HW 6 Solutions

1

$$u = s^{\alpha}(y - tx - sp)^{1-\alpha}$$

$$\frac{\partial u}{\partial s} = \alpha \cdot \frac{\cancel{u}}{s} - (1-\alpha)\cdot\frac{\cancel{u}}{y - tx - sp}\cdot p = 0$$

$$\frac{\alpha}{s} = \frac{1-\alpha}{y - tx - sp}\cdot p$$

$$\frac{\alpha}{p}\cdot(y - tx) - \cancel{s\alpha} = s - \cancel{s\alpha}$$

$$s(p) = \frac{\alpha}{p}\cdot(y - tx) \qquad z = y - ps(p) = (1-\alpha)(y - tx)$$

a

$$v(p, y - tx) = \left[\frac{\alpha}{p}(y - tx)\right]^{\alpha}\left[(1-\alpha)(y - tx)\right]^{1-\alpha}$$

$$v(p, y - tx) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(y - tx)\cdot p^{-\alpha}$$

$$\bar{u} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(y - tx)\,p^{-\alpha}$$

Solve for p...
$$\boxed{\bar{p}(x) = \alpha\underbrace{\left[\frac{(1-\alpha)^{1-\alpha}\cdot(y - tx)}{\bar{u}}\right]^{1/\alpha}}}$$

b

$$\bar{s}(x) = s[\bar{p}(x)] = \frac{\cancel{\alpha}\cdot(y - tx)}{\cancel{\alpha}\,[\ldots]} = \left[\frac{\bar{u}}{(1-\alpha)^{1-\alpha}\cdot(y - tx)^{1-\alpha}}\right]^{1/\alpha}$$

2

a

$$f(k) = k^{\beta} \rightarrow k(f) = f^{1/\beta}$$

so we have
$$\pi = \underbrace{f\cdot p}_{TR} - \underbrace{k\cdot c_k}_{TC} = f\cdot p - f^{1/\beta}\cdot c_k$$

$$\text{FOC: } \frac{\partial \pi}{\partial f} = 0 = p - \frac{1}{\beta}\, f^{\frac{1-\beta}{\beta} = \frac{1}{\beta} - 1}\, c_k$$

$$f^{*}(p) = \left(\frac{\beta}{c_k}\cdot p\right)^{\frac{\beta}{1-\beta}}$$

b

$$\pi^{*}(p) = f^{*}(p)\cdot p - f^{*}(p)^{1/\beta}\, c_k$$

$$= \left(\frac{\beta}{c_k}\cdot p\right)^{\frac{\beta}{1-\beta}}\cdot p - \left(\frac{\beta}{c_k}p\right)^{\frac{1}{1-\beta}}\cdot c_k$$

$$= \left(\frac{\beta}{c_k}\right)^{\frac{\beta}{1-\beta}} p^{\frac{1}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\cdot p^{\frac{1}{1-\beta}}\, c_k^{-\frac{\beta}{1-\beta}}$$

$$\pi^{*}(p) = \left(\frac{\beta}{c_k}\right)^{\frac{\beta}{1-\beta}}\cdot p^{\frac{1}{1-\beta}}\cdot(1-\beta)$$