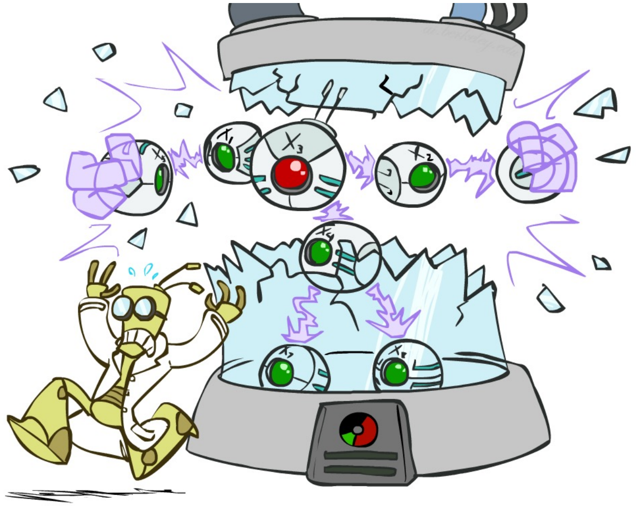
Announcement

- Quiz is up on blackboard
  - Covers MDP and RL. Time set to 1 hour. You don't need that much time. Just be sure to study before doing it
- Homework
  - HW 4 was due today
  - HW 5 will go up soon
- Projects
  - Project 3 is due next Sunday 31
  - Project 4 (last non-optional) will go up in two/three weeks

CS 3568: Intelligent Systems

Bayes’ Nets (Part 4)



Instructor: Tara Salman

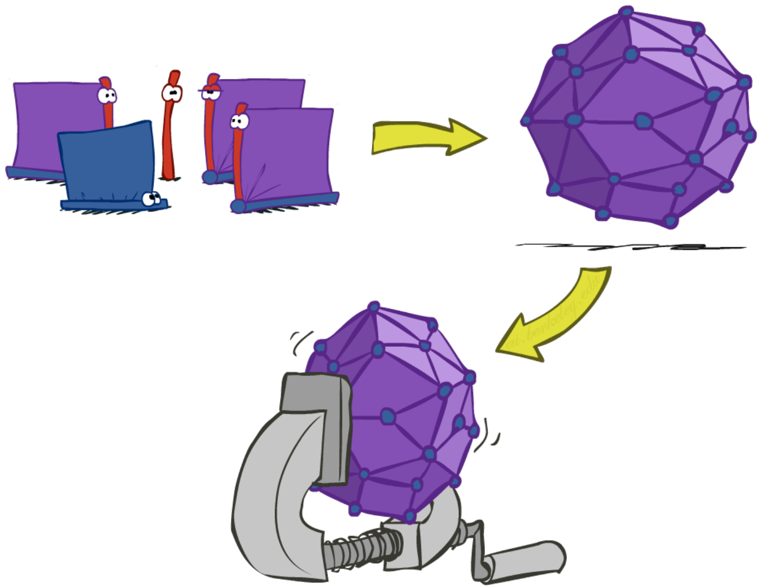
Texas Tech University

Computer Science Department

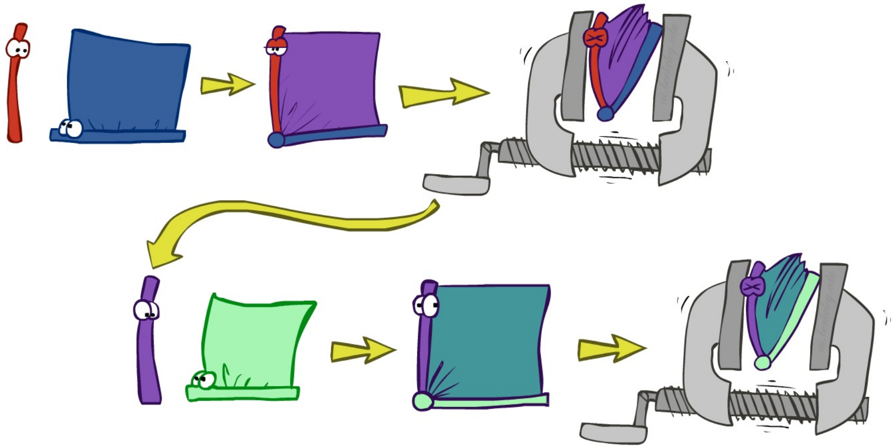
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley (ai.berkeley.edu).]

Inference by Enumeration vs. Variable Elimination

- Why is inference by enumeration so slow?
  - You join up the whole joint distribution before you sum out the hidden variables



- Idea: interleave joining and marginalizing!
  - Called "Variable Elimination"
  - Still NP-hard, but usually much faster than inference by enumeration



Inference by Enumeration: Procedural Outline

- Track objects called factors
- Initial factors are local CPTs (one per node)

$P(R)$

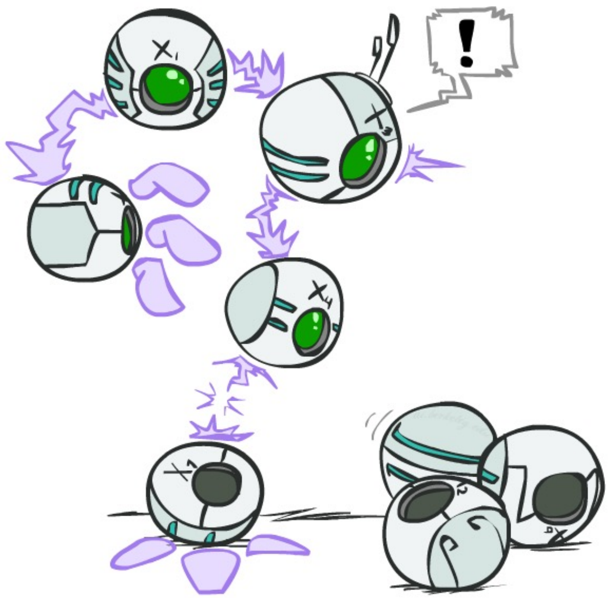
+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9



- Any known values are selected
  - E.g. if we know $L = +\ell$, the initial factors are

$P(R)$

+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

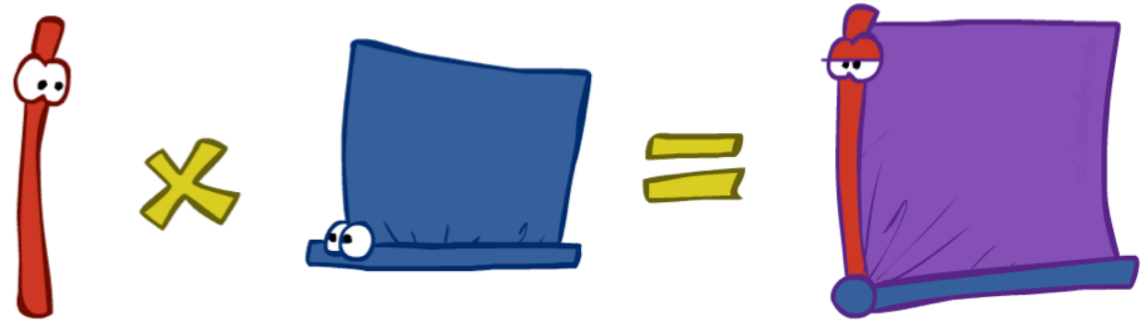
$P(+\ell|T)$

+t	+l	0.3
-t	+l	0.1

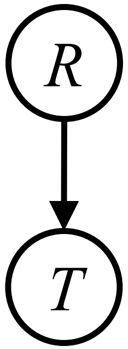
- Procedure: Join all factors, eliminate all hidden variables, normalize

Operation 1: Join Factors

- First basic operation: joining factors
- Combining factors:
  - Just like a database join
  - Get all factors over the joining variable
  - Build a new factor over the union of the variables involved



- Example: Join on R



$$P(R) \times P(T|R) \rightarrow P(R,T)$$

$P(R)$

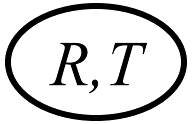
+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

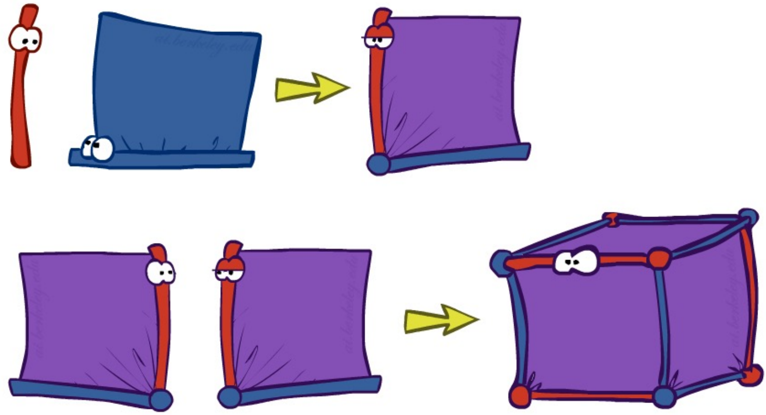
$P(R,T)$

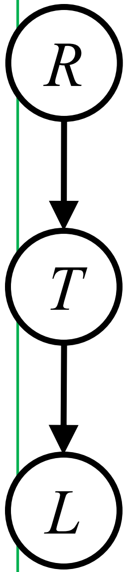
+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81



  - Computation for each entry: pointwise products $\forall r,t: \quad P(r,t) = P(r) \cdot P(t|r)$

Example: Multiple Joins





$P(R)$

+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

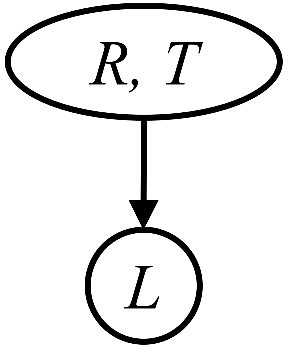
Join R

$P(R,T)$

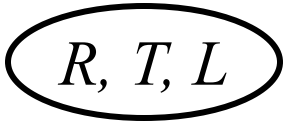
+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9



Join T



$P(R,T,L)$

+r	+t	+l	0.024
+r	+t	-l	0.056
+r	-t	+l	0.002
+r	-t	-l	0.018
-r	+t	+l	0.027
-r	+t	-l	0.063
-r	-t	+l	0.081
-r	-t	-l	0.729

Operation 2: Eliminate

- Second basic operation: marginalization
- Take a factor and sum out a variable
  - Shrinks a factor to a smaller one
  - A projection operation
- Example:

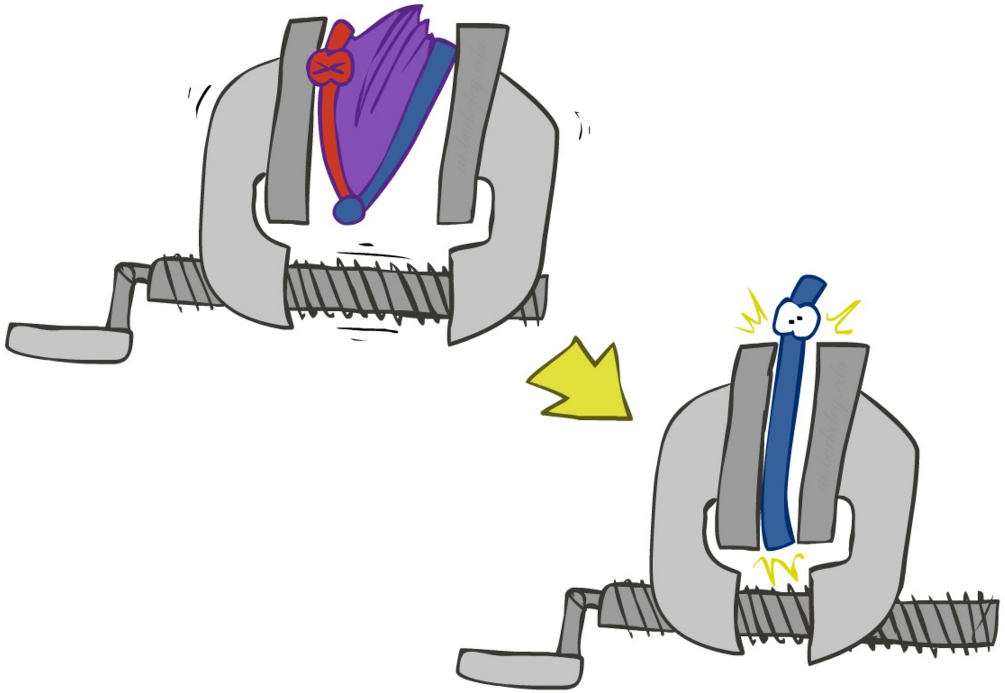
$P(R,T)$

+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81

$\xrightarrow{\text{sum } R}$

$P(T)$

+t	0.17
-t	0.83



Multiple Elimination

$P(R, T, L)$ R, T, L

+r	+t	+l	0.024
+r	+t	-l	0.056
+r	-t	+l	0.002
+r	-t	-l	0.018
-r	+t	+l	0.027
-r	+t	-l	0.063
-r	-t	+l	0.081
-r	-t	-l	0.729

Sum out R

T, L

$P(T, L)$

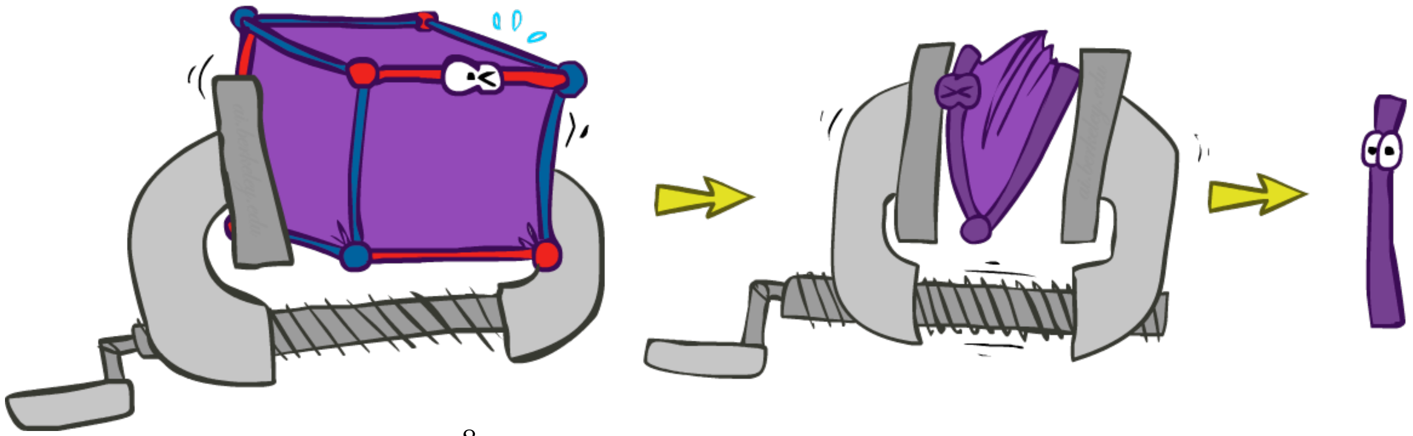
+t	+l	0.051
+t	-l	0.119
-t	+l	0.083
-t	-l	0.747

Sum out T

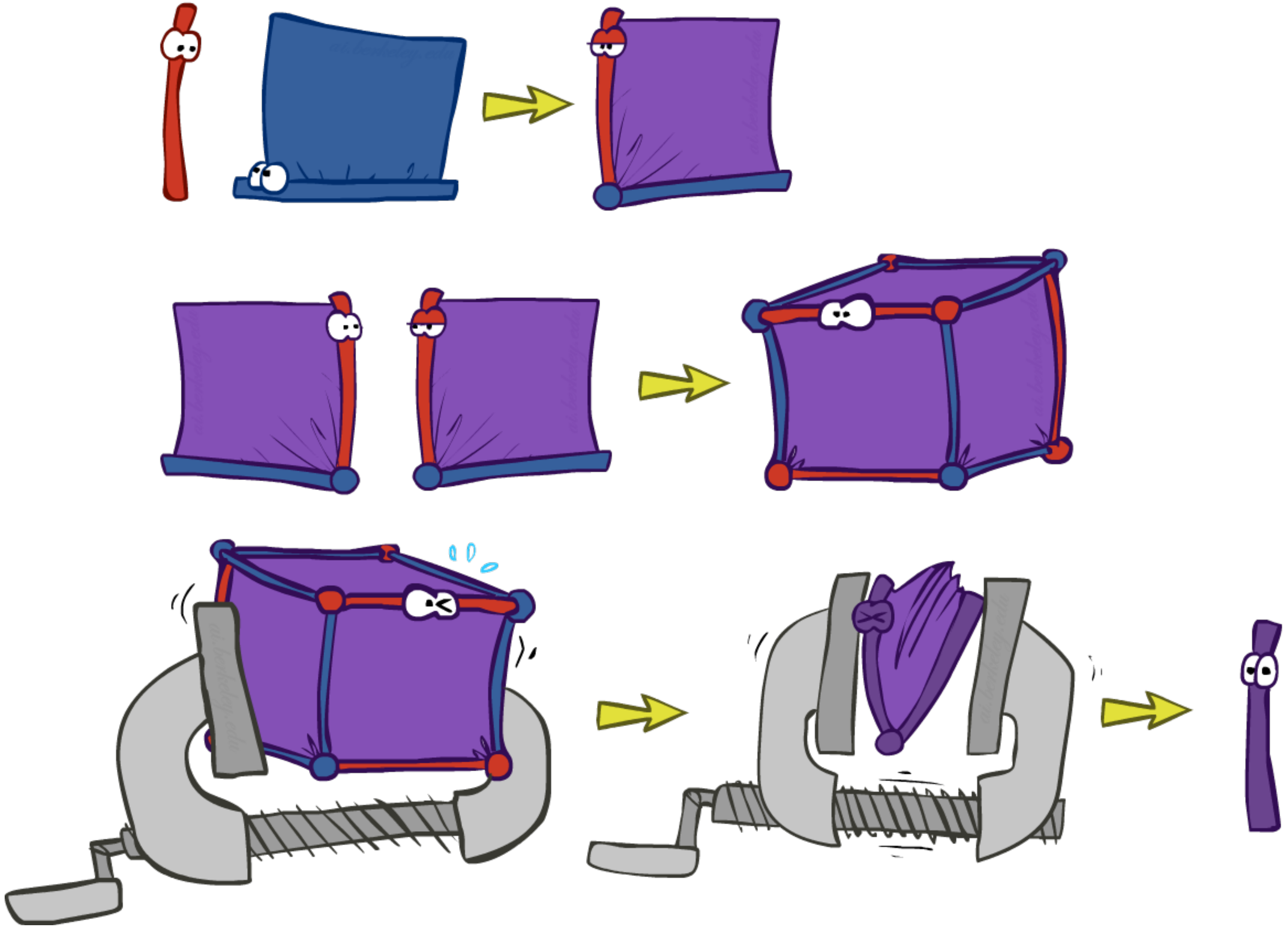
L

$P(L)$

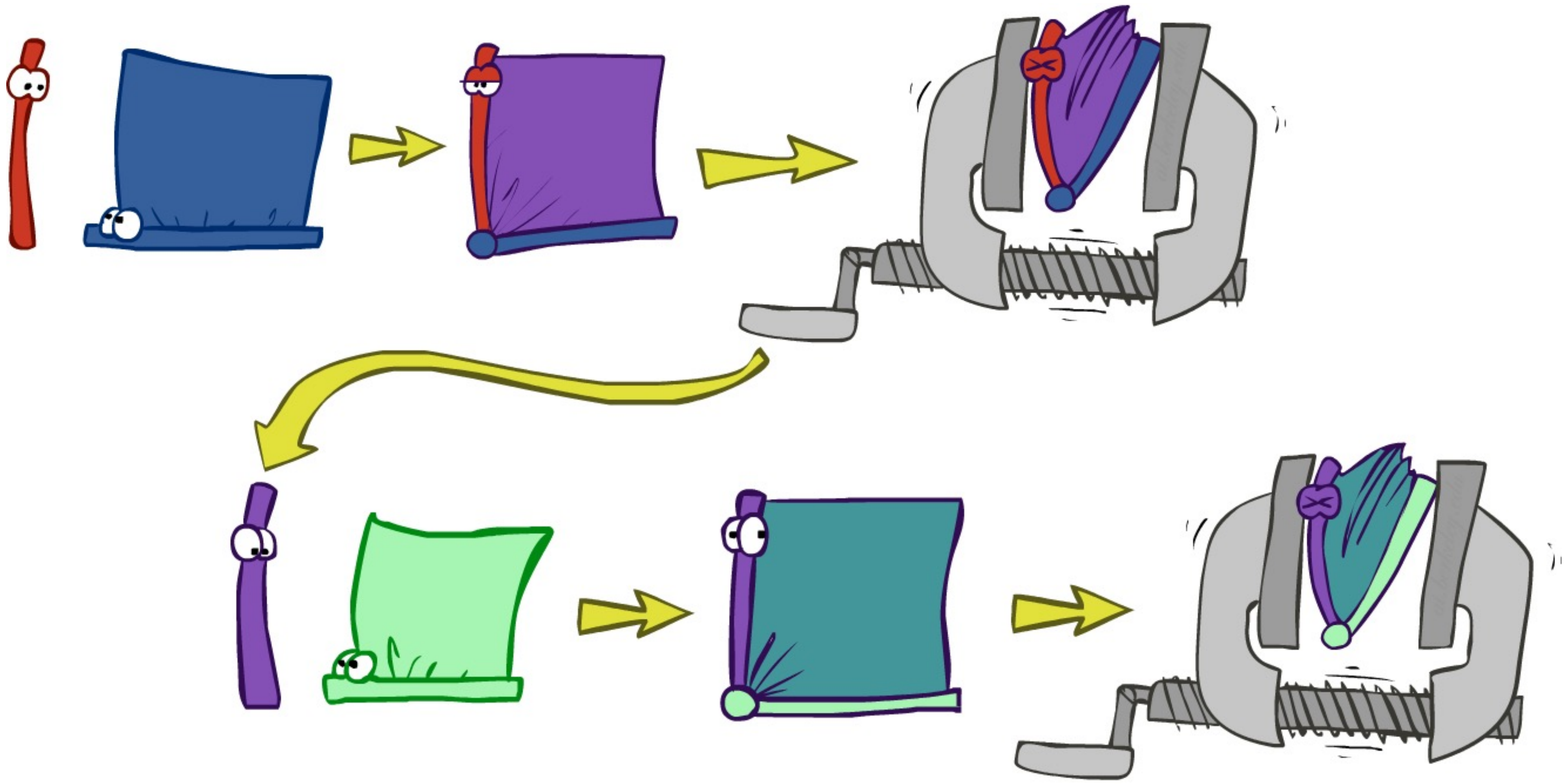
+l	0.134
-l	0.886



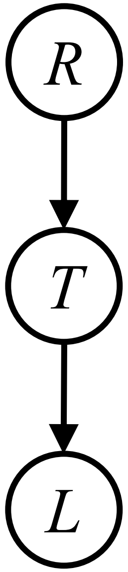
Thus Far: Multiple Join, Multiple Eliminate (= Inference by Enumeration)



Marginalizing Early (= Variable Elimination)



Traffic Domain



$$P(L) = ?$$

- Inference by Enumeration

$$= \underbrace{\sum_t \underbrace{\sum_r \underbrace{P(L|t)\underbrace{P(r)P(t|r)}_{\text{Join on r}}}_{\text{Join on t}}}_{\text{Eliminate r}}}_{\text{Eliminate t}}$$

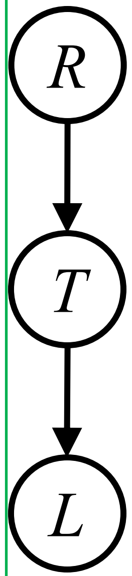
- Variable Elimination

$$= \underbrace{\sum_t \underbrace{P(L|t) \underbrace{\sum_r \underbrace{P(r)P(t|r)}_{\text{Join on r}}}_{\text{Eliminate r}}}_{\text{Join on t}}}_{\text{Eliminate t}}$$

Marginalizing Early! (aka VE)

$P(R)$

+r	0.1
-r	0.9



$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

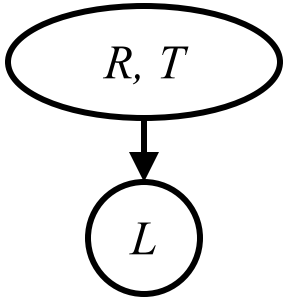
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Join R

$P(R,T)$

+r	+t	0.08
+r	-t	0.02
-r	+t	0.09
-r	-t	0.81



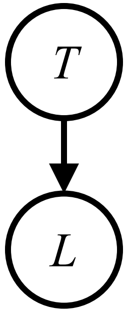
$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

Sum out R

$P(T)$

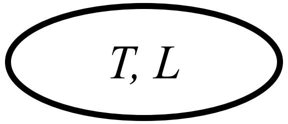
+t	0.17
-t	0.83



$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

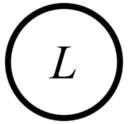
Join T



$P(T,L)$

+t	+l	0.051
+t	-l	0.119
-t	+l	0.083
-t	-l	0.747

Sum out T



$P(L)$

+l	0.134
-l	0.866

Evidence

- If evidence, start with factors that select that evidence
  - No evidence uses these initial factors:

$P(R)$

+r	0.1
-r	0.9

$P(T|R)$

+r	+t	0.8
+r	-t	0.2
-r	+t	0.1
-r	-t	0.9

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9

  - Computing $P(L|+r)$, the initial factors become:

$P(+r)$

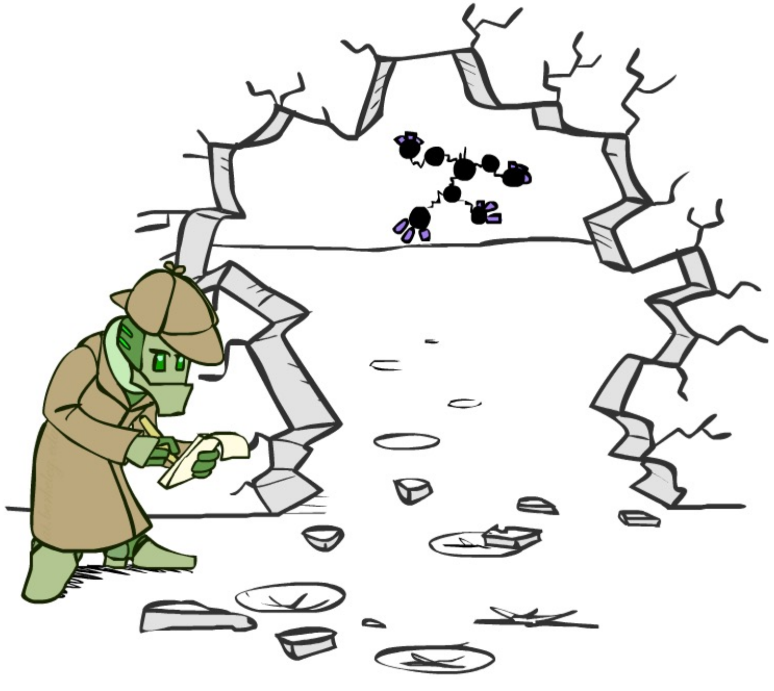
+r	0.1

$P(T|+r)$

+r	+t	0.8
+r	-t	0.2

$P(L|T)$

+t	+l	0.3
+t	-l	0.7
-t	+l	0.1
-t	-l	0.9



- We eliminate all vars other than query + evidence

Evidence II

- Result will be a selected joint of query and evidence
  - E.g. for P(L | +r), we would end up with:

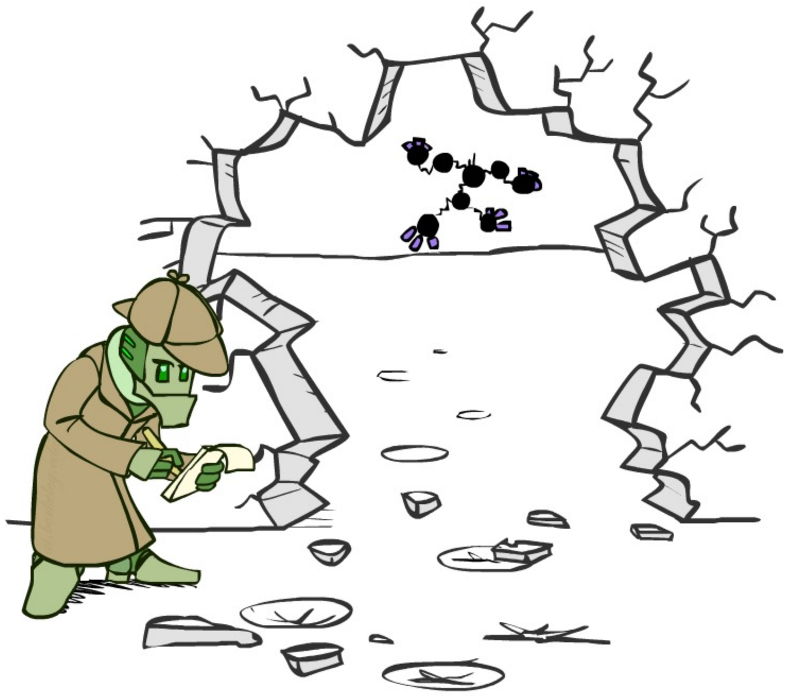
$P(+r, L)$

+r	+l	0.026
+r	-l	0.074

Normalize

$P(L|+r)$

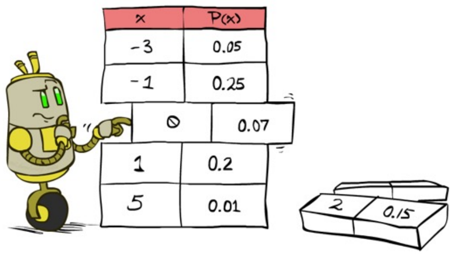
+l	0.26
-l	0.74



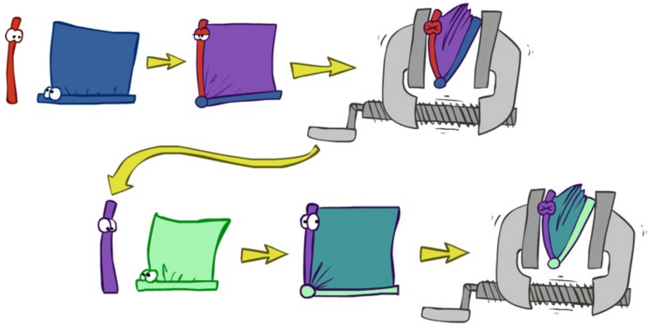
- To get our answer, just normalize this!
- That's it!

General Variable Elimination

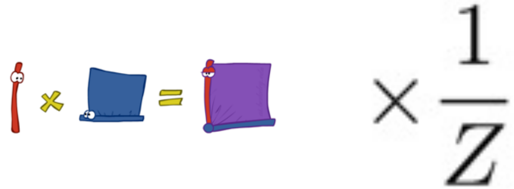
- Query: $P(Q|E_1 = e_1, \ldots E_k = e_k)$



- Start with initial factors:
  - Local CPTs (but instantiated by evidence)



- While there are still hidden variables (not Q or evidence):
  - Pick a hidden variable H
  - Join all factors mentioning H
  - Eliminate (sum out) H

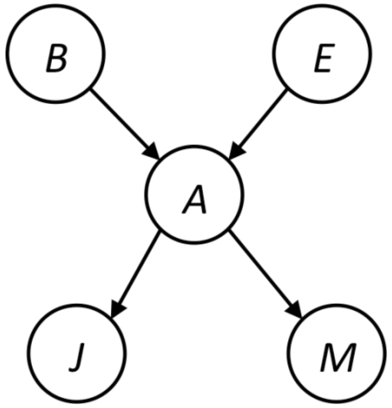


- Join all remaining factors and normalize

Example

$$P(B|j,m) \propto P(B,j,m)$$

$$\boxed{P(B) \qquad P(E) \qquad P(A|B,E) \qquad P(j|A) \qquad P(m|A)}$$

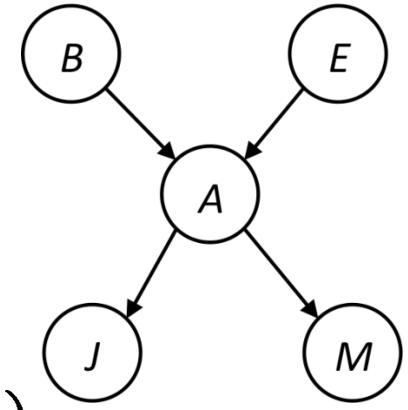


Choose A

$$P(A|B,E)$$
$$P(j|A) \quad \xrightarrow{\times} \quad P(j,m,A|B,E) \quad \xrightarrow{\sum} \quad P(j,m|B,E)$$
$$P(m|A)$$

$$\boxed{P(B) \qquad P(E) \qquad P(j,m|B,E)}$$

Example



$$\boxed{P(B) \qquad P(E) \qquad P(j, m|B, E)}$$

Choose E

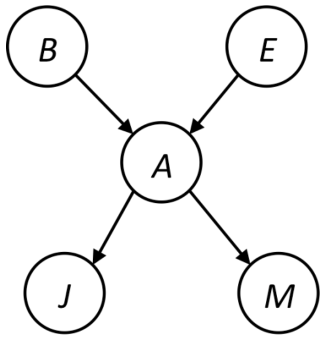
$$\left.\begin{matrix} P(E) \\ P(j, m|B, E) \end{matrix}\right. \xrightarrow{\times} P(j, m, E|B) \xrightarrow{\Sigma} P(j, m|B)$$

$$\boxed{P(B) \qquad P(j, m|B)}$$

Finish with B

$$\left.\begin{matrix} P(B) \\ P(j, m|B) \end{matrix}\right. \xrightarrow{\times} P(j, m, B) \xrightarrow{\text{Normalize}} P(B|j, m)$$

Same Example in Equations



$$P(B|j,m) \propto P(B,j,m)$$

$$\boxed{P(B) \qquad P(E) \qquad P(A|B,E) \qquad P(j|A) \qquad P(m|A)}$$

$$\begin{aligned}
P(B|j,m) &\propto P(B,j,m) \\
&= \sum_{e,a} P(B,j,m,e,a) \\
&= \sum_{e,a} P(B)P(e)P(a|B,e)P(j|a)P(m|a) \\
&= \sum_{e} P(B)P(e)\sum_{a} P(a|B,e)P(j|a)P(m|a) \\
&= \sum_{e} P(B)P(e)f_1(B,e,j,m) \\
&= P(B)\sum_{e} P(e)f_1(B,e,j,m) \\
&= P(B)f_2(B,j,m)
\end{aligned}$$

marginal obtained from joint by summing out

use Bayes' net joint distribution expression

use x*(y+z) = xy + xz

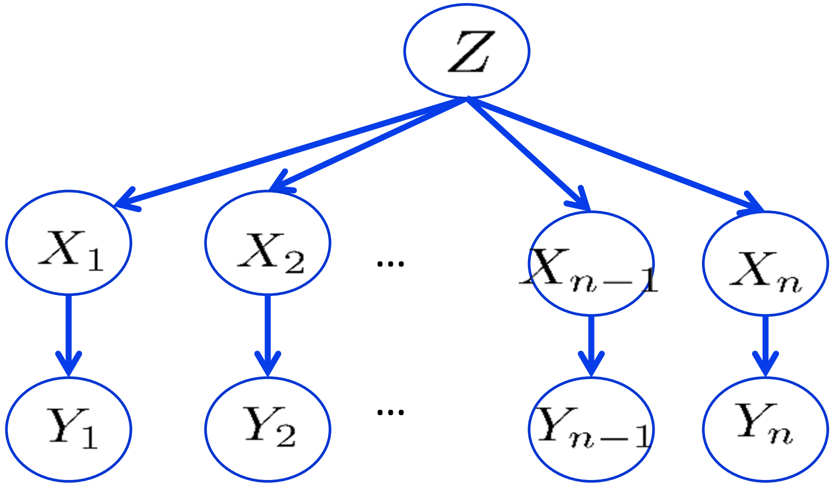
joining on a, and then summing out gives f_1

use x*(y+z) = xy + xz

joining on e, and then summing out gives f_2

Variable Elimination Ordering

- For the query $P(X_n|y_1,\ldots,y_n)$ work through the following two different orderings as done in previous slide: $Z, X_1, \ldots, X_{n-1}$ and $X_1, \ldots, X_{n-1}, Z$. What is the size of the maximum factor generated for each of the orderings?



- Answer: 2^{n+1} versus 2^2 (assuming binary)
- In general: the ordering can greatly affect efficiency.

VE: Computational and Space Complexity

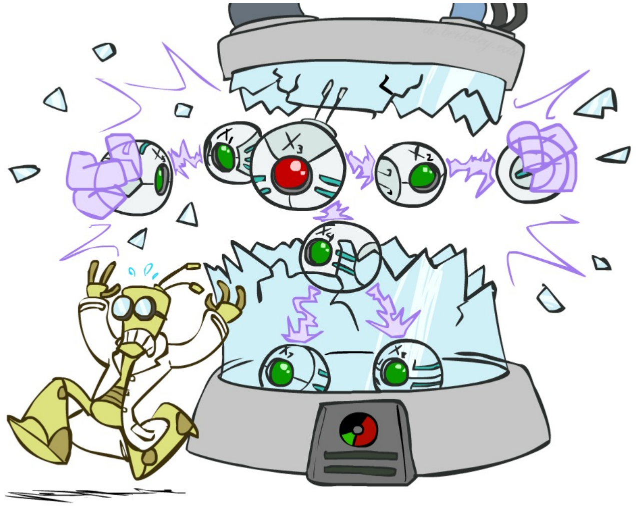
- The computational and space complexity of variable elimination is determined by the largest factor

- The elimination ordering can greatly affect the size of the largest factor.
  - E.g., previous slide's example 2^n vs. 2

- Does there always exist an ordering that only results in small factors?
  - No!

CS 3568: Intelligent Systems

Decision Networks and Value of Information



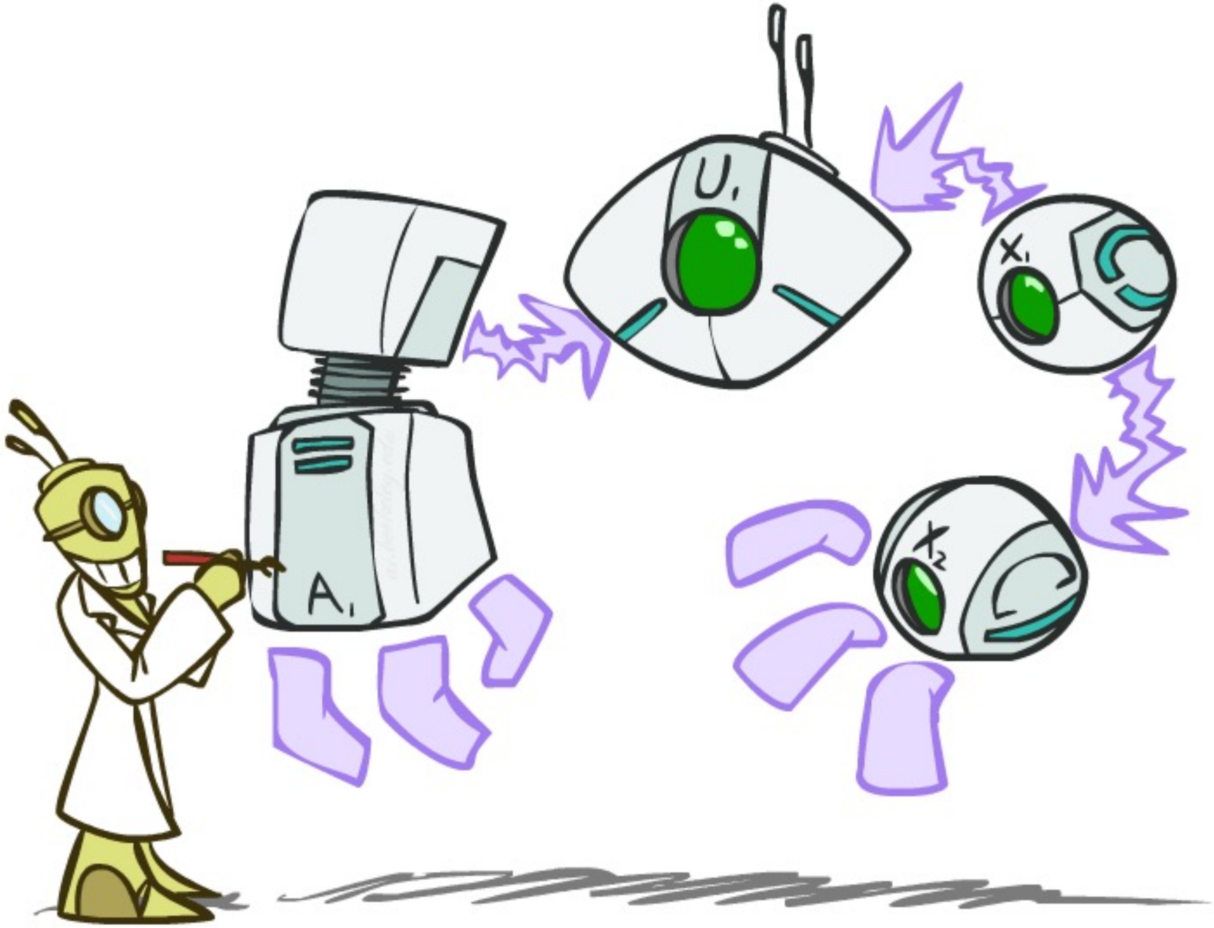
Instructor: Tara Salman

Texas Tech University

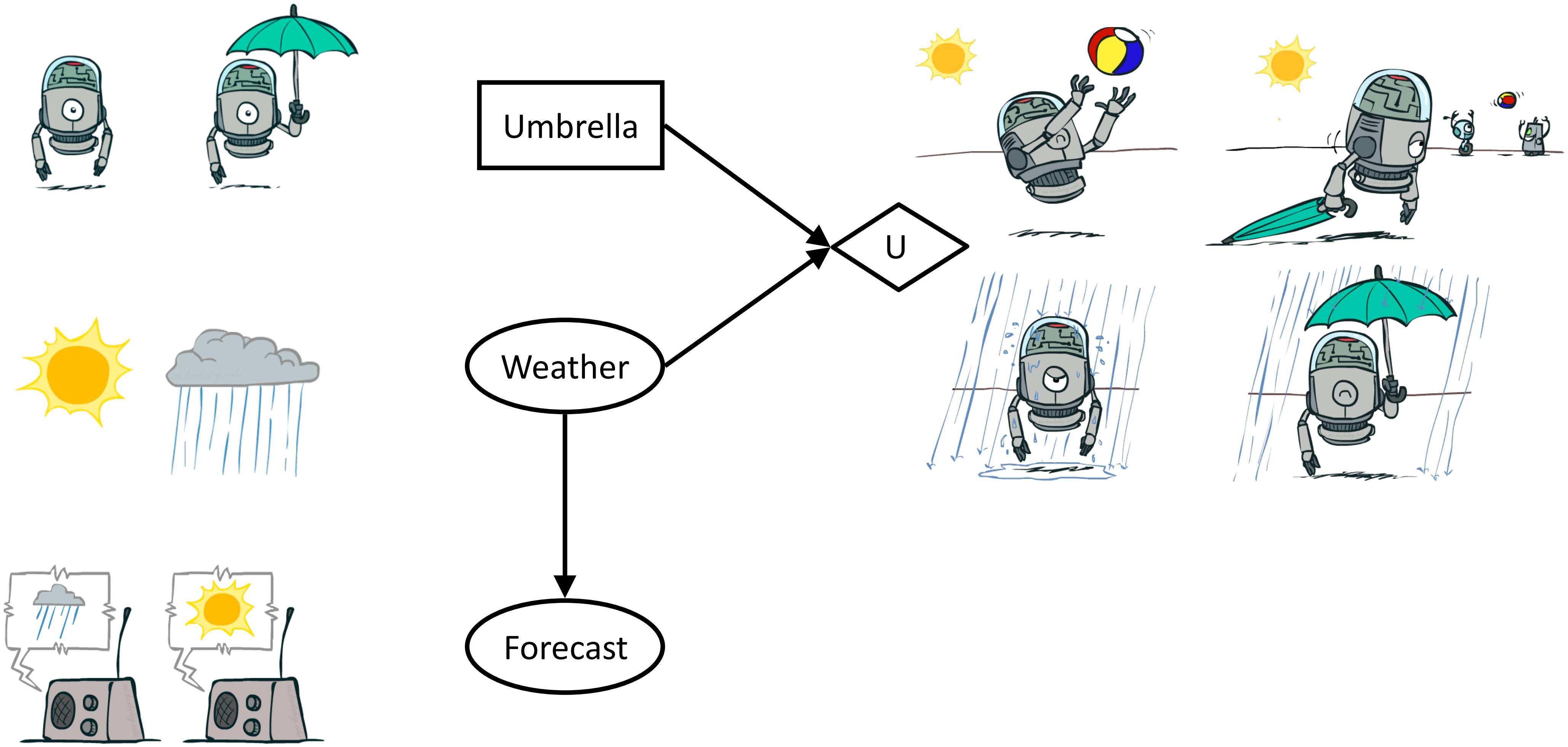
Computer Science Department

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley (ai.berkeley.edu).]

Decision Networks



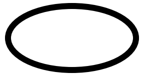
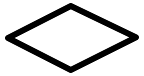
Decision Networks

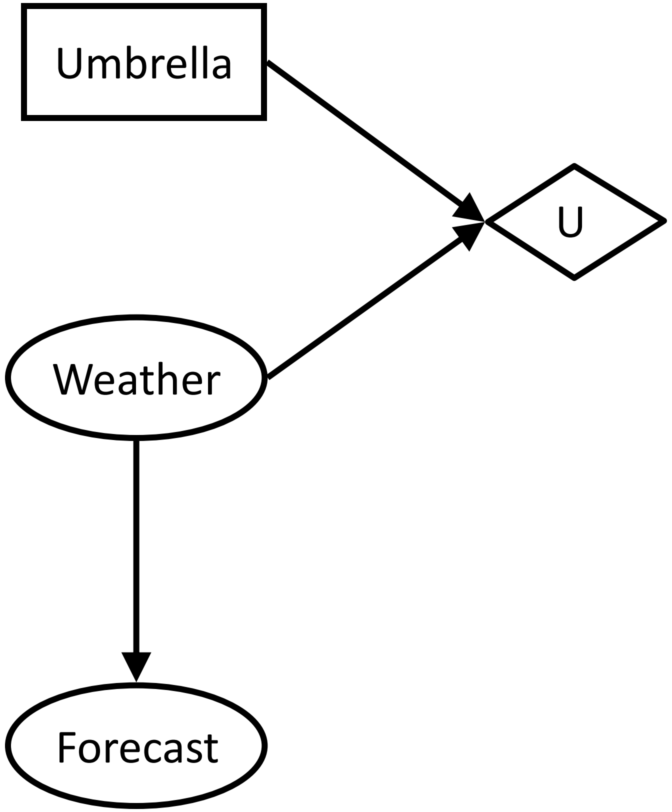


Decision Networks

- **MEU: choose the action which maximizes the expected utility given the evidence**

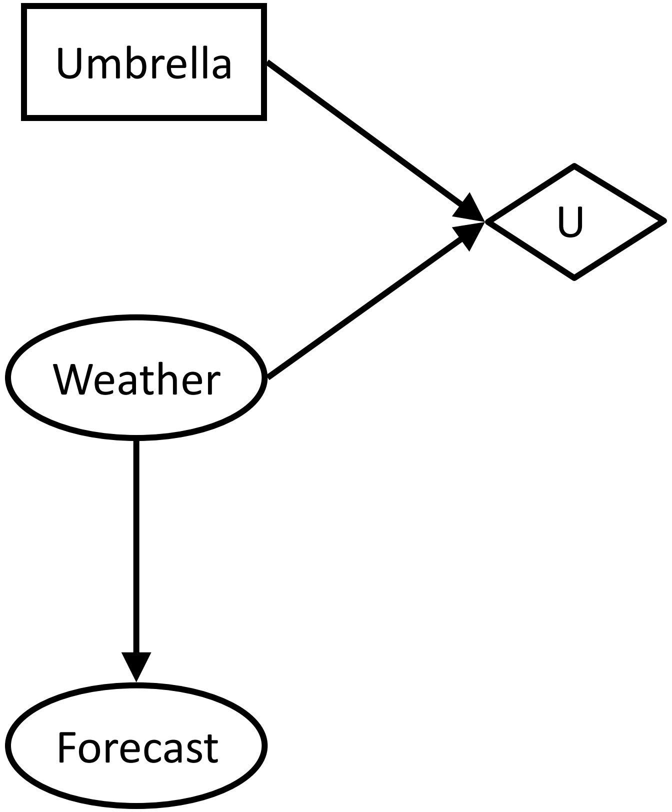
- Can directly operationalize this with decision networks
  - Bayes nets with nodes for utility and actions
  - Lets us calculate the expected utility for each action

- New node types:
  - Chance nodes (just like BNs) 
  - Actions (rectangles, cannot have parents, act as observed evidence) 
  - Utility node (diamond, depends on action and chance nodes) 



Decision Networks

- Action selection
  - Instantiate all evidence
  - Set action node(s) each possible way
  - Calculate posterior for all parents of utility node, given the evidence
  - Calculate expected utility for each action
  - Choose maximizing action



Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w) U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

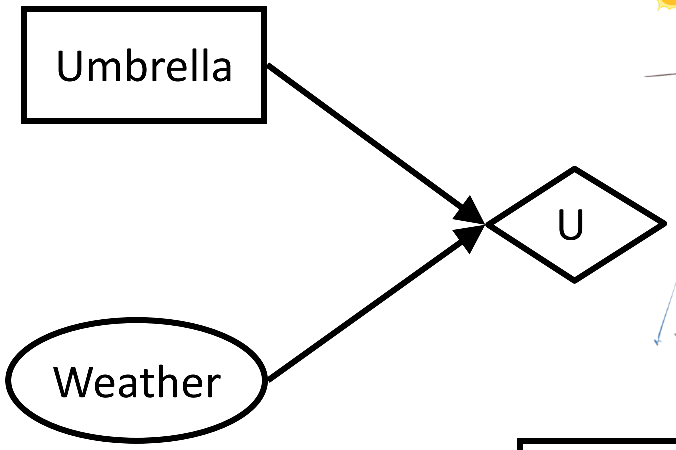
Umbrella = take

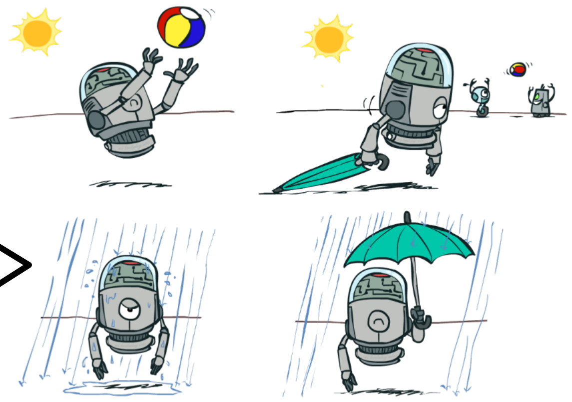
$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w) U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\emptyset) = \max_a \mathrm{EU}(a) = 70$$

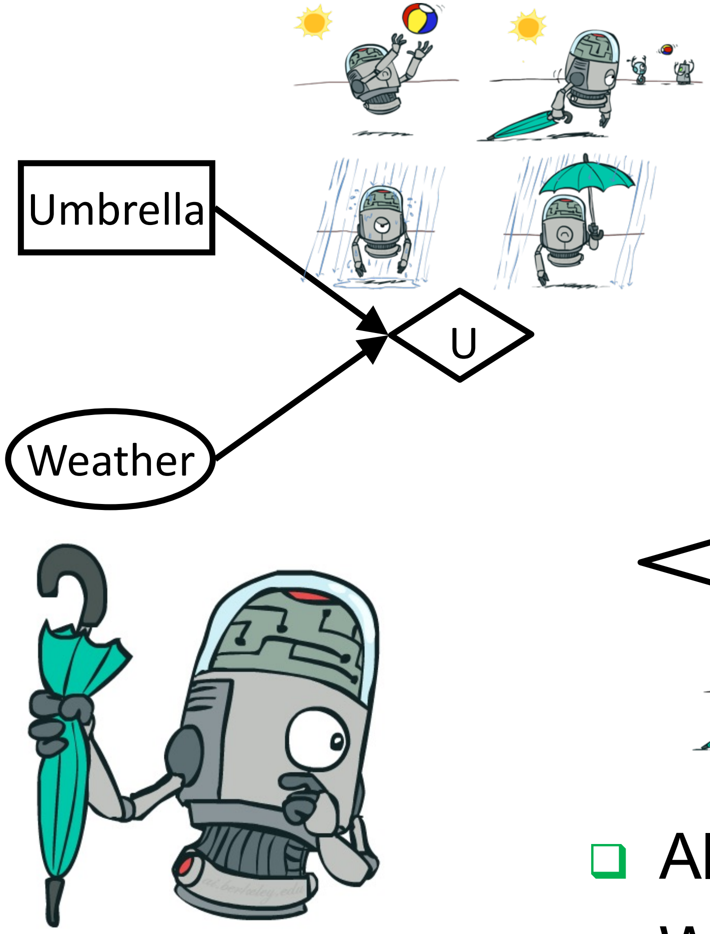


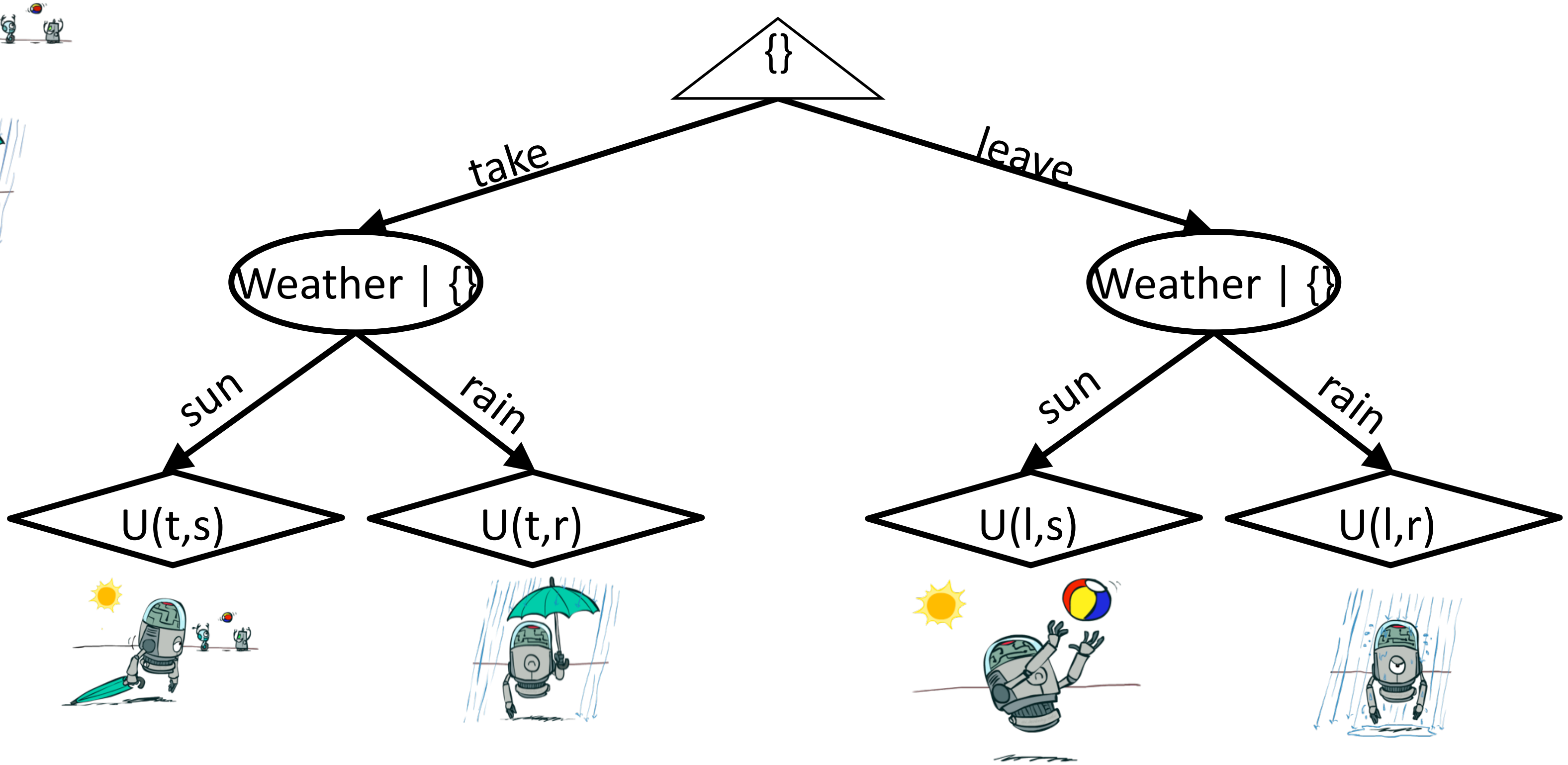


W	P(W)
sun	0.7
rain	0.3

A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

Decisions as Outcome Trees





- Almost exactly like expectimax / MDPs
- What's changed?

Example: Decision Networks

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

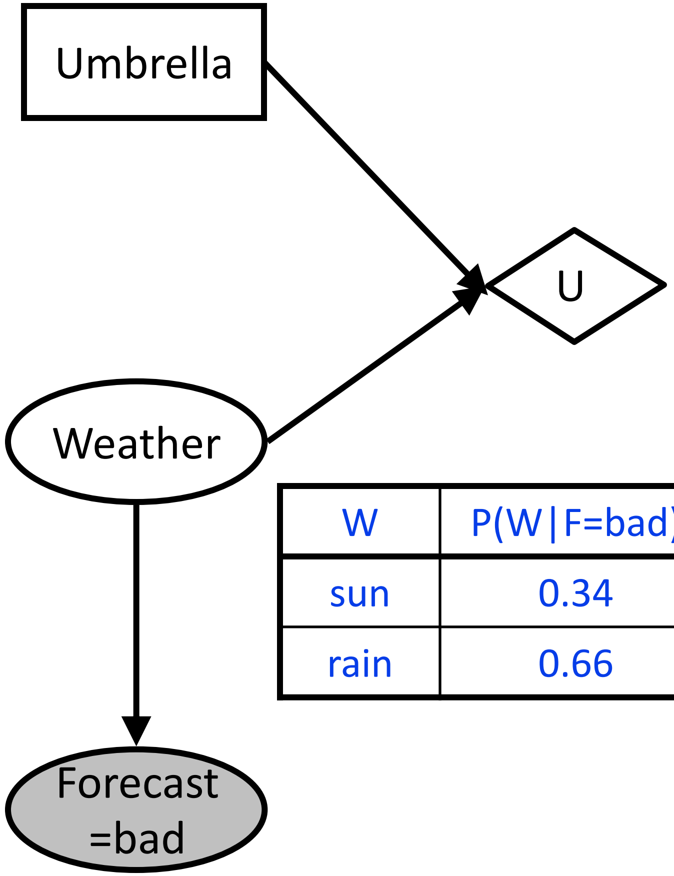
Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

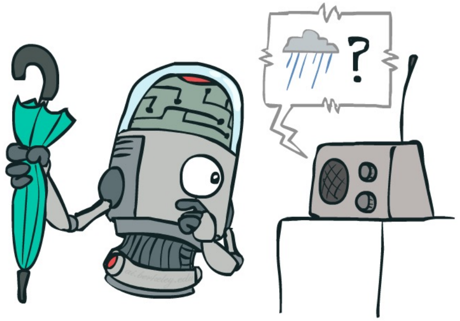
Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

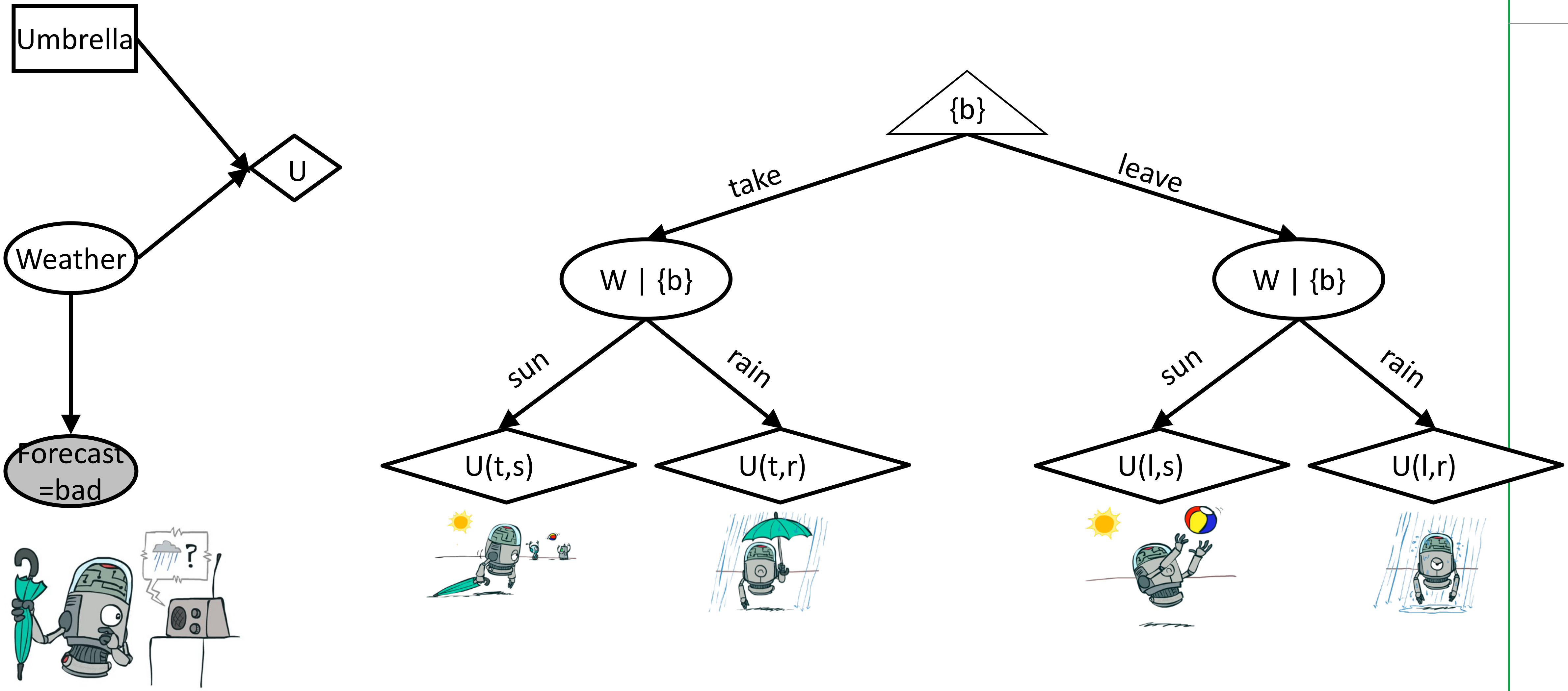


A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

W	P(W|F=bad)
sun	0.34
rain	0.66

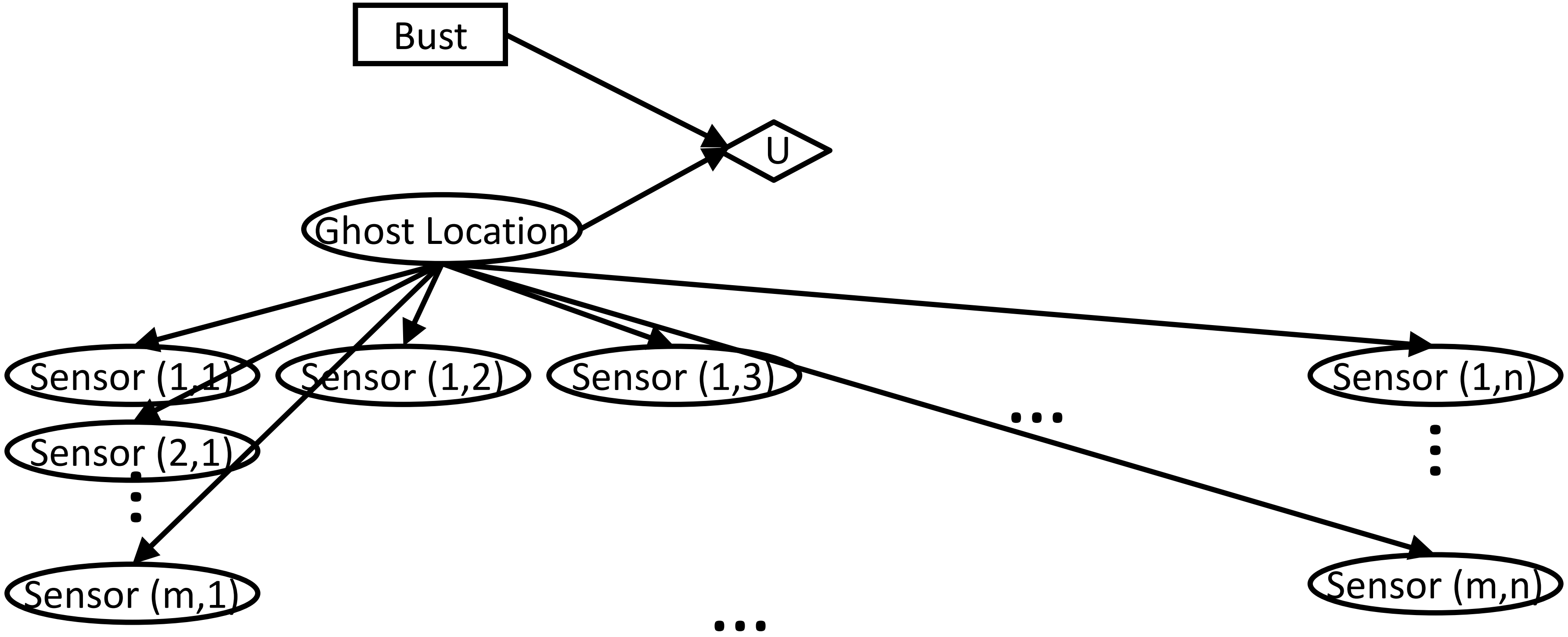


Decisions as Outcome Trees

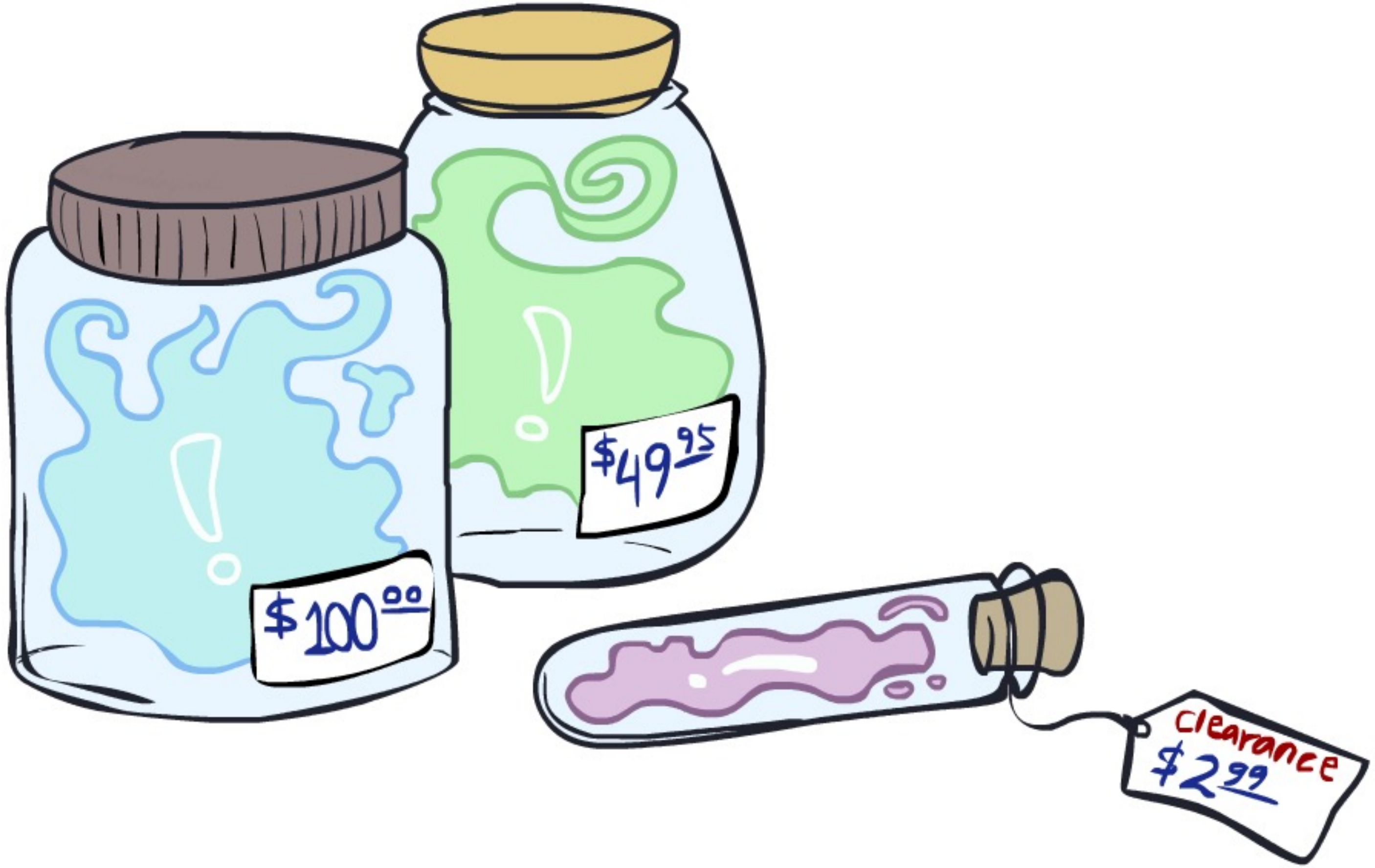


Video of Demo Ghostbusters with Probability

Ghostbusters Decision Network



Value of Information



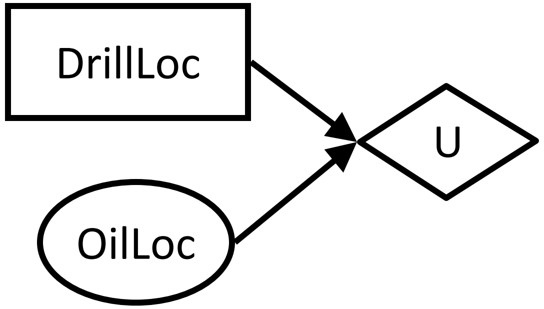
Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

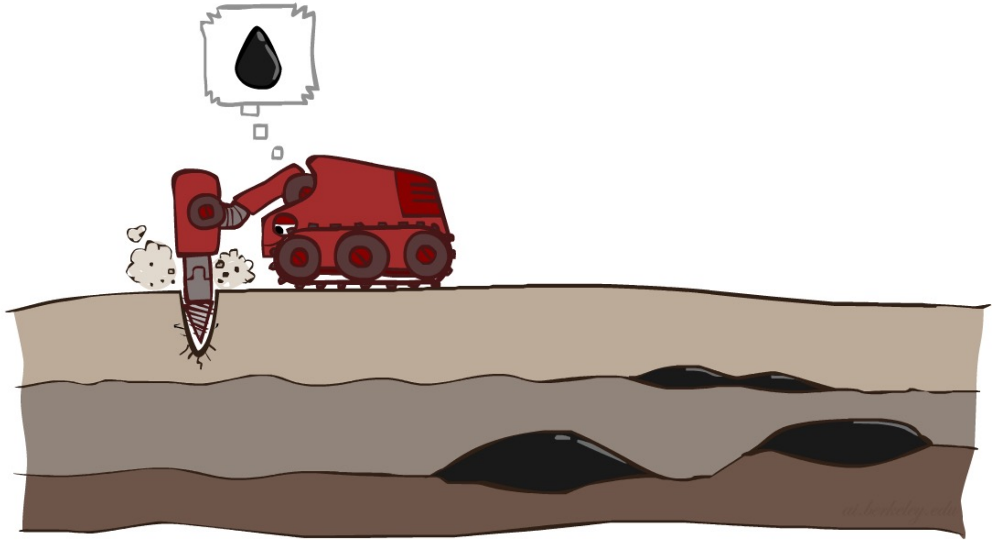
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b", prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2

O	P
a	1/2
b	1/2



D	O	U
a	a	k
a	b	0
b	a	0
b	b	k



VPI Example: Weather

MEU with no evidence

$$\mathrm{MEU}(\phi) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

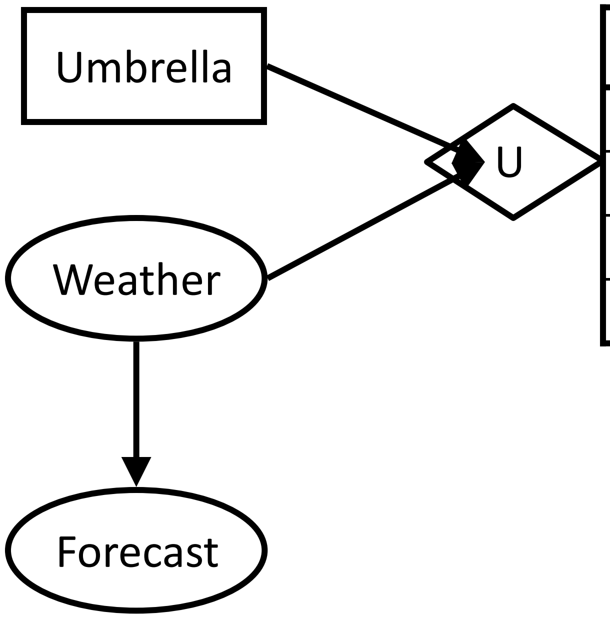
$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

Forecast distribution

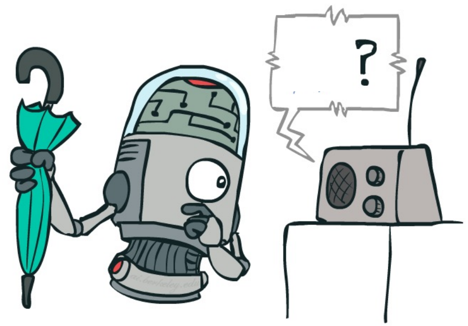
F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$

$$77.8 - 70 = 7.8$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



$$\mathrm{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\mathrm{MEU}(e, e')\right) - \mathrm{MEU}(e)$$

Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Assume we see that E' = e'. Value if we act then:

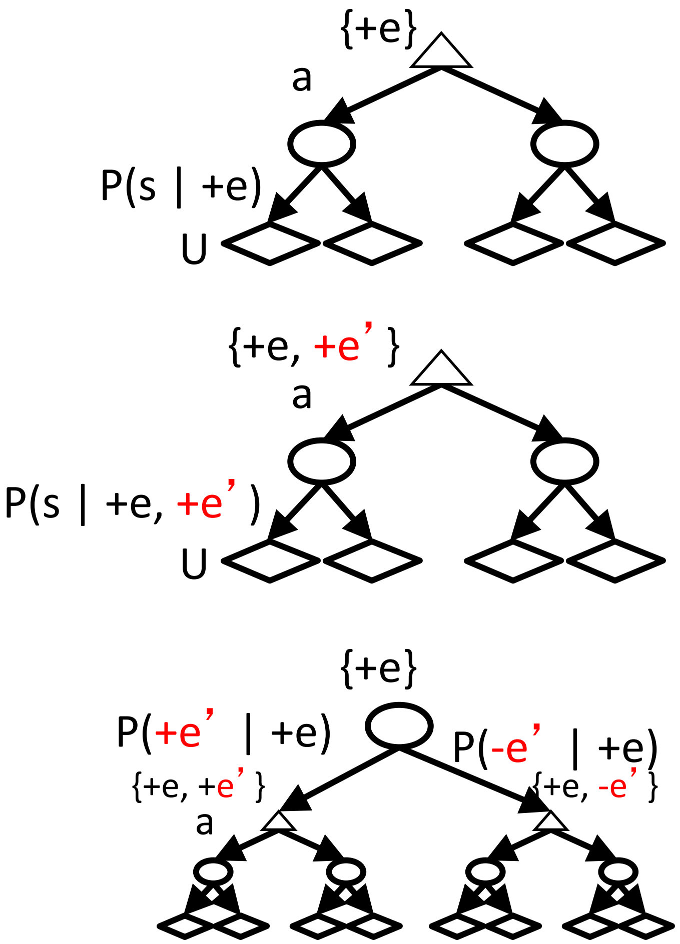
$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

- BUT E' is a random variable whose value is unknown, so we don't know what e' will be
- Expected value if E' is revealed and then we act:

$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: how much MEU goes up by revealing E' first then acting, over acting now:

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$



VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

- Nonadditive

  (think of observing E_j twice)

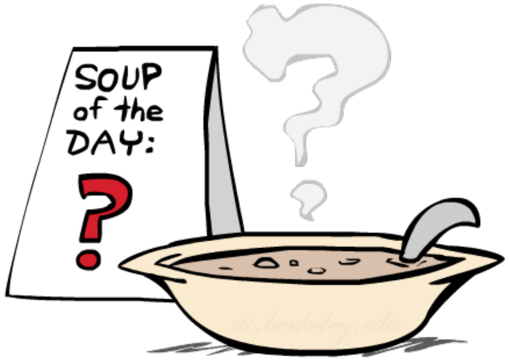
$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

- Order-independent

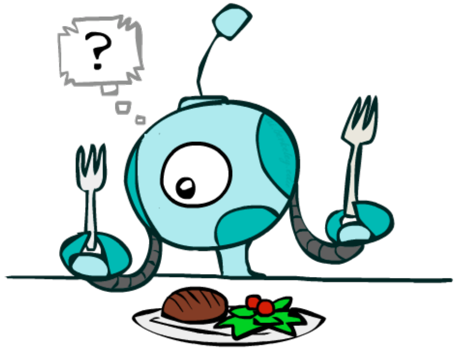
$$\begin{aligned}\text{VPI}(E_j, E_k|e) &= \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j) \\ &= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)\end{aligned}$$

Quick VPI Questions

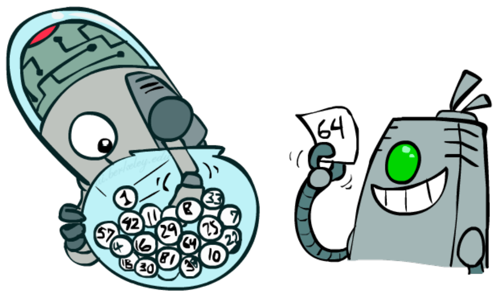
- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?



- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?



- You're playing the lottery. The prize will be \$0 or \$100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



Value of Imperfect Information?

- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout | ScoutingReport) ?
- Generally:

  If $\quad Parents(U) \perp\!\!\!\perp Z \mid CurrentEvidence$

  Then $\quad VPI(Z \mid CurrentEvidence) = 0$