Quiz 9 solution

① $\iint\limits_{R} x^2 - y \, dy \, dx$

$R: \begin{array}{l} -1 \leq x \leq 1 \\ -x^2 \leq y \leq x^2 \end{array}$

$= \int\limits_{-1}^{1} \int\limits_{-x^2}^{x^2} x^2 - y \, dy \, dx$

$= \int\limits_{-1}^{1} \left[x^2 y - \frac{y^2}{2} \right]_{-x^2}^{x^2} dx$

$= \int\limits_{-1}^{1} \left(x^4 - \frac{x^4}{2} \right) - \left(-x^4 - \frac{x^4}{2} \right) dx$

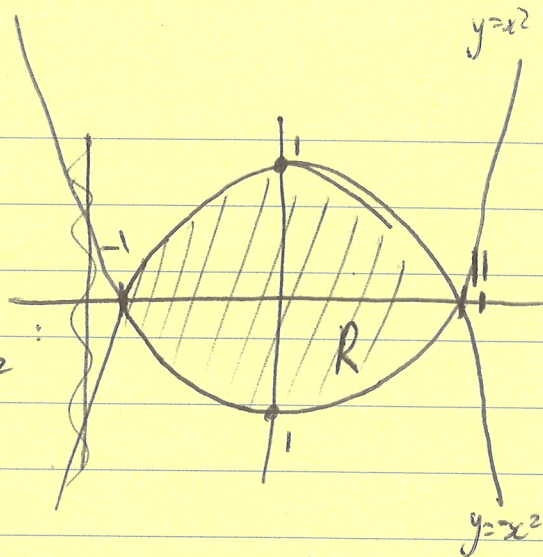$= \int\limits_{-1}^{1} \frac{x^4}{2} + \frac{3x^4}{2} \, dx$

$= \int\limits_{-1}^{1} 2x^4 \, dx$

$= \left. \frac{2x^5}{5} \right|_{-1}^{1}$

$= \frac{2}{5} - -\frac{2}{5}$

$= \frac{4}{5}.$

② $\dfrac{dy}{dx} = \dfrac{x^2 + 5}{2y - 1}$; $y(0) = 11$

$(2y - 1)\,dy = (x^2 + 5)\,dx$

$\int (2y - 1)\,dy = \int (x^2 + 5)\,dx$

$y^2 - y = \dfrac{x^3}{3} + 5x + C.$

when $x = 0$, $y = 11$.

$121 - 11 = 0 + 0 + C$

$110 = C.$

so   soln is: $y^2 - y = \dfrac{x^3}{3} + 5x + 110.$