

$$\begin{aligned}
 (a) \quad \gcd(1266, 888) &= \gcd(888, 378) \quad \text{since } 1266 \% 888 = 378 \\
 \gcd(888, 378) &= \gcd(888, 132) \quad \text{since } 888 \% 378 = 132 \\
 \gcd(378, 132) &= \gcd(132, 114) \quad 378 \% 132 = 114 \\
 \gcd(132, 114) &= \gcd(114, 18) \\
 \gcd(114, 18) &= \gcd(18, 6) \\
 \gcd(18, 6) &= \gcd(6, 0)
 \end{aligned}$$

$$(b) \text{ Let } u > v \Rightarrow u \bmod v < u/2$$

$$u \bmod v \Leftrightarrow u = kv + r \text{ where } r \in [0, v)$$

$$\text{Because } k \geq 1 \text{ (because } u > v)$$

$$\text{and } r < v, \text{ so } r < kv$$

$$\text{Then: } r \leq kv, r \leq r$$

$$\Rightarrow r + r \leq kv + r \Rightarrow 2r \leq kv + r$$

$$\Rightarrow \frac{r}{2} \leq \frac{kv}{2}$$

$$\Rightarrow r \leq kv$$

$$\text{Since } k \geq 1 \text{ and } u > r \text{ (K at least 1 because } u > v)$$

$$\therefore u \bmod v < \frac{u}{2}$$

$$(c) \text{ assume } a > b, \text{ then after } \gcd(a, b)$$

$$\text{also } a \text{ is at most } a/2 \text{ based on part (b)}$$

$$\text{also the } \gcd(b, a) \text{ need most } \log_2 b \text{ times, since}$$

$$\text{it always reduce at least half of the origin number.}$$

$$\text{so } \gcd(a, b) \text{ needs } 1 + \log_2 b \text{ times.}$$

$$\text{since } k \leq \log_2 b,$$