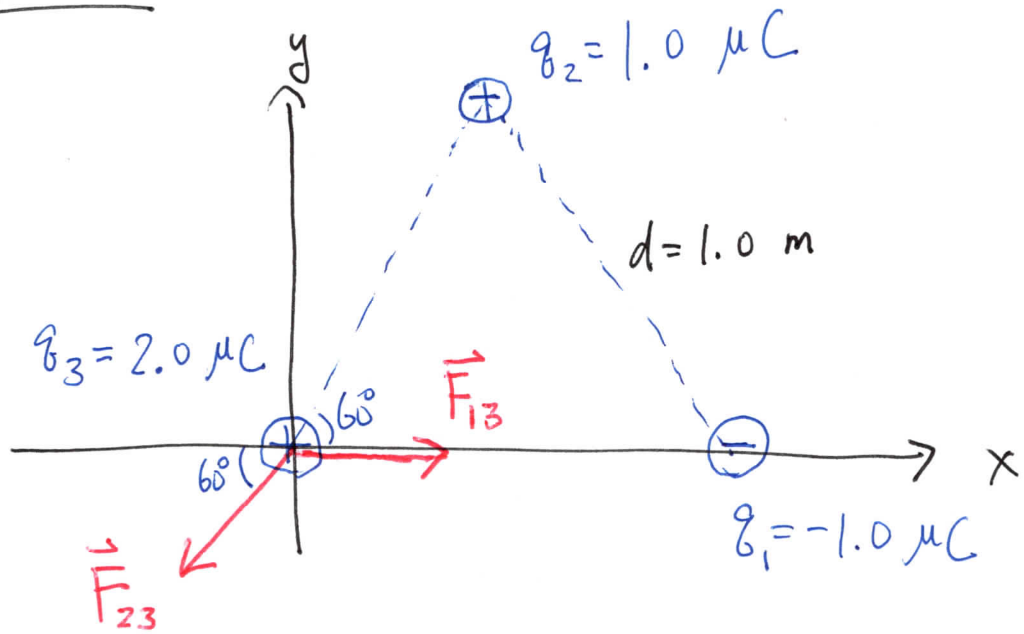
Solutions to Sample Problems

SP 1.1



Here, I only show the free-body diagram for the charge q_3. And the ~~the~~ forces acting on the other two charges are "suppressed" in the diagram for clarity.

- The magnitude of $\vec{F}_{13}$ is:

$$F_{13} = |\vec{F}_{13}| = k_e \frac{|q_1 q_3|}{d^2}$$

$$\therefore F_{13} = k_e \frac{|q_1 q_3|}{d^2}$$

$$= \left(9.0\times10^{9}\ N\cdot m^2/C^2\right)\cdot\frac{|(-1.0)\cdot(2.0)|}{(1.0)^2\cdot m^2}\times10^{-6}\times10^{-6}\ C^2$$

$$= \frac{9.0\cdot|(-1.0)\cdot(2.0)|}{(1.0)^2}\times10^{9-12}\quad(N)$$

$$= 18\times10^{-3}\quad(N)$$

$$F_{13} = \boxed{1.8\times10^{-2}\ (N)}$$

SP 1.2

From the diagram, we see that $\vec{F}_{13}$ points to the right.

$$\boxed{\vec{F}_{13} = 1.8\times10^{-2}\ \hat{i}\quad(N)}$$

(Go on to the next page...)

SP. 1.3

The magnitude of $\vec{F}_{23}$ is:

$$F_{23} = |\vec{F}_{23}| = k_e \frac{|q_2 q_3|}{d^2}$$

$$= \left(9.0 \times 10^9 \ N \cdot m^2/C^2\right) \cdot \frac{|(1.0)\cdot(2.0)|}{(1.0)^2 \ m^2} 10^{-6} \cdot 10^{-6} \ C^2$$

$$= \frac{(9.0)\cdot |(1.0)\cdot(2.0)|}{(1.0)^2} \times 10^{9-12} \ (N)$$

$$= 18 \times 10^{-3} \ (N)$$

$$\boxed{F_{23} = 1.8 \times 10^{-2} \ (N)}$$

SP 1.4

From the diagram, we see that $\vec{F}_{23}$ points to the third quadrant.

SP 1.5 & 1.6

The resultant electric force acting on q_3 is:

$$\vec{F_3} = \vec{F_{13}} + \vec{F_{23}} \, ,$$

where

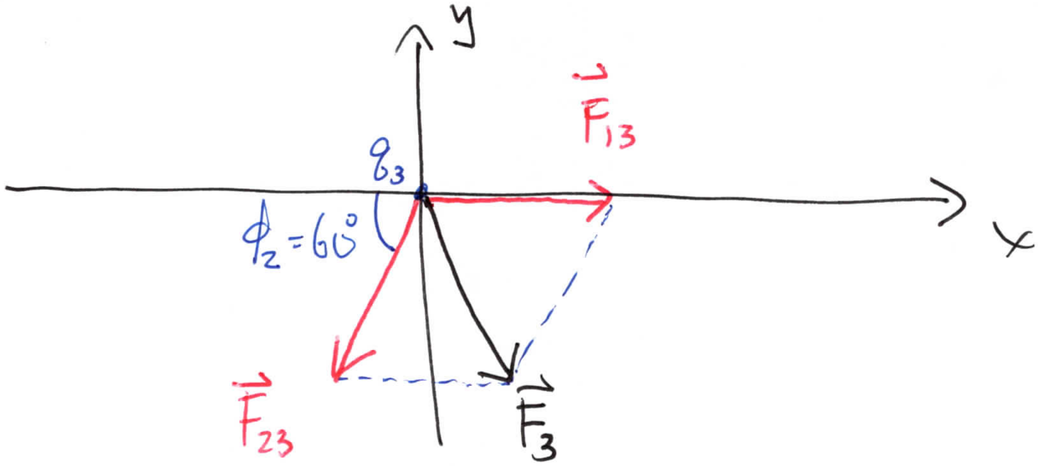
$$\begin{cases} \vec{F_{13}} = F_{13} \, \hat{i} \\ \vec{F_{23}} = -F_{23} \cos\phi_2 \, \hat{i} - F_{23} \sin\phi_2 \, \hat{j} \end{cases}$$

(Here, F_{13} denotes the magnitude of $\vec{F_{13}}$, and F_{23} denotes the magnitude of $\vec{F_{23}}$.)

$$\vec{F_3} = \vec{F_{13}} + \vec{F_{23}}$$

$$= \left(F_{13} - F_{23}\cos\phi_2\right)\hat{i} - F_{23} \sin\phi_2 \, \hat{j}$$

Before we start to compute $\vec{F_3}$, we can observe the direction of $\vec{F_3}$ from the free-body diagram of q_3.



We have shown that $F_{13} = F_{23} = 1.8 \times 10^{-2}$ (N).

(They are equal in magnitude.)

Besides, we know that $\phi_2 = 60^\circ$

From the free-body diagram shown in above, we use the parallelogram rule to see that the resultant electric force $\vec{F}_3$ points to the fourth quadrant.

Now, we start to compute $\vec{F}_3$

$$\vec{F}_3 = \left(\vec{F}_{13} + \vec{F}_{23}\right) = \left(F_{13} - F_{23}\cos\phi_2\right)\hat{i} - F_{23}\sin\phi_2\,\hat{j}$$

$$\therefore \vec{F}_3 = \left[1.8\times10^{-2} - 1.8\times10^{-2}\cdot\cos(60^\circ)\right]\hat{i}$$
$$- 1.8\times10^{-2}\cdot\sin(60^\circ)\,\hat{j} \quad (N)$$

$$= 1.8\times10^{-2}\left(1-\frac{1}{2}\right)\hat{i} - 1.8\times10^{-2}\cdot\frac{\sqrt{3}}{2}\hat{j} \quad (N)$$

$$\vec{F}_3 = 1.8\times10^{-2}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) \quad (N)$$

Since $F_{3,x} > 0$ and $F_{3,y} < 0$, we confirm that $\vec{F}_3$ points to the fourth quadrant.

- The magnitude of $\vec{F}_3$ is:

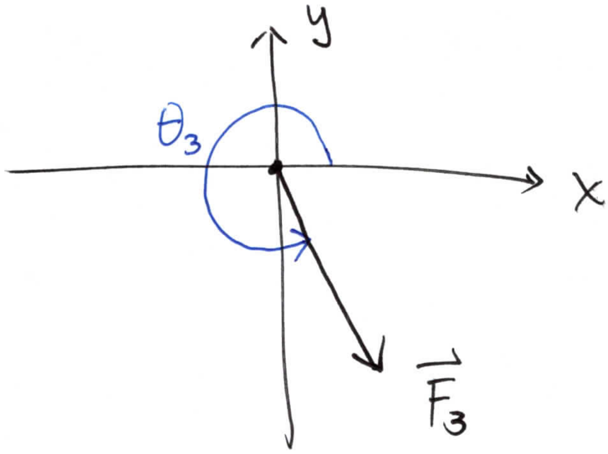
$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2}$$
$$= \sqrt{(1.8\times10^{-2})^2\cdot\left(\frac{1}{2}\right)^2 + (1.8\times10^{-2})^2\cdot\left(-\frac{\sqrt{3}}{2}\right)^2}$$
$$= 1.8\times10^{-2}\sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}$$
$$= 1.8\times10^{-2}\sqrt{\frac{1}{4}+\frac{3}{4}} = 1.8\times10^{-2}\ (N)$$

∴ The magnitude of $\vec{F_3}$ is

$$\boxed{F_3 = 1.8 \times 10^{-2} \ (N)}$$

Remark:

We can also find the angle that is associated with the resultant electric force $\vec{F_3}$



Define the angle in Standard position as θ_3

$$\tan\theta_3 = \frac{F_{3,y}}{F_{3,x}} = \frac{(1.8\times10^{-2})\cdot\left(\frac{-\sqrt{3}}{2}\right) \ N}{(1.8\times10^{-2})\cdot\frac{1}{2} \ N.}$$

$$= -\sqrt{3}$$

$$\therefore \tan\theta_3 = -\sqrt{3}$$

We can see that

$$\tan^{-1}(-\sqrt{3}) = -60^\circ$$

Thus, 60° is the reference angle that is associated with θ_3.

- Recall that θ_3 is the angle in standard position.

$$\therefore \theta_3 = 360^\circ - 60^\circ = 300^\circ$$

$\therefore$ $\boxed{\theta_3 = 300^\circ}$ is the direction of $\vec{F_3}$