Finite Automata and Regular Expression

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DFA,NFA & REG

- We have learned
  - Deterministic Finite Automata
  - Nondeterministic Finite Automata
  - Regular expression

DFA,NFA & REG

- We have learned
  - Deterministic Finite Automata
  - Nondeterministic Finite Automata
  - Regular expression
- We define a language regular if a DFA or NFA recognizes it
- We will show a language is regular if and only if a regular expression describes it.

Equivalent definition

DFA,NFA & REG

- Q: Show a language is regular if and only if a regular expression describes it.

  The language defined by regular expression, L_{REG}, equals the language defined by NFA, L_{NFA}

DFA,NFA & REG

- Target: $L_{REG} = L_{NFA}$

  - $L_{REG} \subseteq L_{NFA}$, i.e., any regular expression is recognized by some NFA
  - $L_{NFA} \subseteq L_{REG}$, i.e., any language recognized by some NFA can be described by some regular expression

$L_{REG} \subseteq L_{NFA}$

- How do we show $L_{REG} \subseteq L_{NFA}$?

  - For any regular expression r, construct an NFA that accepts exactly the language $L(r)$

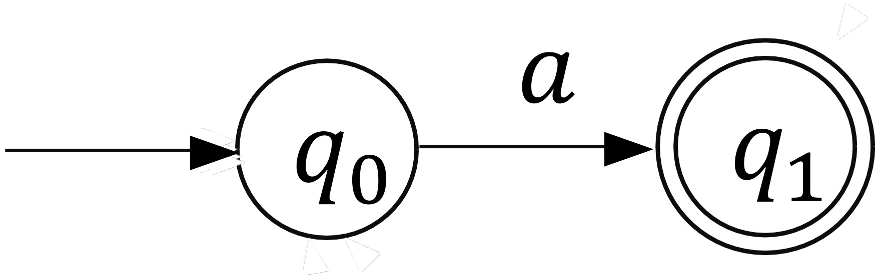
  - Regular languages are defined inductively, we prove $L_{REG} \subseteq L_{NFA}$ by induction

Regular expression

- The regular expressions of Σ^* are all strings over $\Sigma \cup \{(,), \emptyset, +, \star\}$ that can be obtained through the following operations:
  - $\emptyset$ and every member of Σ is a regular expression
  - If α and β are regular expressions, then so is $(\alpha\beta)$
  - if α and β are regular expressions, then so is $(\alpha + \beta)$
  - if α is a regular expression, then so is α^*
  - Nothing else is a regular expression

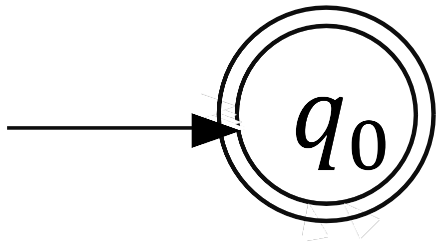
$L_{REG} \subseteq L_{NFA}$

- Base case
  - $\mathrm{L}(r) = \{a\}, a \in \Sigma$



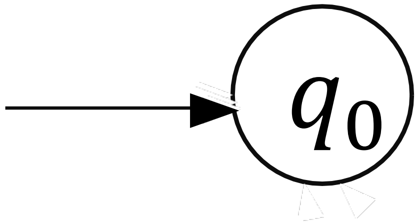
$L_{REG} \subseteq L_{NFA}$

- Base case
  - $\mathrm{L}(r) = \{e\}$



$L_{REG} \subseteq L_{NFA}$

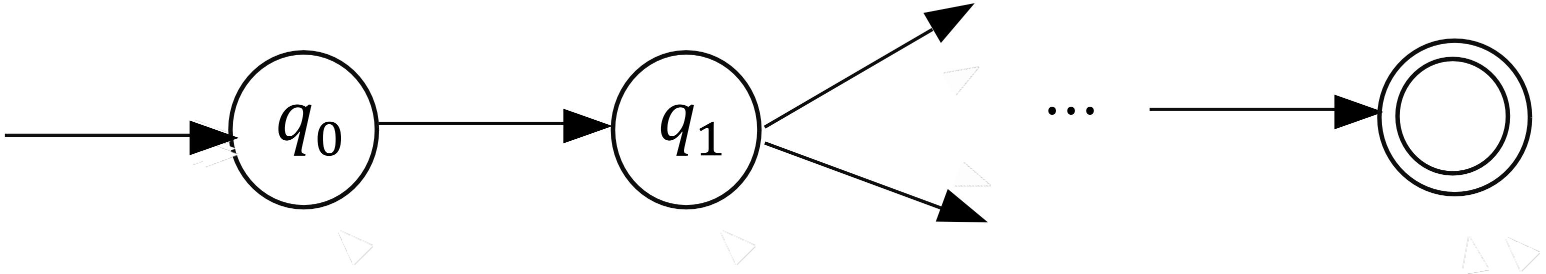
- Base case
  - $\mathrm{L}(r) = \emptyset$



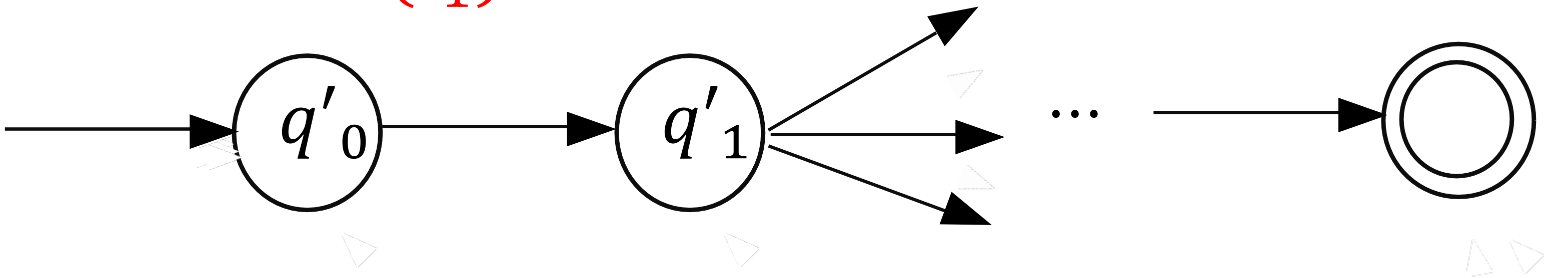
$L_{REG} \subseteq L_{NFA}$

- Recursive definition of Regular Language
  - $\mathrm{L}(r) = L(r_1 r_2)$

NFA for $L(r_1)$



NFA for $L(r_1)$



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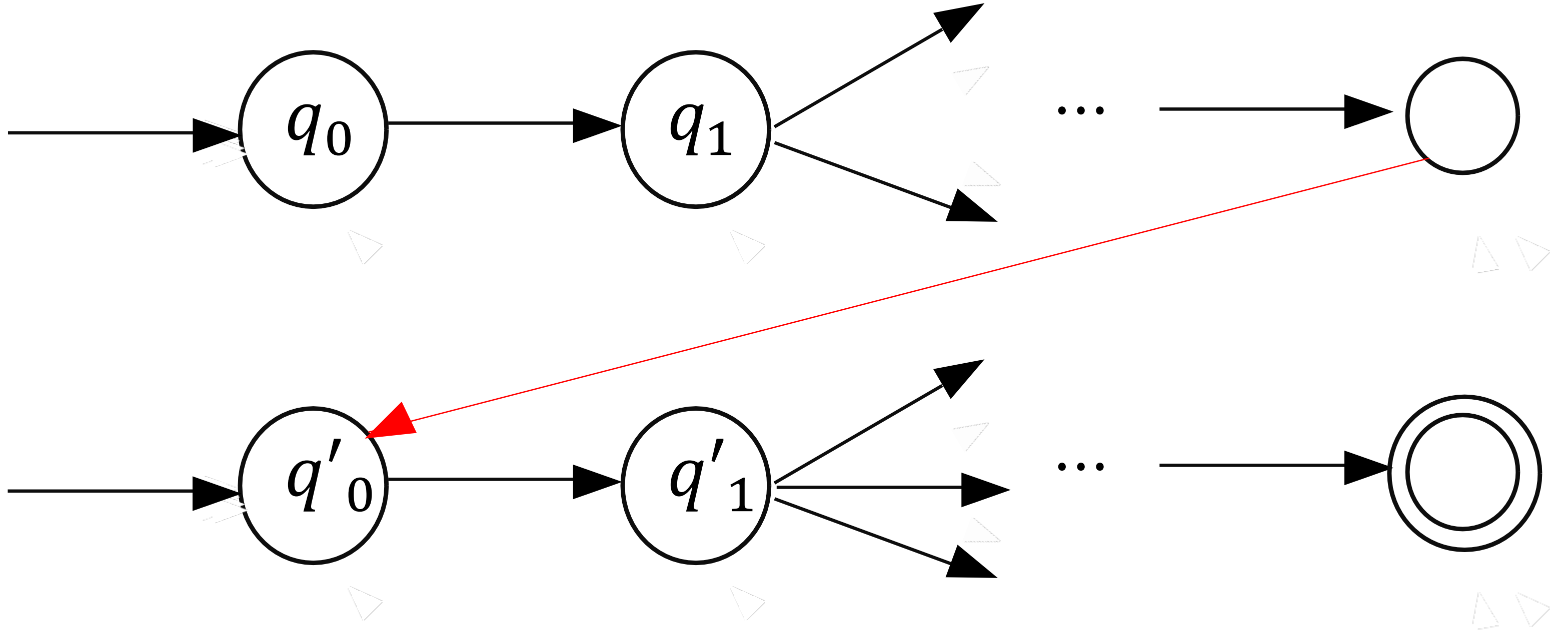

$L_{REG} \subseteq L_{NFA}$

- Recursive definition of Regular Language

  - $\mathrm{L}(r) = L(r_1 + r_2)$

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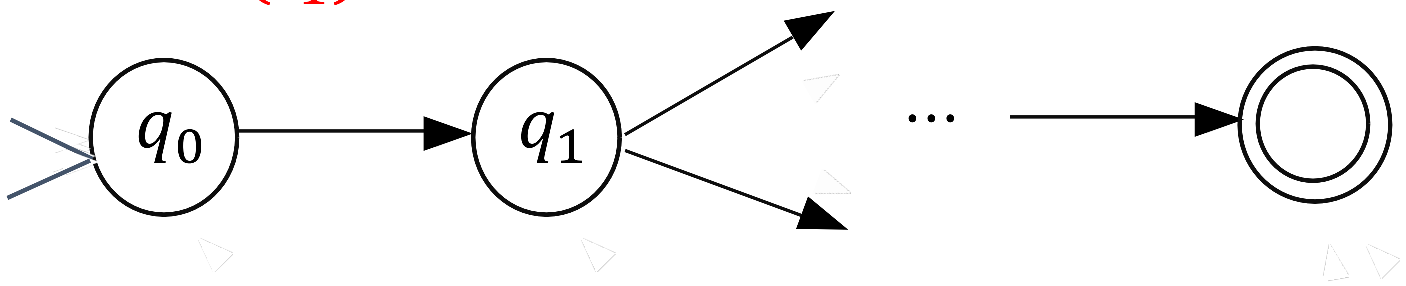
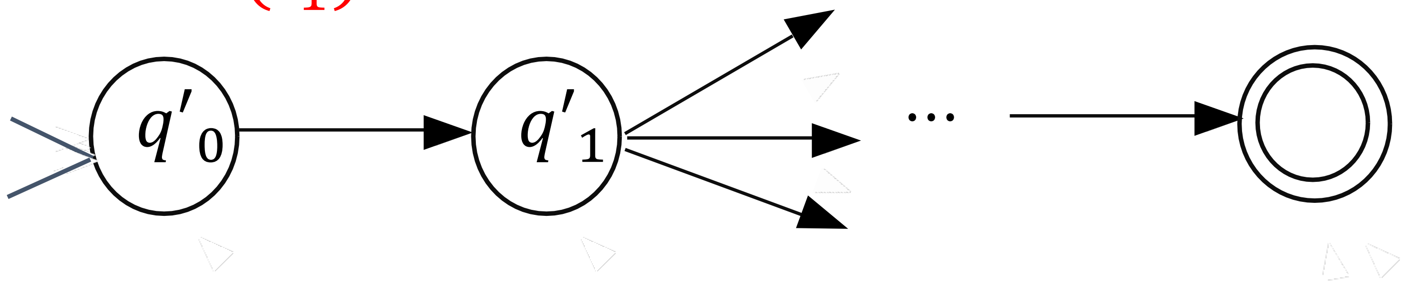


NFA for $L(r_1)$



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- Recursive definition of Regular Language
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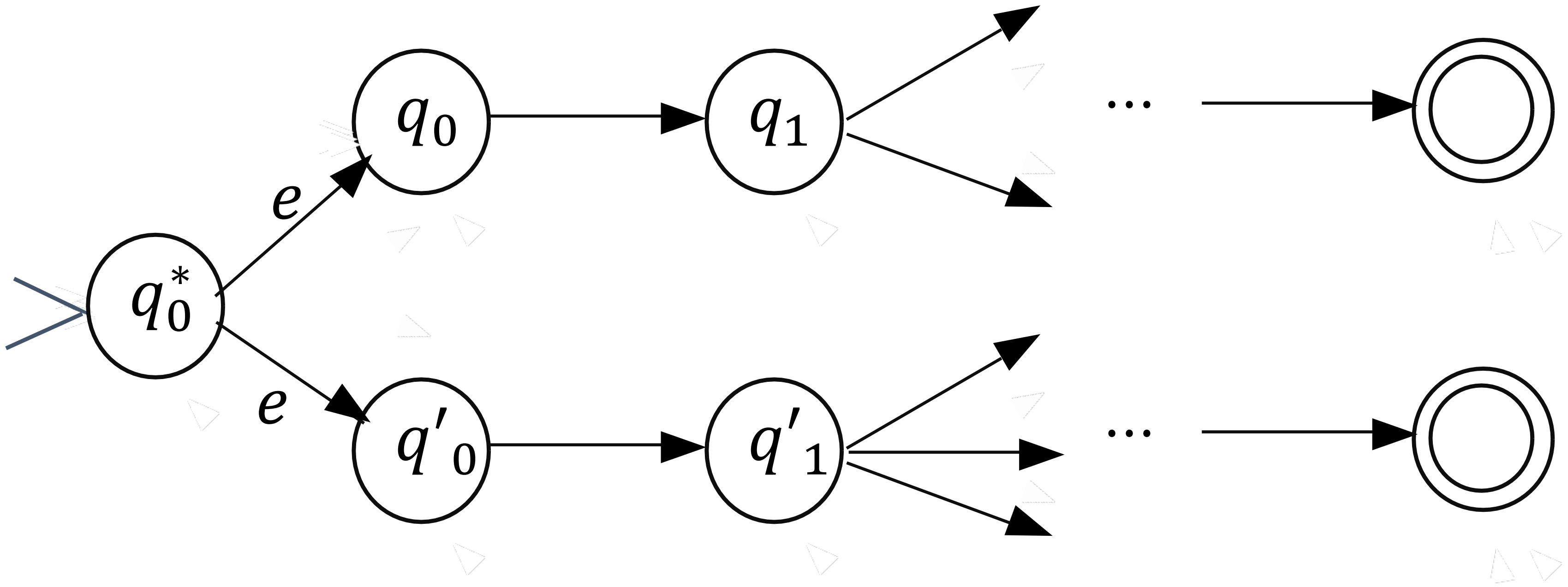
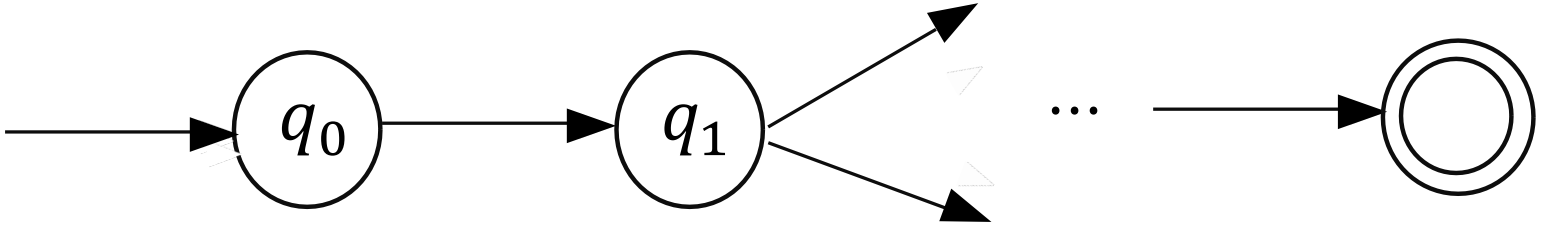
NFA for $L(r_1 + r_2)$



$L_{REG} \subseteq L_{NFA}$

- Recursive definition of Regular Language
  - $\mathrm{L}(r) = L(r_1^*)$

NFA for $L(r_1)$



$L_{REG} \subseteq L_{NFA}$

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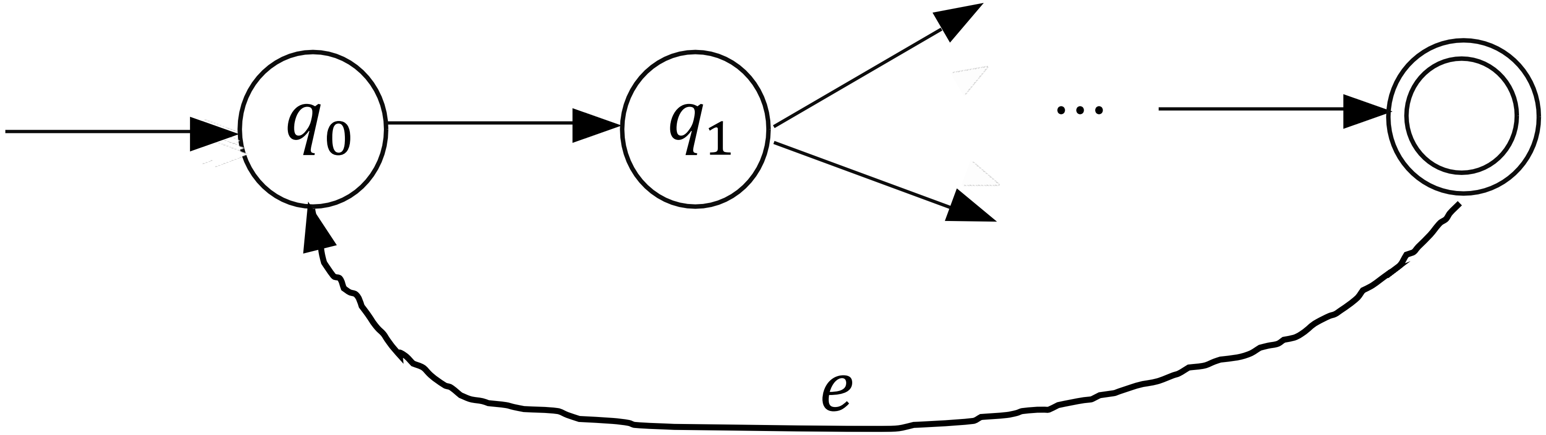
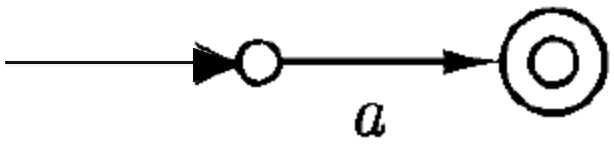
  - $L(r) = L(r_1^*)$

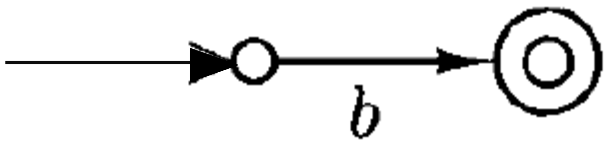
NFA for $L(r_1^*)$



$L_{REG} \subseteq L_{NFA}$

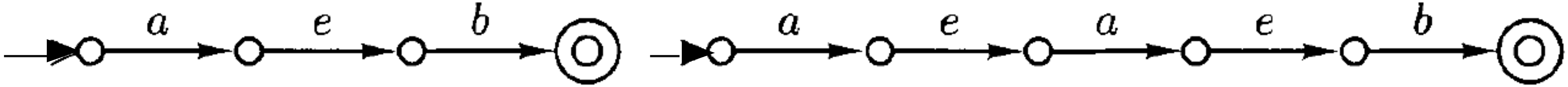
- Example
  - a, b





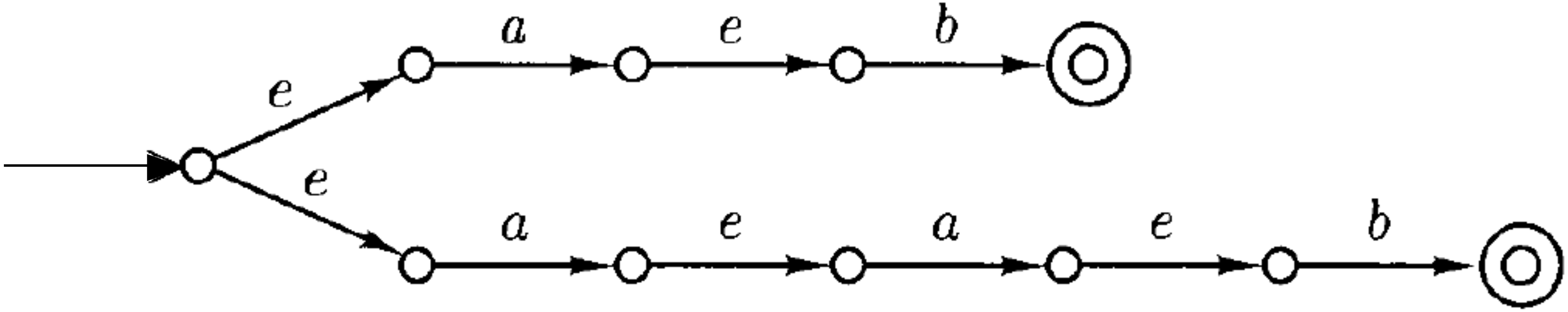
$L_{REG} \subseteq L_{NFA}$

- Example
  - ab, aab



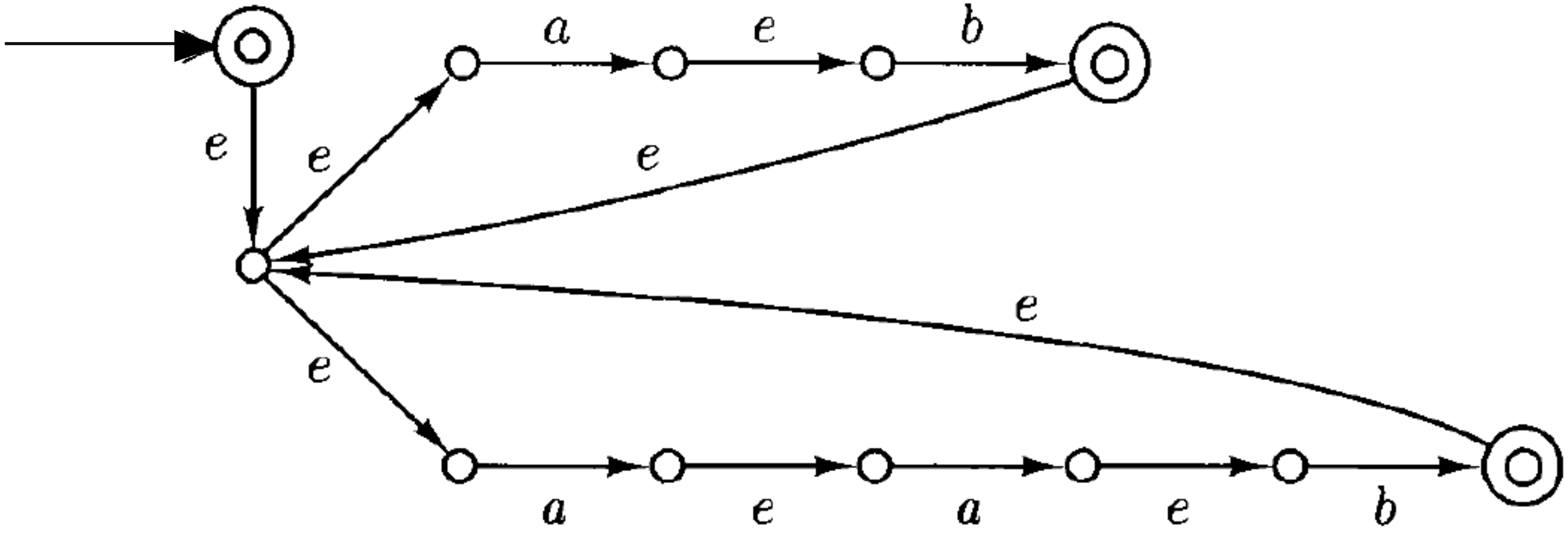
$L_{REG} \subseteq L_{NFA}$

- Example
  - $ab \cup aab$



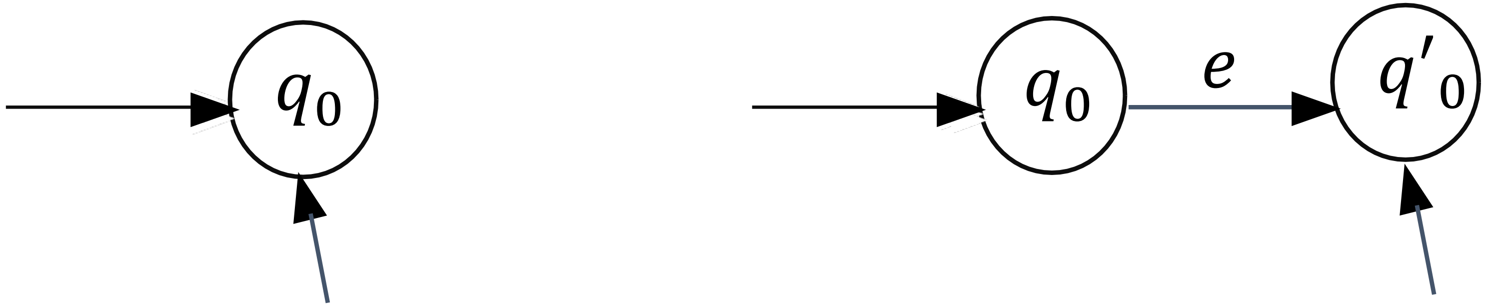
$L_{REG} \subseteq L_{NFA}$

- Example
  - $(ab \cup aab)^*$



$L_{NFA} \subseteq L_{REG}$

- Start with a specialized NFA
  - No transition into the start state



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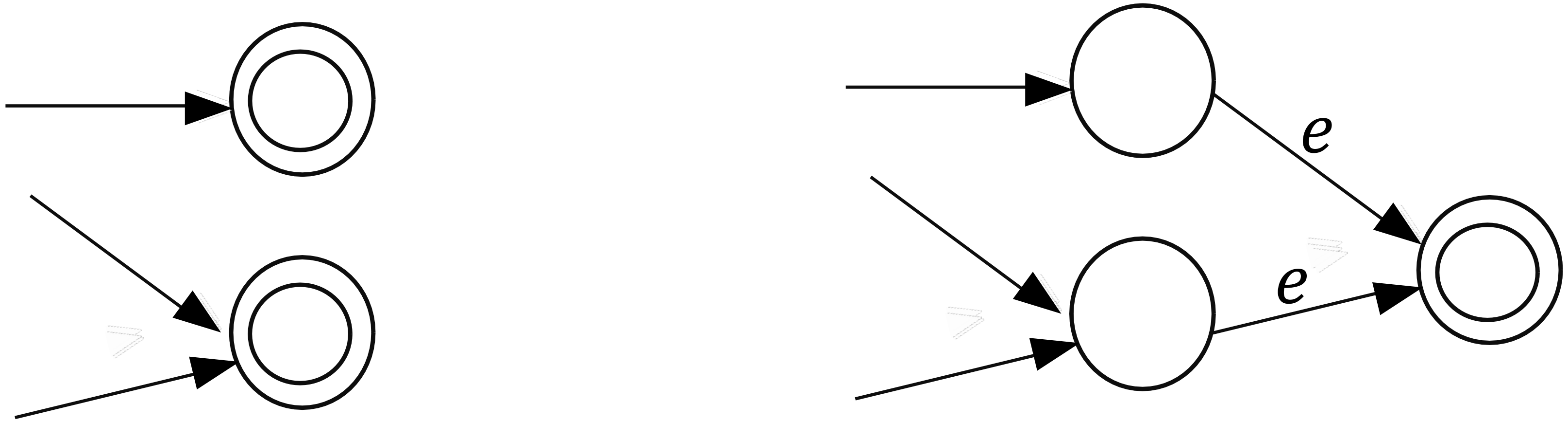
  - Single final state



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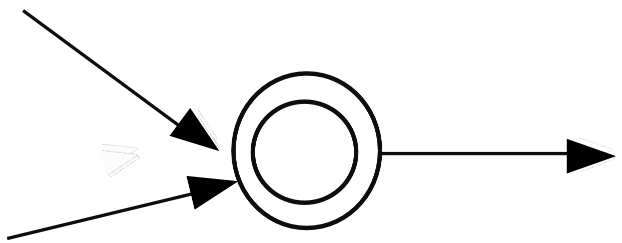
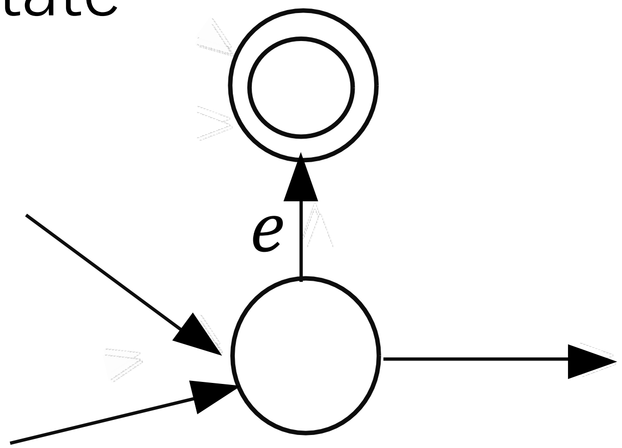
- Start with a specialized NFA
  - No transition into the start state
  - Single final state
  - No transition out of the final state





$L_{NFA} \subseteq L_{REG}$

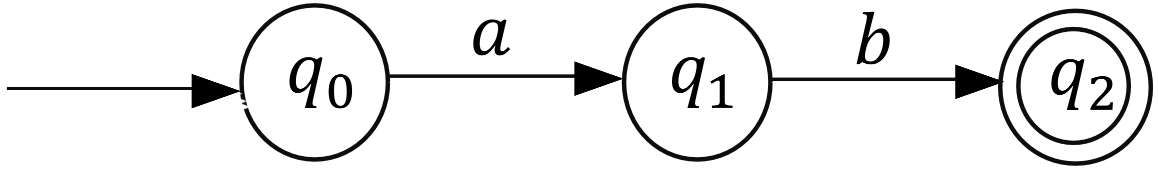
- Transitions will be labeled with regular expressions
- If there is a transition from state q_1 to state q_2 labeled with regular expression r, then any string generated by r can move the machine from q_1 to q_2
- Remove states, relabeling transitions so that the language defined by the machine does not change

$L_{NFA} \subseteq L_{REG}$

- Proof by induction on the "length" of the path
- Use example to illustrate the basic idea (example≠proof!)

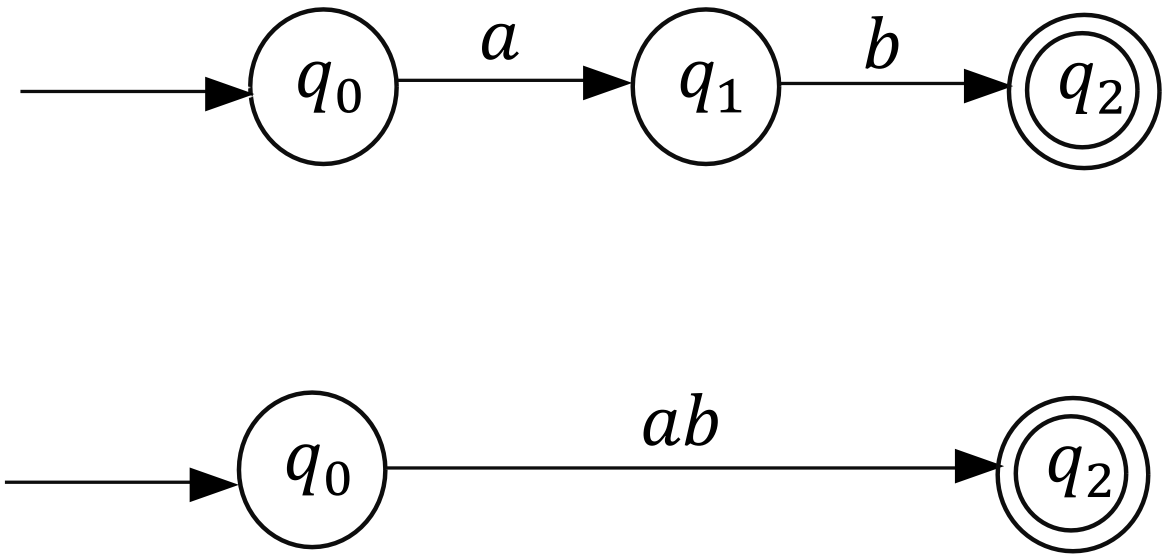
$L_{NFA} \subseteq L_{REG}$

- Example:



$L_{NFA} \subseteq L_{REG}$

- Example:
  - Remove state q_1



$L_{NFA} \subseteq L_{REG}$

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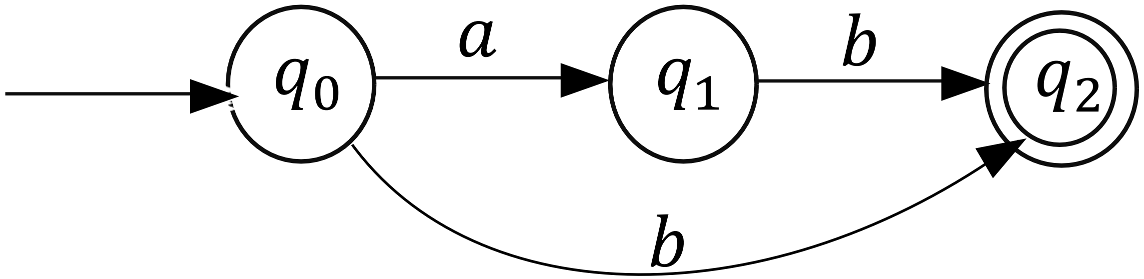

$L_{NFA} \subseteq L_{REG}$

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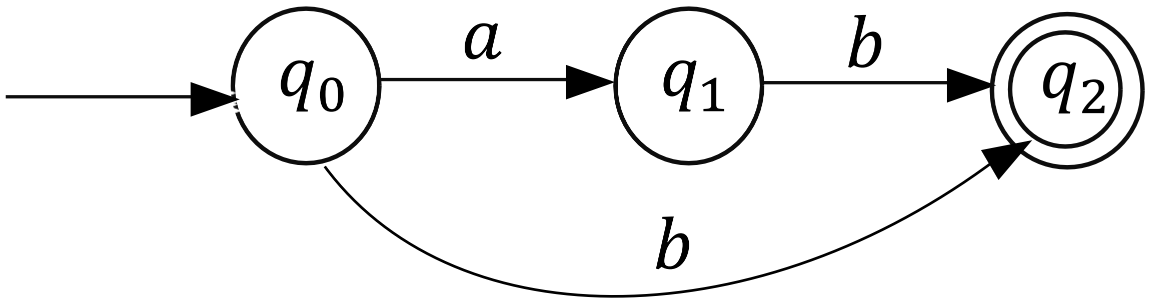
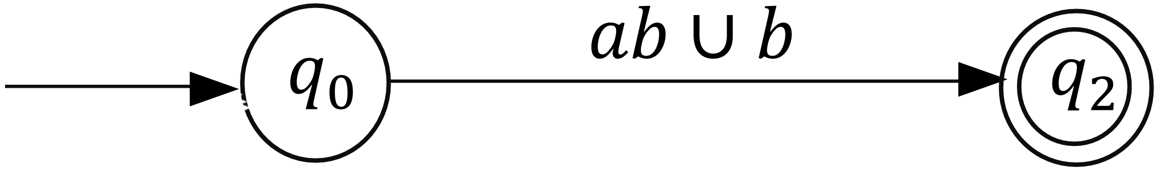
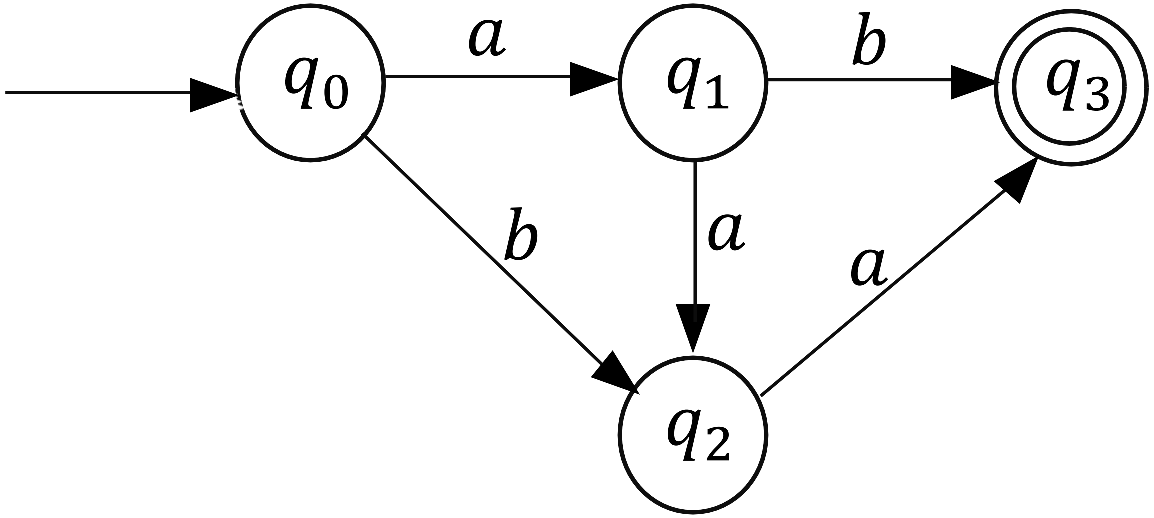




$L_{NFA} \subseteq L_{REG}$

- Example:



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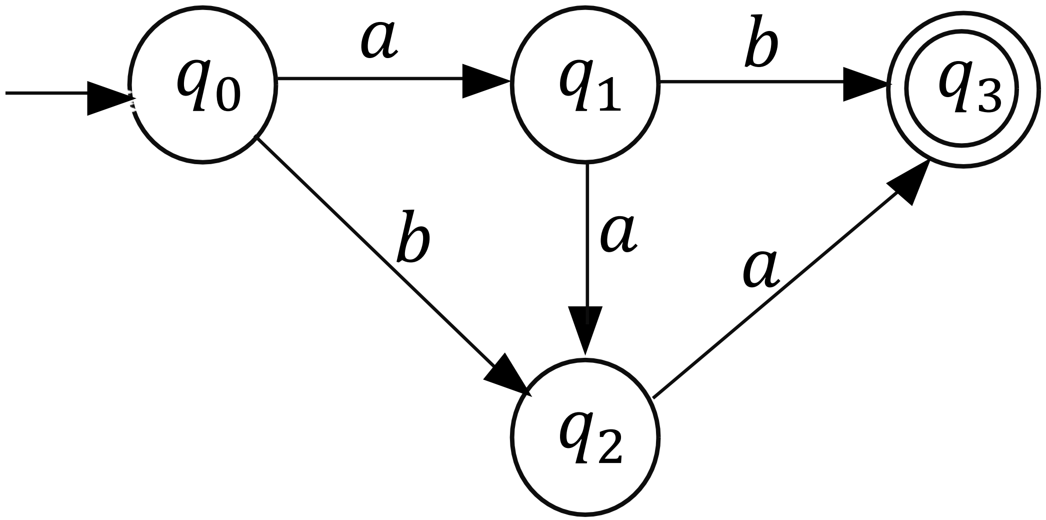

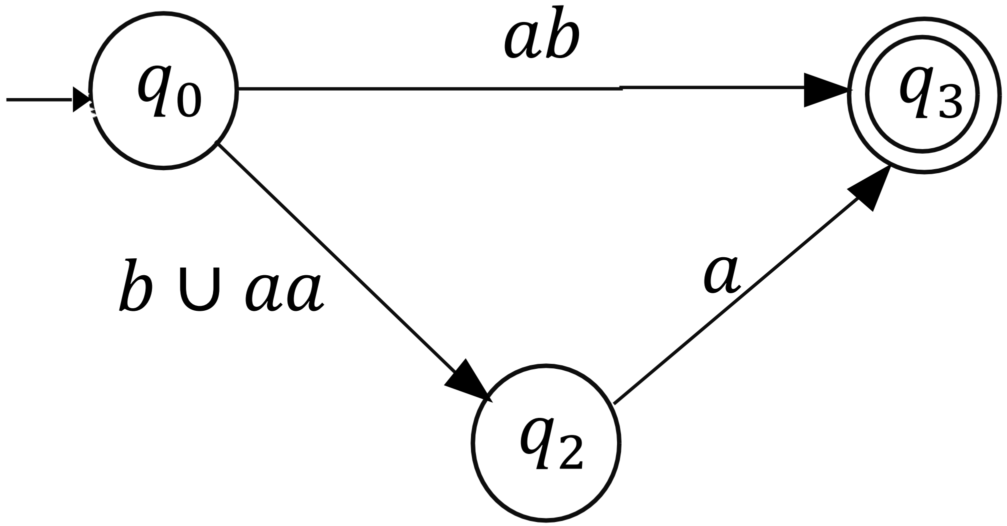

$L_{NFA} \subseteq L_{REG}$

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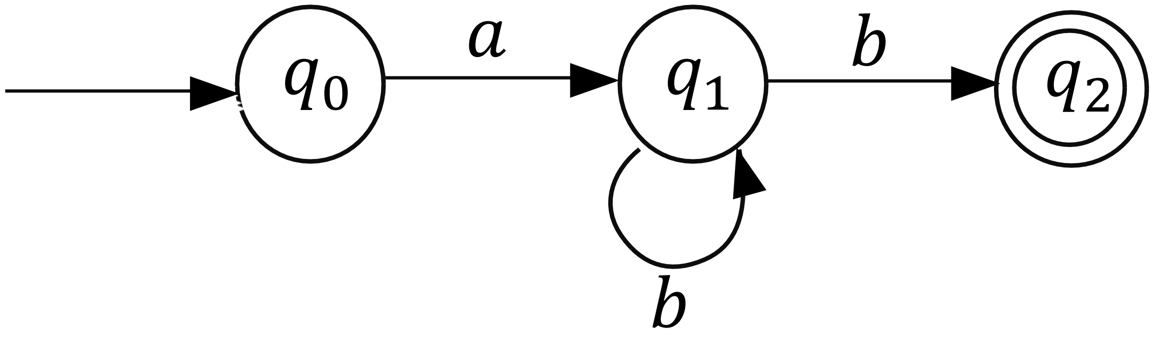

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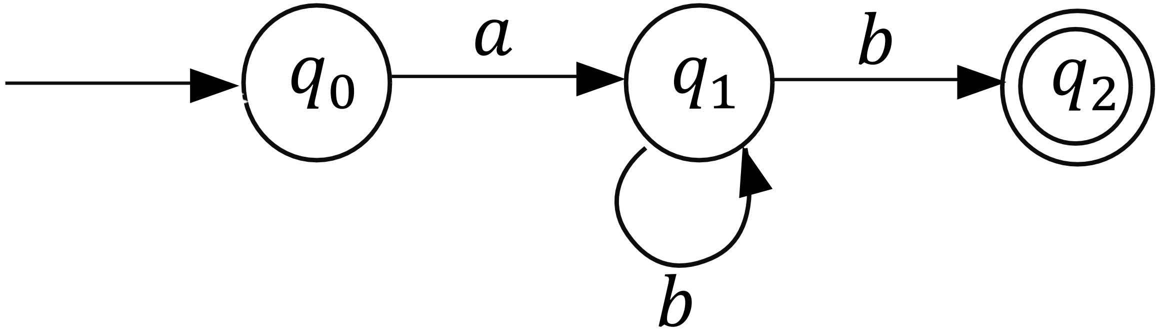
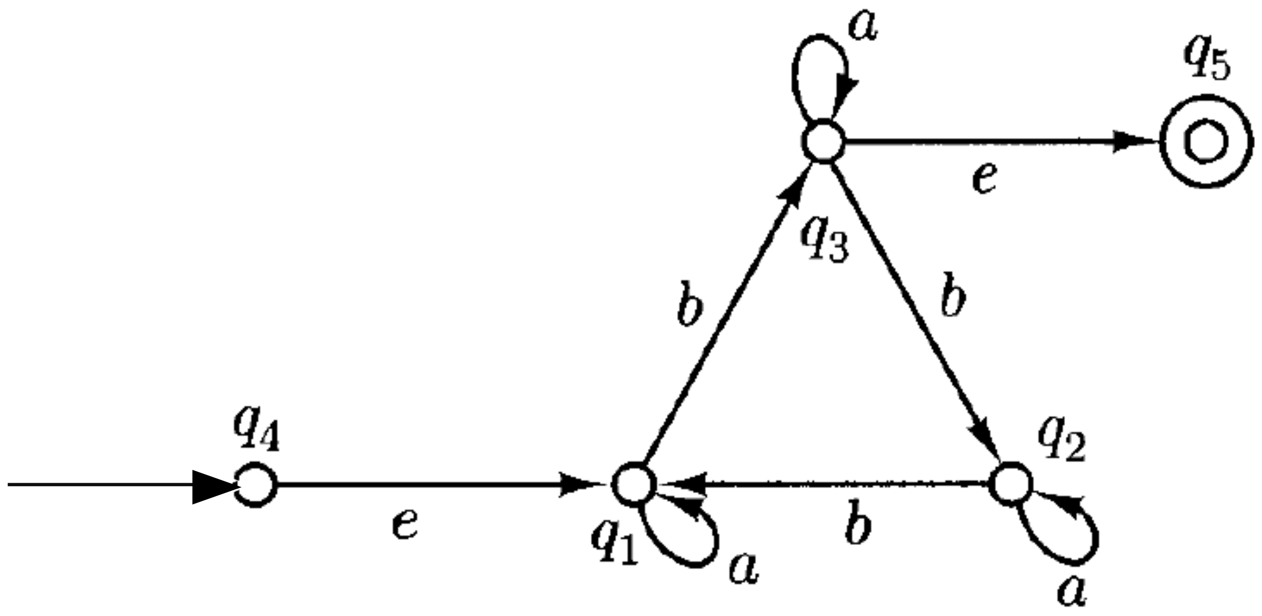


$L_{NFA} \subseteq L_{REG}$

- More example:



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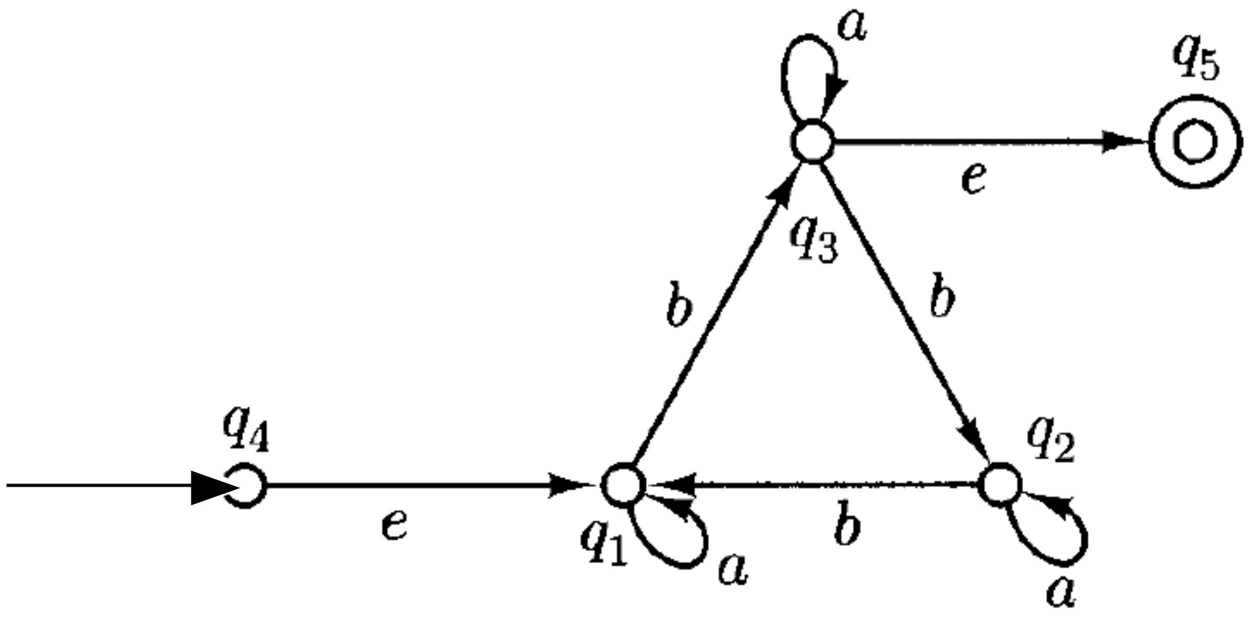

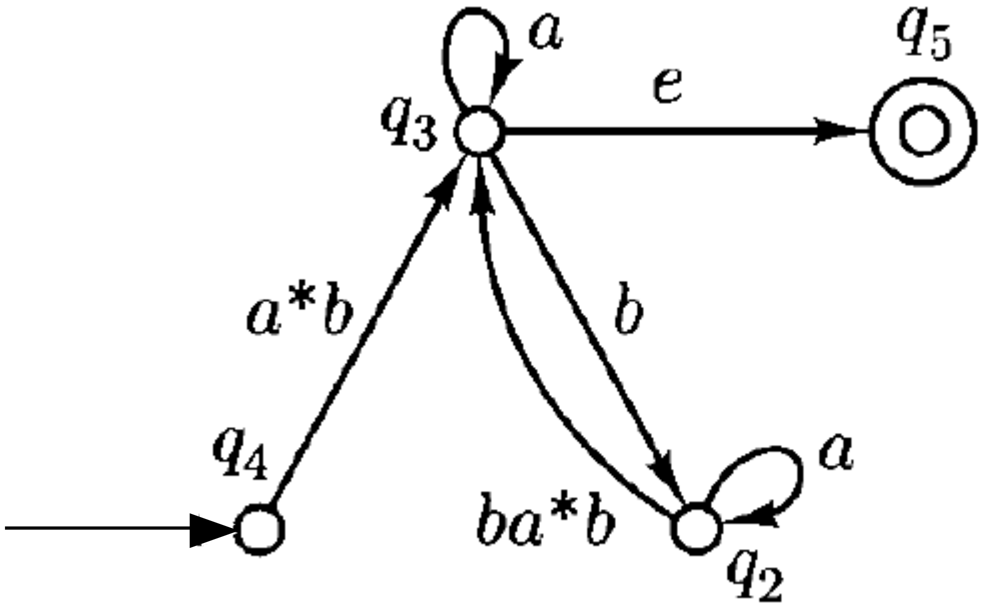

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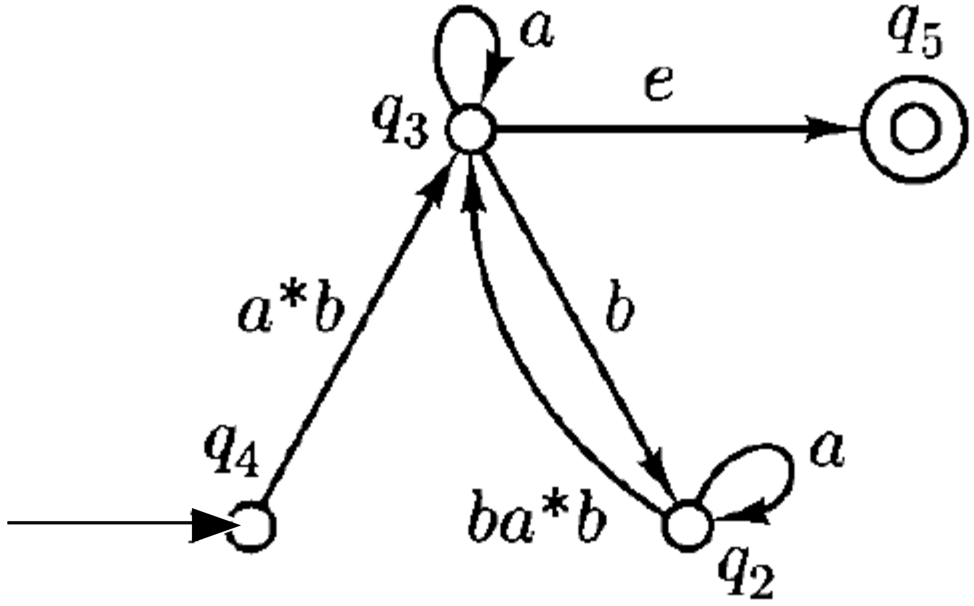

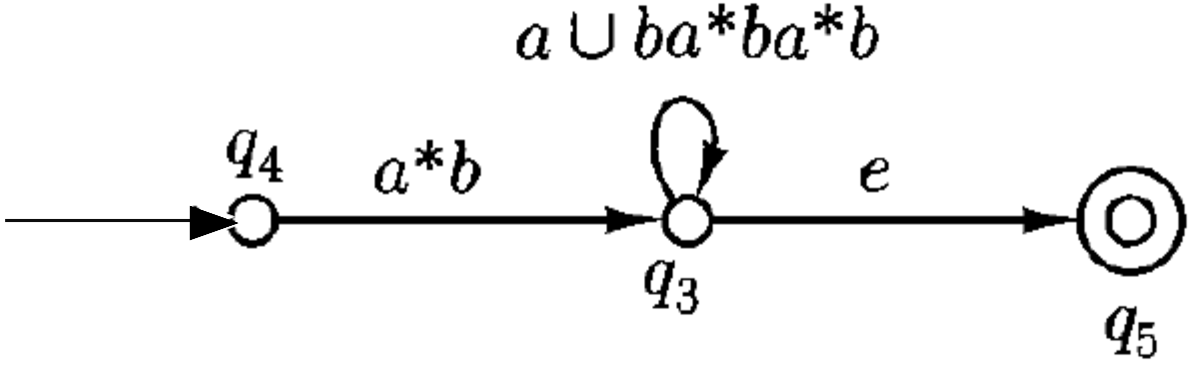

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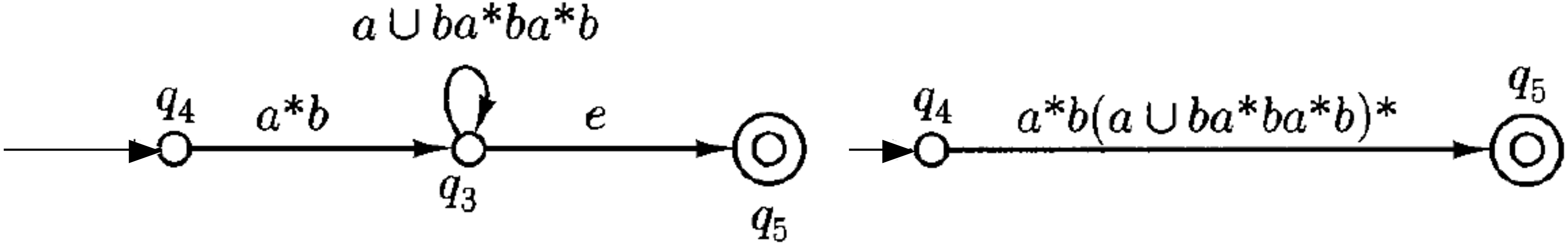

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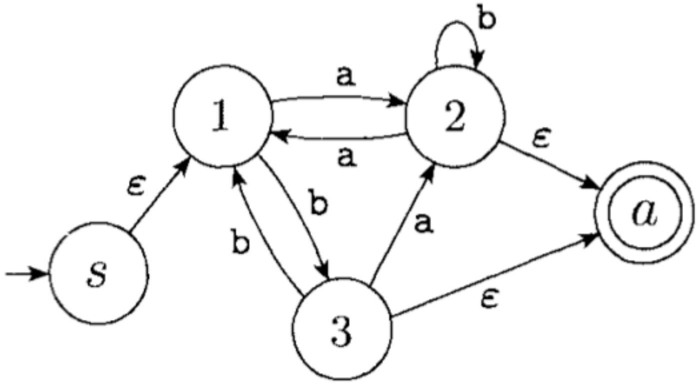

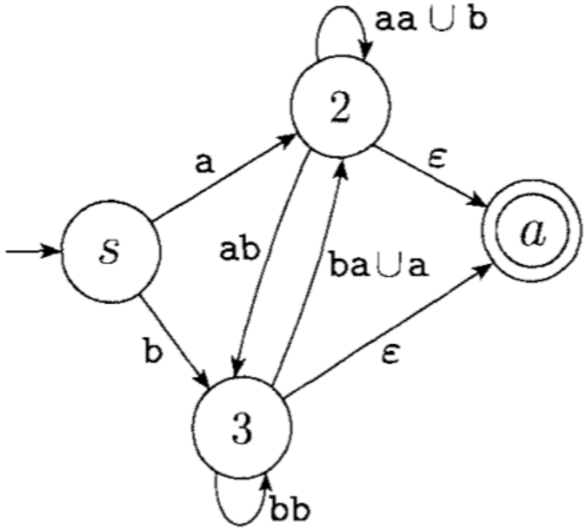

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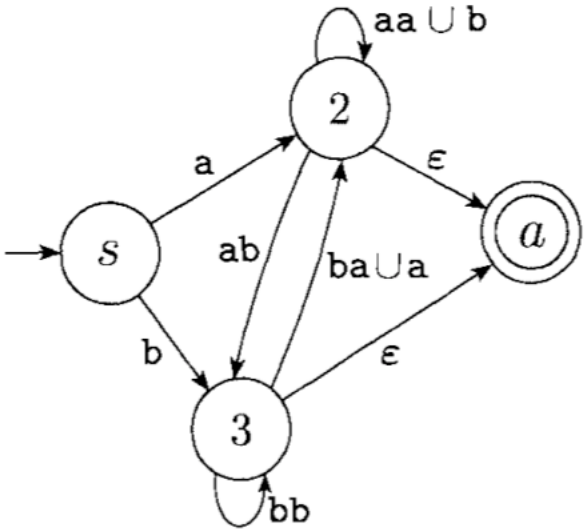

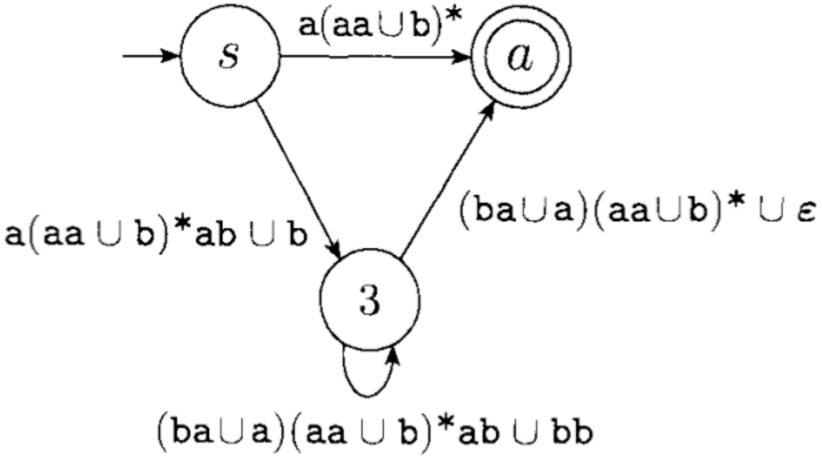

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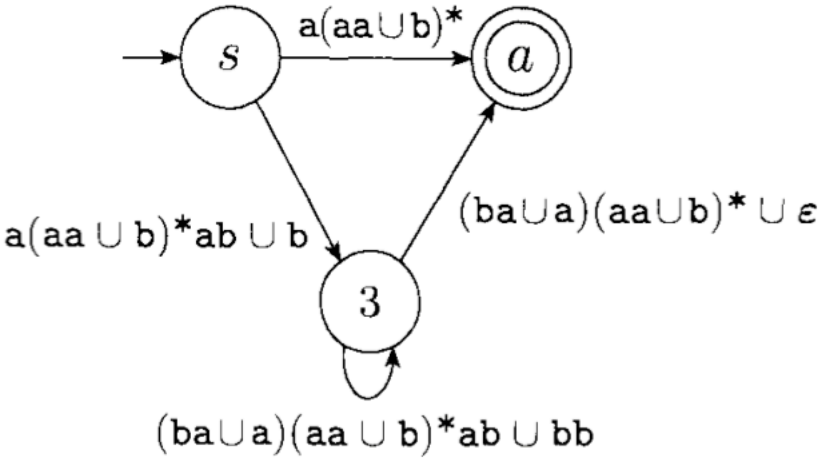

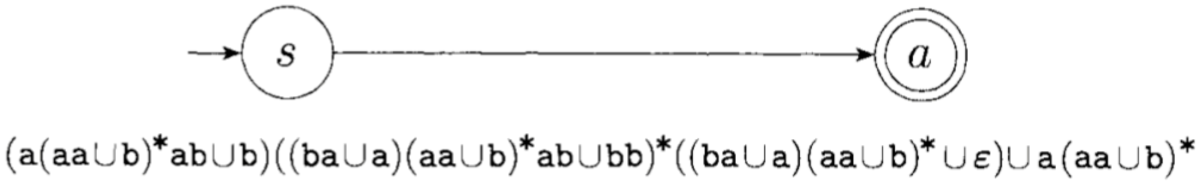

$L_{NFA} \subseteq L_{REG}$

- Proof by induction on the "length" of the path
  - Let $K = \{q_1, q_2, \cdots, q_n\}$ and $s = q_1$
  - Define $R(i, j, k)$ as the set of strings that can drive M from q_i to q_j without passing any state numbered $k + 1$ or higher

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- What is the language accepted by M

$$L(M) = \bigcup \{R(1, j, n) : q_j \in F\}$$

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- We prove $R(i,j,k)$ is regular for any i,j,k

$$R(i,j,k) = R(i,j,k-1) \cup R(i,k,k-1)R(k,k,k-1)^*R(k,j,k-1)$$