Steam Drive

May 18, 2016

Lewis Li, Daniel Brito

1 Introduction

In this part of the project, we are assuming that steam is being injected into a well at a constant pressure, rate and quality. We will evaluate the project success as a function of time, by estimating the changes in thermal efficiency, area of steam zone, and cumulative oil production. We will apply this based on the numbers from the Kern River project, which assumes a project life of 20 years. The reservoir parameters are assumed to be well understood, and is given below:

Symbol	Units	Value	Description
T_r	F	90	Reservoir temperature
phi	%	0.32	Reservoir porosity
h	ft	55	Net pay thickness
S_{or}	%	0.15	Residual oil saturation
ΔS_o	%	0.4	Oil saturation variation
M_s	BTU/cu.ft-F	42	Overburden volumetric heat capacity
M_t	BTU/cu.ft-F	35	Total formation volumetric heat capacity
K_s	BTU/cu.ft-F-hr	1.2	Overburden thermal conductivity

```
In [1]: Tr = 90;                    % Reservoir temperature (F)
        phi = 0.32;                 % Porosity
        h = 55;                     % Net pay thickness(ft)
        Sor = 0.15;                 % Residual oil saturation
        deltaSo = 0.4;              % Oil saturation variation
        Ms = 42;                    % BTU/cu.ft-F
        Mt = 35;                    % BTU/cu.ft-F
        Ks = 1.2;                   % BTU/cu.ft-F-hr
```

The steam injection conditions and steam properties are given as:

Symbol	Units	Value	Description
P_{bh}	psig	100	Bottom hole injection pressure
rate	bbl/day	360	bbl/day CWE (i.e., the rate on a condensed water basis)

SymbolUnits		Value	Description
X_{bh}	ft	0.5	Steam quality
A_{well}	acres	2.5	Drainage area
T_s	F	338	Steam temperature
Δ_H^v	n/a	880	delta-H for vapour
$HwTs$	BTU/lb	310	enthalpy of water @ steam temperature
$HwTr$	BTU/lb	58	enthalpy of water @ reservoir temperature
C_w	BTU/lb-F	$\frac{HwTs-HwTr}{T_s-T_r}$	heat capacity of water over the temperature range

In [2]:
```
% Bottom hole injection pressure (psig)
Pbh = 100;
% bbl/day CWE (i.e., the rate on a condensed water basis)
rate = 360;
% Steam quality
Xbh = 0.5;
% Drainage area
acresPerWell = 2.5;
% Steam temperature (F)
Ts = 338;
% delta_H for vapour
deltaHv = 880;
% BTU/lb @ steam temp
HwTs = 310;
% BTU/lb @ res temp
HwTr = 58;
% (BTU/lb-F)
Cw = (HwTs - HwTr) / (Ts - Tr);
```

The mass injection rate can be just evaluated as follows:

In [3]:
```
%Mass injection rate
DaysInYear = 365;
VolInBarrel = 5.615; %ft3/B
Density = 62.4; %lb/ft^3
% Rate of injection (lb/year)
mi = rate * VolInBarrel * Density * DaysInYear;
```

Our next step is to convert time into a dimensionless unit which is given as:

$$t_d = 4t\left(\frac{M_s}{M_t}\right)^2\left(\frac{\alpha_s}{h^2}\right)$$

```
In [4]: %% Thermal diffusivity
        alphaSs = 1.2*24/42;              % Thermal diffusivity (ft^2/day)


        %% Descritize time
        t = [0.02098296, 0.10491482, 0.20982964, 1.34290969, ...
        2.09829639, 4.19659278,6.29488917, 8.39318557, ...
        10.491482, 20.9829639, 41.9659278, 62.9488917];
        t = t * DaysInYear;

        %% Calculate dimensionless time
        td = 4 * t * ( Ms / Mt )^2 * ( alphaSs / h^2 );
        ti = t / DaysInYear;
```

The heat injection rate is evaluated as:

$$Q_i = (\dot{m}_i C_W \Delta T + X \dot{m}_i \Delta H_w)\, t$$

```
In [5]: Qi = ( mi * Cw * (Ts - Tr)  + Xbh * mi * deltaHv ) * ti;
```

We need to evaluate the reservoir heat efficiency. We will use two different approaches: 1) Marx-Langenheim and 2) Myhill-Stegemeier

2 Marx-Langenheim

In Marx-Langenheim, the reservoir heat efficiency is evaluated as:

$$E_h = \frac{2\sqrt{\frac{t_D}{\pi}} - 1 + e^{t_d} erfc\sqrt{t_D}}{t_D}$$

```
In [6]: EhMarx = (2*sqrt(td/pi) - 1 + exp(td).* erfc( sqrt(td)))./td;
```

The steam volume is then just:

$$V_s = \frac{Q_i E_h}{M_T \Delta T}$$

```
In [7]: VsMarx = Qi.*EhMarx / (Mt * (Ts - Tr));
```

The cumulative oil production is:

$$N_p = V_s \phi \Delta S_o$$

```
In [8]: NpMarx = VsMarx .* phi * deltaSo / VolInBarrel;
```

The OSR (Oil Steam Ratio) is determined by:

$$OSR = \frac{N_p}{V_{s,eq}}$$

```
In [9]: OSRMarx= NpMarx./(mi*(t/DaysInYear)/(Density*VolInBarrel));
```

The area covered by the steam is:

$$Acres = \frac{V_s}{h}$$

```
In [10]: AcresMarx = VsMarx ./ h;
```

3 Myhill-Stegemeier

The second approach we will use to evaluate the reservoir heat efficiency is the Myhill-Stegemeier technique. The first step is to evaluate the critical time using the expression:

$$e^{t_{cD}} erfc\sqrt{t_{cD}} = \frac{1}{1+h_D}$$

where h_D is the ratio of latent heat to sensible heat:

$$h_D = \frac{X\Delta H_v}{C_w \Delta T}$$

```
In [11]: hd = Xbh * deltaHv / (Cw *(Ts - Tr) );  % latent heat / sensible heat
         tcd = fsolve(@(tcD) exp(tcD).*erfc(sqrt(tcD)) -1/(1+hd),[1], ...
         optimset('Algorithm','levenberg-marquardt','Display','iter'));
         tcd = tcd(end);
```

```
First-Order                    Norm of
 Iteration  Func-count    Residual        optimality      Lambda           step
     0           2       0.00402232        0.00866         0.01
     1           4       0.00076832        0.00284        0.001        0.302283
     2           6      2.28396e-05       0.000403       0.0001        0.246897
     3           8      2.58217e-08        1.3e-05        1e-05       0.0558744
     4          10      1.27913e-13       2.89e-08        1e-06      0.00198069
     5          12      3.04758e-21       4.47e-12        1e-07     4.42068e-06
     6          14      1.11242e-30       8.53e-17        1e-08     6.82449e-10

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the default value of the function tolerance, and
the problem appears regular as measured by the gradient.
```

We next evaluate the variable rate of G:

$$G(t_D) = \left(2\sqrt{\frac{t_d}{\pi}} - 1 + e^{t_D} erfc\sqrt{t_D}\right)$$

```
In [12]: G = 2 * sqrt ( td / pi ) - 1 + exp( td ).* erfc( sqrt(td));
```

The reservoir heat efficiency is then evaluated as:

$$E_h(t_D) = \frac{1}{t_D}\left\{G(t_D) + \frac{U(t_D - t_{cD})}{\sqrt{\pi}(1+h_D)}\left[2\sqrt{t_D} - \frac{2\sqrt{t_D - t_{cD}}}{(1+h_D)} - \int_0^{t_cD} \frac{e^x erfc\sqrt{x}}{\sqrt{t_D - x}}dx - \sqrt{\pi}G(t_D)\right]\right\}$$

```
In [13]: N = length(td);
         EhMyHill = zeros (1,N);

         for i = 1:N
```

```
    errorFun = @(u) exp(u).*erfc(sqrt(u))./sqrt(td(i) - u);
    EhMyHill(i) = 1/td(i) * ( G(i) + ( (td(i) > tcd)/...
    ( sqrt(pi)*(1+hd))*(2*sqrt(td(i))-...
    ( 2*sqrt(td(i) - tcd) )/( 1 + hd )- ...
    integral(errorFun, 0, tcd) - sqrt(pi) * G(i) ) );
end
```

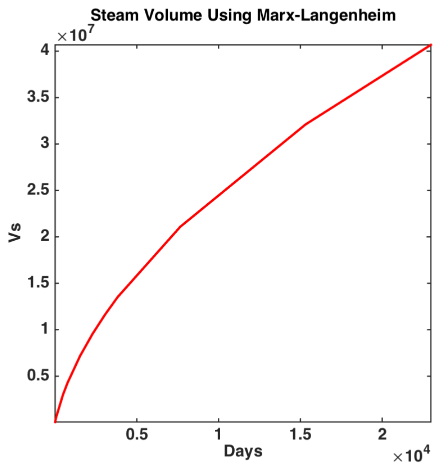
The steam volume, cumulative production, OSR and area can be calculated using the same expressions as above.

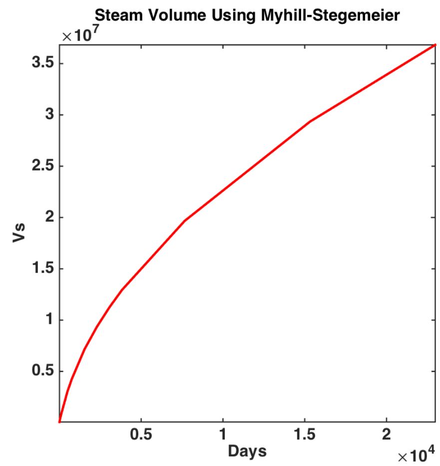
```
In [14]: % Steam volume (ft^3)
         VsMyHill = Qi.*EhMyHill / (Mt * (Ts - Tr));
         % Np (bbl)
         NpMyHill = VsMyHill .* phi * deltaSo / VolInBarrel;
         % oil/steam ratio
         OSRMyHill= NpMyHill./(mi*(t/DaysInYear)/(Density*VolInBarrel));
         % Area (acres)
         AreaMyHill = VsMyHill ./ h;
```

4 Results

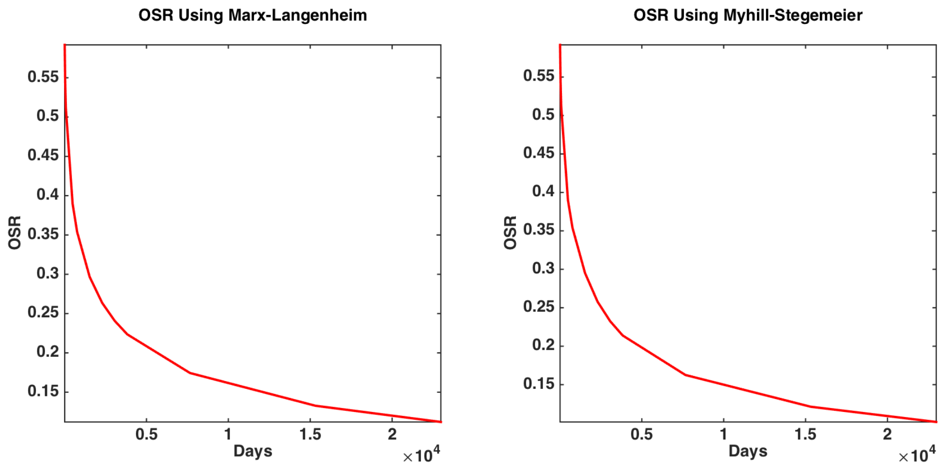
The results for both approaches are plotted below:

```
In [15]: %plot inline -s 2000,1000
         subplot(1,2,1);
         plot(t,VsMarx,'r-','LineWidth',2);
         xlabel('Days','FontSize',14,'FontWeight','bold')
         ylabel('Vs','FontSize',14,'FontWeight','bold')
         title({'Steam Volume Using Marx-Langenheim';''},'FontSize',14);
         set(gca,'FontSize',14,'FontWeight','bold');
         axis tight; axis square;

         subplot(1,2,2);
         plot(t,VsMyHill,'r-','LineWidth',2);
         hold on;
         xlabel('Days','FontSize',14,'FontWeight','bold')
         ylabel('Vs','FontSize',14,'FontWeight','bold')
         set(gca,'FontSize',14,'FontWeight','bold');
         title({'Steam Volume Using Myhill-Stegemeier';''},'FontSize',14);
         axis tight; axis square;
```

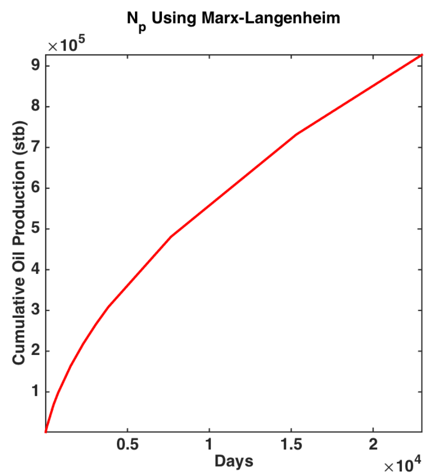




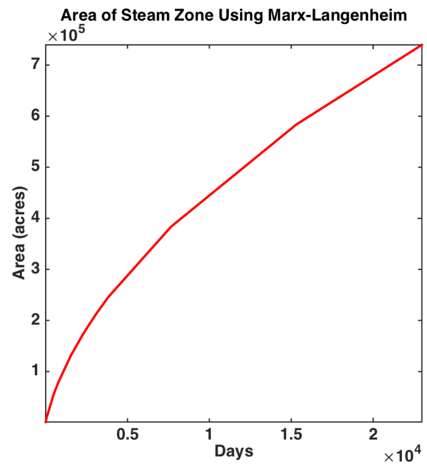
In [16]:
```
%plot inline -s 2000,1000
subplot(1,2,1);
plot(t,OSRMarx,'r-','LineWidth',2);
xlabel('Days','FontSize',14,'FontWeight','bold')
ylabel('OSR','FontSize',14,'FontWeight','bold')
title({'OSR Using Marx-Langenheim';''},'FontSize',14);
set(gca,'FontSize',14,'FontWeight','bold');
axis tight; axis square;

subplot(1,2,2);
plot(t,OSRMyHill,'r-','LineWidth',2);
hold on;
xlabel('Days','FontSize',14,'FontWeight','bold')
ylabel('OSR','FontSize',14,'FontWeight','bold')
set(gca,'FontSize',14,'FontWeight','bold');
title({'OSR Using Myhill-Stegemeier';''},'FontSize',14);
axis tight; axis square;
```

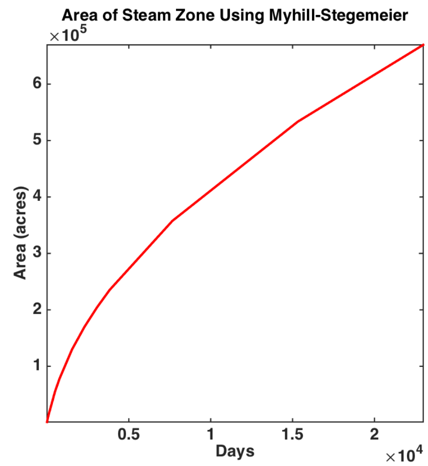


In [17]:
```
%plot inline -s 2000,1000
subplot(1,2,1);
plot(t,NpMarx,'r-','LineWidth',2);
xlabel('Days','FontSize',14,'FontWeight','bold')
ylabel('Cumulative Oil Production (stb)',...
'FontSize',14,'FontWeight','bold')
title({'N_p Using Marx-Langenheim';''},'FontSize',14);
set(gca,'FontSize',14,'FontWeight','bold');
axis tight; axis square;

subplot(1,2,2);
plot(t,NpMarx,'r-','LineWidth',2);
hold on;
xlabel('Days','FontSize',14,'FontWeight','bold')
ylabel('Cumulative Oil Production (stb)',...
'FontSize',14,'FontWeight','bold')
set(gca,'FontSize',14,'FontWeight','bold');
title({'N_p Using Myhill-Stegemeier';''},'FontSize',14);
axis tight; axis square;
```




```
In [18]: %plot inline -s 2000,1000
         subplot(1,2,1);
         plot(t,AcresMarx,'r-','LineWidth',2);
         xlabel('Days','FontSize',14,'FontWeight','bold')
         ylabel('Area (acres)','FontSize',14,'FontWeight','bold')
         title({'Area of Steam Zone Using Marx-Langenheim';''},...
         'FontSize',14);
         set(gca,'FontSize',14,'FontWeight','bold');
         axis tight; axis square;

         subplot(1,2,2);
         plot(t,AreaMyHill,'r-','LineWidth',2);
         hold on;
         xlabel('Days','FontSize',14,'FontWeight','bold')
         ylabel('Area (acres)','FontSize',14,'FontWeight','bold')
         set(gca,'FontSize',14,'FontWeight','bold');
         title({'Area of Steam Zone Using Myhill-Stegemeier';''},...
         'FontSize',14);
         axis tight; axis square;
```






```
In [19]: Results = [td' t' EhMarx' Qi' VsMarx' AcresMarx' NpMarx' OSRMarx'];
         format shortEng
         format compact

         display('Results for Marx-Langenheim are as follows:');
         display('                  tD                   t                Ehs                 Qi'
         display(Results(:,1:4));
         display('                  Vs              Acres                Np                 OSR'
         display(Results(:,5:end));
```

```
Results for Marx-Langenheim are as follows:
              tD                t               Ehs               Qi
ans =
    10.0000e-003      7.6588e+000   929.4897e-003   668.5018e+006
    50.0000e-003     38.2939e+000   853.8003e-003     3.3425e+009
   100.0000e-003     76.5878e+000   804.0326e-003     6.6850e+009
   640.0000e-003    490.1620e+000   612.1936e-003    42.7841e+009
     1.0000e+000    765.8782e+000   555.9627e-003    66.8502e+009
     2.0000e+000      1.5318e+003   465.9866e-003   133.7004e+009
     3.0000e+000      2.2976e+003   413.9171e-003   200.5506e+009
     4.0000e+000      3.0635e+003   378.0385e-003   267.4008e+009
     5.0000e+000      3.8294e+003   351.0918e-003   334.2509e+009
    10.0000e+000      7.6588e+003   273.8826e-003   668.5019e+009
    20.0000e+000     15.3176e+003   208.4739e-003     1.3370e+012
    30.0000e+000     22.9763e+003   176.0585e-003     2.0055e+012
              Vs            Acres                Np              OSR
ans =
    71.5859e+003      1.3016e+003     1.6319e+003   591.8689e-003
   328.7829e+003      5.9779e+003     7.4950e+003   543.6723e-003
```

```
   619.2366e+003    11.2588e+003    14.1162e+003   511.9819e-003
     3.0175e+006    54.8642e+003    68.7879e+003   389.8250e-003
     4.2818e+006    77.8513e+003    97.6088e+003   354.0190e-003
     7.1777e+006   130.5039e+003   163.6238e+003   296.7251e-003
     9.5635e+006   173.8821e+003   218.0107e+003   263.5690e-003
    11.6461e+006   211.7465e+003   265.4845e+003   240.7227e-003
    13.5199e+006   245.8164e+003   308.2008e+003   223.5638e-003
    21.0934e+006   383.5170e+003   480.8477e+003   174.3996e-003
    32.1118e+006   583.8510e+003   732.0233e+003   132.7494e-003
    40.6782e+006   739.6029e+003   927.3026e+003   112.1084e-003
```

```
In [20]: Results = [td' t' EhMyHill' Qi' VsMyHill' AreaMyHill' NpMyHill' OSRMyHill'
         format shortEng
         format compact

         display('Results for Myhill-Stegemeier are as follows:');
         display('              tD                t              Ehs                Qi'
         display(Results(:,1:4));
         display('              Vs            Acres                Np               OSR'
         display(Results(:,5:end));
```

```
Results for Myhill-Stegemeier are as follows:
              tD                t              Ehs                Qi
ans =
    10.0000e-003     7.6588e+000   929.4897e-003   668.5018e+006
    50.0000e-003    38.2939e+000   853.8003e-003     3.3425e+009
   100.0000e-003    76.5878e+000   804.0326e-003     6.6850e+009
   640.0000e-003   490.1620e+000   612.1936e-003    42.7841e+009
     1.0000e+000   765.8782e+000   555.9627e-003    66.8502e+009
     2.0000e+000     1.5318e+003   463.5257e-003   133.7004e+009
     3.0000e+000     2.2976e+003   405.0423e-003   200.5506e+009
     4.0000e+000     3.0635e+003   365.3067e-003   267.4008e+009
     5.0000e+000     3.8294e+003   336.0558e-003   334.2509e+009
    10.0000e+000     7.6588e+003   255.3770e-003   668.5019e+009
    20.0000e+000    15.3176e+003   190.4923e-003     1.3370e+012
    30.0000e+000    22.9763e+003   159.3830e-003     2.0055e+012
              Vs            Acres               Np               OSR
ans =
    71.5859e+003     1.3016e+003     1.6319e+003   591.8689e-003
   328.7829e+003     5.9779e+003     7.4950e+003   543.6723e-003
   619.2366e+003    11.2588e+003    14.1162e+003   511.9819e-003
     3.0175e+006    54.8642e+003    68.7879e+003   389.8250e-003
     4.2818e+006    77.8513e+003    97.6088e+003   354.0190e-003
     7.1398e+006   129.8148e+003   162.7597e+003   295.1581e-003
     9.3585e+006   170.1539e+003   213.3363e+003   257.9178e-003
    11.2538e+006   204.6152e+003   256.5433e+003   232.6155e-003
    12.9409e+006   235.2890e+003   295.0017e+003   213.9894e-003
```

```
19.6682e+006   357.6037e+003   448.3579e+003   162.6158e-003
29.3420e+006   533.4916e+003   668.8834e+003   121.2993e-003
36.8253e+006   669.5506e+003   839.4722e+003   101.4899e-003
```

```
In [ ]:
```