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Cooperative Game Theory
Problem Set 3

Problem 1 Notation: the strategies adopted by both players will be given by (p, q), where p denotes the probability the row player picks the first option and q the probability the column player picks the first.

(a) The zero sum game for this negotiation is $\begin{pmatrix} 1 & 4 \\ 0 & -3 \end{pmatrix}$. Because $p = 1$ is a dominant strategy for player 1 (row), the nash equilibrium for this game is given by the threats $(1, 1)$, with $D(1, 1) = (3, 2)$. $\sigma = 6$, with $(1, 0)$, so $R = (3.5, 2.5)$ with player 1 paying 1.5 to 2.

(b) The zero sum game for this negotiation is $\begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$. Solving the equations $q - 3(1 - q) = 4 - 4(1 - q)$ and $p = -3p + r(1 - p)$ gives the disagreement point $D(1/2, 7/8) = (1.5, 1)$. The TU solution involves playing $(0, 0)$, and player 1 pays player 2 1.75 for $R = (3.25, 2.75)$.

(c) The zero sum game is $\begin{pmatrix} 3 & -6 \\ -1 & -1 \end{pmatrix}$. 0 is a dominant strategy for the column player, giving us $D(0, 0) = (5, 6)$, which is also the profit maximizing set of strategies, thus no transfers need to be made.

(d) The zero sum game is $\begin{pmatrix} -1 & 3 \\ -3 & 7 \end{pmatrix}$. 1 is a dominant strategy for 2, thus we have $D(1, 1) = (-1, 0)$. The TU involves playing $(0, 0)$, with 1 paying 4 for $R = (3, 4)$.

Problem 2 The matrix for the zero sum game is:

$$\begin{pmatrix} -2 & 2 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

$(3, 3)$ is a nash equilibrium for this game (denoting the row/column of the game), and it is also the profit maximizing move. Thus $D = R = (3, 3)$. The graph is as follows:

Problem 3 False: consider the counterexample with the game

$$\begin{pmatrix} 3,3 & 0,1 \\ 0,2 & 0,1 \end{pmatrix}.$$

The zero sum game for this is

$$\begin{pmatrix} 0 & -1 \\ -2 & -1 \end{pmatrix},$$

with a nash equilibrium at $D(0,0) = (0,1)$. With this disagreement point, the TU value is actually $(2.5, 3.5)$.