MATH 3350-011/012 Exam #3

Directions: Provided on this sheet are the seven problems that you need to complete for this exam. This exam is open-book and open-note, but do **NOT** use the Internet. You will find that to be a fruitless effort on several of these problems. Please **neatly** and **clearly** show **ALL** of your work. This exam is due by **11:59 PM today, 24 April 2020**. You do **NOT** need to print out this exam sheet in your submission. Work each problem on a **separate** page and scan/submit your solutions to the assignment page on Blackboard where you downloaded this exam.

Notation: The unit step function $:= u(t-a)$. The dirac delta function $:= \delta(t-t_0)$.

Prob. 1 For $F(s) = \frac{3s+\beta}{(s-2)^2}$, find $\mathcal{L}^{-1}[F(s)]$. (β is an unknown constant and not a variable.)

Prob. 2 Express $f(t) = \begin{cases} 0 & 0 \le t < \frac{\pi}{2} \\ \sin(t) & \frac{\pi}{2} \le t < \pi \\ 0 & \pi \le t \end{cases}$ in terms of only t, the unit step function, and numerical constants, as appropriate. Your expression cannot be explicitly piecewise like the above definition.

Prob. 3 For $f(t) = \sin(t) * u\left(t + \frac{\pi}{2}\right)$, find $\mathcal{L}[f(t)]$.
(Note: The $*$ in this Problem signifies multiplication, not convolution.)

Prob. 4 Using Laplace and Inverse Laplace Transforms, solve $ty'' - y' = 2t^2$, where $y(0) = 0$. Show all steps.

Prob. 5 Consider the function $f(t) = \cos(t) + \int_0^t e^{-\tau} f(t-\tau)d\tau$. Using Laplace and Inverse Laplace Transforms, express $f(t)$ in terms of only t, the unit step function, and numerical constants, as appropriate.

Prob. 6 Using Laplace and Inverse Laplace Transforms, solve $y'' + y = u(t-2\pi)$, where $y(0) = 1$ and $y'(0) = 0$. Show all steps.

Prob. 7 Using Laplace and Inverse Laplace Transforms, solve $y'' + y = \delta(t-2\pi)$, where $y(0) = 1$ and $y'(0) = 0$. Show all steps.