

(2) let  $\lim_{n \rightarrow \infty} x_n = z$

Then  $\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} \left[ \frac{1}{2} \left( x_{n-1} + \frac{y}{x_{n-1}} \right) \right]$

Become:  $z = \frac{1}{2} \left[ \lim_{n \rightarrow \infty} x_{n-1} + y \lim_{n \rightarrow \infty} \frac{1}{x_{n-1}} \right]$

$$z = \frac{1}{2} [z + y/z]$$

$$2z = z + y/z$$

$$z = y/z$$

$$z^2 = y$$

$$z = \sqrt{y}$$

$$\therefore \lim_{n \rightarrow \infty} x_n = \sqrt{y}$$

(3) Since  $y \geq 1$ , then  $a_{n-1} > a_n > a_{n+1}$

So, the sequence is a monotonic decrease.

Because,  $x_n = \frac{1}{2} \left( x_{n-1} + \frac{y}{x_{n-1}} \right)$

$$a_n = \frac{1}{2} (a_{n-1} + \frac{y}{a_{n-1}}) \quad a_{n+1} = \frac{1}{2} (a_n + \frac{y}{a_n})$$

$$a_n > a_{n+1}$$

$$\frac{1}{2} (a_{n-1} + \frac{y}{a_{n-1}}) > \frac{1}{2} (a_n + \frac{y}{a_n})$$

Also Based on part 2, the sequence bounded at  $\sqrt{y}$ , it has a lower boundary.

So, If a sequence monotonic decrease, and has a lower boundary, Its limit exist.