Binary black hole detections from LIGO-VIRGO runs 1 and 2

2663452m (University of Glasgow)

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Abstract

In this report the first ten Binary Black Hole merger events and the first neutron star merger event as detected by the LIGO-VIRGO collaboration group are analysed and discussed obtaining estimates for the mass pairs of each event, and distance, time and phase of coalescence estimates and the associated errors. Starting with event GW150914 with mass pairs of 36.06 $M_{\odot}$ and 35.76 $M_{\odot}$ and distance, time and phase estimates of 1128 $\pm$ 82 Mpc, 3.198 $\pm$ 0.000 s and 0.03 $\pm$ 0.16 radians. Continuing the analysis for the rest of the events, handling the neutron star merger seperately as it has different requirements and taking a closer look at the properties of the group of events.

Introduction and Background

Gravitational waves as first predicted by Albert Einstein in 1915 in his paper on special and general relativity, are ripples in the fabric of space-time due to the acceleration of large masses and have been notoriously hard to detect. That was until the LIGO Michelson interferometer in Hanford and Livingston was complete in 2015. A Michelson interferometer is a device that uses the interference of two beams of light to detect small changes in the path distance of the two. A diagram of one can be seen in Figure 1.

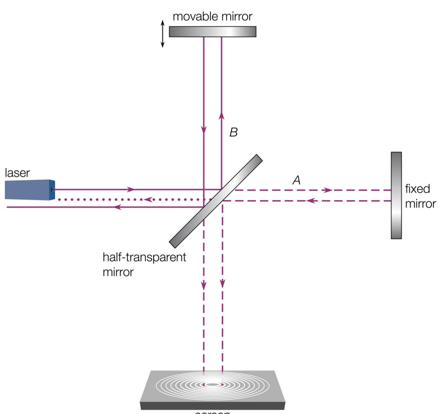

Figure 1: Diagram of a michelson interferometer as used in LIGO.[1]

By using a Michelson interfermoeter in the LIGO experiment the small changes in distance that are required can be detected and measured. These distances can be on the order of 10^{-21}m this is called the strain of the wave and is the amount of stretching over the original length and can be approximated as

$$h \approx \frac{GM}{c^2 d}\left(\frac{v}{c}\right)^2 \tag{1}$$

where G is the gravitational constant, M is the mass of the source, c is the speed of light, d is the distance to the source and v is the velocity of the system.
This is caused by the passing of gravitational waves moving at the speed of light through the interferometer arms (which results in a shift in the interference pattern of the light beams). The first detection of a gravitational wave was on the 14th of September 2015, just 100 years after the publication of Einsteins paper. The first detection was of a binary black hole merger, these mergers commonly release a large amount of energy in the form of gravitational waves. This happens because as the two black holes accelerate towards each other they warp the space-time around them, and as they approach the point of coalescence the amplitude of these waves massively increases, thus allowing them to be detected over the Background noise. The ringdown after merging is extremely quick and thus leaving a distinct peak at the time of coalescence. In this report the first 10 detections of gravitational waves as a result of binary black hole mergers will be analysed and discussed alongside the first detection of a binary neutron star merger that will be dealt with seperately starting with GW150914.

Event GW150914

To be able to carry out the analysis of the gravitaional waves it was necessary to set up our workspace to be able to use some provided functions and packages written by the LIGO collaboration. This was done by first installing the LIGO lalsuite package for Python 3.11 and then importing the packages into our notebook. The package contains a number of useful functions that will be discussed in more detail later in this report. For the first detection of gravitational waves GW150914 the data was provided by the university through the Jupyter Hub. Once this data was loaded in the first thing to do was identify by eye the peak of the gravitational wave. The plot used to do this is shown in Figure 2

From Figure 2 it can be seen that the peak of the gravitaional wave occurs at around 3.2 seconds. Knowing this time we can use the SCIPY package to generate a spectrogram of strain and

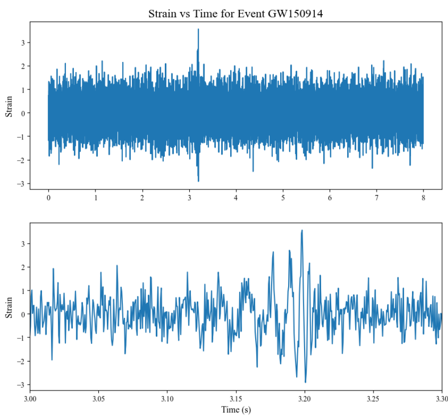

Figure 2: top: Plot of the strain against time for GW150914 bottom: limited to shorter time to resolve the peak more.

frequency and from this a color plot can be created which visualizes the amount of energy in the gravitational wave at a given frequency and time. This plot is shown in Figure 3.

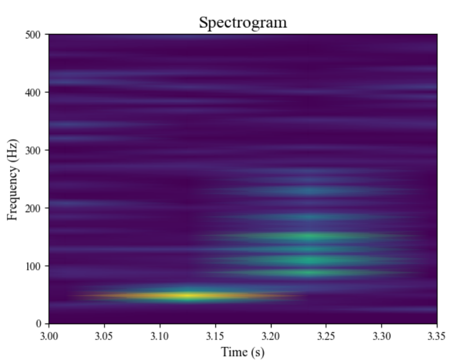

Figure 3: Spectrogram of GW150914, showing the 'chirp' track of the gravitational wave.

In this plot the point where the energy is high across multiple frequencies correlates with the same time as the peak in the strain in Figure 2, This confirms that the correct peak has been identified in the given data and more analysis can be carried out on it.

Now that we have visualised the actual data from the gravitational wave, it would now be useful to be able to compare this to the theorectical prediction of the event, this is necessary as it allows us to use some estimating functions to determine values for the distance from earth that these events occured at as well as some details about the coalescence, this is discussed below. This can be generated using the make template function as supplied by the LIGO collaboration. This function takes in the masses of the two black holes and the time, frequency, distance and uncertainty on the data. The function then returns a strain and time array that can be used to plot the theorectical predictions. This produces an ideal signal as seen in Figure 4, this is easier to see the signal as it no longer has any noise present in it.

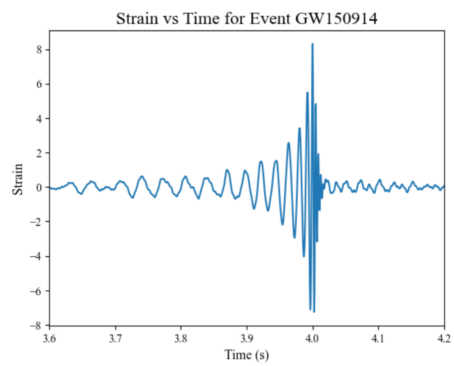

Figure 4: Theoretical prediction for GW150914

Later this template will be overlayed on the true data to determine the goodness of fit.

Detecting the signals for the ten Black hole Mergers

In this next section the detection of the signals will be looked at using some of the techniques as discussed previously.

To determine the signal of the data for the ten black hole mergers the signal to noise ratio (SNR) was used to determine the strength of the signal and its location in time. This could be done using the function given in the lalsuite package. This function returns a list of signal to noise ratios for each time in the data. If the SNR is low then the template that was put into the function is badly aligned or the masses of the black holes that were put into the template function are incorrect, and if the SNR is high then the template is aligned well and the masses are correct. An example SNR time series is shown in Figure 5. This SNR at the peak is important

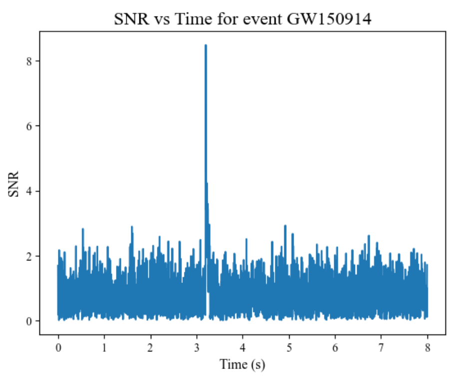

Figure 5: SNR time series for GW150914, with the peak at 3.2 seconds which tells us it is correctly aligned as this is the same time as obtained above

as it can be used later to determine the masses of the black holes as well as other important information. So this value was stored to be used later.

Determining the masses of the black holes

To determine the masses of the black hole pairs, some assumptions were made about the system. The first assumption was that m1 the mass of the first black hole is larger than m2 the mass of the second black hole. And the second assumption was that m1/m2 is less than 8 as this is the maximum mass ratio expected and it helps lower the number of possible mass pairs for the next section. With these assumptions the mass of the black holes can be determined by iterating over 10,000 possible mass pairs and calculating the SNR for each pair, then storing this SNR and the corresponding mass pair. This was done for the 10 binary black hole mergers (the binary neutron star merger was not included in this section as it has different mass ranges, and thus will be analysed seperately later). With the SNR's and the mass pair options stored a color plot was created for each event, where the intensity of the color is proportional to the SNR and the x and y axis are the mass of each black hole. An example of one of these plots is shown in Figure 6.

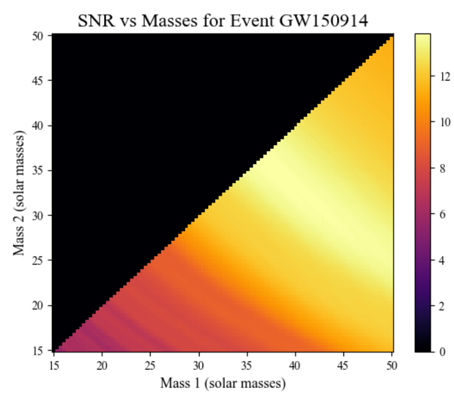

Figure 6: Mass plot for GW150914, showing the mass pairs that give the highest SNR

The black upper triangle is due to the conditional statement that mass one must be greater than mass two. Where the color is the highest is where the SNR is highest and thus the mass pair that is most likely to be correct. From this plot and as discussed above the mass pair that is most likely to be correct is 36.06 ± 1.4 and 35.74 ± 4.3 solar masses. This is within estimated error of the masses published by the LIGO collaboration of 35.6 and 30.6 solar masses. This method was repeated for the ten black hole mergers considered in this report and the results are shown in

Table 1: Estimated values for the distance, time and phase of coalescence and the mass pairs for the ten binary black hole mergers

Event	Distance (Mpc)	Time (s)	Phase (rad)	Mass one ($M_\odot$)	Mass two ($M_\odot$)
GW150914	1128 ± 82	3.198 ± 0.000	0.03 ± 0.16	36.06	35.76
GW151012	3783 ± 1677	2.703 ± 0.001	0.00 ± 0.82	35.40	11.72
GW151226	1812 ± 552	2.067 ± 0.000	0.00 ± 0.46	12.10	9.70
GW170104	2119 ± 340	1.975 ± 0.000	1.79 ± 0.32	35.86	25.35
GW170608	664 ± 81	3.794 ± 0.000	2.67 ± 0.18	11.27	8.36
GW170729	3150 ± 590	3.539 ± 0.001	2.89 ± 0.51	75.15	29.70
GW170809	2146 ± 444	2.641 ± 0.001	2.78 ± 0.42	46.41	15.10
GW170814	2268 ± 385	3.335 ± 0.000	0.00 ± 0.34	30.20	29.85
GW170818	1596 ± 417	2.161 ± 0.000	0.73 ± 0.40	12.79	9.91
GW170823	4111 ± 1121	4.456 ± 0.001	0.00 ± 0.64	55.25	26.26

Table 1.

Determining estimates for the distance, time and phase of coalescence

To find estimates for the distance, time and phase of coalescence the make template function was used again to generate a template for each event but this time using the masses as found above as this would produce the most accurate templates for the sets of data. From this the SCIPY curve fit function could be used as this allow for estimates of unknown quantities depending on the how close the true data/plot is to the template. However for this particlar application it was necessary to provide some upper and lower bounds that these value could take as this reduces the possible solutions to the problem. These bounds were determined by looking at the peak and its location as well as an overall generic distance range for the events. To help visualize this fitting of the data the plot of the generated template using these best estimates overlayed on the raw data has bee included and can be seen in Figure 7. How well these templates fit the raw data varied greatly for all the events as some

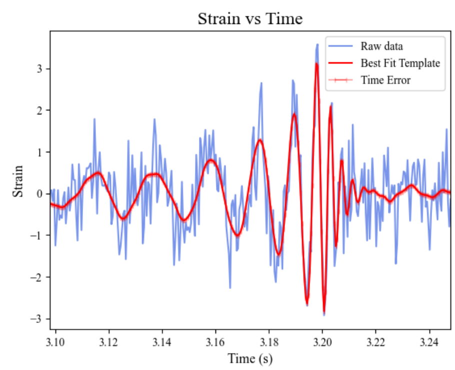

Figure 7: The template data as generated with the estimates for all values overlayed on the raw data

couldn't identify the correct values or the associated errors on these values were so large that it produced a graph that was severly off (these estimates were likely to be off as the SNR for some events was too little to easily identify the true signal.)

The curve fit function also provides a covariance matrix which can be used to determine the error on the estimated values. This was repeated for each event and the results are shown in Table 1 along with the mass estimates from the previous section. It is worth noting here that the distance measurements as obtained in this analysis are incorrect when compared to the values published by the LIGO-VIRGO collaboration group, however this appears to be a systematic error as for every event the distance estimate is off by around 2-3 times the 'true' value. This is most likely the result of some systematic error in the analysis that was completed, either a step was not included as it was beyond the skill level of this lab, or otherwise this was due to not taking into consideration the inclination angle of the events, as this would also throw off the distance (for instance if a gravitational wave propagates towards earth directly its strength will be at 100%, whereas if the inclination angle as taken from the observing plane is at 45 degrees this may be reduced to something like 50% and if inclination angle is not considered this could appear to be a result of the distance therefore giving an estimate that is too large.)

The Binary Neutron star merger GW170817

As mentioned above the binary neutron star merger was not included in the previous analysis as it has a different mass range that is required to be searched over as well as some tighter parameters on the distance time and phase of coalescence as it has an overall weaker signal and therefore a worse SNR than the binary black hole mergers, making it more difficult to detect and requiring it to occur a lot closer to earth. However, it is still possible to use the same methods as above to determine the masses of the neutron stars and the distance time and phase of coalescence. From this the following values were found as shown in Table 2.

From these values the mass ranges for neutron stars is obviously a lot lower then that for the black hole mergers in the range of fractions of a solar mass up to about 5 solar masses, and for this particular merger the masses were 1.63 and 1.16 solar masses, these values are very close to the published values by the LIGO collaboration of 1.46 and 1.27 solar masses. For the results collected in this analysis the errors on the distance time and phase are higher than the general error as seen for the black hole mergers, this as mentioned previously is because of the lower SNR and thus it is more difficult to determine where the signal actually occurs in the data and also to make accurate estimates using the same analysis techniques as for the black hole mergers.

Table 2: Estimated values for the distance, time and phase of coalescence and the mass pairs for the binary neutron star merger GW170817

GW170817	Value	Error $\pm$	Units
Distance	92.0	17	Mpc
Time	1.7	0.2	s
Phase	0.0	1.2	rad
Mass one	1.63	0.00	$M_\odot$
Mass two	1.16	0.00	$M_\odot$

Properties of the events

In this next section the 10 binary black hole events are going to be focused on and the binary neutron star merger is once again going to be ignored as it will only skew the results as for a neutron star merger to be detected it had to be extremely close to earth due to the weakness f its signal and the masses in the pair are going to be significantly lower then that of the black hole merger pairs.

The mass distribution of events

When looking at the mass candidates for the binary black hole mergers, the individual masses are genrally in the region of 30-40 $M_\odot$ with some outliers as high as 76 $M_\odot$ and as low as 8.4 $M_\odot$, this upper limit is because as the masses increse the frequency of the emitted waves decreases and on earth interferometers are not sensitive enough

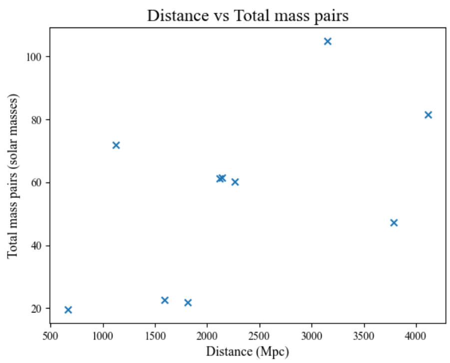

Figure 8: Total mass of a merger as a function of its distance from earth

to be able to detect this. And the lower mass limit is due to the formation of black holes as opposed to neutron stars, at around the 5-8 $M_{\odot}$ a star may collapse into either a neutron star or a black hole, This limit alos arises due to the lower masses having weaker signals and therfore being harder to detect. The average total mass across the ten events is 55.22 $M_{\odot}$ with a lowest mass pair of 19.63 $M_{\odot}$ and a highest mass pair of 104.85 $M_{\odot}$ the total distribution can be seen in Figure 8. The general trend of this plot is that for an increase in distance the mass pair is greater ($d \propto m$ where d is the distance to the source and m is the total mass of the system.) This relation arises from the ability to detect these events, as for closer events it is more likely that lower mass candidates can be detected due to the noise being lower over the shorter distance, where, as the distance increases these low mass pairs will be obscured by the background noise and only higher mass pairs will be detectable.

Distance range of detected events

Of the events discussed in this report the distance range that they occured in is relatively small only being a few thousand mega parsecs, detections closer to earth are unlikely however as binary black hole systems form most commonly in regions of high stellar density which are generally far away, at least beyond the scope of the milky way galaxy. the far distance limit however is most likely because of the decrease in the strength of the signal at such distances making these events more difficult to detect and the ground based interferometers are not sensitive enough for this.

Conclusion

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