

FUNCTIONAL PROGRAMMING IN

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SCALA

## INTRODUCTION

- ▶ The Boost libraries helped develop C++ as a language and helped drive language design.
- ▶ The Typelevel project ([typelevel.org](https://typelevel.org)) has the same effect for functional programming within Scala. Core libraries:
  - ▶ **Cats**, a library for common functional programming abstractions.
  - ▶ **Shapeless**, a high level (functional) generic programming library.
- ▶ Functional programming (FP) has design patterns like 'GoF', in FP these are rooted in category theory & abstract algebra (The theory isn't required!).
- ▶ Little 'OO' design in modern Scala libraries (same could be said for many of the modern 'generic' C++ libraries?)

# LEARNING MATERIALS

- ▶ “Functional Programming in Scala”, Paul Chiusano & Rúnar Bjarnason, Manning.
- ▶ Scala exercises website: “*The path to enlightenment*” (<https://www.scala-exercises.org>)
- ▶ A modern framework for FP applications (<http://frees.io>)
- ▶ Introduction to Scala ([allaboutscala.com](http://allaboutscala.com))

# INTRODUCTION

- ▶ Cats, Shapeless and other libraries provide the functional programming building blocks. These include:
  - ▶ Type Class
  - ▶ Functor
  - ▶ Applicative
  - ▶ Monoid & Semigroup
  - ▶ Monad & Transformer
  - ▶ Free Monad (& Algebra)

# INTRODUCTION

- ▶ These slides will define the most common terms and highlight their uses with some code examples.
- ▶ Not intended to be exhaustive but a collection of collated examples.
- ▶ Chiusano and Bjarnason book is the ideal book for functional programming in Scala.
- ▶ “Learn you a Haskell for great good”, Miran Lipovaca. Well worth reading as it covers the core FP concepts very succinctly and with excellent examples.

## FUNCTIONS

- ▶ A **function**, is a mapping between one set (domain) to another (codomain). In Scala, domain and codomain are the Scala *types*.

```
def square(x: Double): Double = x * 2.0
val square : Double => Double = x => x * x
```

- ▶ function that can take a function as an argument or return a function is a **higher order function**.

```
def cube(x: Int): Int = x * x * x
```

```
def id(x: Int): Int = x
```

```
def sum(f: Int => Int, a: Int, b: Int): Int = {
  if ( a < b) 0
  else f(a) + sum (f, a + 1, b)
}
```

```
def sumInts(a: Int, b: Int) = sum(id, a, b)
```

```
def sumCubes(a: Int, b: Int) = sum(cube, a, b)
```

## FUNCTIONS

- ▶ Functions can define multiple argument lists. If a function is called with a fewer number of parameter lists, the result is a function taking the missing parameter lists as its arguments.

- ▶ Example, **partially apply** the parameters of `f` to give a new function `g`:

```
def filter(xs: List[Int], f: Int ⇒ Boolean): List[Int] ⇒ ???  
def f(n: Int)(x: Int) = ((x%n)==0)  
def g = f(2) _  
  
// g: Int ⇒ Boolean  
>filter(List(1,2,3,4,5,6,7,8),g)
```

- ▶ In a related fashion, we can **curry** a function:

```
def sum(f: Int ⇒ Int): (Int, Int) ⇒ Int = {  
  def sumF(a: Int, b: Int): Int = {  
    if (a > b) 0  
    else f(a) + sumF(a + 1, b) sumF  
  }  
}
```

- ▶ We can now write:

```
def sumInts = sum(x ⇒ x)  
def sumSquares = sum(x ⇒ x * x )  
sumSquares(1,10) + sumInts(10,20)
```

## ALGEBRAIC DATA TYPES

- ▶ An algebraic data type is the composition of *product* or *sum* types.
- ▶ A **product type** is the cartesian cross product on 2 or more types. Usually represented by a case class

```
case AppConfig(k: KafkaConfig, c: CassandraConfig)
```

- ▶ A **sum type** is the **disjoint union** of two or more types (Sometimes called co-product as it is the dual of a product type).
  - ▶ In the simple case of two types, we can use `Either`, which is right biased (for comprehensions) as the left is usually for holding an error value. i.e. `Either[Throwable, Result]`
  - ▶ To represent more than two types in **Scala 2.X** we can use a sealed trait and subtypes or use the union type of Shapeless. In Scala 2.X:

```
trait FooOrBar  
case object Foo extends FooOrBar  
case object Bar extends FooOrBar
```

- ▶ More succinctly, with **Shapeless** we can write:

```
type ISB = Int :+: String :+: Char :+: Boolean :+: CNil  
val isb = Coproduct[ISB]("foo")
```

- ▶ Scala 3.X will directly support union and intersection types (removing the need for the traits).
- ▶ Intersection type is typically something like: 'A with B' this will become 'A & B'. The difference being '&' will be commutative. i.e. A & B is the same type as B & A.



## TYPE CLASSES

- ▶ Suppose we have a list of T, and our list class has a method 'head' that returns the first element. The function will do the same thing for whatever T the list was parameterised with; this is **parametric polymorphism**.
- ▶ **Ad-hoc polymorphism** is bound to the type. Depending on the type, different implementations are invoked.
- ▶ Type class allow for ad-hoc polymorphism.
- ▶ A type class can be thought of a set of types with operations defined on them. Somewhat similar to Java interfaces or Adapter pattern (but cleaner).
- ▶ Example, we can define a 'Showable' interface, together with an apply method that allows the splitting of the definition and different implementations.
- ▶ We have a triple of **trait**, **object** and **instances**.

```
trait Show[A] {  
  def show(a: A): String  
}  
  
object Show {  
  def apply[A: Show]: Show[A] = implicitly // syntactic sugar for creating objects.  
  def show[A: Show](a: A) = Show[A].show(a) // interface method.  
}
```

## TYPE CLASS INSTANCES

- ▶ With the trait on object, we can 'independently' define implementations for 'Show'

```
object ShowInstances {  
  implicit def showForInt: Show[Int] = (i: Int) ⇒ s"My int is: $i"  
}
```

- ▶ To use the interface we bring the instances into scope and invoke the method. As follows:

```
import ShowInstances._  
Show.show(1)
```

- ▶ An alternative approach (to adding interface methods into the object) is to use type enrichment to extend existing types with interface methods.
- ▶ Popular in some Typelevel libraries, by convention this is referred to as the "syntax" as defined similar to as shown below:

```
object ShowSyntax {  
  implicit class ShowOps[A](value: A) {  
    def show(implicit ev: Show[A]): String = ev.show(value)  
  }  
  // ...  
}
```

- ▶ This allows us to write the following:

```
val x = 1234.show // "My int is: 1234"
```

- ▶ For code readability, it is important to follow the coding standards for where in your project to place the instance definitions.

# HIGHER KINDED TYPES

- ▶ A type constructor can be thought of a function that accepts some type and returns a new type.
- ▶ Languages such as Java have basic type constructors, e.g. `ArrayList` can be thought of as type level function that takes one parameter `<T>` and returns new types `ArrayList<Boolean>`, `ArrayList<Double>`, etc...
- ▶ Functional programming languages like Scala have 'higher order type constructors'. That is, a type constructor that takes a type constructor as a parameter.
- ▶ This is a fundamental requirement for functional programming.
  - ▶ Java with lambda expressions is not functional programming.

## HIGHER KINDED TYPES

- ▶ We previously defined a **type constructor** as '*a function that accepts some type and returns a new type*'.
  - ▶ New types can be defined by recursively composing type constructors.
  - ▶ Concretely, a type constructor is a  $n$ -ary type operator, taking as arguments zero or more types and returning a new type.
  - ▶ Currying, we can re-write an  $n$ -ary type operator as a sequence of unary type operators.
- ▶ A **kind** is the type of a type constructor.
- ▶ Examples,
  - \* is the kind of all data types (the set of all types).
  - \*  $\rightarrow$  \* is the kind of all unary type constructors (e.g. `List`).
  - \*  $\rightarrow$  \*  $\rightarrow$  \* is a binary type constructor (via currying) (e.g. `Either`).
  - ( \*  $\rightarrow$  \* )  $\rightarrow$  \* is a higher order type constructor from unary type constructor to proper types.
- ▶ Simply, similar to higher order functions, we have higher order type constructors that take type constructors as arguments.

## TYPE LAMBIDAS

- ▶ In the same way that functions may be partially applied, we can also partially apply types.
- ▶ Example, in the second case below, we define `Result` as partial application of the `Either` type.

```
type Result = Either[Throwable, Double]
type Result[A] = Either[Throwable, A]
```

- ▶ It is not always possible to use this convenient syntax, we sometimes have to use a full type lambda, e.g.

```
({type λ[α] = Either[String, α]})#λ
```

- ▶ Thankfully, there is a compile plugin (Typelevel kind projector) that simplifies the syntax and allows us to write examples such as:

```
Tuple2[?, Double]           // equivalent to: type R[A] = Tuple2[A, Double]
Either[Int, +?]             // equivalent to: type R[+A] = Either[Int, A]
Function2[-?, Long, +?]     // equivalent to: type R[-A, +B] = Function2[A, Long, B]
EitherT[?[_], Int, ?]       // equivalent to: type R[F[_], B] = EitherT[F, Int, B]
```

## FUNCTOR

- ▶ When we apply a function to  $(+2)$  to 3 we get 5. Suppose we have an instance of the `Option` type: `Some(3)`. How do we apply  $(+2)$  ?
  - ▶ Intuitively, we need to take  $(+2)$  “apply it” to `Some(3)` and return `Some(5)`. If the value had be `None`, we would expect the result to be `None` too.
  - ▶ In this case the ‘functor’ is `Option`.
  - ▶ Informally, you apply a function to some ‘wrapped’ value and get back a ‘wrapped’ value.
  - ▶ This can be implemented with a `map` function, in the case of `Option`:

```
def map[A,B](oa: Option[A])(f: A⇒B): Option[B] = oa match {  
  case Some(a) ⇒ Some(f(a))  
  case None ⇒ None  
}
```

- ▶ We can generalise the functor ‘map’ as a trait:

```
trait Functor[F[_]] {  
  def map[A,B](fa: F[A])(f: A⇒B): F[B]  
}
```

## FUNCTOR

- ▶ We have parameterised map on the type constructor  $F[_]$ , which will take type constructors such as `List` or `Option`.
- ▶ That is, a  $F$  is a type constructor of kind  $* \rightarrow *$
- ▶ Example for `List`, we could define the Functor instance as:

```
val listFunctor = new Functor[List] {  
  def map[A,B](a: List[A])(f: A⇒B): List[B] = a map f  
}
```

- ▶ Here we are saying that a type constructor `List` is a functor - and the `Functor[List]` instance is proof that `List` is actually a functor.
- ▶ Functor Laws
  - ▶ Identity -  $\text{map}(f)(x \Rightarrow x) = f$

## NATURAL TRANSFORMATIONS

- ▶ A polymorphic function that maps from one functor  $F[_]$  to another  $G[_]$  is called a natural transformation. Usually represented by the type  $\rightsquigarrow$

```
trait FunctionK[-F[_], +G[_]] {  
  def apply[A](fa: F[A]): G[A]  
}  
type  $\rightsquigarrow$  [-F[_],+G[_]] = FunctionK[F,G]
```

- ▶ Example,

```
val toList = new (Option  $\rightsquigarrow$  List) {  
  def apply[A](fa: Option[A]): List[A] = fa match {  
    case Some(a)  $\Rightarrow$  List(a)  
    case None  $\Rightarrow$  List.empty[A]  
  }  
}
```

- ▶ Example, return the first element of a list or error:

```
type ErrorOr[A] = Either[String, A]  
  
val errorOrFirst: FunctionK[List, ErrorOr]  
  =  $\lambda$ [FunctionK[List, ErrorOr]](_.headOption.toRight("the list is empty."))
```



## APPLICATIVES

- ▶ The informal definition of a functor was:
  - ▶ “you apply a function to some ‘wrapped’ value and get back a ‘wrapped’ value”.
  - ▶ Example,  $(+2)$  applied to `Some(3)` gives a result `Some(5)`.
- ▶ With applicatives we *apply a wrapped function to a wrapped value and get back a wrapped value*.
- ▶ Intuitively, `Some(+2)` “applied to” `Some(3)` returns `Some(5)`.
- ▶ **Lifting** - For some  $X$ ,  $F[X]$  is referred to as the lifted version of  $X$ .
  - ▶ Example,  $x \Rightarrow x * x$  can be lifted into `List` as `List(x  $\Rightarrow$  x*x)`.
- ▶ Concretely, an **Applicative** is a functor that:
  - ▶ Provides a way to apply a lifted function  $F[A \Rightarrow B]$  to some lifted value  $F[A]$  and returns a lifted value  $F[B]$ .
  - ▶ Provides a way to lift a value into the functor.
- ▶ This behaviour can be defined as a trait as follows:

```
trait Applicative[F[_]] extends Functor[F] {  
  def pure[A](a: A): F[A]  
  def ap[A,B](fa: F[A])(f: F[A  $\Rightarrow$  B]): F[B]  
}
```

## APPLICATIVES

- ▶ Applicative Laws

- ▶ Identity -  $\text{ap}(f a)(\text{pure}) = f a$
- ▶ Homomorphism -  $\text{ap}(\text{pure}(a))(\text{pure}(ab)) = \text{point}(ab(a))$
- ▶ Interchange -  $\text{ap}(\text{pure}(a))(\text{point}(ab)) = \text{point}(ab(a))$

## APPLICATIVE EXAMPLE

- We can now write (n.b. usually we would make Applicative a type class with instances):

```
implicit val applicativeOpt = new Applicative[Option] {  
  def pure[A](a: A): Option[A] = Some(a)  
  
  def ap[A,B](fa: Option[A])(of: Option[A⇒B]): Option[B] = fa match {  
    case None ⇒ None  
    case Some(a) ⇒ of match {  
      case Some(g) ⇒ Some(g(a))  
      case None ⇒ None  
    }  
  }  
}  
  
import applicativeOpt._  
val f: Int ⇒ Int = x ⇒ x + 2  
  
ap(Some(3))(Some(f)) // : Some(5)  
ap(ap(Some(3))(Some(f)))(Some(f)) // : Some(7)
```

## SEMIGROUP AND MONOID

- ▶ A **semigroup** can be thought of as a way to combine two values of the same type to form another value of the same type.

- ▶ Addition forms a semigroup over the integers.
- ▶ In Scala this *behaviour* can be defined as a trait:

```
trait Semigroup[T] {  
  def combine(x: T, y: T): T  
}
```

- ▶ A **monoid** is a semigroup that has a unit or 'zero' element.

- ▶ Natural numbers, + and 0 form a monoid.
- ▶ Can be defined simply as:

```
trait Monoid[T] extends Semigroup[T] {  
  def unit: T  
}
```

- ▶ Monoid laws

- ▶ Associativity -  $\text{combine}(x, \text{combine}(y, z)) = \text{combine}(\text{combine}(x, y), z)$
- ▶ Identity -  $\text{combine}(x, \text{unit}) = x$  ,  $\text{combine}(\text{unit}, x) = x$

## MONOID

- ▶ We can write a generic 'combineAll' method as follows:

```
def combineAll[A: Monoid](xs: List[A]): A = xs.foldLeft(Monoid[A].unit)(Monoid[A].combine)
```

- ▶ The above is a generic function that will work for all types that define a monoid instance. For example, if we define for integers:

```
implicit val intMonoid = new Monoid[Int] {  
  def unit: Int = 0  
  def combine(x: Int, y: Int): Int = x + y  
}
```

- ▶ And for strings:

```
implicit val stringMonoid = new Monoid[String] {  
  def unit: String = ""  
  def combine(x: String, y: String): String = x + y  
}
```

- ▶ We can now write (n.b. make Monoid a type class with instances as above):

```
combineAll(List(1,2,3,4)) // :Int = 6  
combineAll(List("hello", " ", "world")) // :String = "hello world"
```

## MONOID

- ▶ Booleans with an 'Or' operator form a Monoid:

```
val booleanOr: Monoid[Boolean] = new Monoid[Boolean] {  
  def unit: Boolean = false  
  def combine(x: Boolean, y: Boolean): Boolean = x || y  
}
```

- ▶ Similarly, Booleans with an 'And' operator for a Monoid:

```
val booleanAnd: Monoid[Boolean] = new Monoid[Boolean] {  
  def unit: Boolean = true  
  def combine(x: Boolean, y: Boolean): Boolean = x && y  
}
```

- ▶ We can now write:

```
val results: List[Boolean] = List(true, true, false, true, false)
```

```
Results.reduce(booleanOr) // : true  
Results.reduct(booleanAnd) // : false
```

## MONOID

- ▶ A homomorphism between two monoids  $(M, *)$  and  $(N, \cdot)$  is a function  $f: M \rightarrow N$  such that

$$f(x * y) = f(x) \cdot f(y) \text{ for all } x, y \text{ in } M$$

$$f(e_M) = e_N$$

where  $e_M$  and  $e_N$  are the identities on  $M$  and  $N$  respectively.

- ▶ A bijective monoid homomorphism is called a monoid isomorphism. Two monoids are said to be isomorphic if there is a monoid isomorphism between them.
- ▶ How is this used? We can choose to implement one monoid in terms of another:

```
def booleanIsomorphism(mb: Monoid[Boolean]): Monoid[Boolean] = new Monoid[Boolean] {  
  def combine(x: Boolean, y: Boolean) = !mb.op(!x, !y)  
  def unit = !mb.zero  
}  
  
val booleanAnd: Monoid[Boolean] = booleanIsomorphism(booleanOr)
```

## FOLD MAP

- ▶ foldMap maps elements of type A to elements of type B.
- ▶ The B elements can be reduced by the monoid B.

```
def foldMap[A, B](xs: List[A], m: Monoid[B])(f: A ⇒ B): B
  = xs.foldLeft(m.zero)((b, a) ⇒ m.op(b, f(a)))
```

- ▶ As a simple example, count and average with one parse through a list:

```
def op(a: (Double, Int), b: (Double, Int)): (Double, Int) = {
  val cCount: Int = a._2 + b._2
  val cAverage: Double = (a._2.toDouble * a._1 + b._2.toDouble * b._1) / cCount
  (cAverage, cCount)
}
```

```
val monoidAverageAndCount: Monoid[(Double, Int)] = new Monoid[(Double, Int)] {
  def op( a: (Double, Int), b: (Double, Int) ): (Double, Int) = ...
  def zero: (Double, Int) = (0.0, monoidIntAddition.zero)
}
```

```
val doubles = foldMap(doubles, monoidAverageAndCount){ (d: Double) ⇒ (d, 1) }
```



## MONAD

- ▶ Monad is an extension of Applicative. Informally,
  - ▶ If I have a value in a context  $M[A]$
  - ▶ An a function that returns a value in a context, e.g.  $A \Rightarrow M[B]$
  - ▶ How do we apply  $A \Rightarrow M[B]$  to  $M[A]$ ?
  - ▶ We need a function that takes a value in a nested context and “joins” the contexts together so that we have a single context.
  - ▶ The function is called the bind or flatMap operator, that has the following signature:

```
def flatMap[A, B](fa: M[A])(f: A  $\Rightarrow$  M[B]): M[B]
```

- ▶ As for applicative and the others we can define this behaviour as a trait:

```
trait Monad[F[_]] extends Applicative[F] {  
  def flatMap[A,B](fa: F[A])(f: A  $\Rightarrow$  F[B]): F[B]  
}
```

- ▶ Monad laws

- ▶ left identity -  $\text{pure}(x).\text{flatMap}(f) = f(x)$
- ▶ right identity -  $x.\text{flatMap}(\text{pure}) = x$
- ▶ associativity -  $x.\text{flatMap}(f).\text{flatMap}(g) = x.\text{flatMap}(y \Rightarrow f(y).\text{flatMap}(g))$

## MONAD

- ▶ Implementing the flatMap interface for List (illustration only, this implementation is compact but very slow) and Option:

```
def flatMap[A, B](xs: List[A])(f: A ⇒ List[B]): List[B]
  = xs.foldLeft(List[B].empty)((ys, y) ⇒ ys ++ f(y))

def flatMap[A, B](oa: Option[A])(f: A ⇒ Option[B]): Option[B]
  = if ( oa.isDefined ) f(oa) else None
```

- ▶ Allows us to write:

```
Monad[Option].pure(3) // : Option[Int] = Some(3)
Monad[Option].flatMap(res0)(a ⇒ Some(a + 2)) // : Option[Int] = Some(5)
Monad[List].pure(3) // : List[Int] = List(3)
Monad[List].flatMap(res2)(x ⇒ List(x, x*10)) // : List[Int] = List(3, 30)
```

- ▶ We can now implement **generic** functions, such as:

```
def sumSquare[A[_] : Monad](a: Int, b: Int): A[Int] = {
  val x = a.pure[A]
  val y = b.pure[A]
  x flatMap (x ⇒ y map (y ⇒ x*x + y*y))
}
sumSquare[Option](3, 4) // : Option[Int] = Some(25)
sumSquare[List](3, 4) // : List[Int] = List(25)
```

## IDENTITY MONAD

- ▶ The identity monad can be thought of as encapsulating the effect of having no effect.
- ▶ We can define a type:

```
type Id[+A] = A
```

- ▶ This Id context has no effect, the monad instance can be defined along the lines of:

```
implicit val id = new Monad[Id] {  
  override def flatMap[A, B](fa: Id[A])(f: (A) ⇒ Id[B]): Id[B] = f(fa)  
  override def pure[A](a: A): Id[A] = a  
  override def ap[A, B](fa: Id[A])(f: Id[(A) ⇒ B]): Id[B] = f(fa)  
  override def map[A, B](fa: Id[A])(f: (A) ⇒ B): Id[B] = f(fa)  
}
```

- ▶ At first glance this monad doesn't seem useful.
- ▶ However, it is fundamental to the concept of monad transformers (any monad transformer applied to the identity monad returns a non-transformer version of that monad) - this is dealt with later.

# WRITER MONAD

- ▶ A writer monad `Writer[L,V]` a monad that allows us to carry a log (L) along with a value (V).
- ▶ It has a number of uses, including:
  - ▶ Delayed logging during concurrent computations, where log messages from different contexts may otherwise be interleaved.
  - ▶ Ensuring that a function is 'pure' in the sense that it has no IO side effect.

## WRITER MONAD

- ▶ Looking at simplified implementation:

```
// WriterT transformer.
final case class WriterT[F[_], L, V](run: F[(L, V)]) {

  def tell(l: L)(implicit functorF: Functor[F], semigroupL: Semigroup[L]): WriterT[F,L,V]
    = mapWritten(_ |+| l)

  def written(implicit ev: Functor[F]): F[L] = ev.map(run)(_. _1)

  def value(implicit ev: Functor[F]): F[V] = ev.map(run)(_. _2)

  def mapBoth[M,U](f: L, V) => (M, U)(implicit ev: Functor[F]): WriterT[F,M,U]
    = WriterT { ev.map(run)(f.tupled) }

  def mapWritten[M](f: L => M)(implicit ev: Functor[F]): WriterT[F, M, V] = mapBoth((l, v) => (f(l), v))
}

// Writer is WriterT partially applied with Id monad.
type Writer[L, V] = WriterT[Id, L, V]

object Writer {
  def apply[L, V](l: L, v: V): WriterT[Id, L, V] = WriterT[Id, L, V]((l, v))
  def value[L:Monoid, V](v: V): Writer[L, V] = WriterT.value(v)
  def tell[L](l: L): Writer[L, Unit] = WriterT.tell(l)
}
```

- ▶ We can see that `Writer[L,V]` is actually `WriterT[Id, L, V]`, `WriterT` is an example of a monad transformer. These are discussed in a later slide.

## WRITER MONAD

- ▶ Whilst we should use `Vector` instead of `List` for efficiency reasons, we can now return log and value from a **series** of computations:

```
def logNumber(x: Int): Writer[List[String], Int] =  
  Writer(List("Got number: " + x.show), 3) // Writer[List[String],Int]  
  
def multWithLog: Writer[List[String], Int] = for {  
    a ← logNumber(3)  
    b ← logNumber(5)  
  } yield a * b  
// Writer[List[String],Int]  
  
multWithLog.run  
// Id[(List[String], Int)] = (List(Got number: 3, Got number: 5),9)
```

- ▶ The logging information and value is returned as `Id` ! From which we can transparently take the tuple of log messages and result value.
- ▶ The `Writer` monad is closely related to the `State` monad, dealt with later. It is more restrictive in that we can only 'append' (Monoid combine operator) rather than read and write state.

# MONAD TRANSFORMERS

- ▶ A monad transformer is a type constructor that takes a monad as an argument and returns a monad as a result.
- ▶ Concretely, we can define a monad transformer as:
  - ▶ A type constructor  $T$  of kind  $(* \rightarrow *) \rightarrow * \rightarrow *$
  - ▶ Provides monad operators 'pure' and 'flatMap'
  - ▶ An additional 'lift' operator, that takes a function and a monadic value and maps it over the monadic value.

```
trait MonadTrans[T[_[_],_]] { ...  
  def liftM[M[_], A](a: M[A])(implicit M: Monad[M]): T[M, A]  
}
```

# MONAD TRANSFORMERS

- ▶ *"Functors and Applicatives compose Monads don't".*
- ▶ We can compose monads of the same type easily.
- ▶ Example, two functions both returning Option:

```
def first: Option[Int] = Some(1)
def second: Option[Int] = Some(2)
```

```
val result : Option[Int] = for{
  x ← first
  y ← second
} yield x+y
```

```
println(result) // Some(3)
```



# MONAD TRANSFORMERS

- ▶ Example, two functions both returning Future:

```
def first: Future[Int] = Future(1)
def second(x: Int): Future[Int] = Future(x+2)
```

```
val result : Future[Int] = for{
  x ← first
  Y ← fb(a)
} yield y
```

```
Await.result(result, Duration.Inf)
println(result) // Future(Success(3))
```

## MONAD TRANSFORMERS

- ▶ Problem, when we try compose Future and Option:

```
def first: Future[Option[Int]] = Future(Some(1))
def second(a: Int): Future[Option[Int]] = Future(Some(a+2))

// won't compile !!!!!
val result = for {
  optionX ← first           // ← future
  y ← optionX               // ← option
  xy ← second(y)           // ← future ... types don't line up!
} yield xy
```

- ▶ Naive way to solve this:

```
val composedAB: Future[Option[Int]] = first.flatMap {
  case Some(a) ⇒ second(a)
  case None    ⇒ Future.successful(None)
}
```

- ▶ But this could become very messy, particularly if we have multiple nested calls!

# MONAD TRANSFORMERS

- ▶ Use a *transformer* to compose `Future` and `Option`.
- ▶ The following shows a minimalist transformer:

```
case class Future0[+A](future: Future[Option[A]]) {  
  def flatMap[B](f: A ⇒ Future0[B]): Future0[B] = {  
    val newFuture = future.flatMap {  
      case Some(a) ⇒ f(a).future  
      case None    ⇒ Future.successful(None)  
    }  
    Future0(newFuture)  
  }  
  def map[B](f: A ⇒ B): Future0[B] = {  
    Future0(future.map(option ⇒ option.map(f)))  
  }  
}
```

- ▶ As we can see in the `flatMap` function, we are simply unpacking the option from the future, if there is a value we call `f(value)`.

# MONAD TRANSFORMERS

► We can now write:

```
def divideEven(n: Int): Option[Int] = if (n % 2 == 0) Some(n/2) else None

val f1 = Future(divideEven(14))
val f2 = Future(divideEven(16))

val fc = for {
  a ← Future0(f1)
  b ← Future0(f2)
} yield a + b
val f = fc.future

f onComplete println // Success(Some(15))
```

## KLEISLI & READER MONAD

- ▶ `Kleisli[F[_], A, B]` is a simple wrapper around a function  $A \Rightarrow F[B]$ , there is also a special case involving the identity monad, `Kleisli[Id, A, B]`.
- ▶ Dependent on `F[_]` we can do different things.

- ▶ Example, composing functions:

```
final case class Kleisli[F[_], A, B](run: A => F[B]) {  
  def compose[Z](k: Kleisli[F, Z, A])(implicit F: FlatMap[F]): Kleisli[F, Z, B] =  
    Kleisli[F, Z, B](z => k.run(z).flatMap(run))  
}
```

- ▶ If we want to wrap a function  $A \Rightarrow B$ , rather than  $A \Rightarrow F[B]$  we can partially apply the Kleisli with the identity monad - the Reader monad.

```
type Id[A] = A  
type Reader[A, B] = Kleisli[Id, A, B]  
object Reader {  
  def apply[A, B](f: A => B): Reader[A, B] = Kleisli[Id, A, B](f)  
}  
type ReaderT[F[_], A, B] = Kleisli[F, A, B]  
val ReaderT = Kleisli
```

- ▶ Informally, we can build up a computation that is a function of context (configuration, etc.) rather than passing the context as an argument of the function.

## READER MONAD

- ▶ We create a `Reader[A,B]` from a function of type  $A \Rightarrow B$  and run it:

```
def square(a: Int): Int = a*a
val squareR: Reader[Int, Int] = Reader(square)
squareR.run(2)
```

```
// :Id[Int] = 4
```

- ▶ What does the `flatMap` implementation of a `Reader` look like?

```
def flatMap[B](f: A  $\Rightarrow$  Reader[E,B]): Reader[E,B] = Reader[E,B] { e  $\Rightarrow$  f(run(e)).run(e) }
```

- ▶ We can therefore combine readers in a for comprehension:

```
val composeReaders: Reader[Int, Int] =
  for {
    x  $\leftarrow$  squareR
    y  $\leftarrow$  cubeR(x)
  } yield x + y
composeReaders.run(10)
```

## READER MONAD

- ▶ Analogous to **dependency injection**:

```
def areaR(r: Int): Reader[Double,Double] = Reader { pi ⇒ pi * r * r }

val areaRR: Reader[Int,Reader[Double,Double]] = Reader { r ⇒ areaR(r) }

def volumeRR(h: Int): Reader[Int,Reader[Double,Double]] =
  areaRR map { areaR ⇒ areaR map { a ⇒ a * h }}

val volumeRRR = Reader { h: Int ⇒ volumeRR(h) }

val computation = volumeRR(2).run(1) // forms the computation.
val valueToInjectIntoCalc = 3.141    // the value we want to inject.

println(computation.run(valueToInjectIntoCalc)) // 6.282
```

## STATE MONAD

- ▶ Imagine a computer game where we have a number of state updates:

```
val (s0, _) = init()
val (s1, _) = nextBlock(s0)
val (s2, moved0) = moveBlock(s1, LEFT)
val (s3, moved1) = if (moved0) moveBlock(s2, LEFT) else (s2, moved0)
```

- ▶ Passing around state objects (s0,s1,...) becomes error-prone boilerplate.
- ▶ A stateful computation is a function that takes some state and returns a value along with some new state.
- ▶ State is a data type that encapsulates a stateful computation  $S \Rightarrow (S,A)$  and forms a monad which passes along the states represented by the type S.
- ▶ When we examine the type of State, we see that it is the partial application of the monad transformer StateT with Eval (a monad that controls evaluation) which emulates a call stack with heap memory to prevent overflow.

```
type State[S, A] = StateT[Eval, S, A]
```



## STATE MONAD

- ▶ The core parts of StateT are defined as:

```
final class StateT[F[_], S, A](val runF: F[S ⇒ F[(S, A)]]) { ... }

object StateT extends StateTInstances {
  def apply[F[_], S, A](f: S ⇒ F[(S, A)])(implicit F: Applicative[F]): StateT[F, S, A]
    = new StateT(F.pure(f))

  def applyF[F[_], S, A](runF: F[S ⇒ F[(S, A)]]): StateT[F, S, A]
    = new StateT(runF)

  /* Run with the provided initial state value */
  def run(initial: S)(implicit F: FlatMap[F]): F[(S, A)] = F.flatMap(runF)(f ⇒
f(initial))
}
```

- ▶ To construct a State value, you pass a state transition function to State.apply

```
def apply[S, A](f: S ⇒ (S, A)): State[S, A] = StateT.applyF(Now((s: S) ⇒ Now(f(s))))
```

- ▶ Next we can show how to use the State monad.

## STATE MONAD

- ▶ Lets define a stack type:

```
type Stack = List[Int]
```

- ▶ With some functions to pop and push from the stack:

```
val pop: State[Stack, Int] = for {  
  s ← State.get[Stack]  
  (x :: xs) = s  
  _ ← State.set[Stack](xs)  
} yield x
```

```
def push(x: Int): State[Stack, Unit] = for {  
  xs ← State.get[Stack]  
  r ← State.set(x :: xs)  
} yield r
```

## STATE MONAD

- ▶ We can now compose stateful computations as follows:

```
def stackComputation: State[Stack, Int] = for {  
  _ ← push(3)  
  a ← pop  
  b ← pop  
} yield(b)  
  
val program = stackComputation.run(List(5, 8, 2, 1))  
// forms a computation  
  
val result = program.value // evaluates the computation.  
// (Stack, Int) = (List(8, 2, 1),5)
```

- ▶ Both push and pop are purely functional and we have avoided boiler-plate passing of state between each of the calls.

# FREE MONAD

- ▶ A free monad is a monad that allows you to build a monad from any functor. *Eh?*
- ▶ What does this mean? Informally,
  - ▶ Free stores a list of functors, wrapped around an initial value.
  - ▶ Functor and Monad instances of Free do nothing other than hand the function down using flatMap.

# FREE MONAD

- ▶ Free monads are commonly used to:
  - ▶ Provide an abstraction for trampolining recursive functions.
  - ▶ Define algebra and co-products of algebra, to form a kind of DSL and target different interpreters using natural transformations.
- ▶ Many other use cases ...

## FREE MONAD

- ▶ First, look at the outline definition of Free:

```
sealed trait Free[F[_],A]
case class Return[F[_],A](a: A) extends Free[F,A]
case class Bind[F[_],I,A](i: F[I], k: I ⇒ Free[F,A]) extends Free[F,A]
```

- ▶ With the implementation of Free as:

```
sealed trait Free[F[_],A] {
  def flatMap[B](f: A ⇒ Free[F,B]): Free[F,B] = this match {
    case Return(a) ⇒ f(a)
    case Bind(i, k) ⇒ Bind(i, k andThen (_ flatMap f))
  }
  def map[B](f: A ⇒ B): Free[F,B] = flatMap(a ⇒ Bind(f(a)))
}
```

- ▶ Together with a lift function:

```
implicit def lift[F[_],A](fa: F[A]): Free[F,A] = Bind(fa, (a: A) ⇒ Return(a))
```

## FREE MONAD

- ▶ Pure builds a Free instance from an A value.
- ▶ Suspend builds a new Free by applying F to the previous Free.
- ▶ We end up with a recursive data structure:

`Suspend(F(Suspend(F(Suspend(F(...(Pure(a))))))))`

- ▶ Can be seen as a sequence of computations, where:
  - ▶ Pure returns an A value and ends the computation.
  - ▶ Suspend is a continuation; it suspends the current computation with the functor F and hands control to the caller. A represents a value bound to this computation.
- ▶ The recursion is *encoded* on the heap rather than the stack.

## FREE MONAD

- ▶ Suppose we have a console application:

```
sealed trait Interact[A]  
case class Ask(prompt: String) extends Interact[String]  
case class Tell(msg: String) extends Interact[Unit]
```

- ▶ We can define a program as:

```
val program: Free[Interact, Unit] = for {  
  first ← Ask("What's your first name?")  
  last  ← Ask("What's your last name?")  
  _     ← Tell(s"Hello, $first $last!")  
} yield ()
```

- ▶ This just encodes a representation of the program ...



## FREE MONAD

- ▶ We have an encoding of the 'program':

```
val prg: Free[Interact, Unit] =  
  Bind(Ask("What's your first name?"),  
    first ⇒ Bind(Ask("What's your last name?"),  
      last ⇒ Bind(Tell(s"Hello, $first $last!"),  
        _ ⇒ Return(())))
```

- ▶ Next, we can implement an interpreter:

```
object Console extends (Interact ⇒ Id) { // Natural transformation  
  def apply[A](i: Interact[A]) = i match {  
    case Ask(prompt) ⇒  
      println(prompt)  
      readLine  
    case Tell(msg) ⇒  
      println(msg)  
  }  
}
```

# FREE MONAD

- ▶ Possible to define alternative interpreters for the same program (e.g. test).
- ▶ Run, via a `foldMap` on `Free`:

```
val result = program.foldMap(Console)
```

- ▶ Can combine with `State` and `Coproduct` to handle stateful interactions and multiple algebra.

## TRAMPOLINE

- ▶ We may have a recursive function that causes a stack overflow.
- ▶ Can define a Trampoline in terms of Free (over Function0) as:

```
type Trampoline[A] = Free[Function0, A]
// Function0 is a function that takes zero arguments

object Trampoline {
  def done[A](a: A): Trampoline[A] = Free.Pure[Function0, A](a)

  def suspend[A](a: ⇒ Trampoline[A]): Trampoline[A]
    = Free.Suspend[Function0, A](() ⇒ a)

  def delay[A](a: ⇒ A): Trampoline[A] = suspend(done(a))
}
```

## TRAMPOLINE

- ▶ Can now express recursive functions (that we can't easily make tail recursive) in terms of suspend and done:
- ▶ Example, trampolining an 'unfold' method:

```
def unfold[T,R](z: T)(f: T ⇒ Option[(R,T)]): Trampoline[Stream[R]] = f(z) match {  
  case None ⇒  
    Trampoline.done(Stream.empty[R])  
  case Some((r,v)) ⇒  
    Trampoline.suspend(unfold(v)(f)).flatMap(s ⇒ Trampoline.done(r #:: s))  
}
```

- ▶ *"The under appreciated unfold"*, Jeremy Gibbons & Geraint Jones (available on-line).

## TRAMPOLINE

- ▶ Here is an example illustrating use, the traversal function takes a 'Directed' (graph) and a starting node and *unfolds* using either a depth or breadth first search direction:

```
def traverse[A](g: Directed[A], l: List[A], dfs: Boolean): Stream[A]
  = unfold( (l, Set.empty[A]) ) { case (current, visited) => current match {
    case w :: Nil =>
      Some((w, (g.successors(w).toList.filterNot(visited.contains),
        visited + w)))
    case w :: vs =>
      val next = if (dfs) g.successors(w).toList ::: vs
      else vs ::: g.successors(w).toList
      Some((w, (next.filterNot(visited.contains), visited + w)))
    case _ => None
  }
}.run
```

# APPENDIX – LAMBDA CALCULUS

- ▶  $\lambda$ -calculus is a functional notion of computation. The basis for functional programming.
- ▶ The Turing machine is a state based model of computation.
- ▶ Introduced by Alonzo Church (Princeton mathematician, who was the Ph.D. supervisor of Alan Turing).
- ▶ The two are equivalent (part of the Church-Turing hypothesis).
  - ▶  $\lambda$ -calculus can be used to simulate any single-taped Turing machine.
- ▶ Concretely,  $\lambda$ -calculus is *"a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution."*

# TYPE INFERENCE

- ▶ Hindley-Milner type system is a type system for the  $\lambda$ -calculus with parametric polymorphism.
- ▶ Built from the algorithm for simply typed  $\lambda$ -calculus by Haskell Curry and Robert Feys.
- ▶ Powerful type inference used in Haskell. Scala doesn't use Hindley-Milner (can't because of sub-typing), but uses a "local type inferencing" algorithm".