Identifying Extra Solar Planets and their Key Features using the Doppler Wobble and Planetary Transits Methods

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Abstract

In this report, the search for extra-solar planets is discussed, with emphasis on using the Doppler Wobble and Planetary Transits methods to identify extra-solar planets and their key features, such as the mass of the planet and the semi-major axis of its orbit. These methods are particular effective when observing planets who's orbits are close to the plane of the observer's line of sight. For the Doppler Wobble method two stars are considered who are known to have extra-solar planets in orbit around them. The radial velocity of the parents stars are measured and thus allowing the mass of the planets and the semi-major axis of their orbits to be calculated. However for the Planetary transits method, only one star is considered and to determine the relevant characteristics of the extra-solar planets, the magnitude of the I-band of the star is measured as a function of time, and creating a phase-folded light curve.

Introduction and Background

The search for extra-solar planets has been a growing field of Astronomy for the past 30 years since the first extra-solar planet was discovered in 1992, this detection was made indirectly by observing gaps in a pulsars emission of radio waves.[1] However it wasnt until 1995 that the first direct detection of an extra-solar planet via the radial velocity method was made.[2] This method is based on measuring the wavelength and intensity of light emitted from a star over a period of time, from this, a pattern can be seen where, when the star is moving towards the observer the wavelength of the light emitted is shifted towards the blue end of the Electromagnetic spectrum (resulting in a lower wavelength) and when the star is moving away from the observer an increase in wavelength is observed (redshift).

By measuring the wavelength and intensity and thus the radial velocity of a star as a function of time, it is possible to determine characteristics about the Planetary companions in orbit such as the mass of the planet using Kepler's third law and Newton's law of Unviersal Gravitation, an equation can derived as seen in equation 1.[3]

$$v_s = \left(\frac{2\pi G}{T}\right)^{1/3} M_s^{-2/3} m_p \tag{1}$$

where v_s is the amplitude of the radial velocity curve (RVC), G is the unviversal gravitational constant, T is the orbital period of the planet, M_s is the mass of the star and m_p is the mass of the planet.

From the mass of the planet the semi-major axis of the orbit of the planet can be calculated using Kepler's third law as seen in equation 2.[3]

$$G(M_s + m_p) = \frac{4\pi^2 a^3}{T^2} \tag{2}$$

where a is the semi-major axis of the orbit, G is the unviversal gravitational and the rest of the variables are the same as in equation 1.

The Doppler Wobble Method of Detecting Extra-solar Planets

For the analysis on data of two stars and the existence of an extra-solar planets around them, the doppler wobble method was used. This method is based on measuring the radial velocity of the stars (HD-28185 and HD-73256) as they move towards and away from the observer. Thus producing a doppler shift in the light emitted from the stars, that can be used to determine the velocity of the stars in the plane of the observer's line of sight.

To determine the radial velocity, observations were made of the stars on different Julian dates, recording the wavelength of light emitted as well as the observed intensity of the light. The radial velocity of the stars can then be calculated using the doppler shifted wavelength and intensity, as provided in the Python Library.[3]

$$\lambda_{obs} = \lambda_{emit}(1 + v/c) \tag{3}$$

$$I_{obs} = \frac{I_{emit}}{(1 + v/c)} \tag{4}$$

where λ_{obs} is the observed wavelength, λ_{emit} is the emitted wavelength, I_{obs} is the observed intensity, I_{emit} is the emitted intensity, v is the radial velocity of the stars and c is the speed of light. From this it was possible to calculate the radial velocity of each star on each date. This was done using the Python SCIPY library.[4] An uncertainty in the radial velocity was assigned to each date of each star of $\pm 15 ms^{-}1$.[3]

When plotting the radial velocity as a function of time, as seen in Figure 1, it was possible to see that there was a periodic sinusoidal pattern in the data, however, it was not very accurate as the time between observations varyed. To correctly plot the radial velocity of the stars over time it was neccessary to calculate the phase of each star through the period of the orbit. In this experiment the phase is defined as the fraction of

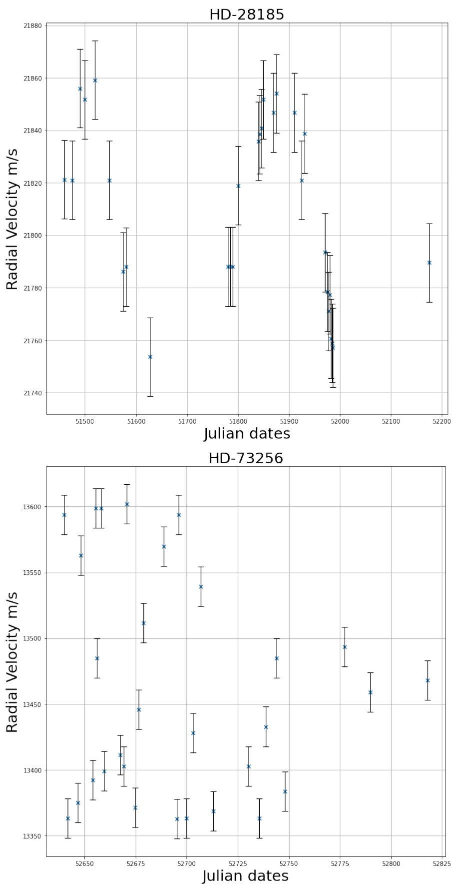

Figure 1: Radial velocity of HD-28185 and HD-73256 as a function of time (Julian Date)

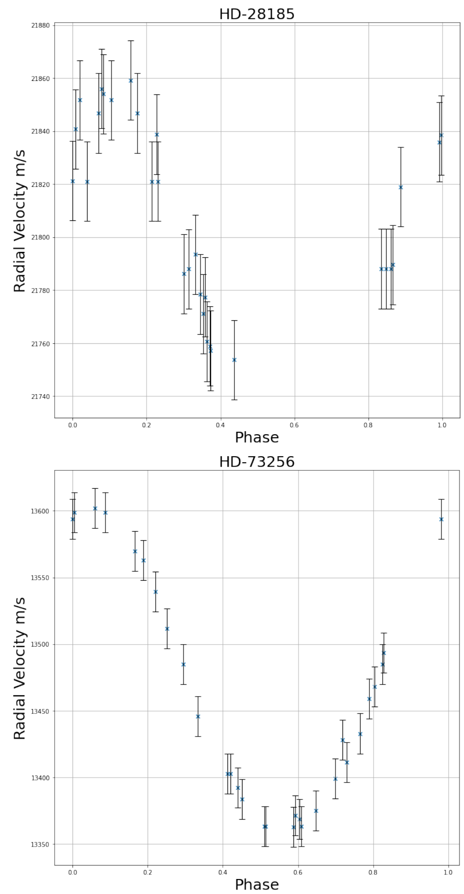

Figure 2: Radial velocity of HD-28185 and HD-73256 as a function of phase

the orbital period that has elapsed. The radial velocity of the stars (HD-28185 and HD-73256) as a function of phase was then plotted, as seen in Figure 2. This allowed for a more accurate representation of the radial velocity of the stars as a function of time, as the time between observations was more consistent.

Now that there was a clear correlation between the radial velocity of the stars and time, it was possible to calculate characteristics about the extra-solar planets in orbit around the stars (HD-28185 and HD-73256) using

$$v_{pred} = v_{mean} + v_s \cos[2\pi(\phi_{obs} - \phi_{max})] \quad (5)$$

By using the data calculated so far in this report it was possible to now calculate the mean radial velocity v_{mean}, the amplitude of the (RVC) v_s and the phase when the RVC is at a maximum ϕ_{max} of the stars. As well as the associated covariance matrix uncertainties for each of these values.

Table 1: Radial velocities and Phase of stars HD-28185 and HD-73256.

	HD-28185	HD-73256
v_{mean}	$2.18 \times 10^4 \pm 4.00 ms^{-1}$	$1.35 \times 10^4 \pm 2.80 ms^{-1}$
v_s	$67.6 \pm 6.16 ms^{-1}$	$1.20 \times 10^2 \pm 3.92 ms^{-1}$
ϕ_{max}	$8.59 \times 10^{-2} \pm 0.01$	$5.73 \times 10^{-2} \pm 0.01$

Table 2: Mass and semi-major axis of planets in orbit around stars HD-28185 and HD-73256.

	HD-28185	HD-73256
m_p	$1.7654 \pm 0.1089 \times 10^{25} kg$	$1.0517 \pm 0.0413 \times 10^{26} kg$
a	$2.2756 \pm 0.1365 \times 10^6 m$	$2.5446 \pm 0.1018 \times 10^7 m$

For the stars (HD-28185 and HD-73256) the values obtained for v_{mean}, v_s and ϕ_{max} were.

Values of ϕ_{max} are dimensionless in this instance as they are fractions of the orbital periods.

Now that values for v_{mean}, v_s and ϕ_{max} of both stars have been calculated, it was possible to determine the mass and semi-major axis of the extra-solar planets in orbit around the stars (HD-28185 and HD-73256). The mass of the planets was calculated using.

$$v_s = \left(\frac{2\pi G}{T}\right)^{1/3} M_s^{-2/3} m_p \tag{1}$$

where the mass of the planet m_p is dependent on the amplitude of the RVC v_s, the mass of the star M_s and the period of the orbit of the planet T. As the mass of the parent star had not been calculated previously it was necessary to make an educated assumption of what this mass might be. Therefore based on the idea that these stars share similar properties to the sun it was assumed that the mass of the parent stars was $M_s = 1.0 M_\odot$. Then by rearrangin equation 2 we can obtain an equation for a the semi-major axis.

$$a = \left(\frac{G(M_s + m_p)T^2}{4\pi^2}\right)^{1/3} \tag{6}$$

it was possible to calculate the semi-major axis of the planets in orbit around the stars (HD-28185 and HD-73256). The values for the mass and semi-major axis of the relevant planets around each star are shown in table 2, with the associated uncertainties progated from the uncertainties in the values of v_{mean}, v_s and T.

Conclusion

From the data collected it is possible to determine that the accuracy of the Doppler wobble method is effective at detecting extra-solar planets, as, when comparing the two stars (HD-28185 and HD-73256) with similar masses, their respective planets have similar masses and semi-major axis of orbit, giving rise to the idea that for a star with a particular mass there is a likely chance that a planet with a similar mass will be in orbit around it.

Introduction and Background

Searching for Extra-solar planets can be difficult however, those orbiting along the plane of the observer can be detected using the method of Planetary Transits (PLT). This method relies on observing the flux from a star and the reduction that can be seen when a planet partially blocks some area of the star, this reduction in Flux is known as a transit. However one of the drawbacks of this method is that it relies on a telescope with a relatively high angular resolution.

From this method we can determine the semi-major axis of the planets orbit and from this the circumference of orbit, the radius of the star and the radius of the planet.

The Planetary Transit Method for Detecting Extra-solar Planets

The Method of Planetary Transits for detecting extra-solar planets around stars first requires the magnitude of light from (in this case) the I-band of the star to be measured over some period of time. By plotting the this magnitude of I-band light as a function of the phase of the planetary orbit, it is possible to observe a significant reduction in the magnitude for a short period of time. And thus this can be deduced to be the transit of a satellite around the star. For the star being measured in this report (OGLE-III-TR56) the magnitude of light was measured over one orbit of the planet (one phase) and the results are shown in Figure 3

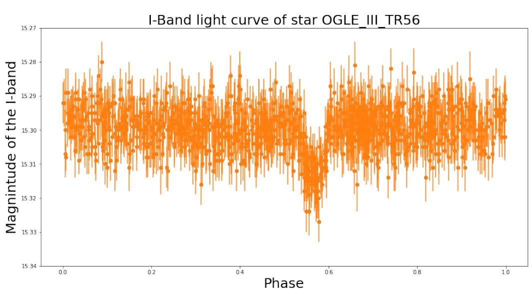

Figure 3: I-band light as a function of phase for star OGLE-III-TR56

To be able to take more accurate data from this graph, it was necessary to limit the graph to the area of interest (the dip in magnitude) as can be seen in Figure 4.

Now that a phase-folded light curve has been obtained, it is possible to start calculating certain properties about the Extra-solar Planet that

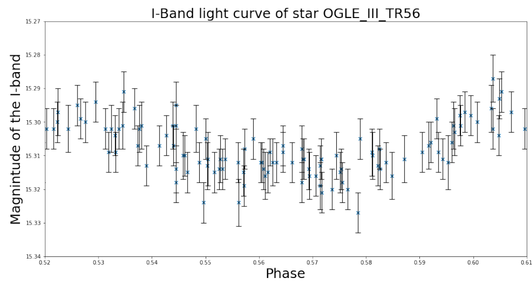

Figure 4: I-band light as a function of phase for star OGLE-III-TR56 limited to the dip in magnitude

has been detected. THe first of these properties is the semi-major axis of the orbit of the planet. This was calculated using the Kepler's Third law of Planetary motion as seen below.

$$G(M_s + m_p) = \frac{4\pi^2 a^3}{T^2} \tag{2}$$

As the star that we are observing is roughly equivalent to 1 solar mass ($1M_\odot$) and other similar characteristics it is reasonable to assume that the mass of the planet is equal to the mass of the earth and therefore is negligible compared to the mass of the star and can be omitted. Rearranging equation 2 for a the semi-major axis gives us:

$$a = \left(\frac{GM_\odot T^2}{4\pi^2}\right)^{1/3} \tag{7}$$

where G is the gravitational constant, $M_\odot$ is the mass of the star, and T is the period of the orbit of the planet. From this we can obtain $1.0737 \pm 0.0179 \times 10^8 m$ for the semi-major axis of orbit (a).

The next property to calculate was the radius of the planet, however in order to do this, it was first necessary to calculate the circumference of the orbit, which can be done when assuming that the orbit is circular and therefore the circumference is equal to $2\pi r$, where r is the radius of the orbit. This gives a value of $6.7460 \pm 0.1125 \times 10^8 m$

for the circumference of orbit. Now by comparing the fraction of the orbital circumference that the planet covers in one orbit, the diameter of the star can be calculated by extrapolating lines on the graph in Figure 4 like shown in Figure 5.

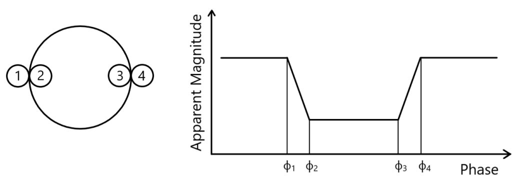

Figure 5: ϕ_1 and ϕ_4 show where the planet intially covers the star as in position 1 and 4 on the left

Then by using equation

$$r_s = \frac{0.071C}{2} \tag{8}$$

where r_s is the radius of the star, C is the circumference of the orbit thus radius of the star is $2.3949 \pm 0.0399 \times 10^7 m$.

Finally the radius of the planet can be calculated, this is simalar to how the radius of the star was calculated but by using when the planet starts to eclipses the star as shown by position ϕ_1 and when the planet fully eclipses the star at ϕ_2 as shown in Figure 5. From Figure 4 it was taken that the planet began eclipsing at 0.545 phase and was fully eclipsing at a phase of 0.553 thus giving a fraction of the orbital circumference equal to the diameter of the planet. using equation 8 where the 0.071 is changed to 0.08 the diameter of the planet can be found to be $5.3986 \pm 0.0900 \times 10^6 m$

Conclusion

References

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