4330 Assignment 3 [1]

For this assignment, you need to submit two files: a Python program for Problem 1, and a PDF for Problems 2 and/or 3 (legibly handwritten and scanned to PDF, or typed - either is fine). The files should be named, respectively:

$$\boxed{\texttt{hw3-}lastname\texttt{.py}}$$

$$\boxed{\texttt{hw3-}lastname\texttt{.pdf}}$$

Please: name your files in exactly this way; lowercase 'hw', a dash (not an underscore), and **NO SPACES** in the filename!

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(1) (10 points) Suppose $y \geq 1$ is a real number, and consider the sequence $\{x_n\}$ defined by

$$
\begin{aligned}
x_0 &= 1, \\
x_n &= \frac{1}{2}\left(x_{n-1} + \frac{y}{x_{n-1}}\right), \quad \text{for } n \geq 1.
\end{aligned}
$$

Choose a value of y and compute x_3 by hand, to make sure you understand this sequence, and to give you a test case for the program you will write. Then write a Python program which does the following:

(i) Input a floating point number `y` greater or equal 1 and a positive integer `N` from the user.

(ii) Compute the `N`-th term x_N of the sequence defined above, for the given values of y and N, and print it out with 8 decimal digits. Do this using a loop and variables `prev_x` and `next_x` to represent the previous and next terms in the sequence. Do NOT use a list or other data structure to store every term in the sequence.

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(2) (10 points) Prove that if $\lim\limits_{n \to \infty} x_n$ exists, then it equals $\sqrt{y}$.

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(3) (Extra credit 10 points) Prove that $\lim\limits_{n \to \infty} x_n$ exists.