Wellbore and Surface Line Heat Losses

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1 Computing Wellbore and Surface Line Heat Losses

1.1 Surface Heat Losses

We will calculate the steady state heat losses per year per 100ft as steam is injected through a pipe. We will consider 4 inch tubing in a N-80 pipe which is shown below in Figure 1. The insulation used is calcium silicate, and the thermal properties of both the pipe and insulation are shown in Table 1.

Figure 1. Schematic representation of idealized thermal resistances in buried surface lines (from [1]).

Symbol	Units	Value	Description
r_i	ft	0.1478	inner radius of pipe
r_o	ft	0.1667	outer radius of pipe
r_{ins}	ft	0.4167	external radius of insulation
λ_p	Btu/ft-D-F	600	thermal conductivity of pipe
λ_{ins}	BTU/ft-D-degF	0.96	thermal conductivity of insulation
h_f	Btu/sq ft-D-F	48000	film coefficient of heat transfer btw fluid inside the pipe and pipe wall
h_{fc}	Btu/sq ft-D-F	154	film coefficient of heat transfer due to forced convection
h_{Pi}	n/a	∞	coefficient of heat transfer across scale deposits
h_{Po}	Btu/sq ft-D-F	48000	coeffcient of heat transfer btw pipe and insulation
T_b	F	550	steam temperature
T_a	F	60	ambient temperature
L	ft	100	length of pipe
t	days	365	time period for evaluating losses

We will first specify the constants:

```
In [1]: ri = 0.1478;      % ft
```

```
ro = 0.1667;        % ft
r_ins = 0.4167;     % ft
lamda_I = 0.96;     % Btu/ft-D-F
lamda_P = 600;      % BTU/ft-D-degF
hf = 48000;         % Btu/sq ft-D-F
hfc = 154;          % Btu/sq ft-D-F
hpi = inf;          % Btu/sq ft-D-F
hpo = 48000;        % Btu/sq ft-D-F
Tb = 550;           % F
Ta = 60;            % F
```

1.1.1 Without Insulation

In the first case, we assume that the pipe is not insulated. A 20 mph wind (v_W) is present normal to the pipe, therefore the conventive heat transform coefficient is estimated using the empirical relation from B.8 in (Prats,1985)

$$h_{fc} r_{ins} = 18 v_W^{0.6} r_{ins}^{0}.6$$

```
In [2]: v_w = 20;
        hfc = 18 * (v_w^0.6) * (ro^0.6) / ro;
```

For a bare pipe, the effective coefficient of heat transfer h_{Po}^{bare} is estimated by the sum of 110Btu/sq ft-D-F from Table B.14 (Prats,1985) and the coefficient of heat transfer due to the forced convection h_{fc}.

```
In [3]: hrc = 110; % Btu/sq ft-D-F
        h_bare = hfc + hrc;
```

Since there is no insulation, the specific heat resistance is evaluated as:

$$R_h = \frac{1}{2\pi}\left[\frac{1}{h_f r_i} + \frac{1}{h_{pi} r_i} + \frac{1}{\lambda_p}\ln\frac{r_o}{r_i} + \frac{1}{h_{po}^{bare} r_o}\right]$$

```
In [4]: R_h = (1 / (hf * ri) + 1 / (hpi * ri) + ...
        (1 / lamda_P) * log(ro/ri) + 1 / ( (h_bare) * ro ))/(2*pi)

R_h =

    0.0029
```

To compute the heat loss we need to know the steam temperature and ambient temperature. We will assume the steam is T_b 500F, and ambient T_a is 60F. The heat loss per unit length is then:

$$Q_{loss} = \frac{T_b - T_a}{R_h}$$

```
In [5]: Qls = (Tb - Ta) / R_h; %Btu/ft-D
```

For a pipe of length L 100 ft over a period of 365 days t, the loss is:

$$Q_l = Q_{loss} L t$$

In [6]:
```
L = 100; % ft
t = 365; % days

Ql = Qls * L * t; % BTU
```

In [7]:
```
display('For an uninsulated pipe:');
display(['Specific thermal resistance was ' ...
num2str(R_h,3) ' BTU/ft-D'])
display(['Heat loss per unit length was ' ...
num2str(Qls,3) ' BTU/ft-D'])
display(['Heat loss from ' num2str(L)  ...
' of pipe over ' num2str(t) ' days was ' ...
num2str(Ql,3) ' BTU'])
```

```
For an uninsulated pipe:
Specific thermal resistance was 0.00293 BTU/ft-D
Heat loss per unit length was 1.67e+05 BTU/ft-D
Heat loss from 100 of pipe over 365 days was 6.11e+09 BTU
```

1.1.2 With Insulation

When the pipe is insulated, we need to account for the heat loss across dart on outside of the pipe, across the insulation, and losses due to atmosphere from forced convection. Thus, the expression for estimating specific heat resistance becomes:

$$R_h = \frac{1}{2\pi}\left[\frac{1}{h_f r_i} + \frac{1}{h_{pi} r_i} + \frac{1}{\lambda_p}\ln\frac{r_o}{r_i} + \frac{1}{h_{po} r_o} + \frac{1}{\lambda_{ins}}\ln\frac{r_{ins}}{r_o} + \frac{1}{h_{fc} r_{ins}}\right]$$

In [8]:
```
R_h = (1 / (hf * ri) + 1 / (hpi * ri) + ...
 (1 / lamda_P) * log(ro/ri) + 1 / ( (hpo) * ro ) + ...
 (1 / lamda_I)*log(r_ins/ro) + 1 / (hfc * r_ins) )/(2*pi);
```

The heat loss per unit length is computed in the same manner as before:

In [9]:
```
Qls = (Tb - Ta) / R_h; %Btu/ft-D
Ql = Qls * L * t; % BTU
```

In [10]:
```
display('For an insulated pipe:');
display(['Specific thermal resistance was ' ...
num2str(R_h,3) ' BTU/ft-D'])
display(['Heat loss per unit length was '...
num2str(Qls,3) ' BTU/ft-D'])
display(['Heat loss from ' num2str(L)  ' of pipe over ' ...
num2str(t) ' days was ' num2str(Ql,3) ' BTU'])
```

```
For an insulated pipe:
Specific thermal resistance was 0.154 BTU/ft-D
Heat loss per unit length was 3.19e+03 BTU/ft-D
Heat loss from 100 of pipe over 365 days was 1.16e+08 BTU
```

1.2 Wellbore Losses

We now look at the heat losses within a wellbore. Steam is injected into a well that is insulated with calcium silicate. The following properties are used in this analysis.

Symbol	Units	Value	Description
T_b	F	600	steam temperature
T_a	F	100	ambient temperature in subsurface
t	days	21	time period for evaluating losses
d	ft	1000	depth of well
r_o	ft	0.1458	outer radius of tubing
r_{ins}	ft	0.2292	insulation radius
r_{ci}	ft	0.3556	casing inner radius
r_{co}	ft	0.401	casing inner radius
r_w	ft	0.5	wellbore radius
α_E	sq-ft/D	0.96	thermal diffusivity of the earth
ϵ_{ins}	n/a	0.9	emissivity of insulation surface
ϵ_{ci}	n/a	0.9	emissivity of casing surface
λ_E	Btu/ft-D-F	24	thermal conductivity of earth
λ_{cem}	Btu/ft-D-F	12	thermal conductivity of cement
λ_{ins}	Btu/ft-D-F	0.96	thermal conductivity of insulation
λ_{air}	Btu/ft-D-F	0.45	thermal conductivity of air
ν_{air}	cP	0.023	air viscosity @ average annulus temperature

The time function for the radiation boundary condition model is given in Prats, M. 1982 and is included with the data.

In [11]:
```
clear all;
% Temperature of steam (F)
Tb = 600;
% Ambient temperature of subsurface
Ta = 100;
% Time (days)
time = 21;

% Time function f(td) for the radiation boundary condition model
% Table 10.1 (Prats, M. 1982)
load('../../data/Table_10pt1_Prats.mat');

tableVertical = [0.1, 0.2, 0.5, 1.0, 2.0, ...
5.0, 10.0, 20.0, 50.0, 100.0];
tableHorizontal = [100.0, 50.0, 20.0, 10.0, ...
5.0, 2.0, 1.0, 0.5, 0.2, 0.1,...
0.05, 0.02, 0.01, 0.0];

% Well depth (ft)
depth = 1000;
% Tubing outer radius (ft)
```

```
ro = 0.1458;
% Insulation radius (ft)
rIns = 0.2292;
% Casing inner radius(ft)
rci = 0.3556;
% Casing outer diameter (ft)
rco = 0.401;
% Wellbore radius (ft)
rw = 0.5;
% Thermal diffusivity of the earth (sq-ft/D)
alphaE = 0.96;
% Emissivity of insulation surface
epsilonIns = 0.9;
% Emissivity of casing surface
epsilonCi = 0.9;
% Thermal conductivity of earth (Btu/ft-D-F)
lambdaE = 24;
% Thermal conductivity of cement (Btu/ft-D-F)
lambdaCem = 12;
% Thermal conductivity of insulation (Btu/ft-D-F)
lambdaIns = 0.96;
% Air viscosity @ average annulus temperature (Figure B.41)
viscosityAir = 0.023;
% Thermal conductivity of air (Btu/ft-D-F) (Figure B.72)
lambdaAir = 0.45;
```

The first step requires making an assumption such as the sum of all the thermal resistance is twice that due to the insulation.

$$R_h = \frac{2}{2\pi}\left[\frac{\ln(r_{ins}/r_0)}{\lambda_{ins}}\right]$$

```
In [12]: RhGuess = (1/pi)*(log(rIns/ro))/lambdaIns;
         Rprime = RhGuess
```

```
Rprime =

    0.1500
```

The next step is to evaluate $f(t_D)$ at 21 days. The dimensionalness time is evaluated using:

$$t_D = \frac{\alpha_E t}{r_w^2}$$

If this value is less than 100, we evaluate the value of $f(t_D)$ by interpolating Table 10.1 in Prats, else we evaluate the Ramey function

$$f(t_D) \approx \frac{1}{2}\ln(t_D) + 0.403$$

```
In [13]: % Calculate dimensionless time
         tD = alphaE*time/(rw*rw);

         if (tD > 100.0)
             % Eqn 10.10 for tD < 100
             func = 0.5*log(tD) + 0.403;
         else
             lookupVal = 2*pi*Rprime*lambdaE;
             % Interpolate table 10.1
             func = interp2(tableHorizontal, tableVertical,...
                 Table_10pt1, lookupVal, tD);
         end
```

We next evaluate the temperature at the outer radius of the annulus using Equation B.68 from Prats:

$$T_{ci} = T_A + \frac{T_b - T_A}{2\pi R_h}\left[\frac{\ln(r_{co}/r_{ci})}{\lambda_P} + \frac{\ln(r_w/r_{co})}{\lambda_{com}} + \frac{\ln(r_{Ea}/r_w)}{\lambda_{Ea}} + \frac{f(t_D)}{\lambda_E}\right]$$

```
In [14]: % Step 3: Calculate Tci (temperature @ inner surface of casing)
         % Eqn B.68 (neglecting 1st and 3rd terms in parenthesis)
         Tci = Ta + ( (Tb - Ta)/(2*pi*Rprime) ) * ...
             (log(rw/rco)/lambdaCem + func/lambdaE);
```

We next calculate the temperature at the outer face of insulation using equation B.70 but only the 5th term.

$$T_{ins} = T_b + \frac{T_b - T_A}{2\pi R_h}\left[\frac{\ln r_{ins}/r_o}{\lambda_{ins}}\right]$$

```
In [15]: % Step 4: Calculate Tins (temperature @ outer face of insulation)
         % Eqn B.70 (only 5th term in parenthesis)
         Tins = Tb - ((Tb - Ta)/(2*pi*Rprime))*...
             (log(rIns/ro)/lambdaIns);
```

The next step is to calculate the hRcAn coefficient of annular heat transfer due to radiation and convection. The average temperature in the annulus is hneeded to estimate air properties:

$$\overline{T_{an}} = 1/2(T_{ins} + T_{ci})$$

The air in the annulus:

$$\rho_a = 0.076\left(\frac{460 + 60}{460 + \overline{T_{an}}}\right)$$

```
In [16]: % Average temperature @ annulus
         TavgAnnulus = 0.5*(Tins + Tci)
         densityAir = 0.076*(460 + 60)/(460 + TavgAnnulus)
```

```
TavgAnnulus =

  258.6986
```

```
densityAir =

    0.0550
```

We next calculate the isobaric thermal coefficient of volume expansion for gas

$$\beta_R = \frac{1}{T} + \frac{1}{z}\left(\frac{dz}{dT}\right)_p$$

(Assuming that air is an ideal gas.) The Grashof Number N_{Gr} is needed to evaluate the effective thermal conductivity of the air:

$$N_{Gr} = 7.12e7\frac{(r_{ci} - r_{Ins})^3\rho_a^2\beta_G(T_{ins} - T_{ci})}{\nu_a^2}$$

```
In [17]: % Isobaric thermal coefficient of volume
         % expansion for gas (Assuming that air is
         % an ideal gas.)
         betaG = 1.0/(460 + TavgAnnulus);
         % Eqn B.66: Grashof number
         Grashof = 7.12e7*((rci - rIns)^3*densityAir^2*betaG*(Tins - ...
             Tci))/(viscosityAir*viscosityAir)
```

```
Grashof =

   2.0882e+05
```

Assuming a Prandtl number $(N_{Pr})$of 0.92, the apparent thermal conductivity of air in the annulus is evaluated as:

$$\lambda_{a,a} = 0.049\lambda_a N_{Gr}^{0.333} N_{Pr}^{0.407}$$

The radiation temperature function $F(T_{ins}, T_{ci})$ is evaluated as:

$$F(T_{ins}, T_{ci}) = \left[(T_{ins} + 460)^2 + (460 + T_{ci})^2\right](920 + T_{ins} + T_{ci})$$

The coefficient of heat transfer by radiation and convection in the annulus is given by Willhite as:

$$h_{\zeta c,an} = 4.11 \times 10^{-8}\left[\frac{1}{\epsilon_{ins}} + \frac{r_{ins}}{r_{ci}}\left(\frac{1}{\epsilon_{ci}} - 1\right)\right]^{-1} F(T_{ins}, T_{ci}) + \frac{1}{r_e}\frac{\lambda_{a,a}}{ln(\frac{r_{ci}}{r_{ins}})}$$

```
In [18]: % Prandtl number (assumption)
         Prandtl = 0.92;
         % Eqn B.65: Apparent thermal conductivity of air in the annulus
         lambdaAAn = 0.049*lambdaAir*(Grashof^0.333)*(Prandtl^0.407)
         % Eqn B.64: Radiation temperature function
         F = ((460 + Tins)^2 + (460 + Tci)^2)*(920 + Tins + Tci)
         % Eqn B.63: Coefficient of annular heat transfer
         hRcAn = (4.11e-8 / (1/epsilonIns + (rIns/rci)*...
             (1/epsilonCi - 1)) ) * F+(1/rIns)*...
             lambdaAAn/log(rci/rIns)
```

```
lambdaAAn =

    1.2594


F =

   1.5089e+09


hRcAn =

   64.9439
```

Finally we compute R_h using Equation 10.6:

$$R_h = \frac{1}{2\pi}[\frac{1}{h_f r_i} + \frac{1}{h_{Pi} r_i} + \frac{\ln(r_o/r_i)}{\lambda_P} + \frac{1}{h_{Po} r_o} + \frac{\ln(r_{ins}/r_o)}{\lambda_{ins}} + \frac{1}{h_{\zeta c,an} r_{ins}} + \\ + \frac{\ln(r_{co}/r_{ci})}{\lambda_p} + \frac{\ln(r_w/r_{co})}{\lambda_{cem}} + \frac{\ln(r_{Ea}/r_w)}{\lambda_{Ea}} + \frac{f(t_D)}{\lambda_E}]$$

```
In [19]: % Step 6: Calculate Rh using Eqn 10.6
         % Heat loss across insulation
         Rh1 = log(rIns/ro) / lambdaIns;
         % Heat loss through radiation and convection across annulus
         Rh2 = 1.0 / (hRcAn * rIns );
         % Heat loss across cement
         Rh3 = log(rw/rco) / lambdaCem;
         % Heat loss related to thermal resistance of earth
         Rh4 = func / lambdaE;
         % Eqn 10.6: Overall coefficient of heat loss
         Rh = (1./(2*pi))* (Rh1 + Rh2 + Rh3 + Rh4)
```

```
Rh =

    0.1059
```

Since the initially assumed and calcuated values of R_h in steps 1 and 6 do not agree, steps 2 to 6 are repeated until the difference between successive approximations of R_h do not vary much.

```
In [20]: RhGuess = (1/pi)*(log(rIns/ro))/lambdaIns;
         Rprime = RhGuess;
         error = Inf;
         ConvergenceCriterion = 0.001;
         NumIterations = 0;
```

```matlab
% Check for convergence
while (error > ConvergenceCriterion)
    % Step 2: Calculate f(td) (Ramey function)
    % Calculate dimensionless time
    tD = alphaE*time/(rw*rw);

    if (tD > 100.0)
        % Eqn 10.10 for tD < 100
        func = 0.5*log(tD) + 0.403;
    else
        lookupVal = 2*pi*Rprime*lambdaE;
        % Interpolate table 10.1
        func = interp2(tableHorizontal, tableVertical,...
            Table_10pt1, lookupVal, tD);
    end

    % Step 3: Calculate Tci (temperature @ inner
    % surface of casing) Eqn B.68
    % (neglecting 1st and 3rd terms in parenthesis)
    Tci = Ta + ( (Tb - Ta)/(2*pi*Rprime) ) * ...
        (log(rw/rco)/lambdaCem + func/lambdaE);

    % Step 4: Calculate Tins (temperature @ outer
    % face of insulation) Eqn B.70
    % (only 5th term in parenthesis)
    Tins = Tb - ((Tb - Ta)/(2*pi*Rprime))*...
        (log(rIns/ro)/lambdaIns);

    %Step 5: Calculate hRcAn coefficient of annular
    % heat transfer due to radiation and convection
    % Average temperature @ annulus
    TavgAnnulus = 0.5*(Tins + Tci);
    densityAir = 0.076*(460 + 60)/(460 + TavgAnnulus);

    % Isobaric thermal coefficient of volume
    % expansion for gas (Assuming that air is
    % an ideal gas.)
    betaG = 1.0/(460 + TavgAnnulus);
    % Eqn B.66: Grashof number
    Grashof = 7.12e7*((rci - rIns)^3*densityAir^2*betaG*(Tins - ...
        Tci))/(viscosityAir*viscosityAir);
    % Prandtl number (assumption)
    Prandtl = 0.92;
    % Eqn B.65: Apparent thermal conductivity of air
    % in the annulus
    lambdaAAn = 0.049*lambdaAir*(Grashof^0.333)...
    *(Prandtl^0.407);
    % Eqn B.64: Radiation temperature function
```

```
        F = ((460 + Tins)^2 + (460 + Tci)^2)*(920 + Tins + Tci);
        % Eqn B.63: Coefficient of annular heat transfer
        hRcAn = (4.11e-8 / (1/epsilonIns + (rIns/rci)*...
            (1/epsilonCi - 1)) ) * F+(1/rIns)*...
            lambdaAAn/log(rci/rIns);

        % Step 6: Calculate Rh using Eqn 10.6
        % Heat loss across insulation
        Rh1 = log(rIns/ro) / lambdaIns;
        % Heat loss through radiation and convection
        % across annulus
        Rh2 = 1.0 / (hRcAn * rIns );
        % Heat loss across cement
        Rh3 = log(rw/rco) / lambdaCem;
        % Heat loss related to thermal resistance of earth
        Rh4 = func / lambdaE;
        % Eqn 10.6: Overall coefficient of heat loss
        Rh = (1./(2*pi))* (Rh1 + Rh2 + Rh3 + Rh4);

        % Convergence check
        error = abs(Rh - Rprime);
        Rprime = Rh;

        NumIterations = NumIterations + 1;
    end

    display(['Took ' num2str(NumIterations) ...
    ' iterations for convergence']);
```

```
Took 3 iterations for convergence
```

The actual heat loss is evaluated as:

$$\dot{Q}_{ls} = (T_b - T_A)/R_h$$

The heat loss for the given depth of the well and given time period is:

$$\dot{Q}_t = \dot{Q}_{ls} d$$

```
In [21]: %  Eqn 10.1: Heat loss per unit depth of the well (Btu/ft-D)
         Qls = (Tb - Ta) / Rh;
         % Heat loss for the given depth of the well
         % and the given time period
         Qt = Qls * depth;

         display(['Temperature of casing is ' num2str(Tci) ' degrees F']);
         display(['Heat loss rate from ' num2str(depth) ...
             ' ft. deep well after ' num2str(time) ' days  is ' ...
             num2str(Qt,'%4.2e') ' BTU/Day']);
```

```
Temperature of casing is 193.3691 degrees F
Heat loss rate from 1000 ft. deep well after 21 days  is 4.62e+06 BTU/Day
```

```
In [ ]:
```