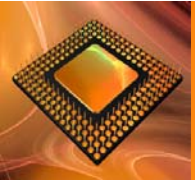


# Modern Digital System Design

ECE 2372 / Fall 2018 / Lecture 07

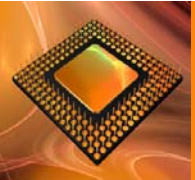
Texas Tech University  
Dr. Tooraj Nikoubin

Quine-McClusky Method



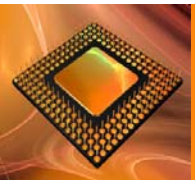
# Quine-McClusky Method

- Phase I : finding Pis  
–Tabular methods: Grouping and combining
- Phase II: Covers minimal PIs



# Quine-McClusky Method

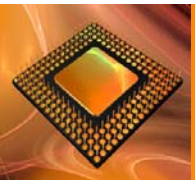
- KMAP methods was practical for at most 6 variable functions
- Larger number of variables: need method that can be applied to computer based minimization
- **Quine-McCluskey** method



# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

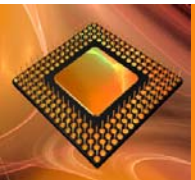
	A	B	C	D
0	0	0	0	0



# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

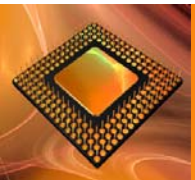
	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0



# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

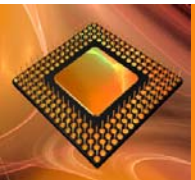
	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0



# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1

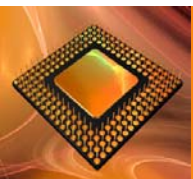


# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1





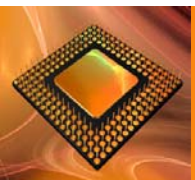
# Quine-McClusky Method

$$F = \sum m(0, 1, 2, 4, 6, 7, 9, 10, 12, 15)$$

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
				D

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	0	1	0	0
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

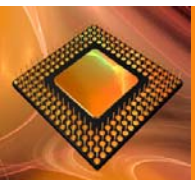


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			D

	A	B	C	D
0,1	0	0	0	-

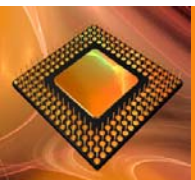


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	D			
	B			

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0

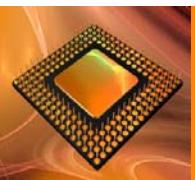


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	D			

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0

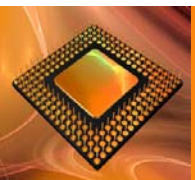


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	C			
	D			

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1

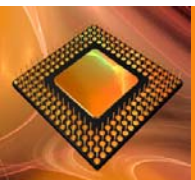


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
				D

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0

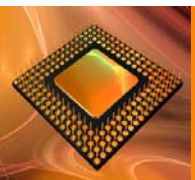


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	D			
C				

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0



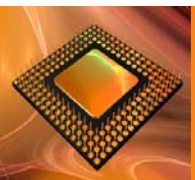
# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	D			
C				

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0



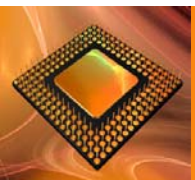


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0

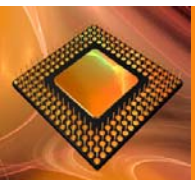


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-

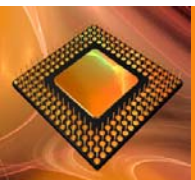


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

		A			
		00	01	11	10
C	00	1	1	1	
	01	1			1
	11		1	1	
	10	1	1		1
		B			

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

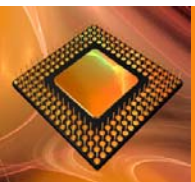


# Quine-McClusky Method

	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
4	0	1	0	0
6	0	1	1	0
9	1	0	0	1
10	1	0	1	0
12	1	1	0	0
7	0	1	1	1
15	1	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	C			
	D			

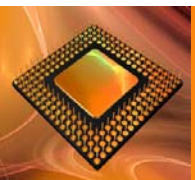
	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1



# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	D			
C				

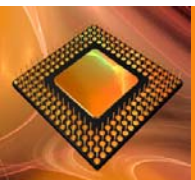


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	C			
	D			

	A	B	C	D
0,2,4,6	0	-	-	0

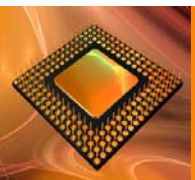


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
				D

	A	B	C	D
0,2,4,6	0	-	-	0



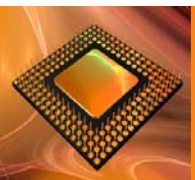
# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,2,4,6	0	-	-	0



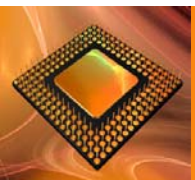


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,2,4,6	0	-	-	0

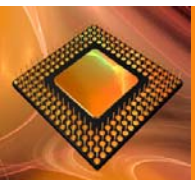


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	C			
	D			

	A	B	C	D
0,2,4,6	0	-	-	0

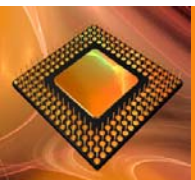


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
	B			
	D			

	A	B	C	D
0,2,4,6	0	-	-	0

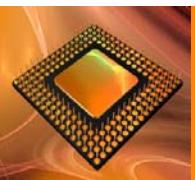


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,2,4,6	0	-	-	0

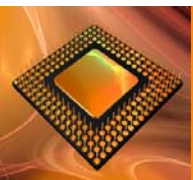


# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A			
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1

	A	B	C	D
0,2,4,6	0	-	-	0

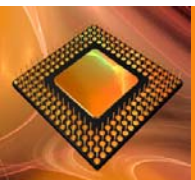


# Quine-McClusky Method

	A	B	C	D	
0,1	0	0	0	-	PIA
0,2	0	0	-	0	
0,4	0	-	0	0	
1,9	-	0	0	1	PIB
2,6	0	-	1	0	
2,10	-	0	1	0	PIC
4,6	0	1	-	0	
4,12	-	1	0	0	PID
6,7	0	1	1	-	PIE
7,15	-	1	1	1	PIF

	A	B	C	D
0,2,4,6	0	-	-	PIG

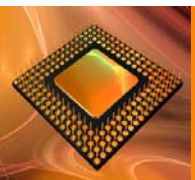
			A	
	00	01	11	10
00	1	1	1	
01	1			1
11		1	1	
10	1	1		1
			B	
				D



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15

0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG

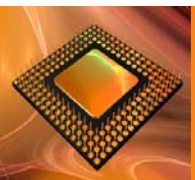


# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								

0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG

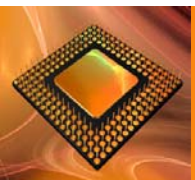




# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			

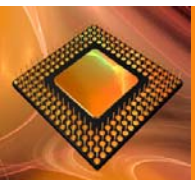
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			
PIC			+					+		

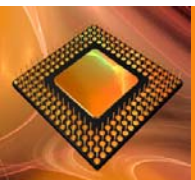
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			
PIC			+					+		
PID				+					+	

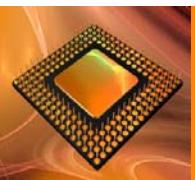
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			
PIC			+					+		
PID				+					+	
PIE					+	+				

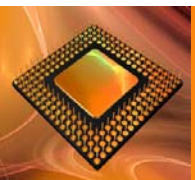
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			
PIC			+					+		
PID				+					+	
PIE					+	+				
PIF						+				+

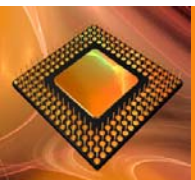
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15
PIA	+	+								
PIB		+					+			
PIC			+					+		
PID				+					+	
PIE					+	+				
PIF						+				+
PIG	+		+	+	+					

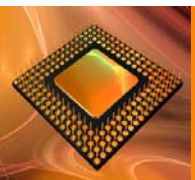
0,1	PIA
0,2	
0,4	
1,9	PIB
2,6	
2,10	PIC
4,6	
4,12	PID
6,7	PIE
7,15	PIF
0,2,4,6	PIG



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15	
PIA	+	+									
PIB		+					+				Main
PIC			+					+			
PID				+					+		
PIE					+	+					
PIF						+				+	
PIG	+		+	+	+						

OUT = B

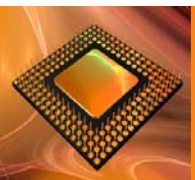


# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15	
PIA	+	+									
PIB		+					+				Main
PIC			+					+			Main
PID				+					+		
PIE					+	+					
PIF						+				+	
PIG	+		+	+	+						

$$\text{OUT} = B + C$$

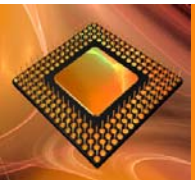




# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15	
PIA	+	+									
PIB		+					+				Main
PIC			+					+			Main
PID				+					+		Main
PIE					+	+					
PIF						+				+	
PIG	+		+	+	+						

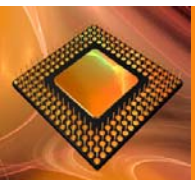
$$\text{OUT} = B + C + D$$



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15	
PIA	+	+									
PIB		+					+				Main
PIC			+					+			Main
PID				+					+		Main
PIE					+	+					
PIF						+				+	Main
PIG	+		+	+	+						

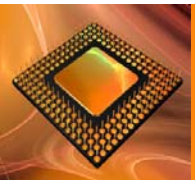
$$OUT = B + C + D + F$$



# Quine-McClusky Method

	0	1	2	4	6	7	9	10	12	15	
PIA	+	+									
PIB		+					+				Main
PIC			+					+			Main
PID				+					+		Main
PIE					+	+					
PIF						+				+	Main
PIG	+		+	+	+						

$$OUT = B + C + D + F + (E \text{ OR } G) + (A \text{ OR } G)$$



# Quine-McClusky Method

	A	B	C	D
0,1	0	0	0	-
0,2	0	0	-	0
0,4	0	-	0	0
1,9	-	0	0	1
2,6	0	-	1	0
2,10	-	0	1	0
4,6	0	1	-	0
4,12	-	1	0	0
6,7	0	1	1	-
7,15	-	1	1	1

	A	B	C	D
0,2,4,6	0	-	-	0

$$Y = F + D + C + B + \left\{ \begin{matrix} E \\ G \end{matrix} \right\} + \left\{ \begin{matrix} A \\ G \end{matrix} \right\}$$

$$Y = BCD + B\bar{C}\bar{D} + \bar{B}C\bar{D} + \bar{B}\bar{C}D + \bar{A}\bar{D}$$



**Thank You**