Modern Digital System Design

ECE 2372 / Fall 2018 / Lecture 05

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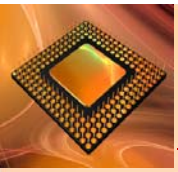
Karnough map simplification

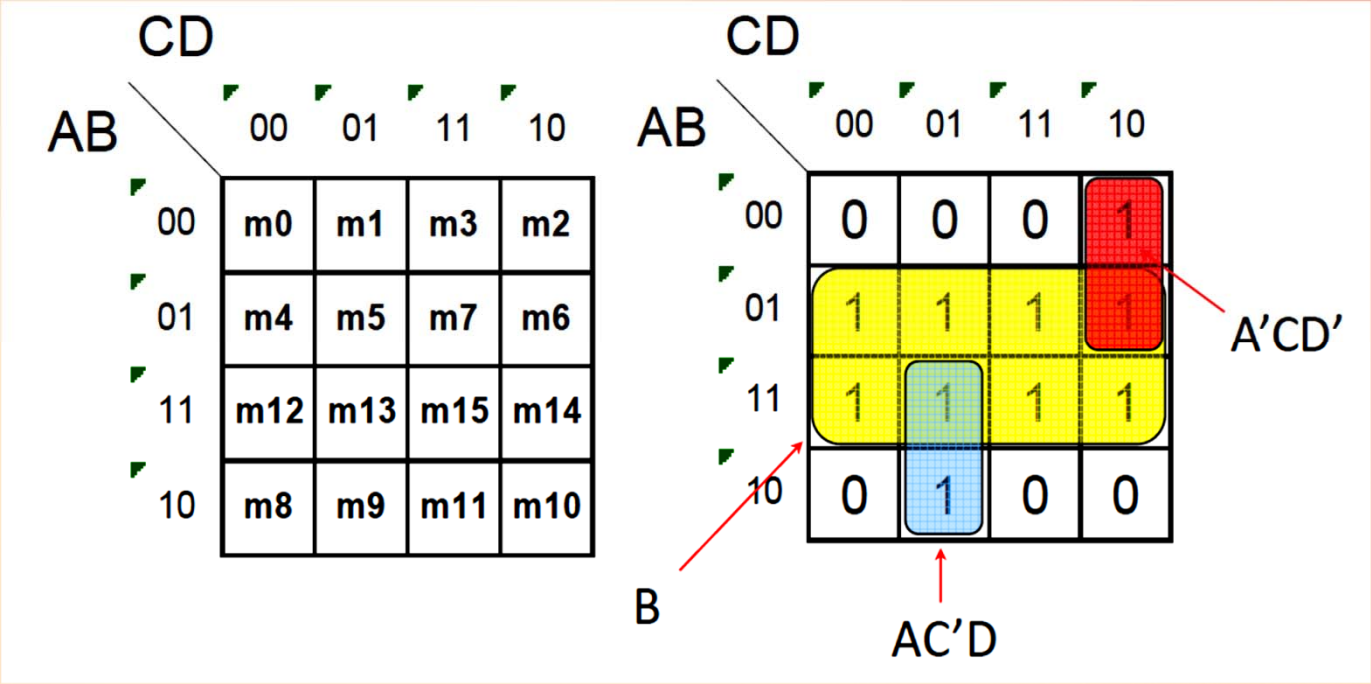
No. Variables	No. cells	No. of adjacent for each cell	No. of variables for one minterm	No. of variables for two Groups	No. of variables for four Groups	No. of variables for eight Groups
1						
2						
3						
4						
5						
6						
n						



Outline

- Karnough map simplification
  - 4 variable karnaugh map
  - Don't care condition
  - Algorithm for better grouping
- Karnaugh map with >= 5 variable

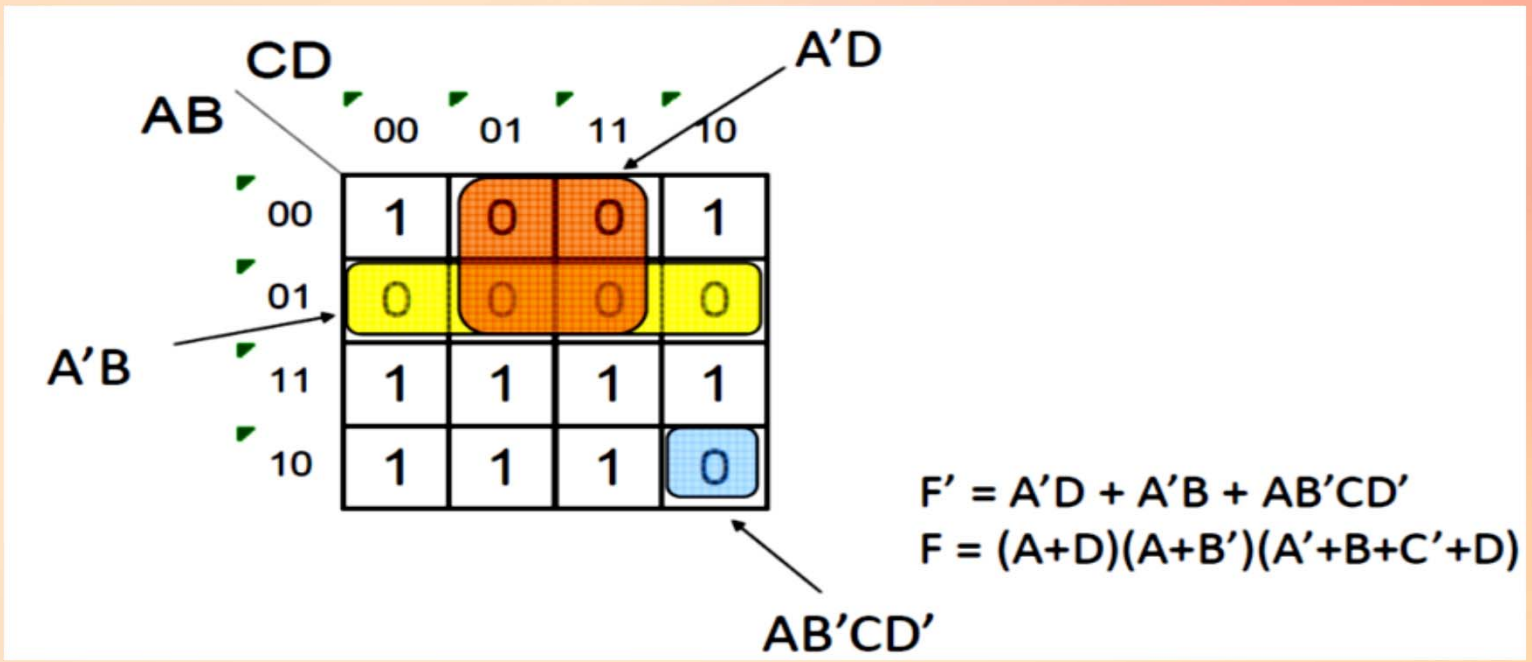
4-Variable Karnaugh Map



F =AC'D + A'CD' + B

Note the row and column orderings. Required for adjacency

Find a POS Solution



Find solutions to groups of 0's to find F' Invert to get F then use DeMorgan's

Don’t Care

- A **don't-care term** is an input to a function that the designer does not care about
- Because that input would never happen
- Example:
  - **BCD number (0-9, A-F) are 4 bits, don’t care about input A-F**

  - **Suppose a system have 5 type of input**

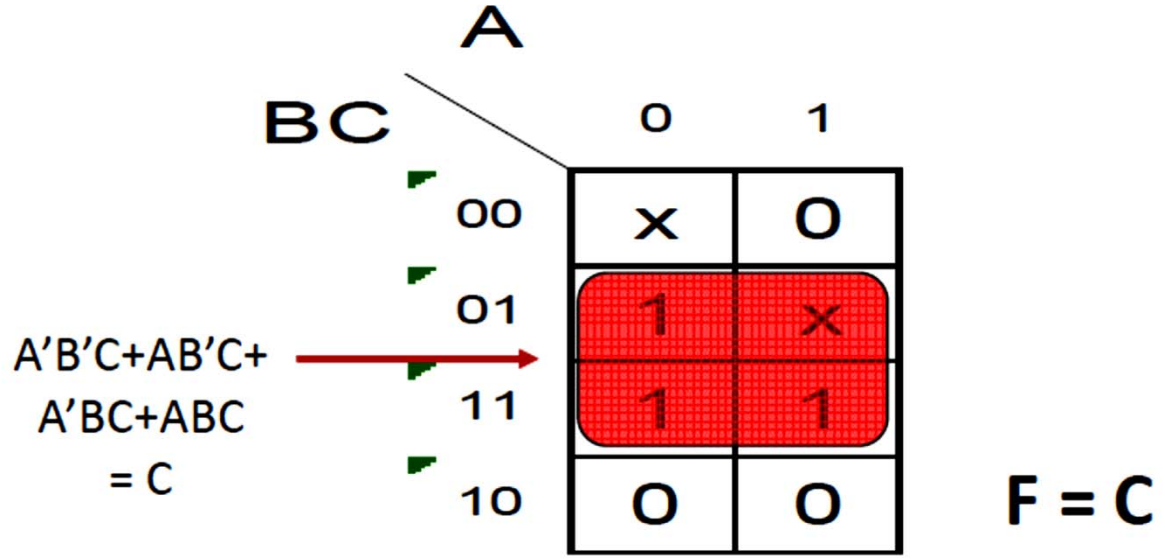
    Unfortunately we can’t have 2 input line

    Make 3 input line and last 3 sequence as don’t care

    S0, S1, S2,S3,S4, X,X,X == > 000, 001….,111

Dealing With Don't Cares

$F = \sum m(1, 3, 7) + \sum d(0, 5)$



Circle the x's that help get bigger groups of 1's
Don't circle the x's that don't

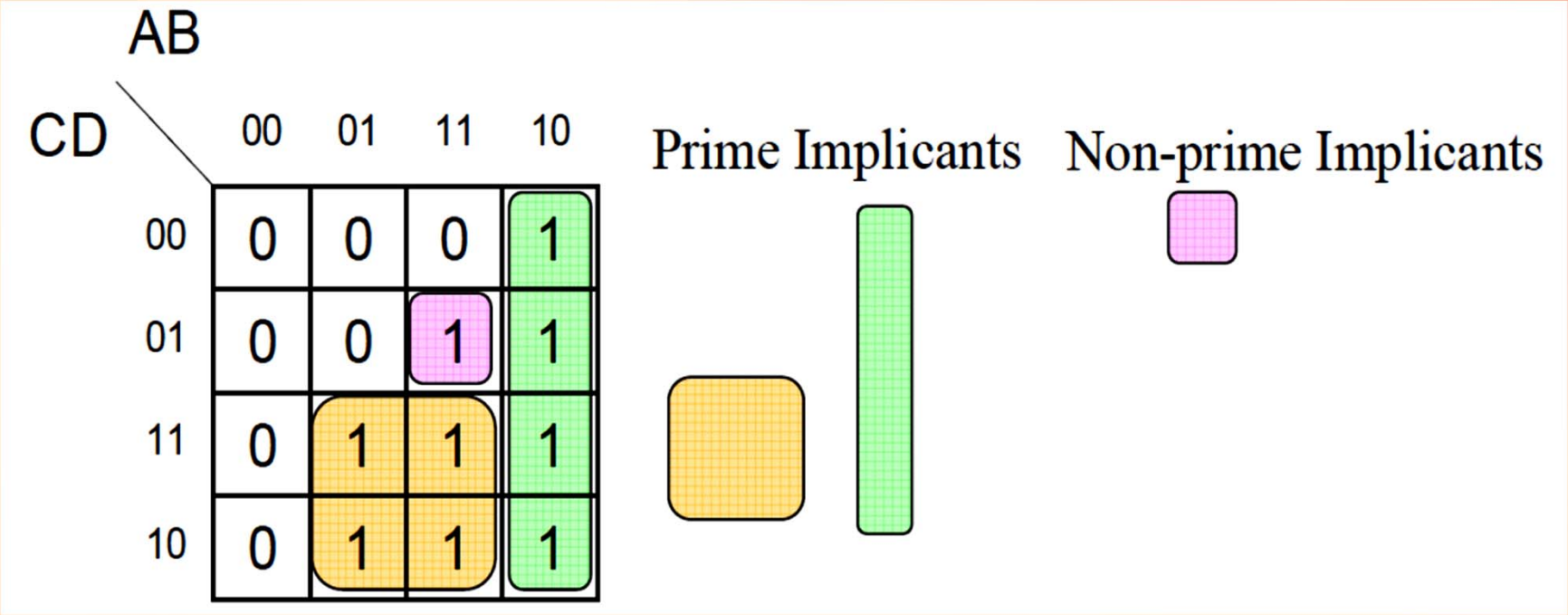
Prime Implicants

• A group of one or more 1's which are adjacent and can be combined on a Karnaugh Map is called an **implicant**.

• The ***biggest*** **group** of 1's which can be circled to cover a given 1 is called a **prime implicant**.

–They are the only implicants we care about.

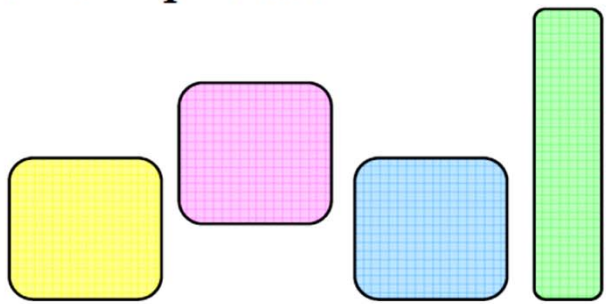
Prime Implicants



Are there any additional prime implicants in the map that are not shown above?

All The Prime Implicants

CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1



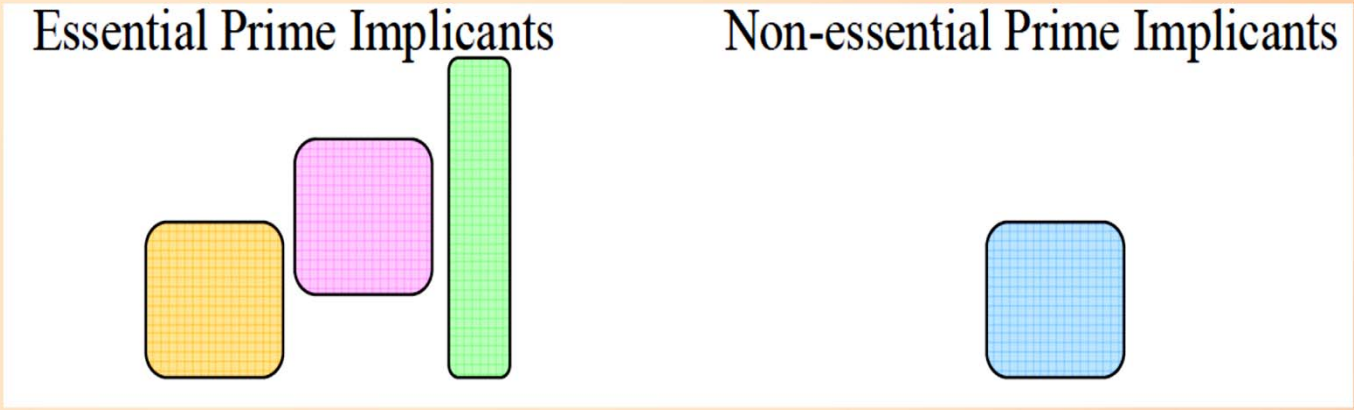
When looking for a minimal solution – *only* circle prime implicants…
A minimal solution will *never* contain non-prime implicants

Essential Prime Implicants

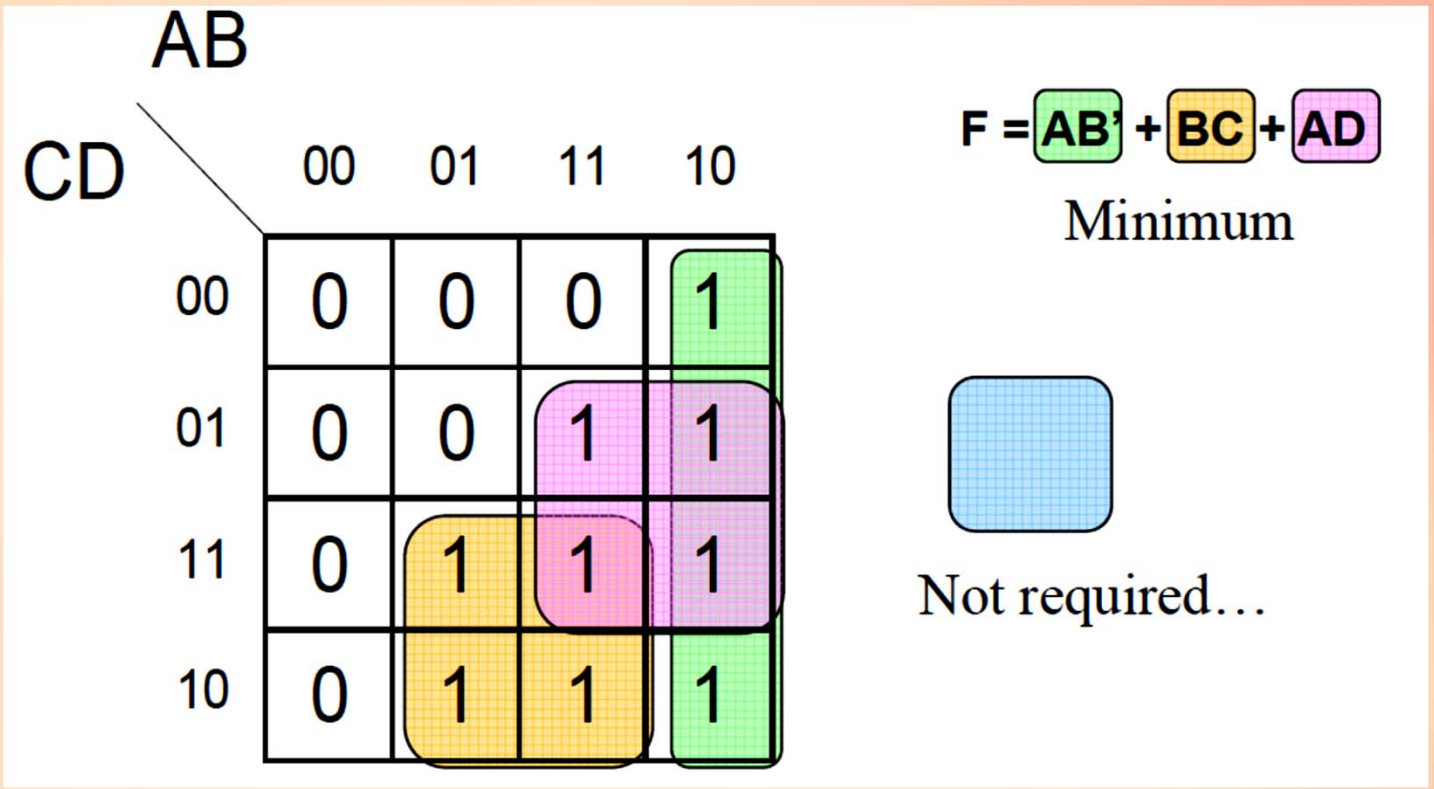
CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1

Not all prime implicants are required

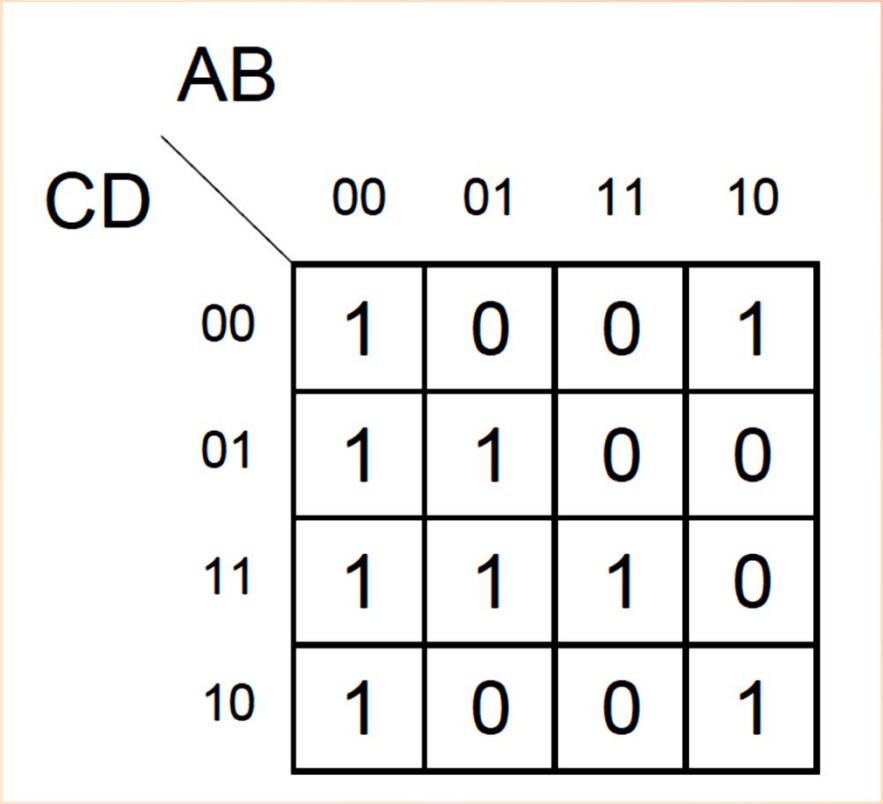
A prime implicant which is the only cover of some 1 is ***essential*** – a minimal solution requires it.



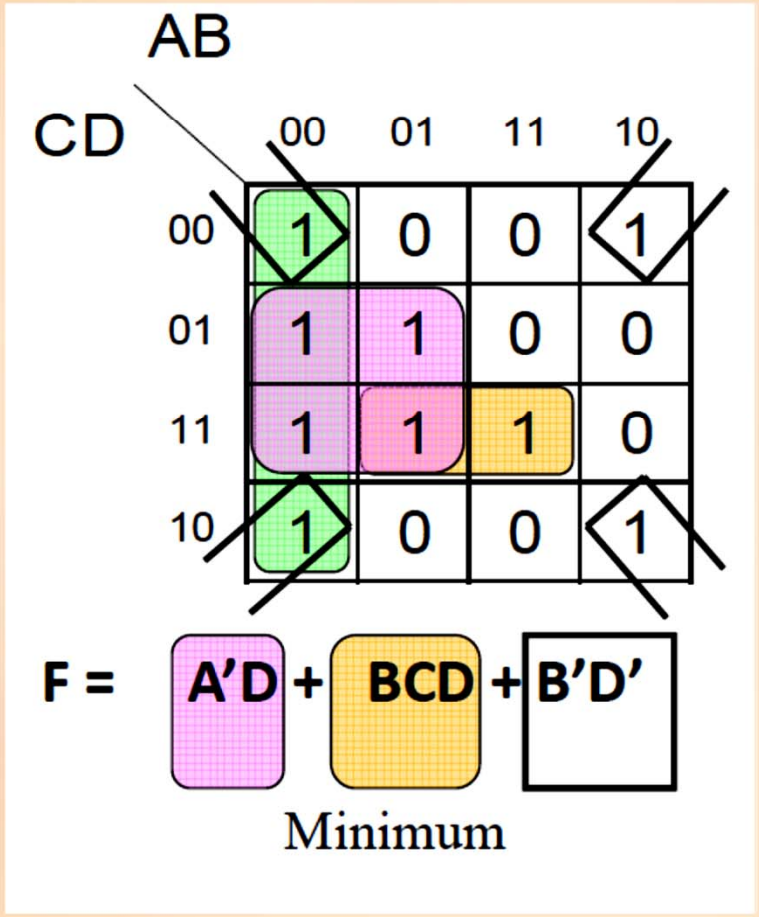
A Minimal Solution Example

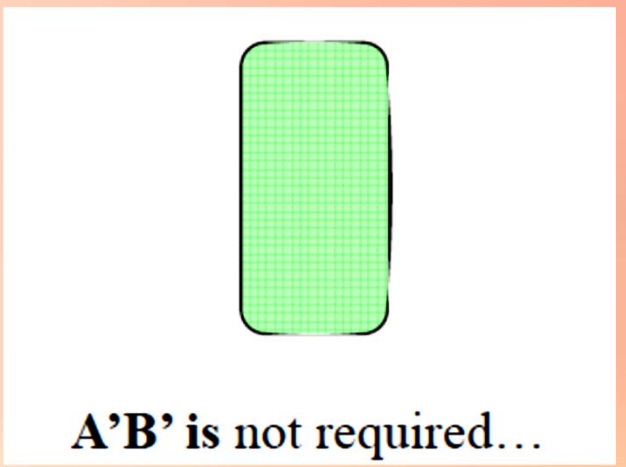


Another Example



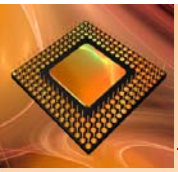
CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	1	0
10	1	0	0	1





A'B' is not required…
Every one of its locations is covered by multiple implicants

After choosing essentials Minimum **essentials**, everything is covered…



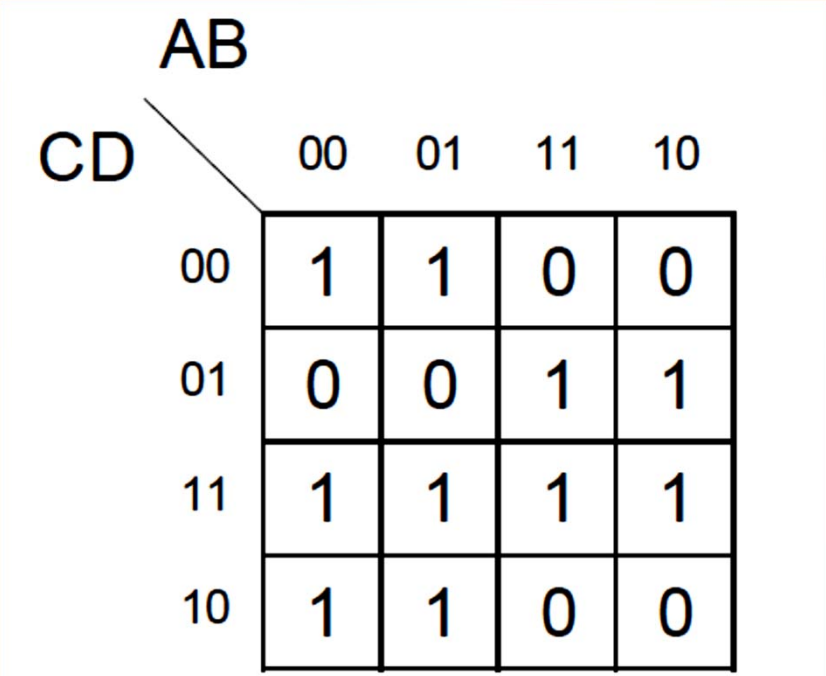
Finding the Minimum Sum of Products

1. Find each essential prime implicant and include it in the solution.

2. Determine if any minterms are not yet covered.

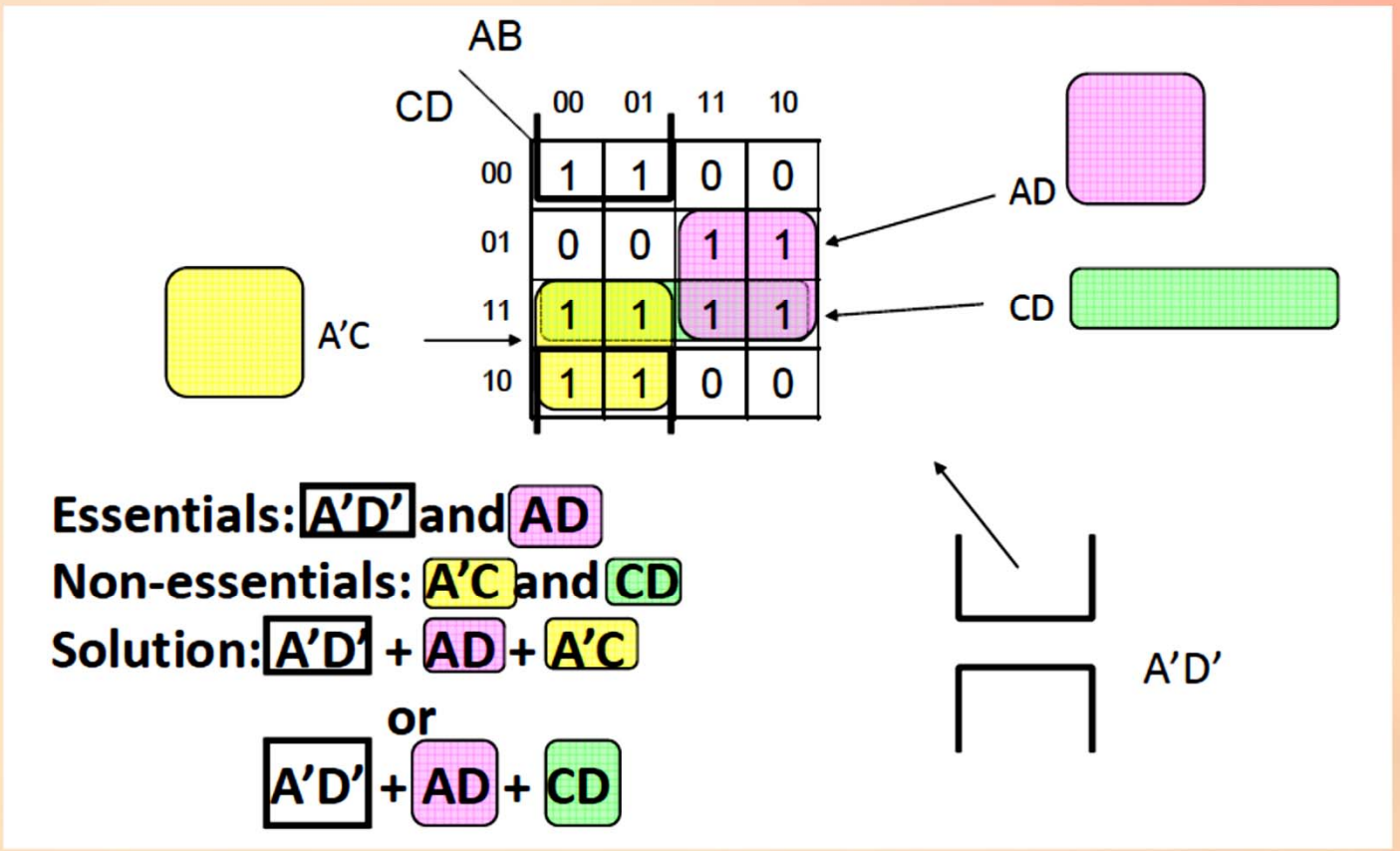
3. Find the minimal # of remaining prime implicants which finish the cover.

Another Example Use of non-essential primes

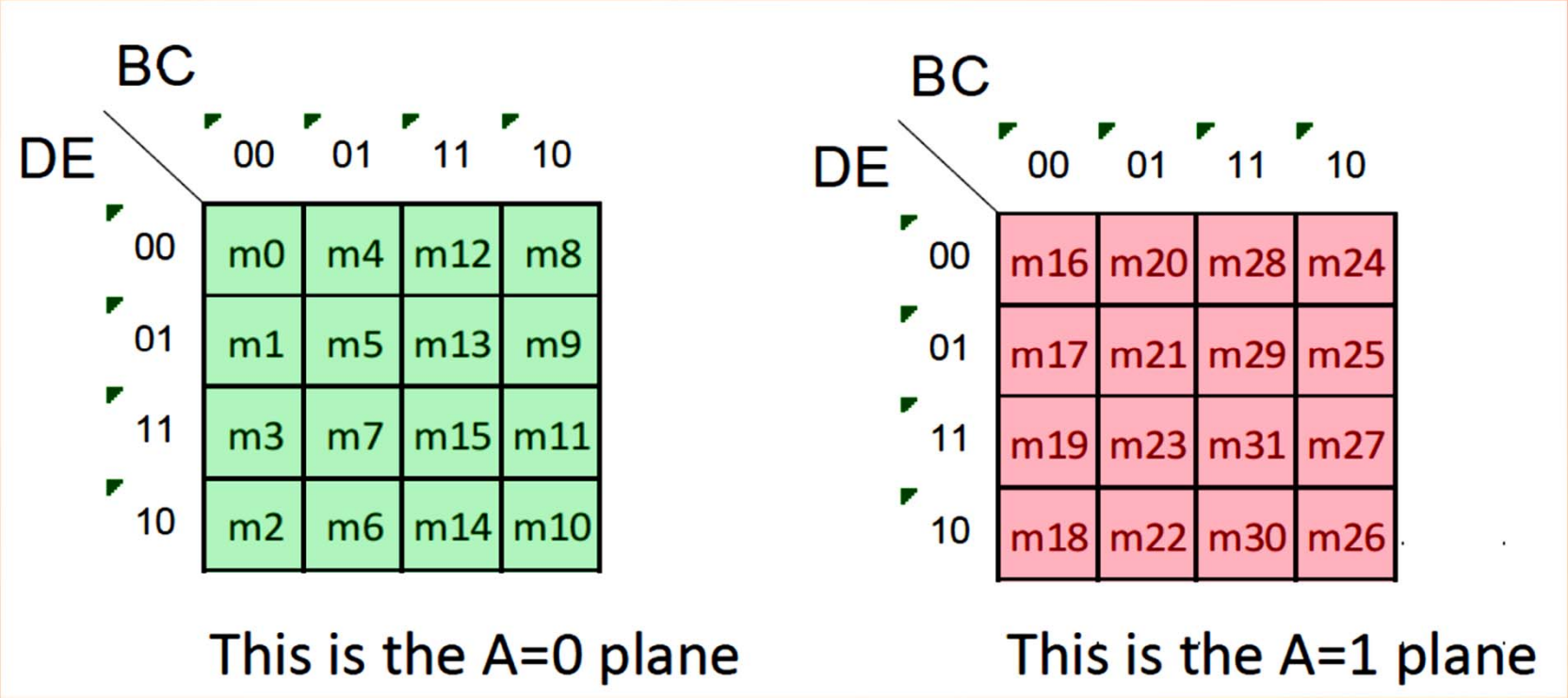


CD \ AB	00	01	11	10
00	1	1	0	0
01	0	0	1	1
11	1	1	1	1
10	1	1	0	0

Another Example Use of non-essential primes

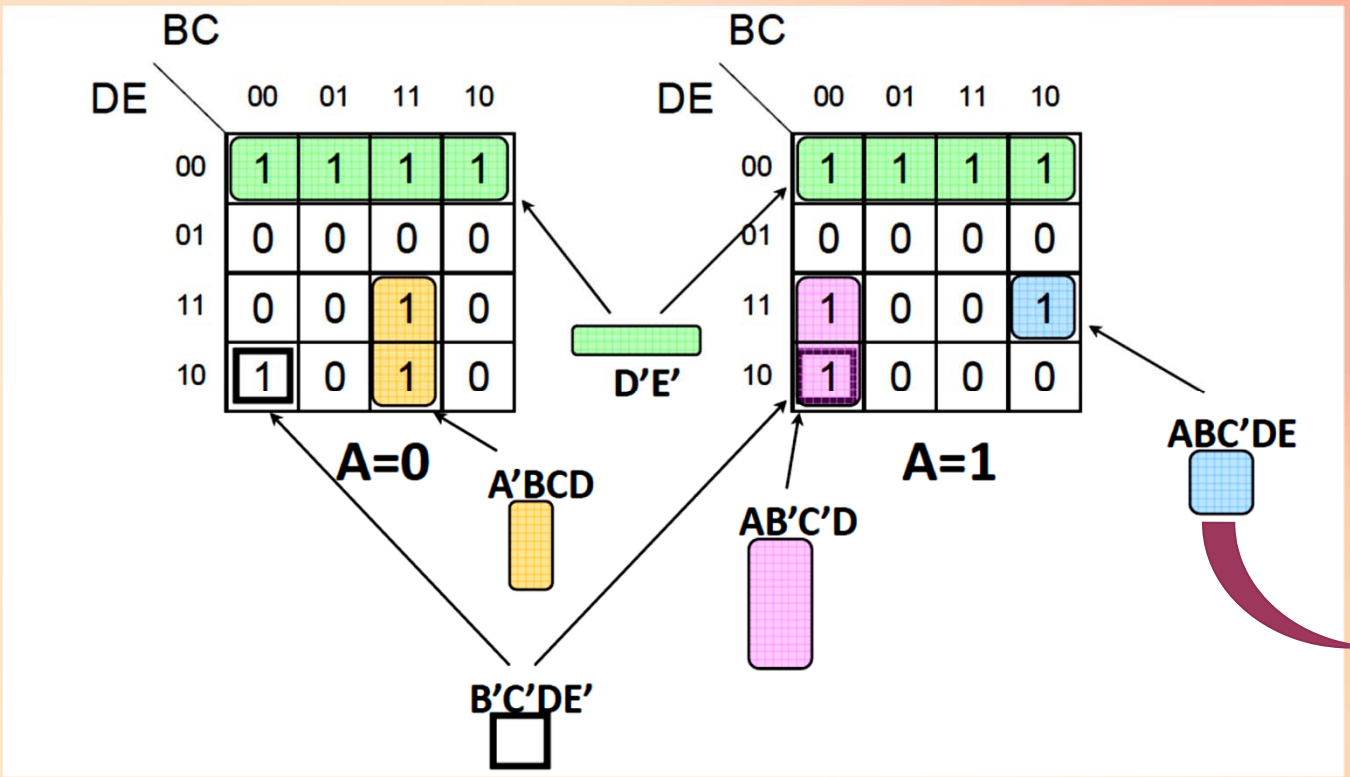


5-Variable Karnaugh Map



The planes are adjacent to one another one is above the other in 3D)

Some Implicants in a 5-Variable KMap

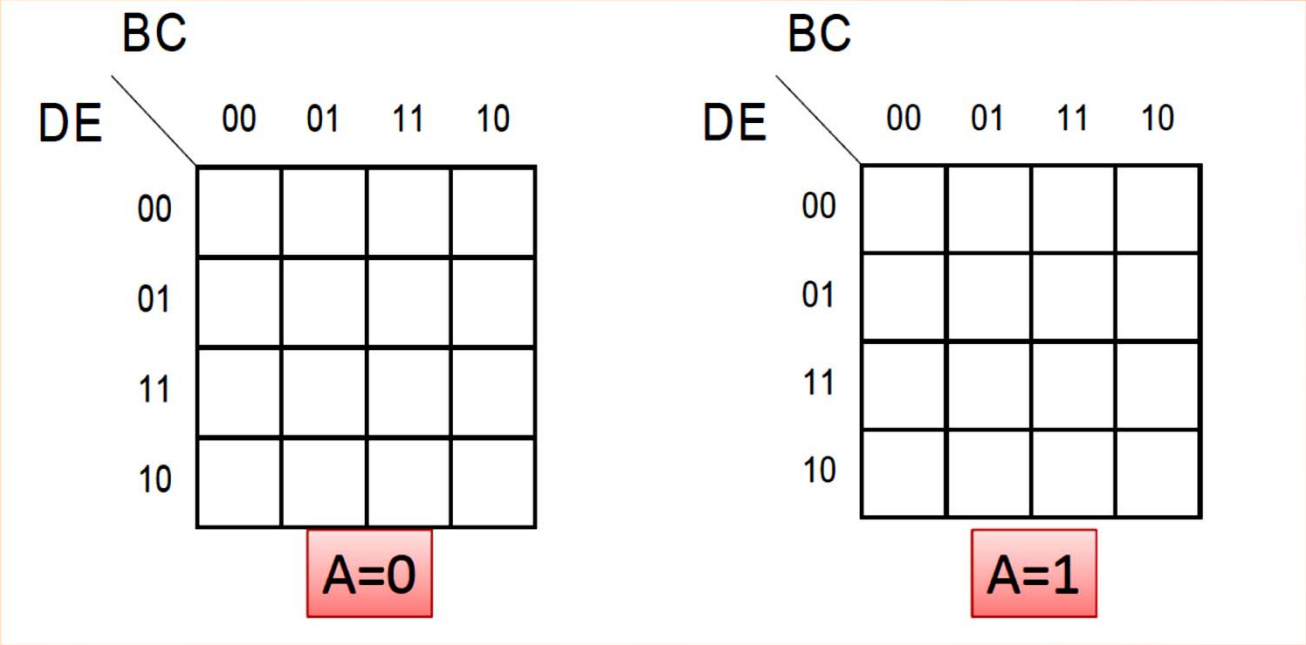


Some of these are not prime…

5-Variable KMap Example

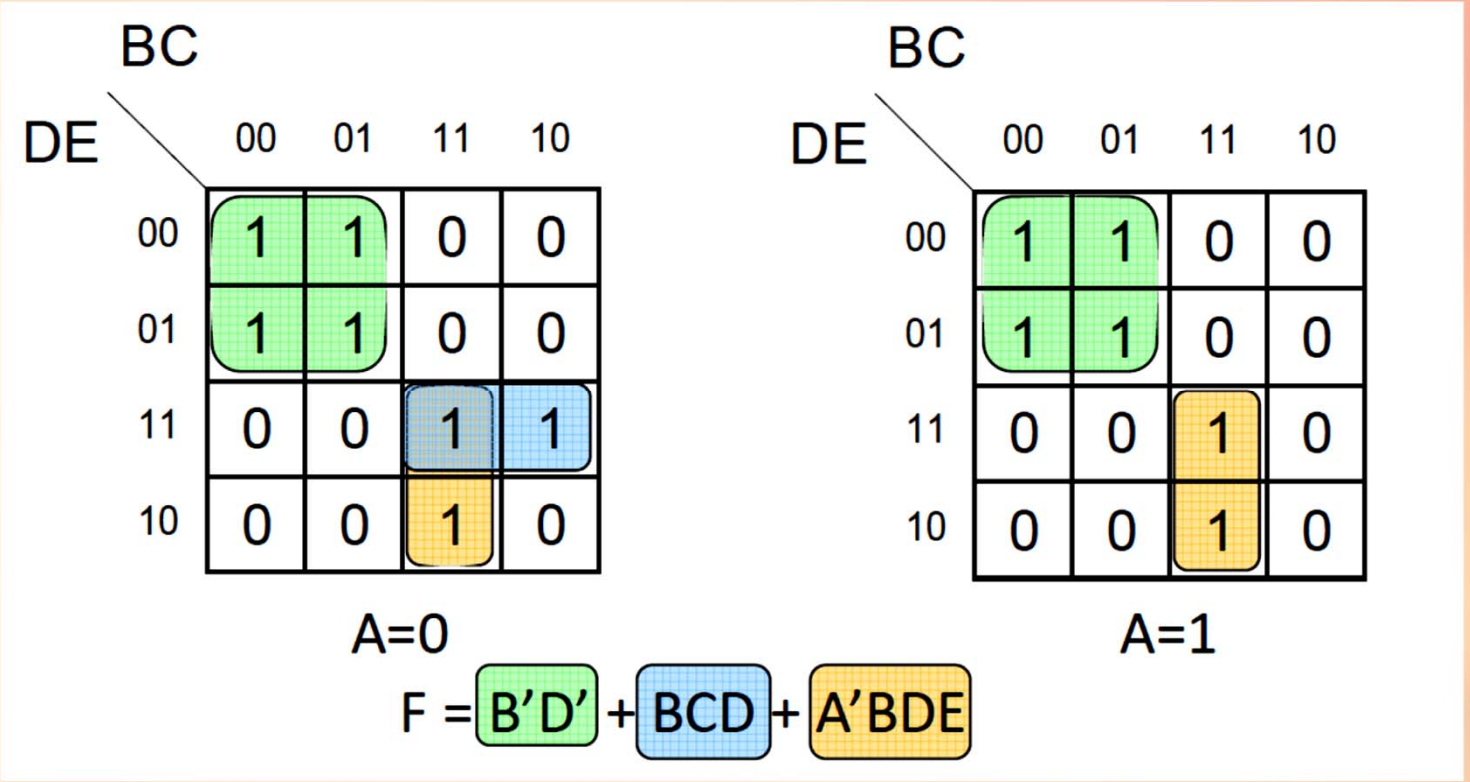
Find the minimum sum-of-products for:

$F = \Sigma m\ (0,1,4,5,11,14,15,16,17,20,21,30,31)$

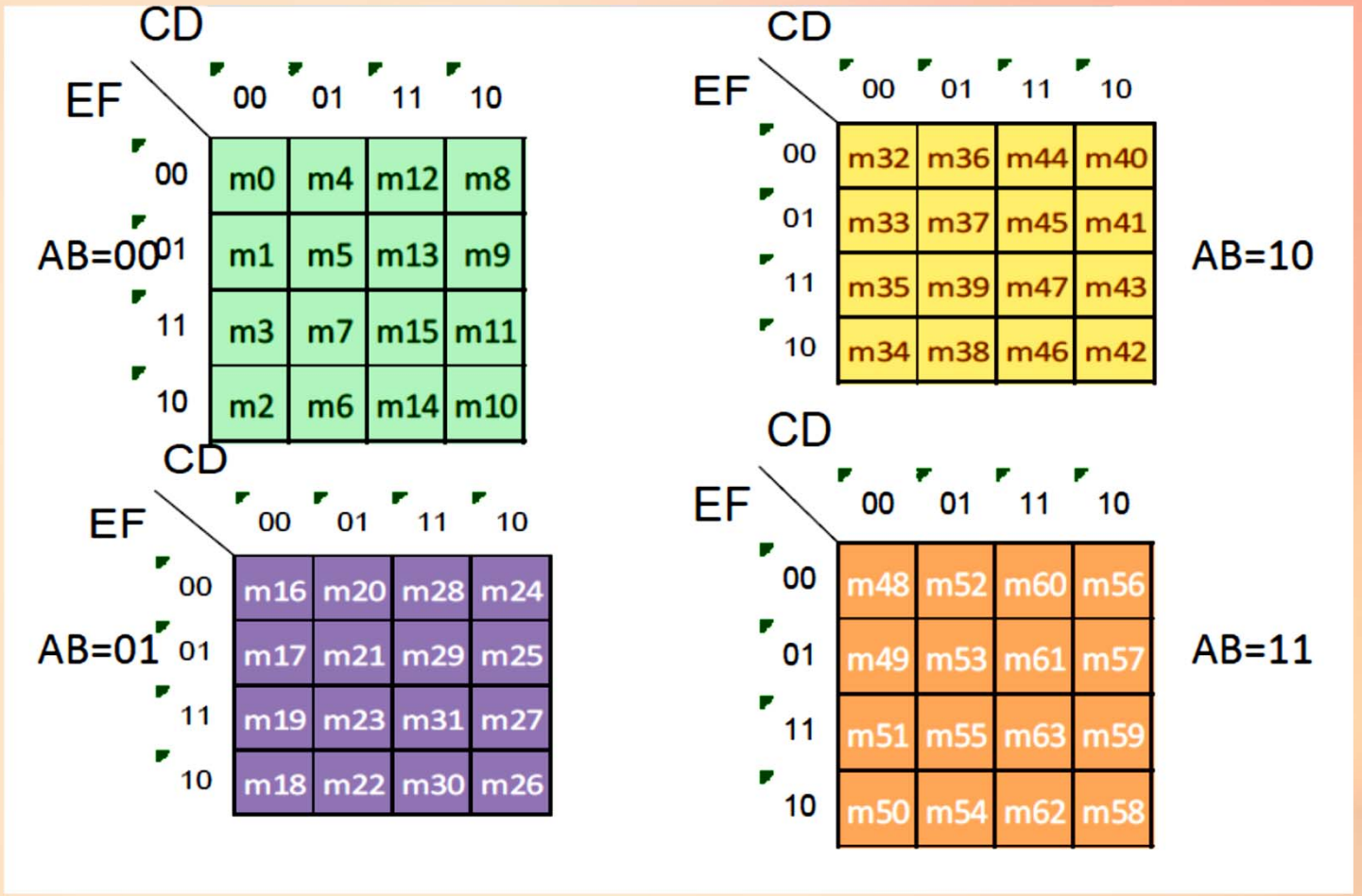


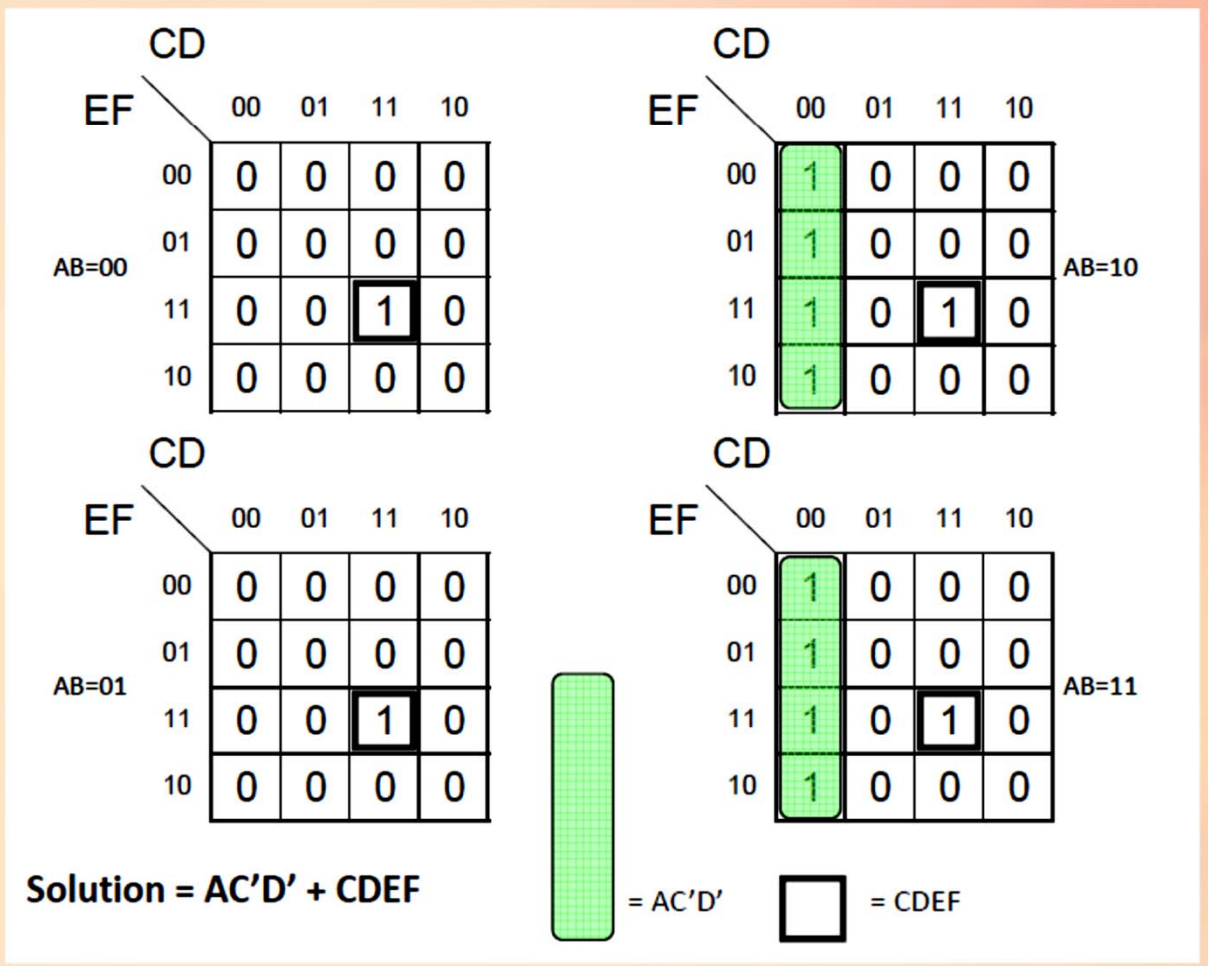
5-Variable KMap Example

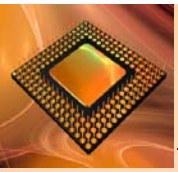
Find the minimum sum-of-products for:
F = Σ m (0,1,4,5,11,14,15,16,17,20,21,30,31)



6-Variable Karnaugh Map




CD
EF
00 01 11 10
00 0 0 0 0
01 0 0 0 0
11 0 0 1 0
10 0 0 0 0
AB=00
CD
EF
00 01 11 10
00 1 0 0 0
01 1 0 0 0
11 1 0 1 0
10 1 0 0 0
AB=10
CD
EF
00 01 11 10
00 0 0 0 0
01 0 0 0 0
11 0 0 1 0
10 0 0 0 0
AB=01
CD
EF
00 01 11 10
00 1 0 0 0
01 1 0 0 0
11 1 0 1 0
10 1 0 0 0
AB=11
Solution = AC'D' + CDEF
= AC'D'
= CDEF

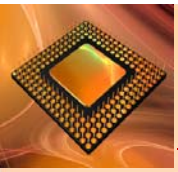


KMap Summary

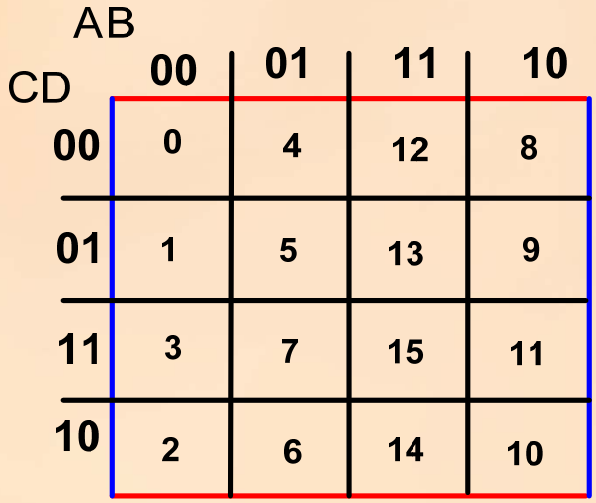
- A Kmap is simply a folded truth table
– where physical adjacency implies logical adjacency

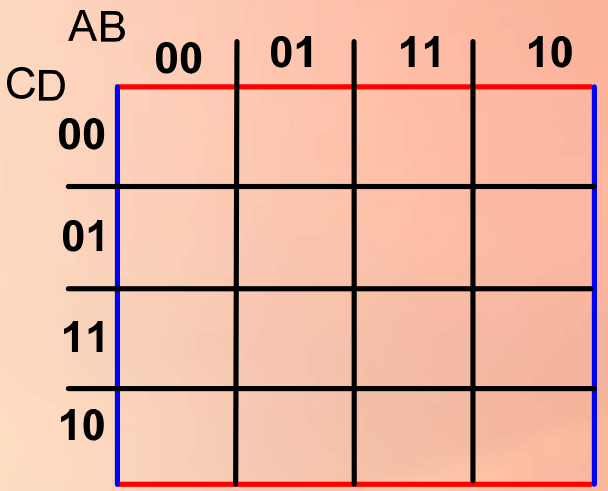
- KMaps are most commonly used hand method for logic minimization

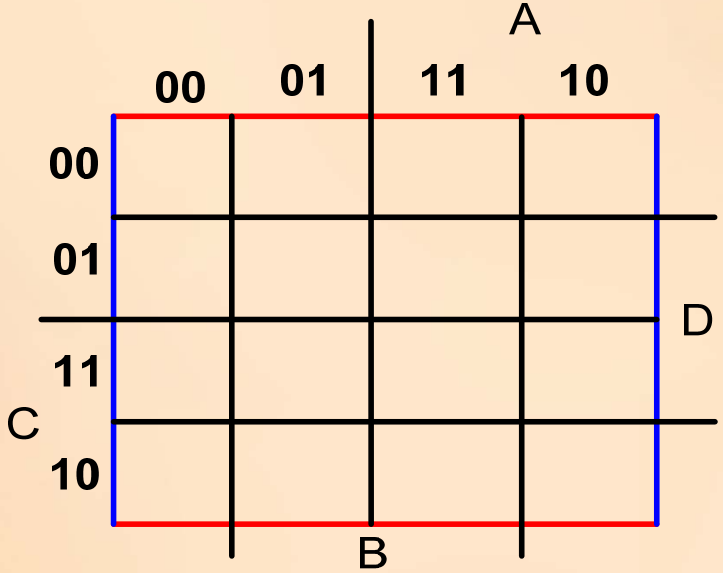
- KMaps have other uses for visualizing Boolean Equations – you may see some later.

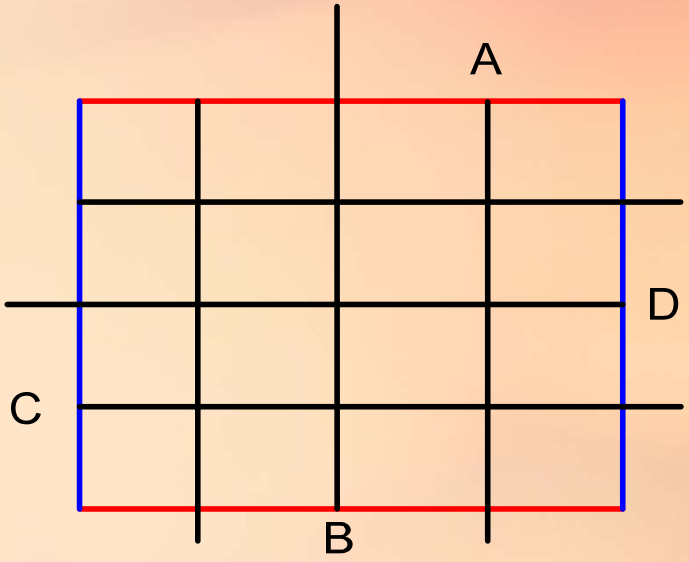


KARNAUGH MAP REVIEW


AB
CD
00
01
11
10
0
4
12
8
1
5
13
9
3
7
15
11
2
6
14
10


AB
CD
00
01
11
10


A
00
01
11
10
D
C
B

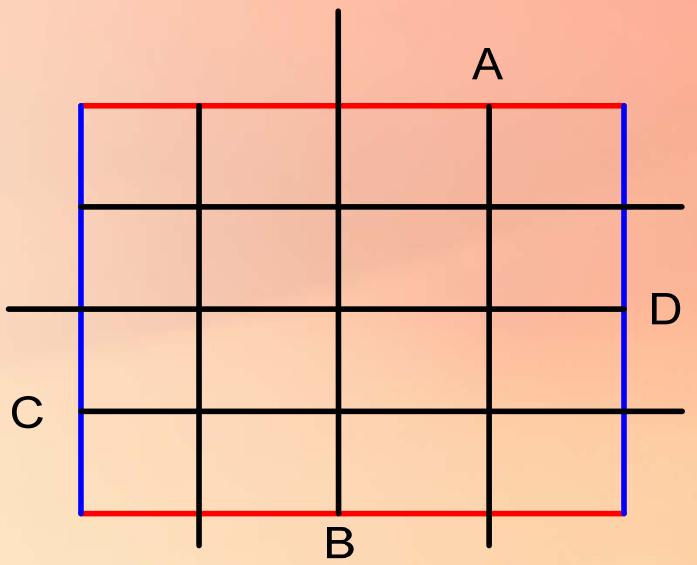

A
D
C
B

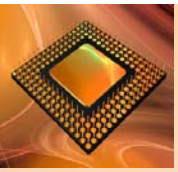
MSB LSB

$Y = F(A, B, C, D)$

$Y = F(8, 4, 2, 1)$

AB / CD	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

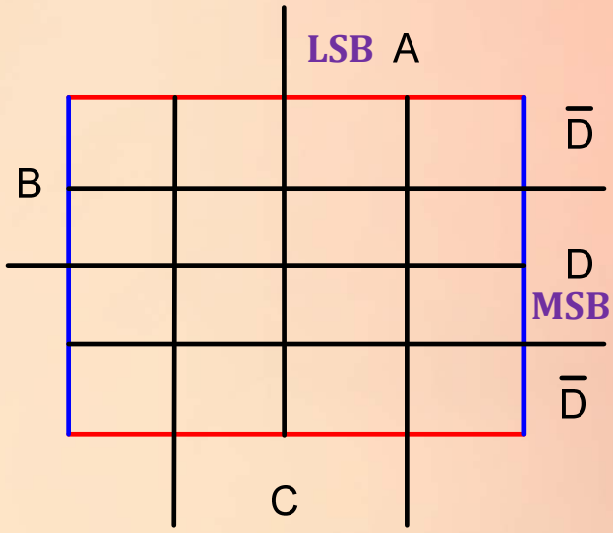


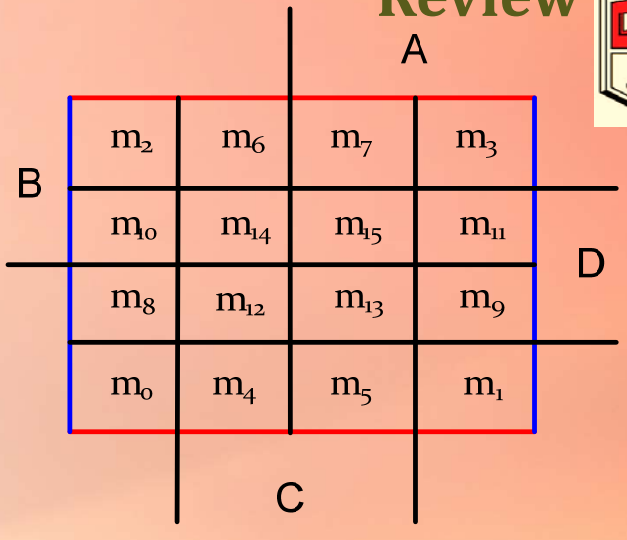


LSB MSB

Y= F(A, B, C, D)

Y= F(1, 2, 4, 8)

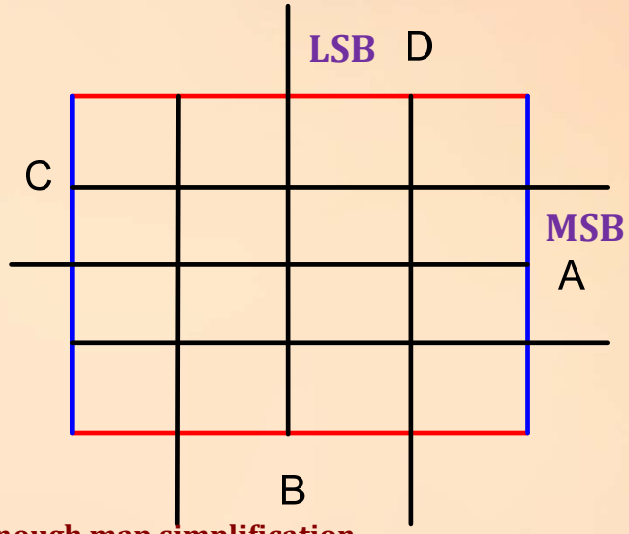


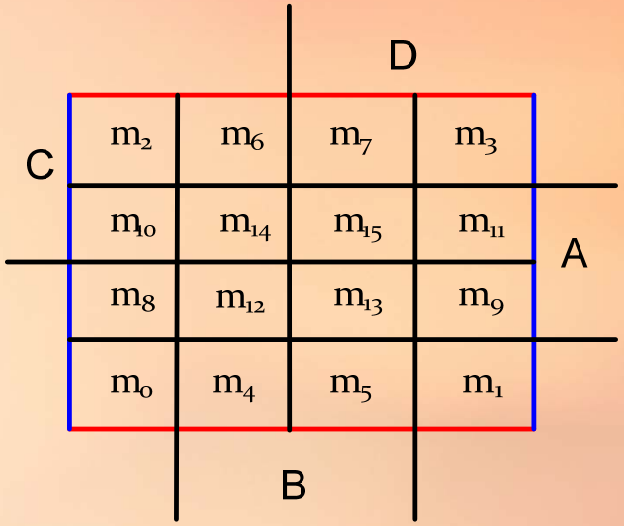


MSB LSB

Y= F(A, B, C, D)

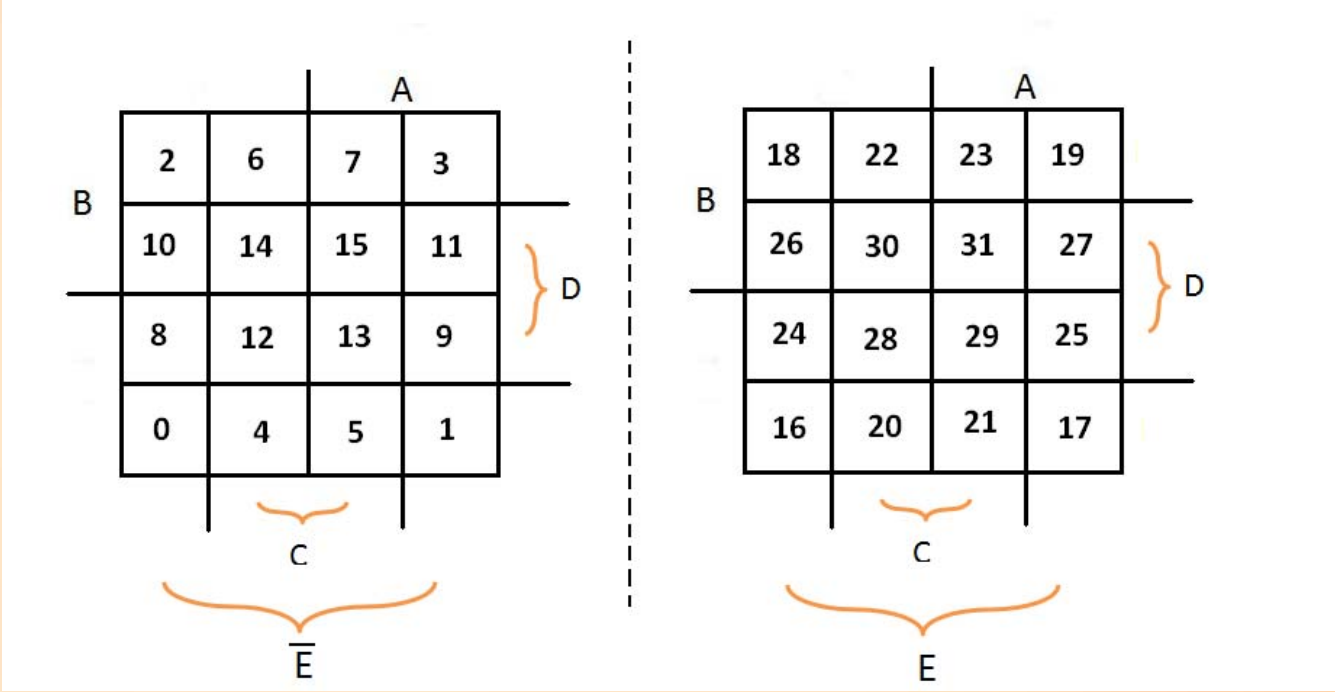
Y= F(8, 4, 2, 1)





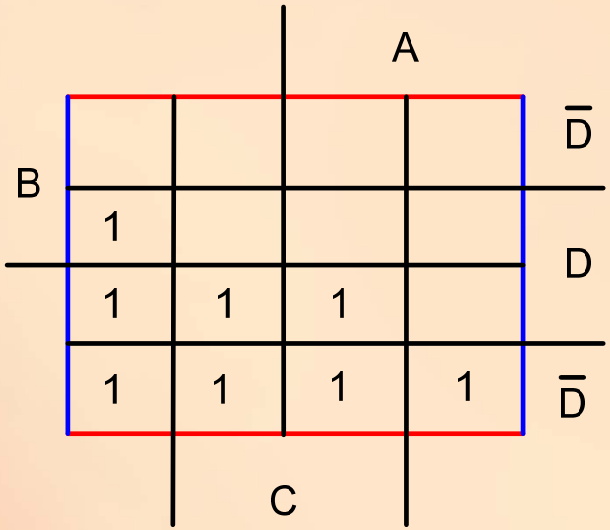
D	0	1	0	1	0	1	0	1
C	0	0	1	1	0	0	1	1
B	0	0	0	0	1	1	1	1
A								
A								

D	0	1	0	1	0	1	0	1
C	0	0	1	1	0	0	1	1
B	0	0	0	0	1	1	1	1
A	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
A	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}

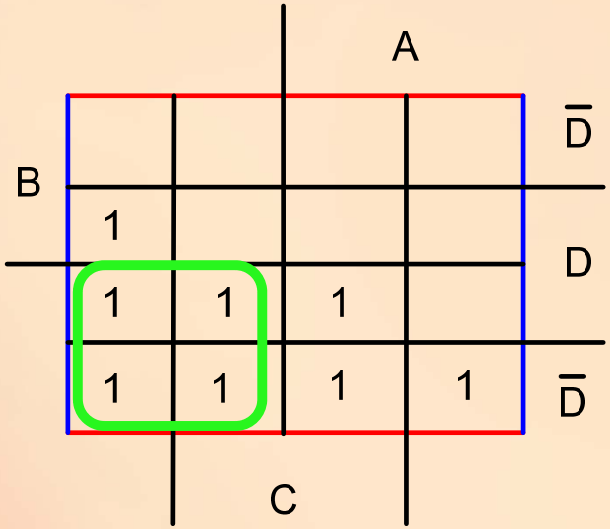

A
B
2
6
7
3
10
14
15
11
8
12
13
9
0
4
5
1
D
C
$\overline{E}$
A
B
18
22
23
19
26
30
31
27
24
28
29
25
16
20
21
17
D
C
E

Mandatory Loop or Mandatory Grope for minimization

Example

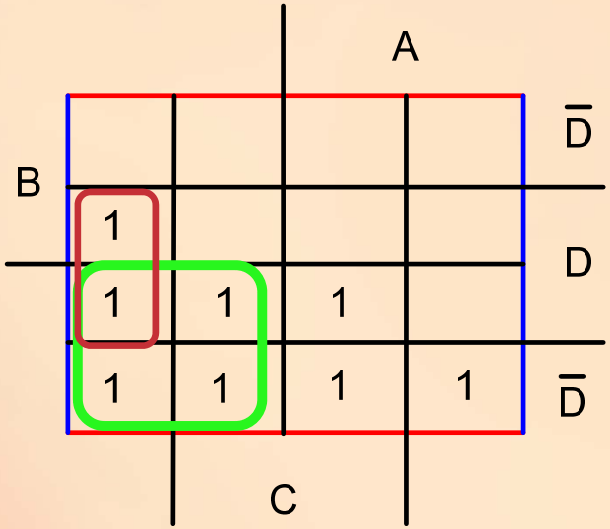


Example



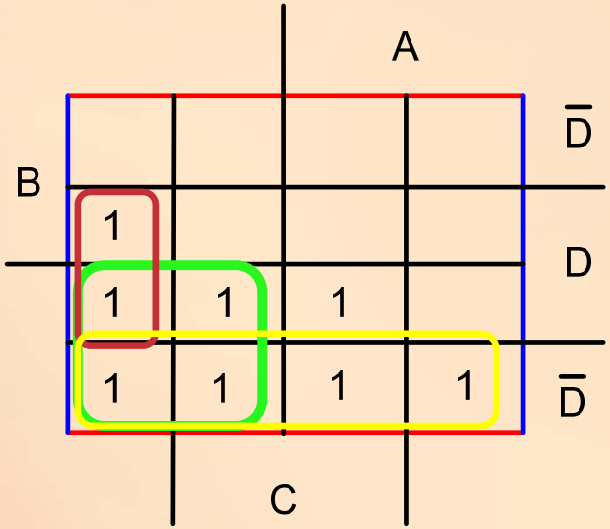
A'B'

Example



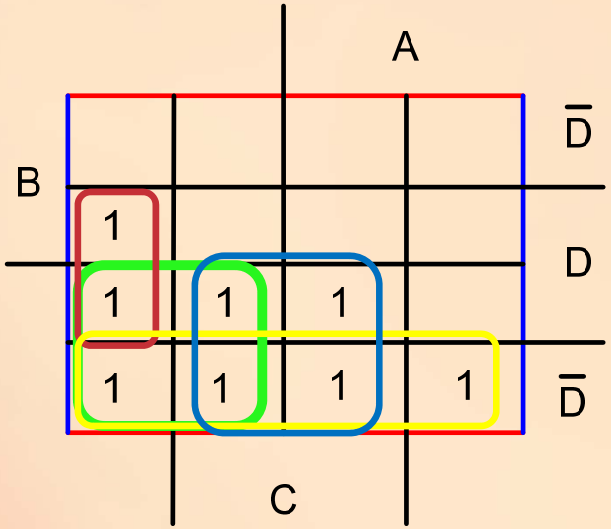
A'B' + A'C'D

Example



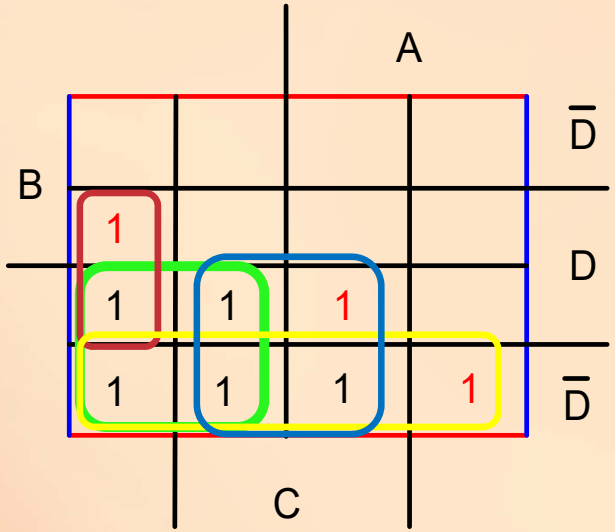
A'B'+ A'C'D + B'D'

Example



A'B'+ A'C'D + B'D'+ CD

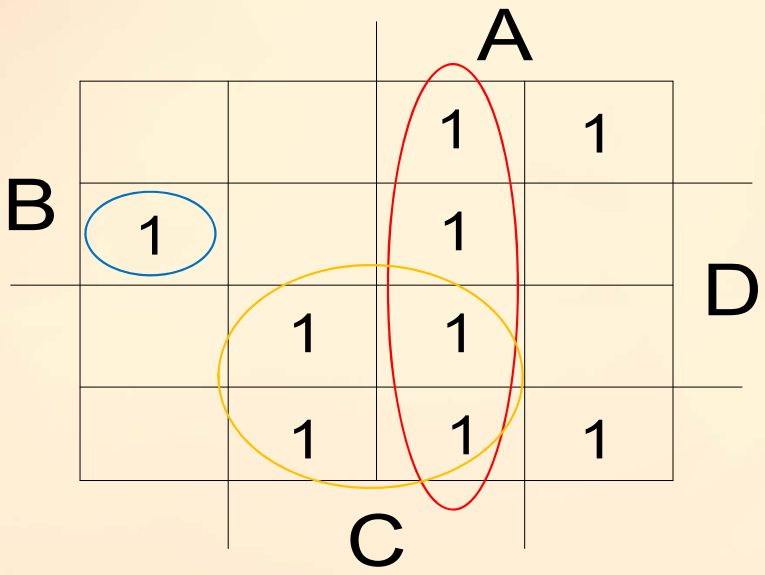
Example



A'B'+ A'C'D + B'D'+ CD

A'C'D + B'D'+ CD

Example

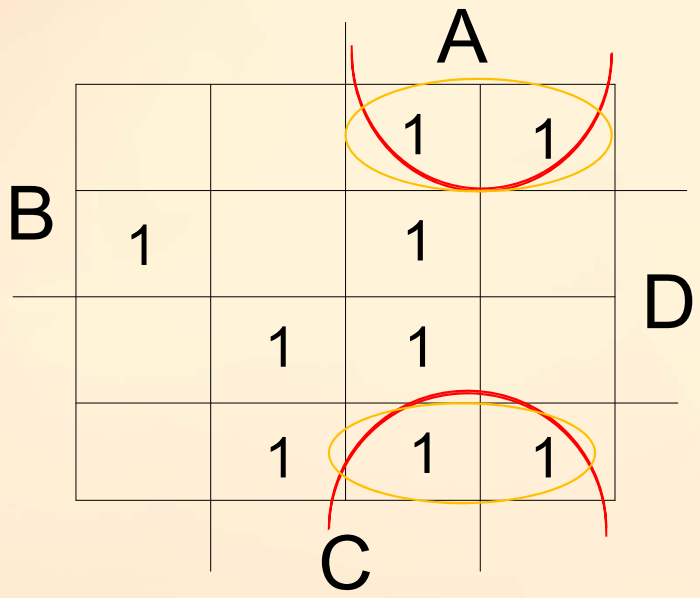


$$Y = \overline{A}\,B\,\overline{C}\,D$$

$$Y = A\,C$$

$$Y = \overline{B\,C}$$

Example

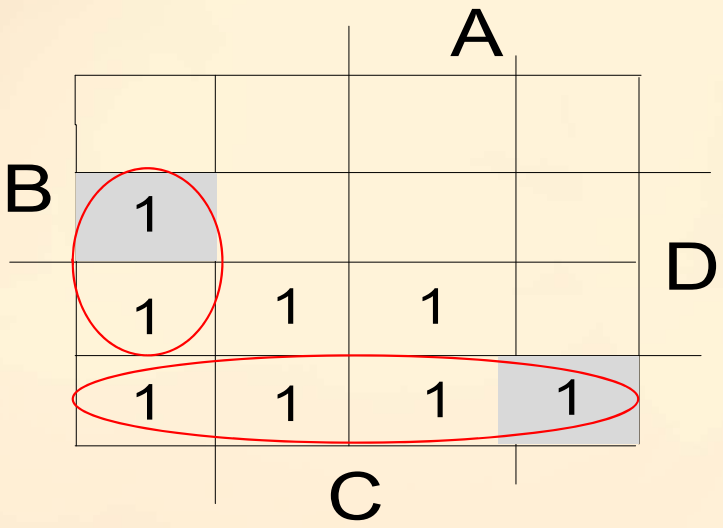


$$Y = A\,B\,\overline{D}$$

$$Y = A\,\overline{B}\,\overline{D}$$

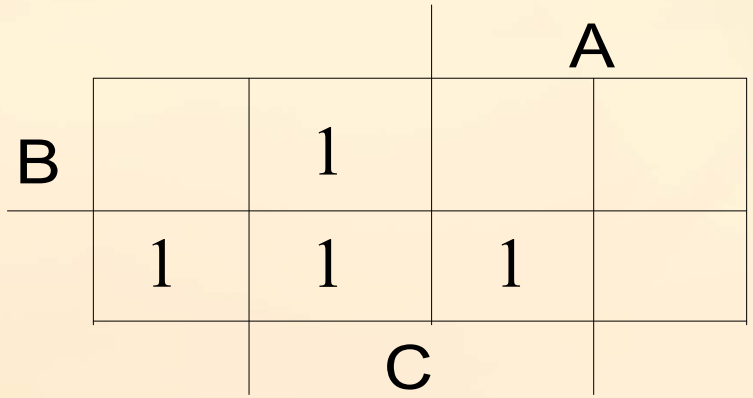
$$Y = A\,\overline{D}$$

Example



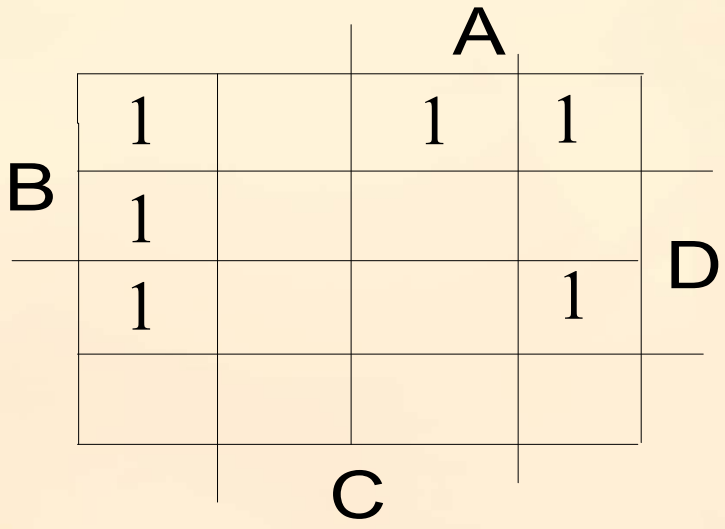
Example

A) $f(A, B, C) = \overline{A}\,\overline{B} + \overline{A}C + \overline{B}C$

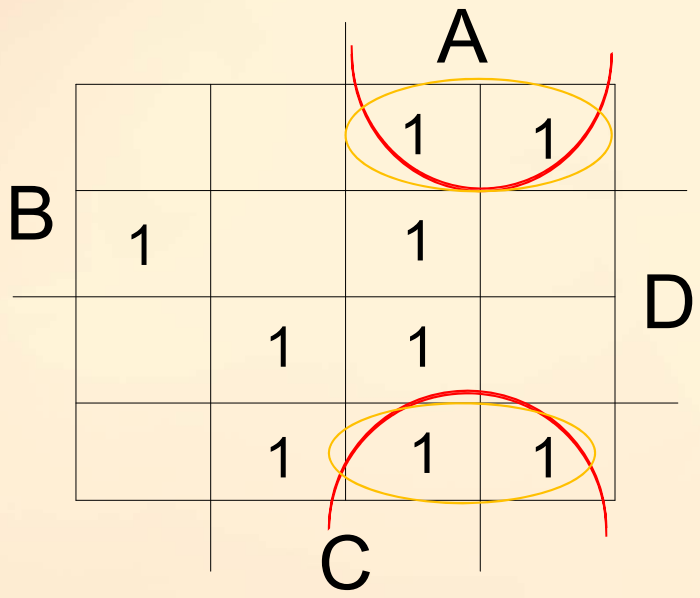


Example

B) $f(A,B,C,D) = AB\overline{D} + \overline{A}B\overline{C} + \overline{B}\overline{C}D$



Example

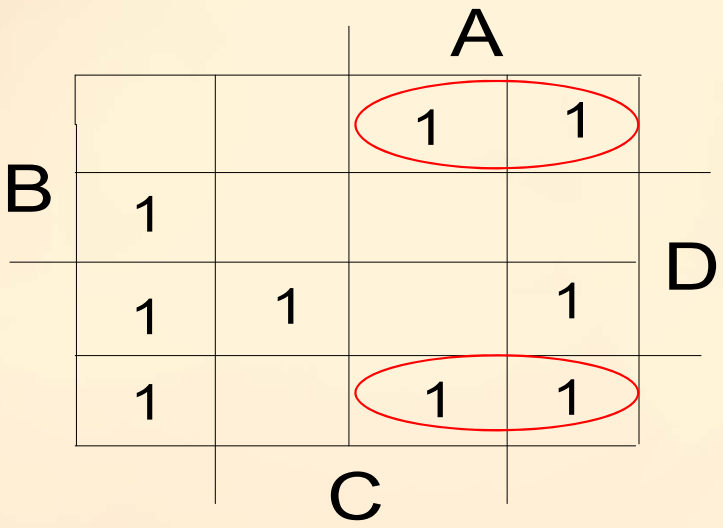


$$Y = A\,B\,\overline{D}$$

$$Y = A\,\overline{B}\,\overline{D}$$

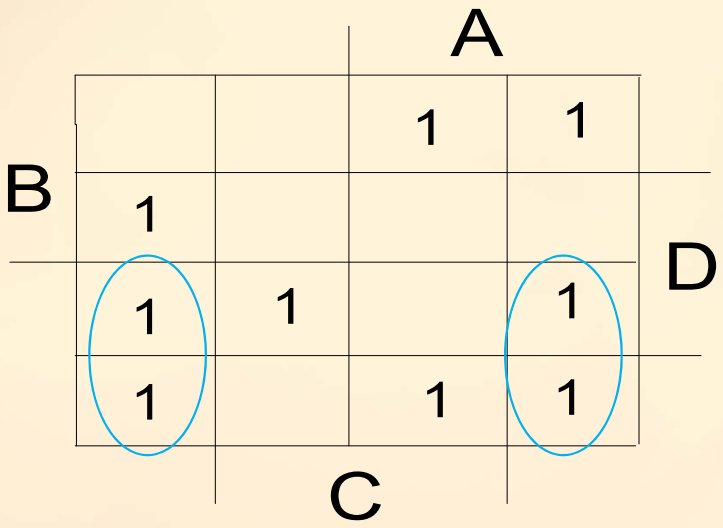
$$Y = A\,\overline{D}$$

Example



$$F = A\,\overline{D}$$

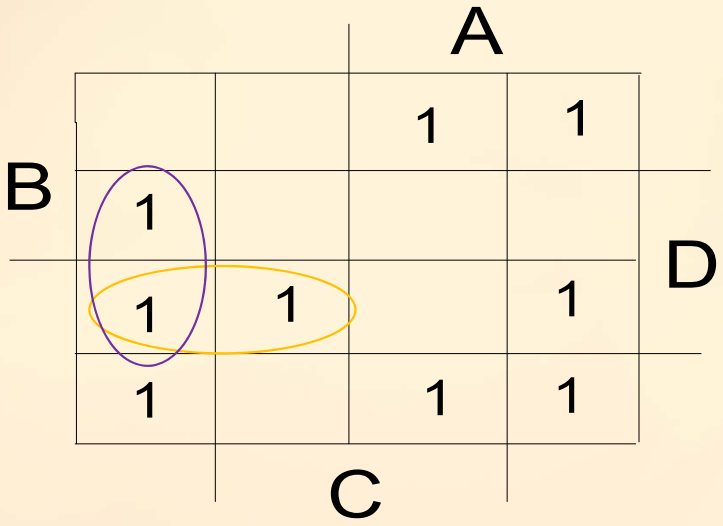
Example



$$F = A\overline{D}$$

$$F = \overline{B}\,\overline{C}$$

Example

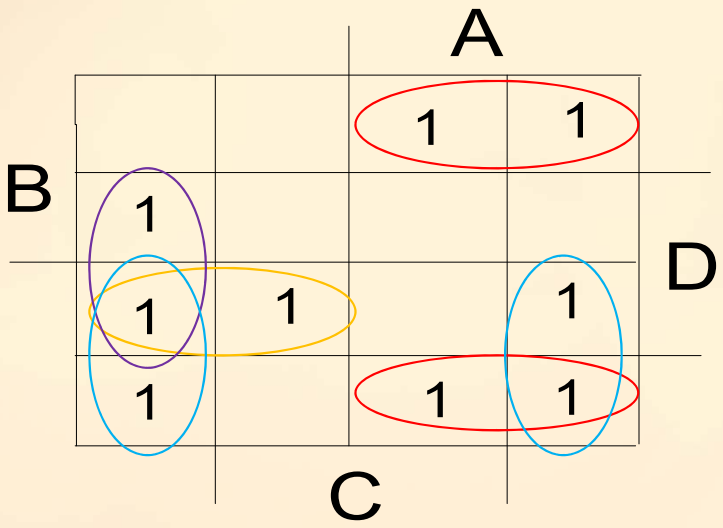


$$F = A\overline{D} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,D$$

$$F = \overline{A}\;\overline{B}\;D$$

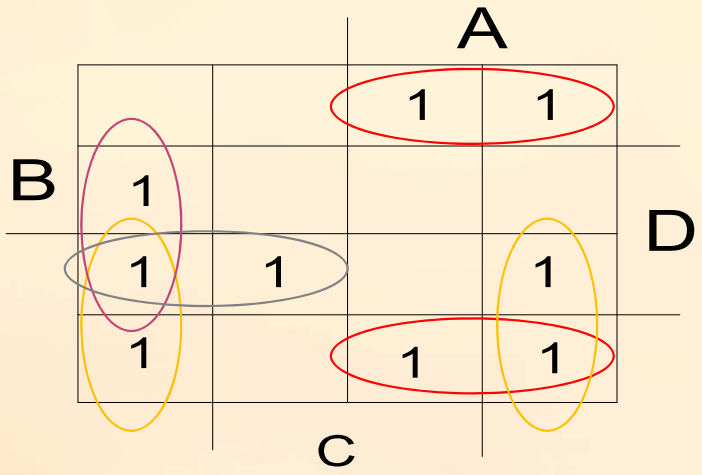
$$F = \overline{A}\;\overline{C}\;D$$

Example



$$F = A\overline{D} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}D + \overline{A}\,\overline{C}D$$

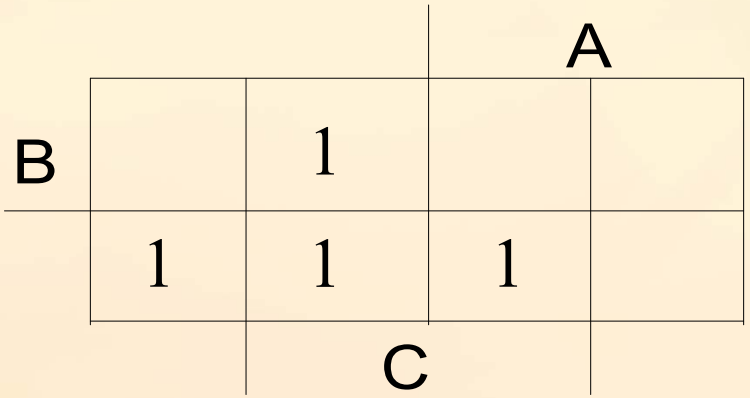
Example



$$Y = A\overline{D} + \overline{B}\,\overline{C} + \overline{A}CD + \overline{A}\,\overline{B}D$$

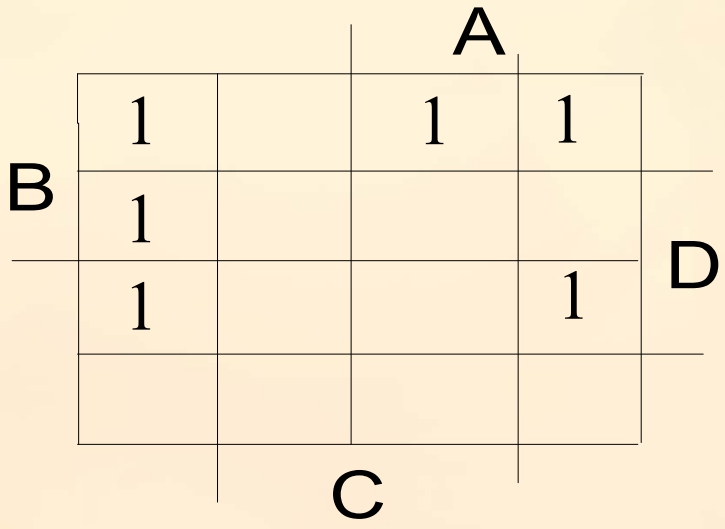
Example

A) $f(A, B, C) = \overline{A}\,\overline{B} + \overline{A}C + \overline{B}C$



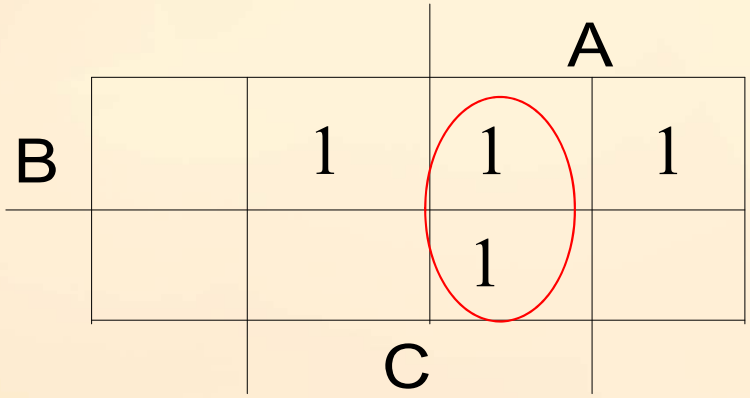
Example

B) $f(A, B, C, D) = AB\overline{D} + \overline{A}B\overline{C} + \overline{B}\overline{C}D$



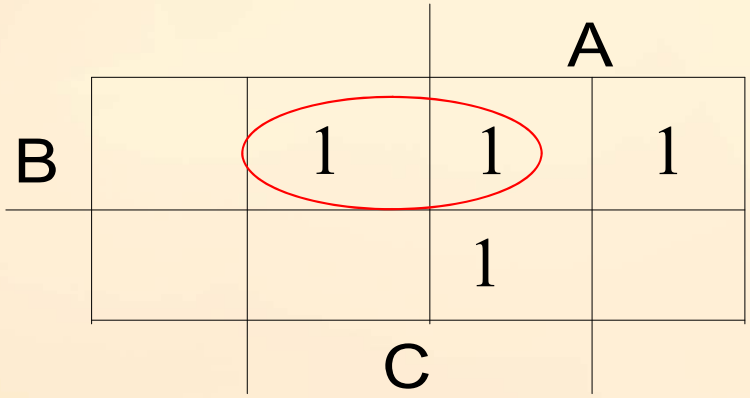
Example

$$F(A, B, C) = A\,C$$



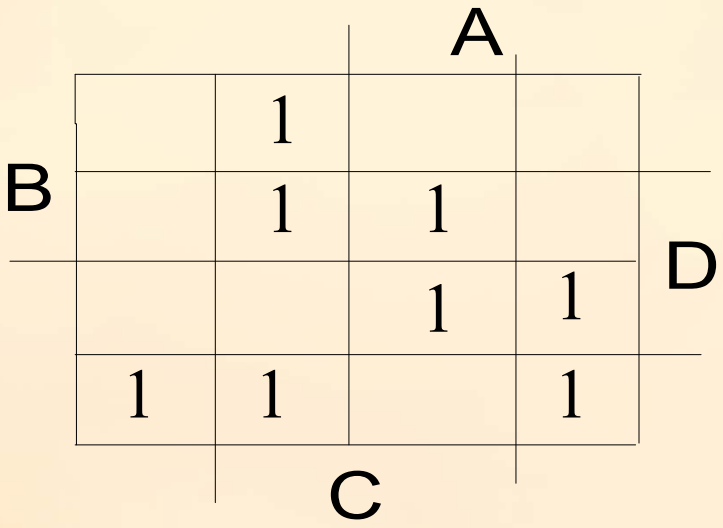
Example

$$F(A, B, C) = A\,C + A\,B + B\,C$$



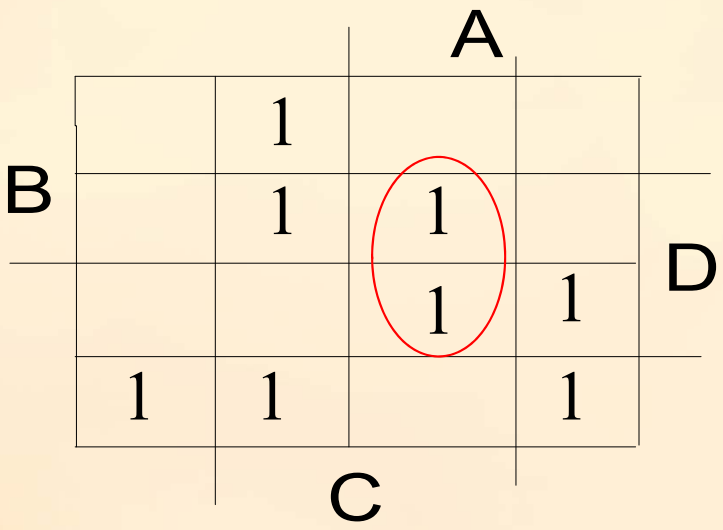
Example

F(A, B, C, D)



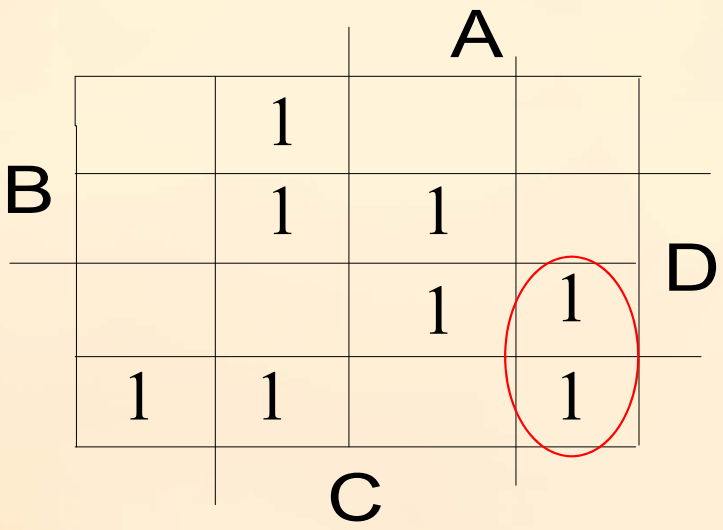
Example

$$F(A, B, C, D) = A\,C\,D$$



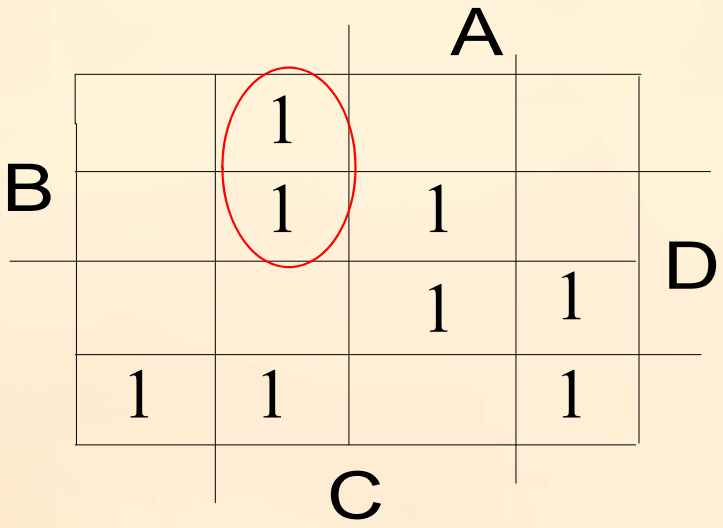
Example

$$F(A, B, C, D) = A\,C\,D + A\,\overline{B}\,\overline{C}$$



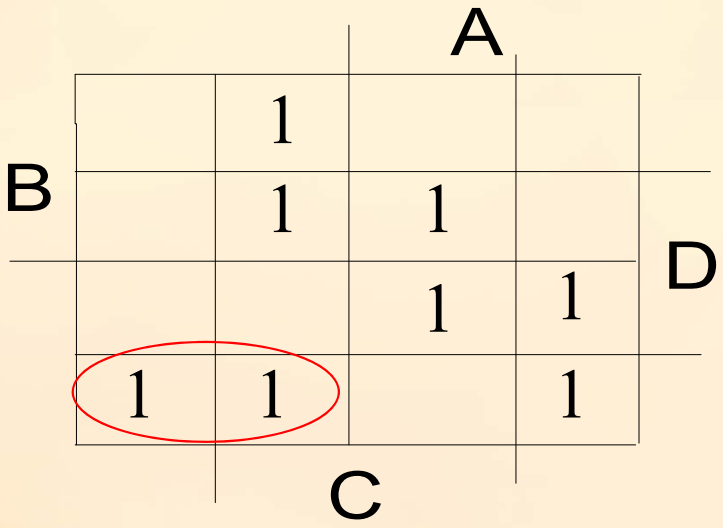
Example

$$F(A, B, C, D) = A\,C\,D + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,C$$

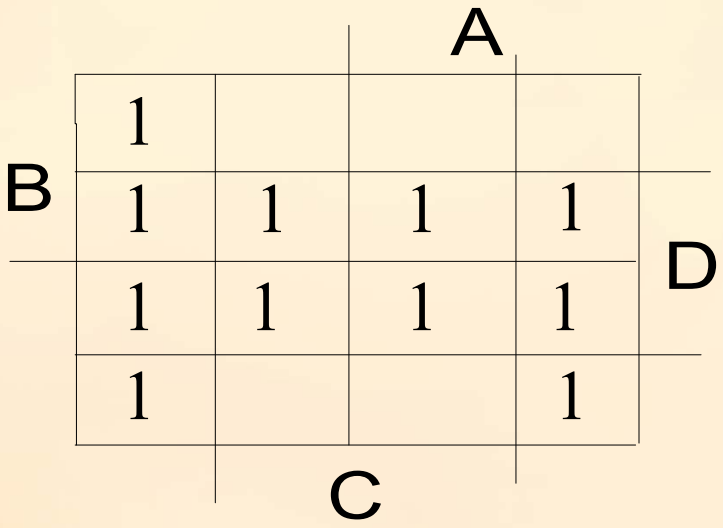


Example

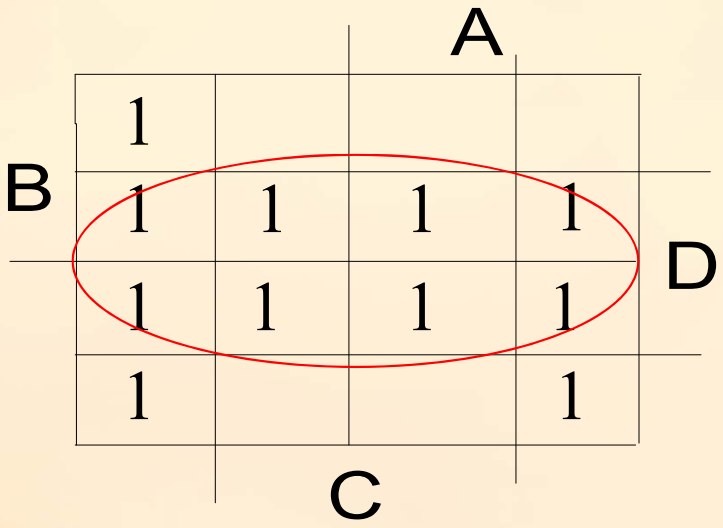
$$F(A, B, C, D) = A\,C\,D + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,\overline{B}\,\overline{D}$$



Example



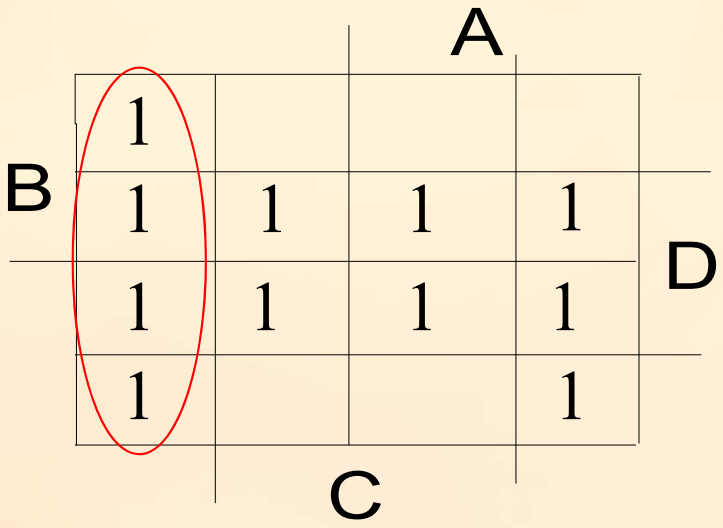
Example



F(A, B, C, D) $= D$

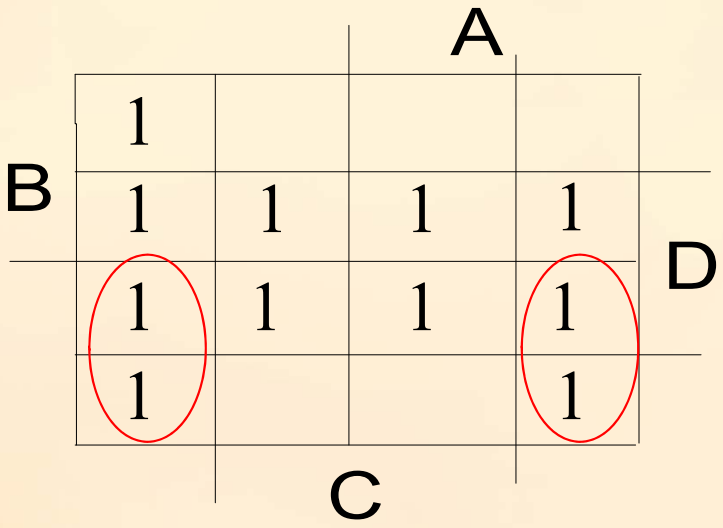
Example

$F(A, B, C, D) = D + \overline{A}\,\overline{C}$

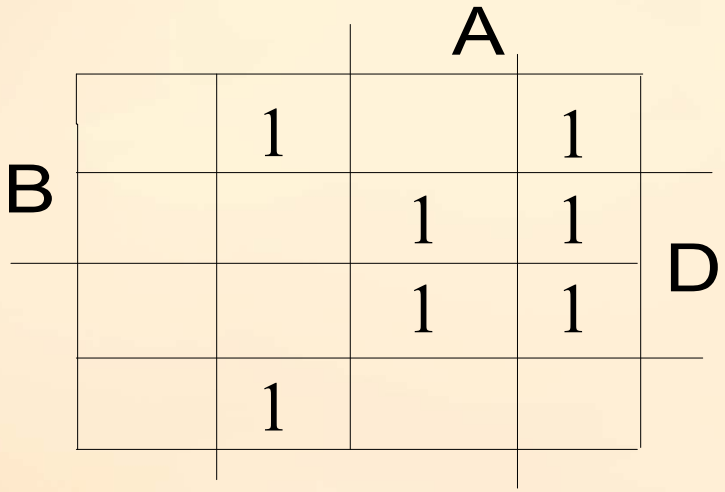


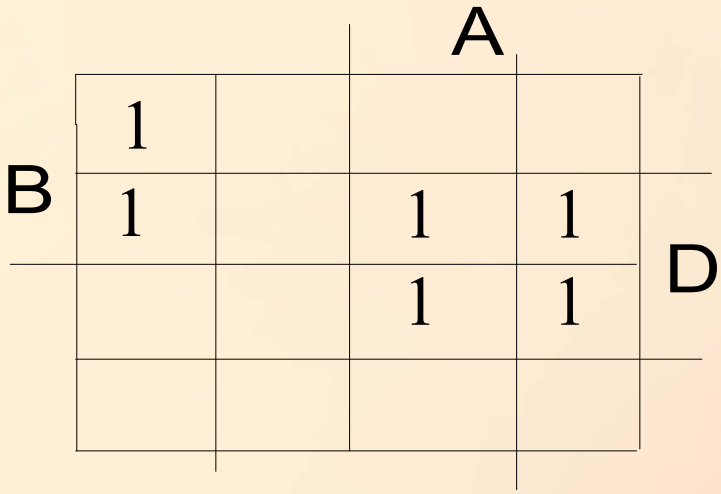
Example

$$F(A, B, C, D) = D + \overline{A}\,\overline{C} + \overline{B}\,\overline{C}$$

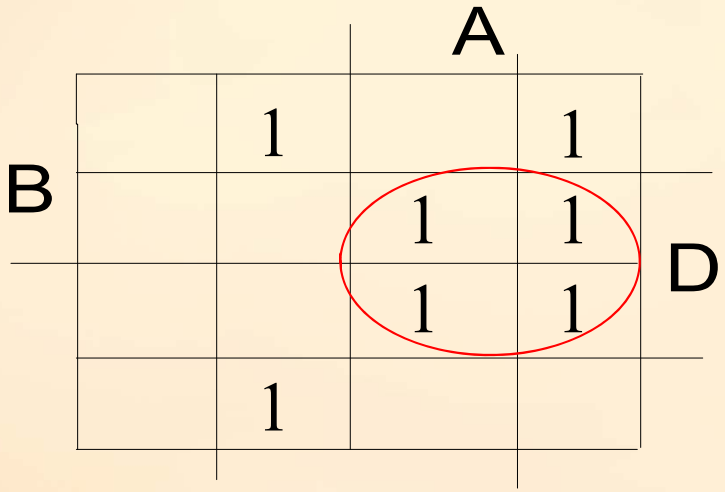


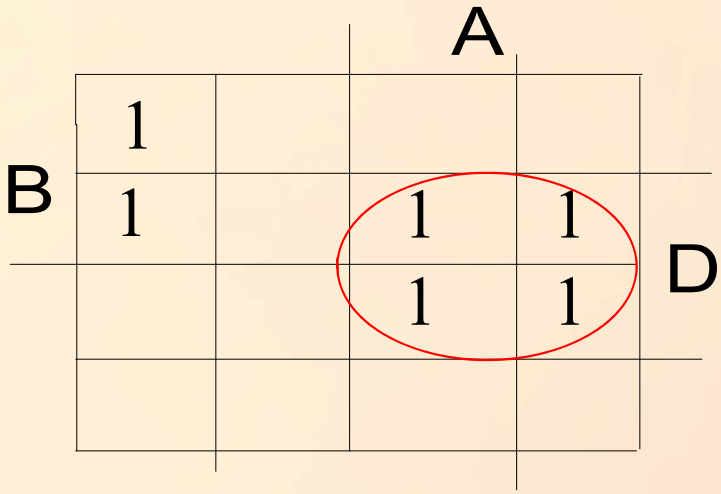
Example





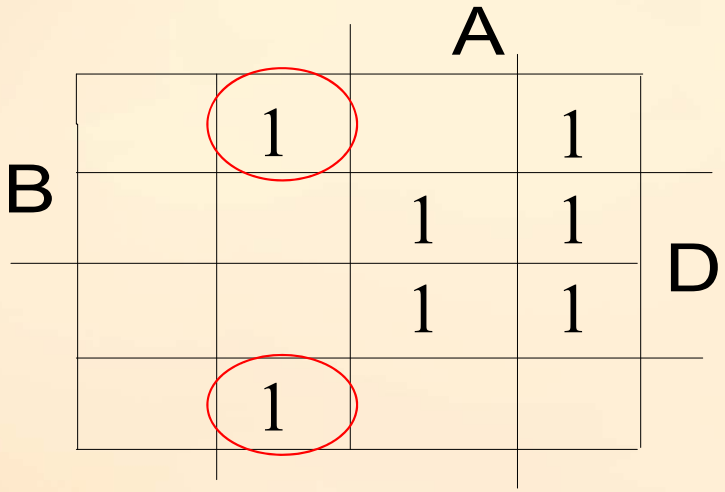
Example

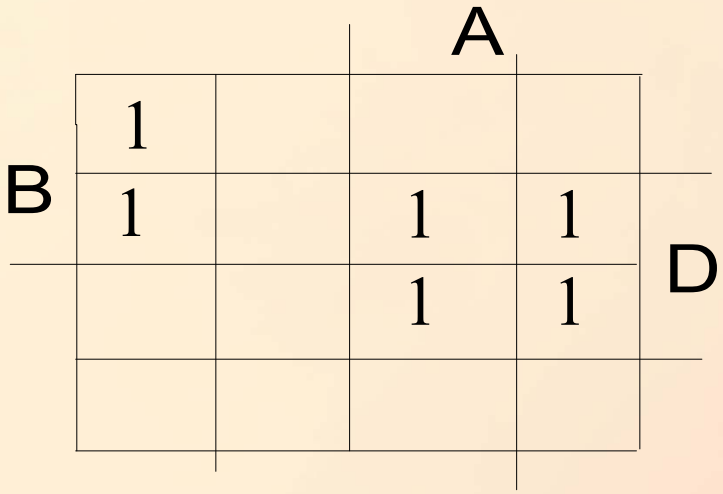




F(A, B, C, D) $= A\,D$

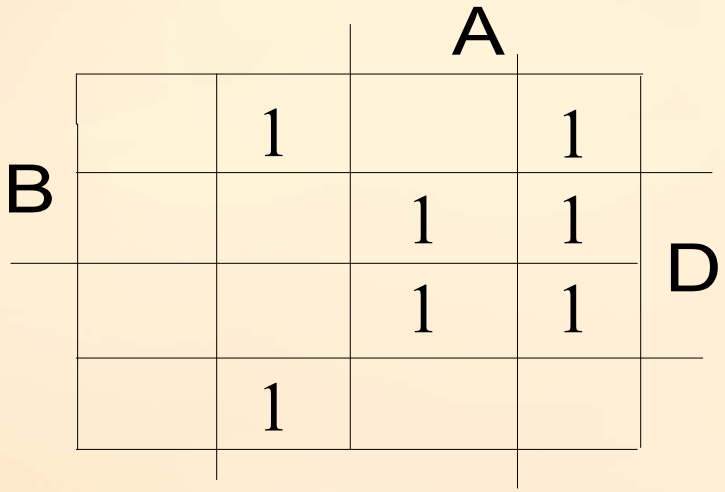
Example

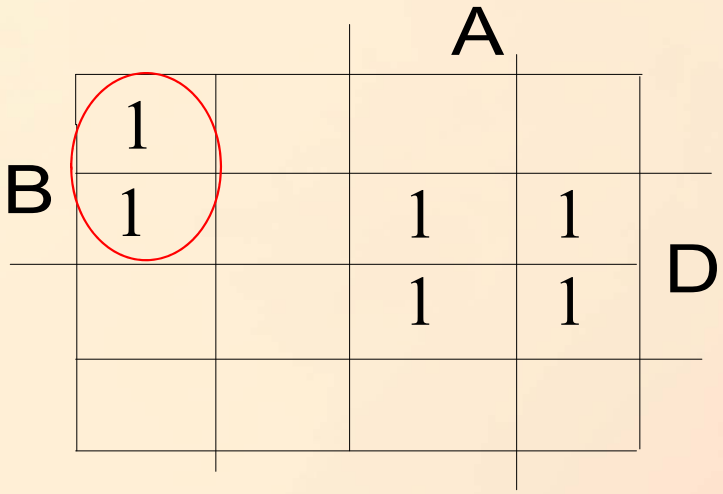




$$F(A, B, C, D) = A\,D + \overline{A}\,C\,\overline{D}\,\overline{E}$$

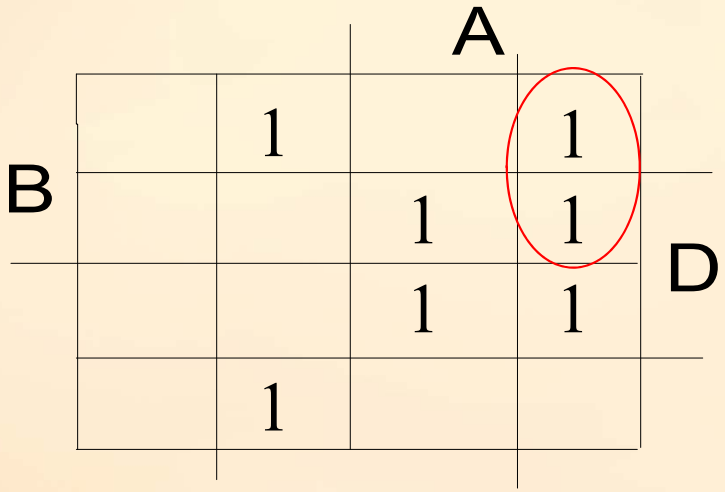
Example

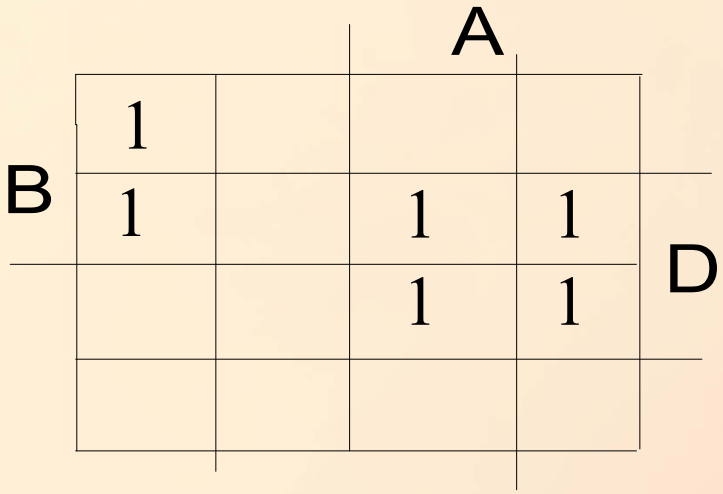




$$F(A, B, C, D) = A\,D + \overline{A}\,C\,\overline{D}\,\overline{E} + \overline{A}\,B\,\overline{C}\,E$$

Example

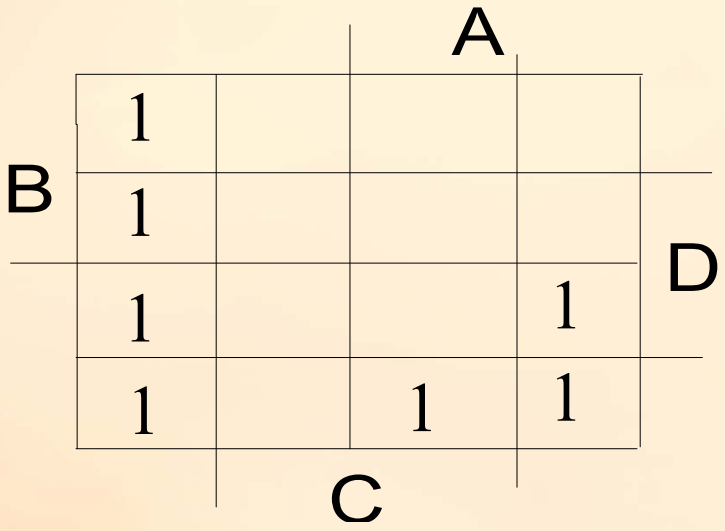




$$F(A, B, C, D, E) = AD + \overline{A}C\,\overline{D}\,\overline{E} + \overline{A}B\,\overline{C}E + AB\,\overline{C}\,\overline{E}$$

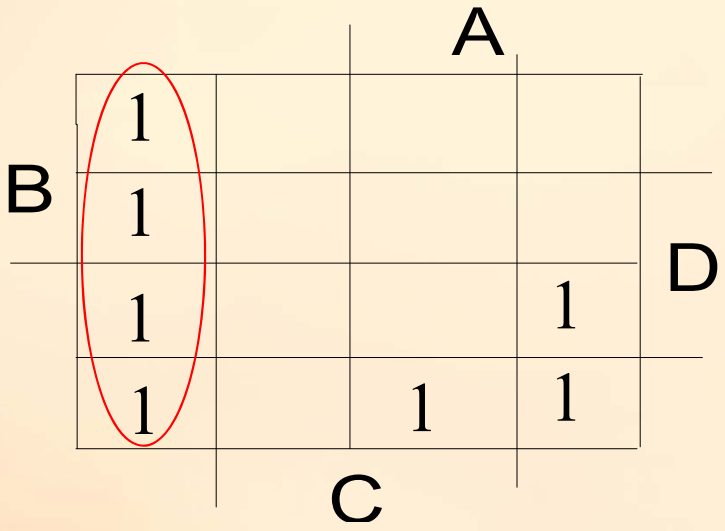
Example

F(A, B, C, D)



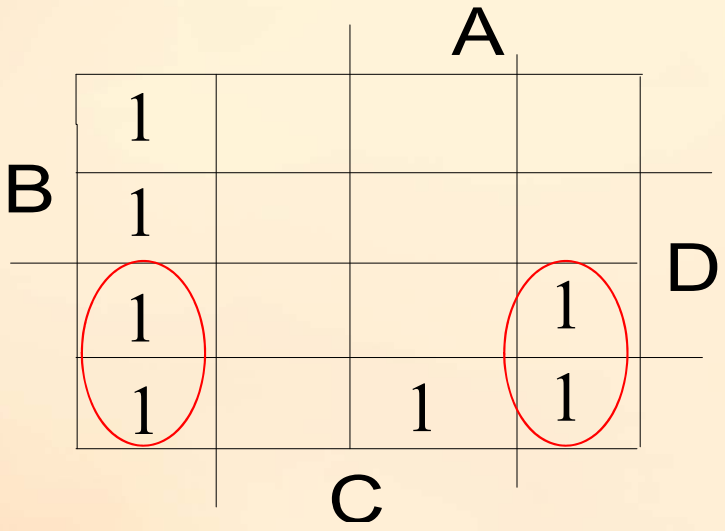
Example

$$F(A, B, C, D) = \overline{A}\,\overline{C}$$



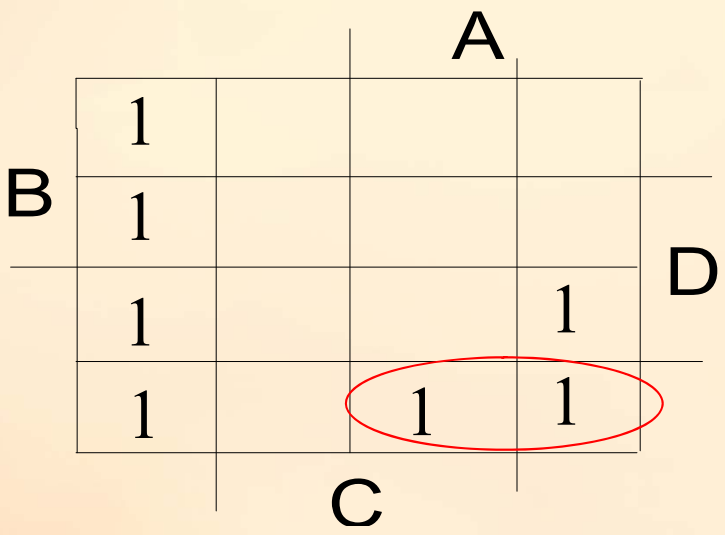
Example

$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C}$$



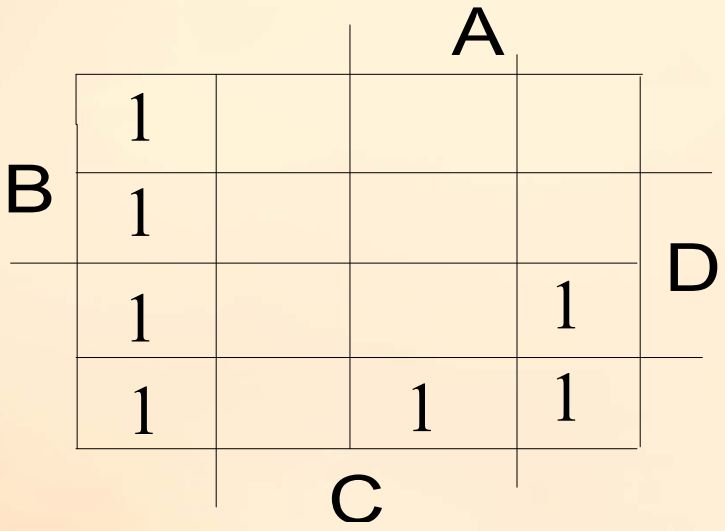
Example

$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D}$$



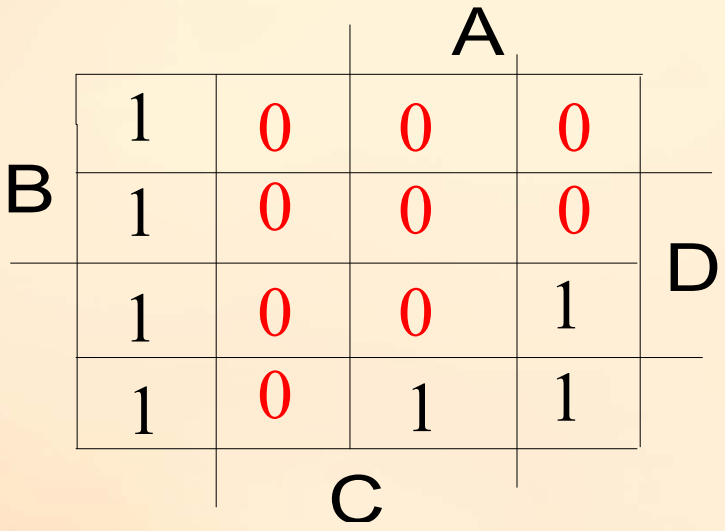
Example

$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D} \Rightarrow SOP$$



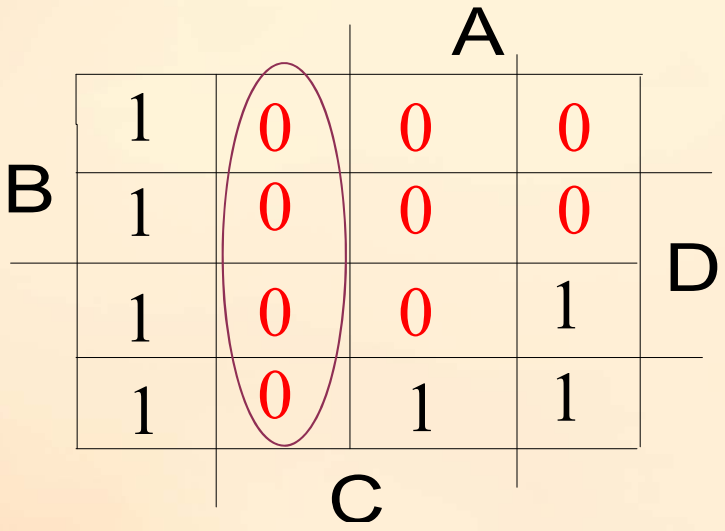
Example

$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D} \Rightarrow SOP$$



Example

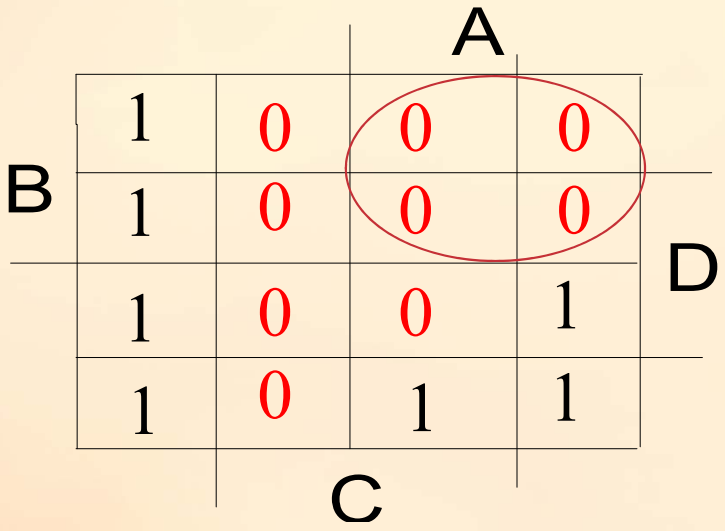
$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D} \Rightarrow SOP$$



$$F' = \overline{A}C$$

Example

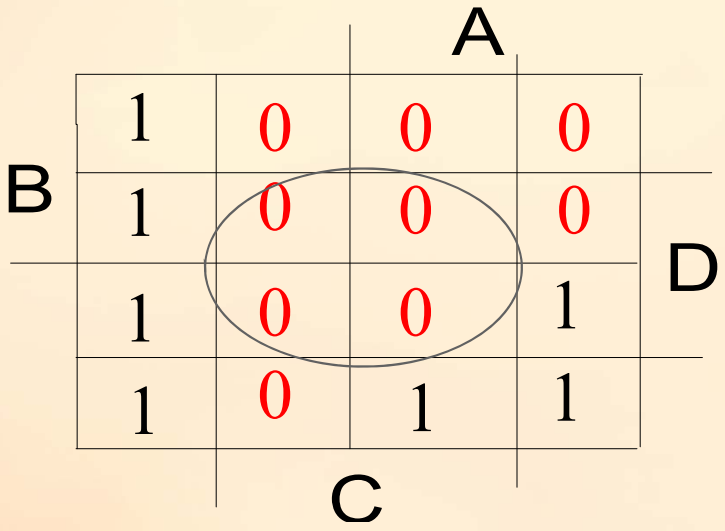
$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D} \Rightarrow SOP$$



$$F' = \overline{A}C + AB$$

Example

$$F(A, B, C, D) = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D} \Rightarrow SOP$$



$$F' = \overline{A}C + AB + CD$$

$$F = (A + \overline{C})(\overline{A} + \overline{B})(\overline{C} + \overline{D})$$

Example

F(A, B, C, D) $= \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,\overline{B}\,\overline{D}$ ⇨ SOP

B \ A	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	0	1
10	1	0	1	1

(C: 00, 01, 11, 10; D: 00, 01, 11, 10)

$$F' = \overline{A}C + AB + CD$$

$$F = (A + \overline{C})(\overline{A} + \overline{B})(\overline{C} + \overline{D})$$

Example

A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$

Example

A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$

B \ A				
				0
		0	0	0
		C		

$$A\ \overline{B}\ +\ \overline{A}\ \overline{B}\ C\ +\ A\ \overline{C}$$

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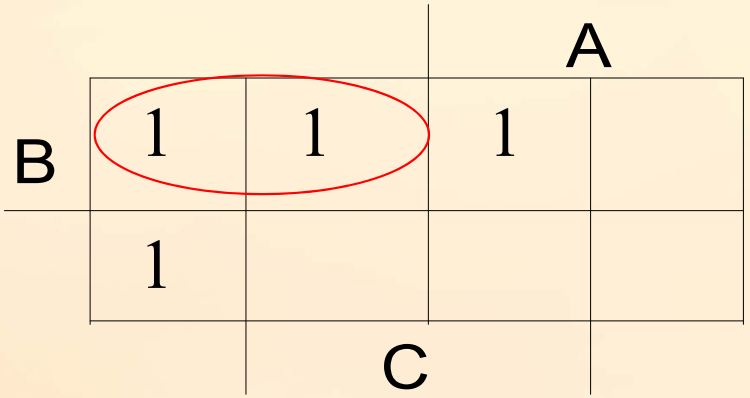
Example

A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$

B \ A				
	1	1	1	
	1			
		C		

Example

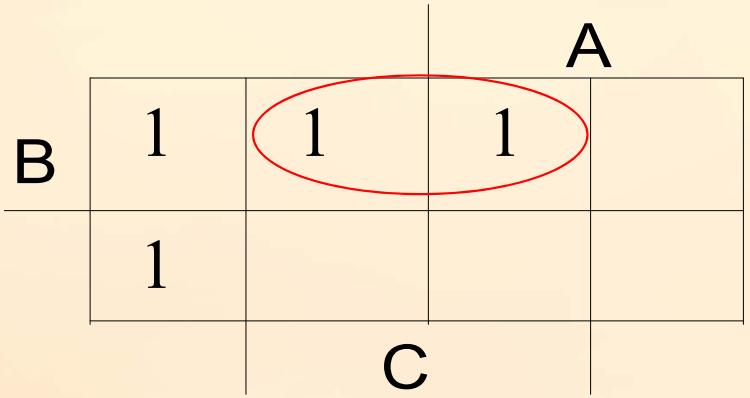
A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$



$$SOP = \overline{A}B$$

Example

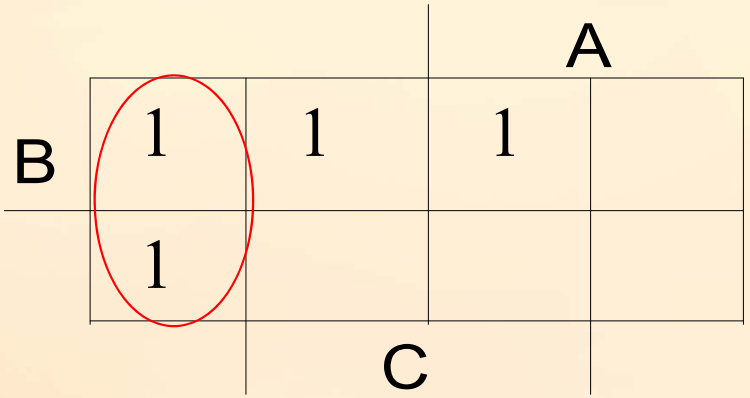
A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$



$$SOP = \overline{A}B + BC$$

Example

A) $f\ (A\ ,B\ ,C\ ) = (\overline{A}\ +\ B\ )(A\ +\ B\ +\overline{C}\ )(\overline{A}\ +C\ )$

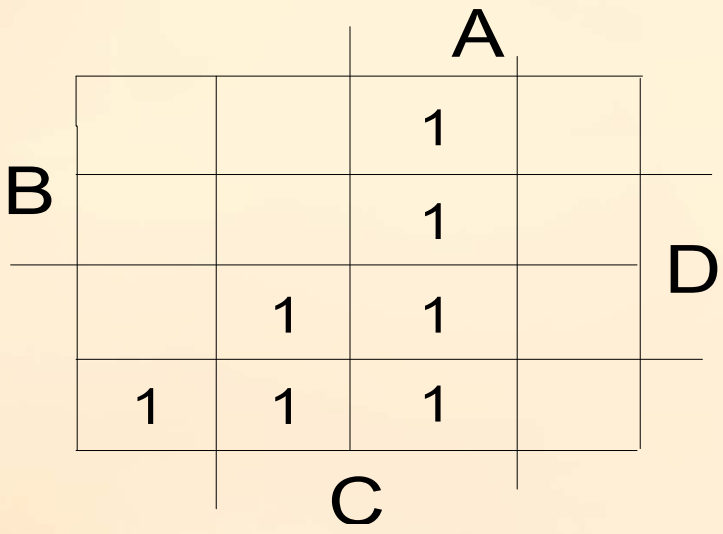


$$SOP = \overline{A}B + BC + \overline{A}\,\overline{C}$$

Example

LSB)

B) $f(A, B, C, D) = (A + \overline{B})(C + \overline{D})(\overline{A} + C)$



$$SOP = AC + C\overline{B} + \overline{A}\,\overline{B}\,\overline{D}$$

Don’t Care

- A **don't-care term** is an input to a function that the designer does not care about
- Because that input would never happen
- Example:
  - **BCD number (0-9, A-F) are 4 bits, don’t care about input A-F**

  - **Suppose a system have 5 type of input**

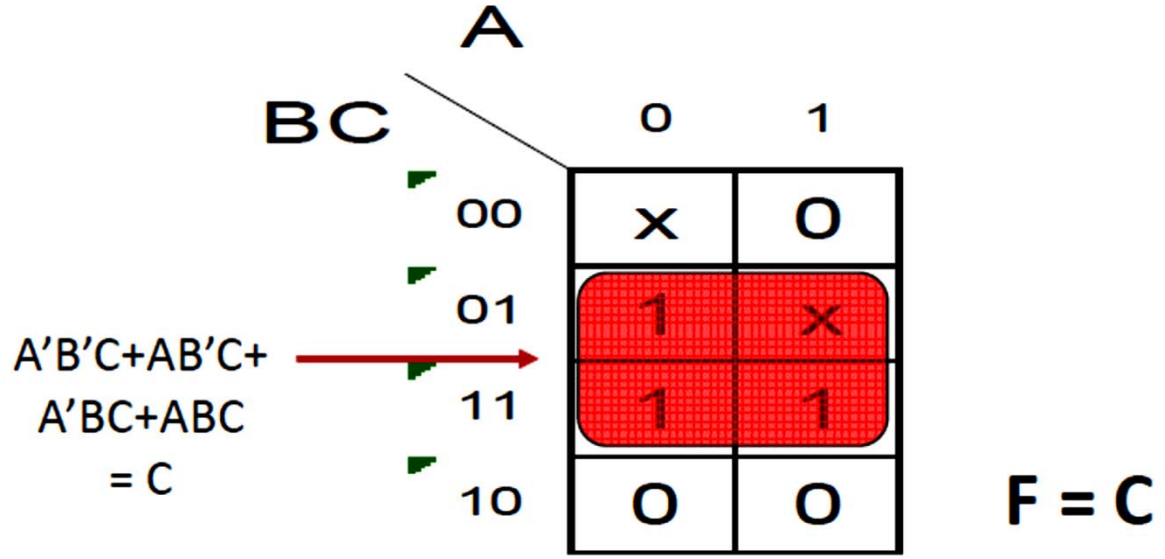
    Unfortunately we can’t have 2 input line

    Make 3 input line and last 3 sequence as don’t care

    S0, S1, S2,S3,S4, X,X,X == > 000, 001….,111

Dealing With Don't Cares

$F = \sum m(1, 3, 7) + \sum d(0, 5)$



Circle the x's that help get bigger groups of 1's
Don't circle the x's that don't

Thank You