Definition of Algorithm

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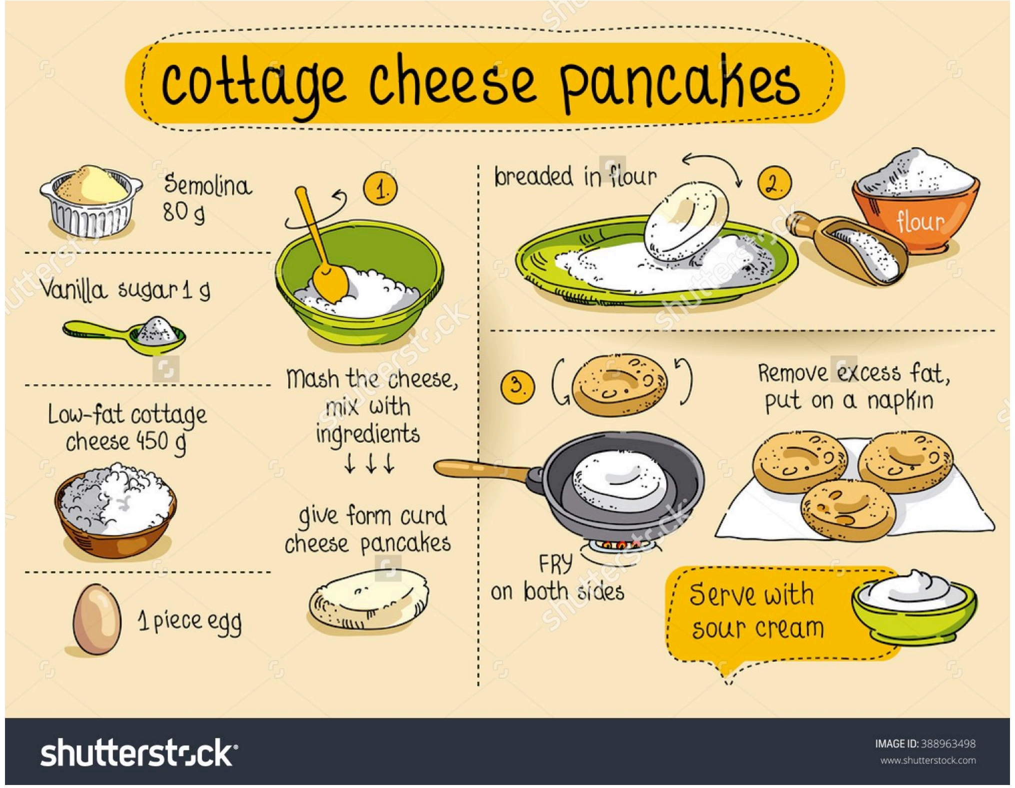
Algorithm

Algorithm is like a recipe

- It consists of finite number of instructions
- Each instruction is expressed by a finite number of symbols
- If it is carried out without error, it produces the same result in finite number of steps
- It demands no insight, intuition, or ingenuity, on the part of the one carrying out it



In principle, we are able to execute the code by hand, the machine just does that for us

Algorithm

Hilbert's 10^{th} problem

Determine whether a given polynomial has an integral root by "*a process according to which it can be determined by a finite number of operations*"

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Polynomials:

$$6x^3yz^2 + 3xy^2 - x^3 - 10$$

Root of a polynomials:

$$x = 5,\ y = 3,\ \text{and } z = 0.$$

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Intuitive notion of algorithms	equals	*Turing machine algorithms*

No Turing machine can decide whether an arbitrary given polynomial has an integral root

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$$D_1 = \{p \mid p \text{ is a polynomial over } x \text{ with an integral root}\}.$$

M_1 = "The input is a polynomial p over the variable x.

1. Evaluate p with x set successively to the values 0, 1, −1, 2, −2, 3, −3, . . . If at any point the polynomial evaluates to 0, *accept*."

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TM M_1 recognizes D_1. Does it decide D_1

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M_1 also decides D_1 because we know when it must terminate: for single variable k-term polynomial with integral coefficients, the integral root has an absolute value at most kc_{max}, if it exists

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For multi-variable polynomials, there is no such an upper bound, and a similar TM only recognizes.

$$x^3 + y^3 + z^3 = k$$

Roger Heath-Brown conjecture (1992): every k unequal to 4 or 5 modulo 9 has infinitely many representations as the sum of three cubes

Is it even true for k within 100? Unknown for 33 and 42 till 2016

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"I feel relieved," Booker said. "In this game, it's impossible to be sure that you'll find something. It's a bit like trying to predict earthquakes, in that we have only rough probabilities to go by. So, we might find what we're looking for with a few months of searching, or it might be that the solution isn't found for another century."

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