Homework 1, EENG 515, Fall 2018

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1. Done

2. Let $R = S \cap T$ where S and T are both convex sets. Take two points s and t within the intersection. We know the points satisfy both
$$\lambda s + (1-\lambda)t \in S$$
and
$$\lambda s + (1-\lambda)t \in T$$
when
$$0 \leq \lambda \leq 1$$
By definition, every point in R is in both S and T. Therefore every point in the intersection also satisfies
$$\lambda s + (1-\lambda)t \in R$$
when
$$0 \leq \lambda \leq 1$$

3. Let $A = [0, 1)$ and let $B = (1, 2]$. If $a = 0$, $b = 2$, and $\lambda = 0.5$, we have
$$\lambda a + (1-\lambda)b = 1$$
when
$$0 \leq \lambda \leq 1$$
Hence,
$$\lambda a + (1-\lambda)b \notin A \cup B$$
when
$$0 \leq \lambda \leq 1$$

4. Let us assume that there exists at least one rational number x such that $x + \sqrt{2}$ is a rational number. This implies that there are a set of integer values a, b, c, and d that satisfy the following.
$$\begin{aligned} \frac{a}{b} + \sqrt{2} &= \frac{c}{d} \\ \sqrt{2} &= \frac{cb - ad}{bd} \end{aligned} \tag{1}$$
Given the products, quotients, and differences of rational numbers are also rational numbers, the last line of (1) indicates that $\sqrt{2}$ is a rational number. Hence, our initial assumption that there is a least one rational number that yields a rational result to the sum $x + \sqrt{2}$ is incorrect.

5. Let m^2 be even. Now suppose m is odd. This means there is an integer a such that $m = 2a + 1$. If this is the case, then the following holds.

$$\begin{aligned} m^2 &= (2a+1)^2 \\ &= 4a^2 + 4a + 1 \\ &= 2(2a^2 + 2a) + 1 \end{aligned} \tag{2}$$

which indicates the m^2 is odd since $2a^2 + 2a$ is an integer. This implies that m must be even.