

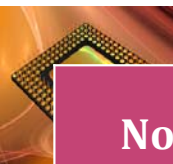


Modern Digital System Design

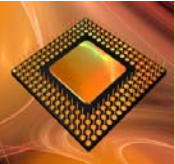
ECE 2372 / Fall 2018 / Lecture 05

Texas Tech University
Dr. Tooraj Nikoubin

Karnough map simplification

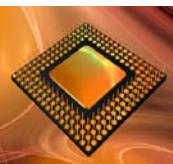


No. Variables	No. cells	No. of adjacent for each cell	No. of variables for one minterm	No. of variables for two Groups	No. of variables for four Groups	No. of variables for eight Groups
1						
2						
3						
4						
5						
6						
n						

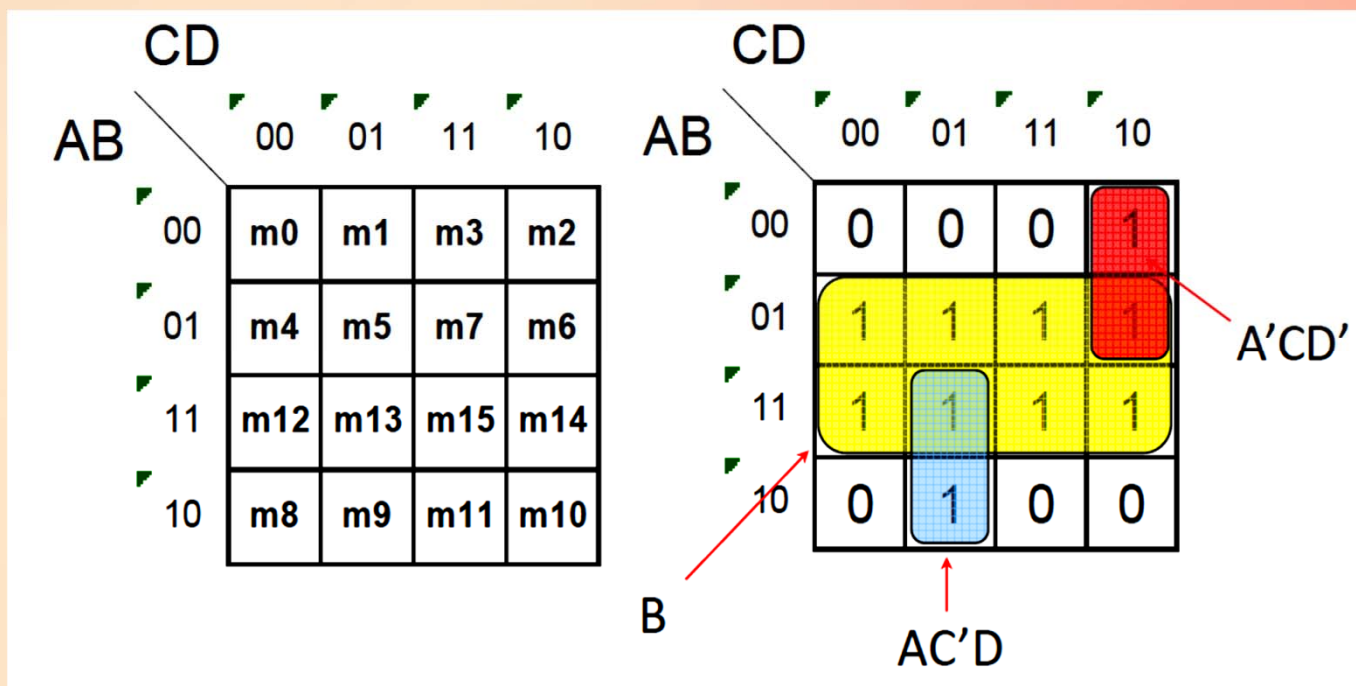


Outline

- Karnough map simplification
 - 4 variable karnaugh map
 - Don't care condition
 - Algorithm for better grouping
- Karnaugh map with ≥ 5 variable

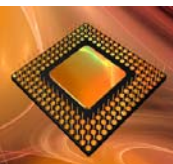


4-Variable Karnaugh Map

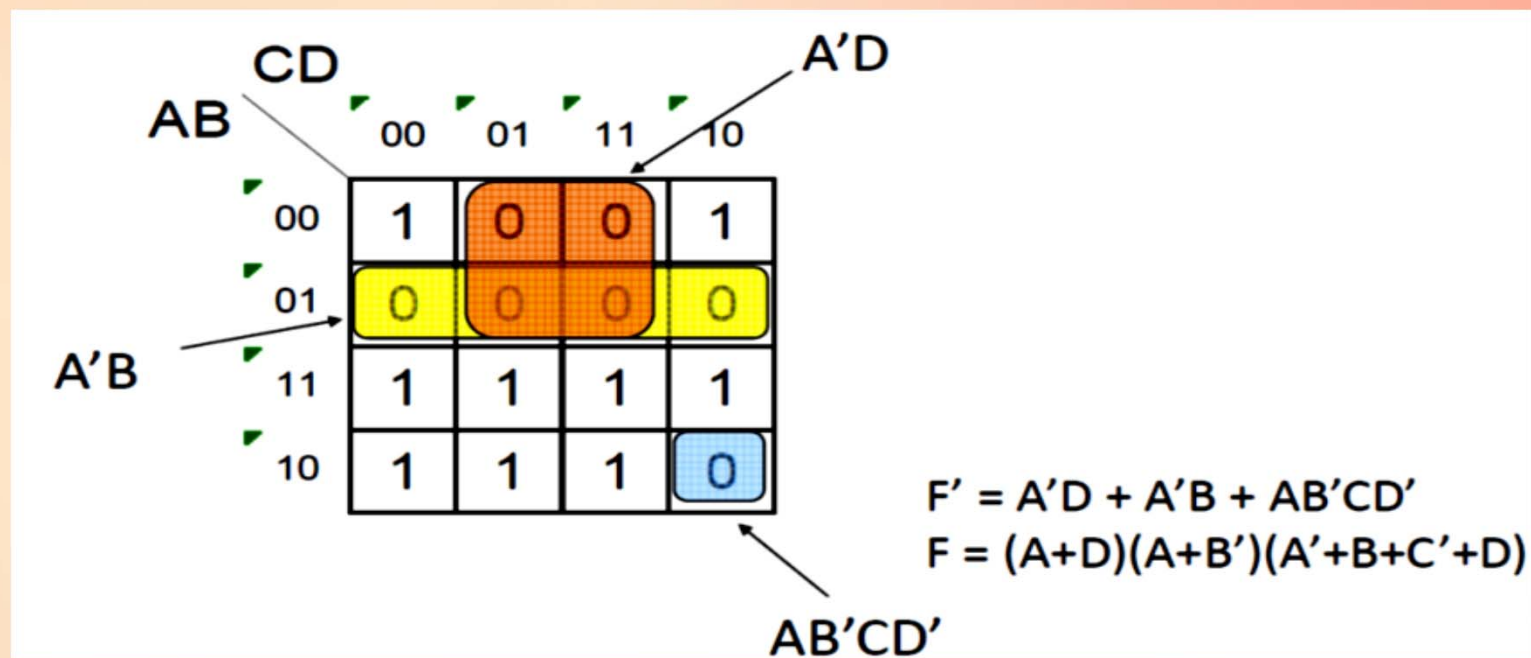


$$F = AC'D + A'CD' + B$$

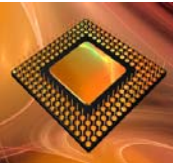
Note the row and column orderings. Required for adjacency



Find a POS Solution



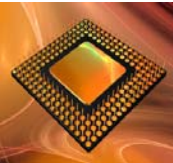
Find solutions to groups of 0's to find F' Invert to get F then use DeMorgan's



Don't Care



- A **don't-care term** is an input to a function that the designer does not care about
- Because that input would never happen
- Example:
 - **BCD number (0-9, A-F) are 4 bits, don't care about input A-F**
 - **Suppose a system have 5 type of input**
Unfortunately we can't have 2 input line
Make 3 input line and last 3 sequence as don't care
 $S_0, S_1, S_2, S_3, S_4, X, X, X \Rightarrow 000, 001, \dots, 111$



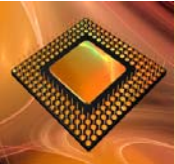
Dealing With Don't Cares

$$F = \sum m(1, 3, 7) + \sum d(0, 5)$$

		A		
		0	1	
BC	00	x	0	
	01	1	x	→ $A'B'C + AB'C + A'BC + ABC = C$
	11	1	1	
	10	0	0	

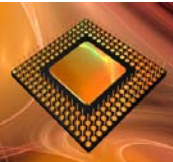
F = C

Circle the x's that help get bigger groups of 1's
Don't circle the x's that don't

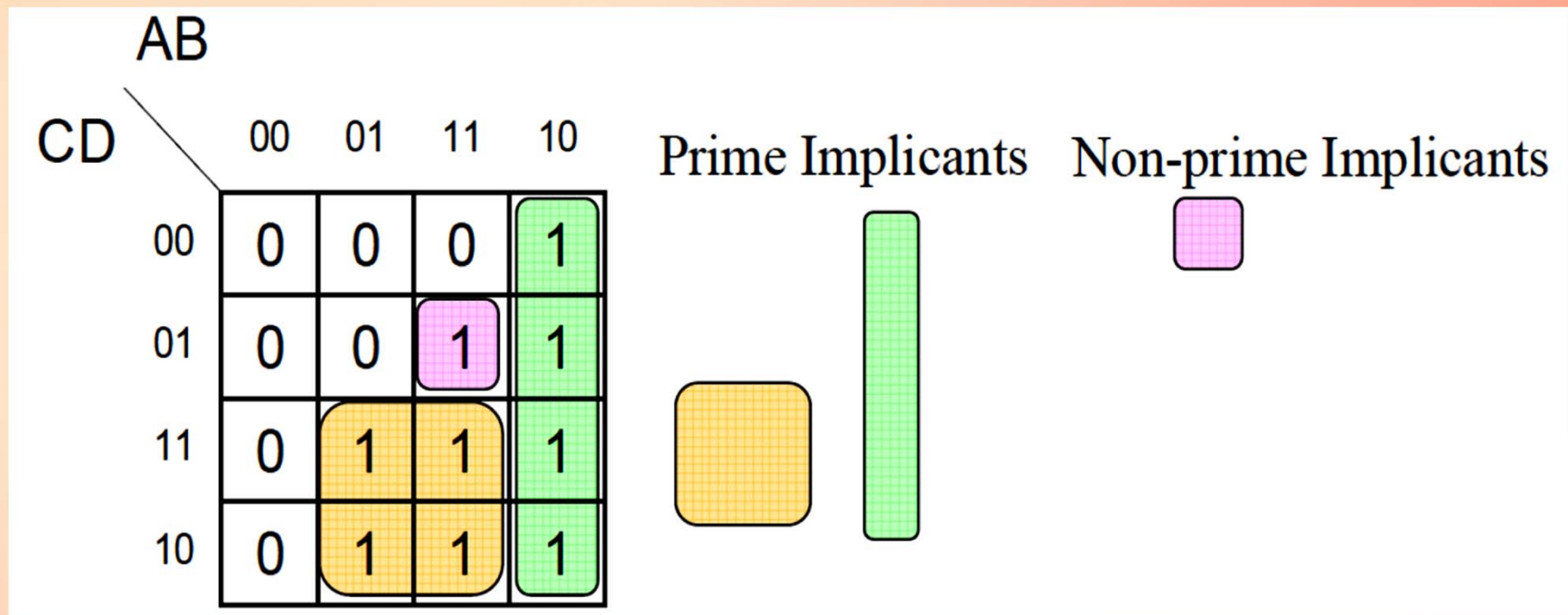


Prime Implicants

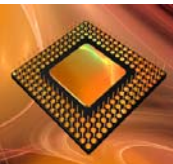
- A group of one or more 1's which are adjacent and can be combined on a Karnaugh Map is called an **implicant**.
 - The **biggest group** of 1's which can be circled to cover a given 1 is called a **prime implicant**.
- They are the only implicants we care about.



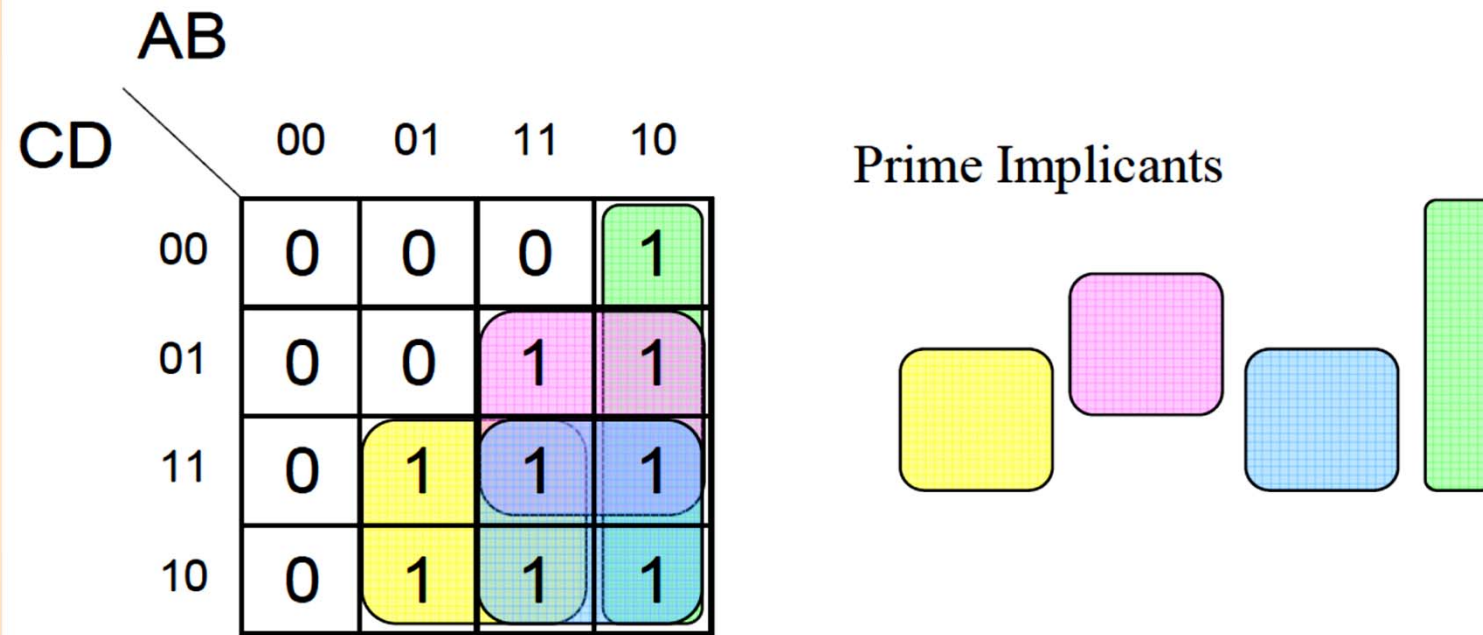
Prime Implicants



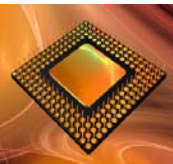
Are there any additional prime implicants in the map that are not shown above?



All The Prime Implicants



When looking for a minimal solution –
only circle prime implicants...
A minimal solution will *never* contain
non-prime implicants



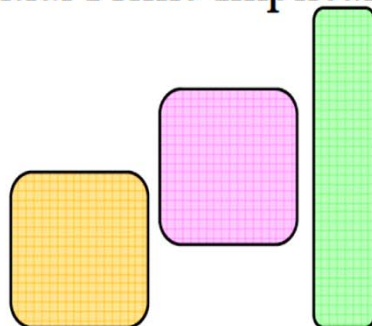
Essential Prime Implicants

		AB			
		00	01	11	10
CD	00	0	0	0	1
	01	0	0	1	1
	11	0	1	1	1
	10	0	1	1	1

Not all prime implicants are required

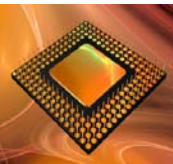
A prime implicant which is the only cover of some 1 is **essential** – a minimal solution requires it.

Essential Prime Implicants



Non-essential Prime Implicants



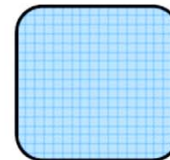


A Minimal Solution Example

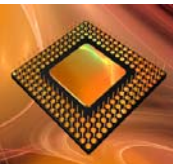
AB \ CD		AB			
CD		00	01	11	10
		00	01	11	10
00		0	0	0	1
01		0	0	1	1
11		0	1	1	1
10		0	1	1	1

$$F = \boxed{AB'} + \boxed{BC} + \boxed{AD}$$

Minimum

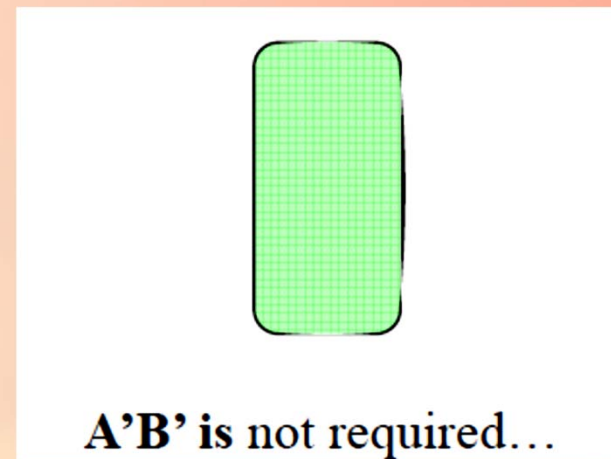
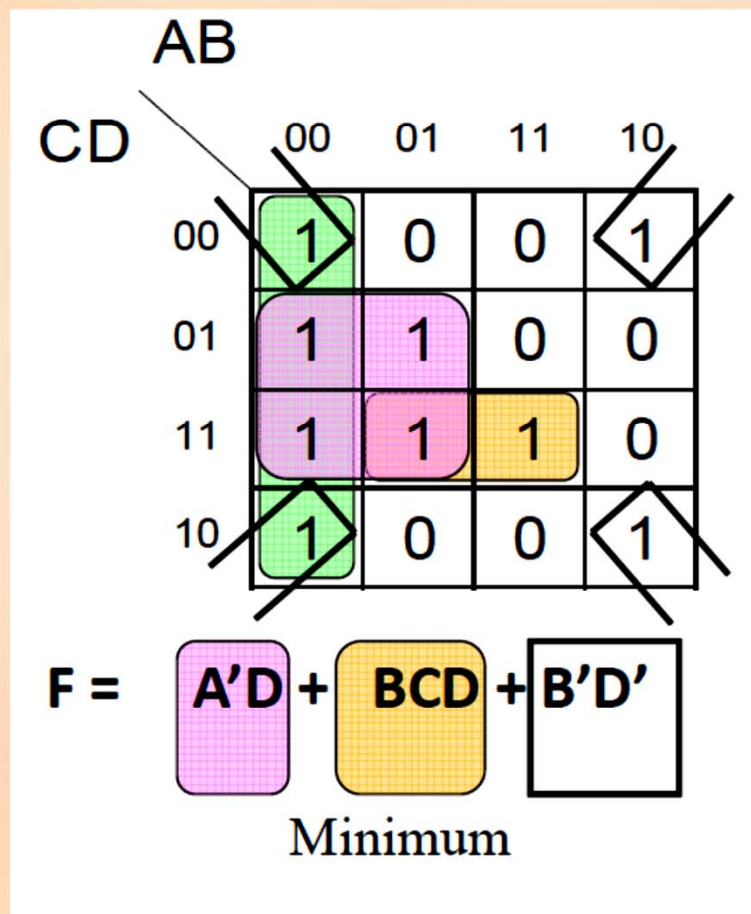
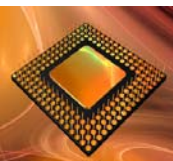


Not required...



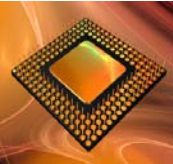
Another Example

		AB			
		00	01	11	10
CD	00	1	0	0	1
	01	1	1	0	0
	11	1	1	1	0
	10	1	0	0	1



$A'B'$ is not required...
Every one of its locations is covered by multiple implicants

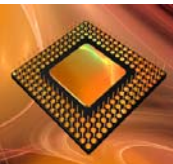
After choosing essentials Minimum **essentials**, everything is covered...



Finding the Minimum Sum of Products



1. Find each essential prime implicant and include it in the solution.
2. Determine if any minterms are not yet covered.
3. Find the minimal # of remaining prime implicants which finish the cover.

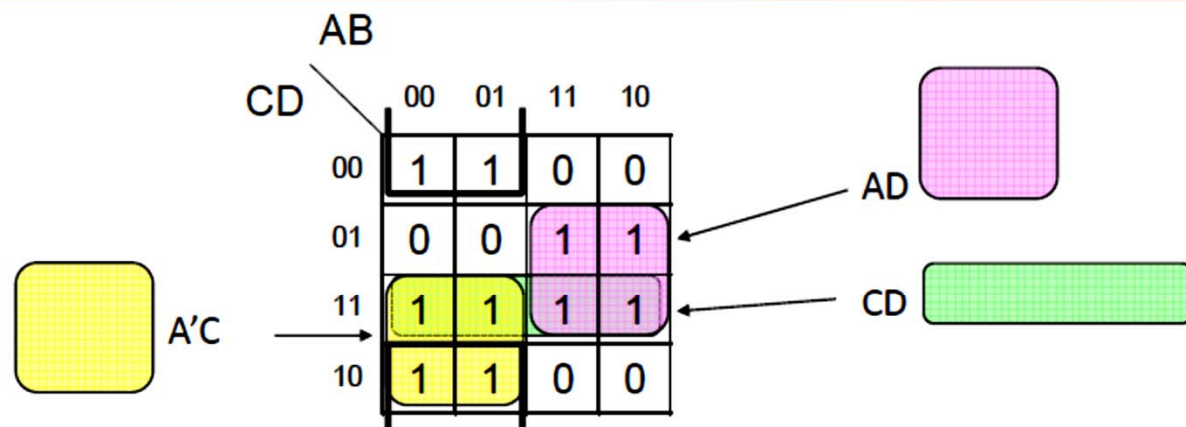


Another Example Use of non-essential primes



		AB			
		00	01	11	10
CD	00	1	1	0	0
	01	0	0	1	1
	11	1	1	1	1
	10	1	1	0	0

Another Example Use of non-essential primes

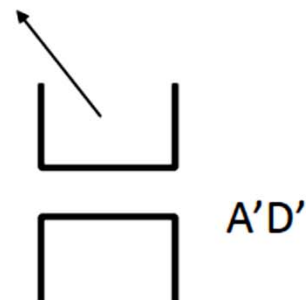


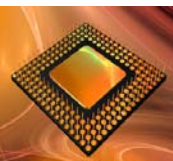
Essentials: $A'D'$ and AD

Non-essentials: $A'C$ and CD

Solution: $A'D' + AD + A'C$

or
 $A'D' + AD + CD$





5-Variable Karnaugh Map

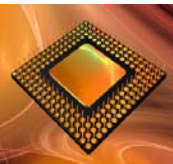
		BC			
		00	01	11	10
DE	00	m0	m4	m12	m8
	01	m1	m5	m13	m9
	11	m3	m7	m15	m11
	10	m2	m6	m14	m10

This is the A=0 plane

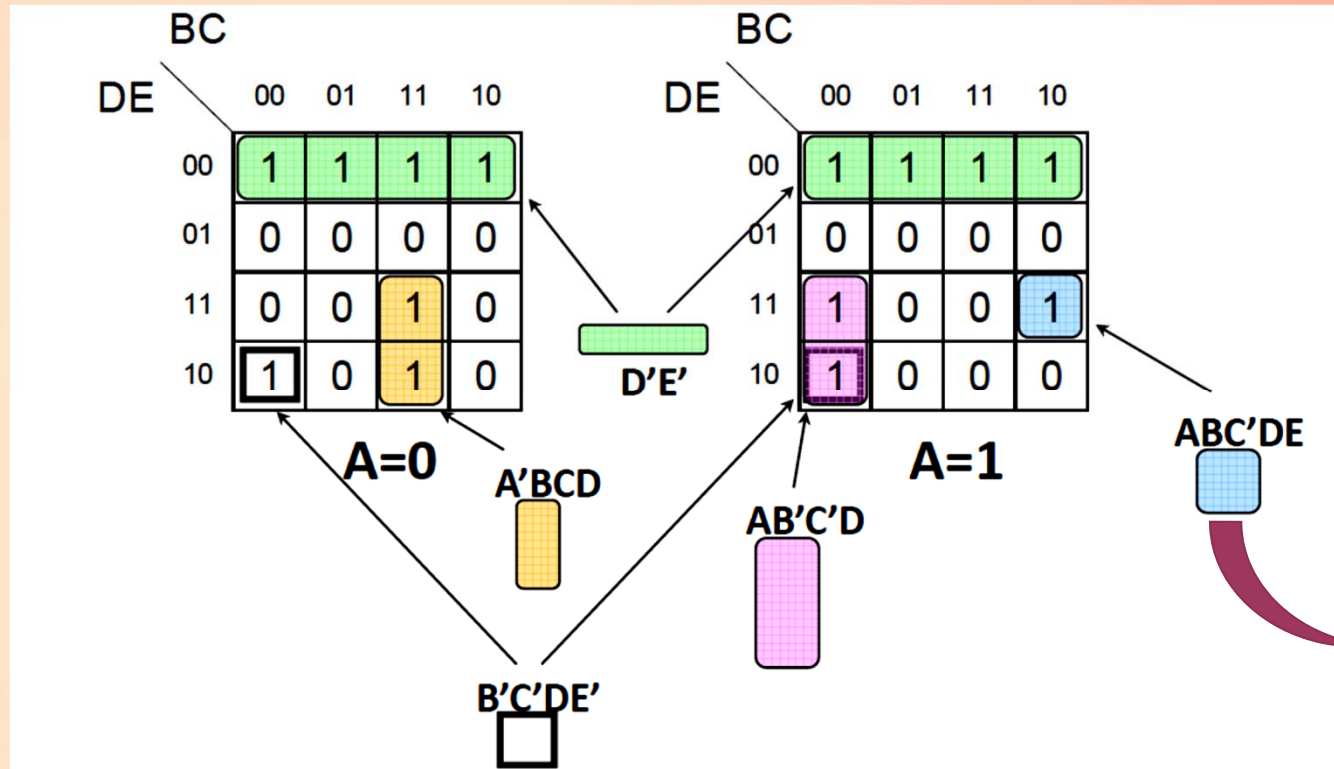
		BC			
		00	01	11	10
DE	00	m16	m20	m28	m24
	01	m17	m21	m29	m25
	11	m19	m23	m31	m27
	10	m18	m22	m30	m26

This is the A=1 plane

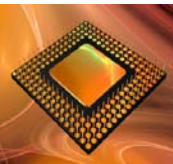
The planes are adjacent to one another one is above the other in 3D)



Some Implicants in a 5-Variable KMap



Some of these are not prime...



5-Variable KMap Example

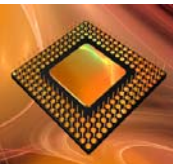
Find the minimum sum-of-products for:
 $F = \sum m (0,1,4,5,11,14,15,16,17,20,21,30,31)$

BC					
DE		00	01	11	10
	00				
	01				
	11				
	10				

A=0

BC					
DE		00	01	11	10
	00				
	01				
	11				
	10				

A=1



5-Variable KMap Example

Find the minimum sum-of-products for:

$$F = \sum m (0,1,4,5,11,14,15,16,17,20,21,30,31)$$

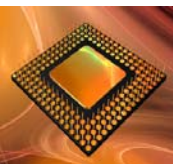
BC					
DE		00	01	11	10
		00	01	11	10
00		1	1	0	0
01		1	1	0	0
11		0	0	1	1
10		0	0	1	0

A=0

$$F = B'D' + BCD + A'BDE$$

BC					
DE		00	01	11	10
		00	01	11	10
00		1	1	0	0
01		1	1	0	0
11		0	0	1	0
10		0	0	1	0

A=1



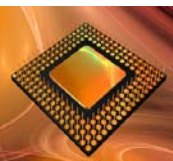
6-Variable Karnaugh Map

		CD			
		00	01	11	10
AB=00	EF	00	01	11	10
	00	m0	m4	m12	m8
	01	m1	m5	m13	m9
	11	m3	m7	m15	m11
	10	m2	m6	m14	m10

		CD			
		00	01	11	10
AB=01	EF	00	01	11	10
	00	m16	m20	m28	m24
	01	m17	m21	m29	m25
	11	m19	m23	m31	m27
	10	m18	m22	m30	m26

		CD			
		00	01	11	10
AB=10	EF	00	01	11	10
	00	m32	m36	m44	m40
	01	m33	m37	m45	m41
	11	m35	m39	m47	m43
	10	m34	m38	m46	m42

		CD			
		00	01	11	10
AB=11	EF	00	01	11	10
	00	m48	m52	m60	m56
	01	m49	m53	m61	m57
	11	m51	m55	m63	m59
	10	m50	m54	m62	m58



		CD			
		00	01	11	10
AB=00	EF				
	00	0	0	0	0
	01	0	0	0	0
	11	0	0	1	0
	10	0	0	0	0

		CD			
		00	01	11	10
AB=01	EF				
	00	0	0	0	0
	01	0	0	0	0
	11	0	0	1	0
	10	0	0	0	0

		CD			
		00	01	11	10
AB=10	EF				
	00	1	0	0	0
	01	1	0	0	0
	11	1	0	1	0
	10	1	0	0	0

		CD			
		00	01	11	10
AB=11	EF				
	00	1	0	0	0
	01	1	0	0	0
	11	1	0	1	0
	10	1	0	0	0

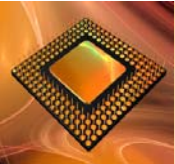
Solution = $AC'D' + CDEF$



= $AC'D'$



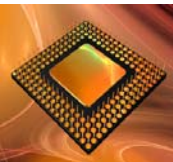
= $CDEF$



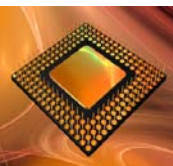
KMap Summary



- A Kmap is simply a folded truth table
 - where physical adjacency implies logical adjacency
- KMaps are most commonly used hand method for logic minimization
- KMaps have other uses for visualizing Boolean Equations – you may see some later.



KARNAUGH MAP REVIEW



Review



CD \ AB	AB			
	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

C \ AB	AB			
	00	01	11	10
00				
01				
11				
10				

A

B

D

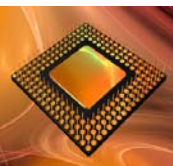
CD \ AB	AB			
	00	01	11	10
00				
01				
11				
10				

C \ AB	AB			
	00	01	11	10
00				
01				
11				
10				

A

B

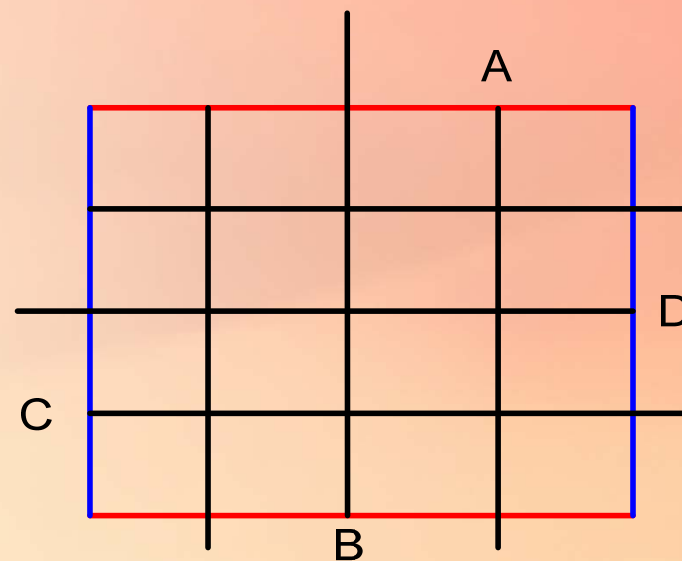
D

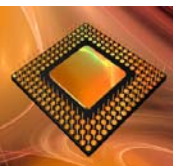


MSB LSB

$$Y = F(A, B, C, D)$$
$$Y = F(8, 4, 2, 1)$$

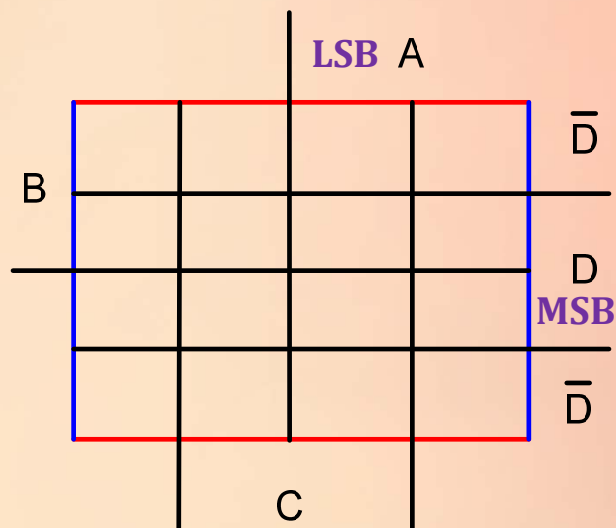
AB		00	01	11	10
CD	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10





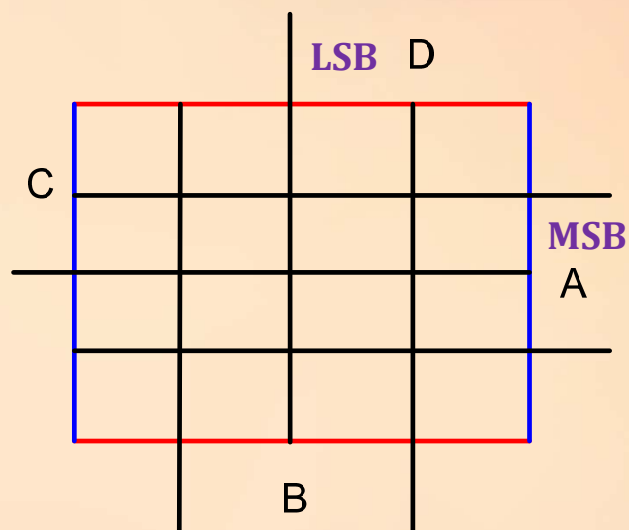
$$Y = F(\overset{\text{LSB}}{A}, B, C, \overset{\text{MSB}}{D})$$

$$Y = F(1, 2, 4, 8)$$

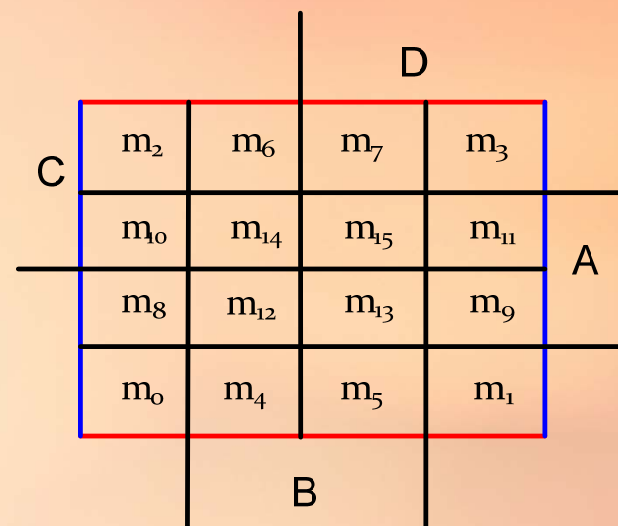
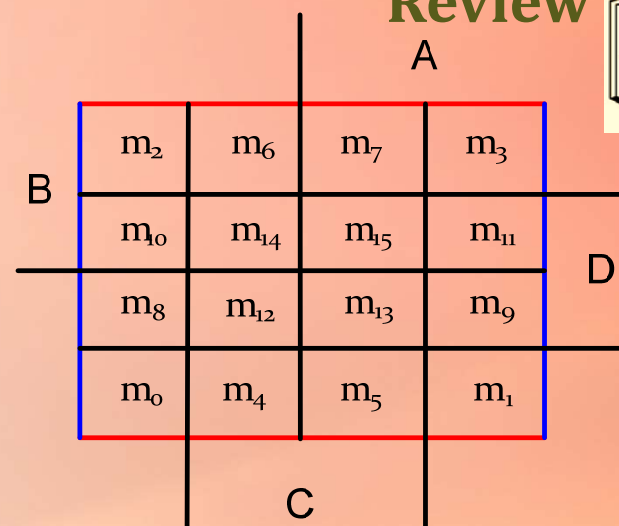


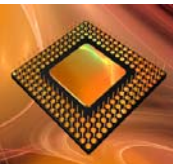
$$Y = F(\overset{\text{MSB}}{A}, B, C, \overset{\text{LSB}}{D})$$

$$Y = F(8, 4, 2, 1)$$



Review



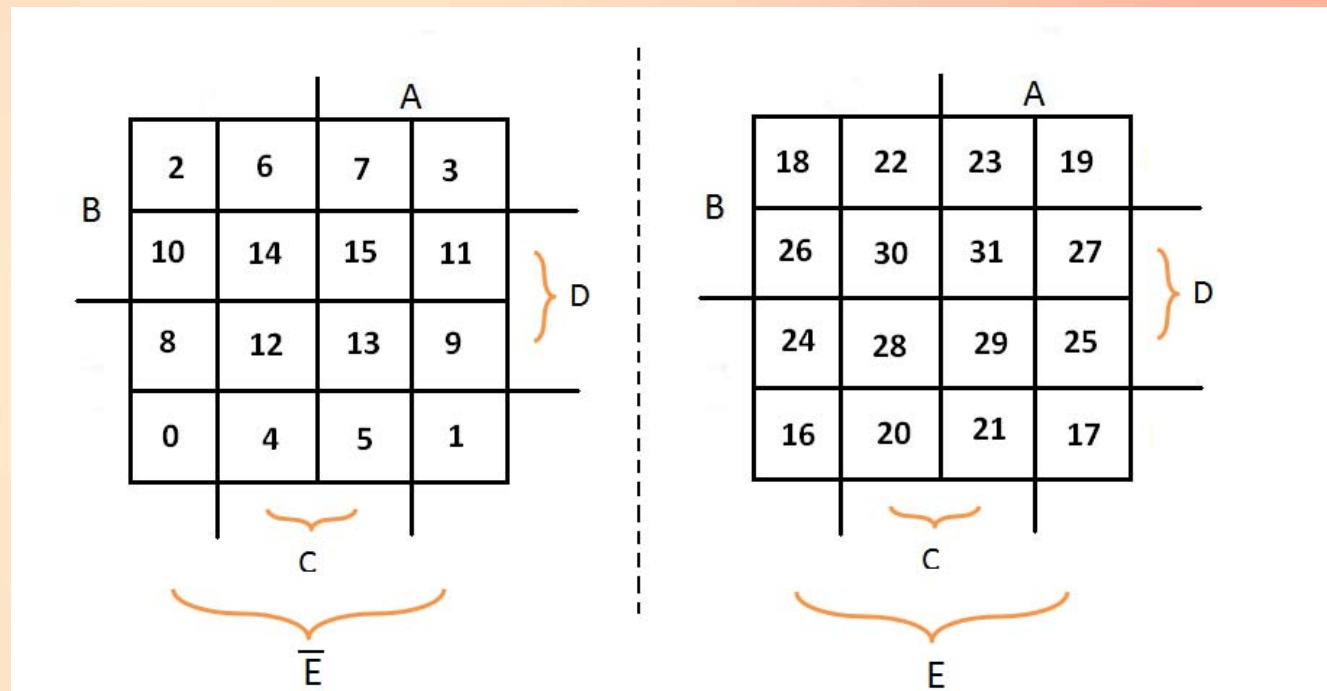
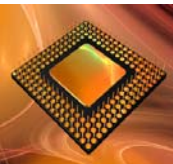


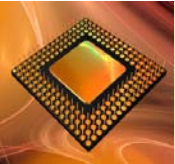
Review



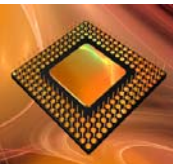
D	0	1	0	1	0	1	0	1
C	0	0	1	1	0	0	1	1
B	0	0	0	0	1	1	1	1
A								
A								

D	0	1	0	1	0	1	0	1
C	0	0	1	1	0	0	1	1
B	0	0	0	0	1	1	1	1
A	m ₀	m ₁	m ₂	m ₃	m ₄	m ₅	m ₆	m ₇
A	m ₈	m ₉	m ₁₀	m ₁₁	m ₁₂	m ₁₃	m ₁₄	m ₁₅





Mandatory Loop or Mandatory Grope for minimization

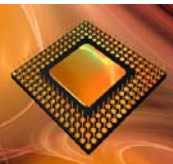


Example

Review



					A
					\bar{D}
B	1				
	1	1	1		D
	1	1	1	1	\bar{D}
					C



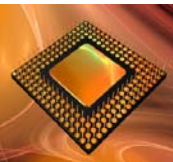
Example

Review



$A'B'$

		A		
B				\bar{D}
	1			
	1	1	1	D
	1	1	1	\bar{D}
		C		

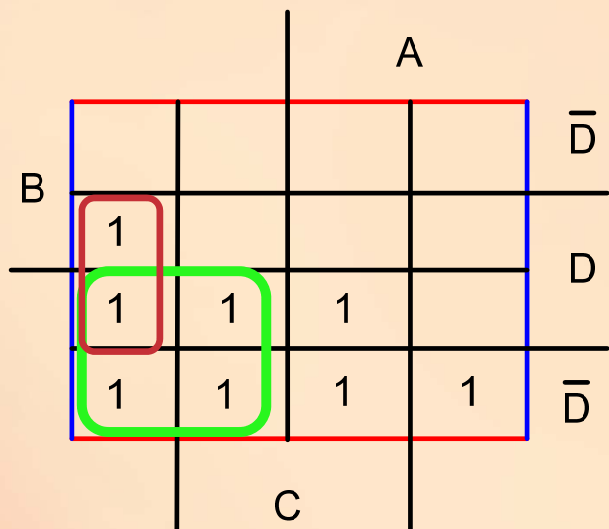


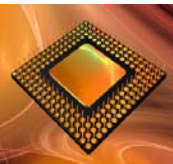
Example

Review



$$A'B' + A'C'D$$



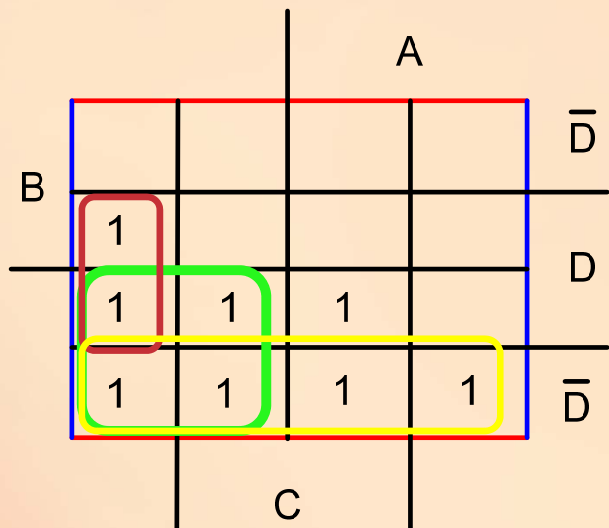


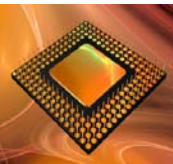
Example

Review



$$A'B' + A'C'D + B'D'$$



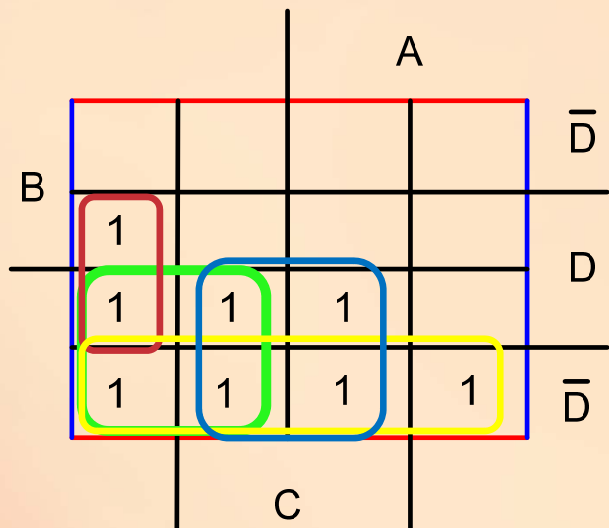


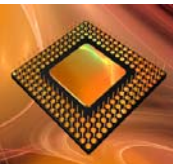
Example

Review



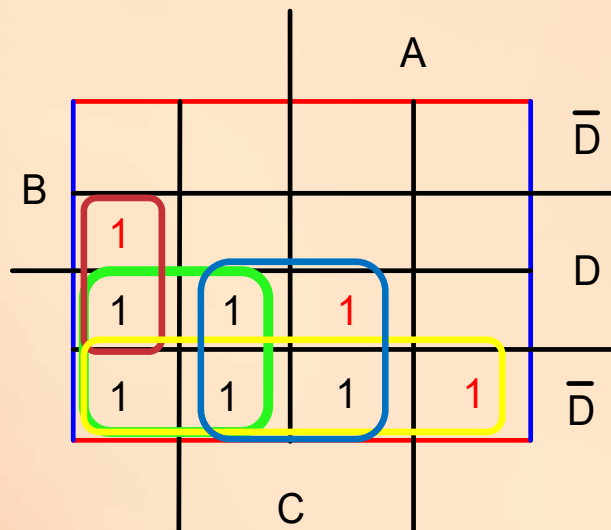
$$A'B' + A'C'D + B'D' + CD$$





Example

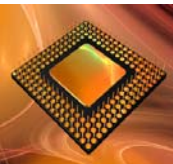
Review



$$A'B' + A'C'D + B'D' + CD$$

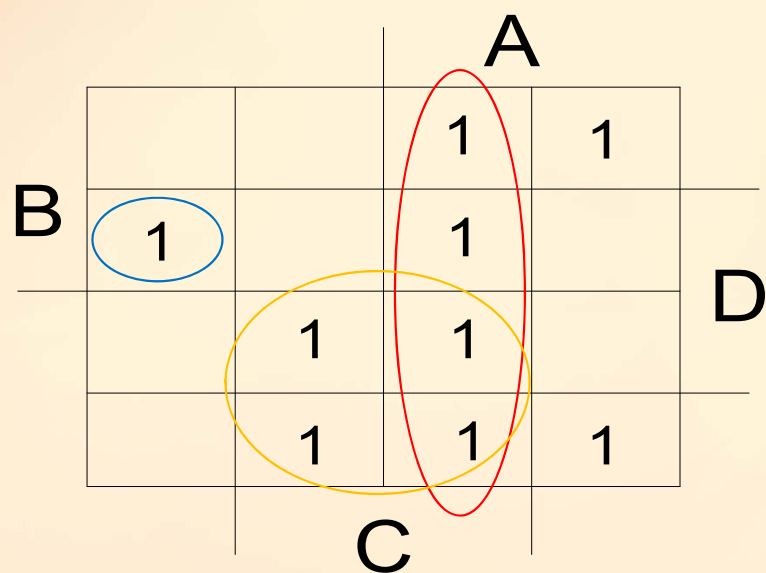


$$A'C'D + B'D' + CD$$



Example

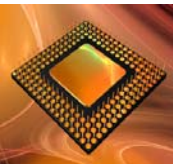
Review



$$Y = \overline{A} B \overline{C} D$$

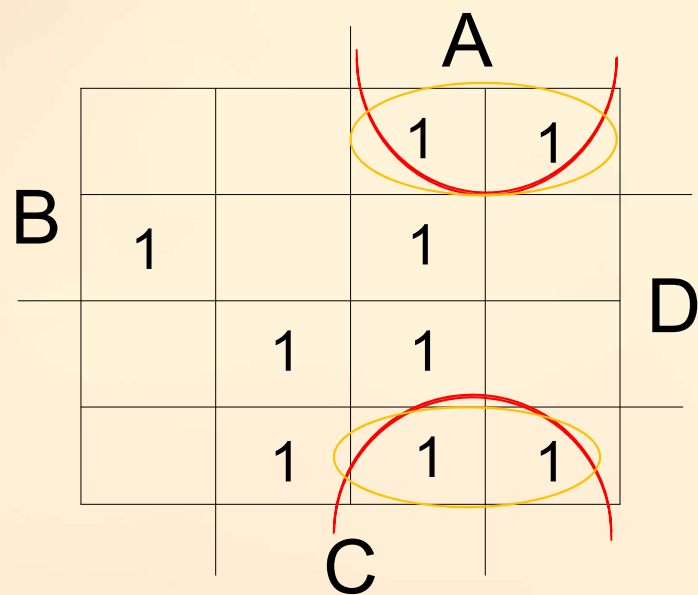
$$Y = A C$$

$$Y = \overline{B} C$$



Example

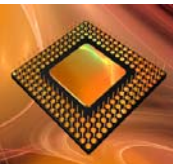
Review



$$Y = A B \overline{D}$$

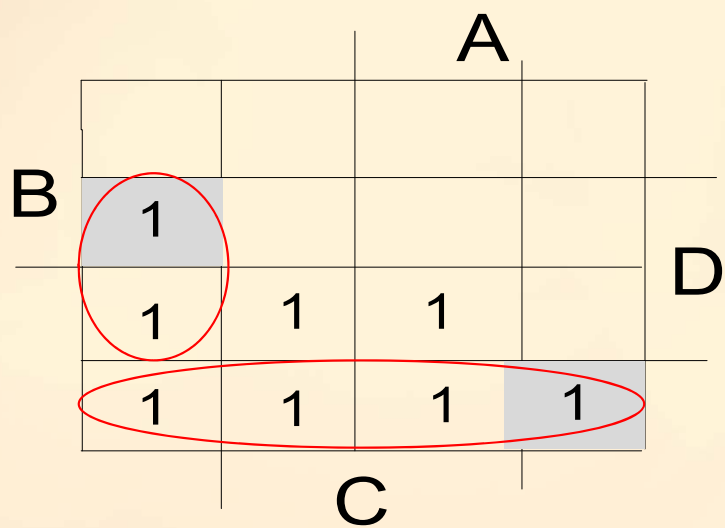
$$Y = A \overline{B} \overline{D}$$

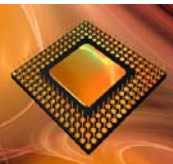
$$Y = A \overline{D}$$



Example

Review





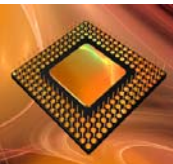
Example

Review



$$A) f(A, B, C) = \overline{A}\overline{B} + \overline{A}C + \overline{B}C$$

		A	
B		1	
		1	
		C	
	1	1	



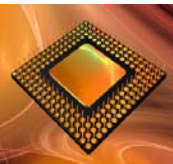
Example

Review



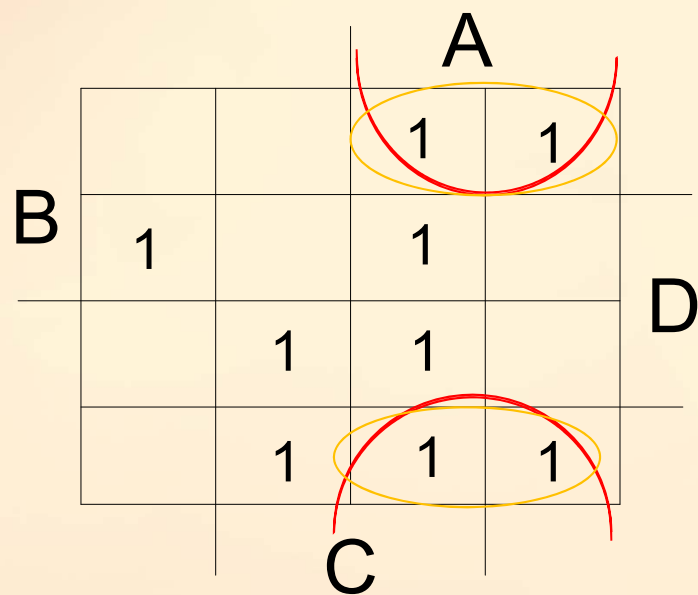
$$B) f(A, B, C, D) = AB\overline{D} + \overline{A}B\overline{C} + \overline{B}\overline{C}D$$

		A		
B	1		1	1
	1			
	1			1
		C		D



Example

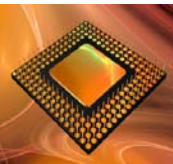
Review



$$Y = A B \overline{D}$$

$$Y = A \overline{B} \overline{D}$$

$$Y = A \overline{D}$$



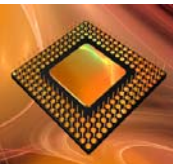
Example

Review



		A	
B		1	1
	1		
	1		1
	1	1	1
		C	

$$F = A \overline{D}$$



Example

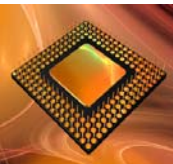
Review



		A	
		1	1
B	1		
	1	1	1
	1	1	1
		C	

$$F = A \overline{D}$$

$$F = \overline{B} \overline{C}$$



Example

Review

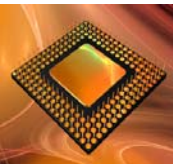


		A	
B		1	1
	1		
	1		1
	1	1	1
		C	

$$F = A \overline{D} + \overline{B} \overline{C} + \overline{A} \overline{B} D$$

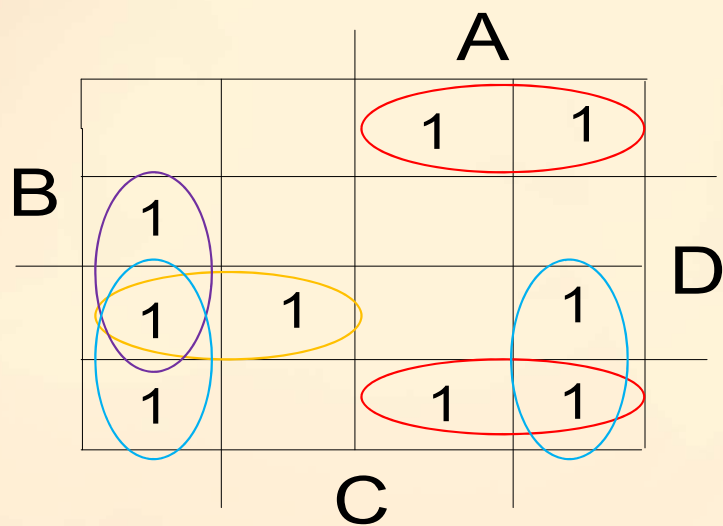
$$F = \overline{A} \overline{B} D$$

$$F = \overline{A} \overline{C} D$$

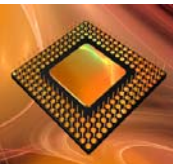


Example

Review

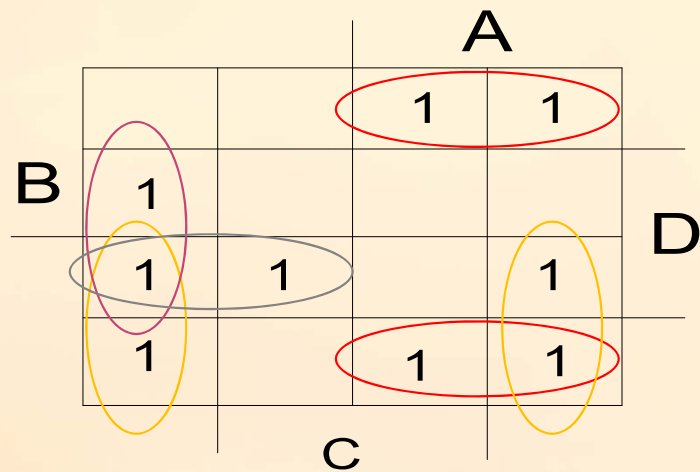


$$F = A \overline{D} + \overline{B} \overline{C} + \overline{A} \overline{B} D + \overline{A} \overline{C} D$$

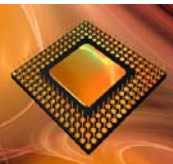


Example

Review



$$Y = A\bar{D} + \bar{B}\bar{C} + \bar{A}\bar{C}D + \bar{A}\bar{B}D$$



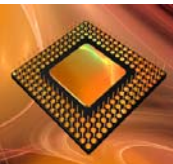
Example

Review



$$A) f(A, B, C) = \overline{A}\overline{B} + \overline{A}C + \overline{B}C$$

		A	
B		1	
	1	1	1
		C	



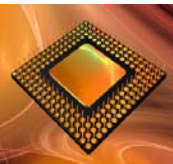
Example

Review



$$B) f(A, B, C, D) = AB\overline{D} + \overline{A}B\overline{C} + \overline{B}\overline{C}D$$

		A		
B	1		1	1
	1			
	1			1
		C		D



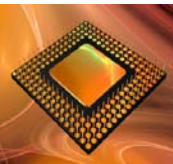
Example

Review



$$F(A, B, C) = A C$$

		A	
B		1	1
		1	1
		C	



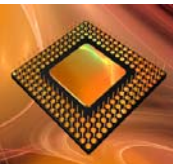
Example

Review



$$F(A, B, C) = A C + A B + B C$$

		A	
B		1	1
			1
		C	



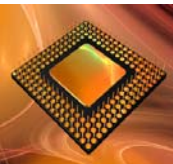
Example

Review



$F(A, B, C, D)$

		A			
B		1			
		1	1		
			1	1	
	1	1		1	
		C		D	



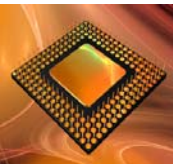
Example

Review



$$F(A, B, C, D) = A C D$$

		A		
B		1		
		1	1	
			1	1
	1	1		1
		C		
				D



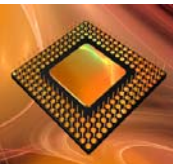
Example

Review



$$F(A, B, C, D) = A C D + A \overline{B} \overline{C}$$

		A			
B		1			
		1	1		
			1	1	D
	1	1		1	
		C			



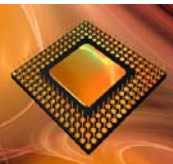
Example

Review



$$F(A, B, C, D) = A C D + A \overline{B} \overline{C} + \overline{A} B C$$

		A			
B	D	1			
		1	1		
	C		1	1	
		1	1		1



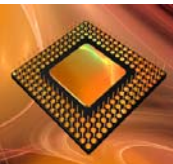
Example

Review



$$F(A, B, C, D) = A C D + A \overline{B} \overline{C} + \overline{A} B C + \overline{A} \overline{B} \overline{D}$$

		A	
B		1	
		1	1
			1
		1	1
		C	
		1	1

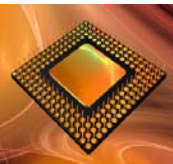


Example

Review



		A		
B	1			
	1	1	1	1
	1	1	1	1
	1			1
		C		
				D



Example

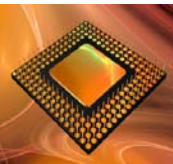
Review



$$F(A, B, C, D) = D$$

		A	
B	1		
	1	1	1
	1	1	1
	1		1
		C	

A red oval highlights the four cells in the second and third rows, where the value of D is 1 for all combinations of A, B, and C.



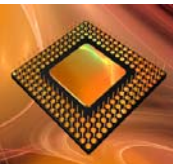
Example

Review



$$F(A, B, C, D) = D + \overline{A}\overline{C}$$

		A			
B	1	1			
	1	1	1	1	1
	1	1	1	1	1
	1	1			1
		C			



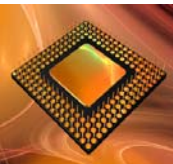
Example

Review



$$F(A, B, C, D) = D + \overline{A}\overline{C} + \overline{B}\overline{C}$$

		A			
B	1				
	1	1	1	1	1
	1	1	1	1	1
	1				1
		C			



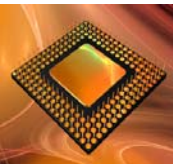
Example

Review



		A		B	D
B	D		1		1
				1	1
				1	1
			1		

		A		B	D
B	D	1			
		1		1	1
				1	1



Example

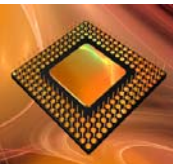
Review



		A	
B	1		1
		1	1
		1	1
	1		
		D	

$$F(A, B, C, D) = AD$$

		A	
B	1		
	1	1	1
		1	1
		D	



Example

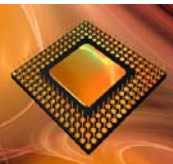
Review



		A			
B		1		1	
			1	1	
			1	1	
		1			
				D	

		A			
B		1			
		1		1	1
				1	1
				D	

$$F(A, B, C, D) = AD + \overline{A}C\overline{D}\overline{E}$$



Example

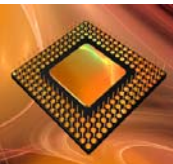
Review



		A		B	D
	1		1		
		1	1		
		1	1		
	1				

		A		B	D
	1				
	1		1		
		1	1		

$$F(A, B, C, D) = AD + \overline{A}C\overline{D}\overline{E} + \overline{A}B\overline{C}\overline{E}$$



Example

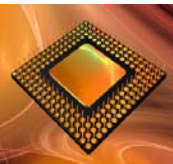
Review



		A	
B		1	1
			1
		1	1
		1	1
		D	

		A	
B	1		
	1	1	1
		1	1
		D	

$$F(A, B, C, D, E) = AD + \overline{A}C\overline{D}\overline{E} + \overline{A}B\overline{C}E + AB\overline{C}\overline{E}$$



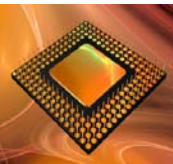
Example

Review



$F(A, B, C, D)$

		A			
B	1				
	1				
	1			1	
	1		1	1	
		C		D	



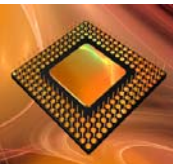
Example

Review



$$F(A, B, C, D) = \overline{A} \overline{C}$$

		A			
B	1				
	1				
	1			1	
	1		1	1	
		C		D	



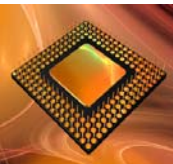
Example

Review



$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C}$$

		A	
B	1		
	1		
	1		
	1		
		D	
	1		
	1		
		C	
	1		
	1		



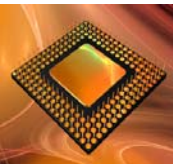
Example

Review



$$F(A, B, C, D) = \overline{A} \overline{C} + \overline{B} \overline{C} + A \overline{B} \overline{D}$$

		A		
B	1			
	1			
	1			1
	1		1	1
		C		D



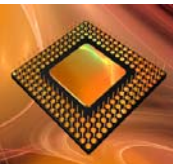
Example

Review



$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

		A		
B	1			
	1			
	1			1
	1		1	1
		C		D



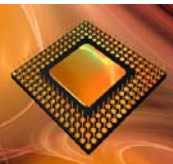
Example

Review



$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

		A			
B	1	0	0	0	D
	1	0	0	0	
	1	0	0	1	
	1	0	1	1	
		C			



Example

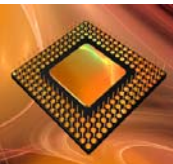
Review



$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

		A			
B	1	0	0	0	D
	1	0	0	0	
	1	0	0	1	
	1	0	1	1	
		C			

$$F' = \overline{A}C$$



Example

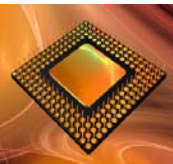
Review



$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

		A	
B	1	0	0
	1	0	0
	1	0	1
	1	0	1
		D	
		C	

$$F' = \overline{A}C + AB$$



Example

Review

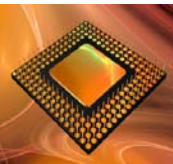


$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

		A			
B	1	0	0	0	D
	1	0	0	0	
	1	0	0	1	
	1	0	1	1	
		C			

$$F' = \overline{A}C + AB + CD$$

$$F = (A + \overline{C})(\overline{A} + \overline{B})(\overline{C} + \overline{D})$$



Example

Review

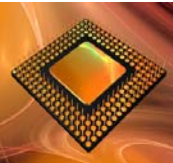


$$F(A, B, C, D) = \overline{A}\overline{C} + \overline{B}\overline{C} + A\overline{B}\overline{D} \Rightarrow SOP$$

					A	
B	1	0	0	0		
	1	0	0	0		
	1	0	0	1		D
	1	0	1	1		
					C	

$$F' = \overline{A}C + AB + CD$$

$$F = (A + \overline{C})(\overline{A} + \overline{B})(\overline{C} + \overline{D})$$

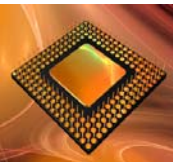


Example

Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$



Example

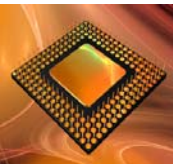
Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$

		A	
B			0
	0	0	0
		C	

$$A \overline{B} + \overline{A} \overline{B} C + A \overline{C}$$



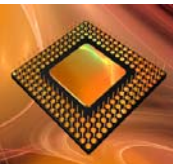
Example

Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$

		A	
B	1	1	1
	1		
		C	



Example

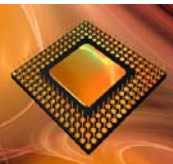
Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$

		A	
B	1	1	1
	1		
		C	

$$SOP = \overline{A}B$$



Example

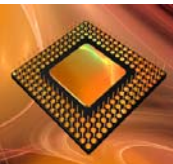
Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$

			A	
B	1	1	1	
	1			
		C		

$$SOP = \overline{A}B + BC$$



Example

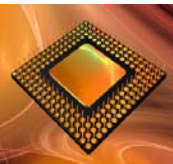
Review



$$A) f(A, B, C) = (\overline{A} + B)(A + B + \overline{C})(\overline{A} + C)$$

		A	
B	1	1	1
	1		
		C	

$$SOP = \overline{A}B + BC + \overline{A}\overline{C}$$



Example

Review

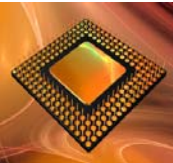


LSB)

$$B) f(A, B, C, D) = (A + \overline{B})(C + \overline{D})(\overline{A} + C)$$

		A	
B		1	
		1	
	1	1	
	1	1	
		C	
		D	

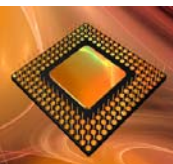
$$SOP = AC + C\overline{B} + \overline{A}\overline{B}\overline{D}$$



Don't Care

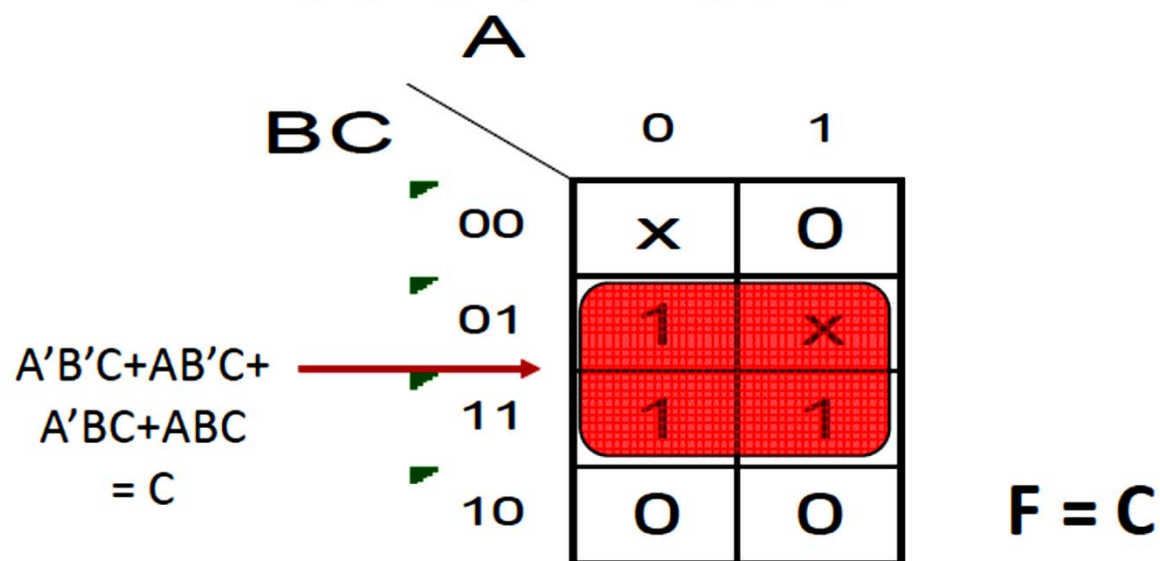


- A **don't-care term** is an input to a function that the designer does not care about
- Because that input would never happen
- Example:
 - **BCD number (0-9, A-F) are 4 bits, don't care about input A-F**
 - **Suppose a system have 5 type of input**
Unfortunately we can't have 2 input line
Make 3 input line and last 3 sequence as don't care
 $S_0, S_1, S_2, S_3, S_4, X, X, X \Rightarrow 000, 001, \dots, 111$



Dealing With Don't Cares

$$F = \sum m(1, 3, 7) + \sum d(0, 5)$$



Circle the x's that help get bigger groups of 1's
Don't circle the x's that don't



Thank You