Test 2 Review Homework

1. For each of the following languages, give a pushdown automaton that recognizes it.

   (a) $\{\omega|\ n_a(\omega) = n_b(\omega)\}$ over {a, b}.

   (b) $\{\omega|\ n_a(\omega) \neq n_b(\omega)\}$ over {a, b}.

   (c) $\{a^n b^{n+m} a^m|\ n, m \geq 0\}$ over {a, b}.

   (d) The language of palindromes over {a, b}.

   (e) The language of even-length palindromes over {a ,b}.

   (f) The language of all non-palindromes over {a, b}.

   (g) $\{a^i b^j c^k|\ i, j, k \geq 0$ and $j = i$ or $j = k\}$ over {a, b, c}.

   (h) $\{\omega|\ n_a(\omega) < n_b(\omega)$ or $n_a(\omega) < n_c(\omega)\}$ over {a, b, c}.

2. Let G_1 be the following grammar:

$$
\begin{aligned}
E &\to E + T \mid T \\
T &\to T \times F \mid F \\
F &\to (E) \mid a
\end{aligned}
$$

   where E is the start variable. Give the parse trees for each of the following strings using G_1:

   (a) a

   (b) a+a

   (c) a+a+a

   (d) ((a))

3. Let G_2 be the following grammar:

$$
\begin{aligned}
\langle STMT\rangle &\to \langle ASSIGN\rangle \mid \langle IF-THEN\rangle \mid \langle IF-THEN-ELSE\rangle \\
\langle IF-THEN\rangle &\to if\ condition\ then\ \langle STMT\rangle \\
\langle IF-THEN-ELSE\rangle &\to if\ condition\ then\ \langle STMT\rangle\ else\ \langle STMT\rangle \\
\langle ASSIGN\rangle &\to a := 1
\end{aligned}
$$

   where $\langle STMT\rangle$ is the start variable. Show that G_2 is ambiguous (i.e. give two different derivations for a string).

4. Let G_3 be the following grammar:

$$S \to SS \mid T$$
$$T \to aTb \mid ab$$

Show that G_3 is ambiguous (i.e. give two different derivations for a string).

5. For each of the following languages, give a context-free grammar that describes it.

   (a) $\{\omega \mid n_a(\omega) = n_b(\omega)\}$ over {a, b}.
   (b) $\{\omega \mid n_a(\omega) \neq n_b(\omega)\}$ over {a, b}.
   (c) $\{a^n b^{n+m} a^m \mid n, m \geq 0\}$ over {a, b}.
   (d) The language of all non-palindromes over {a, b}.
   (e) $\{a^i b^j c^k \mid i, j, k \geq 0$ and $j = i$ or $j = k\}$ over {a, b, c}.
   (f) $\{\omega \mid n_a(\omega) < n_b(\omega)$ or $n_a(\omega) < n_c(\omega)\}$ over {a, b, c}.

6. Select at least 5 of the following problems. Prove whether the language is context-free or non-context-free.

   (a) $\{a^i b^j \mid i \leq j^2\}$ over {a, b}.
   (b) $\{a^i b^j c^k \mid k = ji\}$ over {a, b, c}.
   (c) $\{a^i b^j c^k \mid i < j < k\}$ over {a, b, c}.
   (d) $\{\omega\omega \mid \omega \in \{a, b\}^*\}$.
   (e) $\{a^n \omega\omega^R a^n \mid n \geq 0, \omega \in \{a, b\}^*\}$.
   (f) $\{a^i b^j a^i b^j \mid i, j \geq 0\}$ over {a, b}.
   (g) $\{a^i b^j a^j b^i \mid i, j \geq 0\}$ over {a, b}.
   (h) $\{\omega \mid n_a(\omega) < n_b(\omega) < n_c(\omega)\}$ over {a, b, c}.
   (i) $\{\omega \mid n_a(\omega)/n_b(\omega) = n_c(\omega)\}$ over {a, b, c}.
   (j) $\{a^i b^j \mid i$ is prime or j is prime$\}$ over {a, b}.
   (k) $\{a^n \mid n \geq 0$ and n is a prime number$\}$ over {a}.

7. Write the rules for a Turing machine whose initial tape contains a # followed by a string of ‘a’s and ‘b’s and replaces every second ‘a’ with a ‘c’ (i.e. if counting the ‘a’s, every even numbered ‘a’).

8. Write the rules of a Turning machine whose initial tape contains a # followed by a sting of 1s and halts accepting if the number of 1s is divisible by 3, otherwise it halts rejecting.

9. Write the rules for a Turing machine whose initial tape contains a # followed by a string of ‘a’s and ‘b’s and reverses the string of ‘a’s and ‘b’s.

10. Given the following tapes and sets of rules, what would each tape look like when the Turing machine halts (also indicate where the read/write head is pointing when it halts.

    (a)

    s_0

$\#$	0	1	0	1	0	0	1	1	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

    $(s_0, \#, s_0, \#, R)$ $(s_0, 0, s_1, 0, R)$ $(s_0, 1, s_0, 1, R)$ $(s_0, \sqcup, h_a, \sqcup, R)$
    $(s_1, 0, s_1, 0, R)$ $(s_1, 1, s_2, 1, R)$ $(s_1, \sqcup, h_a, \sqcup, R)$
    $(s_2, 0, s_2, 0, R)$ $(s_2, 1, s_0, 0, R)$ $(s_2, \sqcup, h_a, \sqcup, R)$

    (b)

    s_0

$\#$	a	b	b	a	a	b	a	b	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

    $(s_0, \#, s_0, \#, R)$ (s_0, a, s_0, a, R) (s_0, b, s_1, a, L) $(s_0, \sqcup, h_a, \sqcup, R)$
    $(s_1, \#, s_2, \#, R)$ (s_1, a, s_3, b, R) (s_1, b, s_2, b, R) $(s_1, \sqcup, h_a, \sqcup, R)$
    (s_2, a, s_0, b, R)
    (s_3, a, s_0, a, R)

    (c)

    s_0

$\#$	a	b	b	a	a	b	a	b	$\sqcup$	$\sqcup$	$\sqcup$	$\cdots$

    $(s_0, \#, s_0, \#, R)$ $(s_0, a, s_1, \overline{a}, R)$ (s_0, b, s_0, b, R) $(s_0, \sqcup, h_a, \sqcup, R)$
    (s_1, a, s_1, a, R) (s_1, b, s_1, b, R) $(s_1, \sqcup, s_2, \sqcup, L)$
    (s_2, a, s_0, a, L) (s_2, b, s_3, a, L) $(s_2, \overline{a}, s_2, \overline{a}, L)$ $(s_2, \#, s_4, \#, R)$
    (s_3, a, s_3, a, L) (s_3, b, s_3, b, L) $(s_3, \overline{a}, h_a, b, R)$ $(s_3, \#, s_5, \#, R)$
    (s_4, a, s_4, a, R) (s_4, b, s_4, b, R) $(s_4, \overline{a}, h_a, a, R)$
    (s_5, a, s_5, a, R) (s_5, b, s_5, b, R) $(s_5, \overline{a}, h_a, b, R)$

11. What is the Church-Turing Thesis, and if true, what are the implications?

12. What does it mean for a problem to be Turing Recognizable? What does it mean for a problem to be Turing Decidable? What id the difference?