Gait Scheduling and Foot Placement Policy for MPC on Legged Robots

AME 556 - Robot Dynamics and Control

1 Gait Schedule

A **gait schedule** in legged robot locomotion refers to the timing and sequence of foot placements and movements that a robot's legs follow to achieve a desired type of locomotion. In the context of Model Predictive Control (MPC), the gait schedule is a vector of binary variables that determines when each leg is in either a **stance phase** (in contact with the ground) or a **swing phase** (moving to a new position).

In your assignments and final project, you will primarily deal with *periodic gait* types, meaning the gait cycle is repeated over time.

1.1 Predefined Gait Schedule

A fixed gait schedule is provided to the MPC as an input. The schedule specifies the timing of stance and swing phases for each leg over the prediction horizon.

- Example 1: Double support stance on bipedal robot

  With 10 prediction steps in the MPC, the gait schedule at any time step k for the left and right leg $(\sigma_{\mathrm{L}}, \sigma_{\mathrm{R}})$ can be defined as

$$\begin{bmatrix} \sigma_{\mathrm{L}} \\ \sigma_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} 1,1,1,1,1,1,1,1,1,1 \\ 1,1,1,1,1,1,1,1,1,1 \end{bmatrix} \tag{1}$$

  where each column represents the gait schedule at prediction step i within the MPC window.

- Example 2: Walking gait on bipedal robot

  With 10 prediction steps in the MPC, the gait schedule at time step $k = 0$ for left and right leg $(\sigma_{\mathrm{L}}, \sigma_{\mathrm{R}})$ can be defined as

$$\begin{bmatrix} \sigma_{\mathrm{L}} \\ \sigma_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} 1,1,1,1,1,0,0,0,0,0 \\ 0,0,0,0,0,1,1,1,1,1 \end{bmatrix} \tag{2}$$

  where the gait transition occurs at prediction step 5.

Similarly, at time step $k = 1$,

$$\begin{bmatrix} \sigma_{\mathrm{L}} \\ \sigma_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} 1,1,1,1,0,0,0,0,0,1 \\ 0,0,0,0,1,1,1,1,1,0 \end{bmatrix} \tag{3}$$

The gait schedule table is advanced by one column to reflect the current gait.

With this pattern, you will find, that every 10 time steps, the gait schedule table will reset to the case of $k = 0$, and thus a periodic gait cycle.

1.2 Implementing Gait Schedule in MPC

In legged robot locomotion using MPC, the gait schedule plays a crucial role in activating or deactivating reaction forces at each contact point. For example, in bipedal walking, the swing foot does not exert any ground reaction force. In force-based single rigid-body modeling within MPC, ground reaction forces are often explicitly treated as optimization variables. This allows inequality or equality constraints to be directly applied to enforce the gait schedule.

Consider the force saturation constraint at prediction step i, we have the following inequality constraint:

$$F_{\mathrm{min}} \leq F_{\mathrm{y,i}} \leq F_{\mathrm{max}} \tag{4}$$

where F_{y} is the normal reaction force at the contact point. F_{min} and F_{max} are the saturation limits of reaction forces. To reflect the gait schedule in this constraint, we simply modify it (right leg as an example)

$$\sigma_{\mathrm{R,i}} F_{\mathrm{min}} \leq F_{\mathrm{y,i}} \leq \sigma_{\mathrm{R,i}} F_{\mathrm{max}} \tag{5}$$

where $\sigma_{\mathrm{R,i}}$ is the binary contact schedule (i.e. $\sigma_{\mathrm{R,i}} = 0$ represents right leg in swing phase, and $\sigma_{\mathrm{R,i}} = 1$ represents right leg in stance phase).

By incorporating the force saturation constraint at each prediction step with the consideration of the contact schedule, the periodic gait can be implemented in locomotion MPC.

2 Swing Leg Foot Placement Policy

The foot placement policy we consider in this assignment often is referred to as *the Raibert heuristic*, a method for determining the swing foot placement in legged robots, particularly aimed at maintaining balance and achieving stable locomotion. It was introduced by Marc Raibert in the context of dynamically balancing on legged robots, such as achieving hopping and running behaviors, by simplifying the dynamics of the robot to a Linear Inverted Pendulum (LIP) model with massless legs [5].

2.1 Swing Foot Policy

The core idea of the heuristic focuses on stabilizing the robot's motion by adjusting the **position of the swing foot** based on the **robot's CoM position, velocity, and desired**

velocity. The swing foot placement is computed to ensure the robot remains balanced after the next step and continues moving toward the desired velocity.

The heuristic foot placement location in the x direction follows the simple policy:

$$p^{\text{des}}_{\text{foot,x}} = p_{\text{c,x}} + \frac{1}{2}\Delta t \dot{p}_{\text{c,x}} - k_v(\dot{p}^{\text{des}}_{\text{c,x}} - \dot{p}_{\text{c,x}}), \tag{6}$$

where $p^{\text{des}}_{\text{foot}}$ is the desired foot placement in the world frame. $p_{\text{c,x}}$ and $\dot{p}_{\text{c,x}}$ are the x-direction robot CoM position and velocity feedback. Δt is the time duration of the swing phase. $\dot{p}^{\text{des}}_{\text{c,x}}$ is the desired robot CoM velocity. k_v is a control gain to track desired velocity (normally very small $k_v < 0.1$).

As for the y-direction location (height) of the swing foot, we would like to define a smooth curve for the contact point to follow, One very simple example is a sinusoidal curve,

$$p^{\text{des}}_{\text{foot,y}} = h_{\text{foot}} \sin(\frac{t_s}{\Delta t}\pi), \tag{7}$$

where h_{foot} is the maximum height of the foot swing. t_s is the time duration from the beginning of the swing phase ($0 \leq t_s \leq \Delta t$). In addition, you can also replace this swing curve with the quadratic curve, or an even more advanced Bézier curve for traversing highly complex terrains.

2.2 Cartesian Space Control

Once the swing foot's desired location is established, we can use a simple cartesian PD control law to compute the virtual swing force exerted at the contact,

$$\boldsymbol{F}_{\text{swing}} = K_p(\boldsymbol{p}^{\text{des}}_{\text{foot}} - \boldsymbol{p}_{\text{foot}}) - K_d\dot{\boldsymbol{p}} \tag{8}$$

where K_p and K_d are diagonal matrices defining the control gains. With the assumption of massless legs, we can map the virtual swing force to joint torques $\boldsymbol{\tau}_{\text{j}}$ by world framework contact Jacobian $\boldsymbol{J}_{\text{c,j}}$ of jth leg,

$$\boldsymbol{\tau}_{\text{j}} = \boldsymbol{J}^{\intercal}_{\text{c,j}}\boldsymbol{F}_{\text{swing}}. \tag{9}$$

Students may refer to reference [1, 2, 3, 4] for further information regarding the gait schedule and foot placement policy on legged robot MPC-based control.

References

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