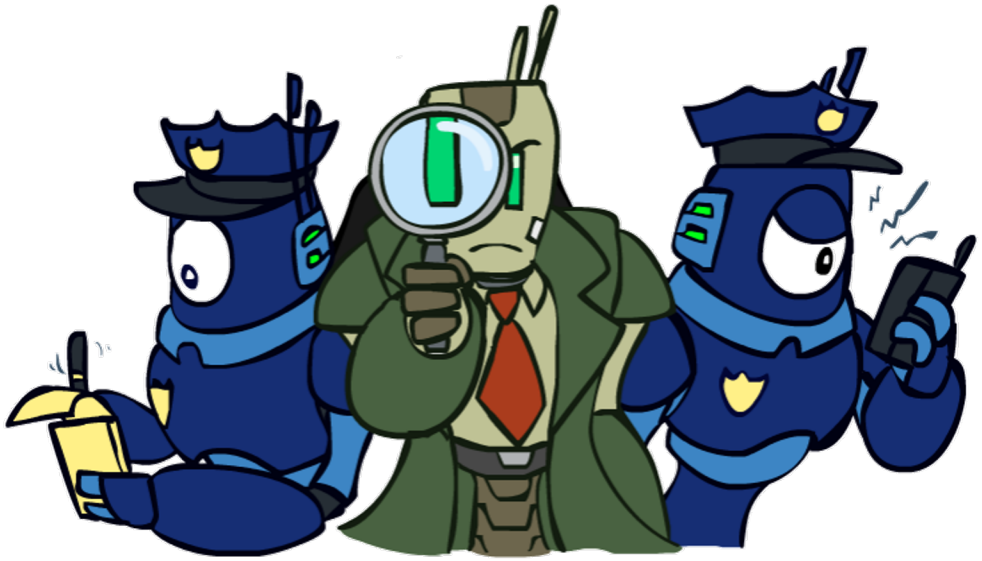
Announcements

- HW0 and Project 0 grades are out
  - Blackboard
  - Project 0 is out of 25 while others will be out of 50. Overall project grade is out of 40
  - HW0 is out of 2. Overall HW grades will be out of 10 (or 12 if you did HW0)
- Note for HWs
  - From homework 1 and forward we will be looking at the score of gradescope. So, be sure that your grade there is what you want
- Final exam date/time
  - Saturday, December 4 7:30 a.m. to 10:00 a.m. (001 and 002)
  - Friday and Saturday (D01)

CS 3568: Intelligent Systems

Constraint Satisfaction Problems (Part 1)



Instructor: Tara Salman

Texas Tech University

Computer Science Department

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley (ai.berkeley.edu).]

Last Lecture

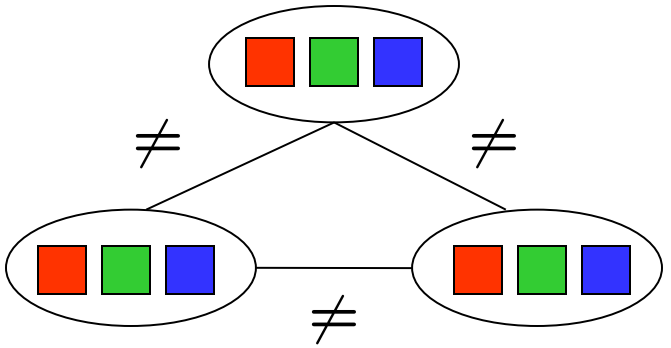
- CSP definitions
- CSP solving
  - Backtracking, Filtering, Arc Consistency, Ordering

Today's Lecture

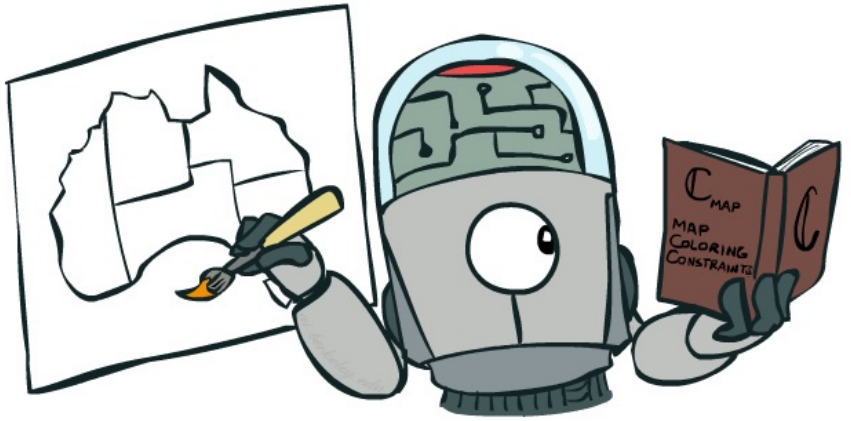
- Ordering
- Structure
- New Topic: Adversarial Search

Reminder: CSPs

- CSPs:
  - Variables
  - Domains
  - Constraints
    - Implicit (provide code to compute)
    - Explicit (provide a list of the legal tuples)
    - Unary / Binary / N-ary



- Goals:
  - Here: find any solution
  - Also: find all, find best, etc.



Reminder: Backtracking Search

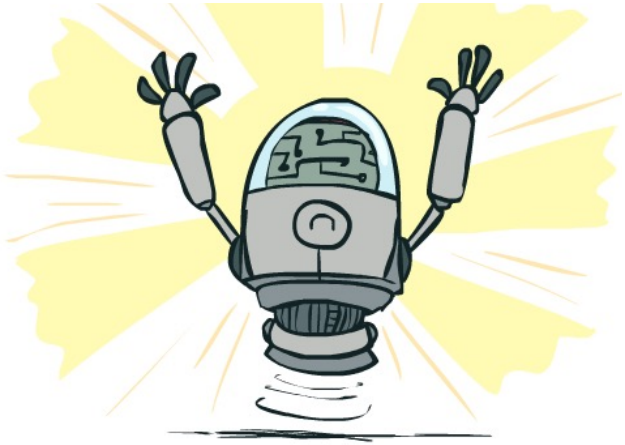
```
function BACKTRACKING-SEARCH(csp) returns solution/failure
    return RECURSIVE-BACKTRACKING({ }, csp)

function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment given CONSTRAINTS[csp] then
            add {var = value} to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

Reminder Improving Backtracking

- Backtracking = DFS + variable-ordering + fail-on-violation
- Improvements

1. Filtering: Can we detect inevitable failure early?
   - Forward checking, Arc consistency, higher order consistency



2. Ordering:
   - Which variable should be assigned next?
   - In what order should its values be tried?

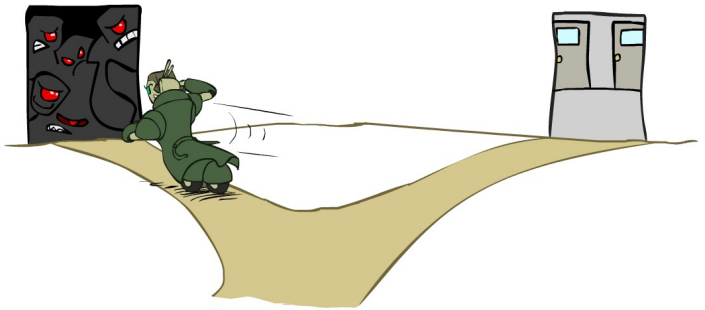
3. Structure: Can we exploit the problem structure?

Ordering

Ordering: Minimum Remaining Values

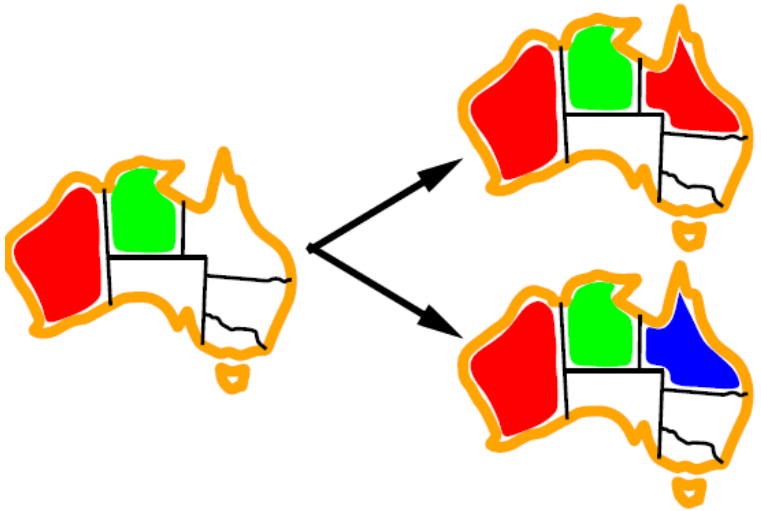
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

- Why min rather than max?
- Also called “most constrained variable”
- “Fail-fast” ordering

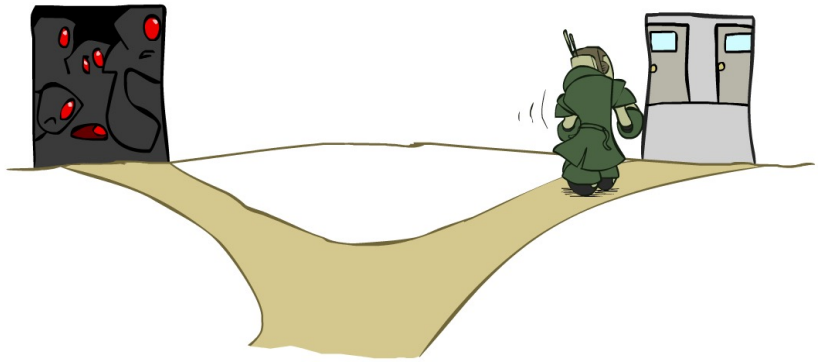


Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)

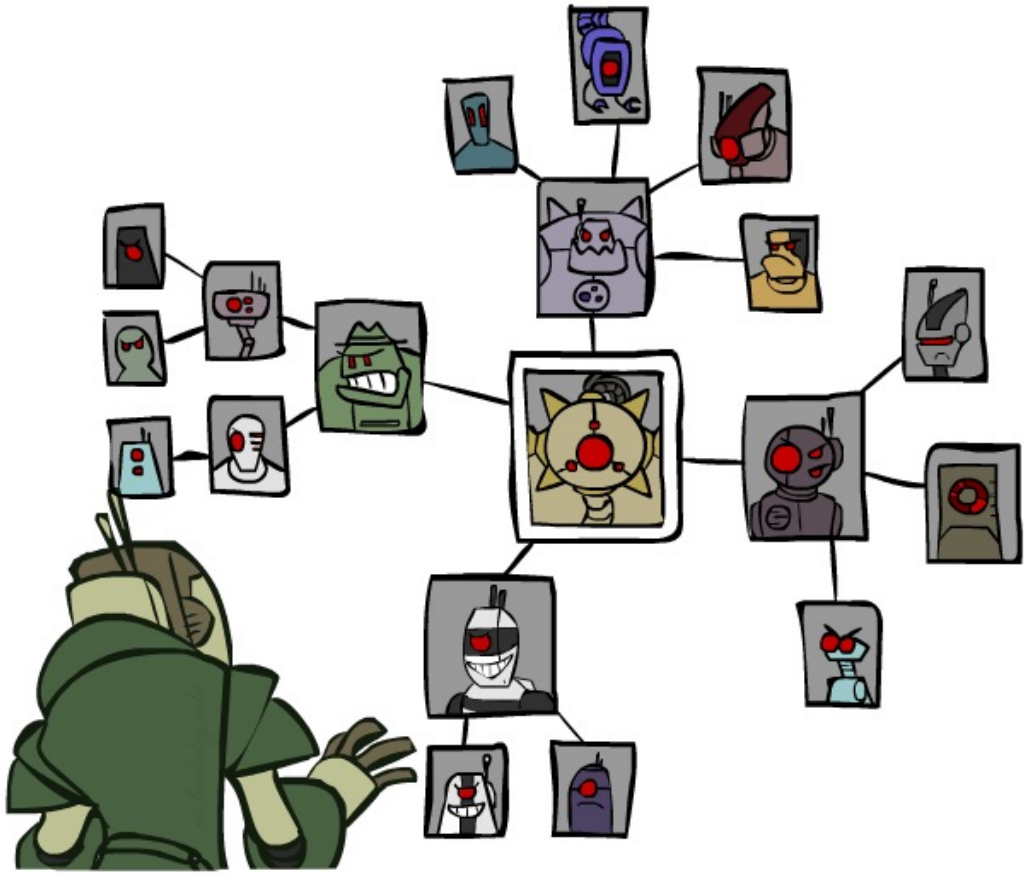


- Why least rather than most?



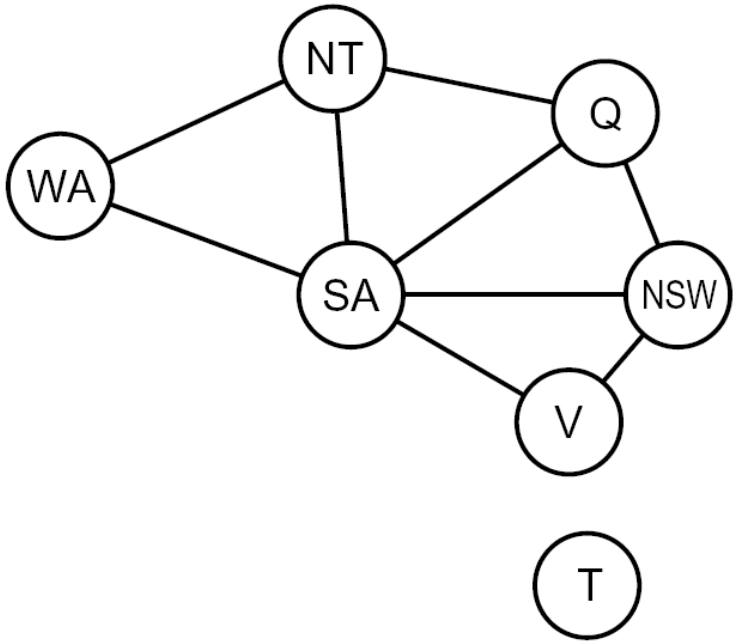
- Combining these ordering ideas makes 1000 queens feasible

Structure

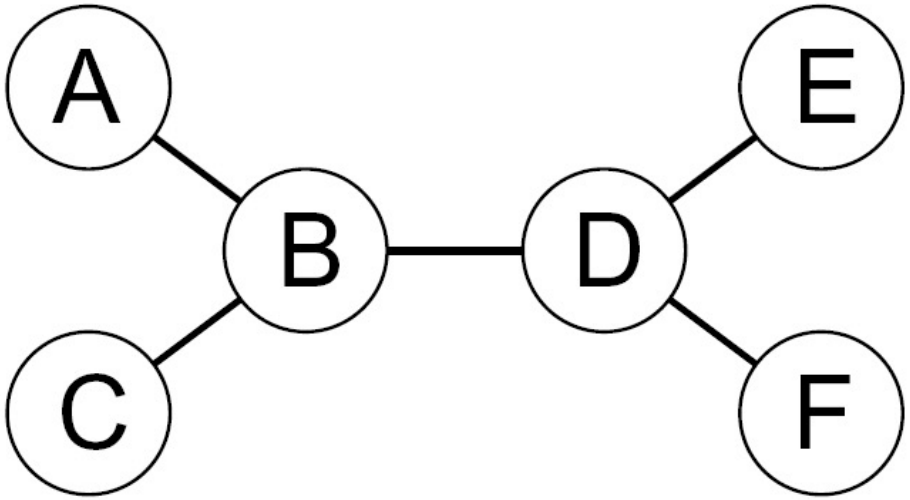


Problem Structure

- Extreme case: independent subproblems
  - Example: Tasmania and mainland do not interact
- Independent subproblems are identifiable as connected components of constraint graph
- Suppose a graph of n variables can be broken into subproblems of only c variables:
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n



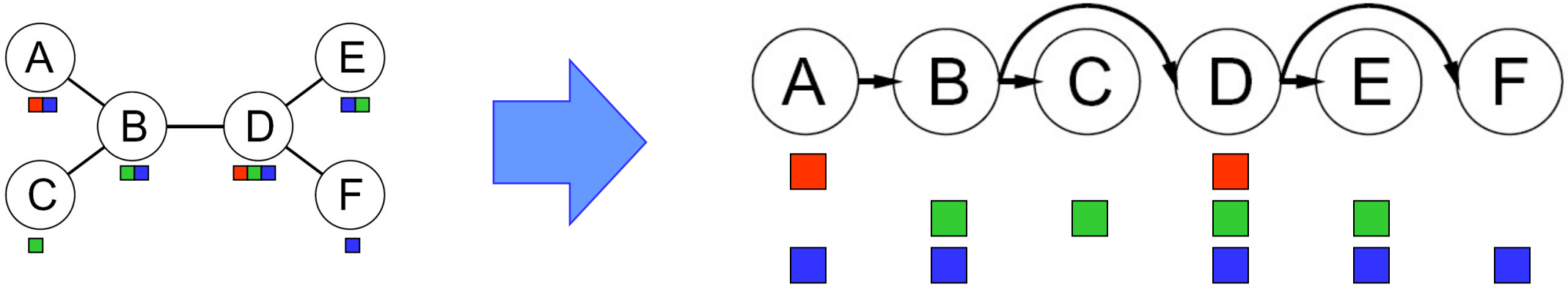
Tree-Structured CSPs



- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$

Tree-Structured CSPs

- Algorithm for tree-structured CSPs:
  - Order: Choose a root variable, order variables so that parents precede children

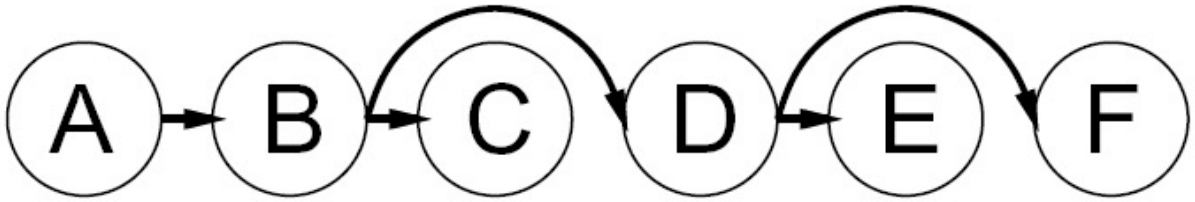


  - Remove backward: For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
  - Assign forward: For i = 1 : n, assign X_i consistently with Parent(X_i)

- Runtime: $O(n\ d^2)$ (why?)

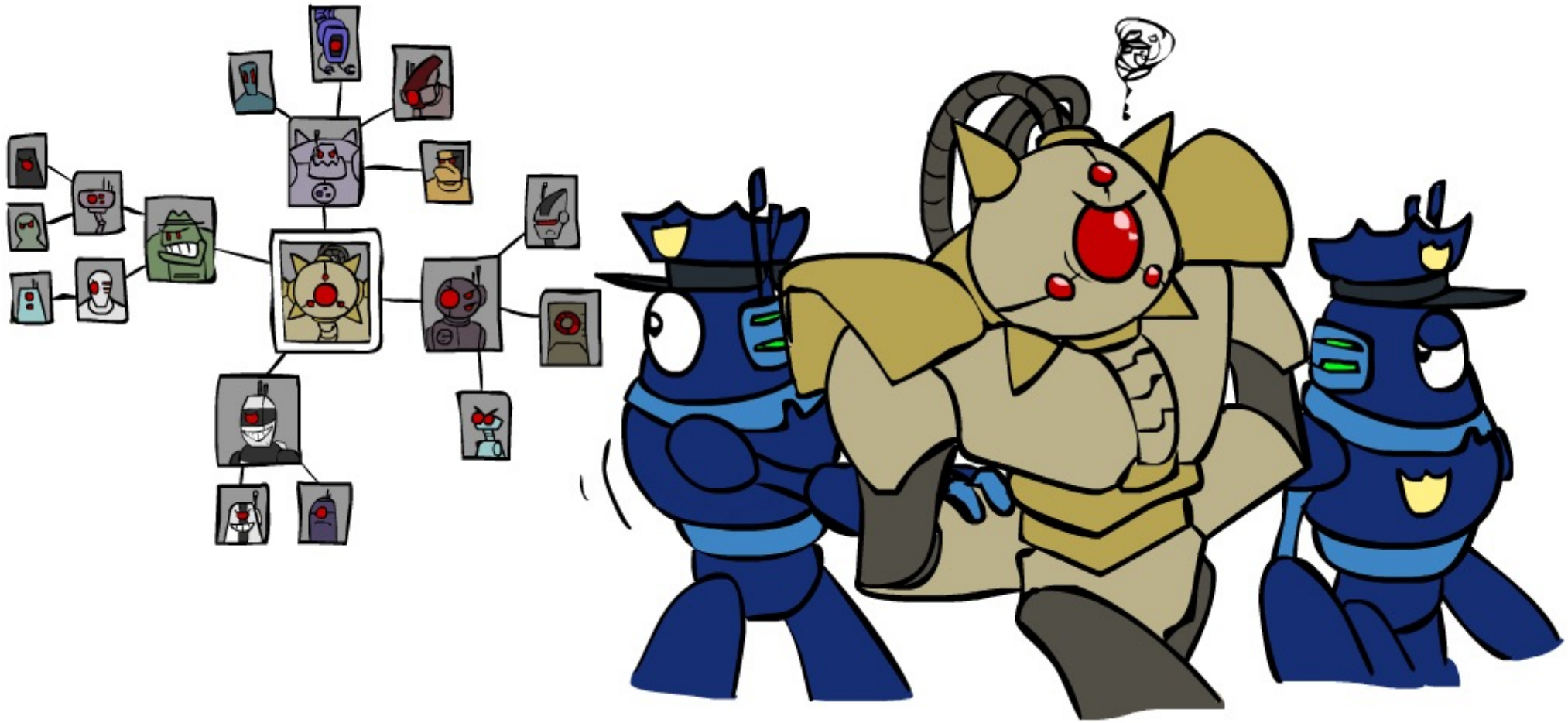
Tree-Structured CSPs

- Claim 1: After backward pass, all root-to-leaf arcs are consistent
- Proof: Each X→Y was made consistent at one point and Y's domain could not have been reduced thereafter (because Y's children were processed before Y)

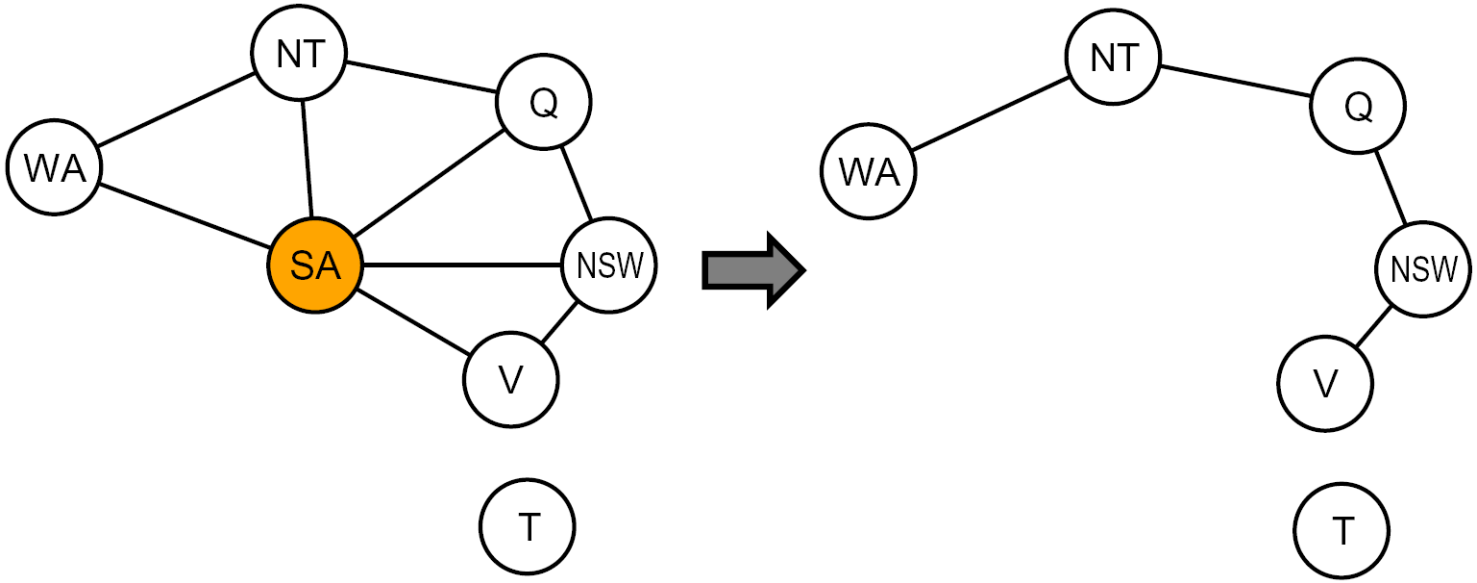


- Claim 2: If root-to-leaf arcs are consistent, forward assignment will not backtrack
  - Proof: Induction on position
- Why doesn't this algorithm work with cycles in the constraint graph?

Improving Structure



Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime O((d^c) (n-c) d^2), very fast for small c

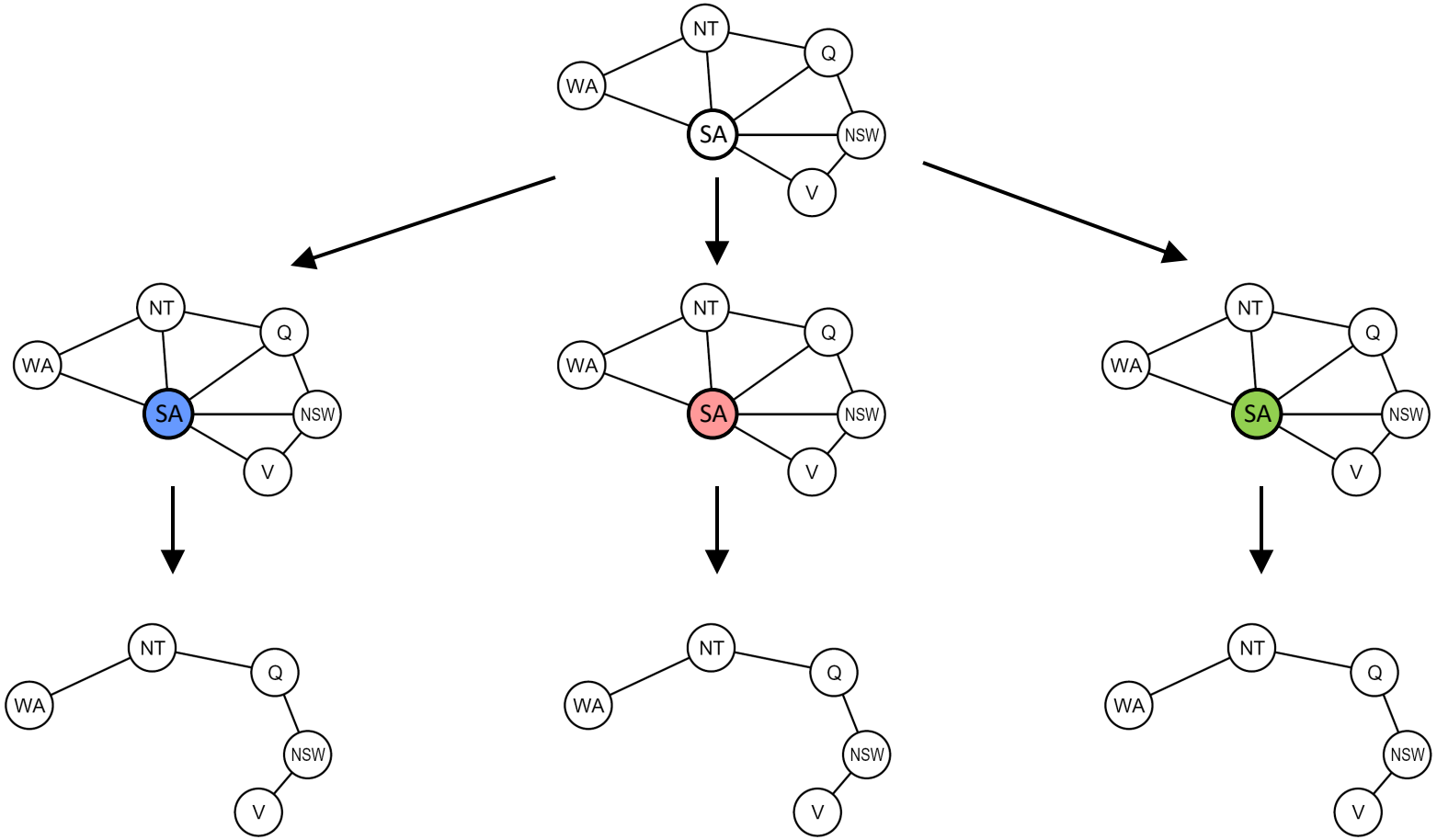
Cutset Conditioning

Choose a cutset

Instantiate the cutset (all possible ways)

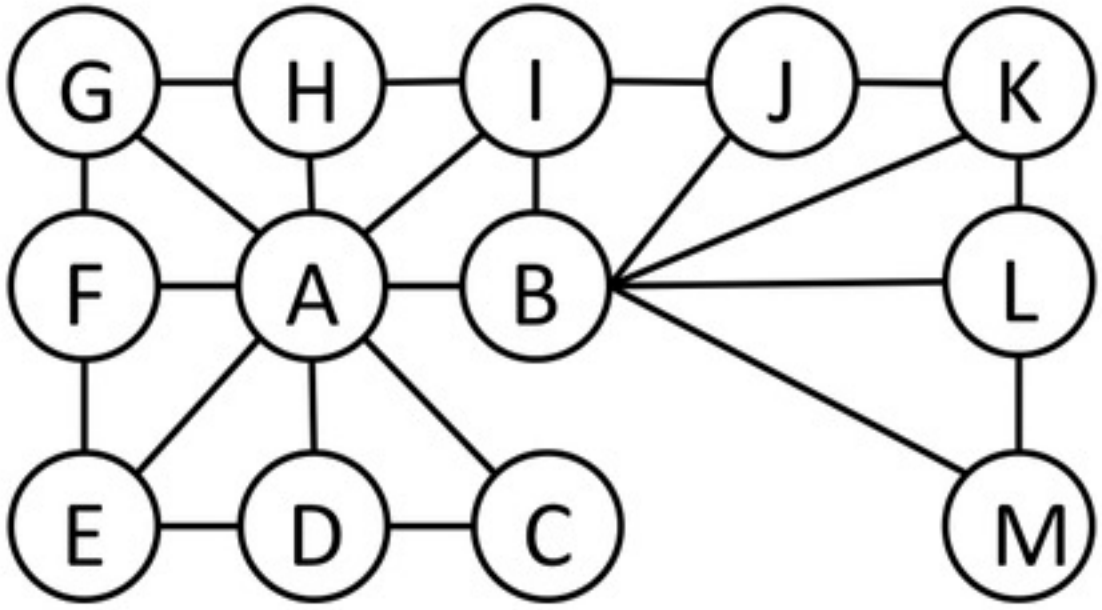
Compute residual CSP for each assignment

Solve the residual CSPs (tree structured)



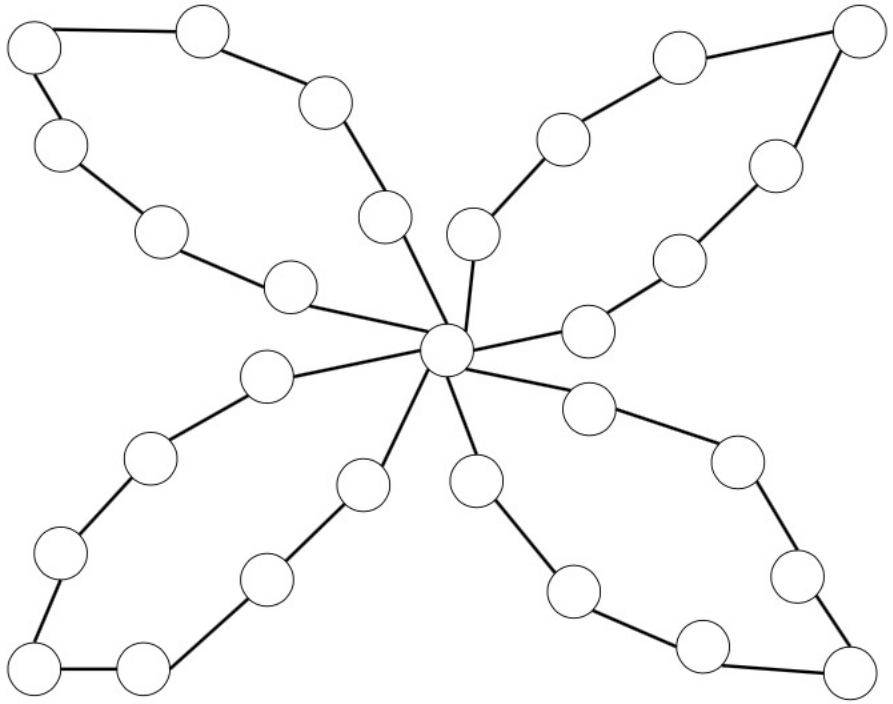
Cutset Quiz

- Find the smallest cutset for the graph below.



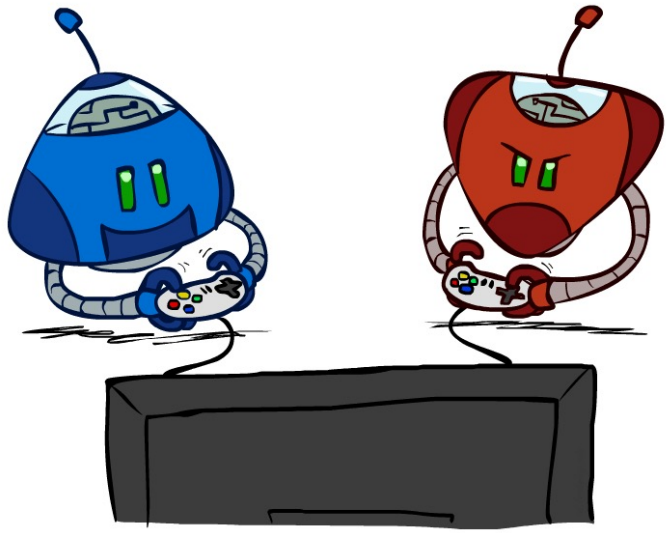
Review Problem

- Consider the graph on the left.
- In two sentences or less, describe a strategy for efficiently solving a CSP with this constraint structure.



CS 3568: Intelligent Systems

Adversarial Search



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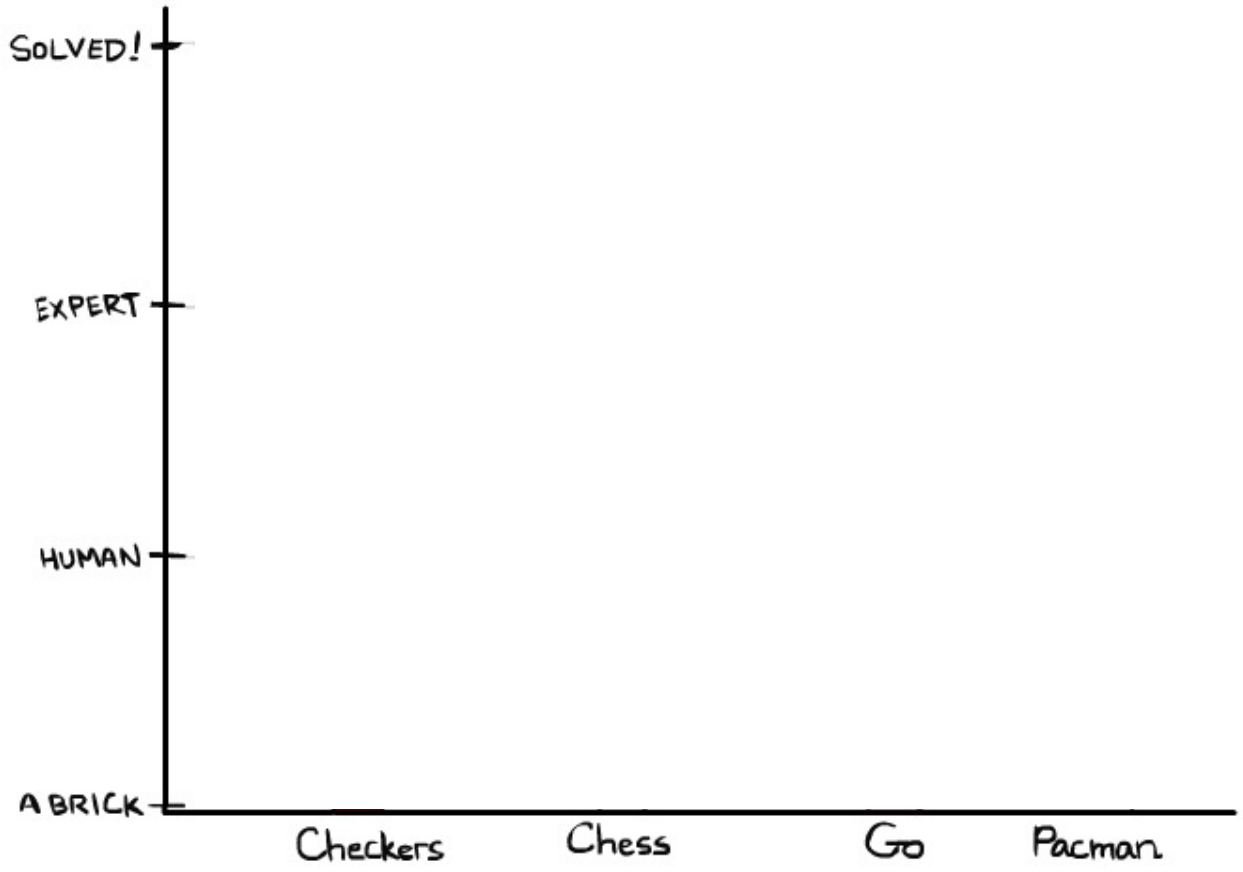
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!
- **Chess:** 1997: Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply. Current programs are even better, if less historic.
- **Go:** Human champions are now starting to be challenged by machines. In go, b > 300! Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.



Game Playing State-of-the-Art

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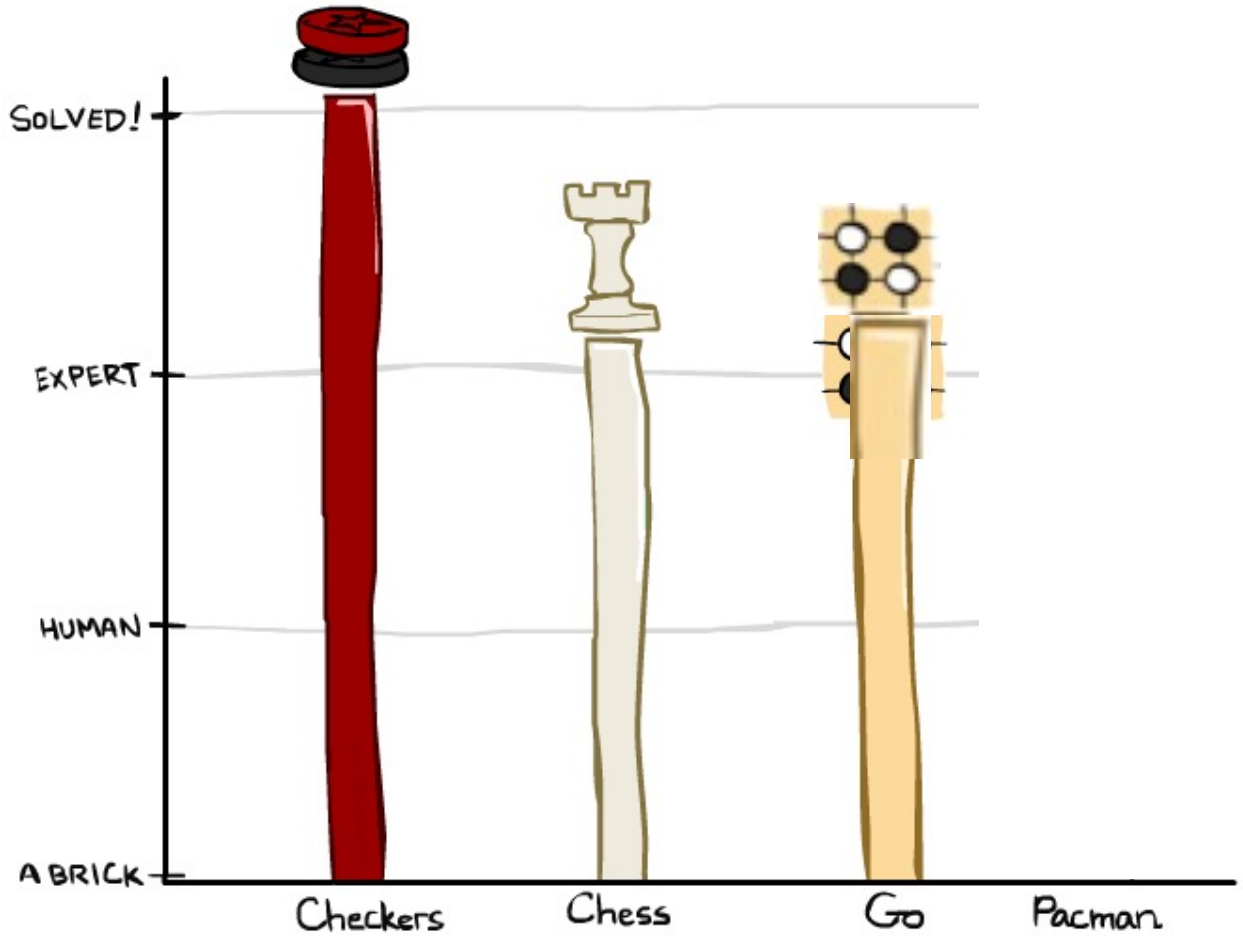
- **Go: 2016: Alpha GO defeats human champion. Uses Monte Carlo Tree Search, learned evaluation function.**
- **Pacman**



Adversarial Games

Types of Games

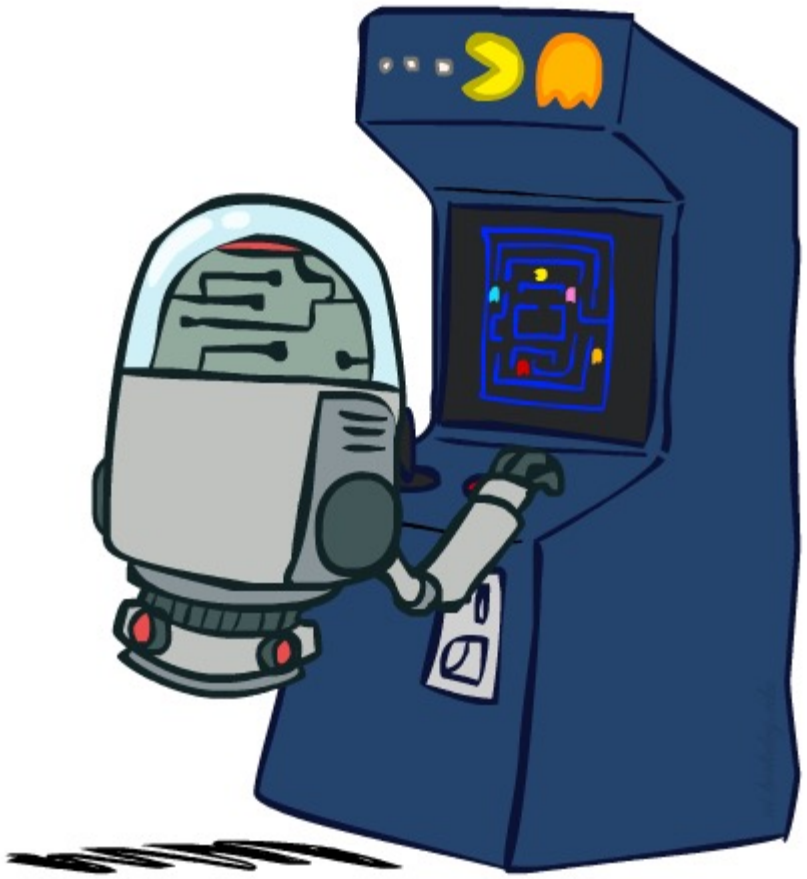
- Many different kinds of games!

- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?

- Want algorithms for calculating a strategy (policy) which recommends a move from each state
  - Not the same as planning

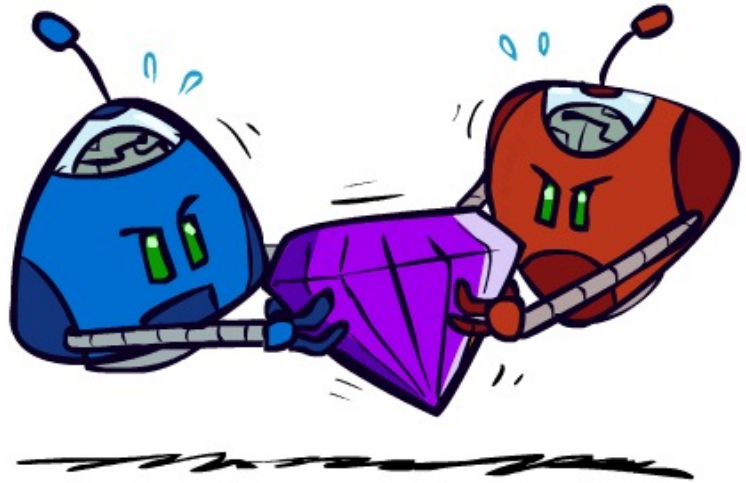
Deterministic Games

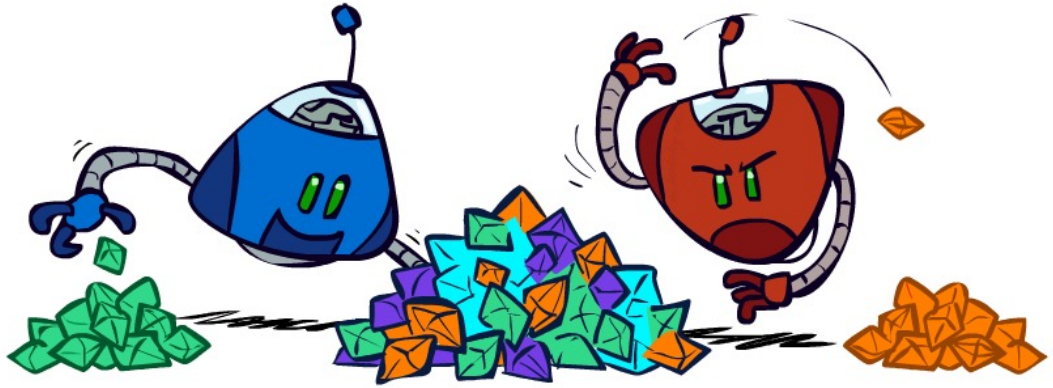
- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R



- Solution for a player is a policy: S $\rightarrow$ A
  - **Policy:** for every possible situation what actions you be taking

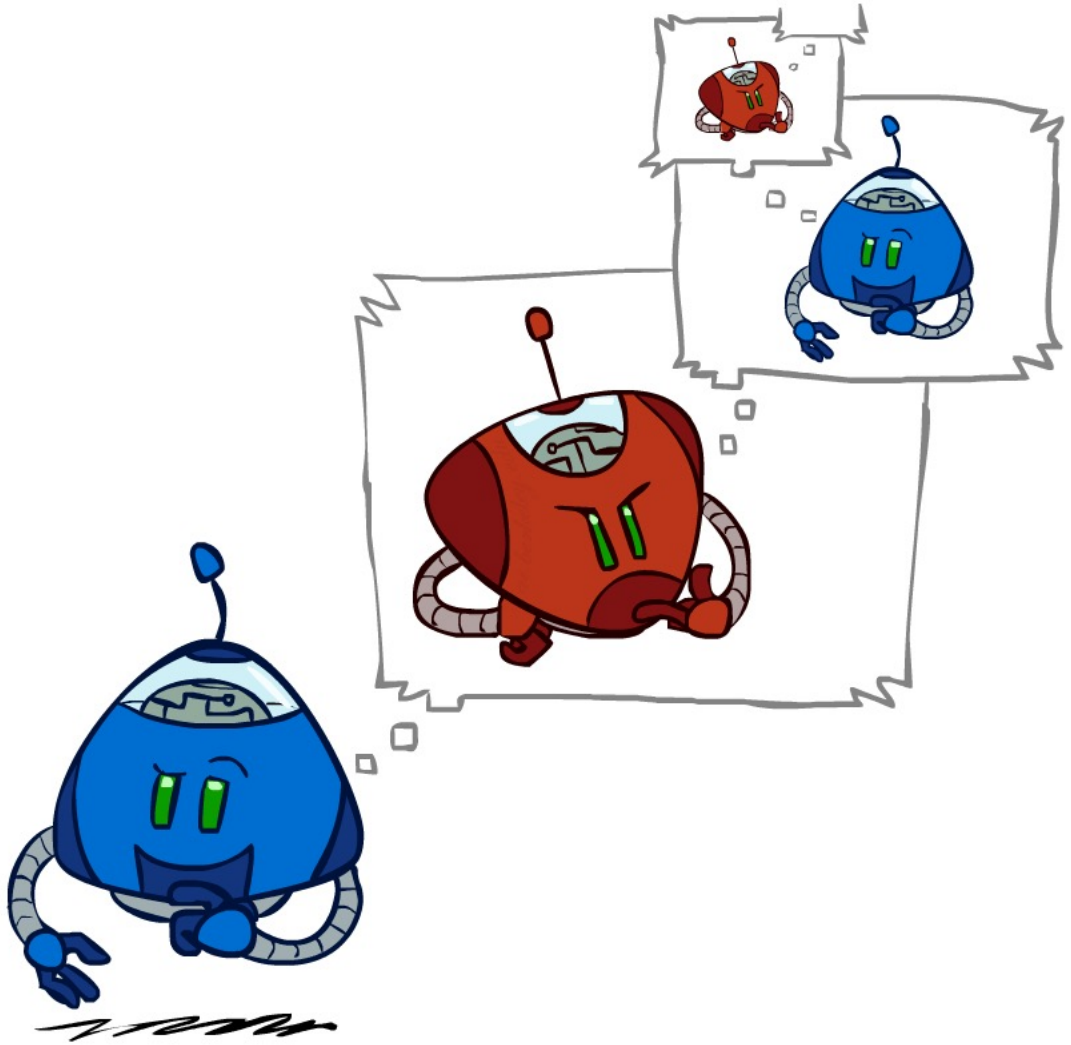
Zero-Sum Games



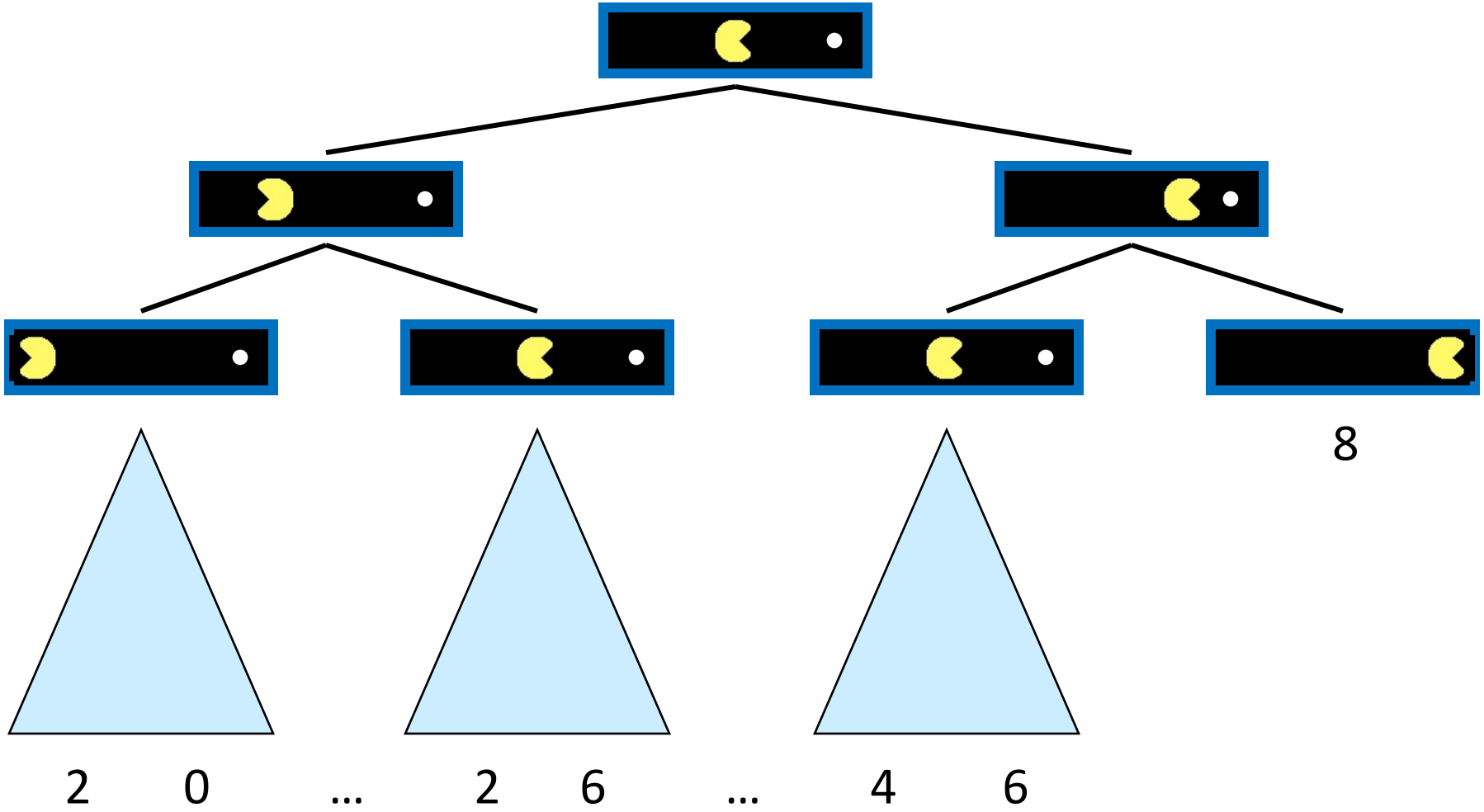


- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Adversarial Search



Single-Agent Trees

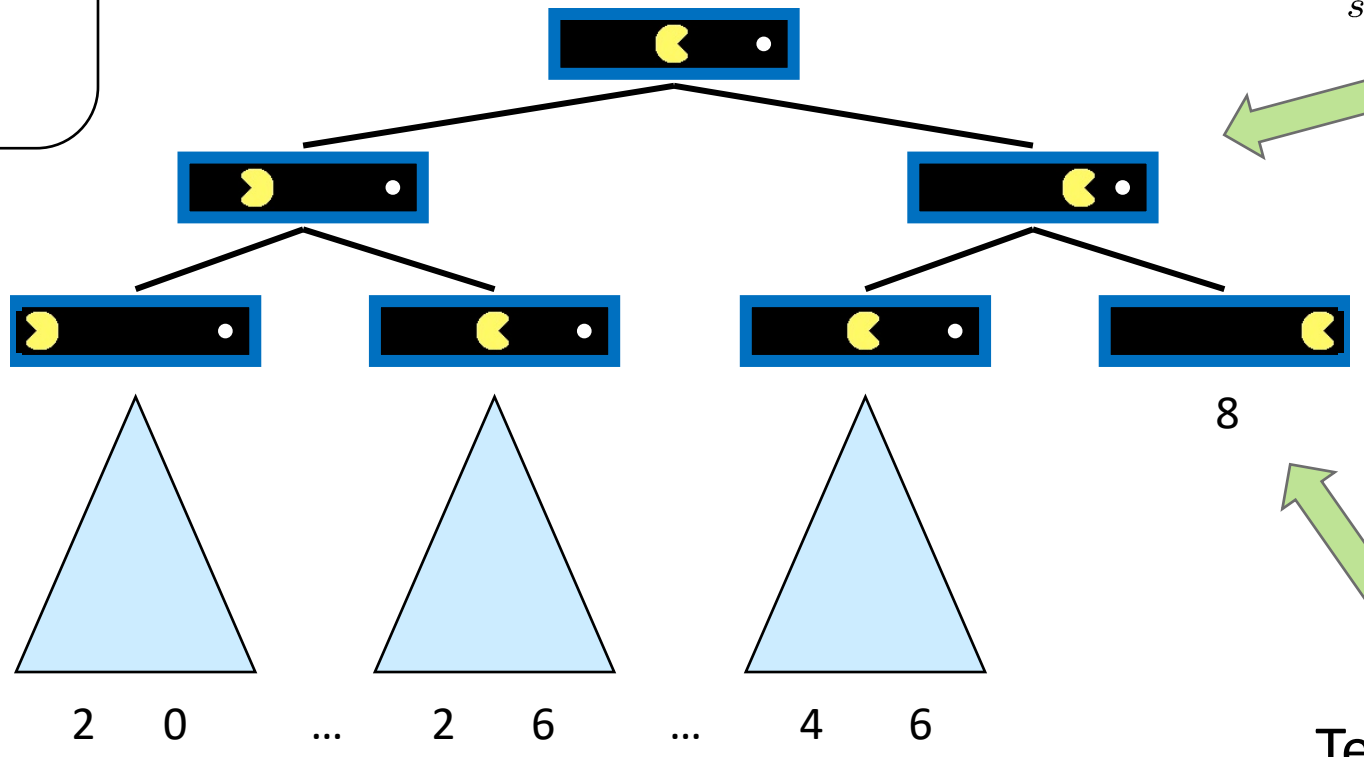


Value of a State

Value of a state:
The best achievable outcome (utility) from that state

Non-Terminal States:

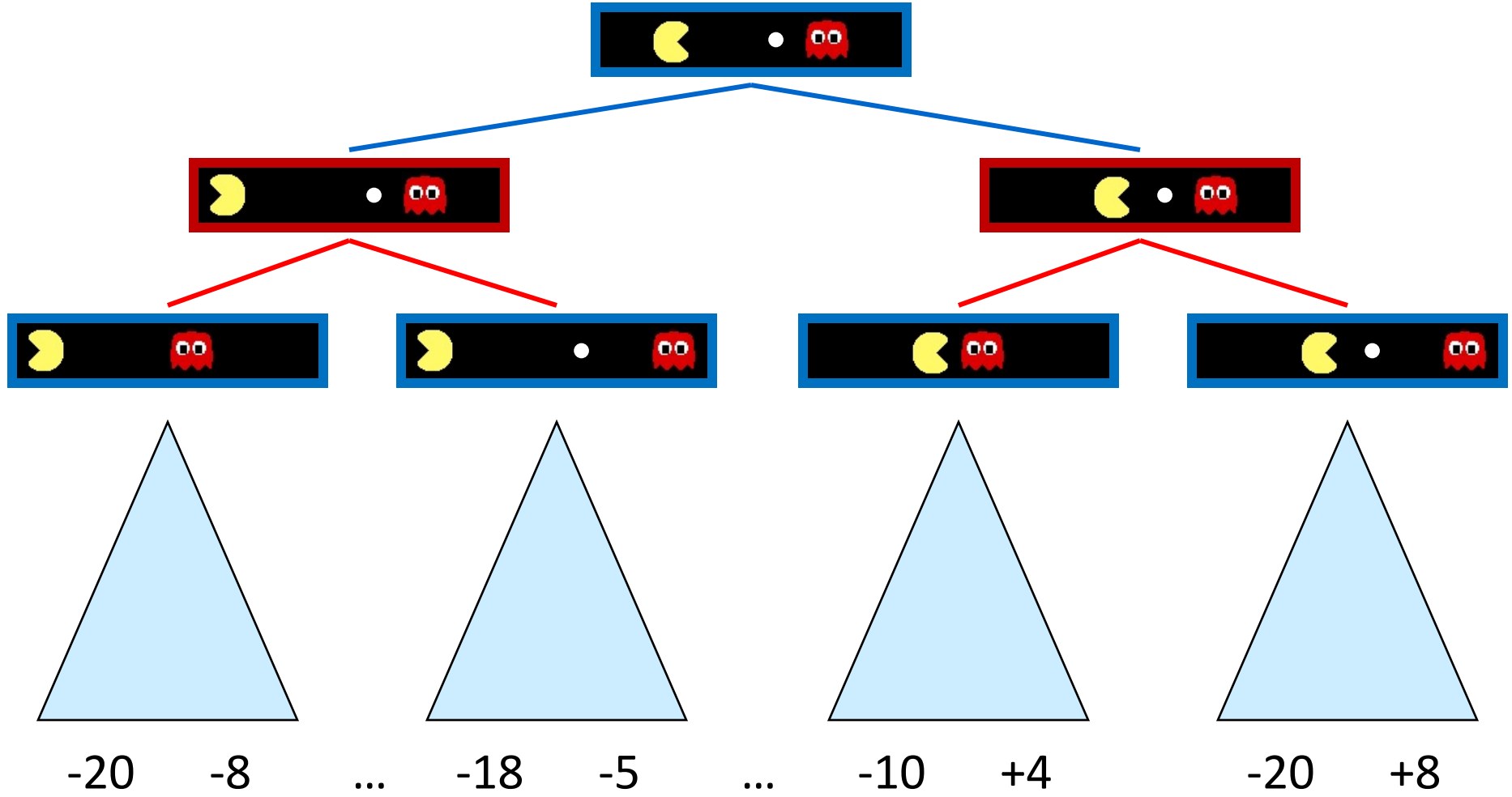
$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

$$V(s) = \text{known}$$

Adversarial Game Trees



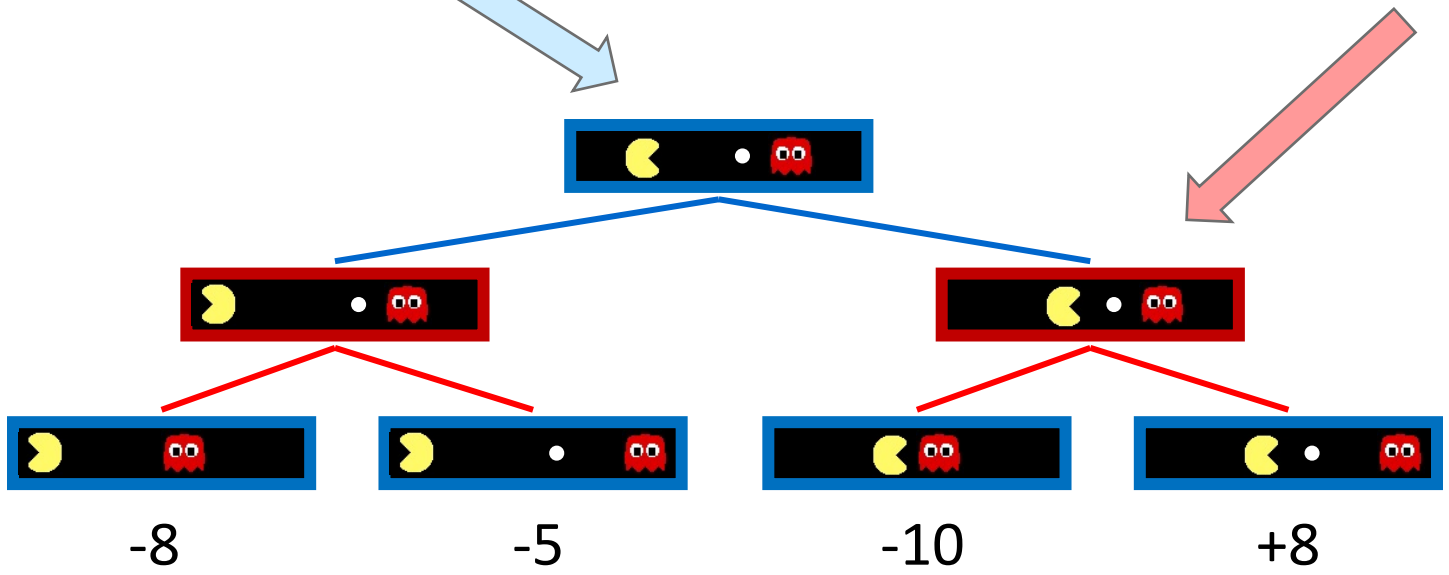
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

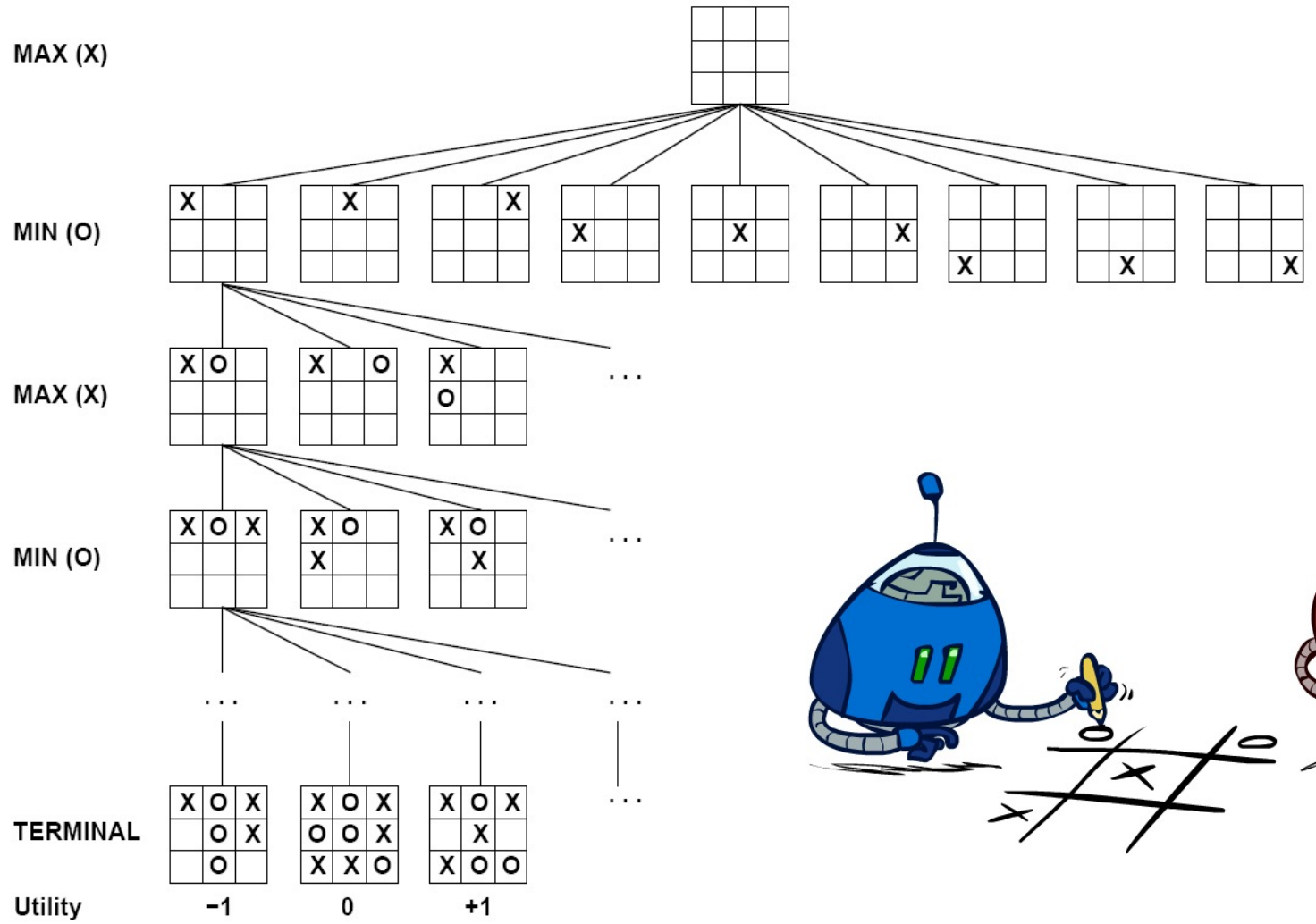
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

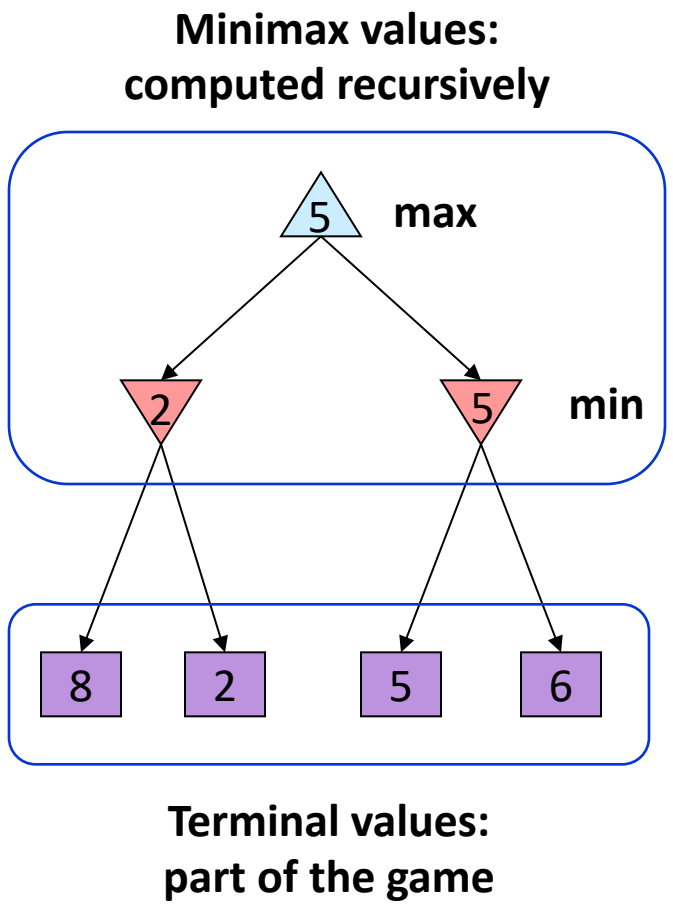
$$V(s) = \text{known}$$

Tic-Tac-Toe Game Tree



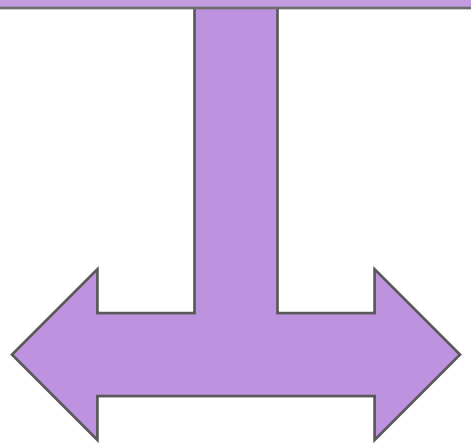
Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Minimax Implementation (Dispatch)

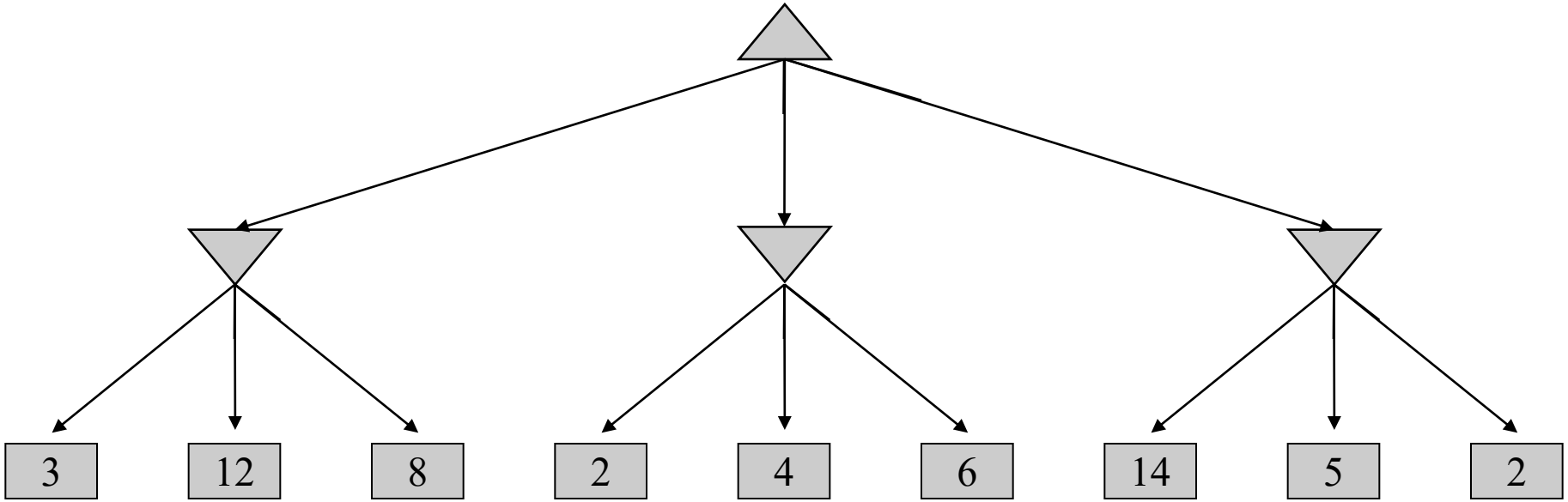
```
def value(state):
        if the state is a terminal state: return the state's utility
        if the next agent is MAX: return max-value(state)
        if the next agent is MIN: return min-value(state)
```



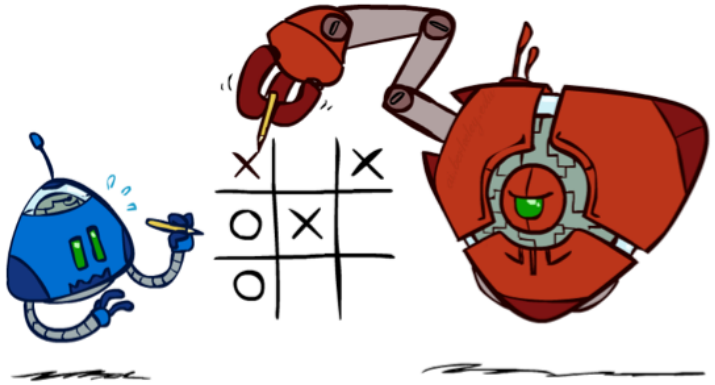
```
def max-value(state):
        initialize v = -∞
        for each successor of state:
                v = max(v, value(successor))
        return v
```

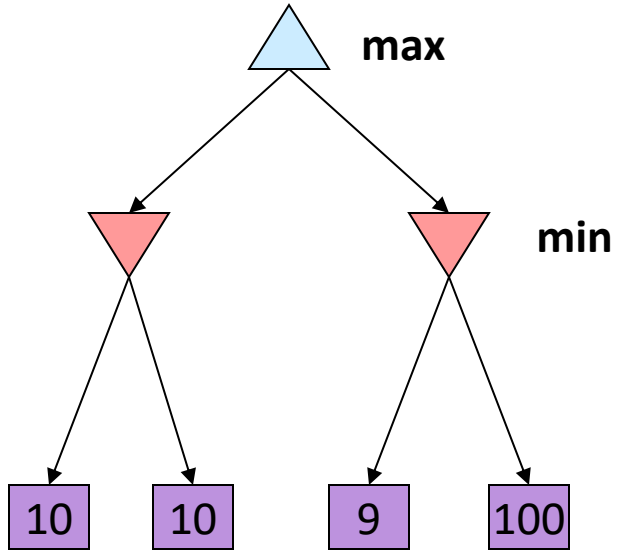
```
def min-value(state):
        initialize v = +∞
        for each successor of state:
                v = min(v, value(successor))
        return v
```

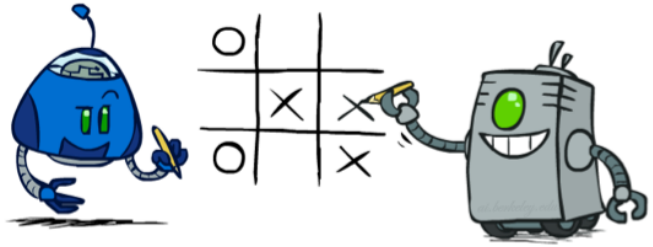
Minimax Example



Minimax Properties

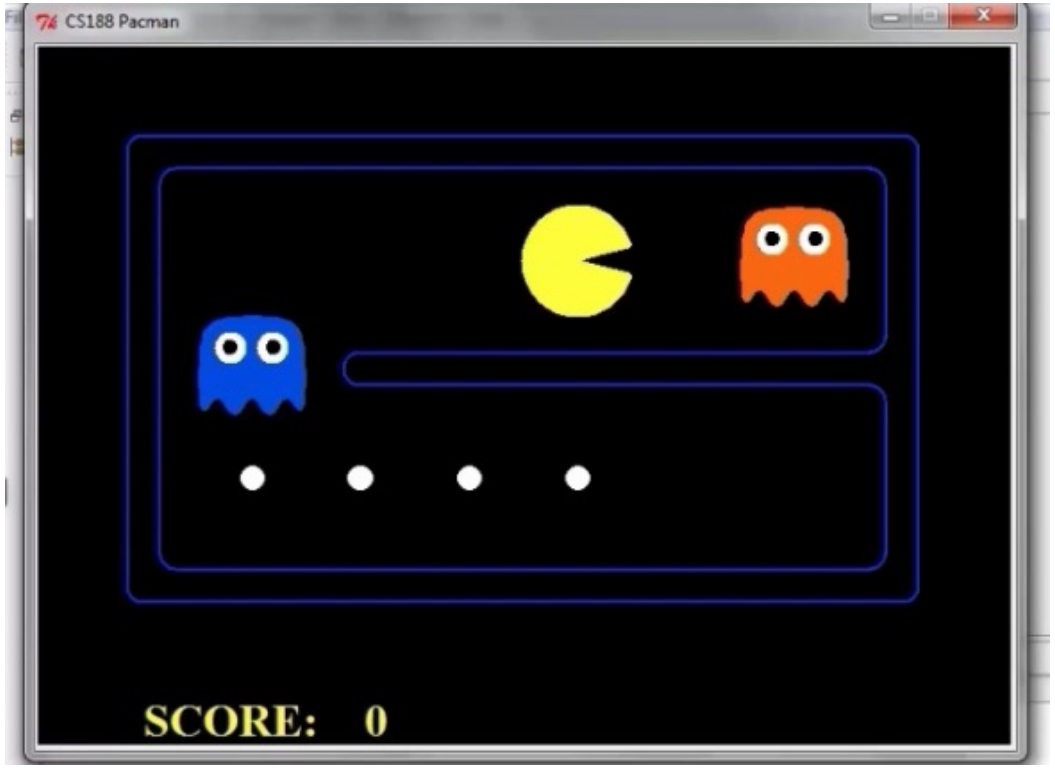




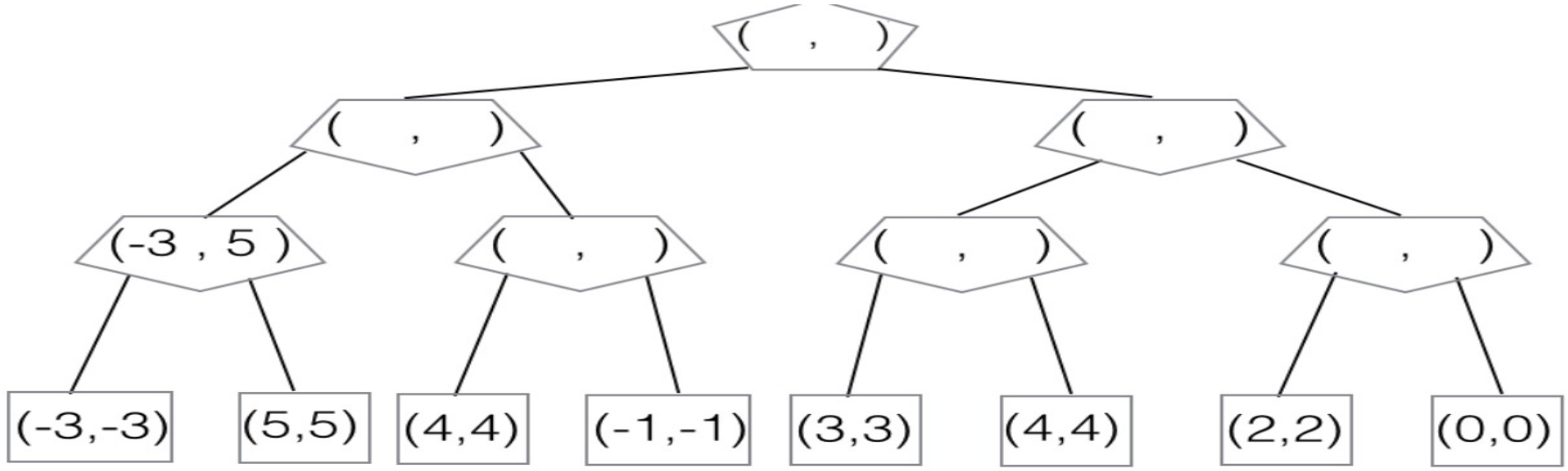


Optimal against a perfect player. Otherwise?

Video of Demo Min vs. Exp (Min)



Review Question



- Similar to the minimax algorithm, where the value of each node is determined by the game subtree hanging from that node, we define a value pair (u,v) for each node: u is the value of the subtree if the power is not used in that subtree; v is the value of the subtree if the power is used once in that subtree. For example, in the below subtree with values (-3, 5), if Pacman does not use the power, the ghost acting as a minimizer would choose -3; however, with the special power, Pacman can make the ghost choose the value more desirable to Pacman, in this case 5.
- Fill in the (u, v) values in the modified minimax tree below. Pacman is the root and there are two ghosts.