CS1382 Discrete Computational Structures

Lecture 15: Trees

Spring 2019

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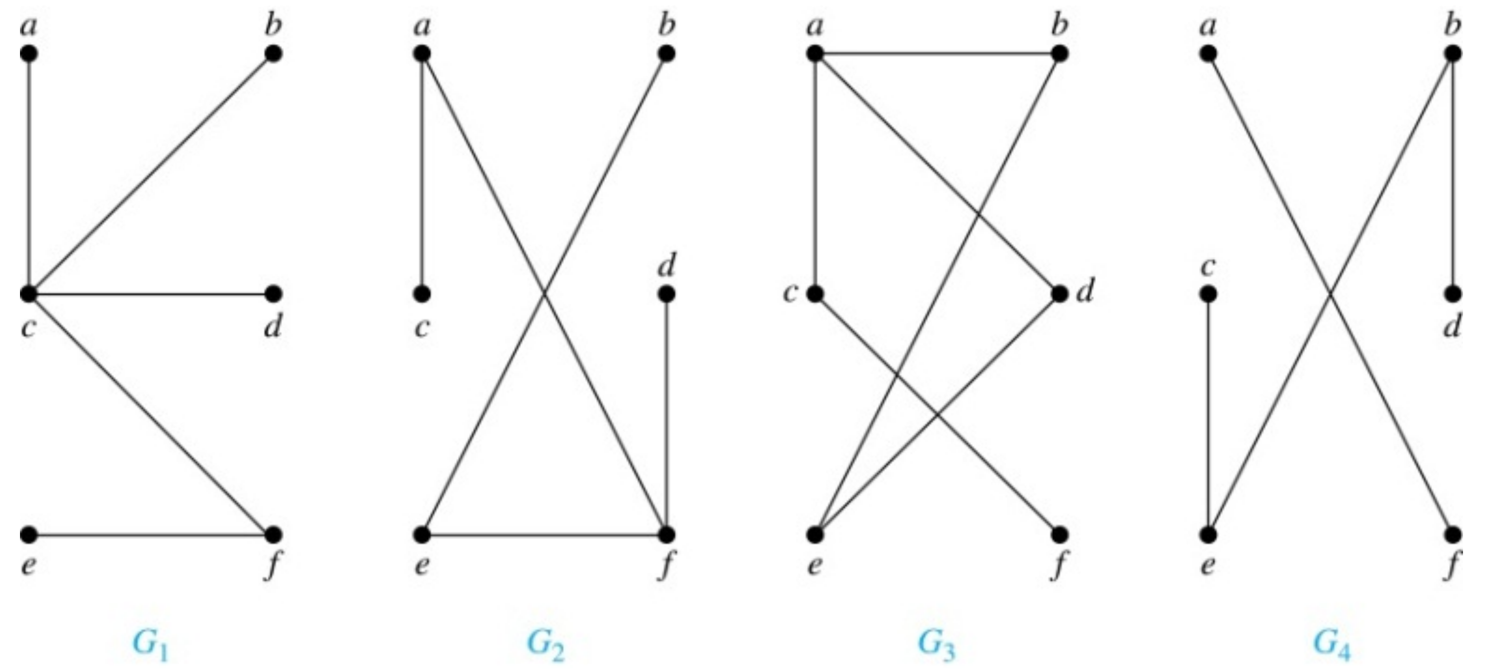
TEXAS TECH UNIVERSITY

Trees

A ***tree*** is a connected undirected graph with no simple circuits.

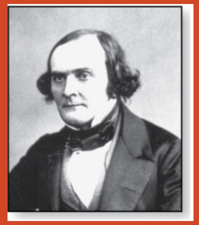


A ***forest*** is a graph that has no simple circuit, but is not connected.

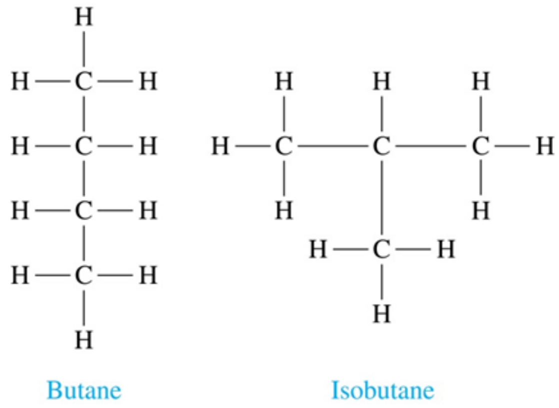
Each of the connected components in a forest is a tree.

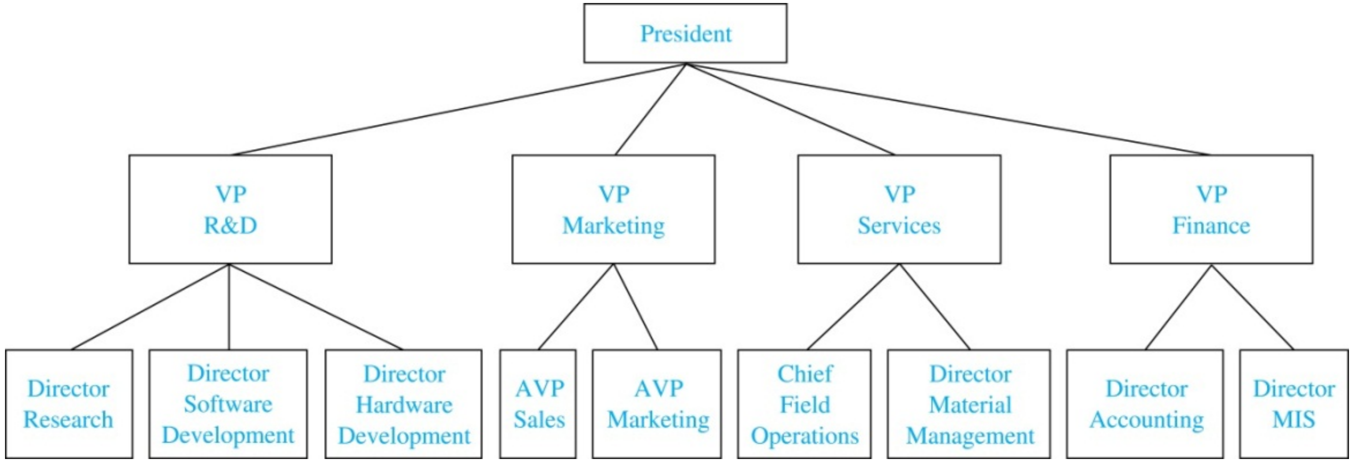
Trees as Models

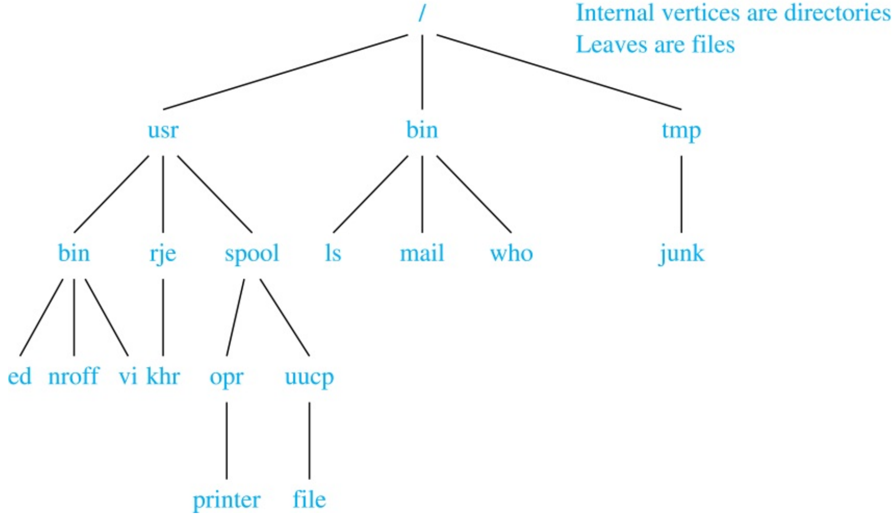
Arthur Cayley
(1821-1895)



Used as models in computer science, chemistry, geology, botany, psychology, and many other areas.



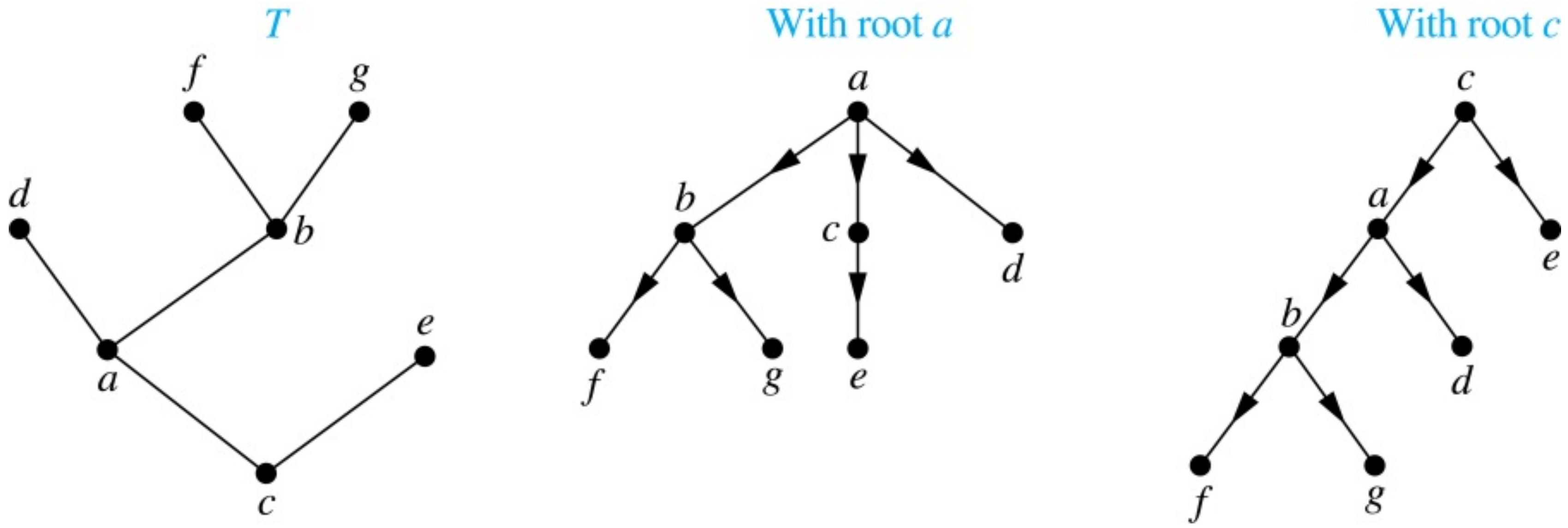




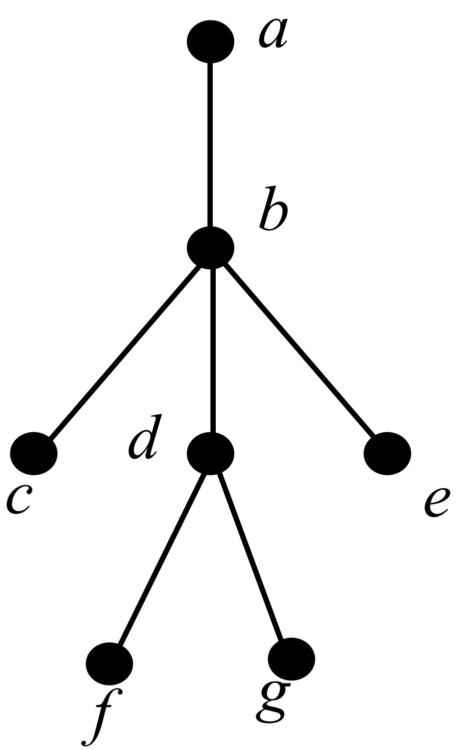
Rooted Trees

A ***rooted tree*** is a tree in which one vertex has been designated as the *root* and every edge is directed away from the root.

An **unrooted** tree is converted into different rooted trees when different vertices are chosen as the root.



Rooted Tree Terminology



a is the **parent** of *b*, *b* is the **child** of *a*,

c, d, e are **siblings**,

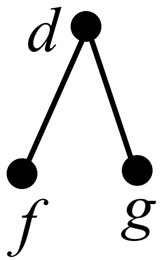
a, b, d are **ancestors** of *f*

c, d, e, f, g are **descendants** of *b*

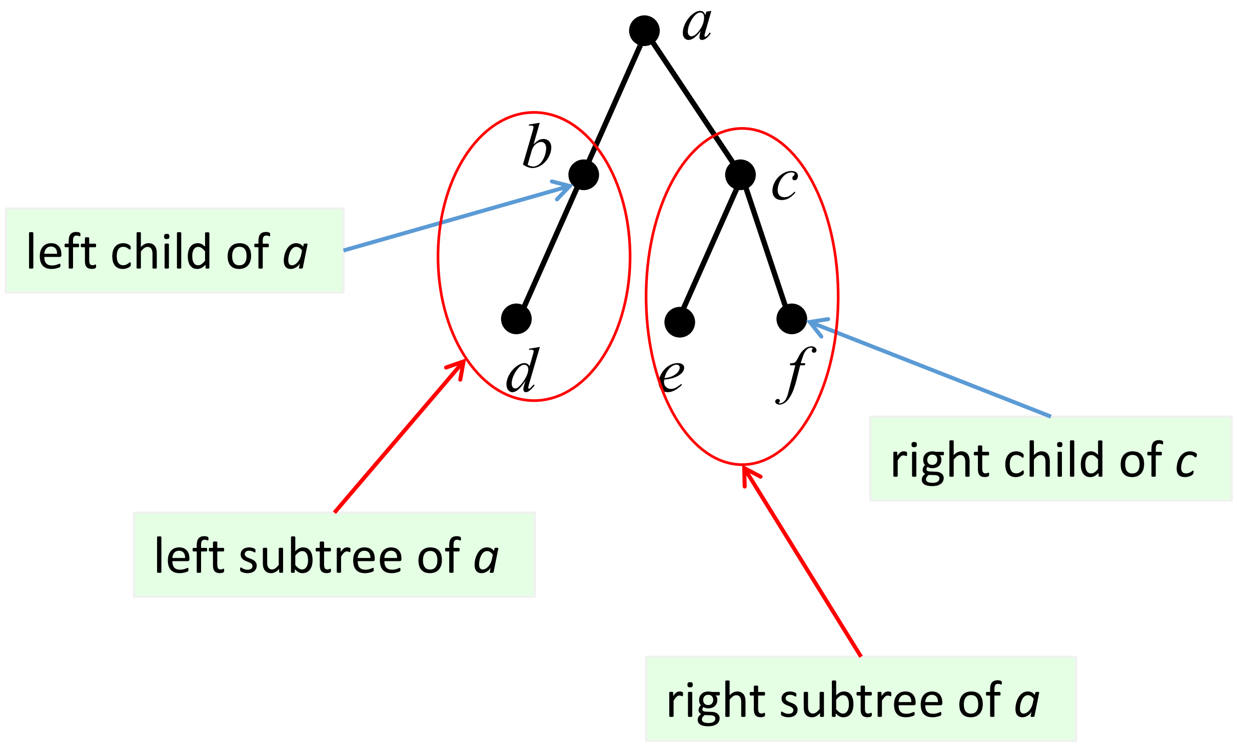
c, e, f, g are **leaves** of the tree (deg=1)

a, b, d are **internal vertices** of the tree (at least one child)

subtree with *d* as its root:

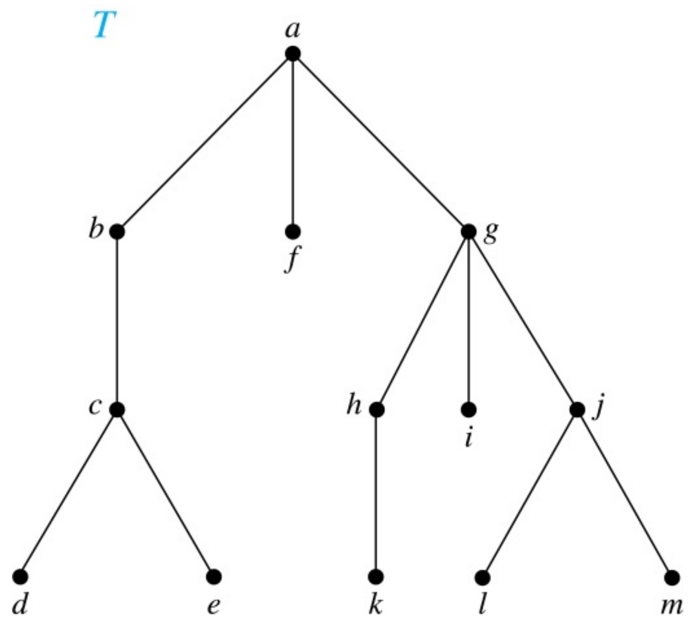


Rooted Tree Terminology

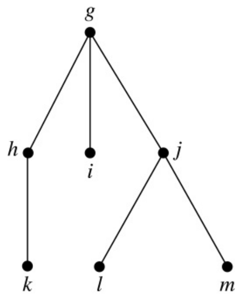


Terminology for Rooted Trees

In the rooted tree T (with root a):



- Find the parent of c, the children of g, the siblings of h, the ancestors of e, and the descendants of b.
- Find all internal vertices and all leaves.
- What is the subtree rooted at G?



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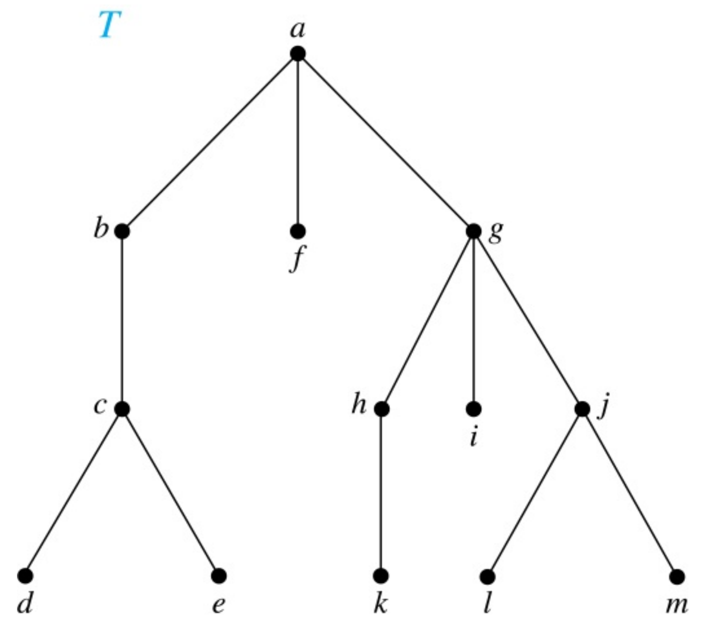
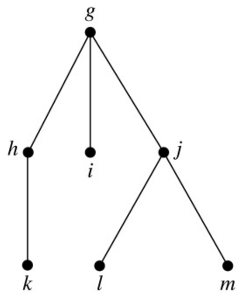
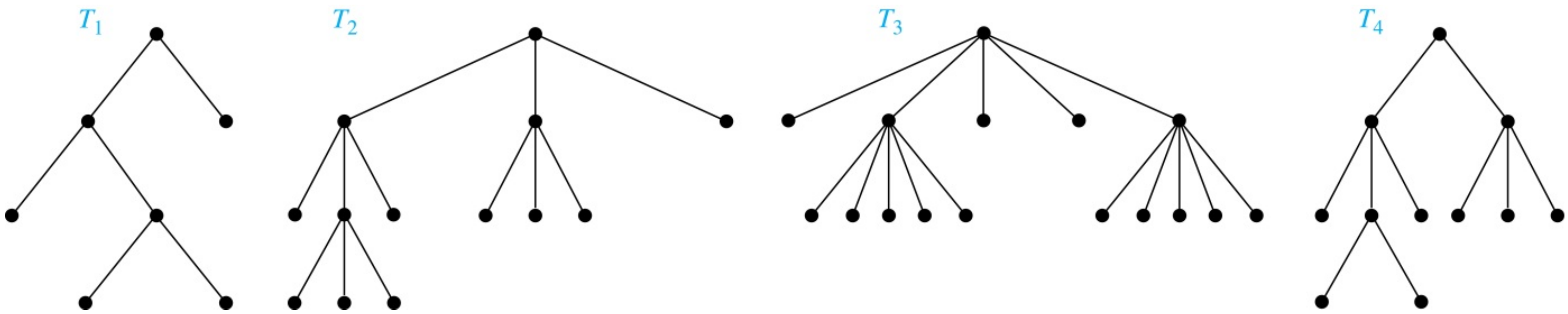




m-ary Rooted Trees

A rooted tree is called an ***m-ary tree*** if every internal vertex has no more than *m* children.

- An *m*-ary tree with *m* = 2 is called a *binary* tree.



Properties of Trees

1. An undirected graph is a tree if and only if there is a **unique simple path** between any two of its vertices.

2. A tree with ***n* vertices** has ***n* – 1 edges**.

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Tree Traversals

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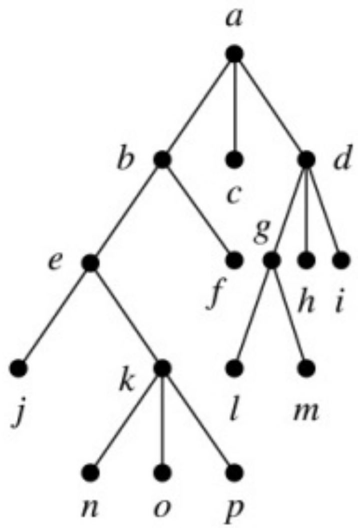
Tree Traversal

Procedures for systematically visiting every vertex of an ordered tree are called ***traversals***.

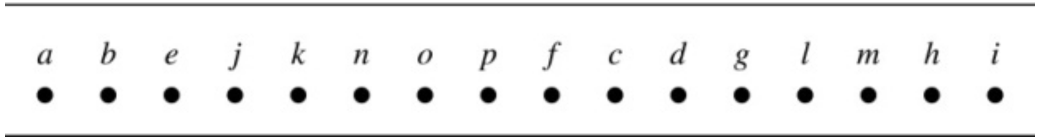
- *Preorder traversal*
- *Inorder traversal*
- *Postorder traversal*.

Preorder Traversal

procedure *preorder* (*T*: ordered rooted tree)
r := root of *T*
list *r*
for each child *c* of *r* from left to right
    T(*c*) := subtree with *c* as root
    preorder(*T*(*c*))

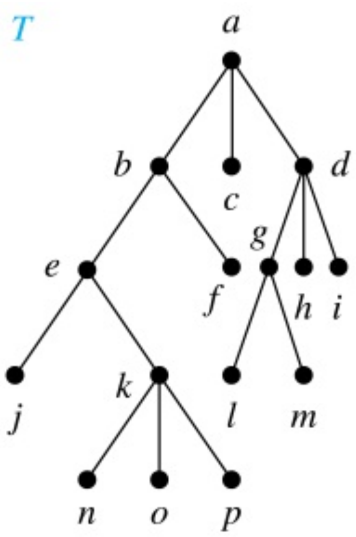


Preorder traversal: Visit root, visit subtrees left to right

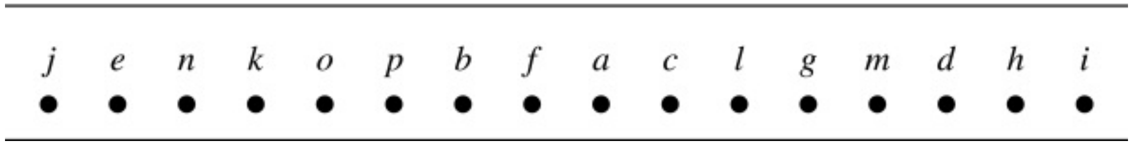


Inorder Traversal (*continued*)

```
procedure inorder (T: ordered rooted tree)
r := root of T
if r is a leaf then list r
else
    l := first child of r from left to right
    T(l) := subtree with l as its root
    inorder(T(l))
    list(r)
    for each child c of r from left to right
        T(c) := subtree with c as root
        inorder(T(c))
```

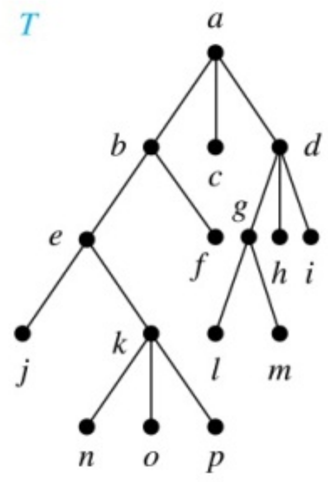


Inorder traversal: Visit leftmost subtree, visit root, visit other subtrees left to right

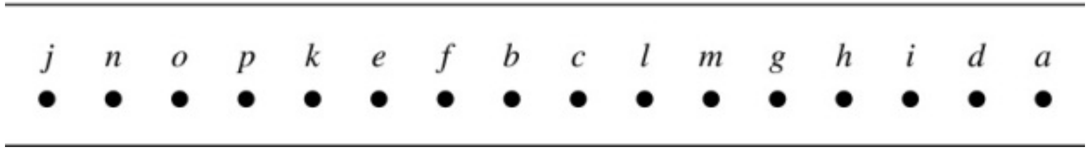


Postorder Traversal (*continued*)

procedure *postordered* (*T*: ordered rooted tree)
r := root of *T*
for each child *c* of *r* from left to right
    T(*c*) := subtree with *c* as root
    postorder(*T*(*c*))
list *r*



Postorder traversal: Visit subtrees left to right; visit root



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Spanning Trees

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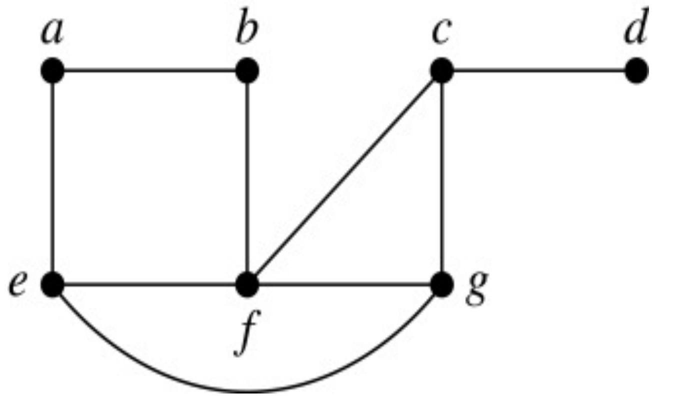
Spanning Trees

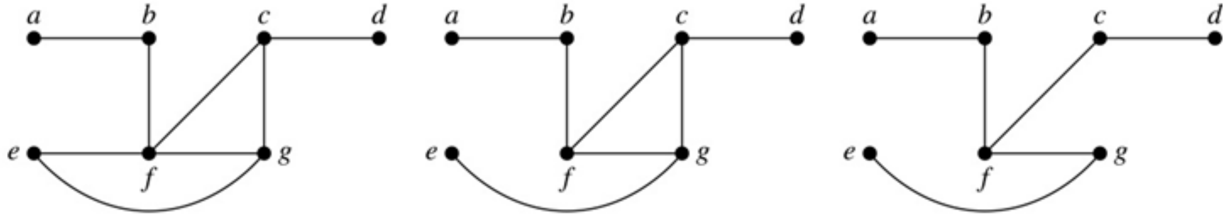
A **spanning tree** of simple graph *G* is a subgraph of *G* that is a tree containing every vertex of *G*.

Applications:

- Communication/PowerGrid Networks
- Data Clustering
- Maze Generation

Find the spanning tree of the simple graph below:



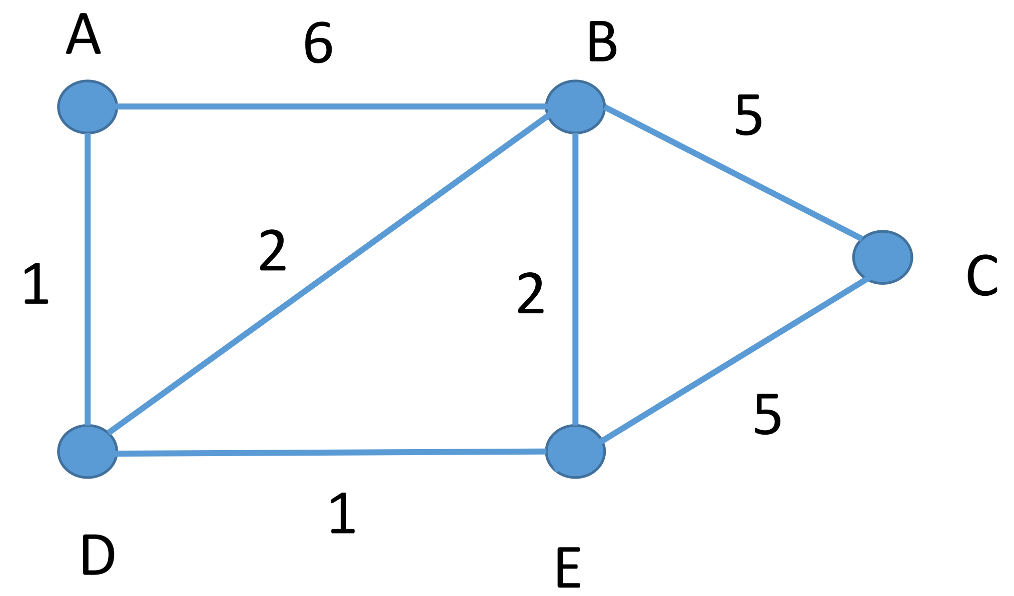


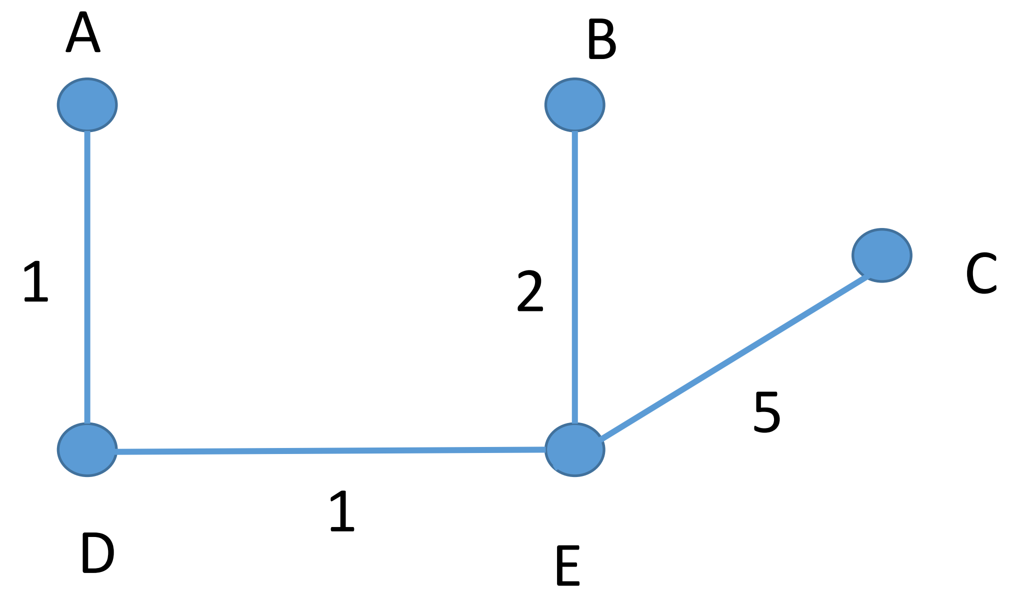
Edge removed: $\{a, e\}$ (a) $\{e, f\}$ (b) $\{c, g\}$ (c)

Minimum Spanning Trees (MST)

MST is a spanning tree whose sum of the weights of edges is minimum

- **Kruskal's Algorithm**
- **Prim's Algorithm**





Questions?

Thank You!