Calculus III

Review Problems for Exam III

We'll do our 3^{rd} Midterm on Thursday, April 25^{th}. Tis test includes section 11.7, 11.8, 12.1, 12.2, 12.3, 12.4, 12.5, 12.7, and 12.8. Here are some potential problems for the exam, but remember to practice examples from lecture notes and homework.

1. Find the critical points of $f(x,y) = -x^3 + 3x - 4y^2$ and classify each point as a relative maximum, a relative minimum, or a saddle point.
2. Find the critical points of $f(x,y) = x^3 + y^3 - 12x^2 - 6$ and classify each point as a relative maximum, a relative minimum, or a saddle point.
3. Maximize $f(x,y) = 16 - x^2 - y^2$ subject to $x + 2y = 6$.
4. Calculate the double integral $\displaystyle\iint_R (4x + 10y + 40)dA$ where R is the region: $0 \leq x \leq 5,\ 0 \leq y \leq 2$.
5. Evaluate the double integral $\displaystyle\iint_D dA$ where D is the triangular region with vertices $(0,0), (1,0), (0,1)$.
6. Find the volume of the solid bounded below by the rectangle $R : 0 \leq x \leq 1,\ 0 \leq y \leq 2$ in the xy-plane and above by the graph of $z = f(x,y) = 5x + 2y$.
7. Evaluate the integral $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1 + x^3}\, dx\, dy$ by reversing the order of the integration.
   Note: You cannot actually evaluate the integral in the given order.
8. Compute the double integral $\displaystyle\iint_D \frac{1}{1+y^2} dA$ where D is the region bounded by $x = 2y,\ y = -x,$ and $y = 2$.
9. Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.
10. Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{x^2+y^2}\, dy\, dx$ by converting to polar coordinates.
11. Find the surface area of the portion of the paraboloid $z = 4 - x^2 - y^2$ above the $xy-$plane.
12. Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 8$ that lies above the cone $z = \sqrt{x^2 + y^2}$.
13. Find the surface area of the surface given by the portion of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 9$.
    (Hint: convert to polar coordinates after setting up the integral).
14. Do problem 9, 13, and 15 from section 12.5 of text book.
15. Find the volume of the solid in the first octant bounded by the plane $2x + y + 2z = 4$ and the coordinate planes.

16. Find the volume of the solid enclosed by the paraboloids $z = 9(x^2 + y^2)$ and $z = 18 - 9(x^2 + y^2)$.
(Hint: Use Cylindrical coordinates to evaluate the integral).

17. Use either cylindrical or spherical coordinates to evaluate the triple integral $\iiint\limits_D z\,dV$, where D is the portion of the ball, $x^2 + y^2 + z^2 \leq 4$, in the first octant, $x \geq 0$, $y \geq 0$ and $z \geq 0$.