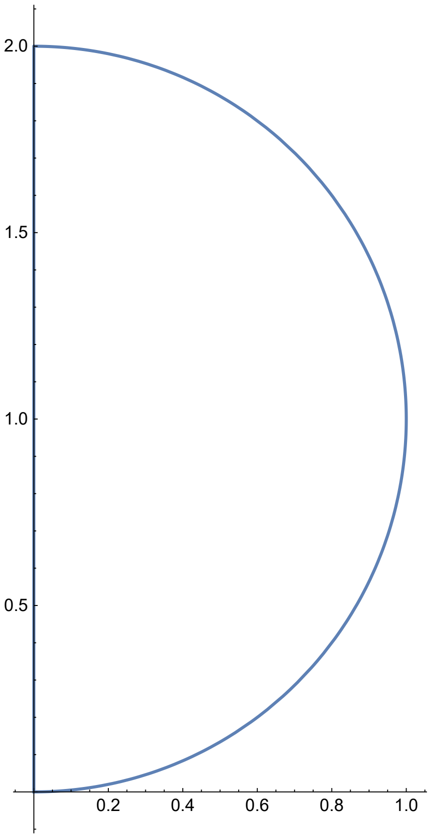
Plot the secant map which is used in the proof of Hopf Umlaufsatz.

[Christian Bär, Elementary differential geometry, P44]

It can be seen from the following plots that the secant map is only discontinuous on the boundary x=y. It is continuous in the interior.

In[∘]:=
```
xcor[s_] := Piecewise[{{Sin[s], 0 ≤ s ≤ π}, {0, π < s ≤ π + 2}}];
ycor[s_] := Piecewise[{{1 - Cos[s], 0 ≤ s ≤ π}, {2 + π - s, π < s ≤ π + 2}}];
ParametricPlot[{xcor[s], ycor[s]}, {s, 0, π + 2}]
```

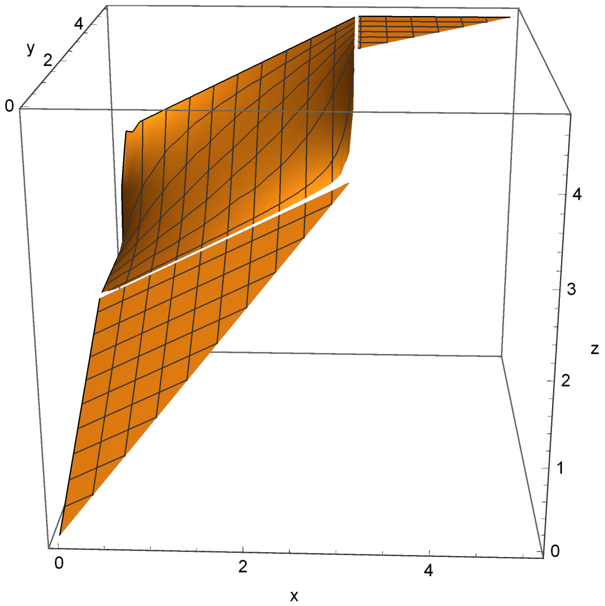
Out[∘]=




```
phi[x_, y_] := Piecewise[{{(x + y)/2, 0 < x < y <= π},
   {π + ArcTan[(y - π + Cos[π - x] - 1)/Sin[π - x]], 0 < x < π < y < π + 2}, {3/2 π, π < x < y < π + 2}}]
  (*secant map of the above arclenght parametrized simple closed
  curve. x and y are the arclength parameters of two points.*)
```

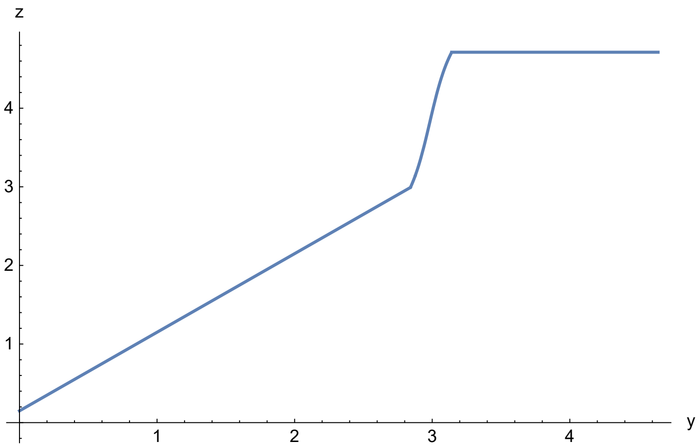
In[∘]:=
```
Plot3D[phi[x, y], {x, 0, π + 2}, {y, 0, π + 2}, RegionFunction → Function[{x, y}, x < y],
 BoxRatios → Automatic, AxesLabel → {"x", "y", "z"}]
```

Out[∘]=



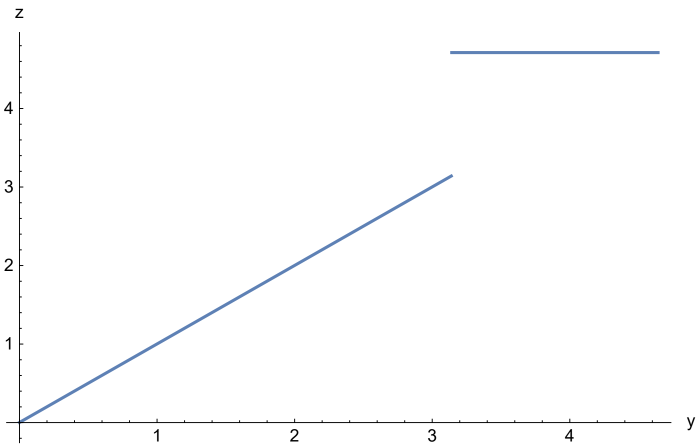
In[∘]:=
```
Plot[phi[x, x + 0.3], {x, 0, π + 1.5}, AxesLabel → {"y", "z"}]
```

Out[∘]=



In[∘]:=
```
Plot[phi[x, x + 0.003], {x, 0, π + 1.5}, AxesLabel → {"y", "z"}]
```

Out[∘]=



In[ ]:=
```
Plot[phi[x, 4], {x, 0, 4}, AxesLabel → {"x", "z"}]
```

Out[ ]=