser removal of tattoos. *American Journal of Clinical Dermatology*, 2(1):21–25, 2001.
- [71] S.H Liew. Laser hair removal. *American Journal of Clinical Dermatology*, 3(2):107–115, 2002.
- [72] C.A Hardaway and E.V Ross. Nonablative laser skin remodeling. *Dermatologic Clinics*, 20(1):97–111, 2002.
- [73] S.M Shapshay, M.S Strong, G.W Anastasi, and C.W Vaughan. Removal of rhinophyma with the carbon dioxide laser: a preliminary report. *Archives of Otolaryngology*, 106(5):257–259, 1980.
- [74] R. Valcavi, F. Riganti, A. Bertani, D. Formisano, and C.M Pacella. Percutaneous laser ablation of cold benign thyroid nodules: a 3-year follow-up study in 122 patients. *Thyroid*, 20(11):1253–1261, 2010.
- [75] M. Hædersdal, F.H Sakamoto, W.A Farinelli, A.G Doukas, J. Tam, and R.R Anderson. Fractional CO<sub>2</sub> laser-assisted drug delivery. *Lasers in Surgery and Medicine: The Official Journal of the American Society for Laser Medicine and Surgery*, 42(2):113–122, 2010.
- [76] M.R Alexiades-Armenakas, J.S Dover, and K.A Arndt. The spectrum of laser skin resurfacing: nonablative, fractional, and ablative laser resurfacing. *Journal of the American Academy of Dermatology*, 58(5):719–737, 2008.
- [77] D.V Widder. *The Heat Equation*, volume 67. Academic Press, 1976.
- [78] N. Ozisik. *Finite Difference Methods in Heat Transfer*. CRC press, 1994.
- [79] Alma Lasers GmbH. *PixelCO2 Operator's Manual*. Alma Lasers GmbH.
- [80] S. Ristov, R. Prodan, M. Gusev, and K. Skala. Superlinear speedup in hpc systems: Why and when? In *Computer Science and Information Systems (FedCSIS), 2016 Federated Conference on*, pages 889–898. IEEE, 2016.
- [81] A.J Welch, M.J.C Van Gemert, et al. *Optical-thermal Response of Laser-irradiated Tissue*, volume 2. Springer, 2011.

- [82] N.T Wright. Quantitative models of thermal damage to cells and tissues. In *Heat Transfer and Fluid Flow in Biological Processes*, pages 59–76. Elsevier, 2015.
- [83] M.H Niemz. *Laser-tissue interactions: fundamentals and applications*. Springer Science & Business Media, 2013.
- [84] F. Petrella, S. Cavaliere, and L. Spaggiari. Popcorn effect. *Journal of Bronchology & Interventional Pulmonology*, 20(2):193–194, 2013.
- [85] R.M Verdaasdonk, C. Borst, and M.J.C Van Gemert. Explosive onset of continuous wave laser tissue ablation. *Physics in Medicine & Biology*, 35(8):1129, 1990.
- [86] W. Husinsky, G. Grabner, I. Baumgartner, F. Skorpik, S. Mitterer, and T. Temmel. Mechanisms of laser ablation of biological tissue. In *Desorption Induced by Electronic Transitions DIET IV*, pages 362–367. Springer, 1990.
- [87] M.S Kitai, V.L Popkov, V.A Semchischen, and A.A Kharizov. The physics of UV laser cornea ablation. *IEEE journal of quantum electronics*, 27(2):302–307, 1991.
- [88] A.A Oraevsky, R.O Esenaliev, and V.S Letokhov. Pulsed laser ablation of biological tissue: Review of the mechanisms. In *Laser Ablation Mechanisms and Applications*, pages 112–122. Springer, 1991.
- [89] A. Vogel and V. Venugopalan. Mechanisms of pulsed laser ablation of biological tissues. *Chemical Reviews*, 103(2):577–644, 2003.
- [90] G. Koren and J.T.C Yeh. Emission spectra, surface quality, and mechanism of excimer laser etching of polyimide films. *Applied Physics Letters*, 44(12):1112–1114, 1984.
- [91] J.E Andrew, P.E Dyer, D. Forster, and P.H Key. Direct etching of polymeric materials using a XeCl laser. *Applied Physics Letters*, 43(8):717–719, 1983.
- [92] A.L McKenzie. Physics of thermal processes in laser-tissue interaction. *Physics in Medicine & Biology*, 35(9):1175, 1990.
- [93] A.L McKenzie. A three-zone model of soft-tissue damage by a CO<sub>2</sub> laser. *Physics in Medicine & Biology*, 31(9):967, 1986.
- [94] B. Majaron, P. Plestenjak, and M. Lukač. Thermo-mechanical laser ablation of soft biological tissue: modeling the micro-explosions. *Applied Physics B*, 69(1):71–80, 1999.
- [95] M. Gerstmann, Y. Linenberg, A. Katzir, and S. Akselrod. Char formation in tissue irradiated with a CO<sub>2</sub> laser: model and simulations. *Optical Engineering*, 33(7):2343–2352, 1994.
- [96] A. Sagi, A. Avidor-Zehavi, A. Shitzer, M. Gerstmann, S. Akselrod, and A. Katzir. Heating of biological tissue by laser irradiation: temperature distribution during laser ablation. *Opt. Eng.*, 31(7):1425–1431, 1992.
- [97] J.A Pearce. Relationship between Arrhenius models of thermal damage and the CEM 43 thermal dose. In *Energy-based Treatment of Tissue and Assessment V*, volume 7181, page 718104. International Society for Optics and Photonics, 2009.
- [98] F.C. Jr Hendriques. Studies of thermal injury; the predictability and the significance of thermally induced rate processes leading to irreversible epidermal injury. *Arch. Pathol.(Chic)*, 43:489–502, 1947.

- [99] S.C Jiang, N. Ma, H.J Li, and X.X Zhang. Effects of thermal properties and geometrical dimensions on skin burn injuries. *Burns*, 28(8):713–717, 2002.
- [100] K.R Diller and L.J Hayes. A finite element model of burn injury in blood-perfused skin. *Journal of Biomechanical Engineering*, 105(3):300–307, 1983.
- [101] J. Zhang and X. Zhang. Dynamic modeling of tissue ablation with continuous wave CO<sub>2</sub> laser. In *2007 1st International Conference on Bioinformatics and Biomedical Engineering*, pages 1057–1060. IEEE, 2007.
- [102] I.V Meglinski and S.J Matcher. Quantitative assessment of skin layers absorption and skin reflectance spectra simulation in the visible and near-infrared spectral regions. *Physiological Measurement*, 23(4):741, 2002.
- [103] B.R Loiola, H.R.B Orlande, and G.S Dulikravich. Thermal damage during ablation of biological tissues. *Numerical Heat Transfer, Part A: Applications*, pages 1–17, 2018.
- [104] V. Casalegno, P. Vavassori, M. Valle, M. Ferraris, M. Salvo, and G. Pintsuk. Measurement of thermal properties of a ceramic/metal joint by laser flash method. *Journal of Nuclear Materials*, 407(2):83–87, 2010.
- [105] E. MacCormack, A. Mandelis, M. Munidasa, B. Farahbakhsh, and H. Sang. Measurements of the thermal diffusivity of aluminum using frequency-scanned, transient, and rate window photothermal radiometry. theory and experiment. *International journal of thermophysics*, 18(1):221–250, 1997.
- [106] Alma Lasers. Pixel CO<sub>2</sub>, 2018.
- [107] M. Lapidoth, S. Halachmi, S. Cohen, and D.B Amitai. Fractional CO<sub>2</sub> laser in the treatment of facial scars in children. *Lasers in Medical Science*, 29(2):855–857, 2014.
- [108] M.A Trelles, M. Shohat, and F. Urdiales. Safe and effective one-session fractional skin resurfacing using a carbon dioxide laser device in super-pulse mode: a clinical and histologic study. *Aesthetic Plastic Surgery*, 35(1):31–42, 2011.
- [109] E. Kohl, J. Meierhöfer, M. Koller, F. Zeman, L. Groesser, S. Karrer, U. Hohenleutner, M. Landthaler, and S. Hohenleutner. Fractional carbon dioxide laser resurfacing of rhytides and photoaged skin—a prospective clinical study on patient expectation and satisfaction. *Lasers in Surgery and Medicine*, 47(2):111–119, 2015.