Error of a Maximum Likelihood Sequence Detector With Viterbi Over an AWGN Channel

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Abstract- This simulation will show the probability of error that a Maximum Likelihood Sequence Detector would incur when receiving a signal that has been modulated with memory. The detector will utilize the Viterbi algorithm to reduce the complexity in the amount of sequences to compare, along with the Monte Carlo Method to find the number of errors per sequence by introducing the a symbol from the demodulator to the detector incrementally.

I. Introduction

The Maximum Likelihood Sequence Detector is useful for detecting sequences which contain symbols that are dependent on the appearance of prior symbols, these modulation systems are called modulation systems with memory, the relationship between symbols can be represented by a trellis or a state diagram. This state diagram must be searched to find the optimal sequence $\hat{x}$. Searching through this sequence can be cumbersome as depending on the length of said sequence and the amount of different transitions and states there are, the amount of sequences to verify is extremely long. Furthermore, this complexity can be reduced by using a trellis search algorithm discovered by Andrew Viterbi.

II. Encoding

This experiment uses a specific state diagram and trellis as shown in figure 1 and 2, however, any encoding can be used that creates inter-symbol dependence. The state diagram illustrates the dependence between symbols, this will be shown through a this example: Given that the initial state is state 1, and the source generates an input of 1, the output symbol would be 3, and the current state is moved to state 2. Notice that when the current state becomes state 2, that not all output symbols are possible at state 2, this is the interdependence between symbols. Furthermore, at state 2, the source generates an input 0, which will result in an output symbol -1, if the source generated input 1, the output will be the -3 symbol. In conclusion, this encoding introduces a dependence that can be exploited by the Viterbi Algorithm to reduce complexity, of the amount of paths that need to be explored.

III. Maximum Likelihood Sequence Detector

A Maximum Likelihood Sequence Detector (MLSD), was created for modulation systems with memory, or a scheme whose signal to be transmitted depends on the prior symbols that have been transmitted. This relationship can be expressed through the use of a mapping from the current set of k bits and the past $(L-1)k$ bits to the set of M = 2^k[2]. Systems with memory utilize dependence between symbols allowing for shaping of the transmitted signal in turn making it possible to match the spectral characteristics of the channel. These modulations systems can be represented as finite-state machines, or a trellis diagrams, which are both graphical representations that show the relationships between states and their respective transitions. The optimal detection rule for a MLSD is to choose symbols that minimize the euclidean distance at each time the detector receives a symbol from the demodulator. For example, if the transition from the starting state is considered as well, observing the trellis shown in figure 2, it is shown that the steady state form is reached after 2 transitions, this is due to the fact that the pattern is repeating. There are two paths entering and leaving each node, the paths entering correspond to symbols. Looking at the right-most state 1 in the figure 2, the trellis diagram, the possible paths are as follows, (0, 0), (0, 0) starting from state 3, (0, 0), or (0, 0) coming from state 4. Note, moving from left to right of the parenthesis shows a transition of oldest to newest input symbols. To find the most likely path, the distance metrics must be found like so $D_0(-3,-3) = (r1-(-3))^2 + (r2-(-3))^2$, where the nearer the received symbol is to the actual symbol, the closer the distance metric will be to 0, meaning this is more likely to be the path than not. This is done for all possible paths, which in total is 4^L paths to consider. In general, if given a sequence of L length, and the modulation scheme is non return to zero, then the detector will have to consider all 2^L messages to find the message with the highest likelihood. Reducing this overhead motivates the use of the Viterbi Algorithm.

IV. Implementation

A. Channel

The channel is an AWGN (Additive White Gaussian Noise) channel. AWGN noise has constant power spectral density, and can be represented as a Gaussian random variable, $n \sim \mathcal{N}(0, N)$, and is added to the transmitted signal,

$$y_i = x_i + n$$

where x_i is the i^{th} transmitted symbol and n is the noise. If the SNR is low, meaning that the noise is louder than the signal, then the received signal may look like another symbol altogether.

V. Algorithm

1) Viterbi

The Viterbi algorithm is a sequential trellis search algorithm that optimizes the MLSD, Viterbi can be utilized to eliminate

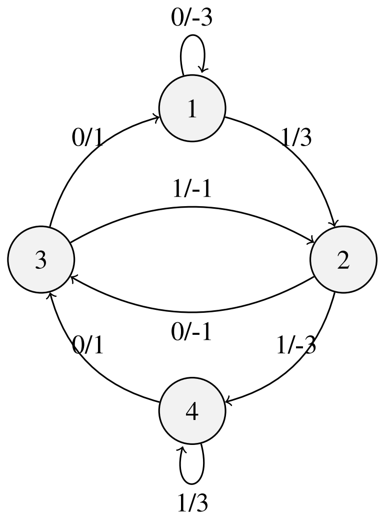

Fig. 1. The Finite State Machine Used shows the interdependence between symbols, and was used in the experiment

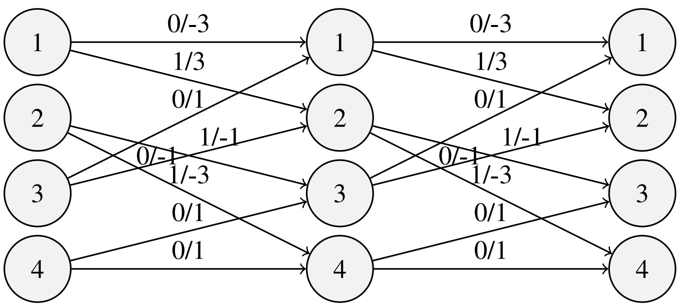

Fig. 2. The Trellis for the respective finite state machine in figure 2

sequences when the detector receives new data from the demodulator. With respect to the Trellis Diagram and the example given in the section above, the Viterbi Algorithm improves the overhead by comparing the 2 metrics per state in the case of the 4 state diagram, or in case of the NRZI encoded example, the 2 metrics, and prunes all paths aside from the lowest metric path. The logic behind only saving the lowest metric branch is as follows, any extension down this path will always have a larger metric than the branch that was saved [2]. Furthermore, on the next iteration of the where the demodulator sends r_3 to the detector, the survivor paths are left over from each state and the new possible paths are found as well as their distance metrics, and the one with the largest metric gets pruned. The Viterbi Algorithm continuously receives transmitted symbols from the demodulator, and saves one possible path per state. Finally, at each end of each stage, half of the possible paths are removed, reducing the complexity at each transmission by a factor of 2.

2) Monte Carlo Method

The algorithm used is the Monte Carlo method. The Monte Carlo method is an algorithm that focuses on independent random sampling to solve deterministic problems. The transmitted symbols and channel noise are randomly sampled and added together which represents the received signal at the detector. The way the random input sequence for finding the outputs to send over the medium was designed aimed to only make possible sequences, meaning that the state diagram relationship had to be transformed in a matrix where the current state and next state could be found.

VI. Results and Conclusion

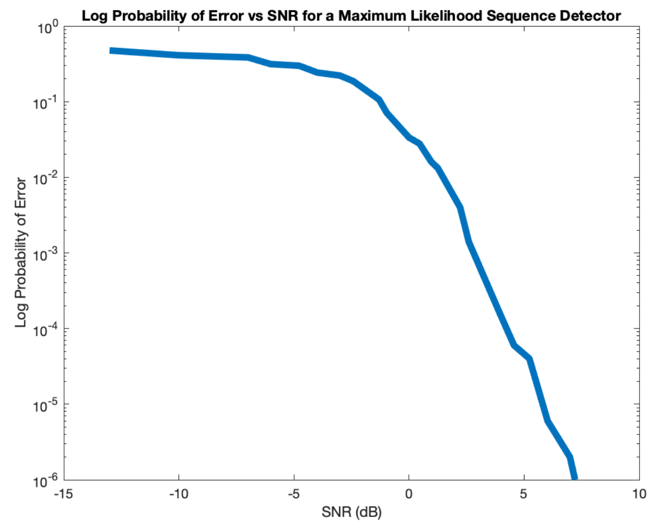

Fig. 3. Log Probability of Error vs SNR for a Maximum Likelihood Sequence Detector

The experiment shows that when the SNR is very small, in other words that the noise is louder than the signal, that the maximum likelihood sequence detector will randomly guess the sequence, where as when the signal gets louder and louder compared to the noise, the probability of error between $\hat{x}$ and x is very low and approaching to 0 the higher the SNR, as shown in figure 3. Introducing the inter-symbol dependence as well as making use of the Viterbi algorithm makes improvements over the prior Maximum Likelihood Detection, which would have not been usable in this case because of said dependence.

References

[1] John Proakis, *Digital Communications*, 5th ed. New York, United States of America: McGraw-Hill Education, 2000, pp 254-300.

[2] John Proakis, Masoud Salehi *Communication Systems Engineering*, 2nd ed. New Jersey, United States of America: Prentice Hall, 2001, pp 254-300.