CS1382 Discrete Computational Structures
Spring 2019 Homework I (10 points)

Due Date: Sun Feb 10, 2019 at 11:59pm
Full points will be awarded for showing all your working.

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1. Use a Venn diagram to determine which relationship, $\subseteq$, =, or $\supseteq$, is true for the pair of sets.
   a. $A \cup B$, $A \cup (B - A)$.
   b. $(A - B) \cup (A - C)$, $A - (B \cap C)$.
   c. $(A - C) - (B - C)$, $A - B$.

2. Suppose U = {1, 2, . . ., 9}, A = all multiples of 2, B = all multiples of 3, and C = {3, 4, 5, 6, 7}. Find C − (B − A).

3. Suppose S = {1, 2, 3, 4, 5}. Find $|\mathcal{P}(S)|$.

4. Suppose $f : \mathbf{R} \to \mathbf{R}$ where f (x) = floor(x/2) .
   a. Draw the graph of f .
   b. Is f 1-1?
   c. Is f onto R ?

5. Suppose $g : A \to B$ and $f : B \to C$ where A = { 1, 2, 3, 4} , B = {a, b, c} , C = { 2, 7, 10} , and f and g are defined by g = { (1, b), (2, a), (3, a), (4, b)} and f = { (a, 10), (b, 7), (c, 2)} .
   a. Find f o g .
   b. Find f^{-1} .

6. For the following, describe each sequence recursively. Include initial conditions and assume that the sequences begin with a_1 .
   a. $a_n = 5^n$.
   b. The Fibonacci numbers.
   c. 0, 1, 0, 1, 0, 1, ...
   d. $a_n = 1 + 2 + 3 + \cdots + n$.
   e. 3, 2, 1, 0, − 1, − 2, ...
   f. $a_n = n!$.

7. Verify that $a_n = 3^{n+4}$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

8. Verify that $a_n = 3^n + 1$ is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$.

9. Suppose inflation continues at three percent annually. (That is, an item that costs \$1.00 now will cost \$1.03 next year.) Let a_n = the value (that is, the purchasing power) of one dollar after n years.

a) Find a recurrence relation for a_n .
b) What is the value of $1.00 after 20 years?
c) If inflation were to continue at ten percent annually, find the value of $1.00 after 20 years.
d) If inflation were to continue at ten percent annually, find the value of $1.00 after 80 years.

10. Find 18 mod 7 and gcd(300, 700)

11. Find gcd(662,414) and express it as a linear combination of 662 and 414

12. Find the inverse of 17 modulo 19

13. Solve the linear congruence $31x \equiv 57 \pmod{61}$

14. Use Fermat's little theorem to find 25^{1202} **mod** 61

15. Encrypt the message NEED HELP by translating the letters into numbers (A=0, B=1, . . . , Z=25), applying the encryption function f(p) = (3p + 7) **mod** 26, and then translating the numbers back into letters.

16. What is the shared key if Alice and Bob use the Diffie-Hellman key exchange protocol with the prime p = 431, the primitive root a = 79 of p = 431, with Alice choosing the secret integer k_1 = 236 and Bob choosing the secret integer k_2 = 334?