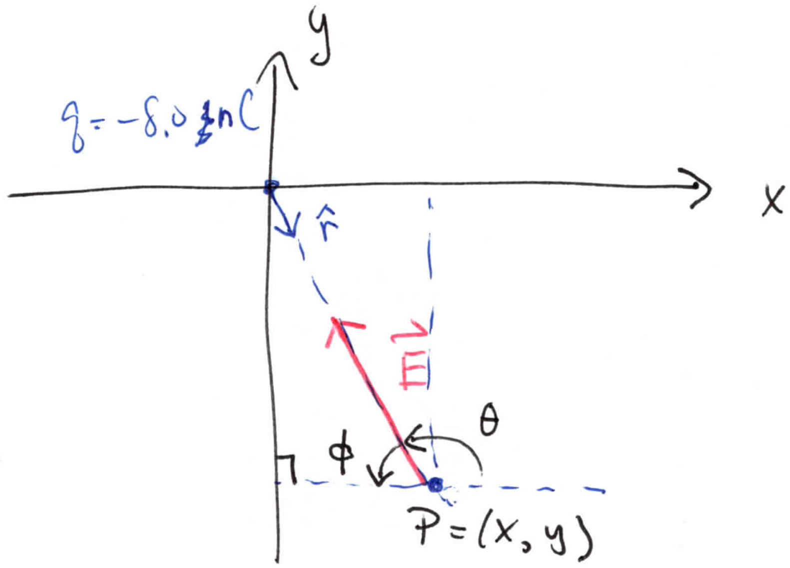
# Solutions to Sample Problems.

## SP. 2.1




<Method 1>

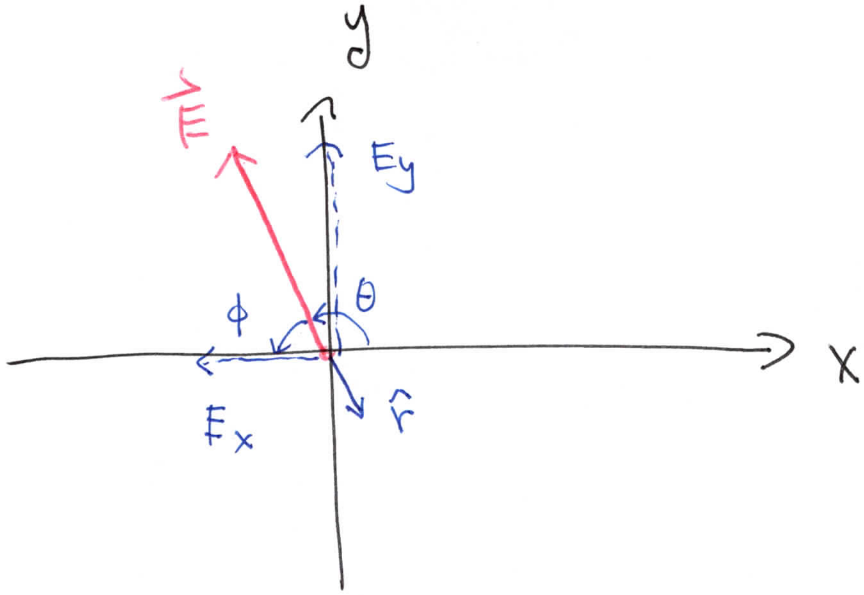
$x = 1.2\ m \qquad y = -1.6\ m$

$\theta$ is the angle in standard position for $\vec{E}$

$\phi$ is the reference angle for $\vec{E}$

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2)^2 + (-1.6)^2} = 2.0\ (m)$$

Shift $\vec{E}$ parallelly so that its tail coincides with the origin, as shown in the following diagram.




Note that $\vec{E}$ and $\hat{r}$ points to opposite directions,

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$= E \cos\theta \hat{i} + E \sin\theta \hat{j}$$

[Observed from the 2nd diagram]

$$= -E\cos\phi \hat{i} + E\sin\phi \hat{j} = E\left(-\cos\phi \hat{i} + \sin\phi \hat{j}\right)$$

$\cos\phi = \frac{x}{r}, \quad \sin\phi = \frac{|y|}{r},$ [Observed from the 1st diagram.]

which can ~~be~~ be seen from the first diagram.

And the magnitude of $\vec{E}$ is:

$$E = k_e \frac{|q|}{r^2} = \left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right) \cdot \frac{|-8.0 \times 10^{-9}|\, C}{(2.0)^2\, m^2} = 18 \left(\frac{N}{C}\right)$$

$$\cos\phi = \frac{x}{r} = \frac{1.2\ \text{m}}{2.0\ \text{m}} = 0.60$$

$$\sin\phi = \frac{|y|}{r} = \frac{|-1.6\ \text{m}|}{2.0\ \text{m}} = 0.80$$

$$\therefore \vec{E} = E\left(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}\right)$$

$$= \left(18\ \text{N/C}\right)\cdot\left(-0.60\,\hat{i} + 0.80\,\hat{j}\right)$$

$$\boxed{\vec{E} \approx \left(-11\,\hat{i} + 14\,\hat{j}\right)\ \text{N/C}}$$

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Remarks: From the 2nd diagram and the above result, we see that

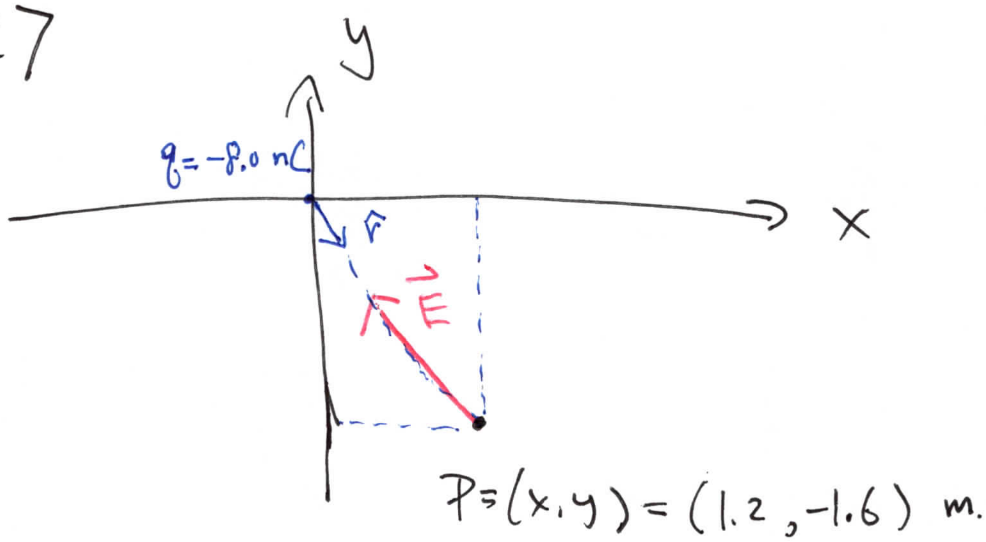
$E_x < 0$ and $E_y > 0$.

Thus, $\vec{E}$ points in the 2nd quadrant.

Note:

→ We ~~always~~ have to shift $\vec{E}$ parallelly so that its tail matches the origin. In this way, the Cartesian components of $\vec{E}$ can be obtained correctly.

<Method 2>




Write down $\vec{E}$ in the plane polar coordinates

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r} \qquad \vec{r} = x\hat{i} + y\hat{j}$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2} = \sqrt{(1.2)^2 + (-1.6)^2}$$

$$= 2.0\ (m)$$

$$\therefore\ \hat{r} = \frac{\vec{r}}{r} = \frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}$$

$$\hat{r} = \frac{(1.2)\ m}{(2.0)\ m}\hat{i} + \frac{(-1.6)\ m}{(2.0)\ m}\hat{j}$$

$$\therefore\ \hat{r} = 0.60\,\hat{i} - 0.80\,\hat{j}$$

$$\therefore \vec{E} = k_e \frac{q}{r^2} \hat{r}$$

$$= \left(9.0 \times 10^9 \ \frac{N \cdot m^2}{C^2}\right) \cdot \frac{-8.0 \times 10^{-9} \ C}{(2.0)^2 \ m^2}$$

$$\times \left(0.60 \ \hat{i} - 0.80 \ \hat{j}\right)$$

$$= \left(18 \ N/C\right) \cdot \left(-0.60 \ \hat{i} + 0.80 \ \hat{j}\right)$$

$$\boxed{\vec{E} = \left(-11 \ \hat{i} + 14 \ \hat{j}\right) \ N/C}$$

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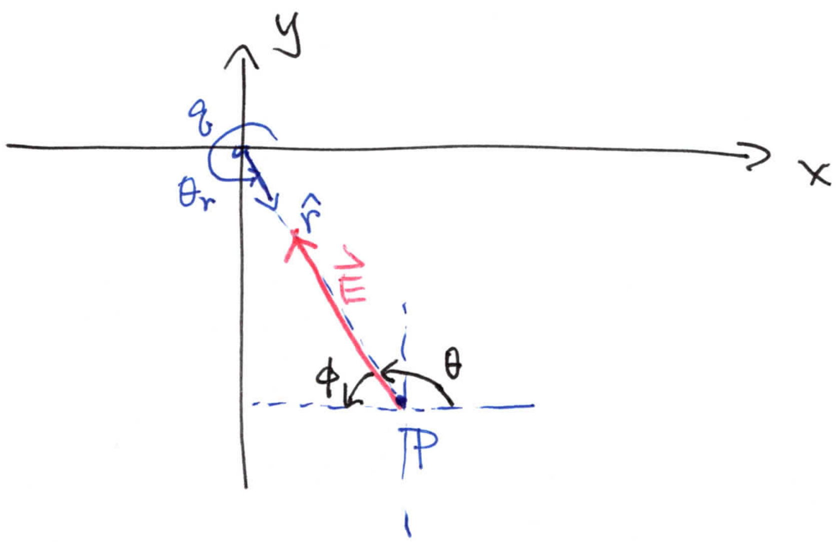
Remark:

Now, we can show that $\vec{E} = k_e \frac{q}{r^2} \hat{r}$

is equivalent to $\vec{E} = E\cos\theta \ \hat{i} + E\sin\theta \ \hat{j}$.

Proof:



Define $\theta_r$ as the angle (in standard position) for $\hat{r}$.

Define $\theta$ as the angle (in standard position) for $\vec{E}$

Since $\vec{E}$ and $\hat{r}$ point in opposite directions,

we have: $\quad \theta_r = \theta + 180^\circ$

$$\hat{r} = \frac{\vec{r}}{r} = (\cos\theta_r)\,\hat{i} + (\sin\theta_r)\,\hat{j}$$

$\left( \because\ q < 0 \text{ in this case} \quad \therefore\ q = -|q| \right)$

$$\cos\theta_r = \cos(\theta + 180^\circ) = -\cos\theta$$

$$\sin\theta_r = \sin(\theta + 180^\circ) = -\sin\theta$$

$$\therefore\ \hat{r} = \cos\theta_r\,\hat{i} + \sin\theta_r\,\hat{j} = -\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$

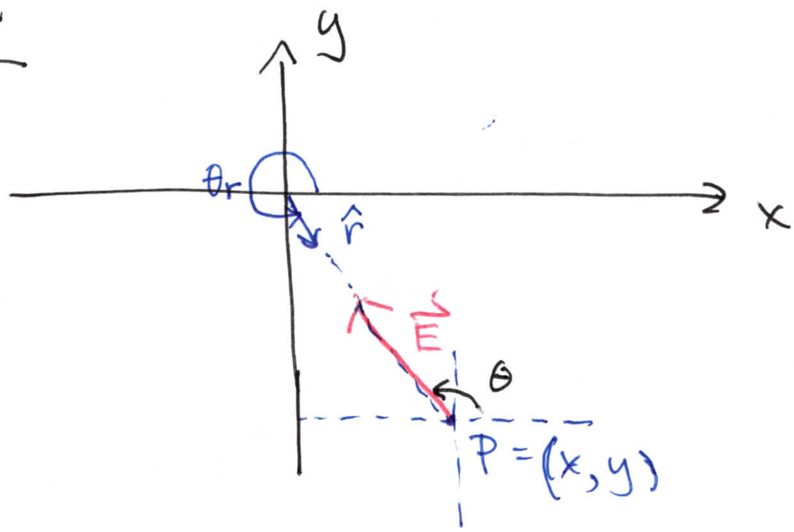
$$\boxed{\therefore\ (-\hat{r}) = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}}$$

$$\therefore\ \vec{E} = k_e \frac{q}{r^2}\hat{r} = k_e \frac{(-|q|)}{r^2}\hat{r} = k_e \frac{|q|}{r^2}(-\hat{r})$$

$$= E(-\hat{r}) = E\cos\theta\,\hat{i} + E\sin\theta\,\hat{j}$$

$$= E_x\,\hat{i} + E_y\,\hat{j}$$

Conclusion:




In this problem, we have: $q = -|q| < 0$

And we have seen that $\theta_r = \theta + 180^\circ$

and that $\hat{r} = \cos\theta_r \, \hat{i} + \sin\theta_r \, \hat{j}$

$= -\cos\theta \, \hat{i} - \sin\theta \, \hat{j}$

$\rightarrow (-\hat{r}) = \cos\theta \, \hat{i} + \sin\theta \, \hat{j}$

$$\therefore \boxed{\begin{aligned} \vec{E} &= k_e \frac{q}{r^2} \hat{r} = k_e \frac{|q|}{r^2} (-\hat{r}) \\ &= E\cos\theta \, \hat{i} + E\sin\theta \, \hat{j} \end{aligned}} \quad , \text{ for } q < 0.$$

where $E = k_e \frac{|q|}{r^2}$ is the magnitude of $\vec{E}$.