## The Hall Effect 2663452m

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GUID: 2663452m

#### 1 Introduction

The Hall effect is the production of a potential difference across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. It was discovered by Edwin Hall in 1879. It commonly occurs in a rectangular conductor, such as a metal sheet or in specific types of semicinductors in this case a combination of a Galium Arsenide and an Aluminium Gallium Arsenide semiconductor, these two combined gives rise to a 2D electron gas or 2DEG that occupies a potential well created by the semiconducting materials. This electron gas as suggested is only able to move with 2 degrees of freedom and is therefore confined to the plane of the semiconductor.

#### 2 Aims

Within this experiment we aim to obtain values for the carrier density and mobility of the 2DEG within the semiconductor at room temperature and close to liquid nitrogen temperature and compare these results. This is done by measuring the Hall voltage and the resistivity through our sample of semiconductor (Hall Bar).

# 3 Equipment

- Digital Multimeter Used to measure the voltage and current through the sample.
- Digital Power Supply Used to supply a voltage through the sample.
- Electromagnet Used to apply a magnetic field perpendicular to the sample.
- Sample The sample is a Hall bar made of a combination of a Galium Arsenide and an Aluminium Gallium Arsenide semiconductor.
- Hall Effect Probe Used to measure the magnetic field applied to the sample.
- Liquid Nitrogen Used to cool the sample to a temperature close to 77K.
- Python 3.10
- HallPy Teach A python package used to automate the data collection from the experiment.

```
[]: import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
import scipy.constants as const
import pickle
```

```
import pandas as pd
import os
# import warnings
# warnings.filterwarnings('ignore')

WIDTH = 100e-6
LENGTH = 300e-6
plt.style.use('../report.mplstyle')
```

#### 4 Manual Data Collection

#### 4.1 Objectives

- To familiarise ourselves with the equipment and the experiment.
- To take measurements of the Hall voltage and the Longitudinal voltage of the Hall bar.
- To take measurements of the magnetic field applied to the Hall bar using the Hall Probe.
- To calculate the carrier density and mobility of the 2DEG within the semiconductor.

#### 4.2 Background Theory

The Hall voltage is given by taking a reading of the potential difference across the Hall bar perpendicular to the supply current and the magnetic field. This can be used to calculate the 2 dimensional Hall coefficient and 2 dimensional carrier density. The equation for the 2D Hall coefficient is given by:

$$R_{H,sq} = \frac{V_H}{IB} \tag{1}$$

where  $V_H$  is the Hall voltage, I is the current through the Hall bar and B is the magnetic field applied to the Hall bar. The equation for the 2D carrier density is given by:

$$n_{sq} = \frac{1}{eR_{H,sq}} \tag{2}$$

where e is the charge of an electron.

The Longitudinal voltage is given by taking a reading of the potential difference across the Hall bar parallel to the supply current. This can be used to calculate the 2 dimensional resistivity and thus the 2 dimensional mobility. The equation for the 2D resistivity is given by:

$$R_{sq} = R \frac{w}{I} \tag{3}$$

where R is the resistance of the Hall bar, w is the width of the Hall bar and l is the length of the Hall bar and the resistance is given by Ohms law  $R = \frac{V}{I}$  where V is the potential difference across the Hall bar and I is the current through the Hall bar The equation for the 2D mobility is given by:

$$\mu = \frac{1}{eR_{sq}n_{sq}} \tag{4}$$

where e is the charge of an electron,  $R_{sq}$  is the 2D resistivity and  $n_{sq}$  is the 2D carrier density.

#### 4.3 Method

The Hall bar was connected to one of the digital power supplies and to both digital multimeters one to measure the voltage and the other to measure the current. The electromagnet was connected to the other power supply. first we took measurements of the longitudinal voltage and current across a varying input voltage of 2V to 12V in 2V increments the plot of this can be seen in Figure 1. The Hall bar was then set up in the Hall voltage configuration and the Hall voltage and current were measured with a constant input voltage and a varying magnetic field of 7mT to 35mT. Measurements were then taken for the longitudinal voltage and current with a constant input voltage and a varying magnetic field of 7mT to 35mT to measure if there were any dependence on the magnetic field. A summary of these results can be seen in Figure 2.

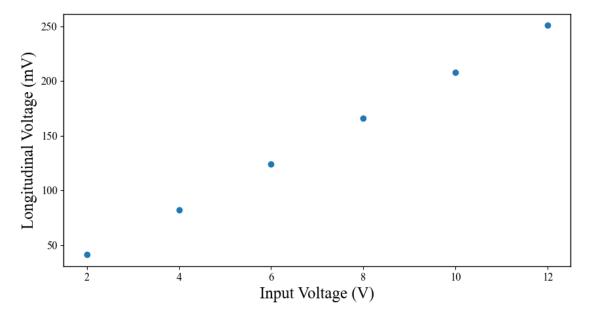


Figure 1 Longitudinal Voltage vs Input Voltage of the Hall Bar sample

From Figure 1 we can see that the longitudinal voltage is linearly proportional to the input voltage.

```
[]: def r_hall_2d(B, I, Vh):
         111
         B is the magnetic field in mT \setminus n
         I is the current in micro A \setminus n
         Vh is the hall voltage in mV
         return Vh/(I*B*1e-3)
     def res(V,I):
         111
         V is the voltage in mV \setminus n
         I is the current in micro A
         111
         return (V*1e-3)/(I*1e-6)
     def r_per_sq(res, width, length):
         res is the resistance in ohms \n
         width is the width of the sample in micro m \n
         length is the length of the sample in micro m
         return (res*width)/length
     def num_per_sq(r_hall_2d):
         111
         r_hall_2d is the function with parameters (B, I, Vh) \n
         return 1/(r_hall_2d*const.e)
     def mobility(r_per_sq, n_per_sq):
         r_per_sq is the function with parameters (res, width, length) n
         n_per_sq is the function with parameters (B, I, Vh) n_s
         return 1/(n_per_sq*r_per_sq*const.e)#
     data_hall = np.loadtxt('data/hall_mag_manual.txt', skiprows=3)
     data_long = np.loadtxt('data/long_mag_manual.txt', skiprows=3)
     hall_err = {
        111
         out\_vol in V
         out_cur in A
         b_field in T
```

```
'out_vol': [5e-6,5e-6,5e-6,5e-6,5e-6],
    'out_cur': [5e-9,5e-9,5e-9,5e-9,5e-9],
    'b_field': [5e-4,5e-4,5e-4,5e-4,5e-4]
    }
plt.figure(figsize=(15,15))
plt.subplot(2,2,1)
plt.text(13.5,-2.35, 'Figure 2 \n Hall and Longitudinal Voltage vs Input,
 ⇔Voltage and Magnetic Field of the Hall Bar sample', ⊔
 ⇔horizontalalignment='center')
plt.plot(data_hall[:, 0], data_hall[:, 1], 'o', label='Data')
plt.xlabel('Input Voltage (V)')
plt.ylabel('Output Voltage (mV)')
plt.title('Hall Voltage')
plt.subplot(2,2,2)
plt.plot(data_long[:, 0], data_long[:, 1], 'o', label='Data')
plt.xlabel('Input Voltage (V)')
plt.ylabel('Output voltage (mV)')
plt.title('Longitudinal Voltage')
plt.subplot(2,2,3)
plt.plot(data_hall[:, 3], data_hall[:, 1], 'o', label='Data')
plt.xlabel('Magnetic Field (mT)')
plt.ylabel('Output Voltage (mV)')
plt.title('Hall Voltage')
plt.subplot(2,2,4)
plt.plot(data_long[:, 3], data_long[:, 1], 'o', label='Data')
plt.xlabel('Magnetic Field (mT)')
plt.ylabel('Output Voltage (mV)')
plt.title('Longitudinal Voltage')
plt.show()
n sq = []
for i in np.arange(len(data_hall[:, 3])):
    n_sq.append(num_per_sq(r_hall_2d(data_hall[i, 3], data_hall[i, 1],_
 odata hall[i, 2])))
n_sq_mean = np.mean(np.array(n_sq))
mu = []
for i in np.arange(len(data_long[:, 2])):
    mu.append(mobility(r_per_sq(res(data_long[i, 1], data_long[i, __
 42]),WIDTH,LENGTH), n_sq_mean))
mu = np.mean(np.array(mu))
```

print(f'The number density in the Hall bar is  $\{n_sq_mean:.4g\}$ , and the mobility  $\{mu:.3f\}$ ')

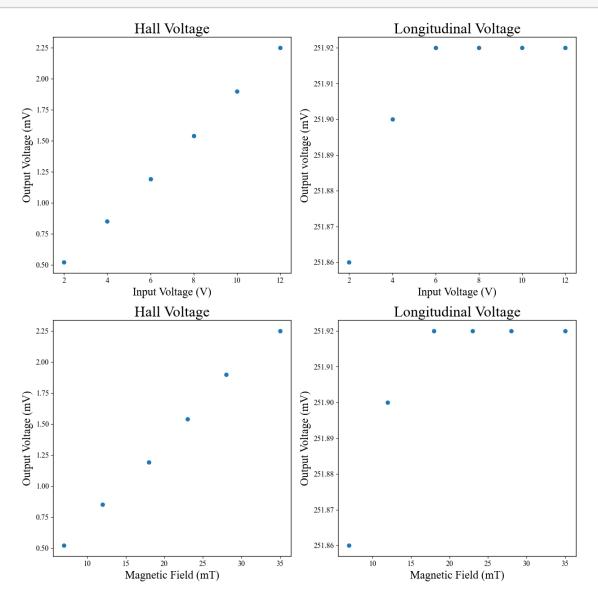


Figure 2 Hall and Longitudinal Voltage vs Input Voltage and Magnetic Field of the Hall Bar sample

#### The number density in the Hall bar is 5.647e+15, and the mobility is 0.491

From Figure 2 we can see that the Hall voltage is linearly proportional to the magnetic field and that the longitudinal voltage is independent of the magnetic field as expected as the magnetic field is perpendicular to the current and the longitudinal voltage is measured parallel to the current. The Hall voltage and Longitudinal voltage were then used to calculate the 2D Hall coefficient and 2D carrier density and the 2D resistivity and 2D mobility respectively. The results for the Density and Mobility can be seen above, no errors have been assigned to these values as they are only rough

indicators for what to expect later on.

### 5 Measuring the Effect of Angle on the Hall Voltage

### 5.1 Objectives

- To take measurements of the Hall voltage and the Longitudinal voltage of the Hall bar at a constant input voltage and magnetic field with a varying angle.
- To determine whether the Hall voltage is dependent on the angle between the magnetic field and the current.
- To determine the angle at which the Hall voltage is at a maximum if it is angle dependent.

#### 5.2 Background Theory

The Hall voltage gives rise to an electric field in the same direction as the Hall voltage and thus the equation relating Electric fields and magnetic fields can be used here to give an idea of when we might expect a maximum Hall voltage. The equation for the electric field is given by:

$$E_t = ev \times B \tag{5}$$

where  $E_t$  is the electric field, e is the charge of an electron, v is the drift velocity of the electrons and B is the magnetic field applied to the Hall bar. From this equation we can see that the electric field is at a maximum when the Hall voltage is perpendicular to the magnetic field, therefore when measuring its dependence in this section we expecT to have a maximum Hall voltage every 180 degrees when the sample is perpendicular to the magnetic field.

#### 5.3 Method

For this section the Hall bar was set up in order to measure the Hall voltage and a constant voltage was supplied to the Hall bar, this voltage was chosen to be 12V has we thought this ould be high enough to give a reasonble change when the angle was varied as we saw the Hall voltage was linearly proportional to the input voltage in the previous section. The magnetic field was also set constant for this section with a drive voltage going into the electromagnet of 30V, this was also chosen to be higher as the higher the magnetic field the higher the Hall voltage when at its maximum. The angle was varied from 0 degrees to 360 degrees in 10 degree increments and the Hall voltage was measured at each angle. The results of this can be seen in Figure 3.

```
[]: data = np.loadtxt('data/angle.txt', skiprows=2)
vol = data[:, 0]
angle = np.deg2rad(data[:, 1])
# x and b are in degrees, a is an amplitude constant

def func(x, a, b):
    return (a*np.cos(2*x + b)+2*a)

popt, pcov = curve_fit(func, angle, vol,p0 = (3.4,0))
angle_range = np.linspace(0, 2*np.pi, 1000)
plt.figure(figsize=(10, 5))
```

```
plt.plot(angle, vol,'o' ,label='Data')
plt.plot(angle_range, func(angle_range, *popt), 'r-', label='fit')
plt.xlabel('Angle (rad)')
plt.ylabel('Hall Voltage (mV)')
plt.legend()
plt.text(3.25,-2, 'Figure 3 \n Output Voltage vs Angle of the Hall Bar sample', uhorizontalalignment='center')
plt.show()
```

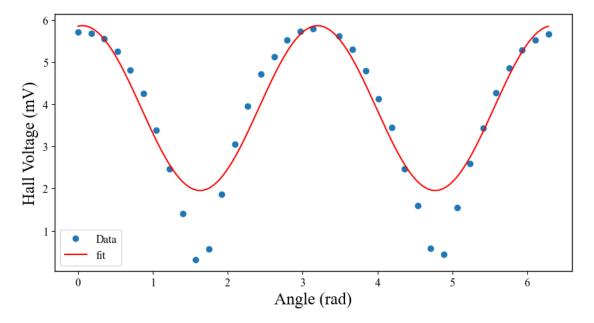


Figure 3
Output Voltage vs Angle of the Hall Bar sample

The point of maximum Hall voltage was determined to be at 0 degrees and 180 degrees this seperation is as predicted above. This was completed so that in the later sections we only ever take readings where the Hall voltage is maximum to help amplify the effects that we want to observe and to provide a higher hall voltage to work with.

### 6 Calibration of the Electromagnet using the Hall Probe

#### 6.1 Objectives

- To calibrate the electromagnet using the Hall probe.
- To investigate the effect of Hysteresis

#### 6.2 Background Theory

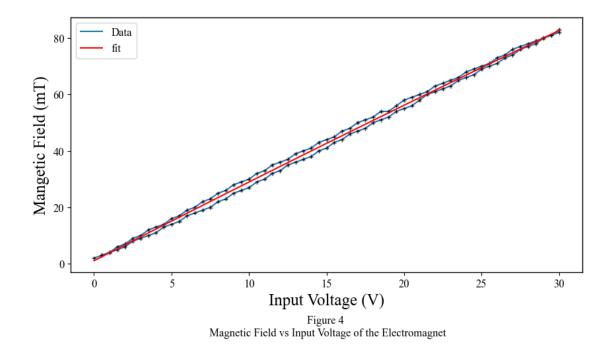
The Voltage and current supplied to the electromagnet is an indirect method of measuring the actual strength of the electromagnet and should be calibrated using a Hall probe or likewise.

Because the electromagnet contains an Iron core the relationship obtained for voltage against magnetic field will not be linear and will have a hysteresis effect (hysteresis is a physical phenomenon in which the physical property of an object has some delay to change compared to the effect causing it, in this case the magnetic field lags behind the changing voltage/current as it is changed up and down).

#### 6.3 Method

The Hall Bar was removed from the electromagnet for this section and the electromagnet was switched on by supplying a voltage for it the voltage was then varied from 0V-30V in 0.5V increments and then decreased in the same increments back to 0V. The magnetic field was measured at each voltage using a Hall probe that had been clamped in the electromagnet and the results can be seen in Figure 4.

```
[]: data =np.loadtxt('data/b_field_calibration.txt', skiprows=2)
    vol = data[:, 0]
    current = data[:, 1]
    b_field = data[:, 2]
    # x and b are in degrees, a is an amplitude constant
    volt_err = 5e-3
    def poly(x, a, b, c):
        return a*x**2 + b*x + c
    pop, pcov = curve_fit(poly, vol, b_field,p0 = (1,1,1),sigma = volt_err*np.
      ones(len(vol)))
    b_err = (np.sqrt(np.diag(pcov))[0])
    vol_range = np.linspace(0, 30, 1000)
    plt.figure(figsize=(10, 5))
    plt.plot(vol, b_field,label='Data')
    plt.errorbar(vol, b_field, yerr=np.sqrt(np.diag(pcov))[0]*100, fmt = __
      plt.errorbar(vol, b_field, xerr=volt_err*np.ones(len(vol)), fmt = 'None',color_
     →= 'black',alpha = 0.5,elinewidth = 1,capsize = 2)
    plt.plot(vol_range, poly(vol_range, *pop), 'r-', label='fit')
    plt.xlabel('Input Voltage (V)')
    plt.ylabel('Mangetic Field (mT)')
    plt.text(15,-25.5, 'Figure 4 \n Magnetic Field vs Input Voltage of the
      →Electromagnet', horizontalalignment='center')
    plt.legend()
    plt.show()
```



# 7 Measurinf the Effect of Temperature on the Hall Voltage and Deteriming Values for the Carrier Density and Mobility

### 7.1 Objectives

- To measure the Hall voltage and longitudinal voltage of the Hall bar for varying input voltages, magnetic fields and at room and liquid nitrogen temperatures.
- To use the calibration of the Electromagnet to determine the magnetic field applied to the Hall bar.
- To determine the effect of temperature on the carrier density and mobility of the 2DEG sample.

#### 7.2 Background Theory

As temperature is decreased it is reasobale to assume the mobility of the electrons will increase as the resistance of the sample will decrease. This is because the electrons will have less thermal energy and will therefore be less likely to collide with the lattice of the semiconductor and the phonons within the lattice. At these low temperature scattering of electrons is primarily due to impurities and defects within the semiconductor. Conversely the carrier density is expected to decrease as the temperature is decreased as the electrons will have less thermal energy and will therefore be less likely to be excited into the conduction band of the semiconductor.

#### 7.3 Method

```
[]: def linear(x, a, b):
        return a*x + b
    hall_data = pd.read_pickle('data/part_D/Hall_data.pickle')
    # print(hall_data)
    files = os.listdir('data/part_D')
    key_list = list(hall_data.keys())
    np.array(key_list)
    mag_volt = []
    mag_volt_li = []
    for i in np.arange(len(key_list)):
        mag_volt.append(float(key_list[i]))
        mag_volt_li.append(key_list[i])
    magvolt = np.array(mag_volt)
    magvolt = np.sort(magvolt)
    ln = 'close to Liquid Nitrogen (78.4K)'
    cold = []
    hot = []
    cold_r = []
    hot_r = []
    colderr = []
    hoterr = []
    cold_rerr = []
    hot_rerr = []
    for file in files:
        volt = []
        current = []
        gradient = []
        errorBI = []
        num = []
        curr = []
        errorlong = []
        if file.endswith('pickle') and file.startswith('Hall'):
            print('----')
            data = pd.read_pickle('data/part_d/'+file)
            plt.figure(figsize=(10,6))
            if file.startswith('Hall_cold'):
                print(f'Hall voltage data at {ln}')
```

```
plt.title(f'Hall Effect at {ln}')
      elif file.startswith('Hall_data'):
           print('Hall voltage data at 300K')
          plt.title('Hall Effect at 300K')
      for i in np.arange(len(mag_volt_li)):
           b_field = (poly(magvolt[i], *pop))*1e-3
           fieldcurrent = b_field * data[(key_list[i])]['supplyCurr']
           error_BI = (np.sqrt(((b_err/b_field)**2)+(((0.005e-6)/

data[(key_list[i])]['supplyCurr'])**2)))*fieldcurrent

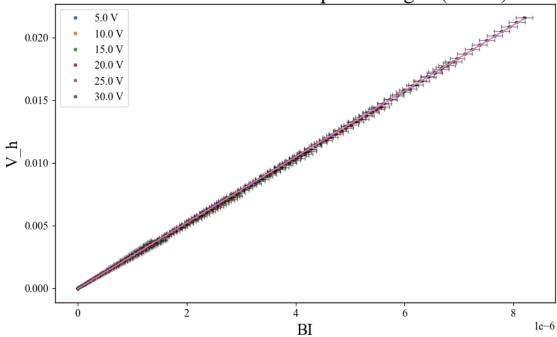
           plt.plot(fieldcurrent,data[(key_list[i])]['hallBarVolt'], '.',__
→label=f'{mag_volt_li[i]} V')
           plt.errorbar(fieldcurrent, data[(key_list[i])]['hallBarVolt'],__
exerr=error_BI, fmt = 'None',color = 'black',alpha = 0.5,elinewidth =
\hookrightarrow1, capsize = 2)
           plt.errorbar(fieldcurrent, data[(key_list[i])]['hallBarVolt'],__
Gyerr=0.005e-3, fmt = 'None',color = 'black',alpha = 0.5,elinewidth = 1
\hookrightarrow 1, capsize = 2)
          plt.xlabel('BI')
          plt.ylabel('V_h')
          plt.legend()
           volt.append(data[(key_list[i])]['hallBarVolt'])
           current.append(fieldcurrent)
           errorBI.append(error_BI)
      volt = np.ndarray.flatten(np.array(volt))
      current = np.ndarray.flatten(np.array(current))
      errorBI = np.ndarray.flatten(np.array(errorBI))
      tot err = np.sqrt((errorBI/current)**2+(0.005e-3/volt)**2)
      popt,pcov = curve_fit(linear, current, volt, p0 = (1,1),sigma = tot_err)
      plt.plot(current, linear(current, *popt), label=f'{mag volt li[i]} V')
      grad_err = np.sqrt(np.diag(pcov))[0]
      gradient = (popt[0])
      num = num_per_sq(gradient)
      num_err = (np.sqrt((grad_err/gradient)**2))*num
      if file.startswith('Hall_cold'):
           print(f'The hall coefficient at {ln} is {gradient} ± {grad_err}_{\sqcup}
→m^3/C')
          print(f'The number of charge carriers per square is {num:4.4g} ±
\rightarrow {num_err:4.4g}')
           cold.append(num)
```

```
colderr.append(num_err)
      elif file.startswith('Hall_data'):
          print(f'The hall coefficient at 300K is {gradient} ± {grad_err}m^3/
GC¹)
          print(f'The number of charge carriers per square is {num:4.4g} ±11
\rightarrow {num_err:4.4g}')
          hot.append(num)
          hoterr.append(num_err)
      plt.show()
  elif file.endswith('pickle') and file.startswith('Long'):
      print('----')
      plt.figure(figsize=(10,6))
      if file.startswith('Long_cold'):
          print(f'Longitudinal data at {ln}')
          plt.title(f'Longitudinal Effect at {ln}')
      elif file.startswith('Long_data'):
          print('Longitudinal data at 300K')
          plt.title('Longitudinal Effect at 300K')
      data = pd.read_pickle('data/part_d/'+file)
      gradient = []
      for i in np.arange(len(mag_volt_li)):
          voltage = data[key_list[i]]['hallBarVolt']
          current = data[key list[i]]['supplyCurr']
          error_long = (np.sqrt(((0.005e-3/voltage)**2)+(((0.005e-6)/voltage)**2))
current)**2)))
          plt.plot(3*current, voltage, '.', label=f'{mag_volt_li[i]} V')
          plt.errorbar(3*current, voltage, xerr=3*current[60:]*1e-2, fmt = 1
plt.errorbar(3*current, voltage, yerr=0.005e-3, fmt = 'None',coloru
→= 'black',alpha = 0.5,elinewidth = 1,capsize = 2)
          plt.xlabel('I')
          plt.ylabel('V')
          plt.legend()
          volt.append(voltage)
          curr.append(current)
          # error2.append(error_hall)
          errorlong.append(error_long)
      volt = np.ndarray.flatten(np.array(volt))
      curr = np.ndarray.flatten(np.array(curr))
      errorlong = np.ndarray.flatten(np.array(errorlong))
      fieldcurrent = np.ndarray.flatten(np.array(fieldcurrent))
```

```
pop_l,pcov = curve_fit(linear, 3*curr, volt, p0 = (1,1),sigma =__
 ⇔errorlong)
        plt.plot(3*curr, linear(3*curr, *pop_1), label='Fit')
        gradient.append(pop_1[0])
        grad_long_err = np.sqrt(np.diag(pcov))[0]
        gradient = np.mean(np.array(gradient))
        if file.startswith('Long_cold'):
            print(f'The Resistance per square at {ln} is {gradient} ±
 →{grad_long_err} ')
            cold_r.append(gradient)
            cold_rerr.append(grad_long_err)
        elif file.startswith('Long data'):
            print(f'The Resistance per square at 300K is {gradient} ±
 →{grad_long_err} ')
            hot_r.append(gradient)
            hot_rerr.append(grad_long_err)
        plt.show()
hot = float(hot[0])
cold = float(cold[0])
hot r = float(hot r[0])
cold_r = float(cold_r[0])
hoterr = float(hoterr[0])
colderr = float(colderr[0])
hot rerr = float(hot rerr[0])
cold_rerr = float(cold_rerr[0])
mu_h = mobility(float(hot), float(hot_r))
mu_c = mobility(cold, cold_r)
mu_h_err = np.sqrt(((hoterr/hot)**2)+((hot_rerr/hot_r)**2))*mu_h
mu_c_err = np.sqrt(((colderr/cold)**2)+((cold_rerr/cold_r)**2))*mu_c
print(f'The mobility at 300K is {mu_h:4.4g} ± {mu_h_err:4.4g}')
print(f'The mobility at {ln} is {mu_c:4.4g} ± {mu_c_err:4.4g}')
```

Hall voltage data at close to Liquid Nitrogen (78.4K)
The hall coefficient at close to Liquid Nitrogen (78.4K) is 2630.0551075677927
± 1.400879552899593 m^3/C
The number of charge carriers per square is 2.373e+15 ± 1.264e+12
C:\Users\lewis\AppData\Local\Temp\ipykernel\_30444\1963443684.py:73:
RuntimeWarning: divide by zero encountered in divide
 tot\_err = np.sqrt((errorBI/current)\*\*2+(0.005e-3/volt)\*\*2)





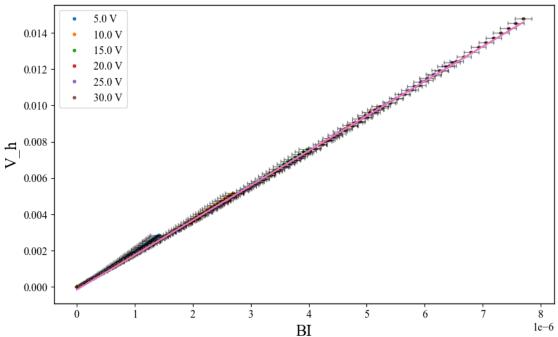
C:\Users\lewis\AppData\Local\Temp\ipykernel\_30444\1963443684.py:73:
RuntimeWarning: divide by zero encountered in divide
 tot\_err = np.sqrt((errorBI/current)\*\*2+(0.005e-3/volt)\*\*2)

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Hall voltage data at 300K

The hall coefficient at 300K is  $1914.9733842462565 \pm 2.135927343450879m^3/C$  The number of charge carriers per square is  $3.259e+15 \pm 3.635e+12$ 

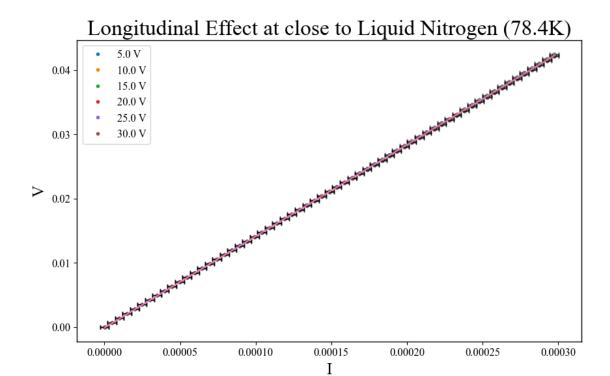
## Hall Effect at 300K



C:\Users\lewis\AppData\Local\Temp\ipykernel\_30444\1963443684.py:110:
RuntimeWarning: divide by zero encountered in divide
 error\_long = (np.sqrt(((0.005e-3/voltage)\*\*2)+(((0.005e-6)/current)\*\*2)))

-----

Longitudinal data at close to Liquid Nitrogen (78.4K) The Resistance per square at close to Liquid Nitrogen (78.4K) is  $142.53188627819716 \pm 0.10547294523892414$ 

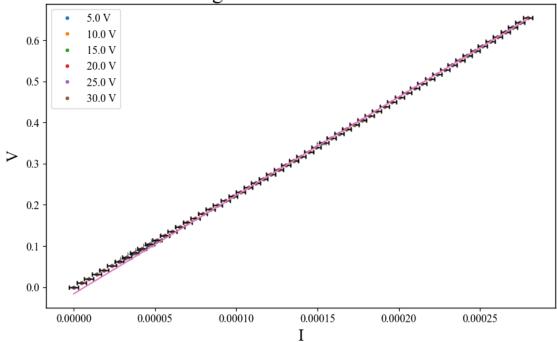


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Longitudinal data at 300K

The Resistance per square at 300K is 2391.7751171731725  $\pm$  1.8281316267225955

# Longitudinal Effect at 300K



The mobility at 300K is 0.8006  $\pm$  0.001083 The mobility at close to Liquid Nitrogen (78.4K) is 18.45  $\pm$  0.01682

	Hall Coefficient $m^3C^{-1}$	Resistance per Square $\Omega m^{-2}$	Carrier per Square $m^{-2}$	Mobility $m^2V^{-1}S^{-1}$
Room temperature 295.1K	$1914.973\pm2.135$	$2391.775 \pm 1.828$	$3.259\mathrm{e}{+15} \pm 0.004\mathrm{e}{+15}$	$0.8006 \pm 0.0011$
Liquid Nitrogen Temperature 78.4K	$2630.055 \pm 1.401$	$142.532 \pm 0.105$	$2.373\mathrm{e}{+15} \pm 0.001\mathrm{e}{15}$	$18.45 \pm 0.02$

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