

# analogue\_filters

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## 1 Analogue Filters

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### 1.1 Abstract

This record details the procedure, results and analysis of several different analogue filters, starting with single pole low pass and high pass filters analysed with an oscilloscope and frequency response analyser, followed by cascaded filters using an operational amplifier as a buffer in between, analysis of a non inverting amplifier and finally an LCR filter with comparable frequency responses allowing for the comparison of the LCR circuit to the single pole low pass filter and the two cascaded low pass filters.

### 1.2 Introduction and Background

Analogue filters are useful control systems that depending on their construction can filter about frequencies of a given value within a circuit this is desirable in a field such as audio technology in which one may want to remove a given frequency range from a speaker input to emphasise other frequencies such that used to create subwoofer or tweeter speakers. This technique can be used more generally to remove noise from a signal particularly noise in the low or high frequency ranges. In this experiment we will be looking at the construction of several different analogue filters and analysing their frequency responses as well as the effects of cascading filters and the use of operational amplifiers as buffers.

### 1.3 Aims of this experiment

The aims of this experiment are to: 1. Derive the transfer function of a single pole low and high pass filter, construct the two circuits and measure the gain and phase response as a function of frequency, and plot these results on bode and nyquist plots. 2. Calculate and obtain an understanding of the corner frequency and slope of the gain response for single and multi pole filters. 3. Construct cascaded active filters using an operational amplifier as a buffer and analyse the frequency response of the cascaded filters. 4. Construct resonant filters (LCR circuits) and analyse the frequency response of the circuit and comparing it to the single pole low pass filter and the cascaded filters.

### 1.4 Equipment

Equipment used in this experiment includes: - Breadboard - MOKU:GO and the MOKU:Lab software - Resistors of various values - Capacitors of various values - Inductor of value  $1\mu H$  - Operational amplifier (TL072) - Multimeter

## 1.5 Task 1 - Single pole low pass filter

### 1.5.1 Task 1.1 - Deriving the transfer function of a single pole low pass filter

To calculate the transfer function ( $G$ ) of a single pole low pass filter we can start from the definition of the transfer function:

$$G = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

and using the circuit diagram of a single pole low pass filter we can see that:

$$Z_1 = R$$
$$Z_2 = \frac{1}{j\omega C}$$

where  $j$  is the imaginary unit and  $\omega$  is the angular frequency of the input signal. Substituting these values into the transfer function we get:

$$G = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

and simplifying this expression we get:

$$G = \frac{1}{j\omega RC + 1}$$

The gain of the circuit is the magnitude of the transfer function and the phase is the argument of the transfer function. The gain is given by:

$$|G| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

and the phase is given by:

$$\angle G = -\arctan(\omega RC)$$

### 1.5.2 Task 1.2 - Constructing the single pole low pass filter

The circuit diagram for the single pole low pass filter is shown below:

using this circuit diagram the single pole low pass filter was constructed on a breadboard picking resistance and capacitance values to give a corner frequency as close to 10kHz as possible (these values were  $R = 15k\Omega$  and  $C = 0.82nF$ ). Thus giving a theoretical corner frequency of

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi * 1.5 \times 10^4 * 0.82 \times 10^{-9}} = 12.9k\Omega$$

### 1.5.3 Task 1.3 - Measuring and analysing the gain and phase response of the single pole low pass filter

To measure the gain and phase response of the single pole low pass filter the MOKU:GO was used with the MOKU:LAB software. The output from this was a peak to peak voltage of the sine wave and a frequency range that the sine wave was swept over as well as a phase offset that was precalculated for each value of frequency by the MOKU:LAB software. These values were obtained using the measurements tab of the software and achieves the same result as reading off the graph. These values were then imported into python and plotted on a bode and nyquist plot. The code to do this is shown below along with the output graphs.

```
[ ]: # Import packages and libraries that will be used throughout the experiment
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy.optimize import curve_fit
from scipy import interpolate
import os
import pandas as pd
import csv
import re
import control
from scipy import signal

plt.style.use('../report.mplstyle') # define a style sheet to use for all plots

[ ]: files = os.listdir('data/') #creates a list of all files in the directory

for file in files: #loops through all files in the directory
    if file.endswith('.csv'): #filters any .csv files into a seperate folder
        os.rename('data/'+file, 'data/moku_csv/'+file)

    elif file.endswith('.png'): #filters any .png files into a seperate folder
        os.rename('data/'+file, 'data/moku_images/'+file)

def float_array_convert(array): #converts strings to floats in an array
    return np.array([float(i) for i in array])

lp_data = pd.read_csv('data/csv/lp_data.csv',sep = ',') #reads in the csv files
    ↳for the low pass and high pass filters
hp_data = pd.read_csv('data/csv/hp_data.csv',sep = ',')

lp_peak_to_peak, lp_phase, lp_freq = lp_data['peaktopeak'], lp_data['phase'],
    ↳lp_data['frequency'] #assigns the columns of the data to variables
hp_peak_to_peak, hp_phase, hp_freq = hp_data['peaktopeak'], hp_data['phase'],
    ↳hp_data['frequency']
```

```

lp_peak_to_peak, lp_phase, lp_freq = float_array_convert(lp_peak_to_peak),
↳float_array_convert(lp_phase), float_array_convert(lp_freq) #converts the
↳strings to floats in each array
hp_peak_to_peak, hp_phase, hp_freq = float_array_convert(hp_peak_to_peak),
↳(float_array_convert(hp_phase))-np.pi/2, float_array_convert(hp_freq)

```

```

[ ]: def arctan(x,r,c): #function to calculate the phase of a filter
    return -np.arctan(x*r*c)

def corner_freq(x,y): #function to calculate the corner frequency of a filter
↳which is defined as the point where the gain is -3dB
    return np.full_like(y,x[np.argmin(np.abs(y+3))])

def gradient(x,y): #function to calculate the gradient of a graph
    return np.gradient(x,y)

lp_resistance = 15000 #resistance and capacitance values for the low pass filter
lp_capacitance = 0.82e-9

log_lp_gain = 20*np.log10(lp_peak_to_peak)
↳#calculates the gain in dB for the low pass filter
lp_omega = (lp_freq*2*np.pi)
↳#converts the frequency to angular frequency
log_lp_omega = np.log10(lp_omega)
↳#calculates the log of the angular frequency
log_lp_theo_gain = 20*np.log10(1/np.
↳sqrt(1+(lp_omega*lp_resistance*lp_capacitance)**2)) #calculates the
↳theoretical gain in dB for the low pass filter
lp_theo_phase = arctan(lp_omega,lp_resistance,lp_capacitance)
↳#calculates the theoretical phase for the low pass filter
lp_corner_freq = corner_freq(lp_freq,log_lp_gain)
↳#calculates the corner frequency for the low pass filter

plt.figure(figsize=(10,10))
↳#plots the low pass filter bode plots of gain and phase
plt.subplot(2,1,1)
plt.plot(lp_freq,log_lp_gain, label = 'Experimental', color = 'blue')
plt.plot(lp_freq,log_lp_theo_gain, label = 'Theoretical', color = 'orange',
↳linestyle = ':', linewidth = 2.5)
plt.plot(lp_corner_freq,log_lp_gain, label = f'Corner Frequency
↳{lp_corner_freq[0]:.2f}Hz', color = 'red', linestyle = '--', linewidth = 1.5)
plt.annotate(f'Gradient = {gradient(log_lp_gain,log_lp_omega)[-1]:.2f}dB/
↳decade', xy = (lp_freq[-2],log_lp_gain[-2]), arrowprops=dict(facecolor =
↳'black', shrink = 0.05,), xytext = (lp_freq[-2]-90000,log_lp_gain[-2]-4.3))
plt.xlabel(r'Frequency (Hz)')

```

```

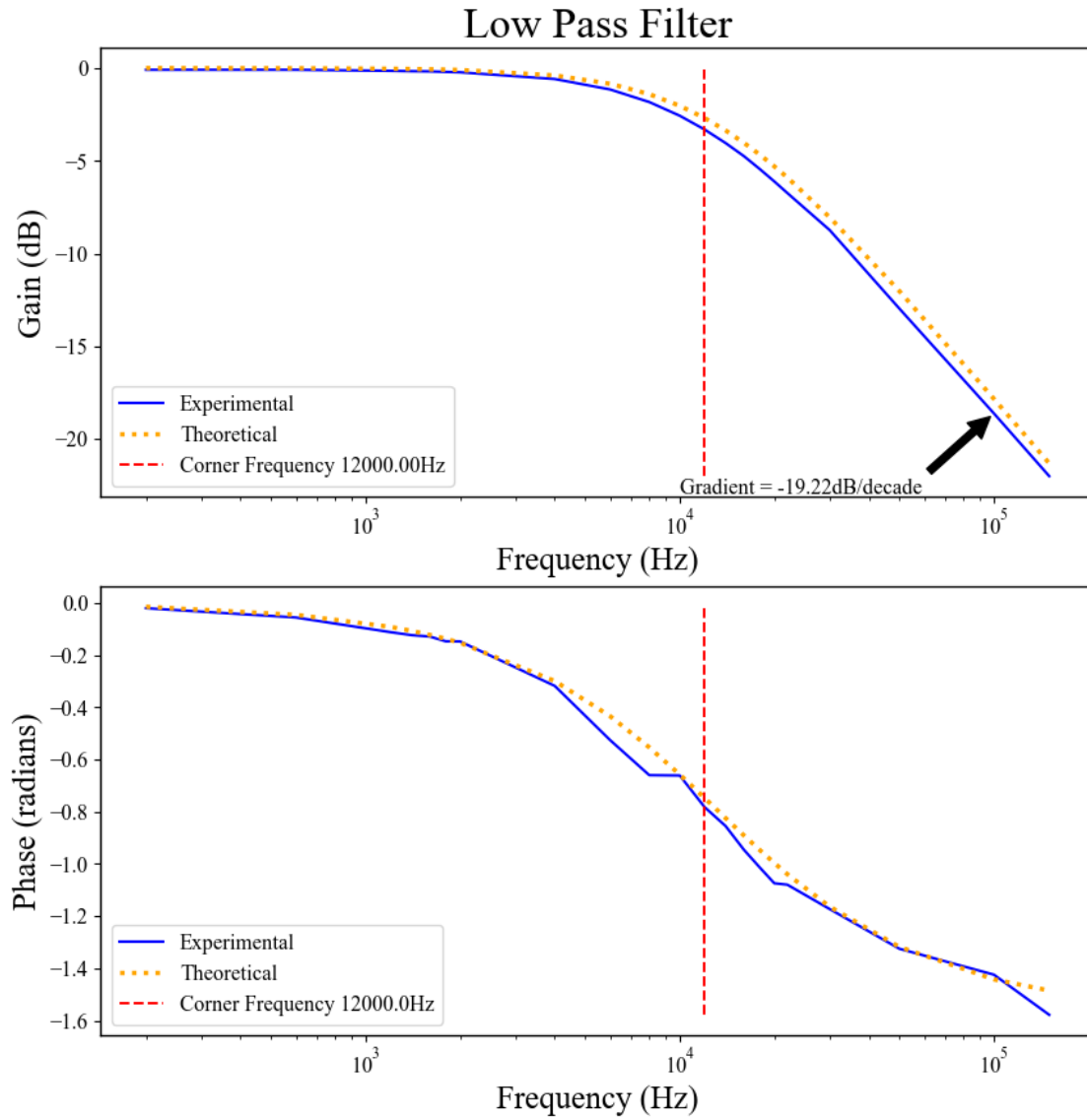
plt.xscale('log')
plt.ylabel('Gain (dB)')
plt.title('Low Pass Filter')
plt.legend()
plt.subplot(2,1,2)
plt.plot(lp_freq,lp_phase, label = 'Experimental', color = 'blue')
plt.plot(lp_freq,lp_theo_phase, label = 'Theoretical', color = 'orange',
    ↳linestyle = ':', linewidth = 2.5)
plt.plot(lp_corner_freq,lp_phase, label = f'Corner Frequency
    ↳{lp_corner_freq[0]}Hz', color = 'red', linestyle = '--', linewidth = 1.5)
plt.xlabel(r'Frequency (Hz)')
plt.xscale('log')
plt.ylabel('Phase (radians)')
plt.legend(loc= 3)
plt.show()

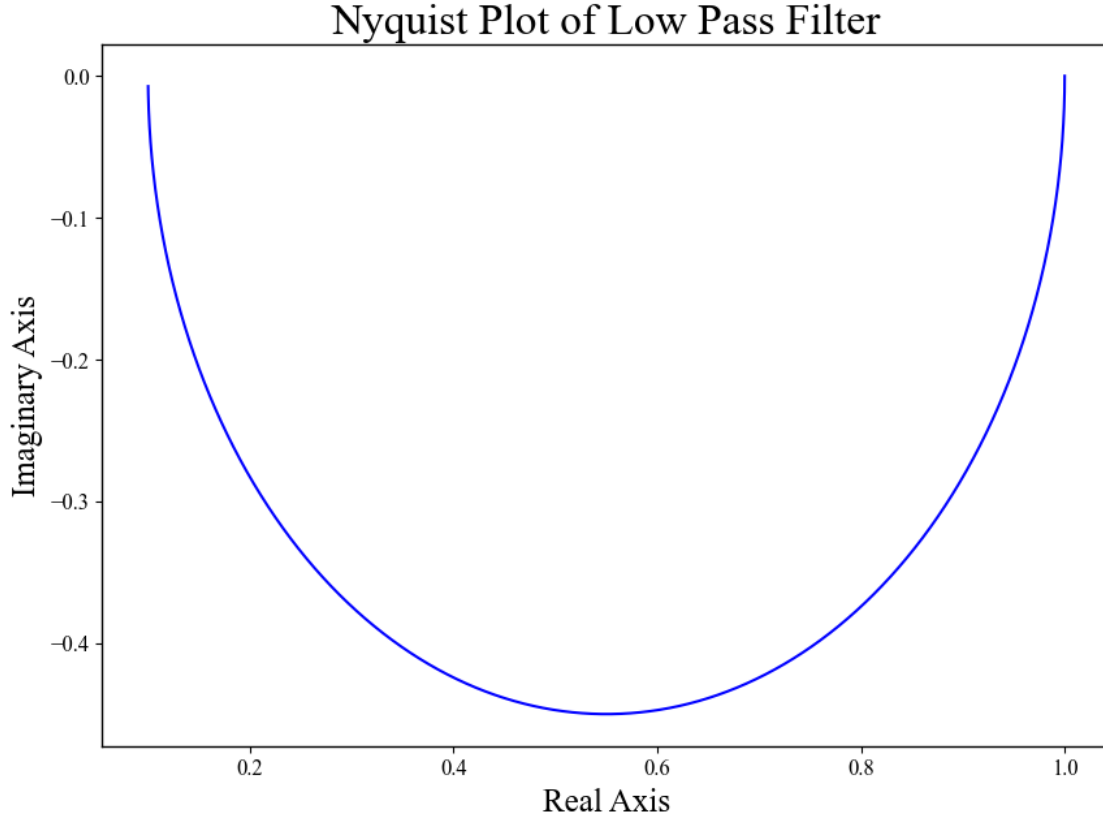
                                # Nyquist Plot of Low Pass Filter
tau = lp_resistance*lp_capacitance;
a = 10;
lp_trans_func = signal.lti([tau, 1],[a*tau, 1])                                #Creating
    ↳Transfer Function for a Linear Time Independent system
w= np.linspace(1e-4,1e6,1000000)                                                #Creating a
    ↳range of frequencies to plot the frequency response

w, H = signal.freqresp(lp_trans_func,w)
plt.figure(figsize=(10,7))
plt.plot(H.real, H.imag, "b")
plt.xlabel("Real Axis")
plt.ylabel("Imaginary Axis")
plt.title("Nyquist Plot of Low Pass Filter")

plt.show()

```





The graph for the gain response of the low pass filter against frequency shows a clear resemblance of the theoretical data with a slight horizontal offset this is due to the experimental data falling away from a gain of 0dB early due to some uncertainty in the component values and the inability to be able to achieve a perfect gain of 0dB up until the corner frequency. The corner frequency as calculated from the experimental data is  $f_c = 12k\Omega$  which is very close to the theoretical value of  $f_c = 12.9k\Omega$ . This discrepancy is due to both uncertainties in the component values, some internal resistance of the circuit that isn't being measured as well as the fact that not enough data points were taken around this corner frequency to get a smoother curve and a more accurate value. Finally for this graph the slope of the gain response past the corner frequency is  $-19.22dB/decade$  which is very close to the theoretical value of  $-20dB/decade$  and once again this is due to the same reasons as the discrepancy in the corner frequency. The phase, frequency plot for the low pass filter is very accurate apart from not being a nice continuous curve due to not enough data points being taken, but this would easily be improved upon had more time been dedicated to this section. And finally the nyquist plot for the low pass filter follows the shape that would be expected for a low pass filter with the phase starting at 0 and decreasing to  $-\pi$  as the frequency increases.

## 1.6 Task 2 - Single pole high pass filter

### 1.6.1 Task 2.1 - Deriving the transfer function of a single pole high pass filter

To calculate the transfer function ( $G$ ) of a single pole high pass filter we can start from the definition of the transfer function:

$$G = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

and using the circuit diagram of a single pole high pass filter we can see that:

$$Z_1 = \frac{1}{j\omega C}$$

$$Z_2 = R$$

where  $j$  is the imaginary unit and  $\omega$  is the angular frequency of the input signal. Substituting these values into the transfer function we get:

$$G = \frac{R}{j\omega RC + 1}$$

The gain of the circuit is the magnitude of the transfer function and the phase is the argument of the transfer function. The gain is given by:

$$|G| = \frac{R}{\sqrt{1 + (\omega RC)^2}}$$

and the phase is given by:

$$\angle G = -\arctan(\omega RC)$$

### 1.6.2 Task 2.2 - Constructing the single pole high pass filter

The circuit diagram for the single pole high pass filter is shown below:

using this circuit diagram the single pole high pass filter was constructed on a breadboard picking resistance and capacitance values to give a corner frequency as close to 1kHz as possible (these values were  $R = 156k\Omega$  and  $C = 0.82nF$ ). Thus giving a theoretical corner frequency of

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi * 1.56 \times 10^5 * 0.82 \times 10^{-9}} = 1.24k\Omega$$

### 1.6.3 Task 2.3 - Measuring and analysing the gain and phase response of the single pole high pass filter

To measure the gain and phase response of the single pole high pass filter the same method was employed as with section 1.3. The code to do this is shown below along with the output graphs.

```
[ ]: hp_resistance = 156000 # resistance and capacitance values for the high pass filter
      hp_capacitance = 1e-9
```



```

log_hp_gain = 20*np.log10(hp_peak_to_peak)
    ↪ #calculates the gain in dB for the high pass filter
hp_omega = (hp_freq*2*np.pi)
    ↪ #converts the frequency to angular frequency
log_hp_omega = np.log10(hp_omega)
    ↪ #calculates the log of the angular frequency
log_hp_theo_gain = 20*np.log10(hp_omega*hp_resistance*hp_capacitance/(np.
    ↪ sqrt(1+(hp_omega*hp_resistance*hp_capacitance)**2))) #calculates the
    ↪ theoretical gain in dB for the high pass filter
hp_theo_phase = arctan(hp_omega, hp_resistance, hp_capacitance)
    ↪ #calculates the theoretical phase for the high pass filter
hp_corner_freq = corner_freq(hp_freq, log_hp_gain)
    ↪ #calculates the corner frequency for the high pass filter

plt.figure(figsize=(10,10))
    ↪ #plots the high pass filter bode plots of gain and phase
plt.subplot(2,1,1)
plt.plot(hp_freq, log_hp_gain, label = 'Experimental', color = 'blue')
plt.plot(hp_freq, log_hp_theo_gain, label = 'Theoretical', color = 'orange',
    ↪ linestyle = ':', linewidth = 2.5)
plt.plot(hp_corner_freq, log_hp_gain, label = f'Corner Frequency
    ↪ {hp_corner_freq[0]:.2f}rad/s', color = 'red', linestyle = '--', linewidth =
    ↪ 1.5)
plt.annotate(f'Gradient = {gradient(log_hp_gain, log_hp_omega)[0]:.2f}dB/
    ↪ decade', xy = (hp_freq[1]-0.165, log_hp_gain[1]-2.5),
    ↪ arrowprops=dict(facecolor = 'black', shrink = 0.05, ), xytext = (hp_freq[1]+0.
    ↪ 1, log_hp_gain[1]-5.4))
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.ylabel('Gain (dB)')
plt.title('High Pass Filter')
plt.legend()

plt.subplot(2,1,2)
plt.plot(hp_freq, hp_phase, label = 'Experimental', color = 'blue')
plt.plot(hp_freq, hp_theo_phase, label = 'Theoretical', color = 'orange',
    ↪ linestyle = ':', linewidth = 2.5)
plt.plot(hp_corner_freq, hp_phase, label = f'Corner Frequency {hp_corner_freq[0]:
    ↪ .2f}rad/s', color = 'red', linestyle = '--', linewidth = 1.5)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.ylabel('Phase (radians)')
plt.legend()
plt.show()

```

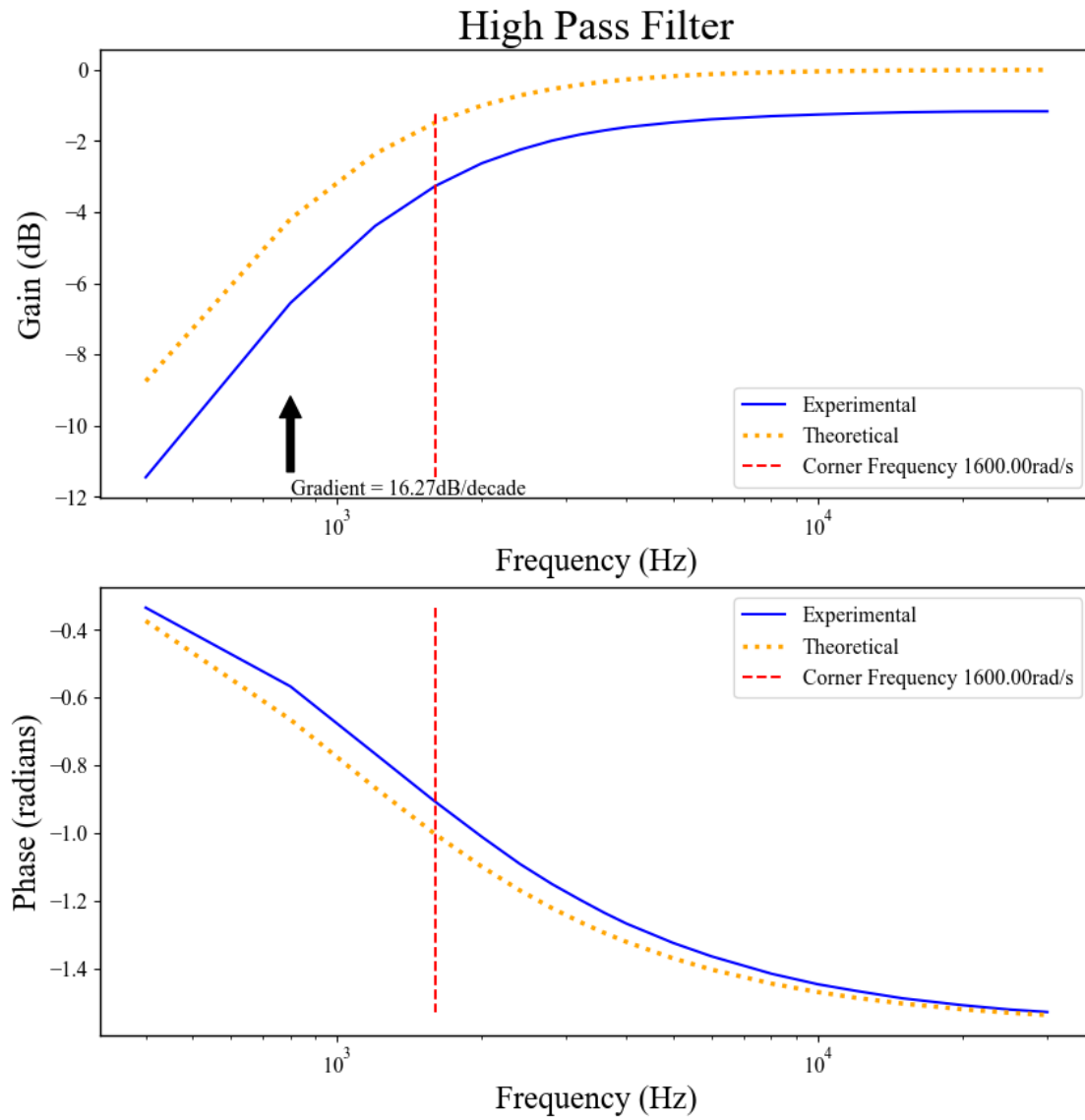
```

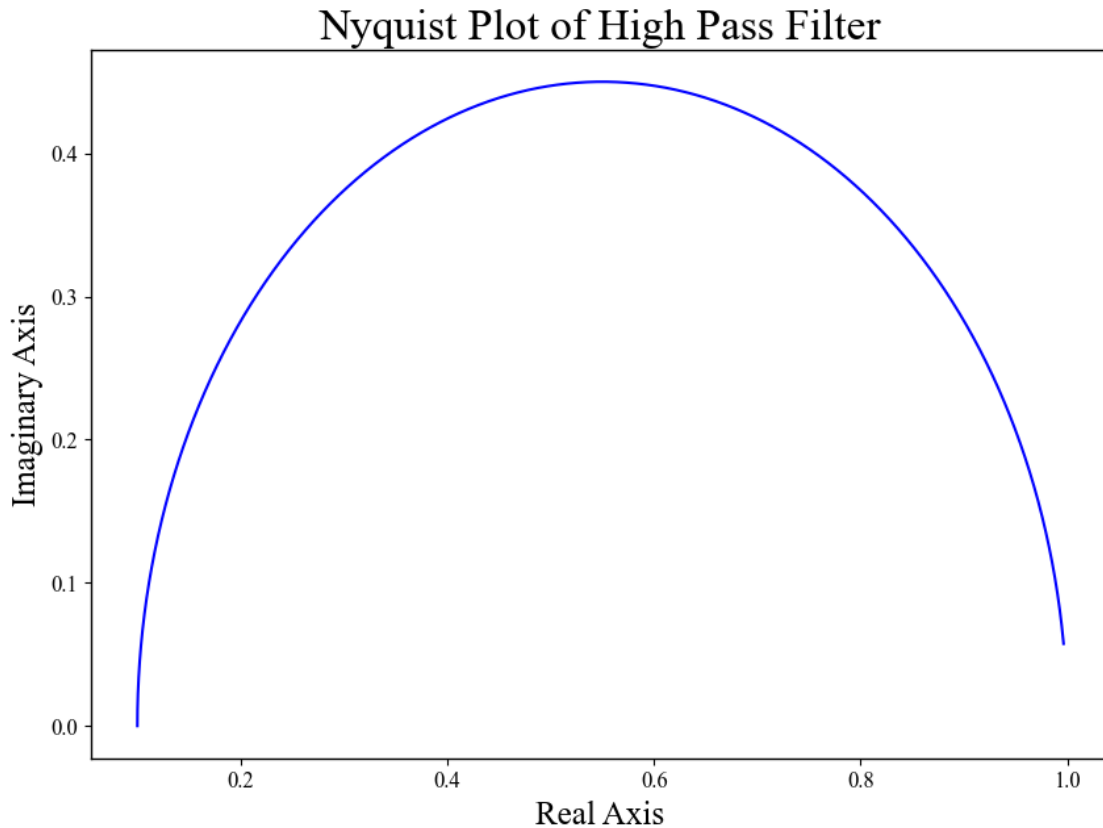
# Nyquist Plot of High Pass
↳Filter
tau = hp_resistance*hp_capacitance;
a = 10;
hp_trans_func = signal.lti([tau, 1],[tau, a]) #Creating
↳Transfer Function for a Linear Time Independent system
w= np.linspace(1e-4,1e6,1000000) #Creating a
↳range of frequencies to plot the frequency response

w, H = signal.freqresp(hp_trans_func,w)
plt.figure(figsize=(10,7))
plt.plot(H.real, H.imag, "b")
plt.xlabel("Real Axis")
plt.ylabel("Imaginary Axis")
plt.title("Nyquist Plot of High Pass Filter")

plt.show()

```





#### 1.6.4 Task 2.4 - Obtaining a calibration curve for the Frequency response analyser of the MOKU:GO

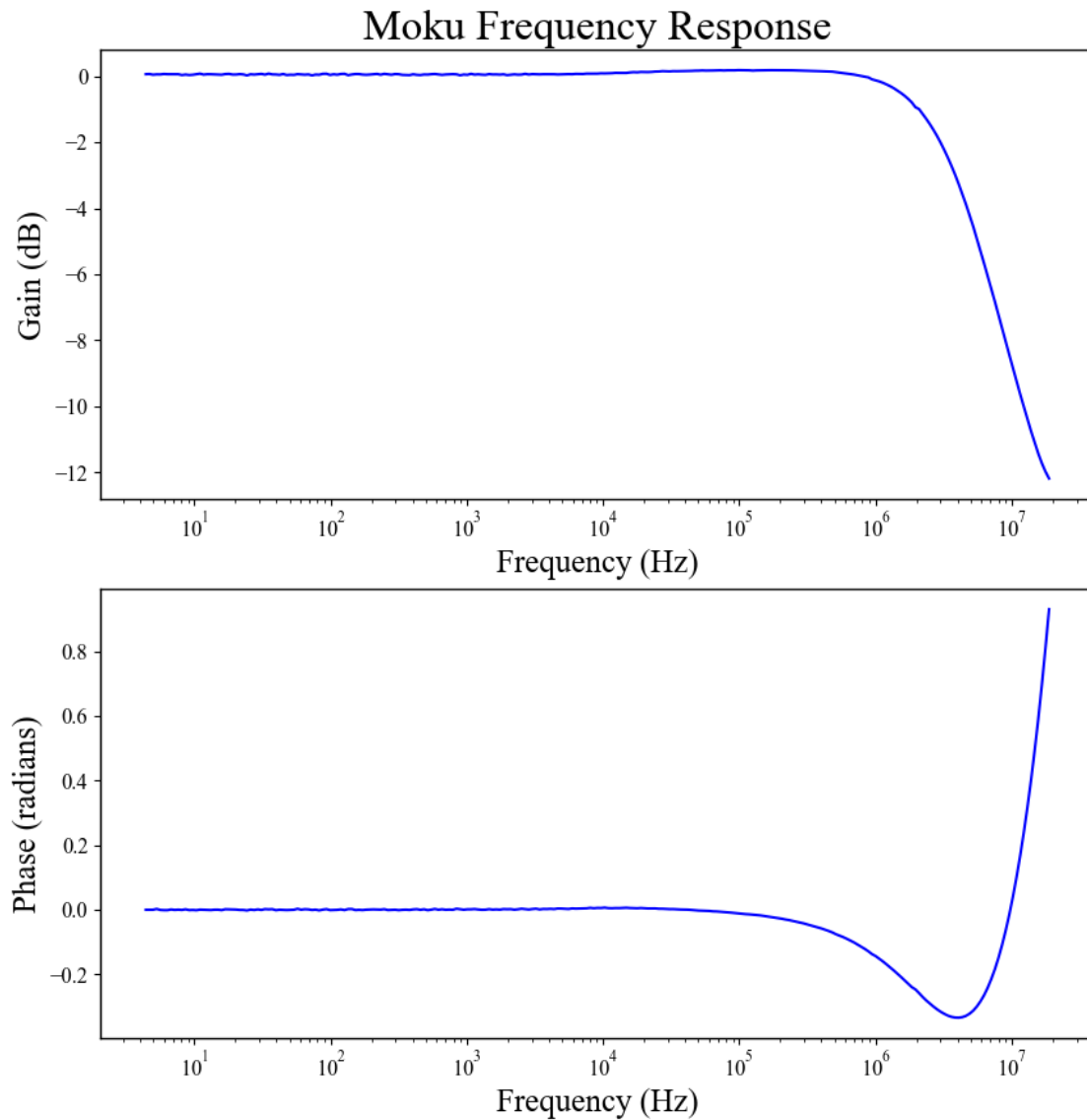
```
[ ]: def load_data(file):    #function to load in the data from a csv file
    data = pd.read_csv(file, delimiter = ',', skiprows = 2)
    data.columns = ['freq', 'gain', 'phase']
    data['phase'] = float_array_convert(data['phase'])*(np.pi/180)
    return data

moku_feedback = load_data('data/moku_csv/moku_itself_data.csv') #loads in the
    ↳data from the moku device that is used to measure any errors that the moku
    ↳device itself introduces

plt.figure(figsize=(10,10))                                     #plots the bode
    ↳plots of gain and phase for the moku device
plt.subplot(2,1,1)
plt.plot((moku_feedback['freq']),moku_feedback['gain'], color = 'blue')
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.ylabel('Gain (dB)')
```

```
plt.title('Moku Frequency Response')

plt.subplot(2,1,2)
plt.plot((moku_feedback['freq']),moku_feedback['phase'], color = 'blue')
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.ylabel('Phase (radians)')
plt.show()
```

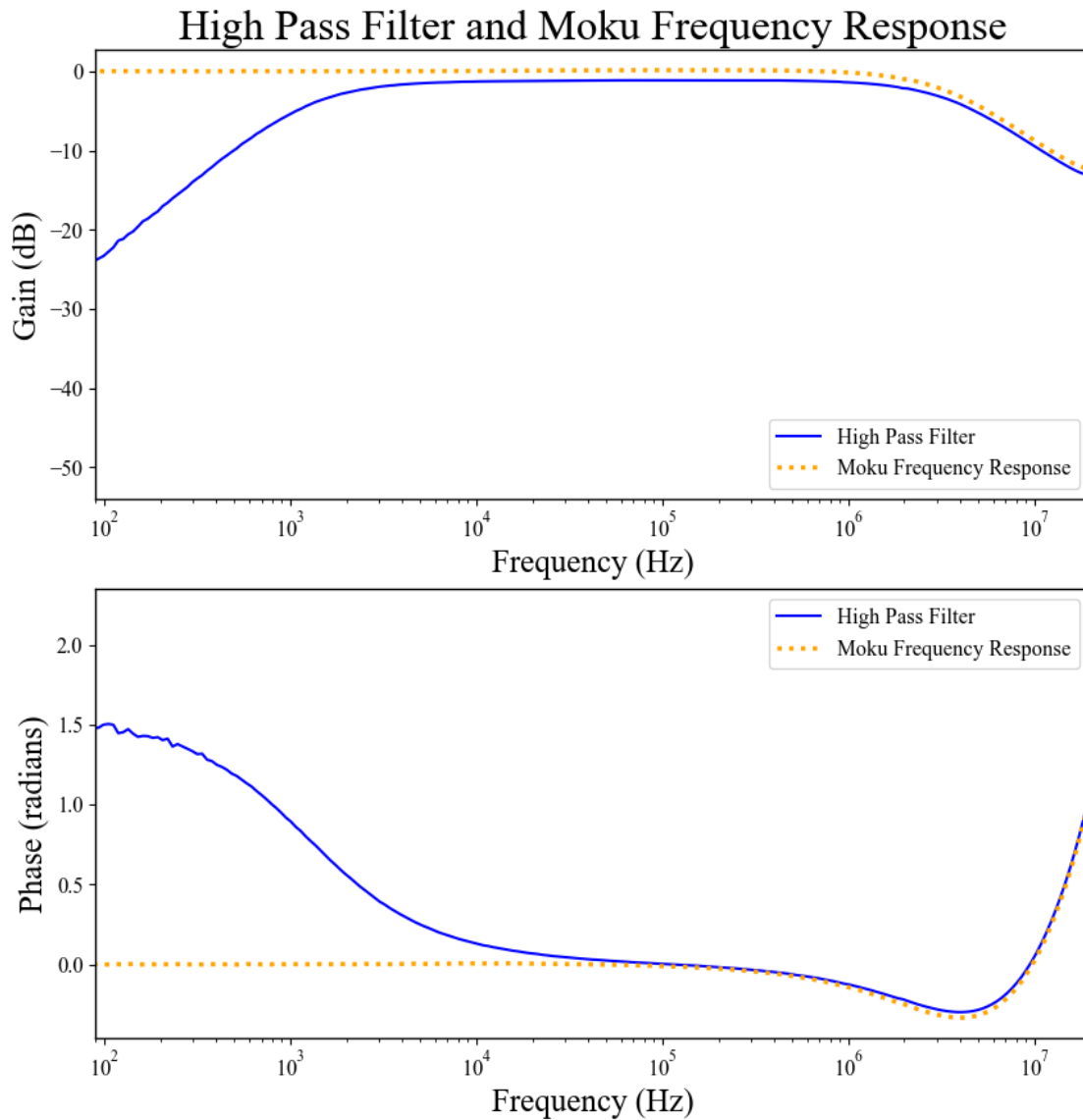


### 1.6.5 Task 2.5 - Comparing the single pole high pass filter analysis using the Oscilloscope and the Frequency Response Analyser

```
[ ]: hp_freq_res = load_data('data/moku_csv/HP_freq_res_data.csv') #loads in the
    ↪data from the moku device that is used to measure the frequency response of
    ↪the high pass filter

plt.figure(figsize=(10,10)) #plots the bode
    ↪plots of gain and phase for the high pass filter and the moku device
plt.subplot(2,1,1)
plt.plot((hp_freq_res['freq']),hp_freq_res['gain'], label = 'High Pass Filter',
    ↪color = 'blue')
plt.plot((moku_feedback['freq']),moku_feedback['gain'], label = 'Moku Frequency
    ↪Response', color = 'orange', linestyle = ':', linewidth = 2.5)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(9e1,2e7)
plt.ylabel('Gain (dB)')
plt.title('High Pass Filter and Moku Frequency Response')
plt.legend(loc =4)

plt.subplot(2,1,2)
plt.plot((hp_freq_res['freq']),hp_freq_res['phase'], label = 'High Pass
    ↪Filter', color = 'blue')
plt.plot((moku_feedback['freq']),moku_feedback['phase'], label = 'Moku
    ↪Frequency Response', color = 'orange', linestyle = ':', linewidth = 2.5)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(9e1,2e7)
plt.ylabel('Phase (radians)')
plt.legend()
plt.show()
```



```
[ ]: Path = 'data/moku_csv/'           #path to the directory of the csv files
plt.figure(figsize=(15,15))
files = os.listdir(Path)               #creates a list of all files in the
    ↪directory

for file in files:                     #loops through all files in the
    ↪directory
    if re.search('high',file):         #filter to only include the cascade low
    ↪and high pass filter files

        data = load_data(Path+file)
```

```

labels = (pd.read_csv(Path+file,nrows = 0)).columns[0]

plt.subplot(2,2,1)                                #plots the bode plots of gain and phase
↳ for the cascaded low and high pass filter
plt.plot((data['freq']),data['gain'], label = labels)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(2e2,6e4)
plt.ylim(-60,0)
plt.ylabel('Gain (dB)')
plt.title('Cascaded Low and High Pass Filters')
plt.legend()

plt.subplot(2,2,3)
plt.plot((data['freq']),data['phase'], label = labels)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(2e2,6e4)
plt.ylim(-np.pi/2,np.pi/2)
plt.ylabel('Phase (radians)')
plt.legend()

elif file.startswith('casc_lowpass'): #filter to only include the 2
↳ cascaded low pass filter files

data = load_data(Path+file)
labels = (pd.read_csv(Path+file,nrows = 0)).columns[0]

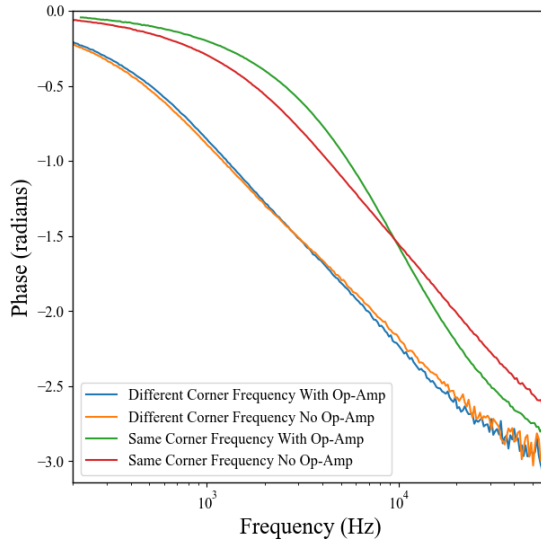
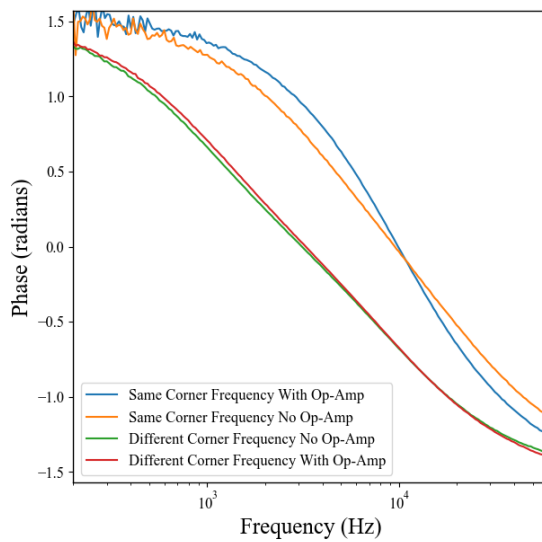
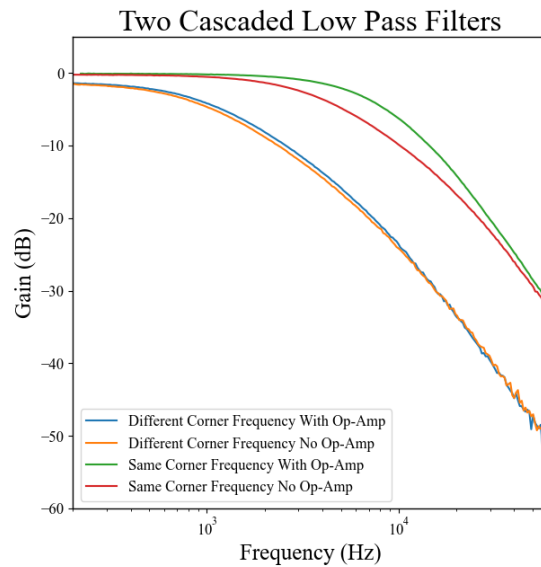
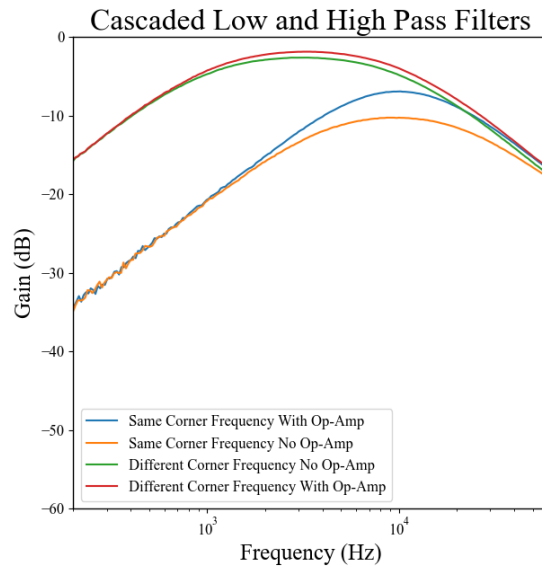
plt.subplot(2,2,2)                                #plots the bode plots of gain and
↳ phase for the 2 cascaded low pass filter
plt.plot((data['freq']),data['gain'], label = labels)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(2e2,6e4)
plt.ylim(-60,5)
plt.ylabel('Gain (dB)')
plt.title('Two Cascaded Low Pass Filters')
plt.legend()

plt.subplot(2,2,4)
plt.plot((data['freq']),data['phase'],label = labels)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(2e2,6e4)
plt.ylim(-np.pi,0)
plt.ylabel('Phase (radians)')

```



```
plt.legend()
plt.show()
```



```
[ ]: plt.figure(figsize=(15,15))
files = os.listdir(Path)

for file in files:
    if file.startswith('non'):
        #loops through all files in the directory
        #filter to only include the non-
        ⇨ inverting amplifier files

    data = load_data(Path+file)
    labels = (pd.read_csv(Path+file,nrows = 0)).columns[0]
```

```

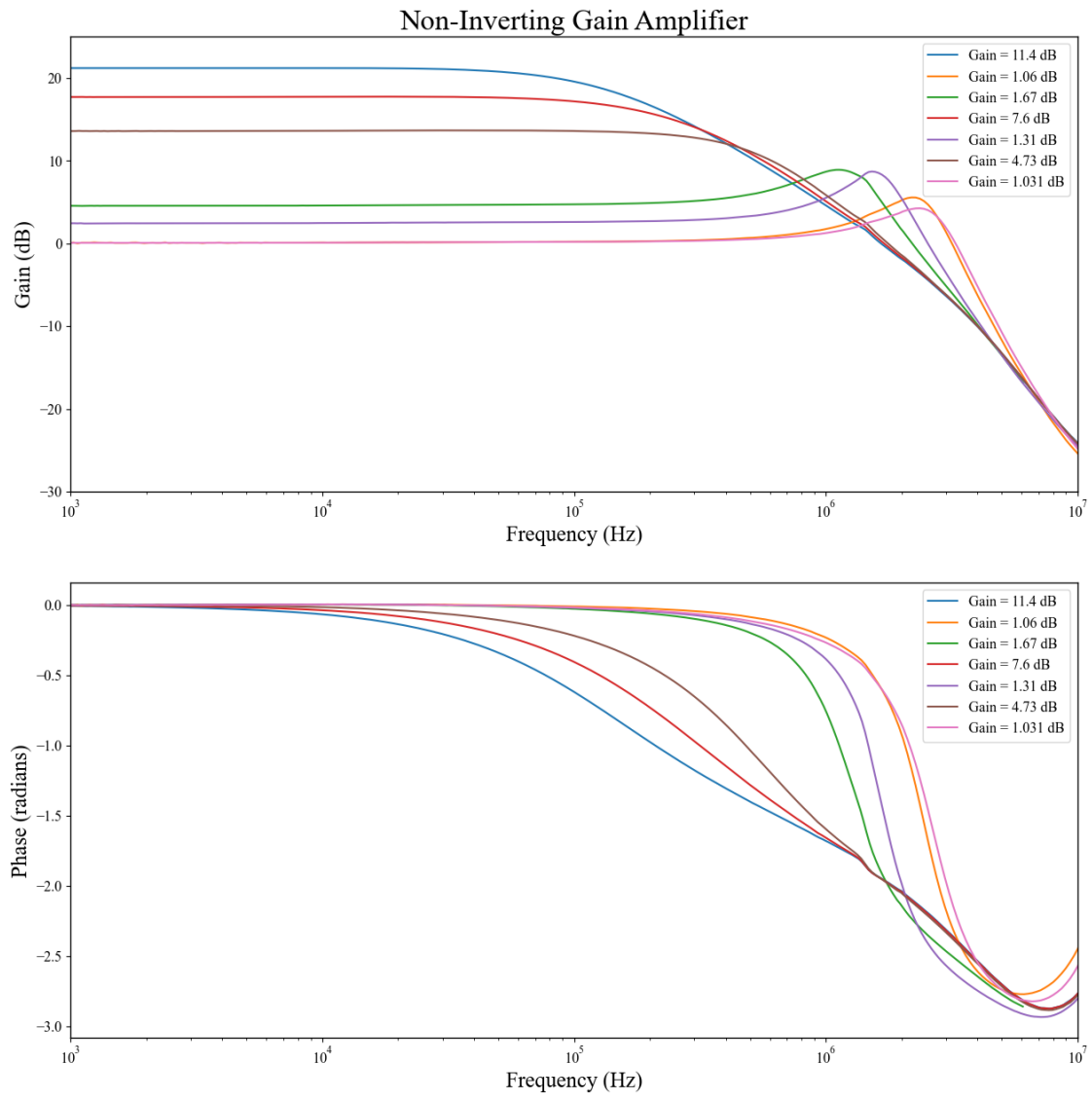
plt.subplot(2,1,1)                                #plots the bode plots of gain and phase
↳for the non inverting amplifier
plt.plot((data['freq']),data['gain'], label = labels)
plt.xlabel('Frequency (Hz)')
plt.xscale('log')
plt.xlim(1e3,1e7)
plt.ylim(-30,25)
plt.ylabel('Gain (dB)')
plt.title('Non-Inverting Gain Amplifier')
plt.legend()

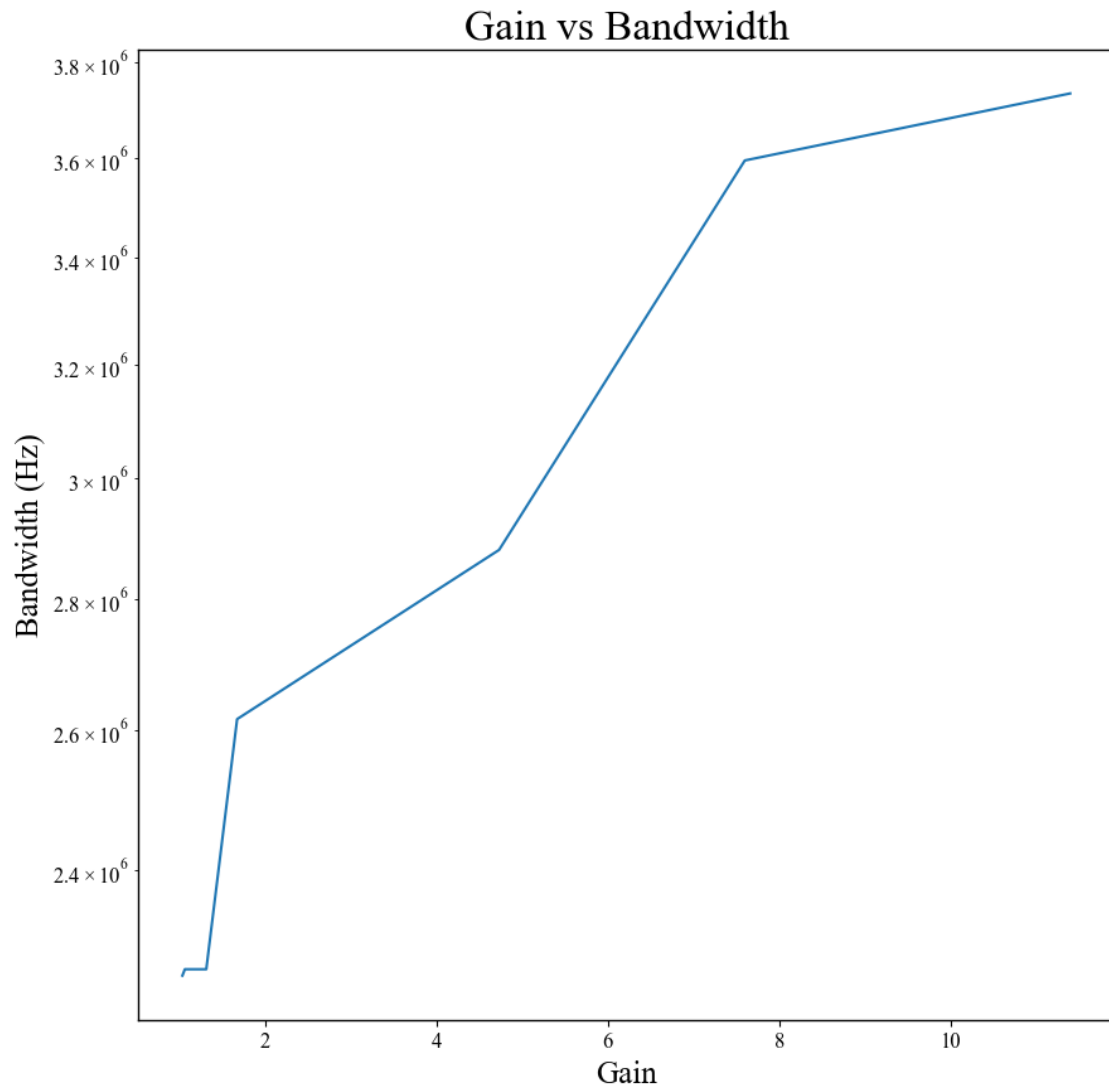
plt.subplot(2,1,2)
plt.plot((data['freq']),data['phase'], label = labels)
plt.xlabel('Frequency (Hz)')
plt.xlim(1e3,1e7)
plt.xscale('log')
plt.ylabel('Phase (radians)')
plt.legend()

def bandwidth_calc(data):                          #function to calculate the bandwidth of
↳a non inverting amplifier
    return data['freq'][np.argmin(np.abs(data['gain']+3))]
bandwidth = []
gain = [11.4,1.06,1.67,7.6,1.31,4.73,1.031]
for file in files:                                #loops through all files in the
↳directory
    if file.startswith('non'):                    #filter to only include the non
↳inverting amplifier files
        data = load_data(Path+file)
        bandwidth.append(bandwidth_calc(data))

plt.figure(figsize=(10,10))                        #plots the bandwidth against the gain
↳for the non inverting amplifier
plt.plot(np.sort(gain),np.sort(bandwidth))
plt.xlabel('Gain')
plt.ylabel('Bandwidth (Hz)')
plt.yscale('log')
plt.title('Gain vs Bandwidth')
plt.show()

```





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