Calculus III **Review Problems for Exam II**

We'll do our 2nd Midterm on Tuesday, March 26^{th}. Tis test includes section 10.2, 10.4, 11.1, 11.2, 11.3, 11.4, 11.5, and 11.6. Here are some potential problems for the exam, but remember to practice examples from lecture notes and homework.

1. Find the position vector $\vec{R}(t)$ given the velocity $\vec{V}(t) = e^t\ \hat{\imath} + t^2\ \hat{\jmath} + cos(3t)\hat{k}$ and the initial position vector $\vec{R}(0) = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$.

2. Find the position vector $\vec{R}(t)$ and velocity vector $\vec{V}(t)$ given the acceleration vector $\vec{A}(t) = t^2\ \hat{\imath} - 2t\ \hat{\jmath} + e^{2t}\hat{k}$, the initial velocity vector$\vec{V}(t) = 2\hat{\imath} + 3\hat{k}$ and the initial position vector $\vec{R}(0) = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$.

3. Find the unit tangent $\vec{T}(t)$ and unit normal $\vec{N}(t)$ for $\vec{R}(t) = 3t\hat{\imath} + cos(2t)\hat{\jmath} + sin(2t)\hat{k}$

4. The position vector of a moving object is $\vec{R}(t) = 2t\ \hat{\imath} + 3sin(3t)\ \hat{\jmath} + (5 - 3cos(3t))\hat{k}$. Find the speed of the object at time t and compute the distance the object travels between times $t = 0$ to $t = 2$.

5. For the curve given by $\vec{R}(t) = \langle 2cos\ t, 2sin\ t, t\rangle$
   (a) Find the curvature when $t = \pi$.
   (b) Find the length of the curve from $t = 0$ to $t = \pi$.

6. Find the curvature of the plane curve $y = 2x^5 + 3x + 10$.

7. Find and sketch the domain of following functions
   (a) $f(x, y) = \sqrt{9 - x^2 - y^2}$
   (b) $f(x, y) = 1 + \sqrt{4 - y^2}$

8. Show that
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4 + y^2}$$
   does not exist.
   (Hint: consider direction $y = mx$ and $y = x^2$).

9. Use polar coordinate to find the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2 + y^2}$$

10. Determine $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$ for the following functions
    (a) $f(x, y) = x^3 + x^2y + xy^2 + y^3$
    (b) $f(x, y) = xe^{xy^3}$
    (c) $f(x, y) = \sqrt{sin(x^2y)}$ (find only f_x and f_y)

11. Find the equation of the tangent plane to the surface $z = x^3 + y^2 + sin(xy)$ at the point $(0, 2, 4)$.

12. If $z = (x + y)sin(x),\ x = 5t,\ y = 1 + t^3$, find $\frac{dz}{dt}$ using the chain rule.

13. If $z = \ln xy,\ x = e^{uv^2},\ y = e^{uv}$, find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

14. Do problem 37 of section 11.5 (page 867).

15. Find the gradient of the function $f(x, y, z) = xe^{y+z^2}$. (Practice more problems from book similar to this)

16. Find the directional derivative of the function $f(x, y) = sin(2x + 3y)$ at the point $(\pi/4, \pi/4)$ in the direction of the vector $\vec{v} = \langle 3, 4\rangle$.

17. Let $f(x, y) = 5x^2 - 2xy^3$. Find the direction in which f is increasing the fasted at the point $(1, -2)$, also find the rate of increase in that direction.

18. A skier is speeding down a mountain path. If the surface of the mountain is modeled by $z = 1 - 3x^2 - \frac{5}{2}y^2$ and the skier begins at the point $(1/4, -1/2, 3/16)$, in what direction should the skier head to descend the mountainside most rapidly?

19. Find equations for the tangent plane and the normal line at the point $P_0(1, -1, 2)$ on the surface given by $x^2y + y^2z + z^2x = 5$.

20. Find equations for the tangent plane and the normal line at the point $P_0(2, -1, 3)$ on the surface given by $ze^{x+2y} = 3$.