Observations with a small radio telescope: the temperature of the Sun and the position of the Observatory

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Abstract. This report describes the use of small radio telescopes in two ways: 1) to observe the Sun and estimate its brightness temperature and 2) to detect, measure the positions of and identify, a number of geostationary telecommunications satellites. With the information thus obtained the latitude and longitude of the observation position on the Earth can be obtained. The brightness temperature of the Sun was found to be approximately 9000 K, which is larger than the photospheric blackbody temperature. The latitude and longitude of the Glasgow University Observatory were found to be $(55.2 \pm 0.4, -4 \pm 6)^\circ$.

(Note: this report, based on an A1 experiment that should be familiar to you. It provides an example of the style, content, standard of writing, presentation, structuring, error analysis and evaluation expected of an A2 lab report. It is on the short side, since it describes an experiment that only took one week to do, rather than the three weeks of an A2 experiment. However I have elaborated on e.g. the error analysis from the A1 lab to bring it up to the A2 standard.)

1. Introduction and Background

The first concrete identification of an astronomical radio source was made by Karl Jansky [1] who, in 1931, established that emission at 20.5 MHz, varying with the sidereal day, originated in the direction of Sagittarius [2]. Radio astronomy research developed rapidly from the 1950s onwards, with the building of large single-dish antennas such as the 250 ft Mark-I dish at Jodrell Bank [3], and also with the linking of antennas into interferometric arrays, enabling radio imaging at long wavelengths.

Since those early days radio astronomy techniques have improved so much that some of the highest-resolution astronomical images available are made in the radio domain using very long baseline interferometry. Time-resolved radio observations of nearby objects such as the Sun and Jupiter reveal varied and dynamic radio emission. Radio astronomy brightness measurements are often expressed in terms of source "brightness" temperature. This equals the source temperature if the source is a blackbody, but it can be very much higher than the blackbody temperature (e.g., for Jupiter). It has been discovered that many astronomical objects emit in the radio domain, and with large international projects such as the Square Kilometre Array [4] on the horizon it is certain that this relatively young science will provide many more exciting discoveries in the future.

Radio antennas are also commonplace in everyday life, fixed to the sides of houses and rooftops in the form of satellite dishes and television antennas. Many of the stations being received by these dishes are broadcast from geostationary satellites, orbiting the Earth in the “Clarke Belt” [5], an equatorial location in geospace where (as first pointed out by Arthur C. Clarke) the orbital period of the satellite exactly matches the rotation period of the Earth [6]. A satellite in the Clarke belt is in geostationary orbit, and is always seen by observers on the Earth in the same position in the sky. All satellites in the Clarke belt form an arc across the sky, the maximum altitude of which depends on the latitude of the observer. Using measurements of the satellite positions and some prior knowledge of the signal strength of each, the position on the Earth of the observer can be measured. This is the main goal of the experiment described in this report.

We describe the equipment used in Section 2 and measurements of the Sun’s brightness temperature in Section 3. Section 4 presents the measurements of the geostationary satellites and the measurements of the latitude and longitude of the point of observation. We end with evaluation and conclusions in Section 5.

2. Equipment

The equipment for this experiment is a small commercial satellite dish, the output of which is connected to an electronic low-noise block (labelled ”LNB” in Figure 1) in which the oscillating voltage signal from the antenna (at approximately 10 GHz) is mixed down to a lower frequency and amplified, then fed to a power meter. The signal can be read from a scale with a moving needle. The output is also converted into a musical tone that rises or falls with the signal strength. The benefit of this is that it allows the observer to rapidly home in on the signal in sometimes difficult outside conditions; thereafter the needle’s movements on the scale allow the source position measurement to be refined.

The dishes are mounted so that they can be moved independently in altitude and azimuth, both of which are read from scales fixed to the mounts. The scale division is in degrees, and is indicated by a moving pointer. An accurate measurement requires that the mounts be level, and this is achieved by means of a spirit level. The mounts have been pre-aligned such that the zero position in azimuth corresponds to North. The possible errors in the alignment are unknown, and constitute systematic errors (i.e., applying to every measurement). A diagram of the equipment is shown in Figure 1.

3. The brightness temperature of the sun

The brightness of the solar emission, compared to that of the ground at the same frequency can be used to estimate the Sun’s temperature. This also requires knowledge of the angular size of the Sun and the angular width of the telescope beam.

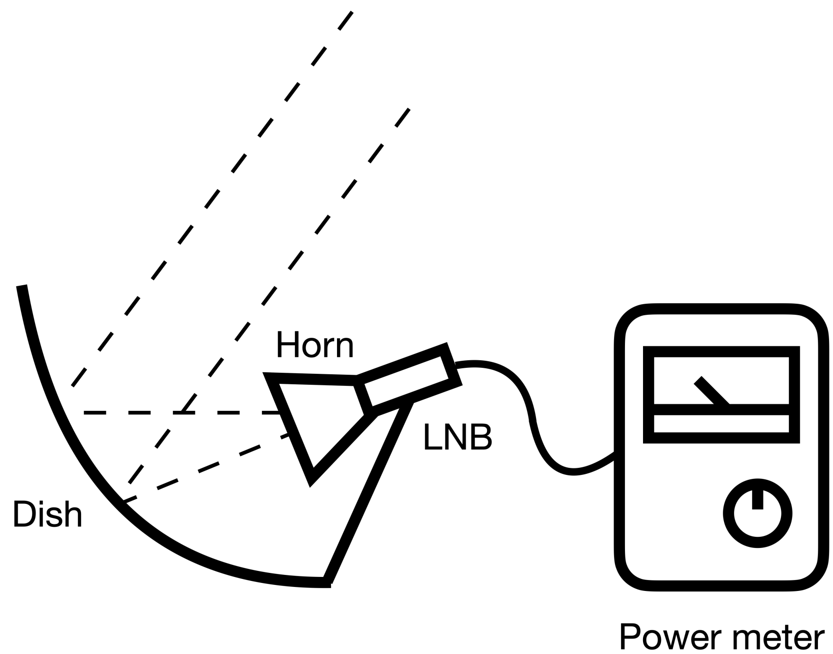

Figure 1. The equipment used in this experiment, showing also rays reflecting from the dish into the radio horn.

The angular width α and the solid angle $\Omega_{\rm beam}$ beam of the telescope beam for a single dish can be obtained by analogy with the angular resolution of an optical telescope. It is given by:

$$\alpha \approx \frac{\lambda}{D} \tag{1}$$

so that

$$\Omega_{\rm beam} \approx \left(\frac{\lambda}{D}\right)^2 \tag{2}$$

where λ is the wavelength of observation, and D is the diameter of the dish. The antenna is sensitive to a narrow range of frequencies centred at 10GHz, and so we take $\lambda = 3.0$ cm as a representative value. The dish is square in shape, and its diagonal measurement is 55.3 ± 0.2 cm, where the error comes from the mean of the two measured values. Therefore using these values,

$$\begin{aligned} \alpha &= \frac{3.0}{55.3} = 0.054 \text{ rad} = 3.11^\circ \\ \Delta\alpha &= \frac{0.2}{55.3} \times 3.11^\circ = 0.01^\circ. \end{aligned}$$

Thus the angular diameter of the telescope beam is $3.11^\circ \pm 0.01^\circ$.

The Sun was low on the horizon in the North West, but was still above the trees. A sweep in azimuth was first made, and the Sun located to within approximately 5° using the audio signal from the power meter. The azimuth was then locked and the altitude co-ordinate varied until the maximum audio signal was again found. Then with

Table 1. Observations of the position and signal strength of the Sun

Date and Time	Altitude (degrees)	Azimuth (degrees)	Meter Reading (arbitrary units)
October 24th 2001 15.24-15.30 UT	15$\pm$ 0.5	218$\pm$0.5	8.5$\pm$0.5

altitude locked, azimuth was again varied and the maximum signal sought. With slow and careful movements of the antenna, the peak needle deflection could be identified, allowing one to home in on the peak signal. However the uncertainties in finding the maximum signal strength were estimated to be $\pm 0.5°$, comparable to the scale-reading error. The results of the solar observations are presented in Table 1.

The telescope was then pointed downwards, so that the ground filled the beam. The meter reading was 7 ± 1, which is close to that from the Sun. However, the meter reading does not vary linearly with the signal strength, therefore these signals can only be used to say that the Sun, which does not fill the beam, and the ground, which does, produce approximately the same radio power. Assuming that the Sun has a significantly higher brightness temperature than the surrounding sky we can write

$$T_{\rm sun}\Omega_{\rm sun} = T_{\rm ground}\Omega_{\rm beam} \tag{3}$$

where $T_{\rm sun}$ and $\Omega_{\rm sun}$ are the solar brightness temperature and solid angle respectively, and $T_{\rm ground}$ is the ground brightness temperature. The angular diameter of the Sun is $0.53°$, so that the ratio of the solid angles of telescope beam and Sun is $\Omega_{\rm beam}/\Omega_{\rm sun} = (3.11°/0.53°)^2 = 34.4$. Therefore

$$T_{\rm sun} = 36T_{\rm ground} = 9400\,{\rm K}$$

where we have assumed a ground temperature of 273 K. In reality, the temperature of the Sun is 5800 K, therefore this calculation gives an overestimate of the solar temperature. The errors on the beam angle are much less than 1%, which cannot explain the difference between observed and actual values. The reasons for the difference between calculated brightness temperature and true solar temperature will be discussed in Section 5.

4. The positions of geostationary satellites

Geostationary satellites orbit the Earth in the Clarke Belt, located at a distance of $R = 6.617$ (in units of Earth radii) from the centre of the Earth. Viewed from the Earth, this circle projects onto the sky forming a portion of an ellipse. By detecting geostationary satellites the ellipse can be plotted, and its properties used to determine the observer's latitude on Earth. Then, by identifying the satellites based on their signal strength, their longitudes and hence the observer's longitude can be determined.

The satellite positions were found by first making a sweep in azimuth at a fixed altitude of around 25° then located more precisely using the technique described for the solar observations in Section 3. Once the first satellite had been identified, it was possible to hop from one satellite to the next by searching the neighbouring region, and knowing that the difference in longitude between satellites (given in the table with the lab notes) is approximately equal to the difference in azimuth. The satellites could be again be located to within $\pm 0.5°$.

4.1. The latitude of the Observatory

The satellite positions are plotted in Figure 2. Superposed on these is an ellipse in azimuth (x) and altitude (y). The ellipse has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{4}$$

where a and b are the semi-major and semi-minor axes of the ellipse. The semi-minor axis b gives the altitude of the peak of the ellipse above the horizon. Using Excel's 'solver' facility, the best fit values for a and b were determined as

$$a = 57.0° \text{ and } b = 27.1°.$$

Also shown on Figure 2 are curves with $b = 26.8°$ (lower dashed curve) and $b = 27.4°$ (upper dashed curve), which show the approximate size of the uncertainty Δb on b. It can be seen that $\Delta b = 0.3°$ is a reasonable error estimate. The latitude ϕ of the observer is then given by the following equation:

$$\cos\phi = \frac{\cos^2 b + \sin b\sqrt{R^2 - \cos^2 b}}{R} \tag{5}$$

where R is the radius of the Clarke belt in units of the Earth's radius, given above. Inserting the values given it is found that the latitude of the Observatory is

$$\phi = 55.2°.$$

Differentiating Equation 5 gives:

$$\frac{\Delta\phi}{\Delta b} \approx \frac{d\phi}{db} = \frac{\cos b}{R\sin\phi}\left[2\sin b + \left(1 + \frac{\sin b}{R^2 - \cos^2 b}\right)\sqrt{R^2 - \cos^2 b}\right]. \tag{6}$$

This allows an error to be calculated for ϕ, knowing the approximate uncertainty on b. The latitude of the Observatory quoted with its error is thus:

$$\phi = 55.2° \pm 0.4°.$$

4.2. The longitude of the Observatory

To establish the longitude of the Observatory requires that each of the satellites is identified, enabling their longitude to be read from the table provided in the experiment notes. This was done knowing that, at the latitude of Glasgow, their difference in azimuth is approximately equal to their difference in longitude, and also using their intensity as a guide. The approximate strengths, identifications and longitudes are listed in Table 2 below.

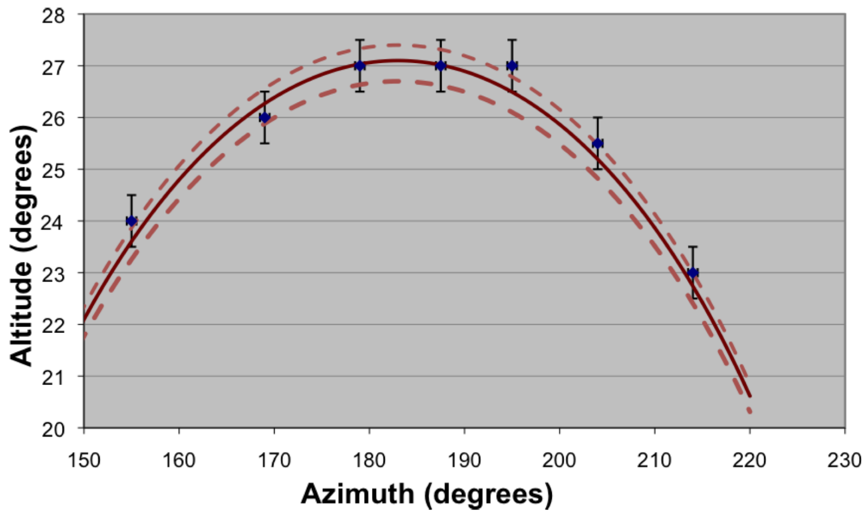

Figure 2. Positions of geostationary satellites as observed from Glasgow. These have been fitted with an ellipse (solid line) with semi-minor axis $b = 27.1° \pm 0.3°$, where the ranges are indicated by the dashed lines.

Table 2. Positions, strengths and identifications of the satellites observed. [*Strength data omitted from this version of the report - hence the * notation*]

Satellite Number	Azimuth (degrees)	Strength	Identification	Longitude (degrees)
1	155.0±0.5	***	Astra 1B	19.2
2	169.0±0.5	**	Eutelsat W3	7
3	179.0±0.5	**	Thor	-0.8
4	187.5±0.5	**	Atlantic Bird	-8.0
5	195.0±0.5	***	Intelsat 901 or Telstar 12	-15 or -18
6	204.0±0.5	**	Telstar 605	-27.5
7	214.0±0.5	*	Telstar 11	-37.5

A graph of the satellite longitude versus azimuth can then be plotted (Figure 3). This gives approximately a straight line, and longitude value at which this line crosses the point where the azimuth equals 180° is the longitude of the Observatory.

Note, the identification of satellite number 5 was uncertain, as there were two strong satellites within 3°, which is approximately the beam angular width. Therefore the radio telescope would have had difficulty in resolving them. On Figure 3, an average longitude value for the position of these two sources was used.

Using Excel's linear least squares straight line-fitting program, the best fit to the data is

$$\lambda = -\left(0.97 \pm 0.02\right) A + \left(171 \pm 2\right)$$

where A is the satellite azimuth in degrees, and the standard errors on the gradient and intercept of the least squares fit have been calculated [7], using the observational error

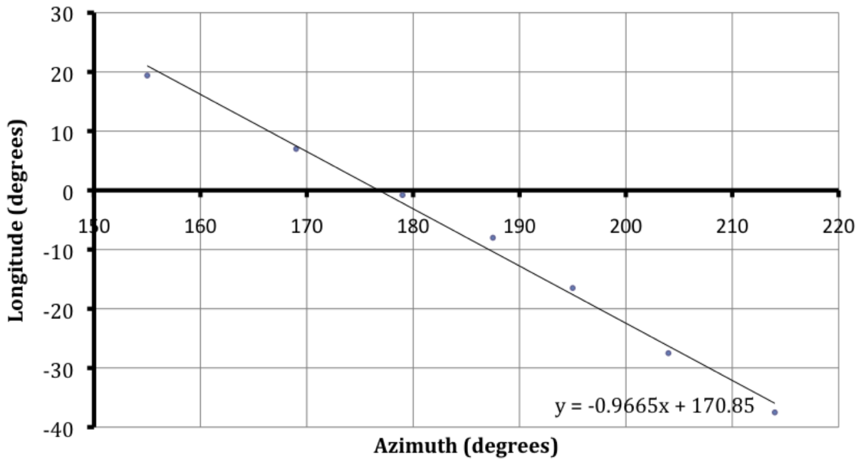

Figure 3. The azimuth versus the longitude of the satellites. Errors in azimuth on this scale are unplottably small. Errors in longitude are assumed to be zero.

in A of 0.5°. Substituting in $A = 180$ gives

$$\lambda = -(3.6 \pm 5.6) = -(4 \pm 6)\,.$$

In summary, the latitude and longitude of the Observatory found by observing geostationary satellites is $(55.2 \pm 0.4, -4 \pm 6)°$.

5. Evaluation and Conclusions

In Section 3, an approximate value of 9400 K was found for the solar brightness temperature. This is of the same order of magnitude of the solar blackbody temperature, but substantially higher. In fact, at 3 cm wavelengths, the Sun does not emit like a blackbody. The temperature calculated above is the brightness temperature, not the blackbody temperature. The brightness temperature is the temperature that a blackbody would have to have to generate the same intensity as is observed. The fact that it is larger than the blackbody temperature indicates that at 3cm the Solar radiation does not follow a blackbody curve.

The experimental values of the Observatory latitude = $(55.2 \pm 0.4)°$ and longitude = $(-4 \pm 6)°$ are close to the true values of latitude = $55°\,54'\,8.29'' = 55.9°$ and longitude = $-4°\,18'\,25.93'' = -4.3°$ [8]. The observed value of longitude is consistent with the actual longitude value within observational errors. This is not surprising since the longitude is well defined once the satellite identifications are established. The error in longitude is produced by the uncertainty in the azimuth values, and the scatter in azimuth about the straight line of the points in Figure 3.

The observed value of latitude is not consistent with the actual value within the errors, however it differs by less than twice the error bar (i.e., to within "2 sigma"). A difference of 0.7° corresponds to 42 nautical miles (one minute of arc = 1 nautical mile on the Earth's surface) so that the position of the Observatory has been identified as

48.33 miles too far north. Since an increased value of the error in semi-minor axis b is not consistent with the data points in Figure 2, or with their random measurement errors, it is likely that the difference between observed and true latitude is due to the telescope platform not being exactly horizontal. An offset of half of a degree from horizontal would lead to an error of half of a degree in each satellite altitude position. This in turn would lead to a systematically shifted value for the latitude. Since the satellites were mounted on wooden plinths (which can warp) sitting on top of gravel, it is likely that the bases were not exactly horizontal to start with (as the small spirit level is unlikely to offer very much precision in horizontal alignment) or were wobbled during the observations.

Overall, the measurements obtained for both the solar temperature and the Observatory position were reasonably close to the true values, and where they differed the source of the differences could be understood in principle.

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