Math 20 Practice Exam

Name: _____

Date: 2 May, 2014

Practice questions for the final exam.
This practice exam contains more questions than the actual final exam does.
If you can do everything in this test you should be pretty well prepared for the final!

1   Integration   ****** points*

- Evaluate the following indefinite integrals:

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  $$\int \frac{(3\ln x + 2)^4}{x}\,dx$$

  —

  $$\int e^{3x^2+4}x\,dx$$

  —

  $$\int x^2 e^{x/2}\,dx$$

- Evaluate the following double integrals over the regions given. If the integral looks impossible to evaluate, you may need to change the order of integration.

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  $$\iint_R xy\,dy\,dx, \quad 0 \le x \le 1, 0 \le y \le 2x$$

  —

  $$\iint_R ye^{y^2+x}\,dy\,dx, \quad 0 \le x \le 1, 0 \le y \le 1$$

  —

  $$\int_0^8 \int_{x/2}^4 \sqrt{y^2+4}\,dy\,dx$$

1

- Determine the value b which makes the following equation true:

$$\int_1^b \frac{16}{x^2}\,dx = 14$$

- Calculate the volume obtained by:

  - Rotating the function $f(x) = x^2/4$ between $y = 0$ and $x = 4$ around the x-axis
  - Integrating underneath the surface $f(x, y) = x\sqrt{x^2 + y}$ between $x = 0$, $x = 1$, $y = 0$ and $y = 1$.

2 Derivatives and Differential equations *****points*

- Find all critical points of the following functions, and classify their nature (maxima, minima, saddle, neither):

  - $f(x, y) = 2x^2 + 4xy + 4y^2 - 3x + 5y - 15$
  - $f(x, y) = 7x^2 + y^2 - 3x + 6y - 5xy$
  - $f(x, y) = y^2 - 2xy + 4x^3 + 20x^2$

- Find the general solution to the following DEs:

  - $y' = 4e^{2x}$
  - $y' = x^3 + 7/x$
  - $y' = (e^x + x)/(y + 1)$
  - $(x \ln x)y' + y = 2x^2$

- Find the value (x, y) which will maximize the function $f(x, y) = 48xy - x^2 - 3y^2$ subject to the constraint $x + y = 52$.

Cotinuous probability distributions

(c) Determine the value of k that will make the function $f(x) = k\sqrt{x}$ a probability density function on

    – $[0, 4]$;
    – $[4, 9]$.

- For the following probability density functions on the given intervals, determine: (i) $P(2 < X < 4)$, (ii) the expected value $\mathbb{E}(X)$, (iii) the variance σ^2, (iv) the *cumulative* distribution function $C(x)$:

    – $f(x) = \frac{2}{9}(x - 2);$ $[2, 5]$
    – $f(x) = 5x^{-6};$ $[1, \infty)$
    – $f(x) = \frac{\sqrt{x}+3}{20\sqrt{x}};$ $[1, 9]$

- Determine the median of the above probability density functions, that is, the value m such that

$$\int_a^m f(x)dx = 0.5.$$

How does this differ from the mean of each distribution?

- Write down expressions for the center and spread for the following distributions:

    – Uniform distribution;
    – Exponential distribution;
    – Normal distribution.

3   Applications   ***** *points*

These will be taken from the book, and testing some combination of the topics covered in the first 3 sections of this practice exam. You should look to the book for sample applications questions!