Probability: Sample Spaces and Events

MATH 3342
Section 2.1

Probability

- The study of randomness and *uncertainty*.
- Important for inferential statistics because random sampling results in *uncertainty*.
- Two random samples from the same population may likely have two different sample means!
  - Probability helps us understand how to work with this.

Experiments

- Also called random processes.
- Exhibit **chance behavior**:
  - Individual outcomes are uncertain, but there is a regular distribution in a large number of repetitions.
  - **Unpredictable in the short-run**, but produce a predictable pattern in the long-run.

Outcomes

- The most basic possible results from random processes.
- Examples:
  - Rolling a 1 on a die.
  - Flipping a head on a coin.
  - Drawing the 2 of diamonds from a deck of cards.

Events

- Outcomes or a collection of outcomes that share some property of interest.
  - **Simple events**: consist of one outcome
  - **Compound events**: consists of more than one outcome
- Examples:
  - Rolling an odd number.
  - Flipping exactly two heads on four coin tosses.
  - Drawing a heart from a deck of cards.

The Sample Space *S*

- The set of ***all*** possible outcomes of a random process.
- Events are *subsets* of the sample space *S*.

Example

[redacted] An urn contains 2 white balls labeled A & B and two black balls labeled C & D. Balls are randomly drawn from the urn until a black ball is obtained. An outcome may be AC where A is drawn first then C is drawn. Find the sample space.

$$\boxed{\mathcal{S} = \{C, D, AC, AD, BC, BD, ABC, ABD, BAC, BAD\}}$$

Example

- Describe the sample space S:

- A new business is started. After two years, it is either still in business or it has closed.

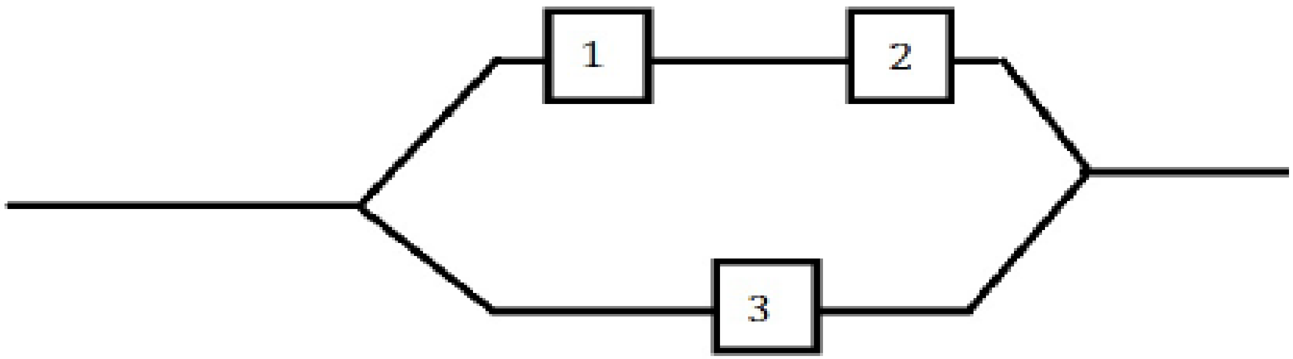
Example

- Describe the sample space S:

- A quality inspector examines four mp3 players and rates each as either **acceptable** or **unacceptable**. You record the number of units rated **acceptable**.

Example

[redacted] Three components are connected to form a system.



- The 1-2 subsystem is in a series, the subsystem works if both 1 and 2 work
- The 1-2 subsystem and 3 are parallel, so the system works if the 1-2 subsystem works or 3 works.

Example

Let

$$S = \text{success (works)}$$
$$F = \text{failue (doesn't work)}$$

An outcome may be

$$SFS \Rightarrow \begin{cases} 1 & \text{works} \\ 2 & \text{fails} \\ 3 & \text{works} \end{cases}$$

Find the sample space.

$$\boxed{\mathcal{S} = \{SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF\}}$$

Example

For the system of components example, List the outcomes of each event.

(i) A = event at least 1 component works.

at least 1 $\Rightarrow$ 1 or more works

$$\Rightarrow A = \boxed{\{SSS, SSF, SFS, FSS, SFF, FSF, FFS\}}$$

Example

(ii) B = event the system works.

Event System works = Event both 1 & 2 work and/or 3 works

$$\Rightarrow B = \boxed{\{SSS, SSF, FSS, SFS, FFS\}}$$

Example

(iii) C = at least 2 components fail

at least 2 fail $\Rightarrow$ 2 or more fail

$$\Rightarrow C = \boxed{\{SFF, FSF, FFS, FFF\}}$$

Important Relationships from Set Theory

- Complements
- Unions
- Intersections

Complements

- The **complement** of event *A* is the set of all outcomes in *S* that are not contained in *A*.
- The event that A does ***not*** occur
- Denoted by *A*'.

Unions

- The **union** of A and B is the event consisting of the outcomes that are *either in A* ***or*** *B* ***or*** *in both*.
- Denoted by $A \cup B$
- Read as "A or B"

Intersections

- The **intersection** of events *A* and *B* is the event consisting of all outcomes that are in ***both A and B***.
- Denoted by $A \cap B$
- Read as "*A and B*"

From Previous Example

$$\boxed{\mathcal{S} = \{SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF\}}$$

$$\Rightarrow A = \boxed{\{SSS, SSF, SFS, FSS, SFF, FSF, FFS\}}$$

$$\Rightarrow B = \boxed{\{SSS, SSF, FSS, SFS, FFS\}}$$

at least 2 fail $\Rightarrow$ 2 or more fail

$$\Rightarrow C = \boxed{\{SFF, FSF, FFS, FFF\}}$$

Example

Find (i) C'

$$C' = \text{all outcomes in } \mathcal{S} \text{ not in } C$$

$$= 1 \text{ or fewer fail}$$

$$\Rightarrow C' = \boxed{\{SSS, SSF, SFS, FSS\}}$$

(ii) $A \cup C$

$$A \cup C = \text{all outcomes in at least one of } AorC$$

$$= \text{all outcomes in which at least 1 component works or at least 2 fails}$$

$$\Rightarrow A \cup C = \boxed{\{SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF\}} = \mathcal{S}$$

Example

(iii) $A \cap B$

$$A \cap B = \text{all outcomes in both A \& B}$$

$$\Rightarrow A \cap B = \boxed{\{SSS, SSF, SFS, FSS, FFS\}} = B$$

The Null Set

- The event consisting of no outcomes.
- Also called the null event.
- Denoted by $\varnothing$
- If $A \cap B = \varnothing$, the events are said to be **disjoint** or **mutually exclusive**.

Venn Diagram: Pictorial representation of events

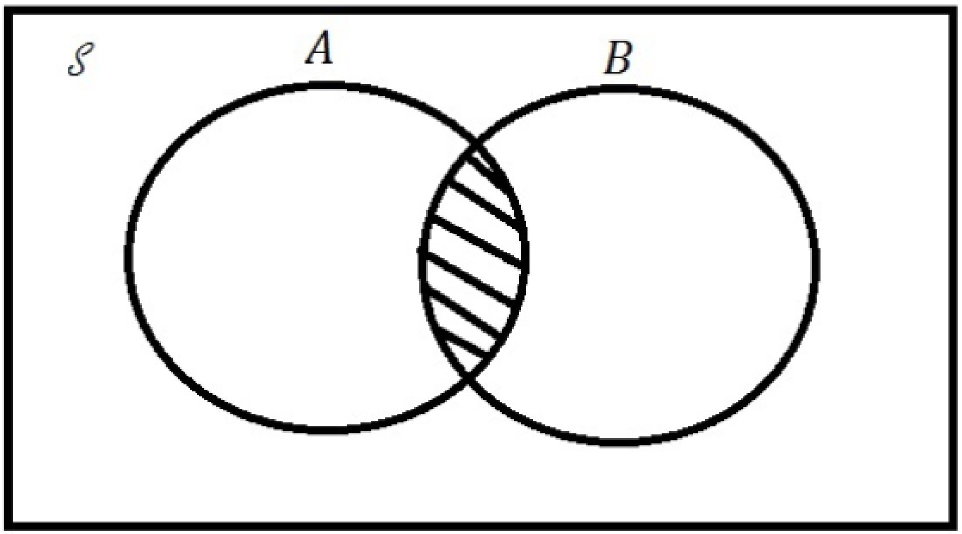


Figure 2.3: $A \cap B$

Venn Diagram: Pictorial representation of events

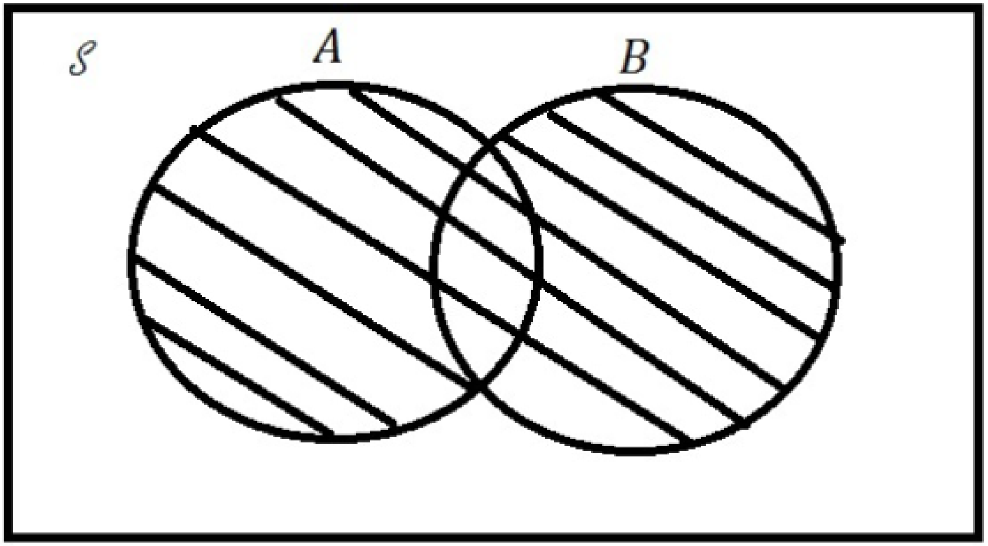


Figure 2.4: $A \cup B$

Venn Diagram: Pictorial representation of events

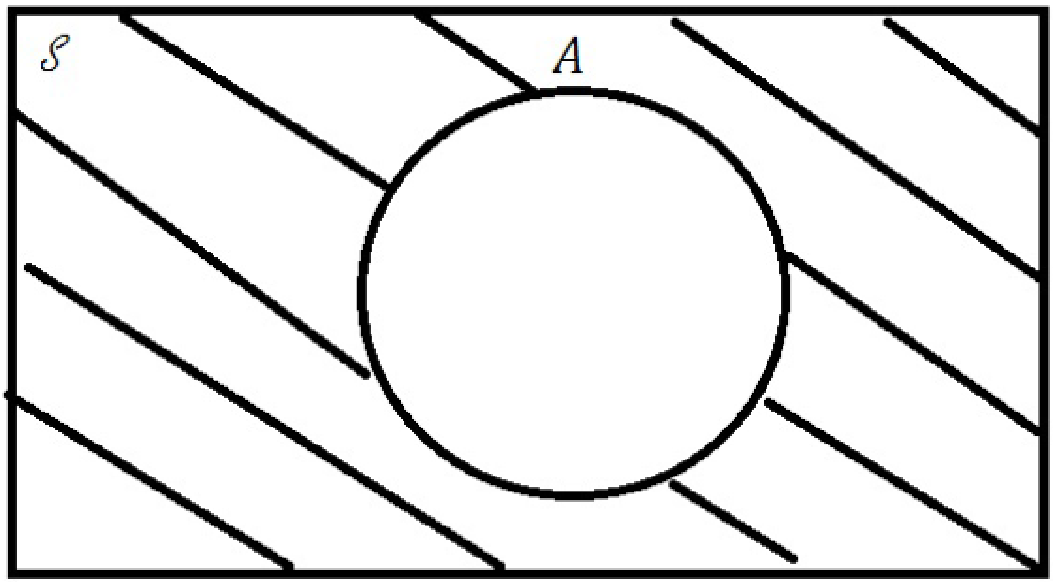


Figure 2.5: A'

Venn Diagram:
Pictorial representation of events

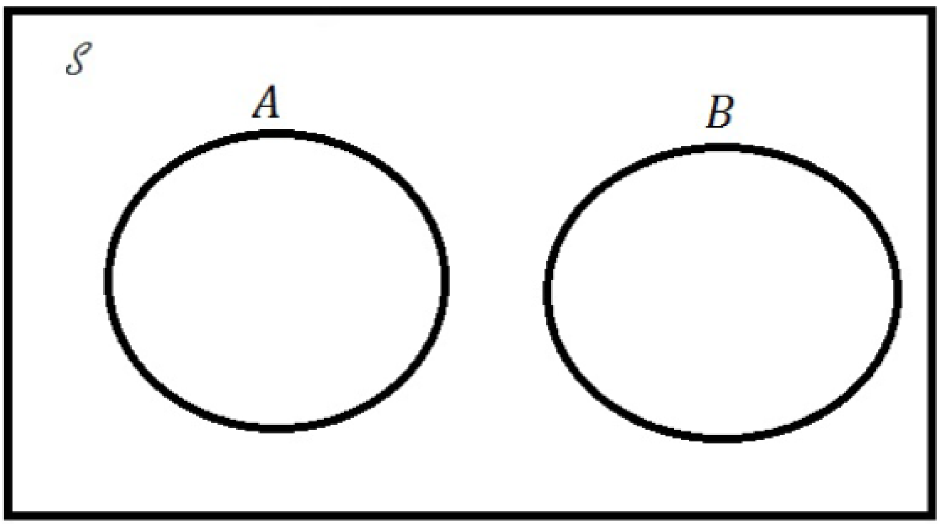


Figure 2.6: A & B mutually exclusive