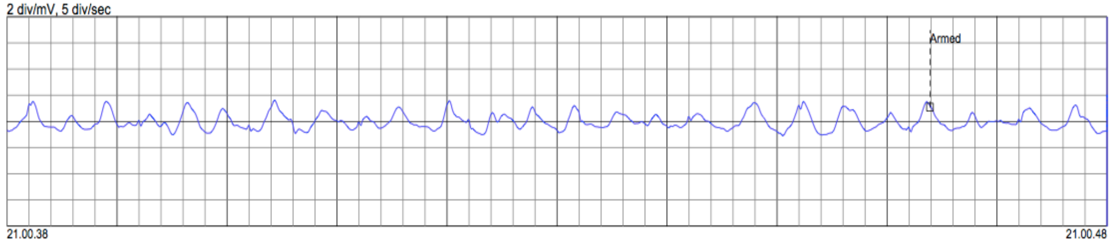
A classifier with guaranteed specificity and sensitivity

Algo Carè,

in collaboration with
Marco C. Campi & Federico A. Ramponi
University of Brescia (Italy)

and many others

A difficult decision



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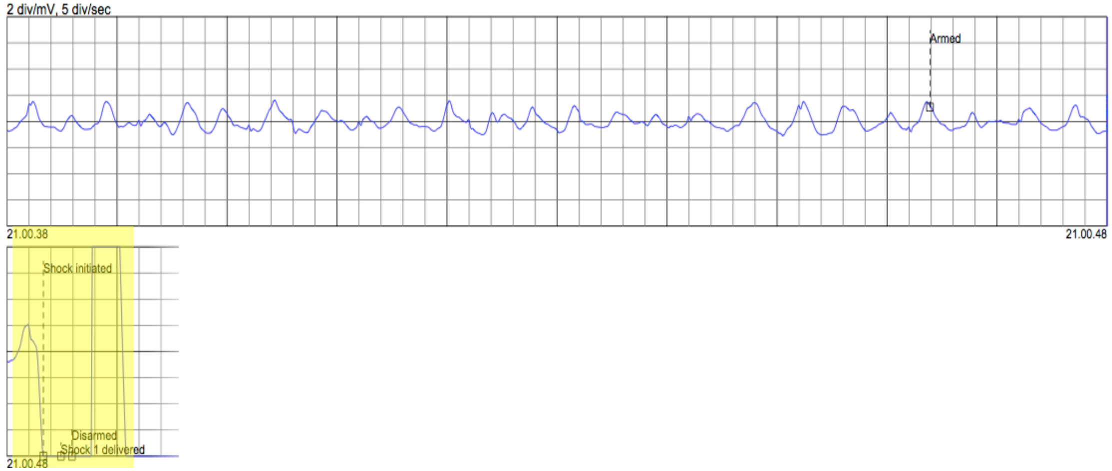

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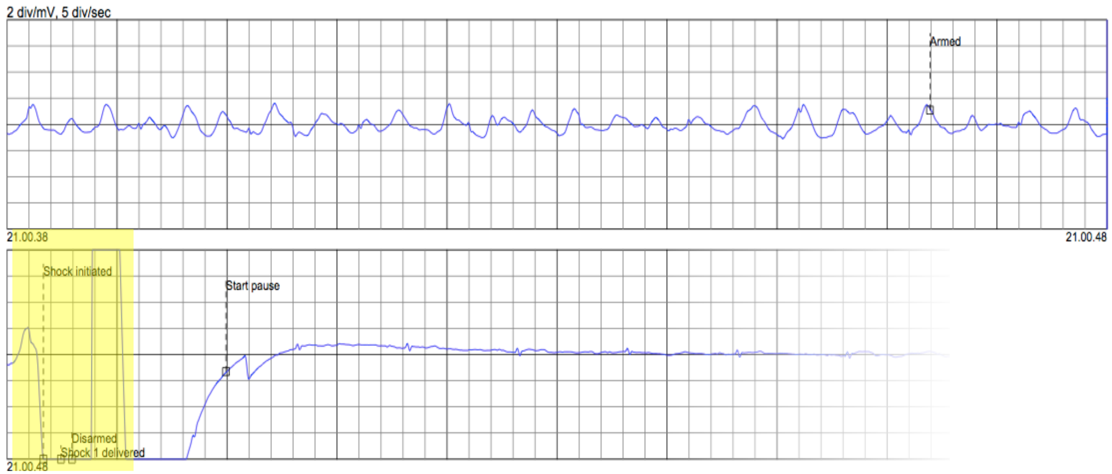

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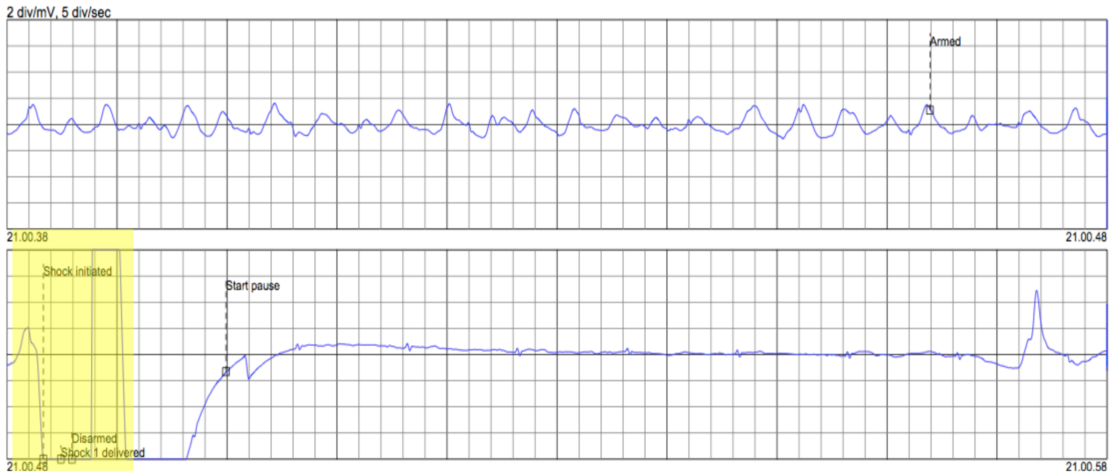

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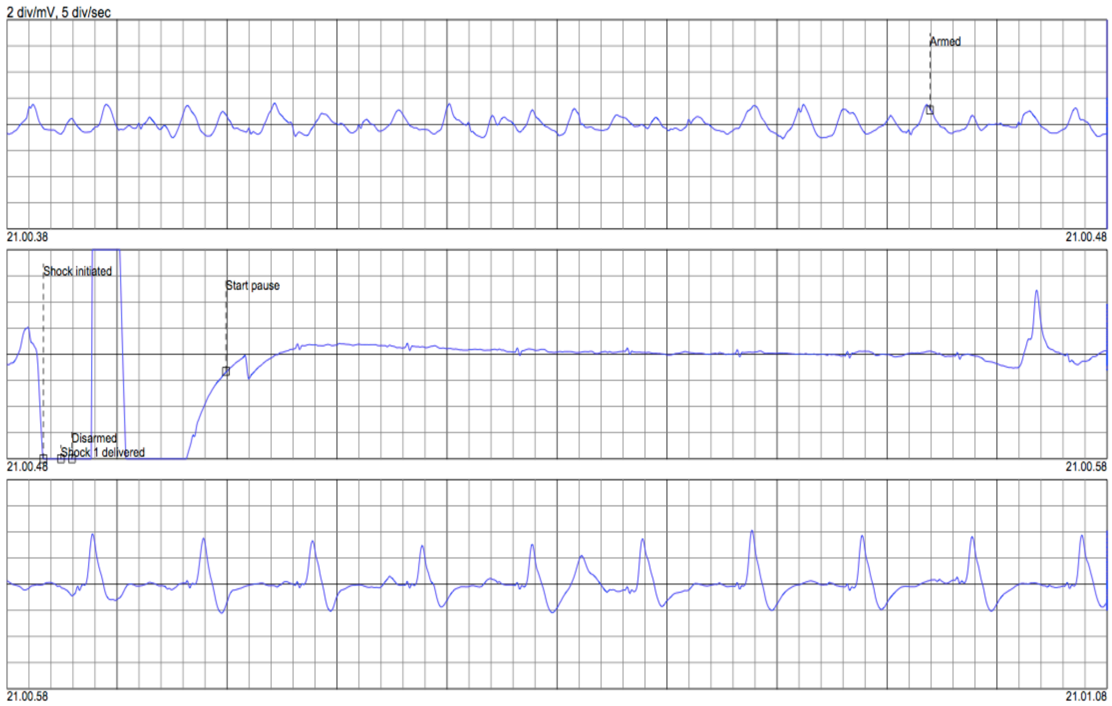

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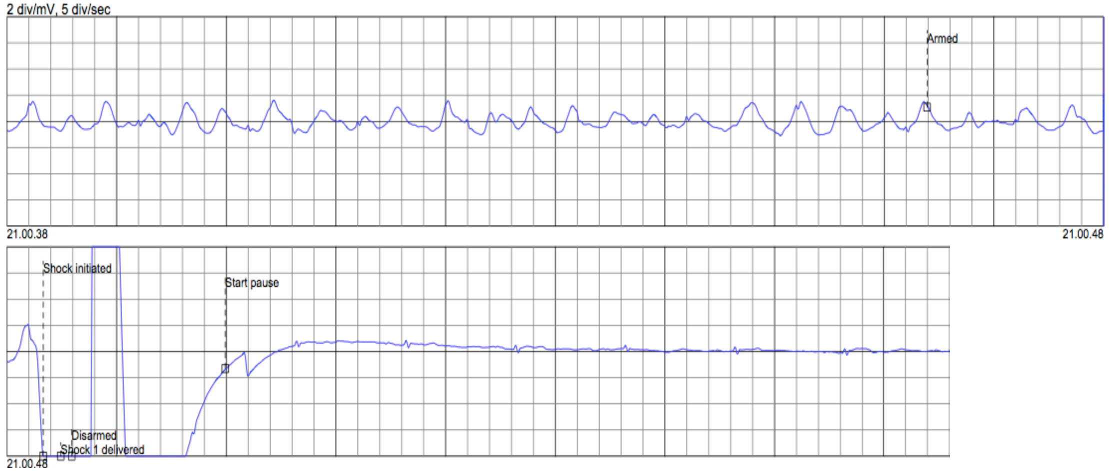

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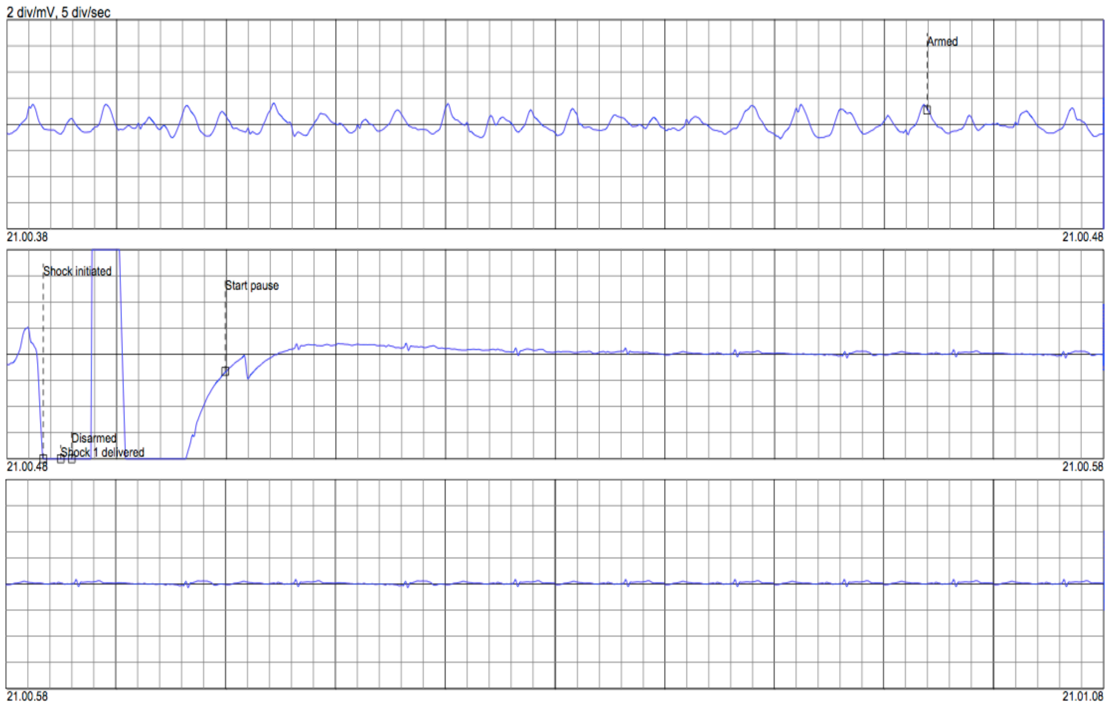

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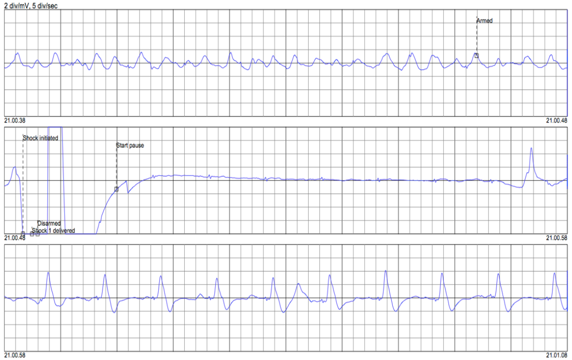

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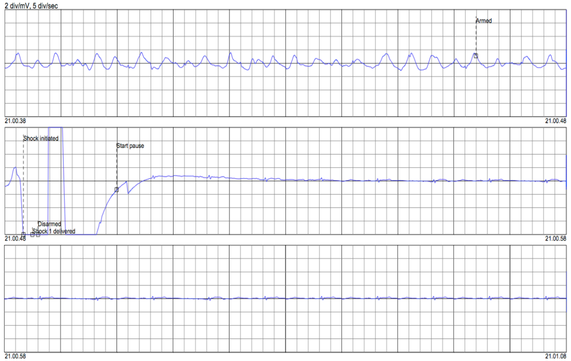
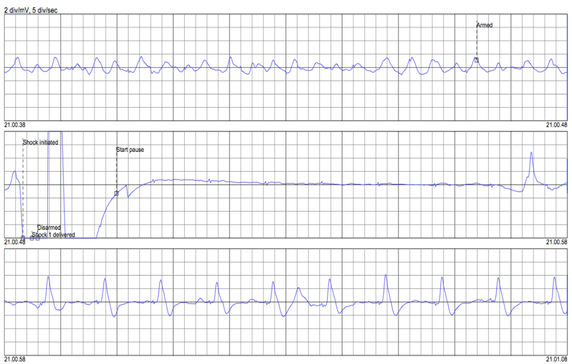


A difficult decision

"each minute of delay before defibrillation
reduces the probability of survival
to discharge by 10-12%"

European resuscitation council guidelines for resuscitation (Resuscitation, 95:100–147, 2015).



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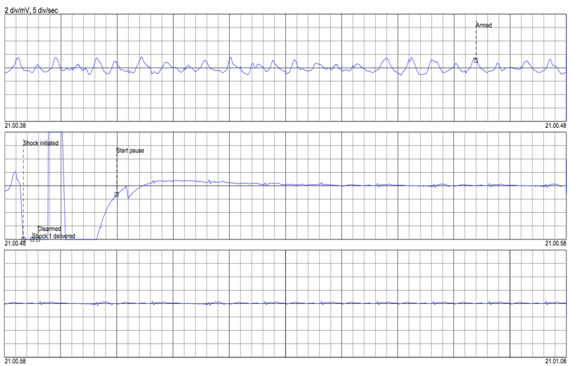

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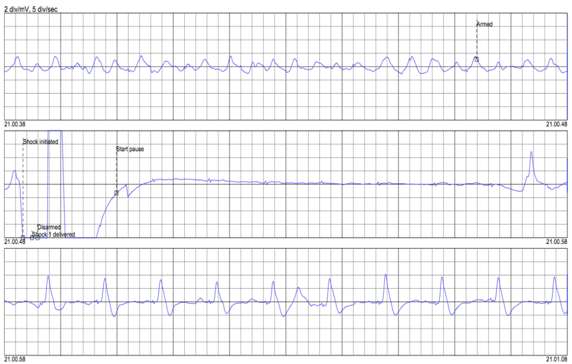

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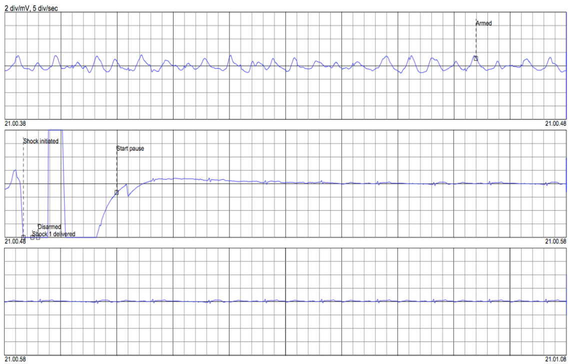
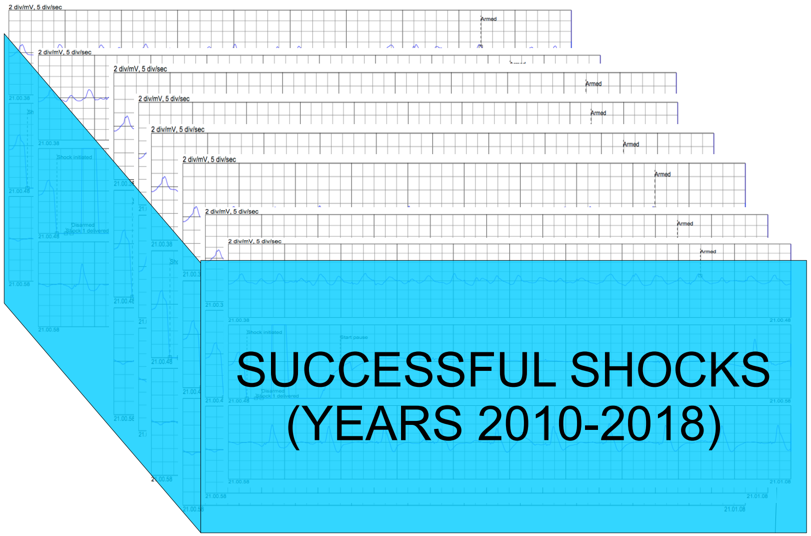
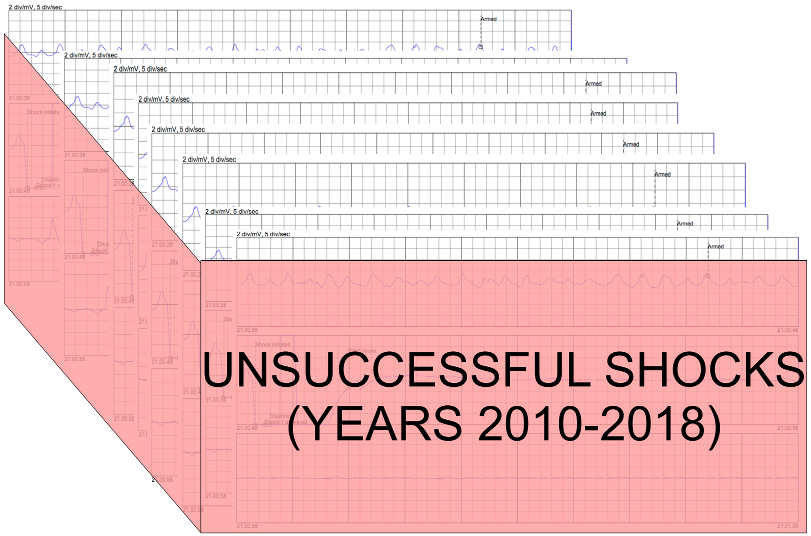


high energy shocks can cause damage
(myocardial injury)

Learning from experience





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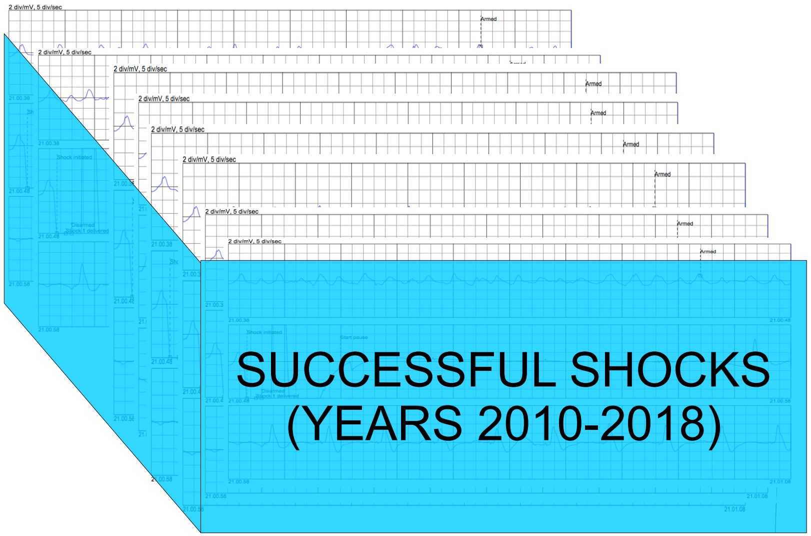
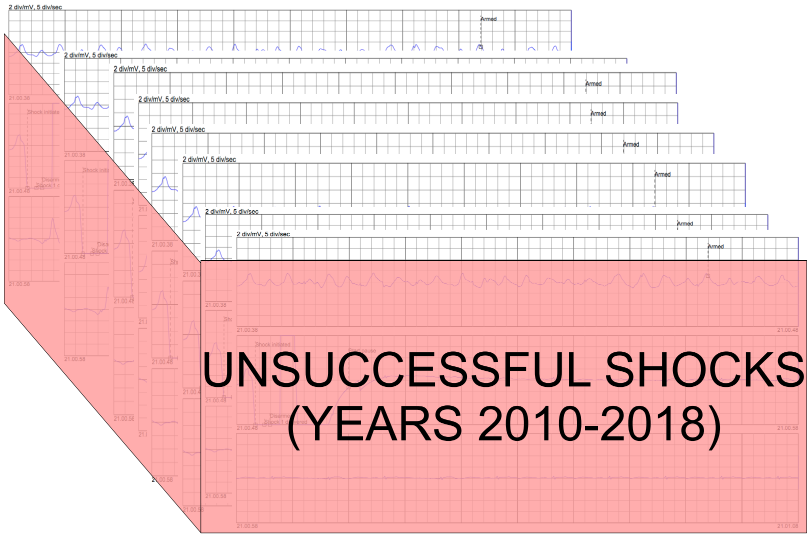
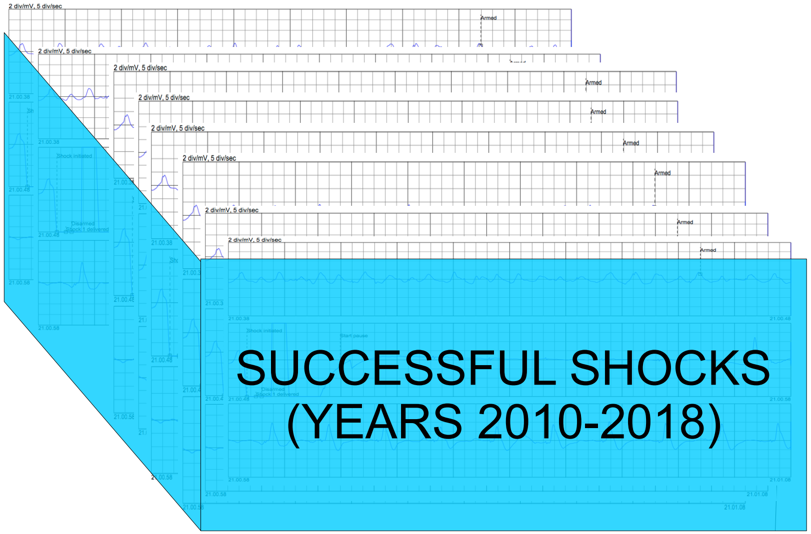
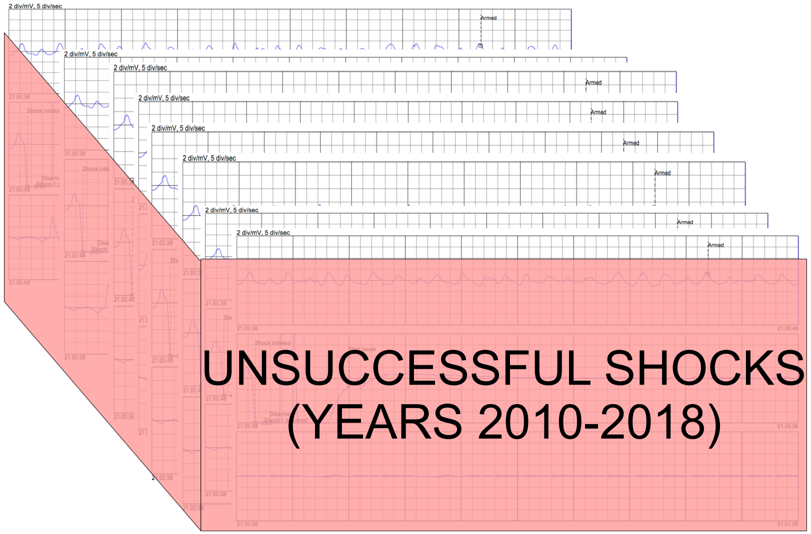




PAST

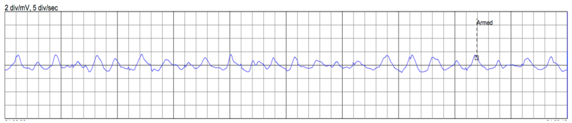
Learning from experience





PAST

PRESENT



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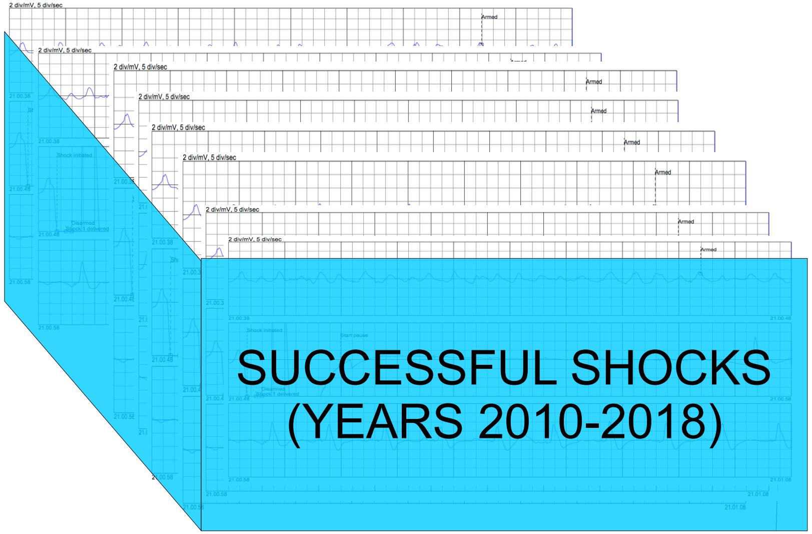

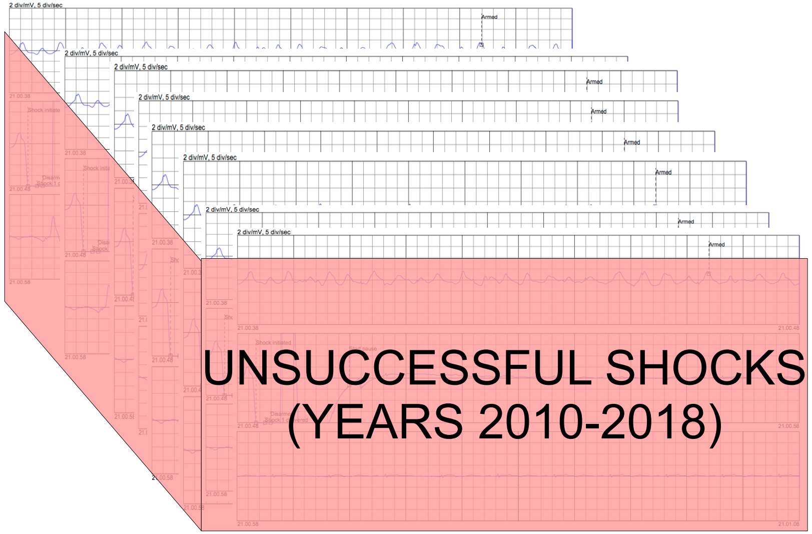

PAST

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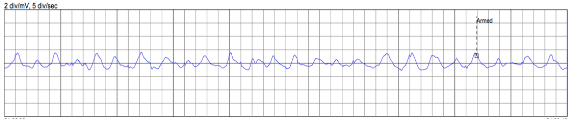
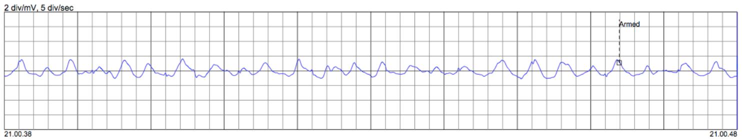


?

SUCCESS

FAILURE

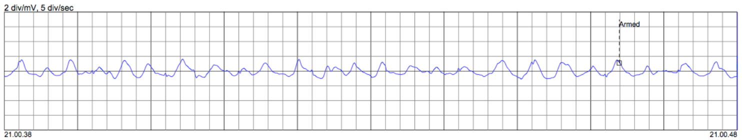
Feature extraction



- PTT (peak to peak amplitude)
- Amax (maximum amplitude)
- Amin (minimum amplitude)
- RMS (root mean square)

- DF (dominant frequency)
- CF (spectral centroid frequency)
- EF (edge frequency)
- AMSA (amplitude spectral area)

Feature extraction



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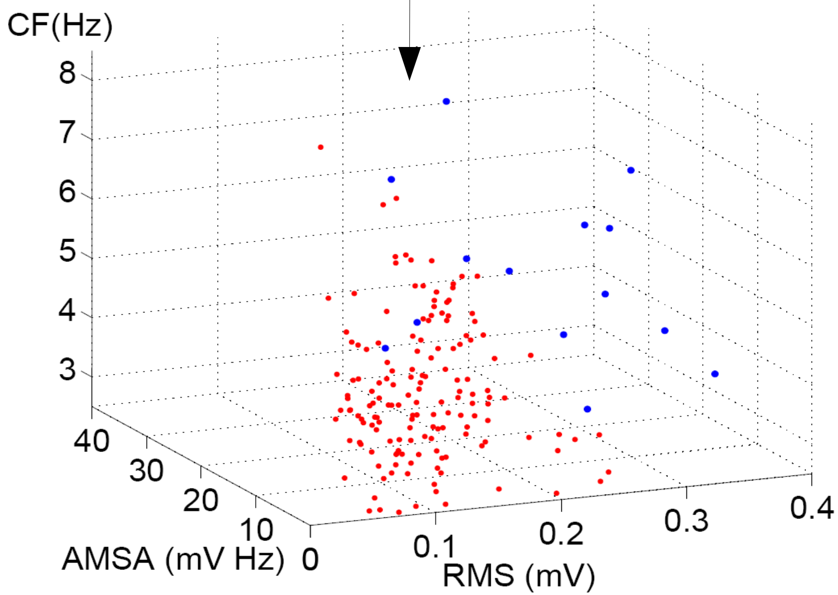
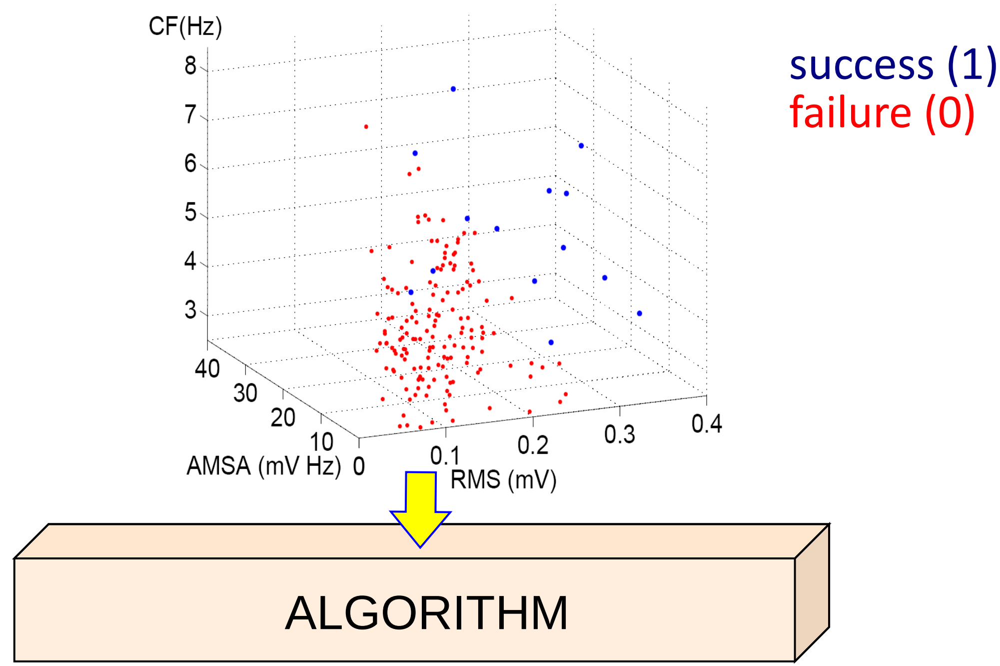
success (1)
failure (0)



Automatic learning



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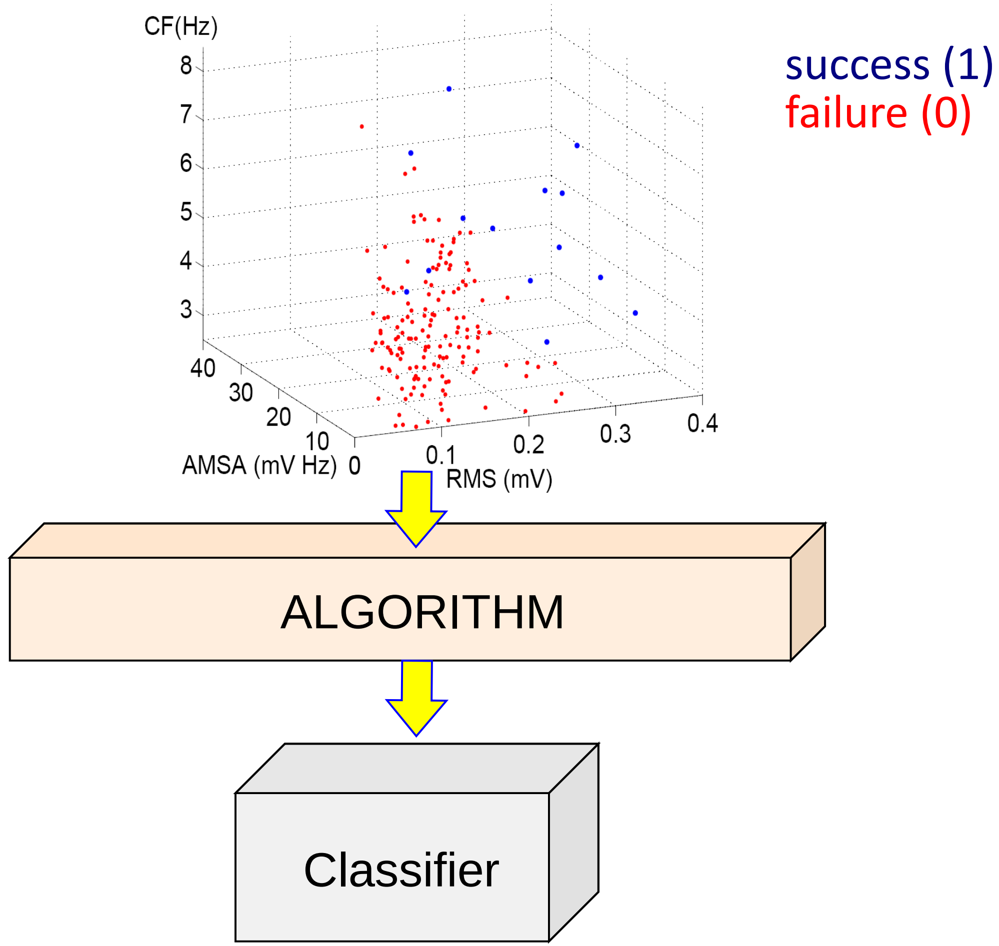

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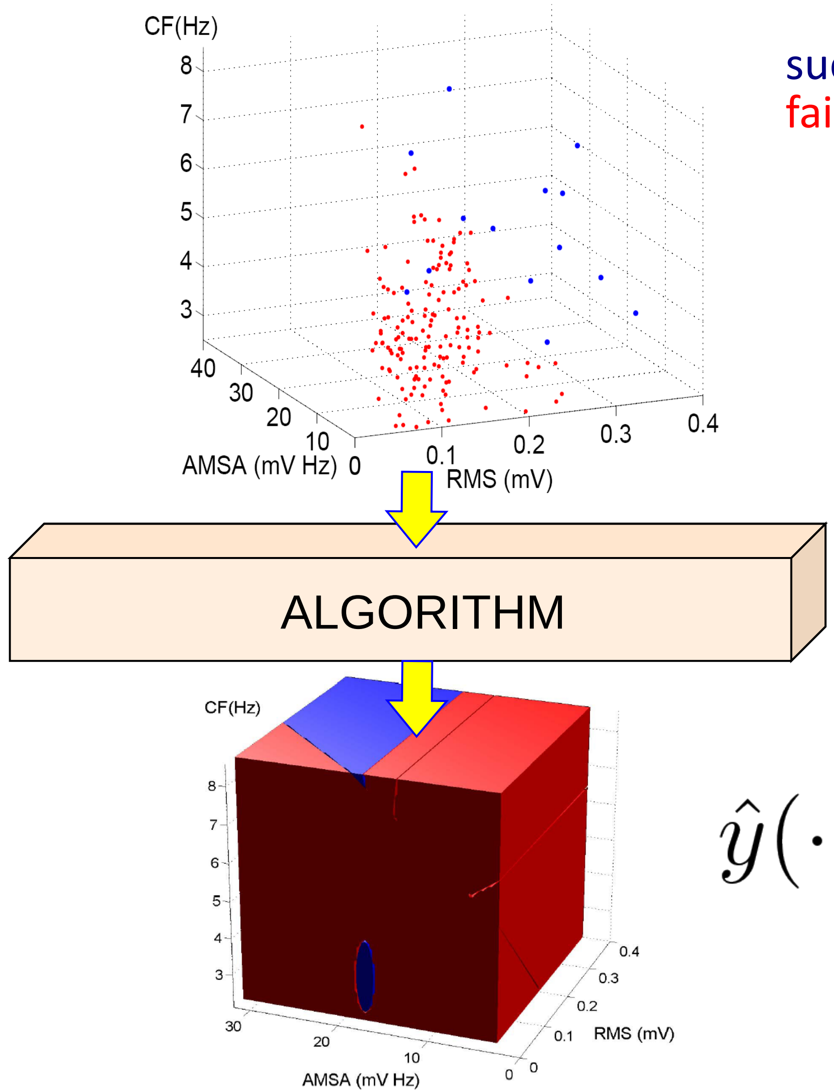

success (1)
failure (0)

$$\hat{y}(\cdot) : \mathbb{R}^3 \to \{1, 0\}$$

Mathematical set-up

$\mathrm{x} \in \mathbb{R}^n$: patient ($n$ features)

$y \in \{1, 0\}$: true outcome

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$(x, y) \sim \mathbb{P}$

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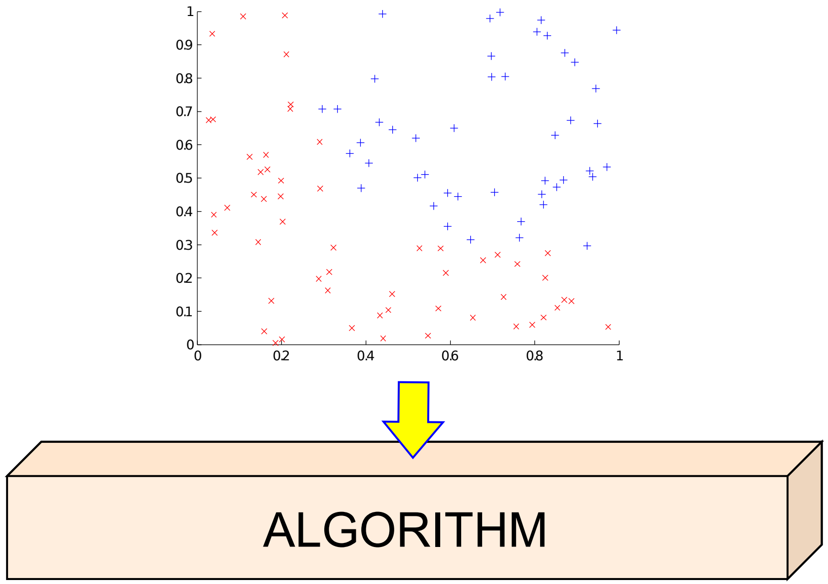
$$PE(\hat{y}) := \mathbb{P}\{\hat{y}(x) \neq y\}$$

Classification through cvx optimisation



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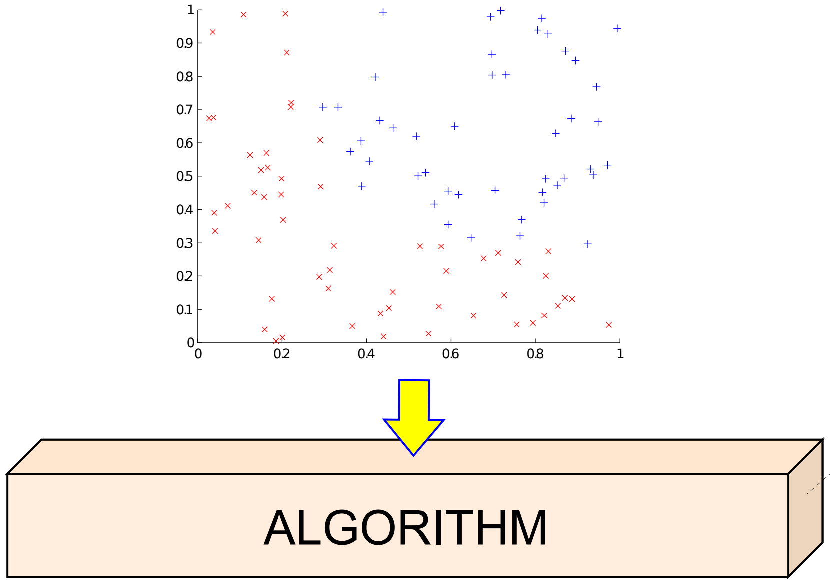
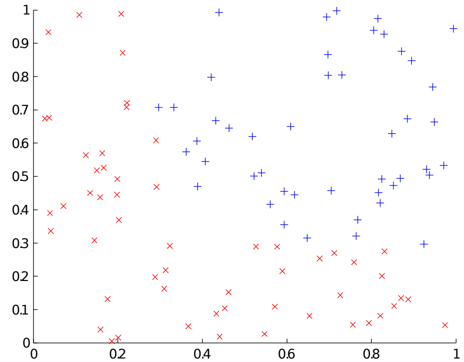
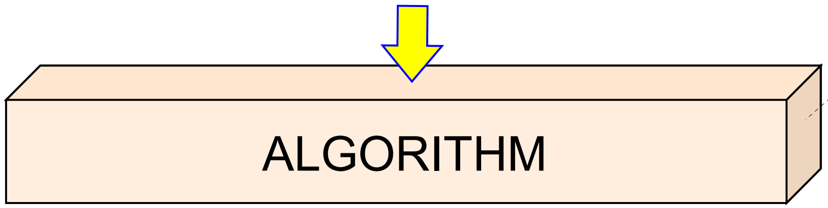
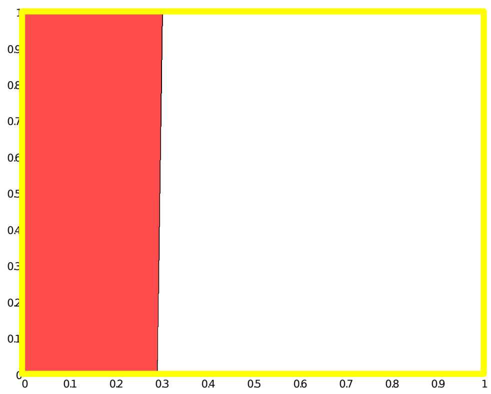


Sequence of
convex optimisation
programs

Classification through cvx optimisation



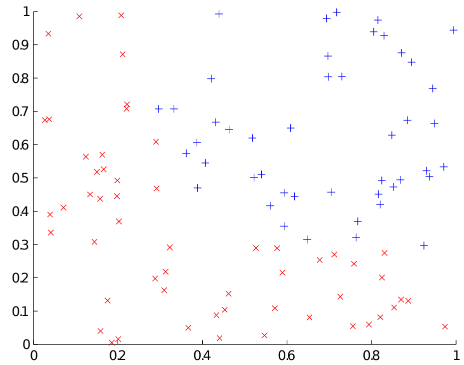


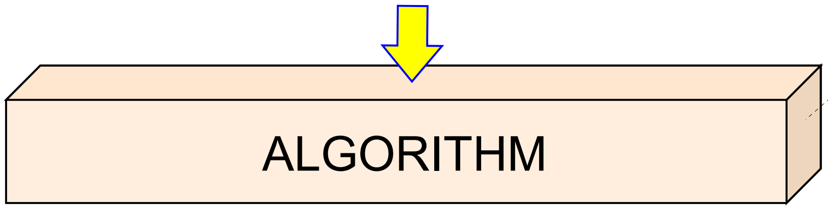


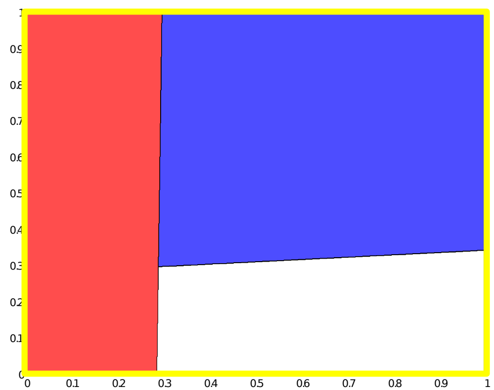
Sequence of convex optimisation programs

$$
\begin{aligned}
&\min_{A=A^T\in\mathbb{R}^{d\times d},\, b\in\mathbb{R}^d} && \mathit{Trace}(A)\\
&\text{subject to:} && (x_i - x_B)^T A (x_i - x_B) + b^T (x_i - x_B) \geq 1,\\
& && \quad \text{for all } i \in P \text{ such that } y_i \neq y(x_B)\\
& && \text{and } A \succeq 0 \quad (A \text{ positive semi-definite}).
\end{aligned}
$$

Classification through cvx optimisation







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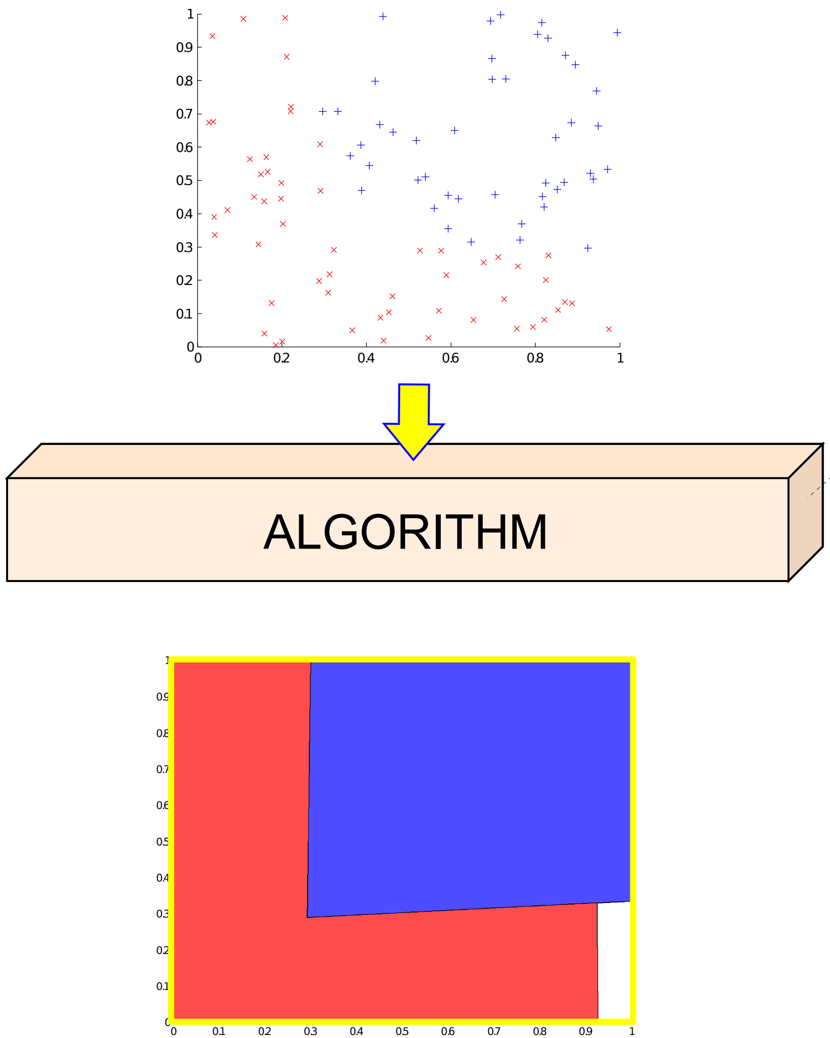

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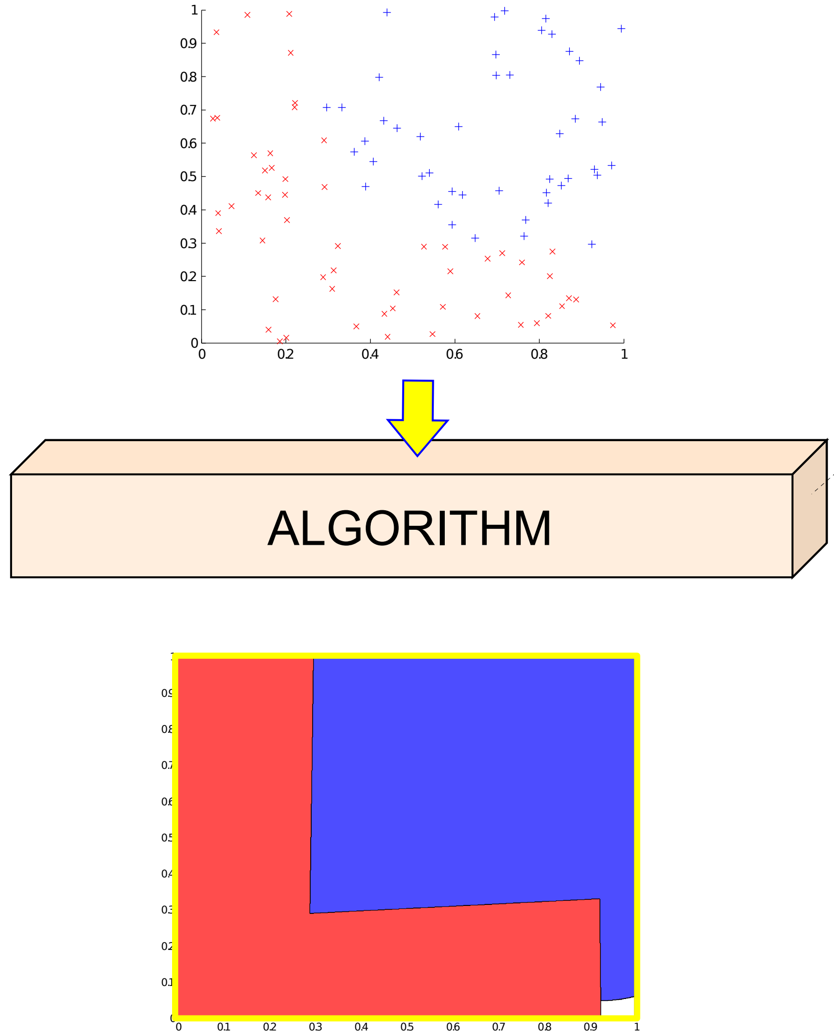

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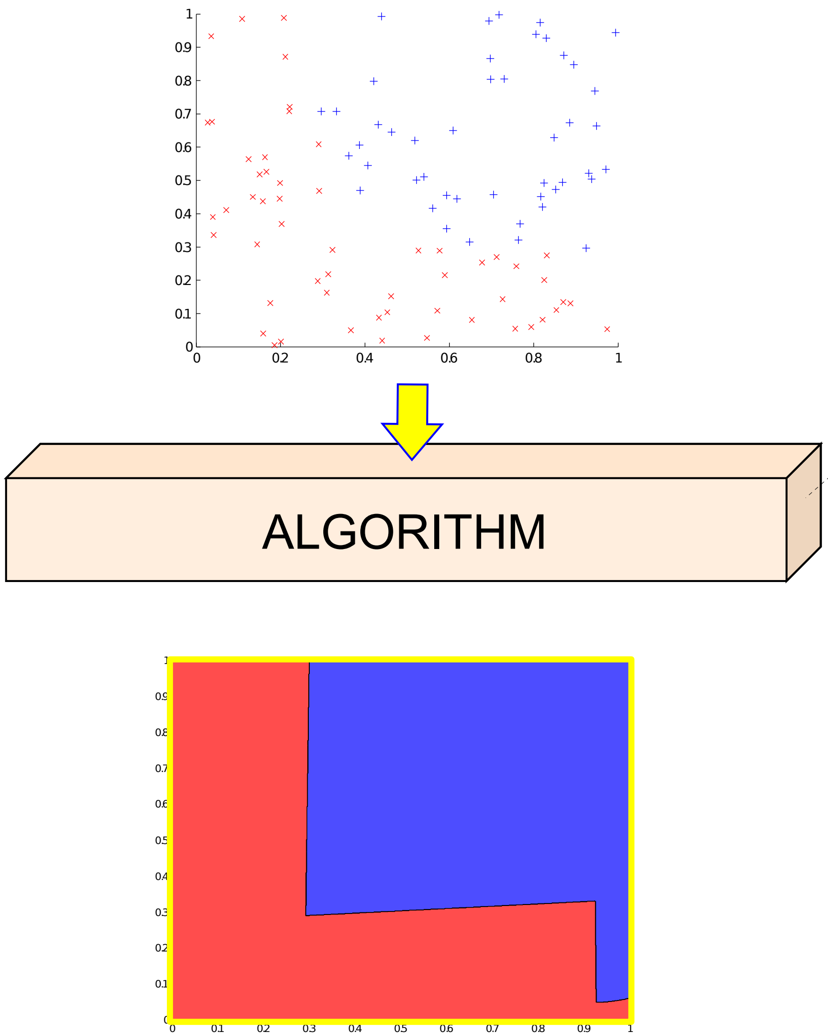

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\end{aligned}$$

•

•

•

$$\begin{aligned}
&\min_{a\geq 0,\, b\in\mathbb{R}^d} \; a\\
&\text{subject to:} \quad a\cdot\|x_i - x_B\|^2 + b^T(x_i - x_B) \geq 1,\\
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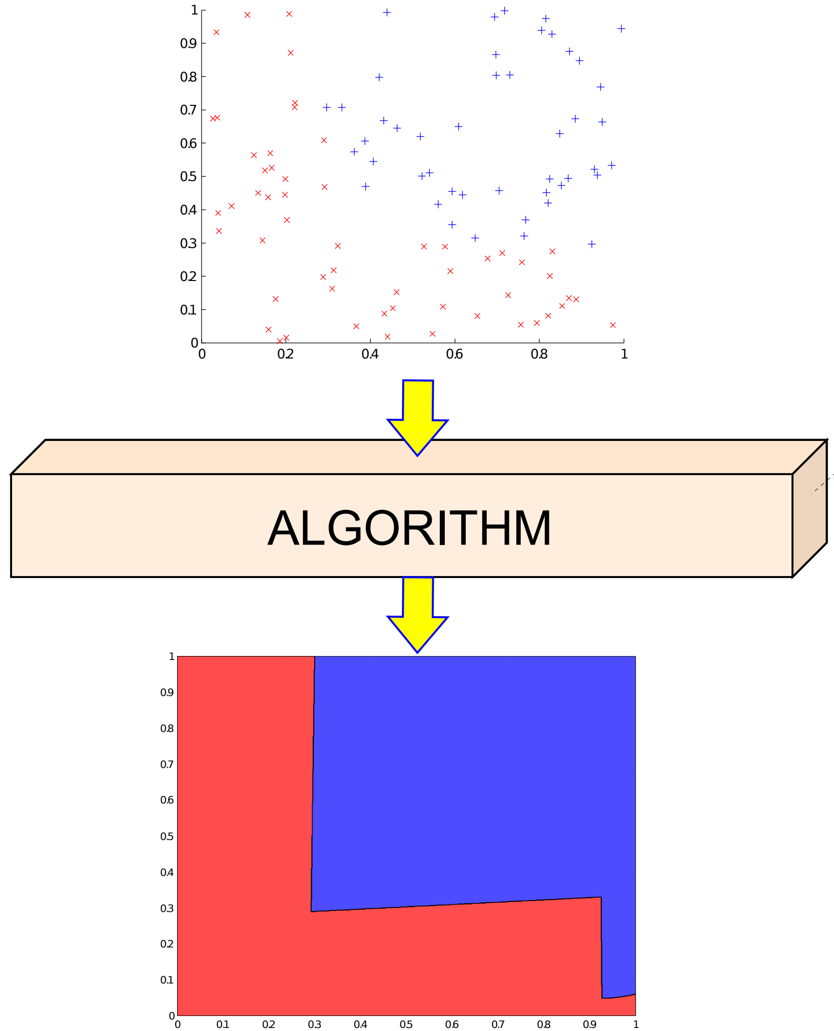
Classification through cvx optimisation



Sequence of
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The theory behind

Theorems for
data-driven optimisation problems

The theory behind

Theorems for
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Classification with

guaranteed probability of error

The theory behind

Theorems for

data-driven optimisation problems

↓

Classification with

user-chosen

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The theory behind

Theorems for
data-driven optimisation problems

Classification with
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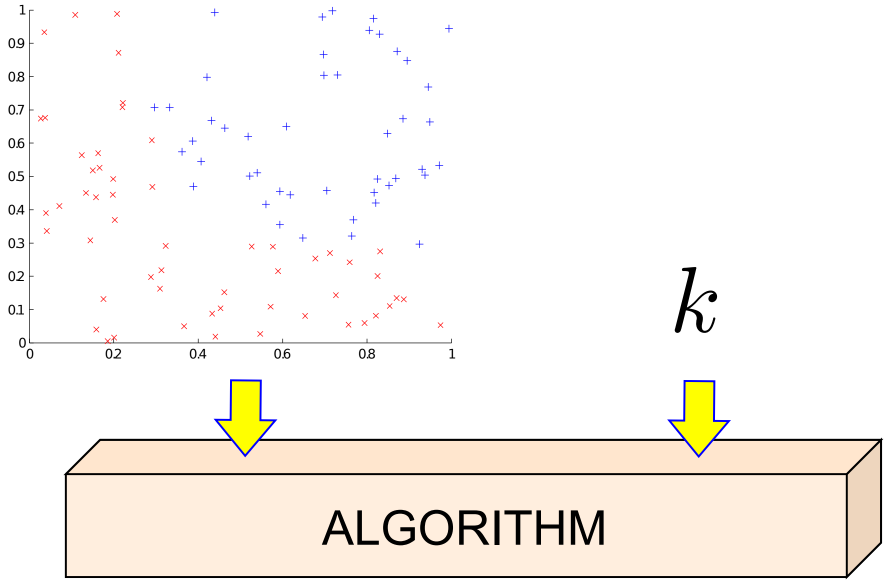
$$\Pr\{\hat{y}(x) \neq y\} \leq 10\%$$

A complexity parameter



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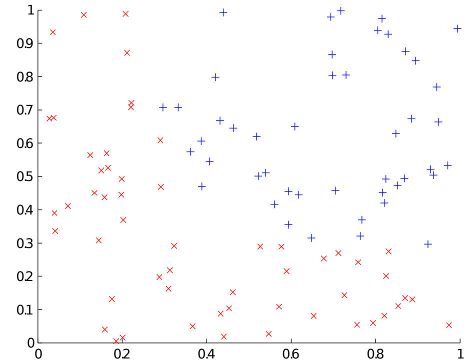
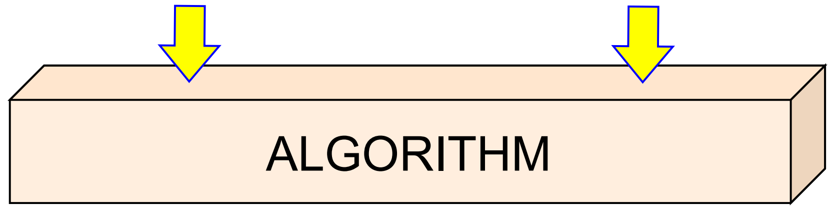


$$k = 5$$



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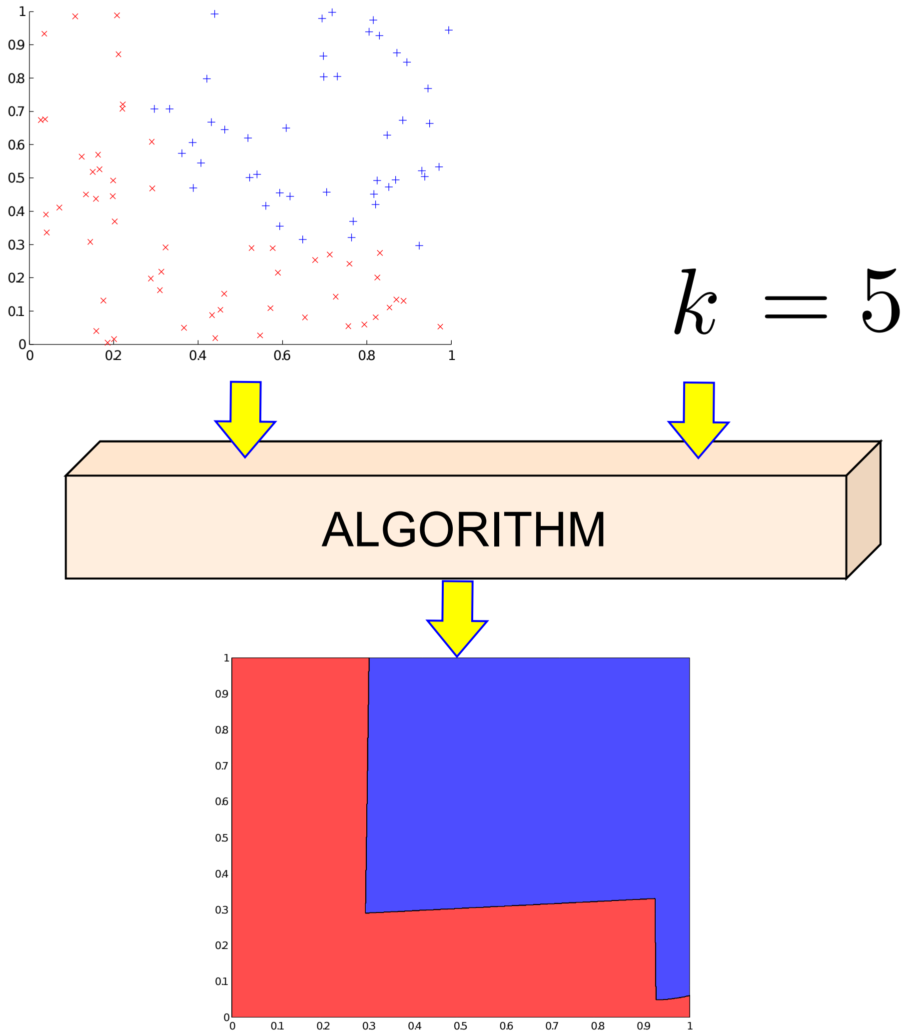

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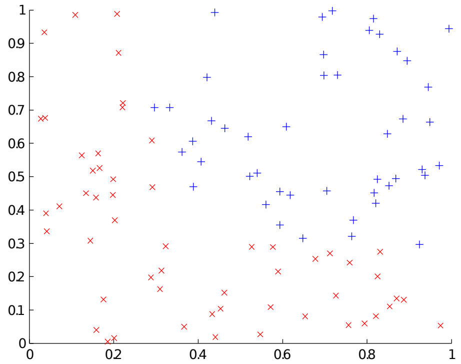
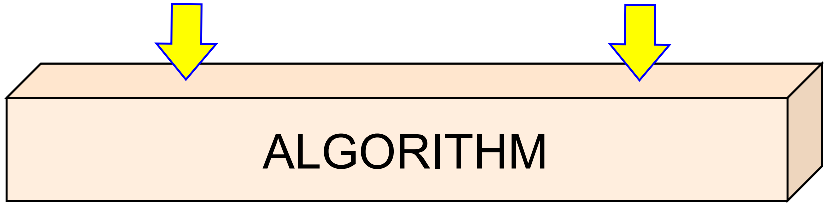


$$k < 5$$



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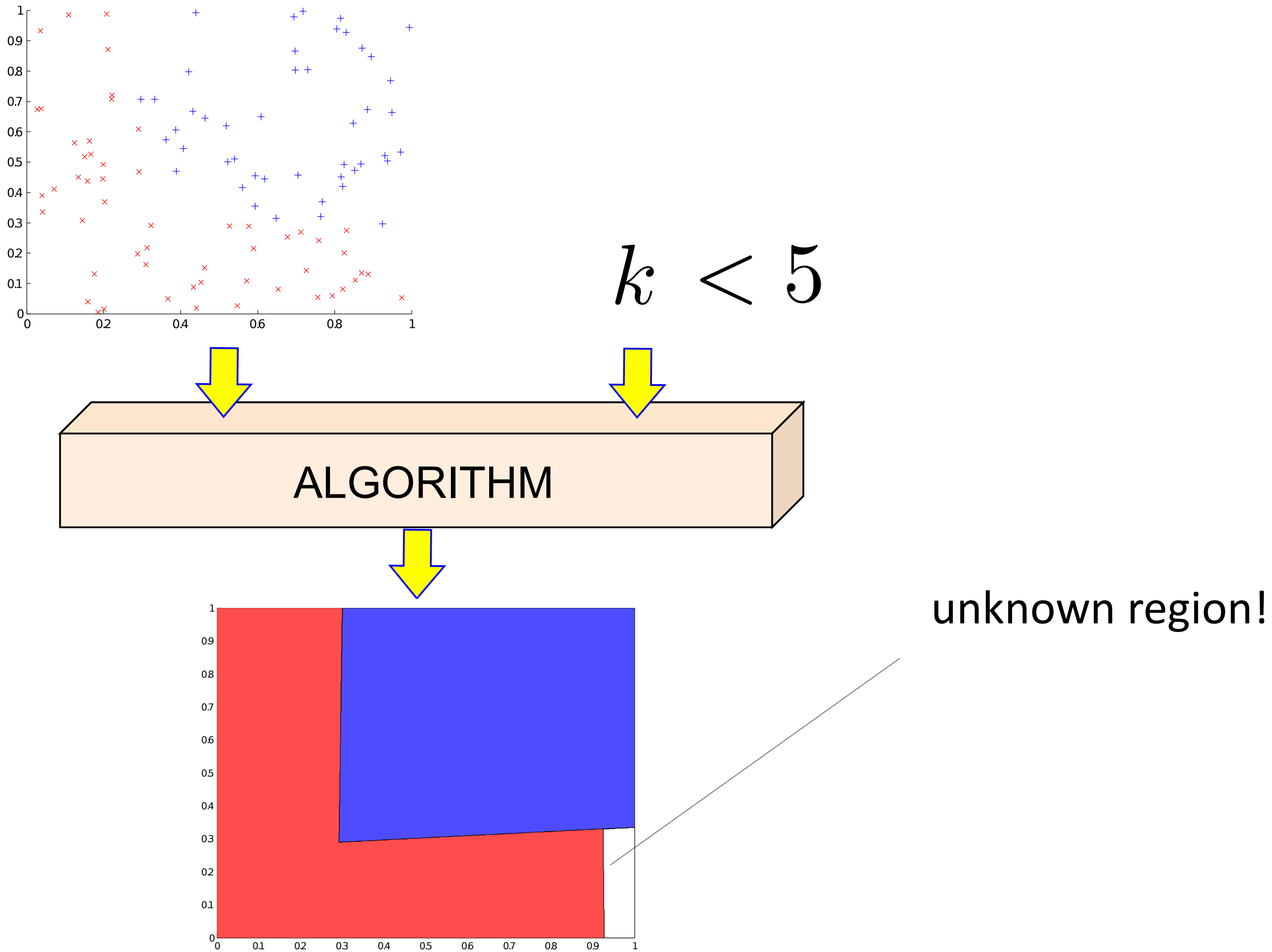

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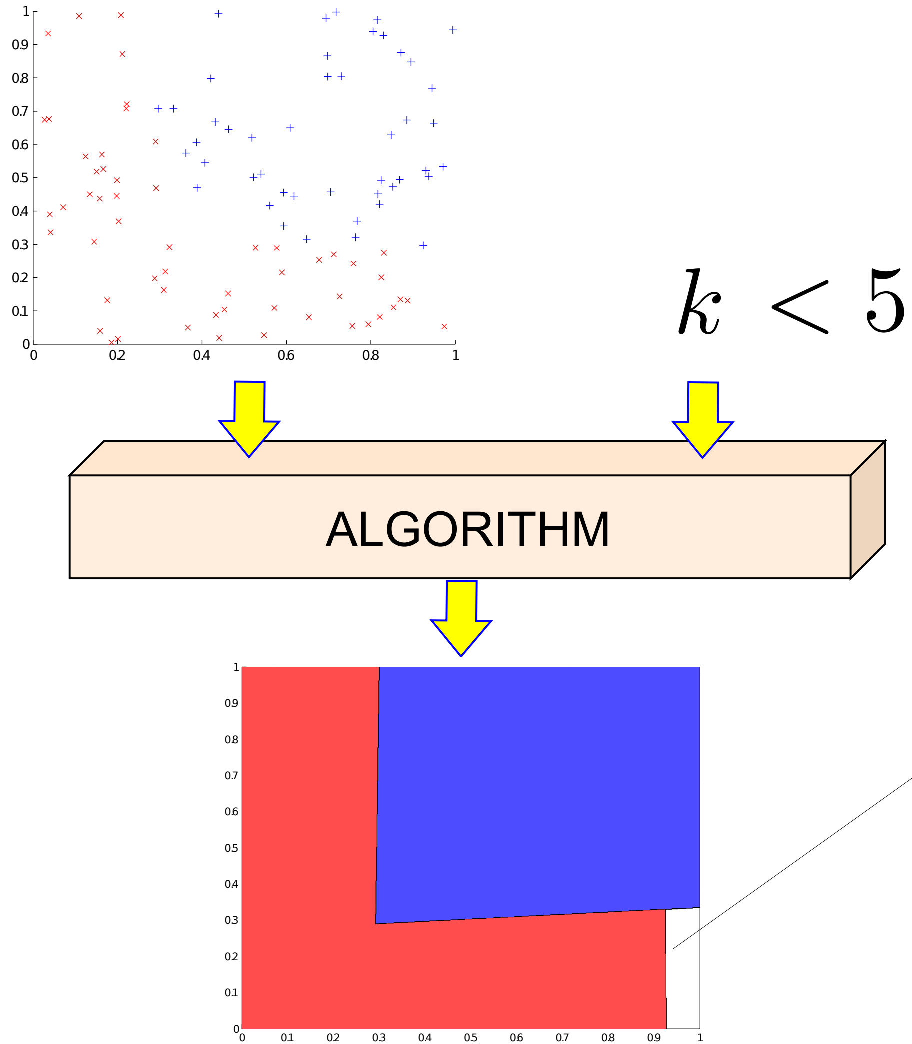

unknown region!

see also Theorem 7.1 in
G. Lugosi, L. Devroye, L. Gyorfi:
”A Probabilistic
Theory of Pattern
Recognition”, 1996.

Classification with Guarantees

N: size of the training set

k: complexity parameter

$\downarrow$

determine the

probability of error

Classification with Guarantees

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$\downarrow$

determine the

(distribution of the)

probability of error

Classification with Guarantees

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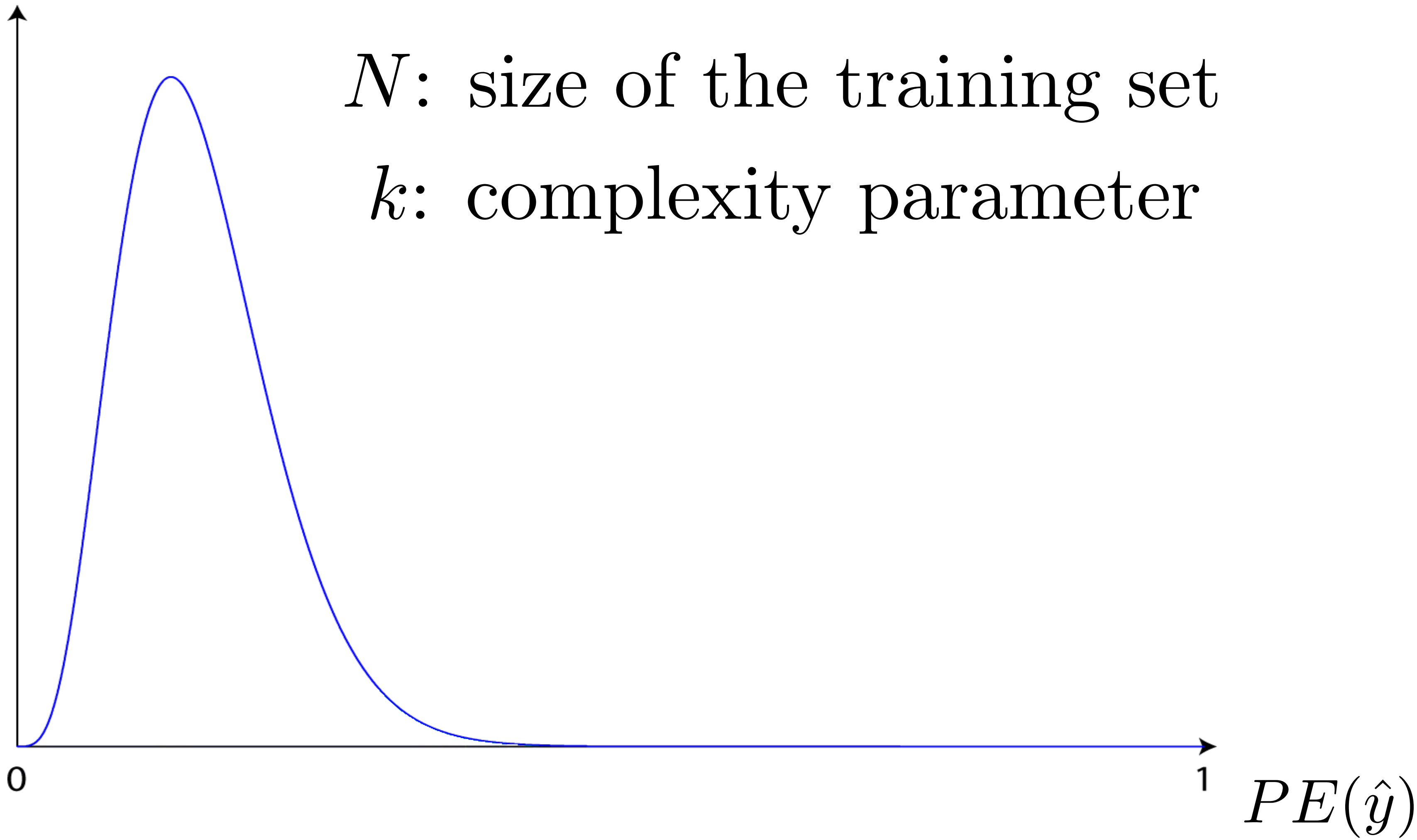
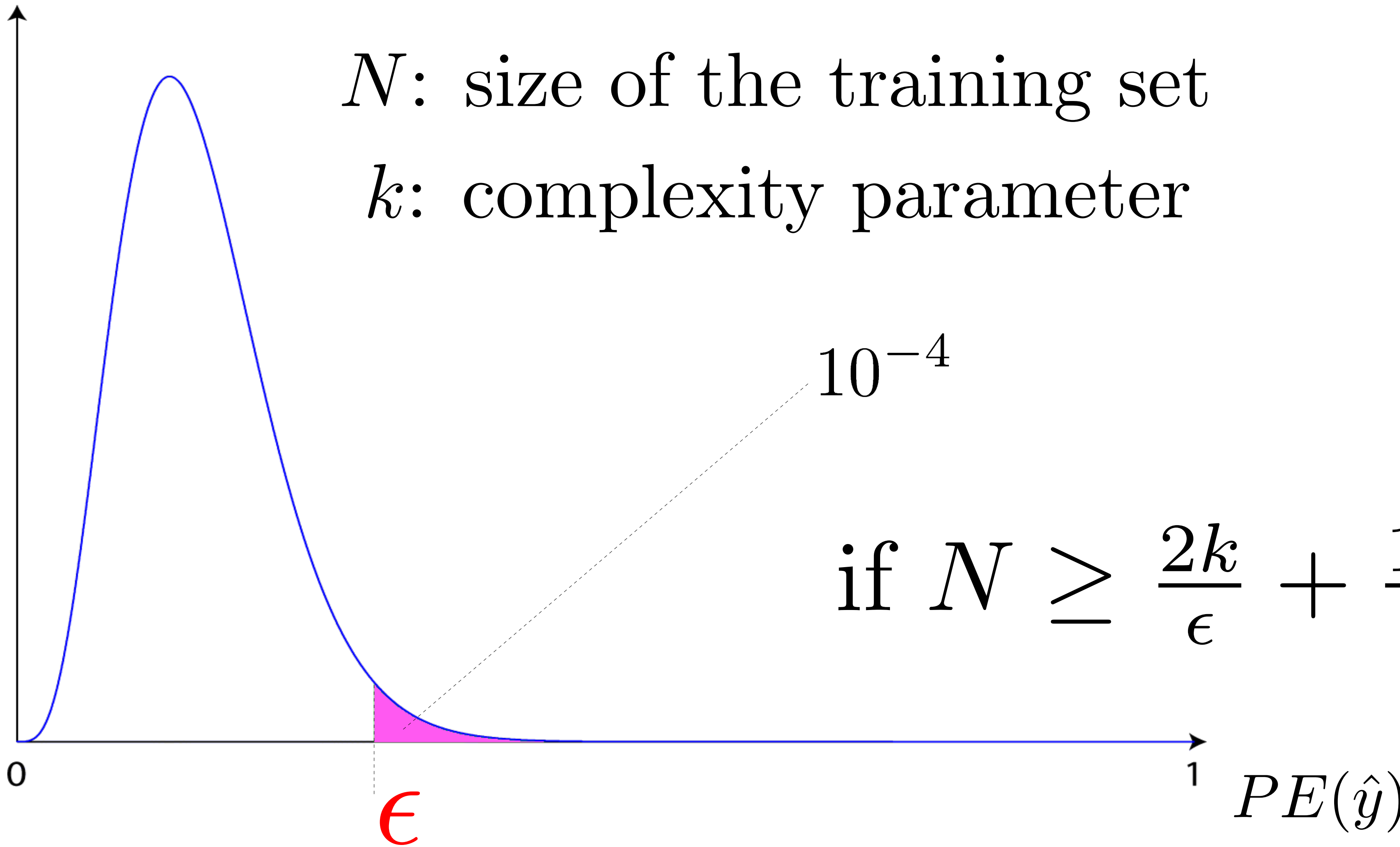


Classification with Guarantees



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$$\text{if } N \geq \frac{2k}{\epsilon} + \frac{19}{\epsilon}$$

Classification with Guarantees

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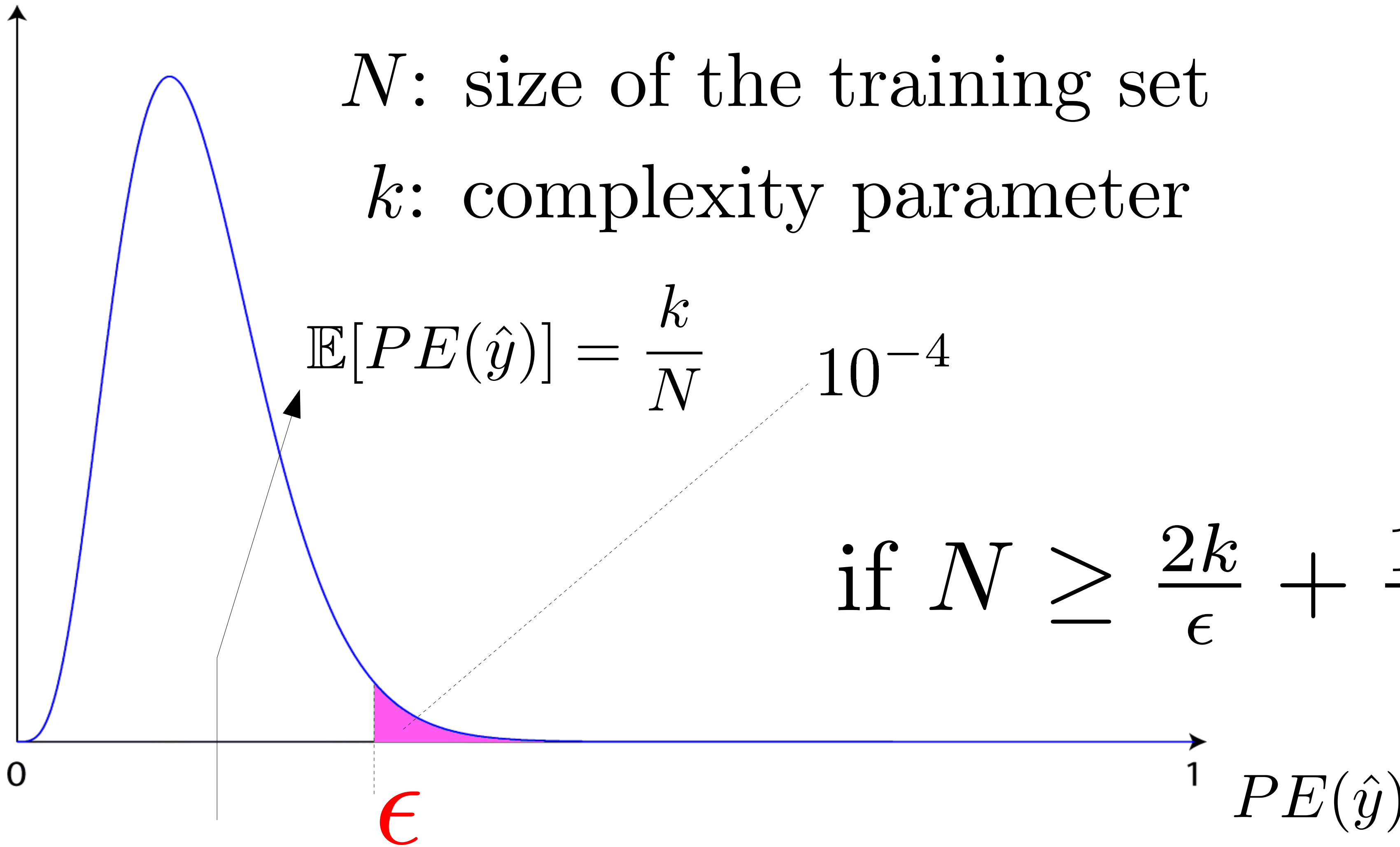
k: complexity parameter



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Alternative Approach: Complete Classification

IDEA:
Choose “k” large enough so that there are no “unknowns”.

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- can be small even when features are many

Very technical caveat:
There is a subtle difference between trying with k=1,2,3,... and stopping when there are no “unknown”s and starting with k=∞ and evaluating the complexity a posteriori. The latter case is more difficult, see “**Wait-and-Judge scenario optimization**”

”Wait-and-Judge scenario optimization”
M.C. Campi, S.Garatti
Mathematical Programming, 2019

A fundamental distinction

Probability of **error**

A fundamental distinction

Probability of **error**

Pr("success" | failure)

Pr("failure" | success)

A fundamental distinction

Probability of **correct classification**

1 – Pr(”success” | failure)

1 – Pr(”failure” | success)

A fundamental distinction

Probability of **correct classification**

1 – Pr(”success” | failure)
=Pr(”failure” | failure)

1 – Pr(”failure” | success)
=Pr(”success” | success)

A fundamental distinction

Probability of **correct classification**

1 – Pr("success" | failure)
=Pr("failure" | failure)

specificity

1 – Pr("failure" | success)
=Pr("success" | success)

sensitivity

A fundamental distinction

Probability of **correct classification**

$1 - \Pr(\text{"success"} \mid \text{failure})$
$= \Pr(\text{"failure"} \mid \text{failure})$

specificity

Target

50%

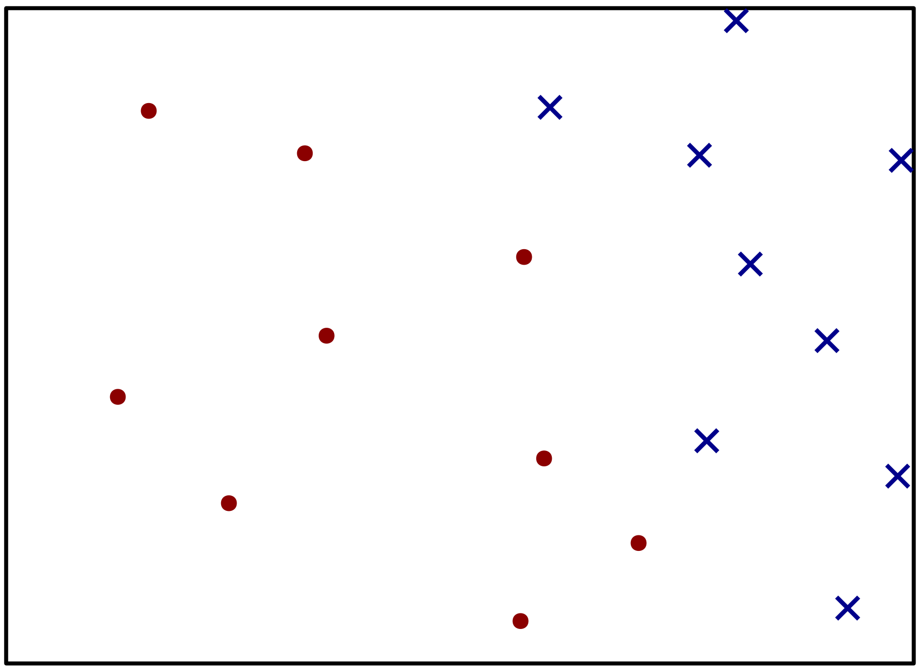
$1 - \Pr(\text{"failure"} \mid \text{success})$
$= \Pr(\text{"success"} \mid \text{success})$

sensitivity

Target

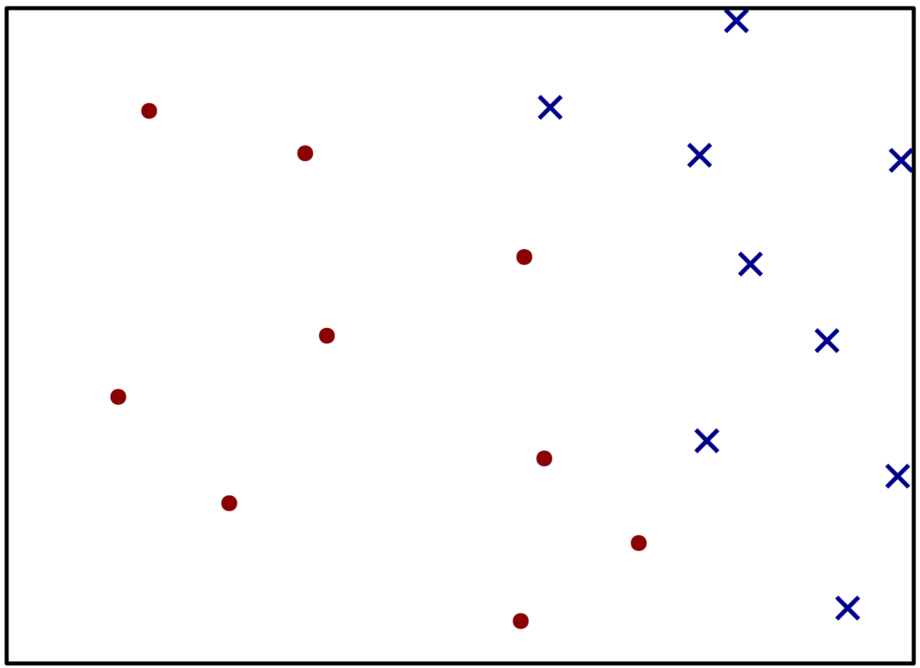
95%

GEM-BALLS



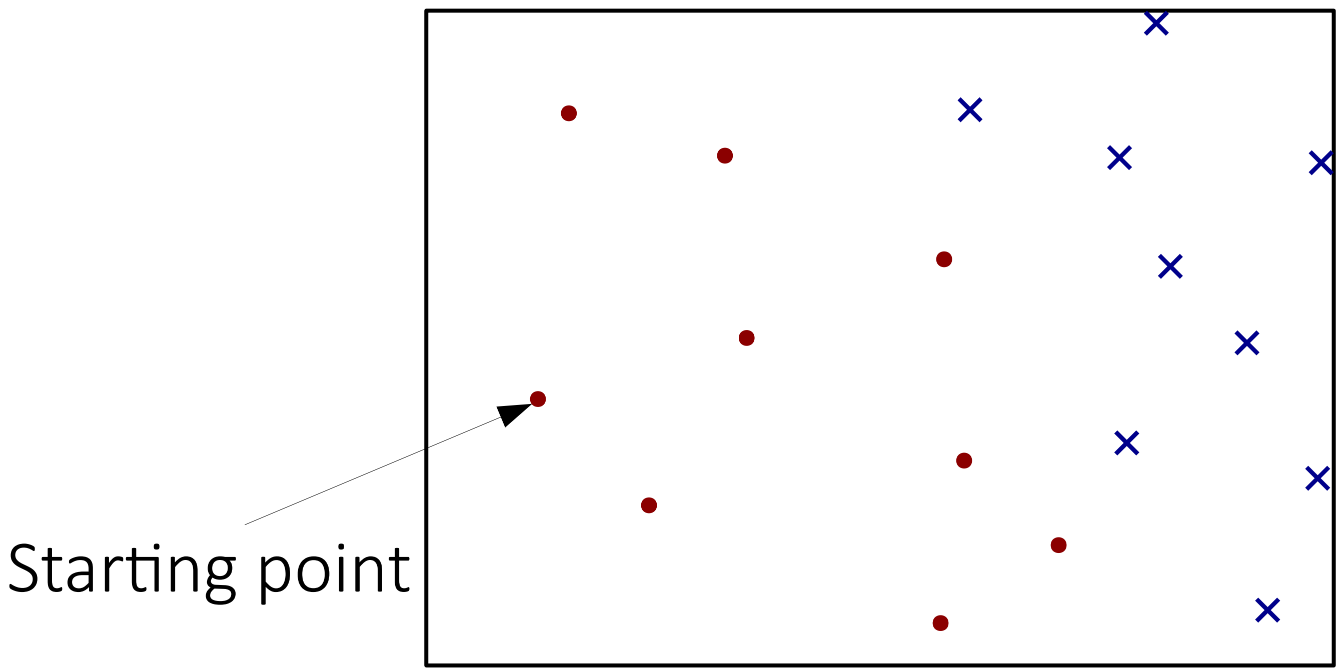
GEM-BALLS

- “0”, “negative”, “failure”
- × “1”, “positive”, “success”



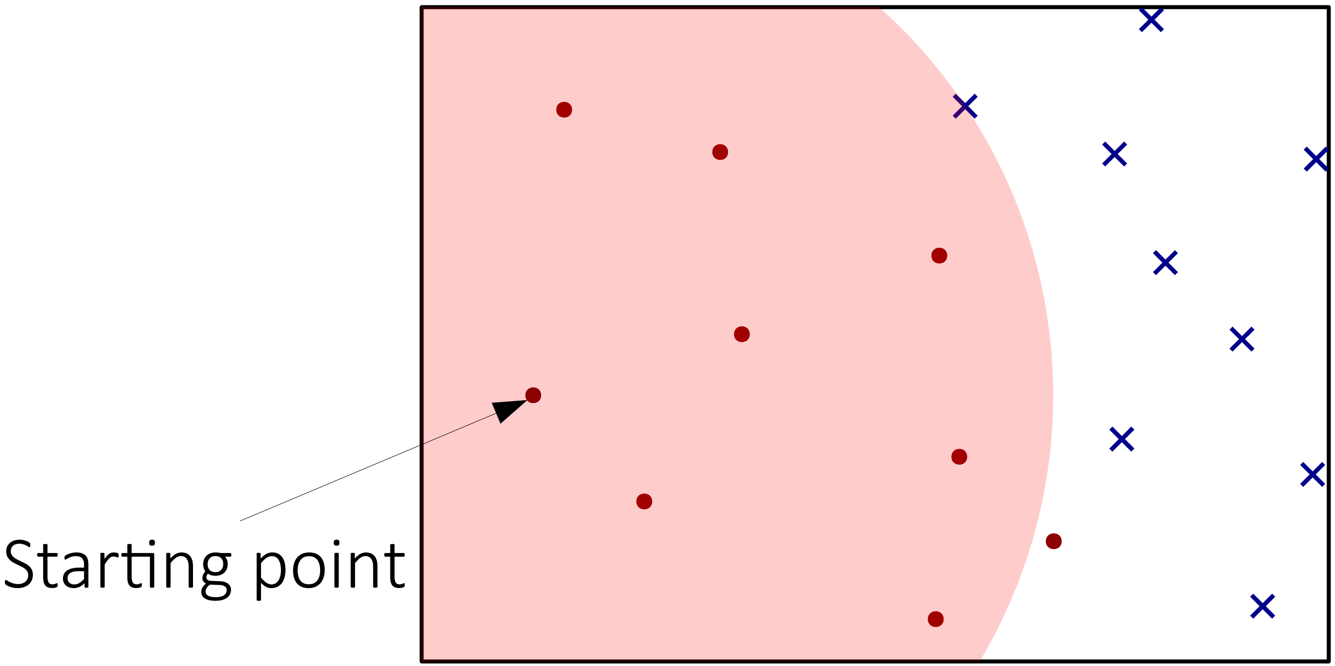
GEM-BALLS

- “0”, “negative”, “failure”
- × “1”, “positive”, “success”

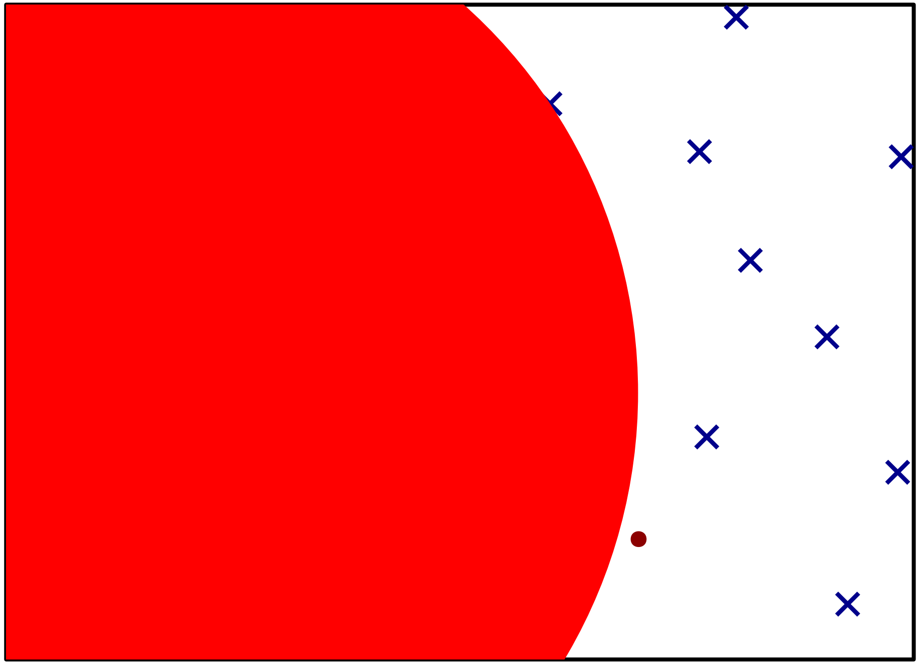


GEM-BALLS

Largest ball that does not include blue points

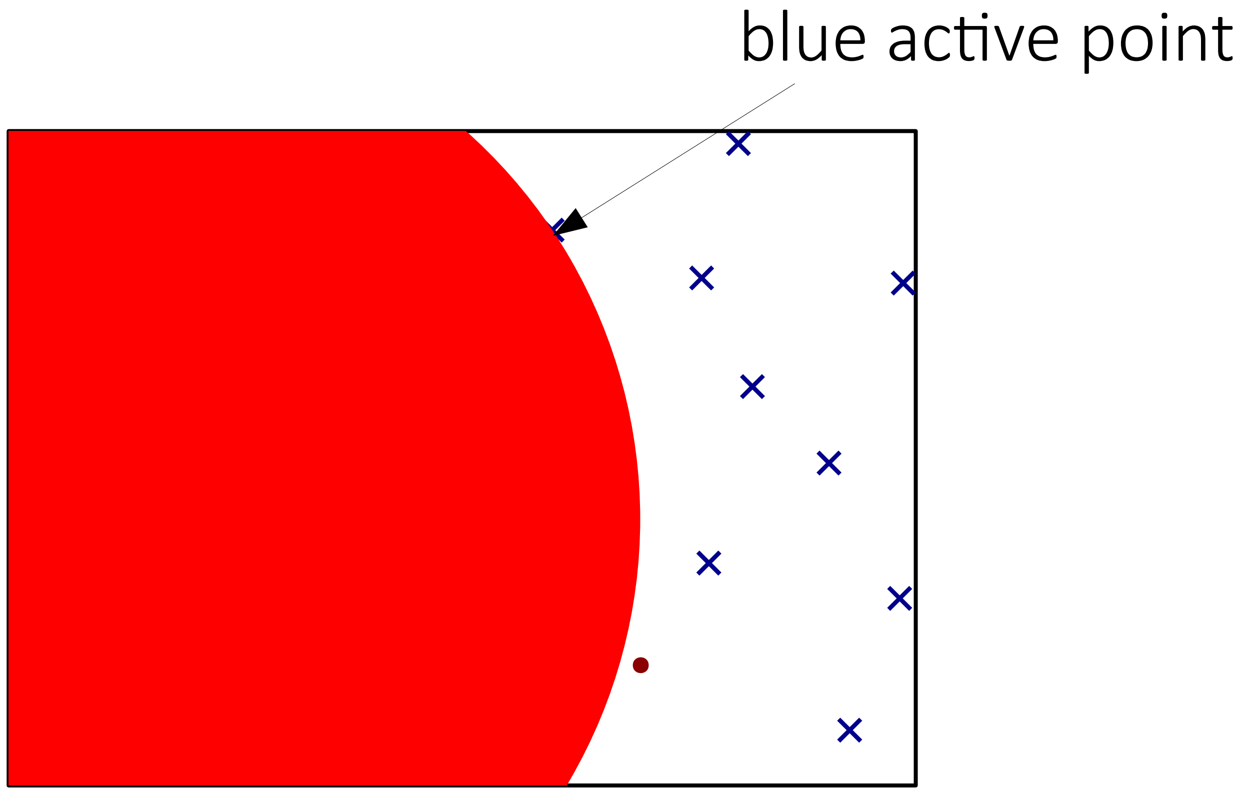


GEM-BALLS



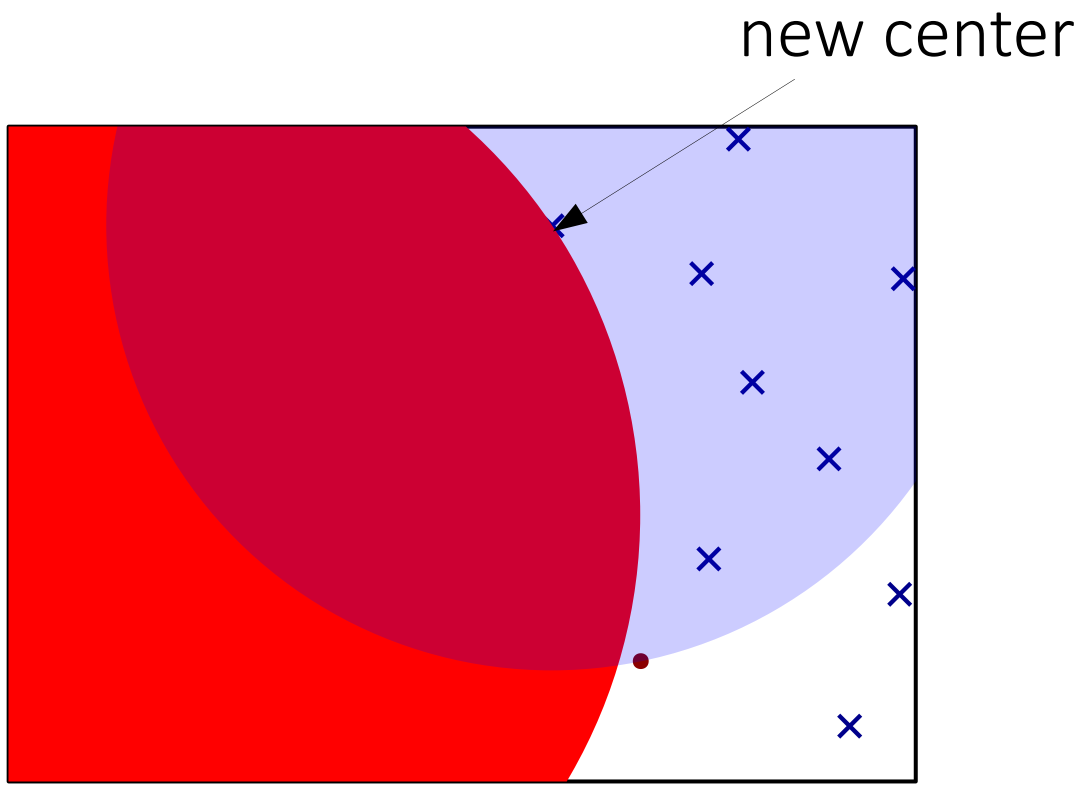
GEM-BALLS

#blue active points so far =1



GEM-BALLS

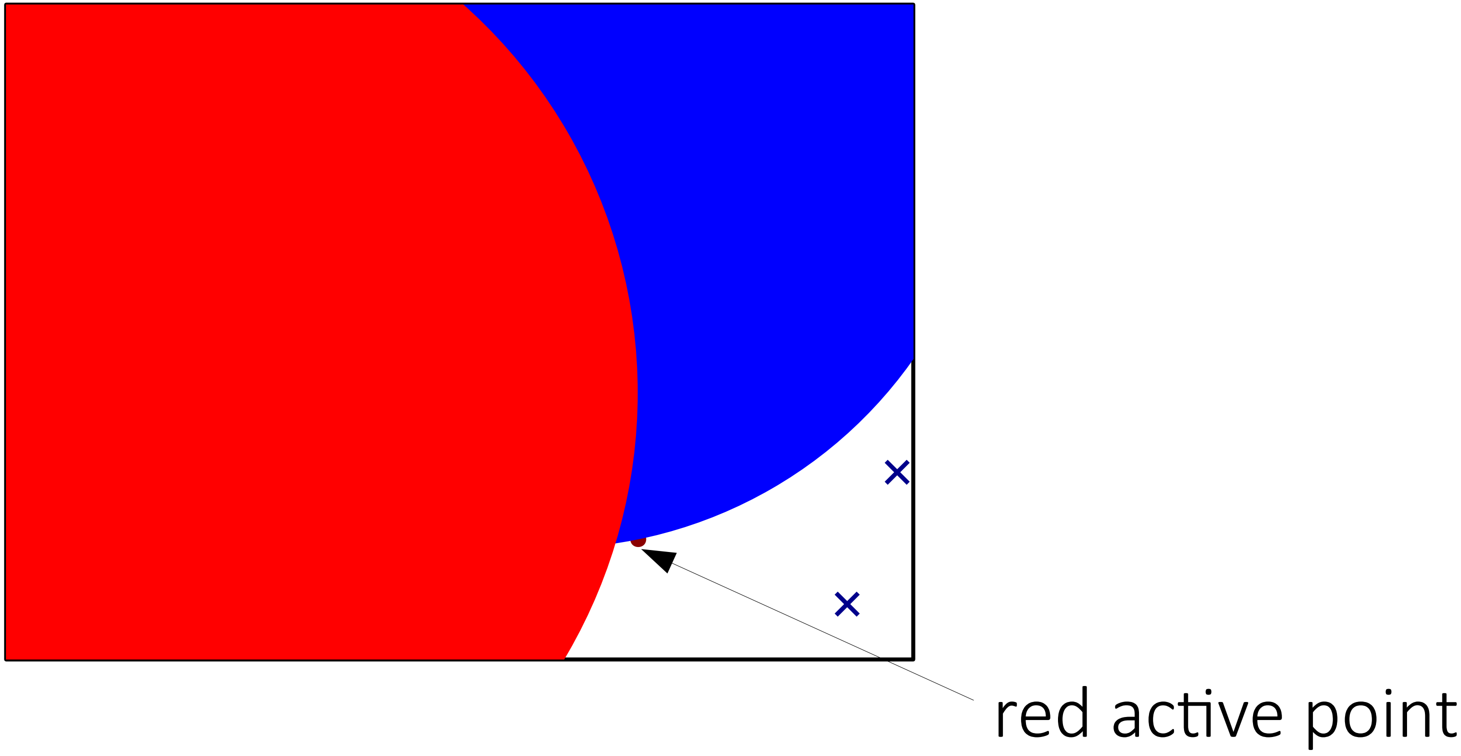
#blue active points so far =1



GEM-BALLS

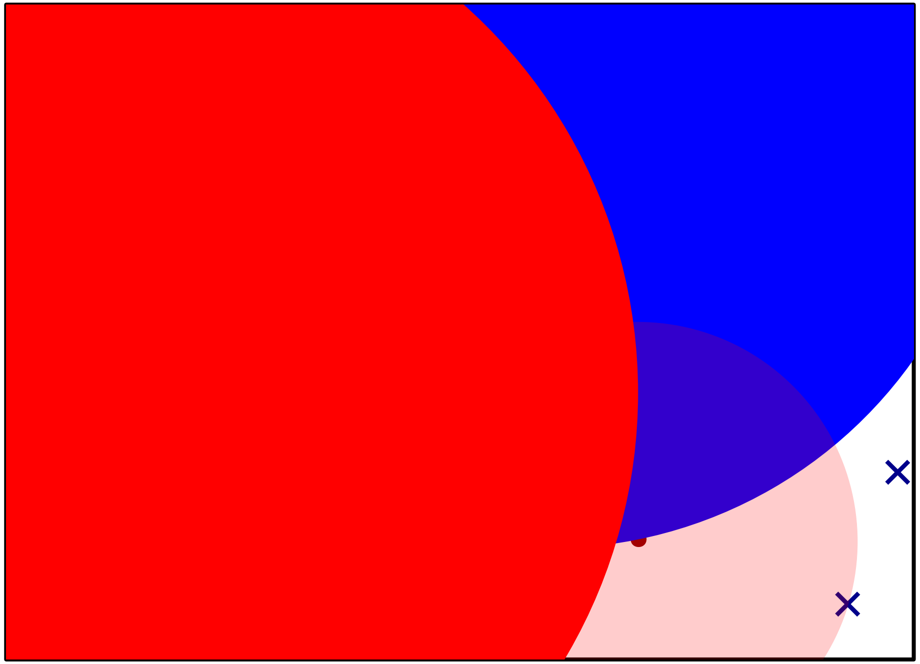
#blue active points so far =1

#red active points so far =1



GEM-BALLS

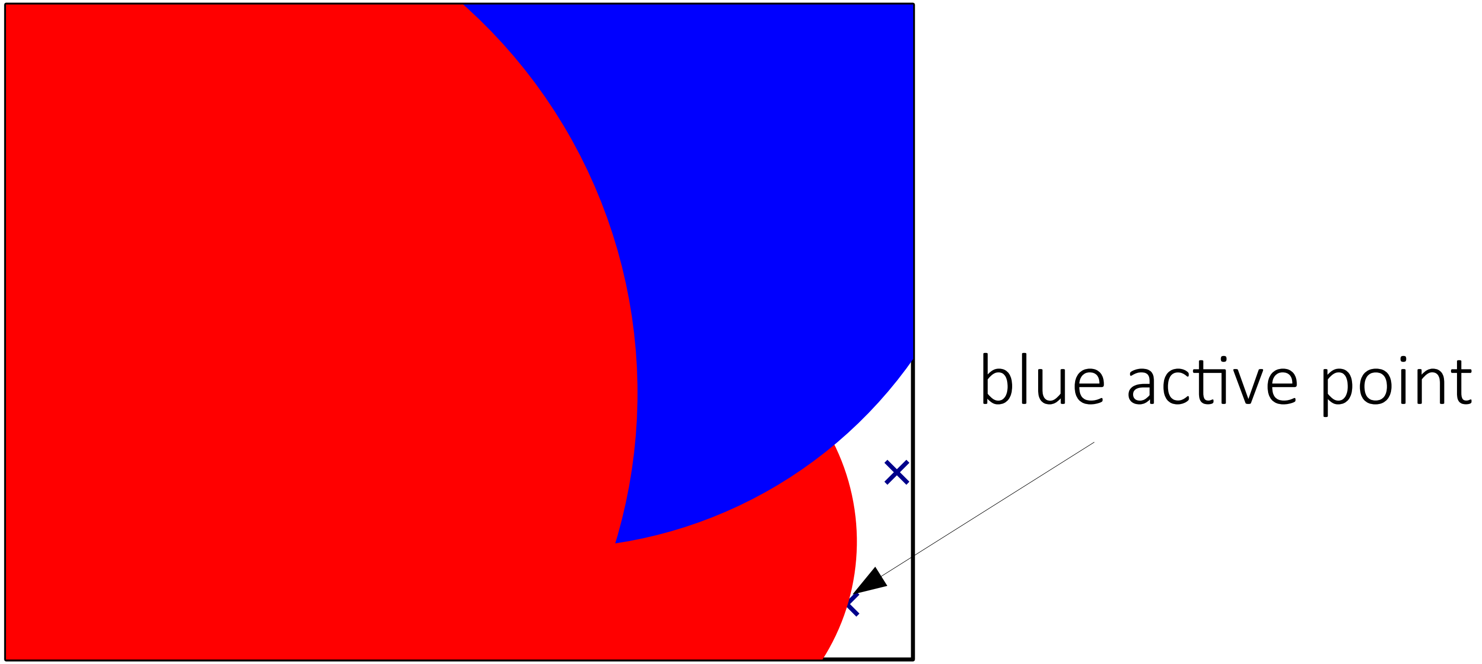
#blue active points so far =1
#red active points so far =1



GEM-BALLS

#blue active points so far =1+1

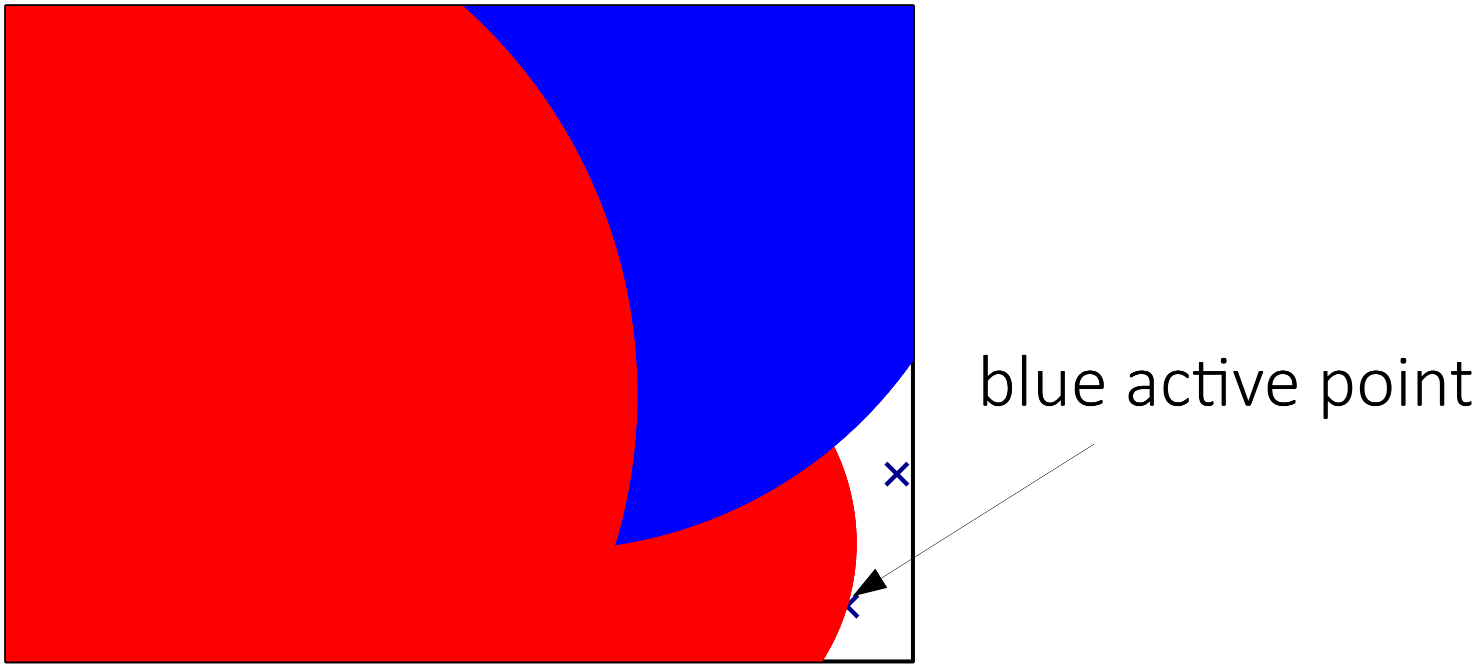
#red active points so far =1



GEM-BALLS

#blue active points so far =2

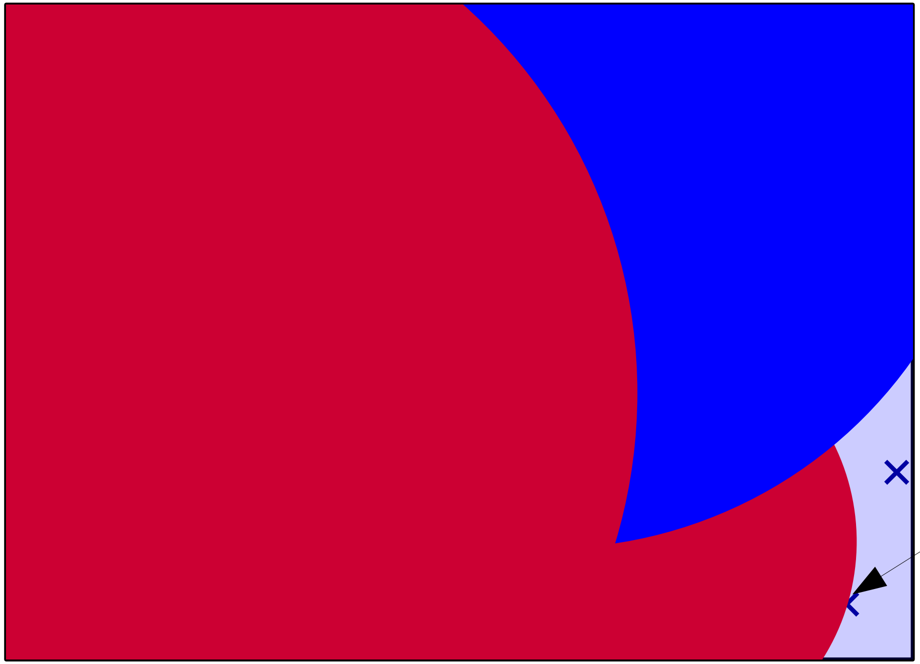
#red active points so far =1



GEM-BALLS

#blue active points so far =2

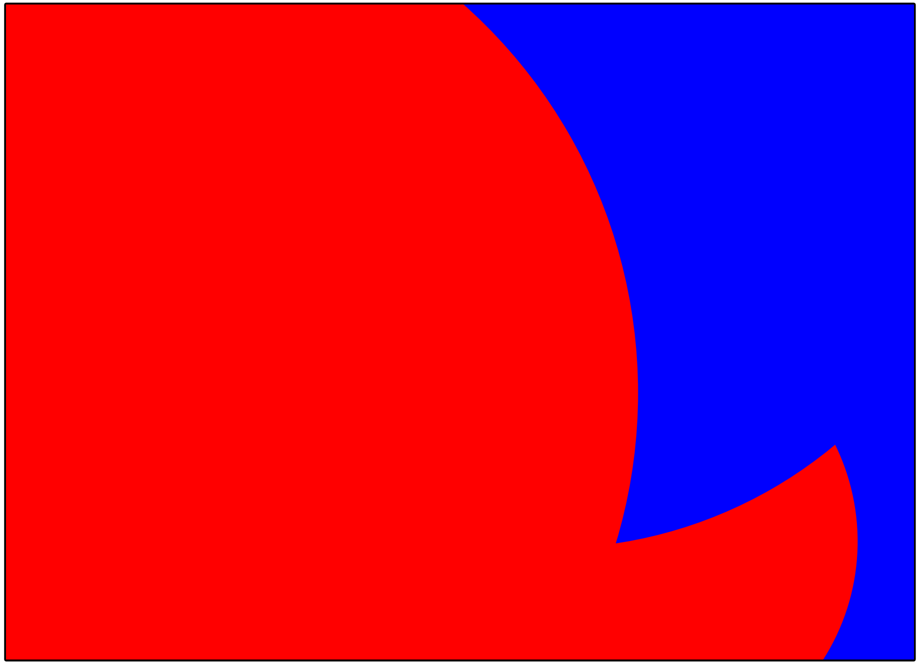
#red active points so far =1



new center

GEM-BALLS

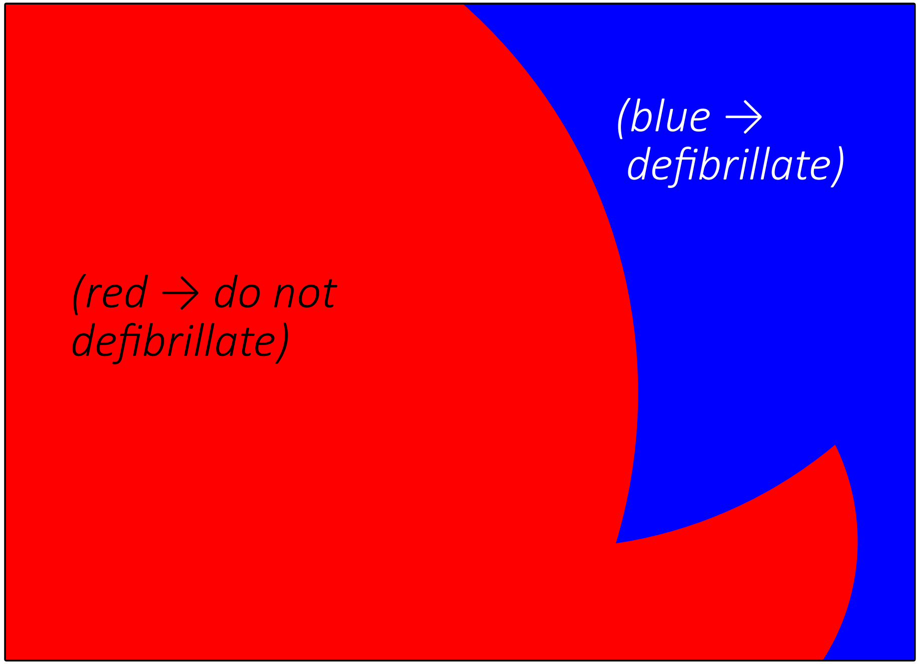
#blue active points so far =2
#red active points so far =1



GEM-BALLS

#blue active points so far =2 → k_1

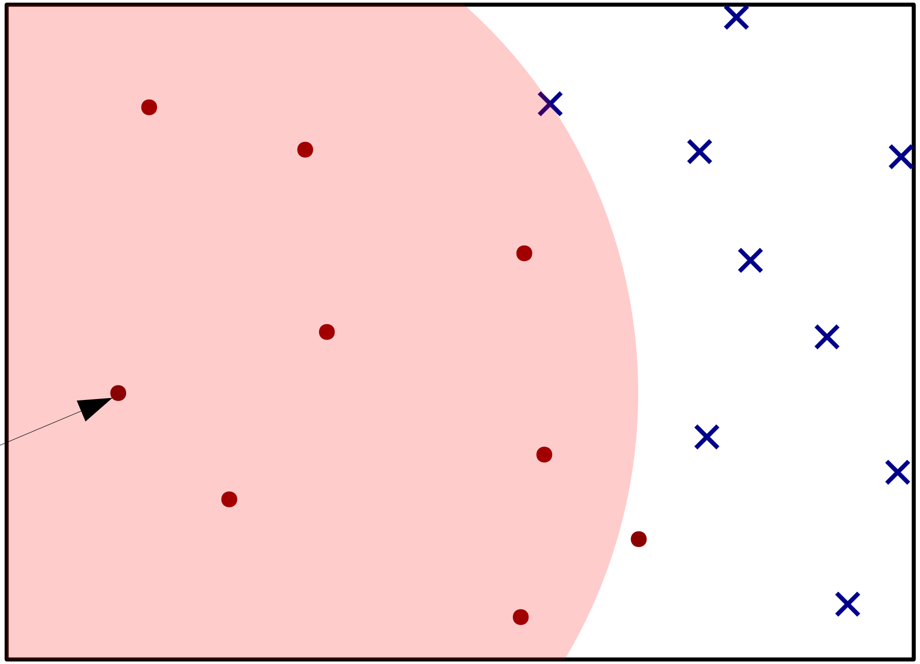
#red active points so far =1 → k_0



The tuning knob

When $C_1=1$

Largest ball that does not include blue points

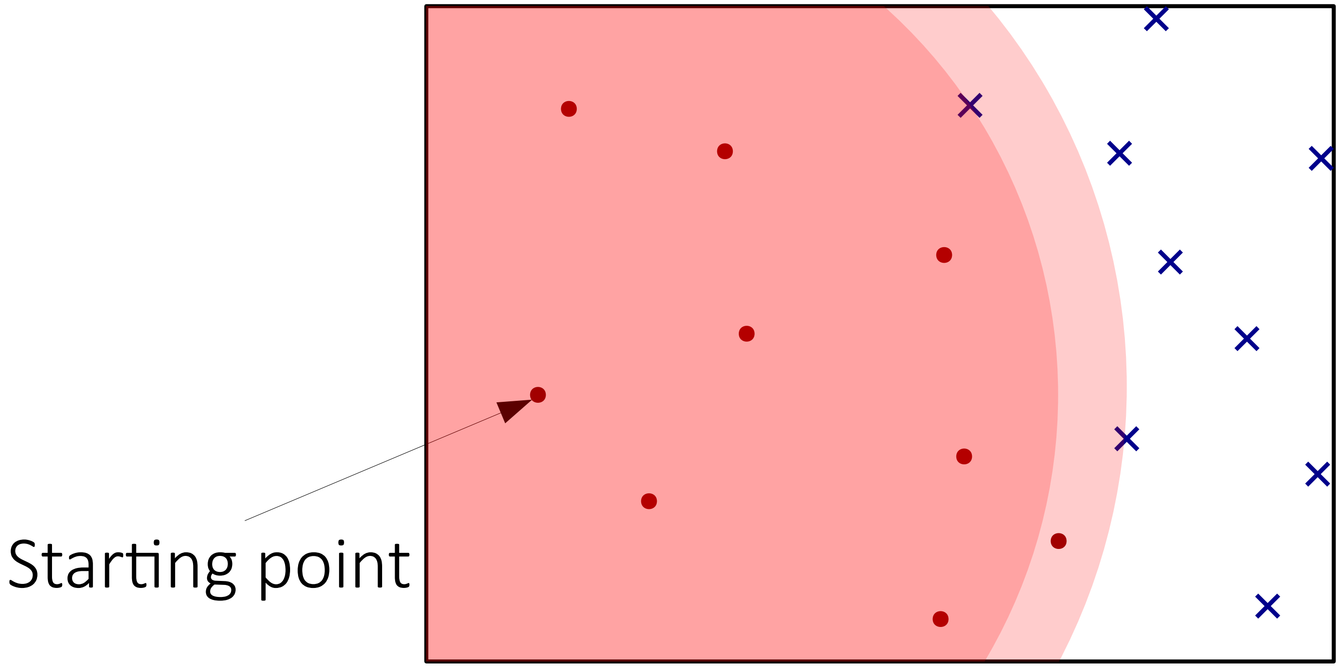


Starting point

The tuning knob

When $C_1=2$

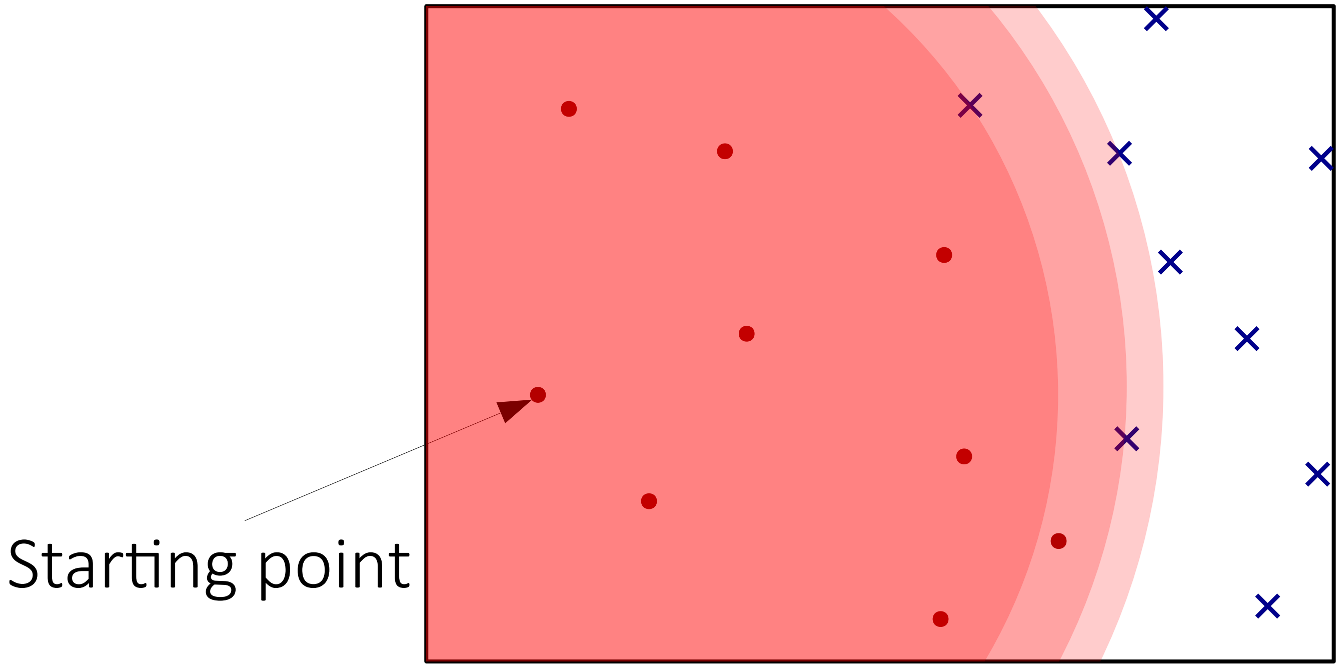
Largest ball that includes less than **2** blue points



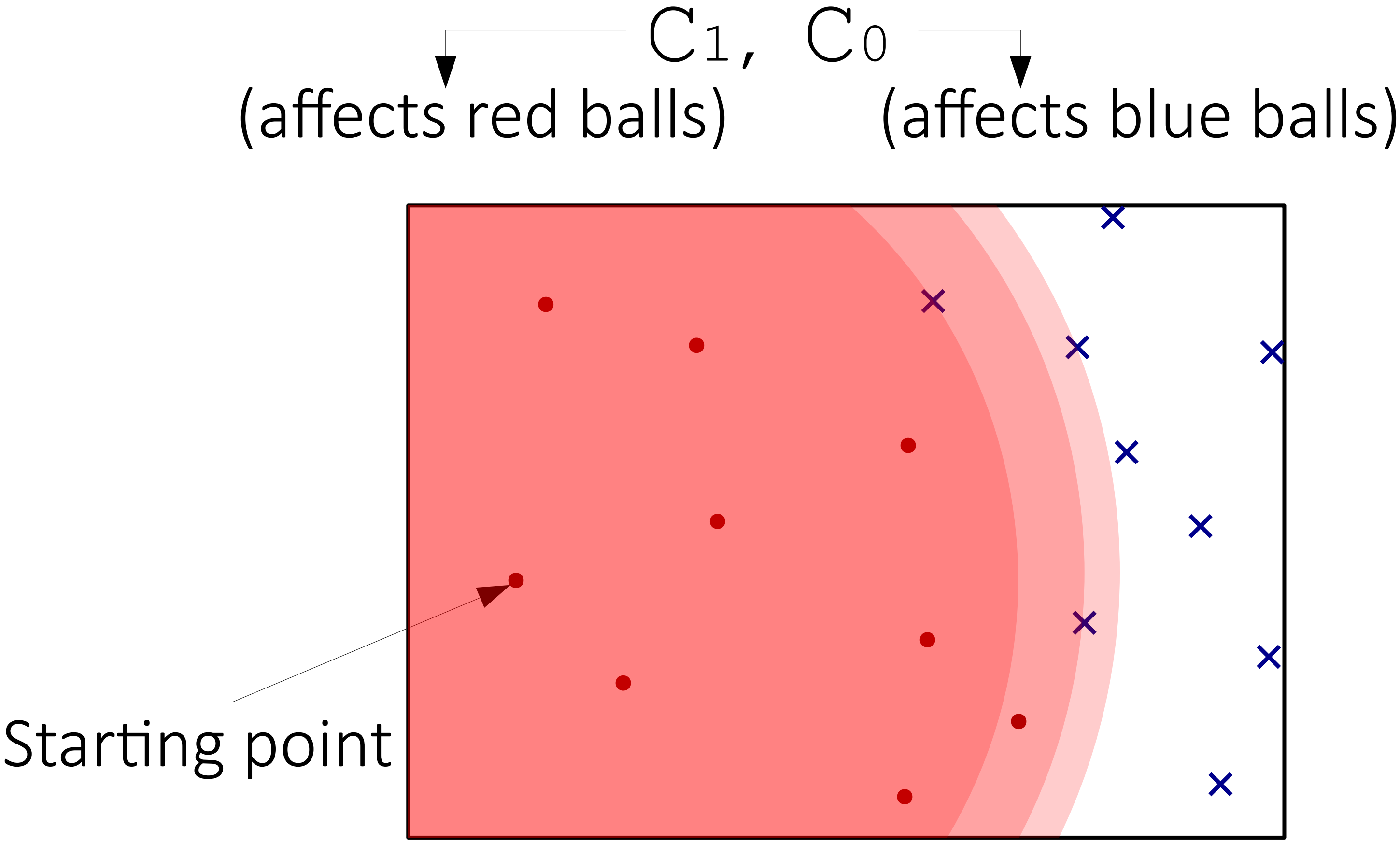
The tuning knob

When $C_1=3$

Largest ball that includes less than **3** blue points



The tuning knob



Theorem - preliminaries

β: small (e.g. 10^{-6}) confidence parameter

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N_1: # of positives

Theorem - preliminaries

β: small (e.g. 10^{-6}) confidence parameter

N_1: # of positives

k_1 : # active positives

Theorem - preliminaries

β: small (e.g. 10^{-6}) confidence parameter

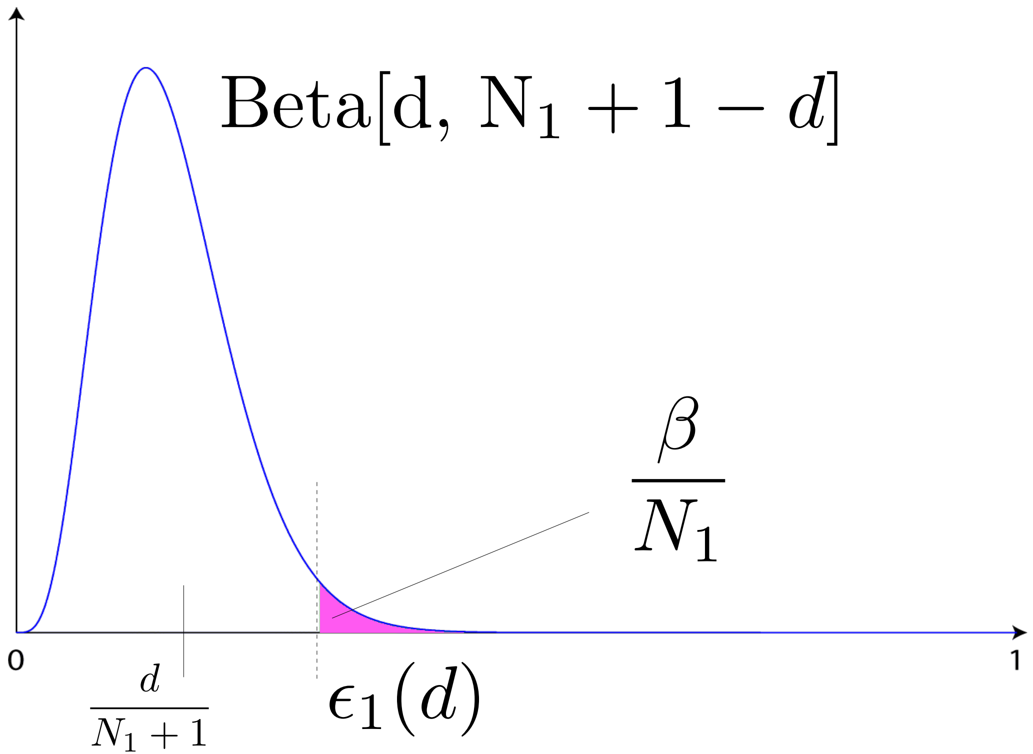
N_1: # of positives

k_1 : # active positives

$\{\epsilon_1(d) : d = 1, \ldots, N_1\}$: are pre-computed thresholds.

Theorem - preliminaries

$\epsilon_1(d)$: for $d = 1, \ldots, N_1$ is the quantile at level $1 - \frac{\beta}{N_1}$ of $\mathrm{Beta}[\mathrm{d},\ \mathrm{N}_1 + 1 - d]$



Theorem - preliminaries

$\hat{y}$ (k_1)

Theorem - preliminaries

$$\mathbb{P}\{\underbrace{\hat{y}(x) = 0}_{\text{“failure”}} \mid \underbrace{y = 1}_{\text{success}}\} \qquad (k_1)$$

Theorem - preliminaries

$$\mathbb{P}\{\overbrace{\hat{y}(x) = 0}^{\text{“failure”}} \mid \overbrace{y = 1}^{\text{success}}\} \qquad \underbrace{\epsilon_1(k_1)}_{\text{precomputed threshold}}$$

Theorem - preliminaries

$$\mathbb{P}\{\overbrace{\hat{y}(x) = 0}^{\text{"failure"}} \mid \overbrace{y = 1}^{\text{success}}\} \leq \underbrace{\epsilon_1(k_1)}_{\text{precomputed threshold}}$$

Theorem - Statement

$$\mathbb{P}\{\overbrace{\hat{y}(x) = 0}^{\text{“failure”}} \mid \overbrace{y = 1}^{\text{success}}\} \leq \underbrace{\epsilon_1(k_1)}_{\text{precomputed threshold}}$$

$$\text{with confidence } 1 - \beta$$

Theorem - Discussion

$$\mathbb{P}\{\underbrace{\hat{y}(x) = 0}^{\text{"failure"}} \mid \underbrace{y = 1}^{\text{success}}\} \leq \underbrace{\epsilon_1(k_1)}_{\text{precomputed threshold}}$$

$\Updownarrow$

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1)$$

$$\text{with confidence } 1 - \beta$$

Theorem - Discussion

$$\mathbb{P}\{\overbrace{\hat{y}(x) = 0}^{\text{“failure”}} \mid \overbrace{y = 1}^{\text{success}}\} \leq \underbrace{\epsilon_1(k_1)}_{\text{precomputed threshold}}$$

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1)$$

$$\text{with confidence } 1 - \beta$$

Theorem - Statement

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1)$$

$$\text{with confidence } 1 - \beta$$

Theorem - Discussion

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1)$$

$$\text{with confidence } 1 - \beta$$

Theorem - Discussion

$$(x_i, y_i) \sim \mathbb{P}$$

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1)$$

Theorem - Statement

$$(x_i, y_i) \sim \mathbb{P}$$

$$\mathbb{P}^N\{\text{sensitivity} \geq 1 - \epsilon_1(k_1)\} \geq 1 - \beta$$

Theorem - Statement

$$(x_i, y_i) \sim \mathbb{P}$$

$$\mathbb{P}^N\{\text{sensitivity} \geq 1 - \epsilon_1(k_1)\} \geq 1 - \beta$$

$$\mathbb{P}^N\{\text{specificity} \geq 1 - \epsilon_0(k_0)\} \geq 1 - \beta$$

Theorem - Statement

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1) \;\&\; \text{specificity} \geq 1 - \epsilon_0(k_0)$$

$$\text{with overall confidence } 1 - 2\beta$$

Theorem - Statement

$$\text{sensitivity} \geq 1 - \epsilon_1(k_1) \ \& \ \text{specificity} \geq 1 - \epsilon_0(k_0)$$

$$\text{with overall confidence } 1 - 2\beta$$

Take-home message:

“GEM-BALLS is a self-testing algorithm!”

Benchmark datasets

BreastW (239 positive instances, 444 negative instances), $\beta = 5 \cdot 10^{-3}$							
$c_1 : c_0$	$1:1$	$1:2$	$1:3$	$1:5$	$1:10$	$10:100$	$1:50$
$\mathbf{k}_1 : \mathbf{k}_0$	$17:17$	$11:21$	$10:28$	$6:28$	$4:33$	$10:17$	$2:66$
$Sens$:$Spec$	83% : 91%	87% : 89%	88% : 87%	90% : 87%	92% : 86%	88% : 91%	94% : 76%

Pima (268 positive instances, 500 negative instances), $\beta = 10^{-3}$							
$c_1 : c_0$	$1:1$	$1:2$	$2:4$	$1:4$	$1:8$	$1:10$	$1:50$
$\mathbf{k}_1 : \mathbf{k}_0$	$125:125$	$88:175$	$96:189$	$61:245$	$38:300$	$33:324$	$9:424$
$Sens$:$Spec$	40% : 65%	54% : 55%	51% : 52%	65% : 41%	75% : 30%	70% : 20%	90% : 9%

Haberman (75 positive instances, 219 negative instances), $\beta = 10^{-3}$					
$c_1 : c_0$	$1:1$	$1:3$	$1:5$	$1:10$	$1:20$
$\mathbf{k}_1 : \mathbf{k}_0$	$46:46$	$28:82$	$23:114$	$14:139$	$9:176$
$Sens$:$Spec$	20% : 67%	41% : 49%	48% : 35%	62% : 24%	71% : 10%

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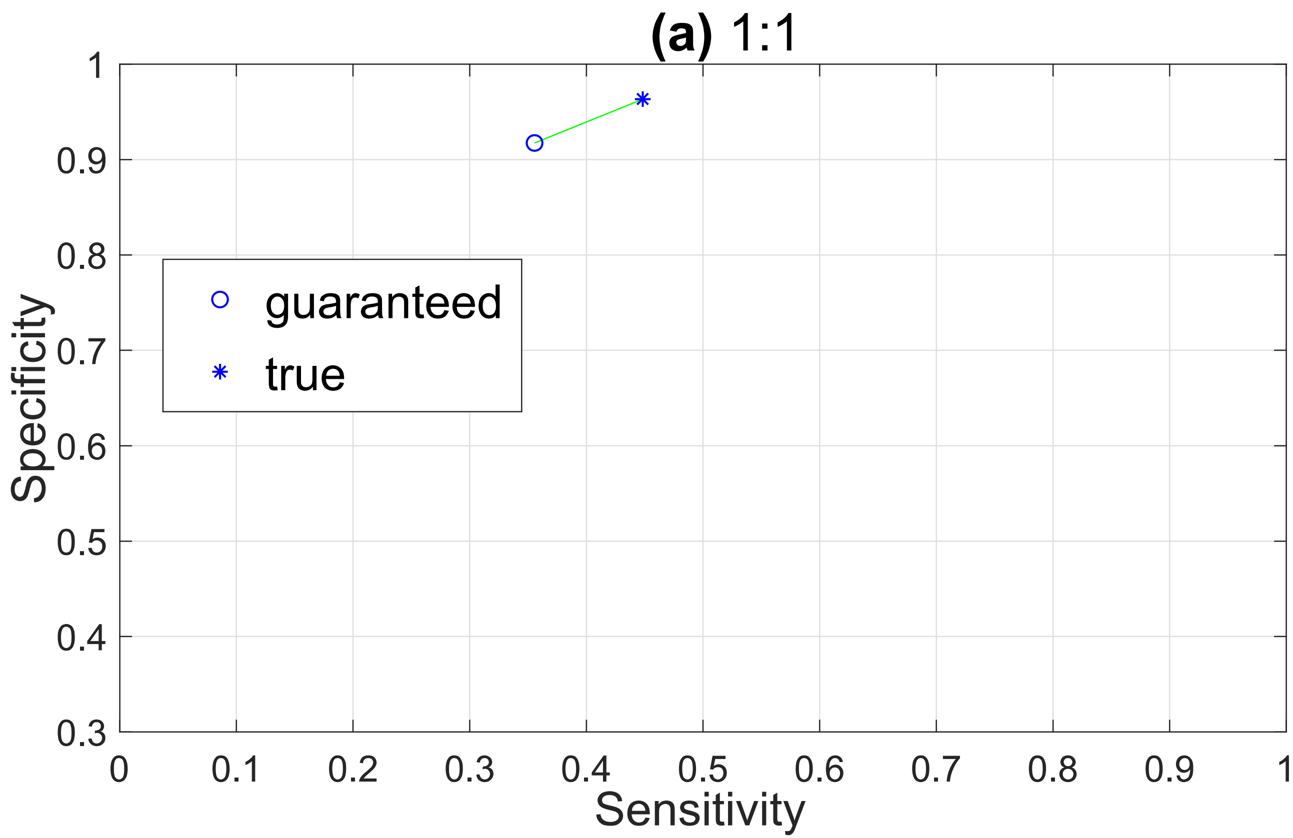
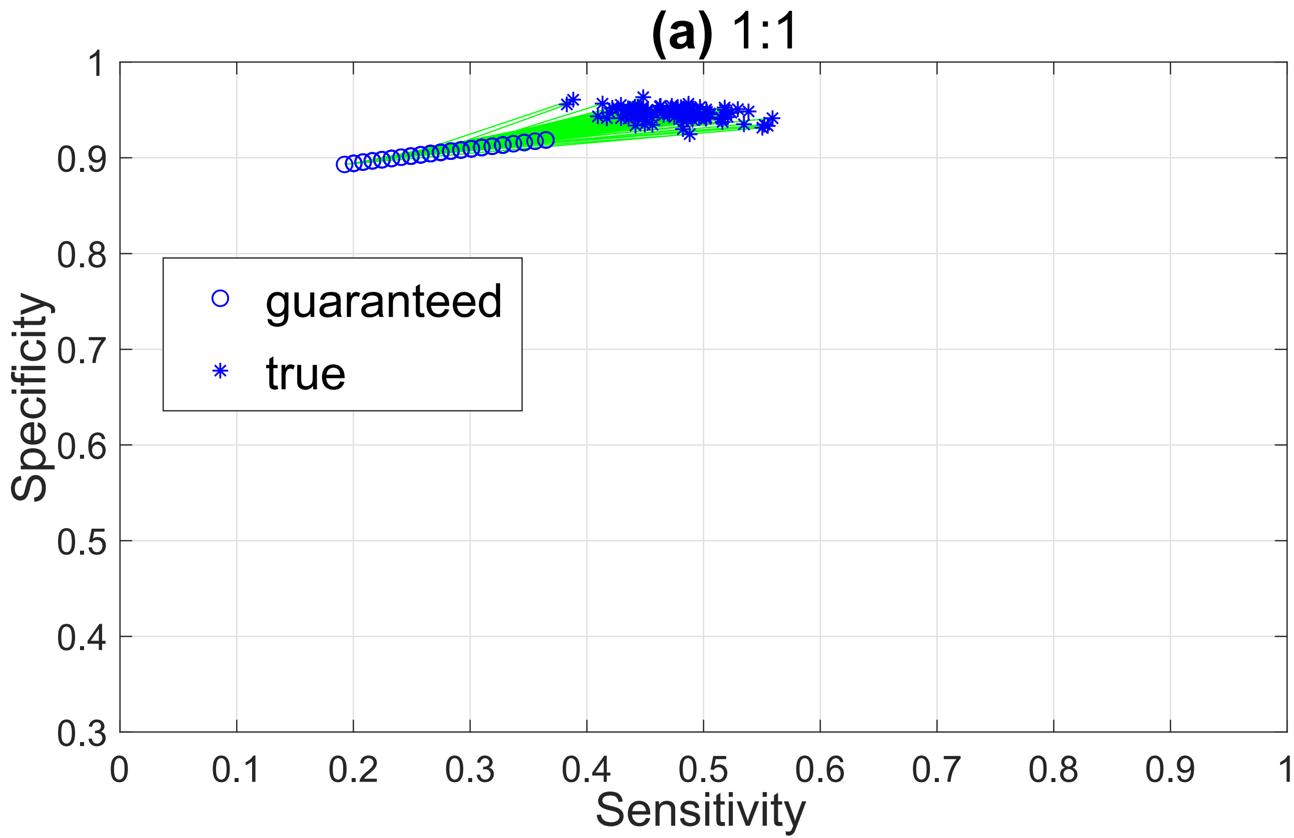
Tests on reproducible simulated data



Labels are generated by using the MATLAB kstest function.
$N_0 = 1000,\ N_1 = 100.\ \beta = 10^{-3}$

Tests on reproducible simulated data



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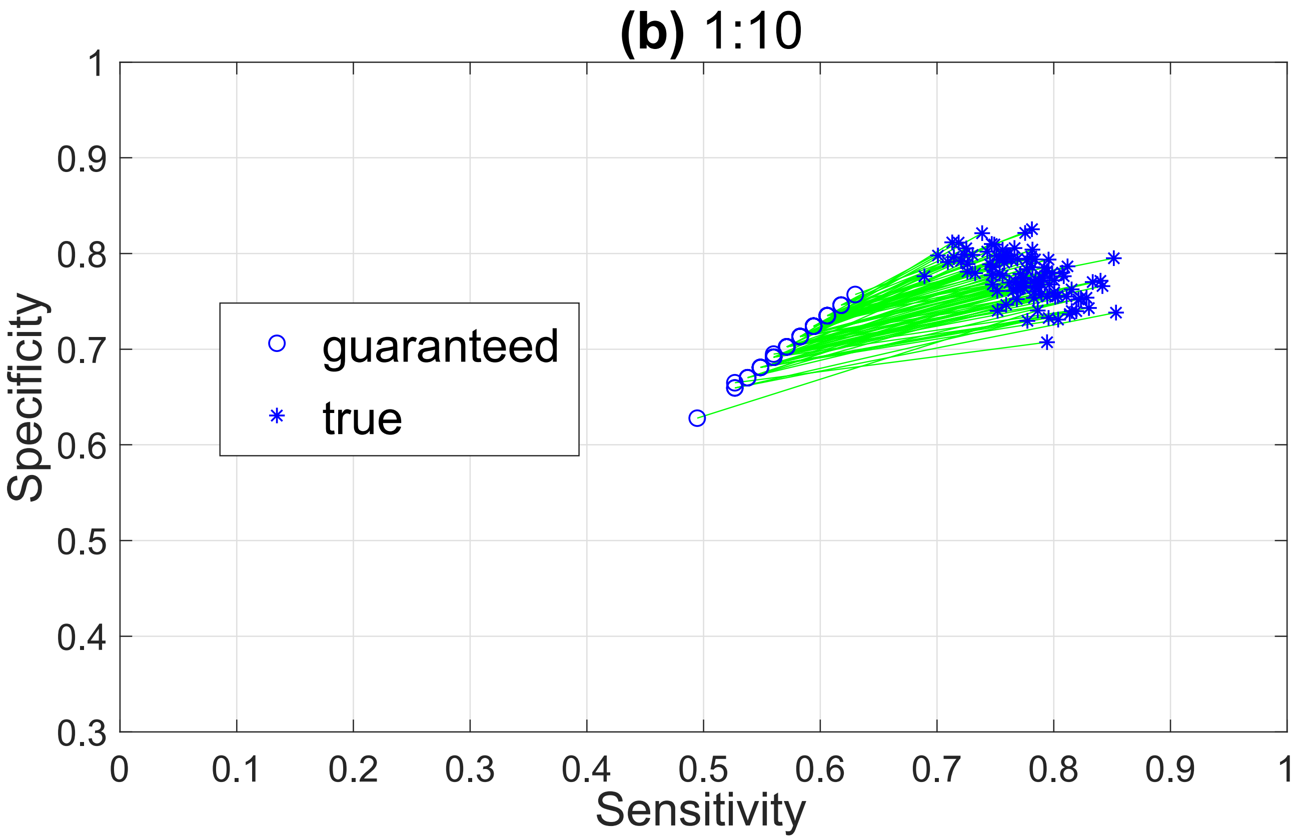

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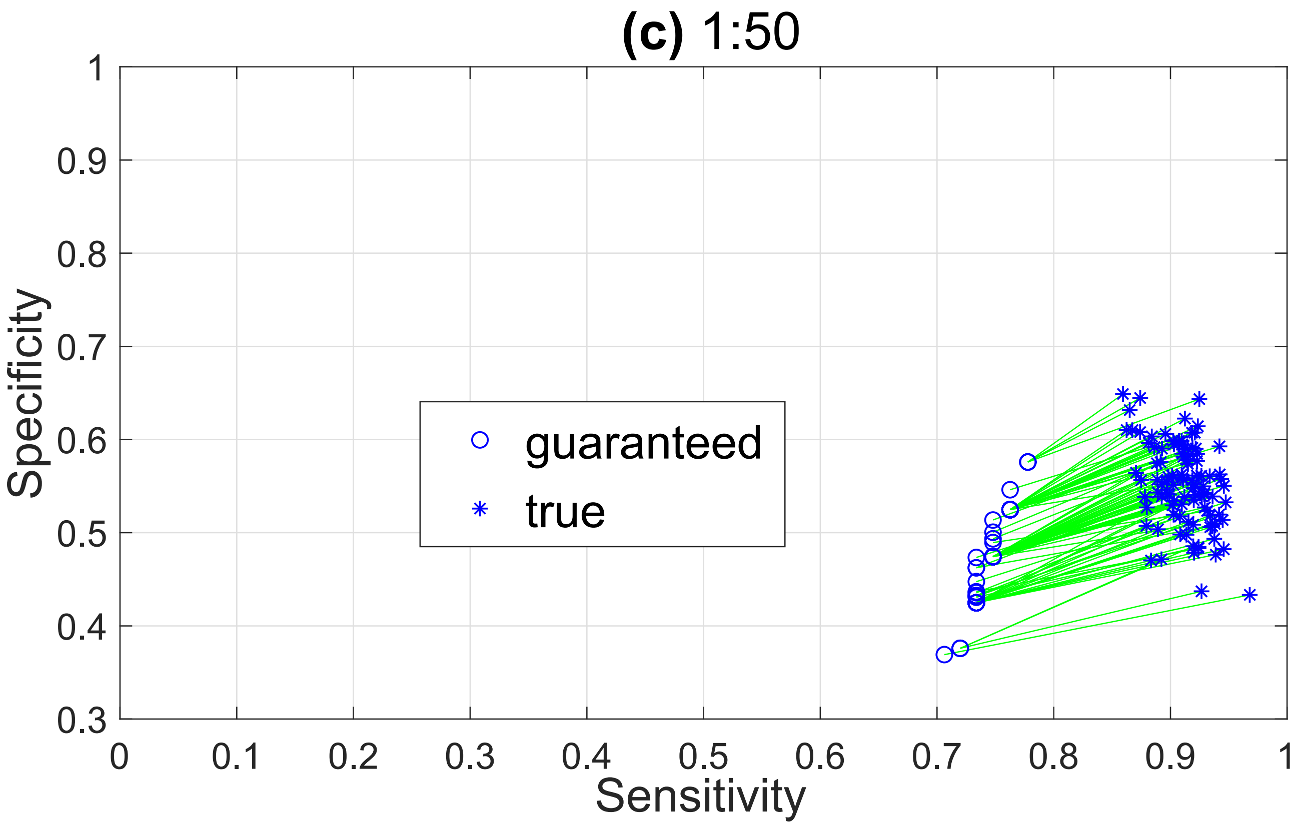

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$N_0 = 1000,\ N_1 = 100.\ \beta = 10^{-3}$

Ventricular Fibrillation (VF) dataset

VF dataset (15 pos, 155 neg), $\beta = 10^{-2}$			
$c_1 : c_0$	$1 : 1$	$1 : 10$	$1 : 80$
$\mathbf{k}_1 : \mathbf{k}_0$	$9 : 9$	$5 : 41$	$2 : 90$
$Sens : Spec$	$11\% : 85\%$	$30\% : 59\%$	$51\% : 28\%$

Ventricular Fibrillation (VF) dataset

VF dataset (15 pos, 155 neg), $\beta = 10^{-2}$			
$c_1 : c_0$	$1:1$	$1:10$	$1:80$
$\mathbf{k}_1 : \mathbf{k}_0$	$9:9$	$5:41$	$2:90$
$Sens : Spec$	$11\% : 85\%$	$30\% : 59\%$	$51\% : 28\%$

Expanded VF dataset (240 pos, 2477 neg), $\beta = 10^{-3}$				
$c_1 : c_0$	$1:1$	$1:10$	$1:80$	$1:240$
$\mathbf{k}_1 : \mathbf{k}_0$	$16:16$	$1:121$	$8:568$	$4:1055$
$Sens : Spec$	$84\% : 98\%$	$86\% : 93\%$	$89\% : 73\%$	$92\% : 53\%$

SMOTE (Synthetic Minority Over-sampling Technique)

New lines of research

Pool of guaranteed classifiers:

Train many guaranteed classifiers simultaneusly.

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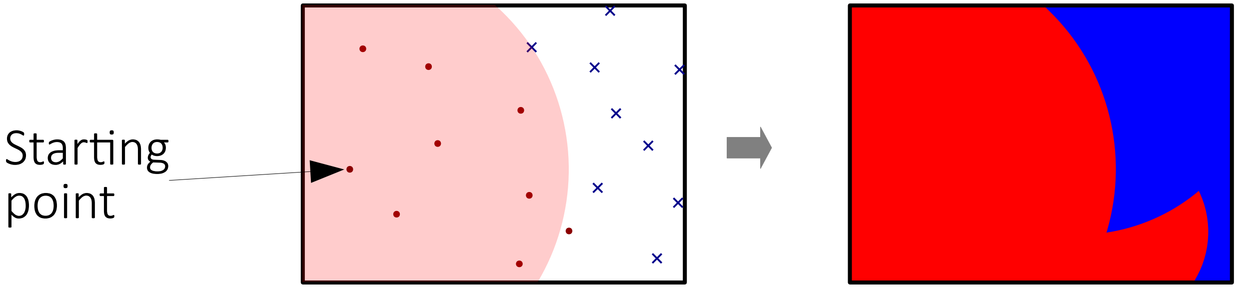

New lines of research

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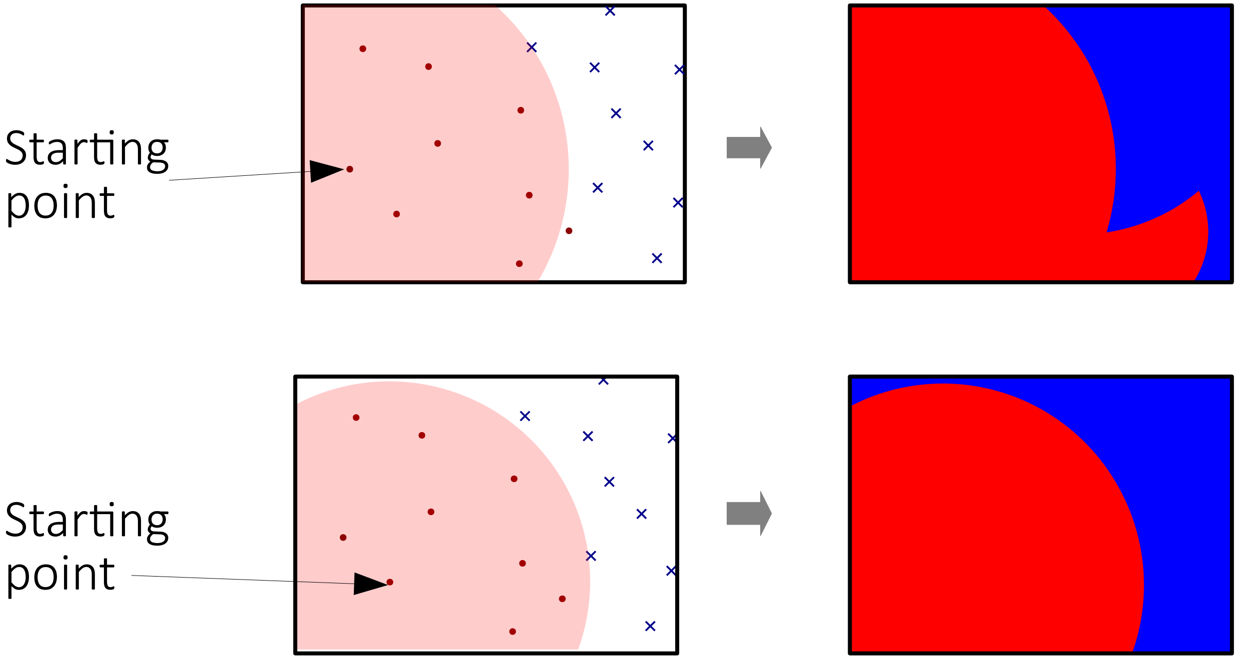

New lines of research

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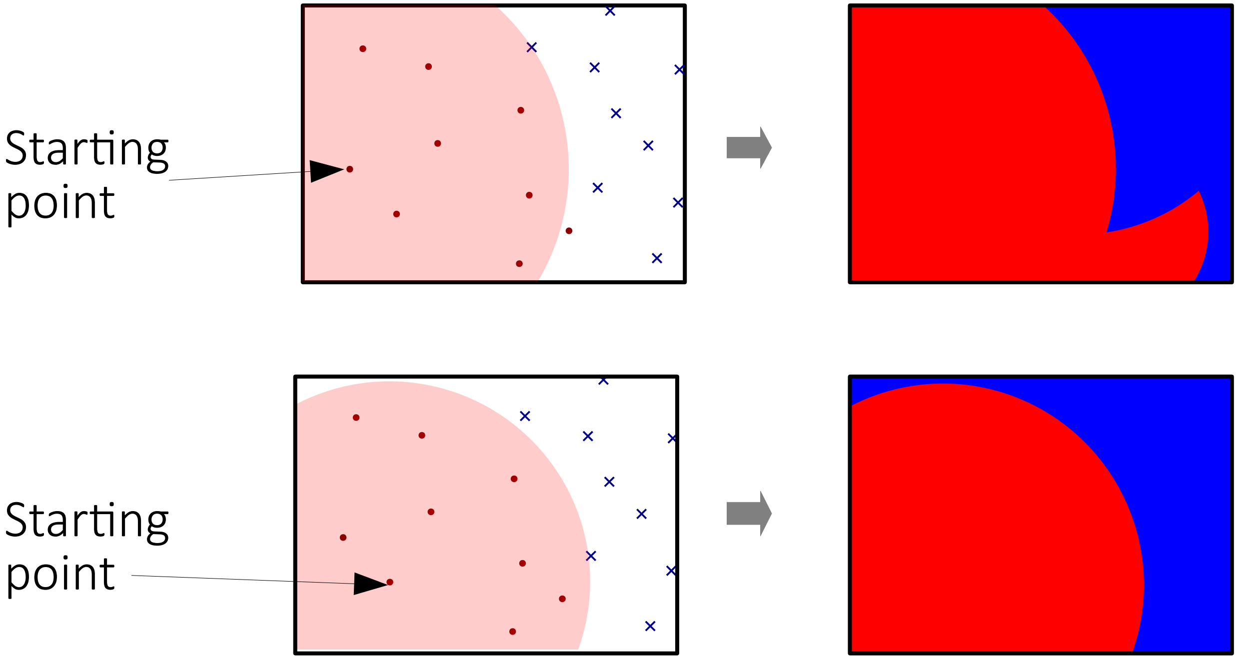


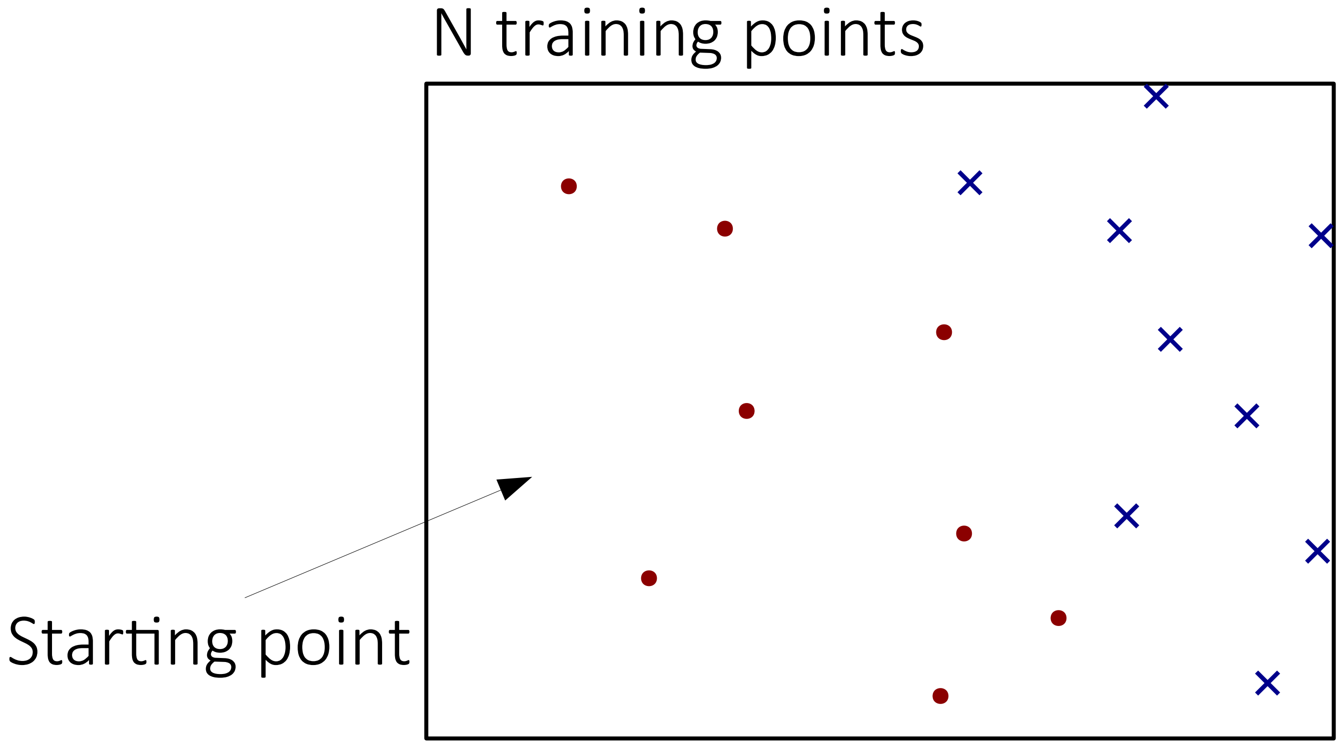
*Can we be **more** confident in the case of agreement?*

A taste of the theory

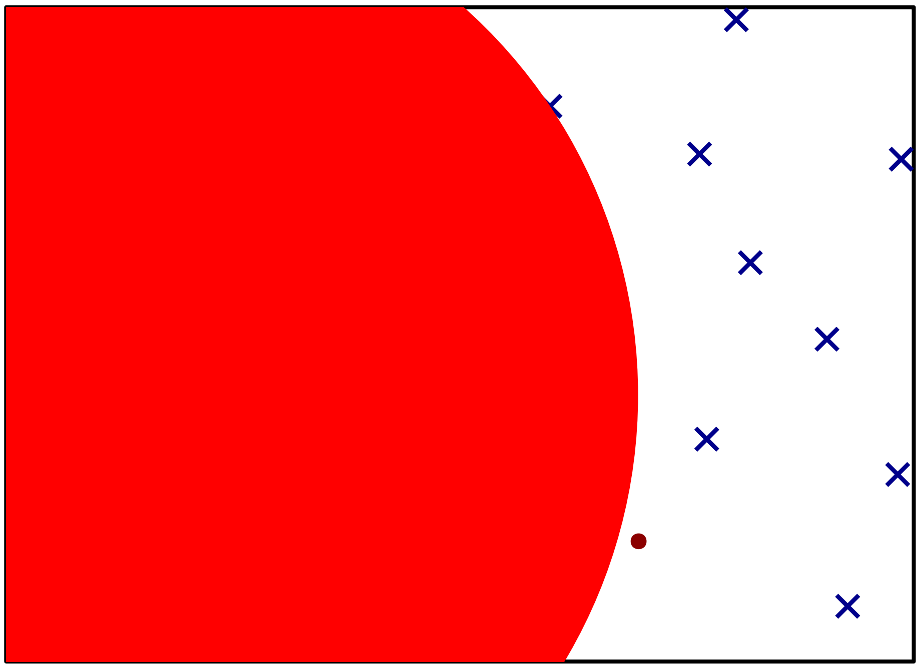
Just one ball

A taste of the theory

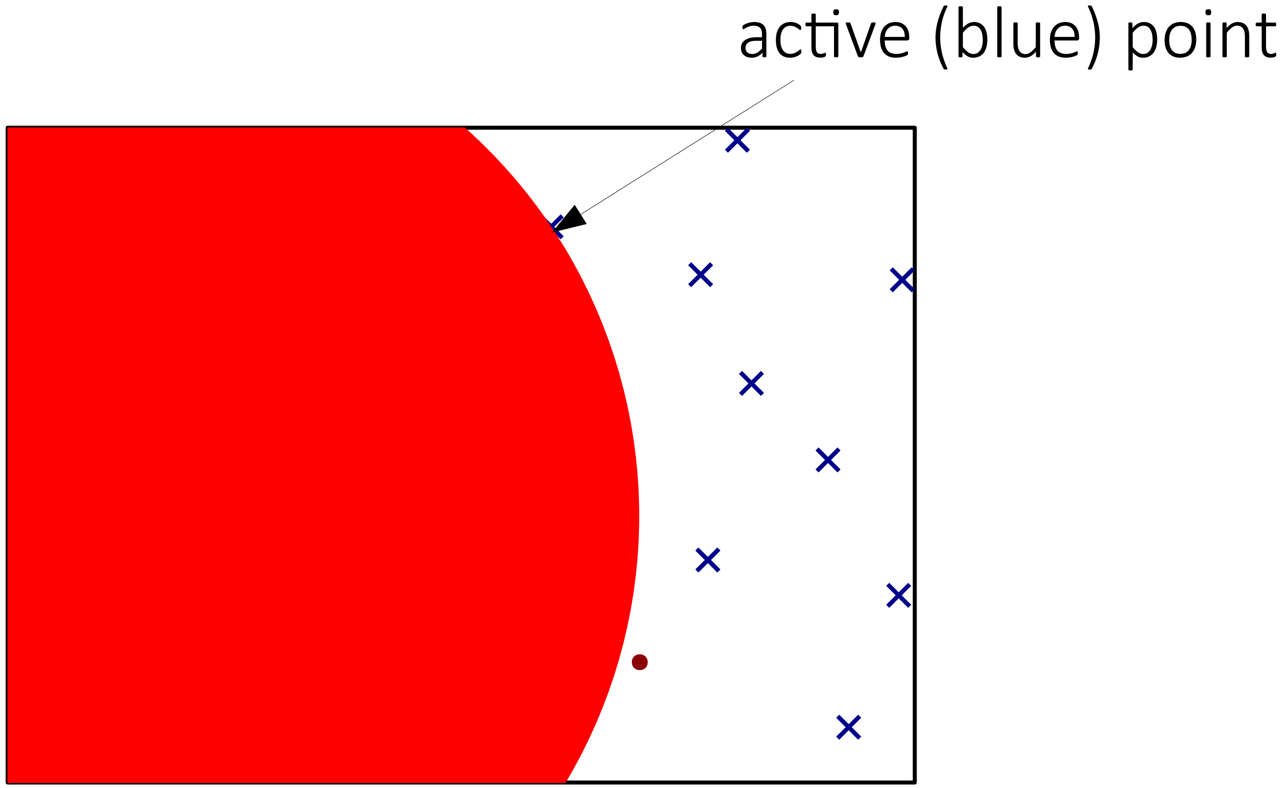
Just one ball



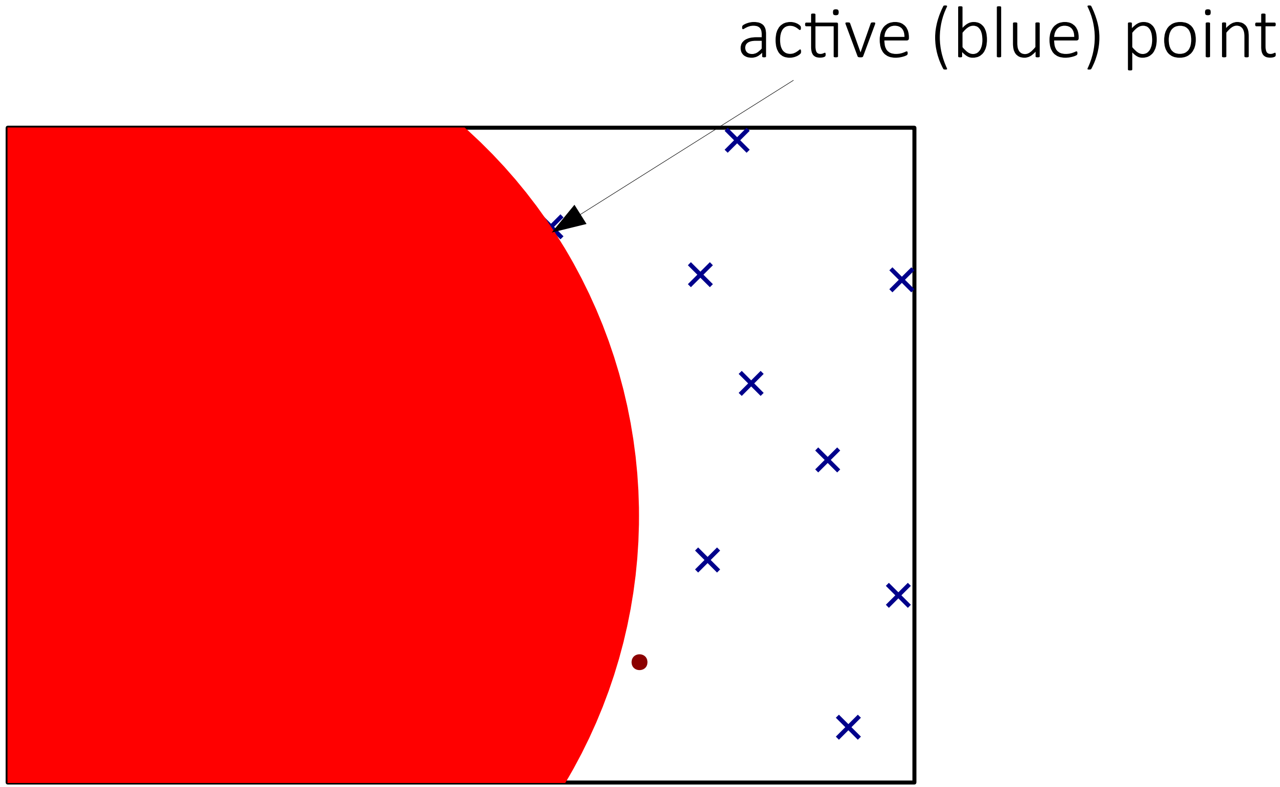
A taste of the theory



A taste of the theory



(1) There is one important point



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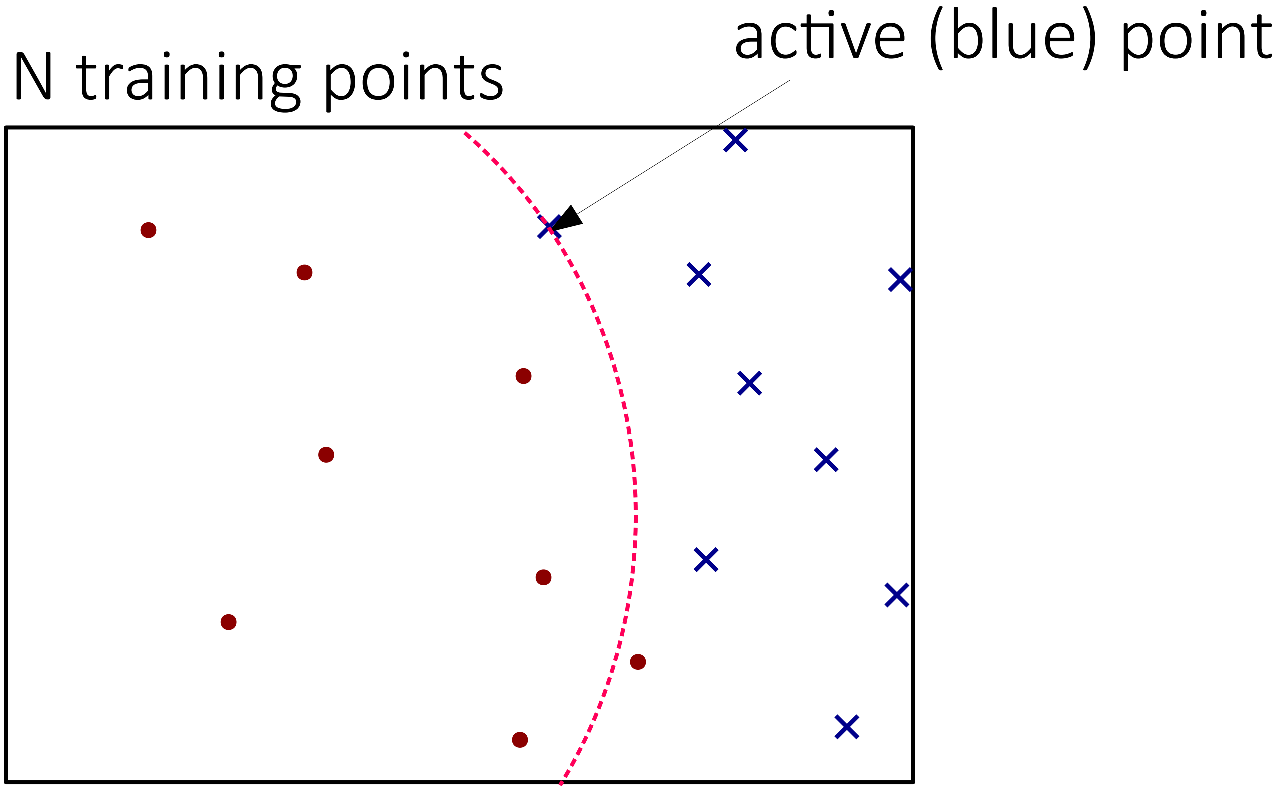

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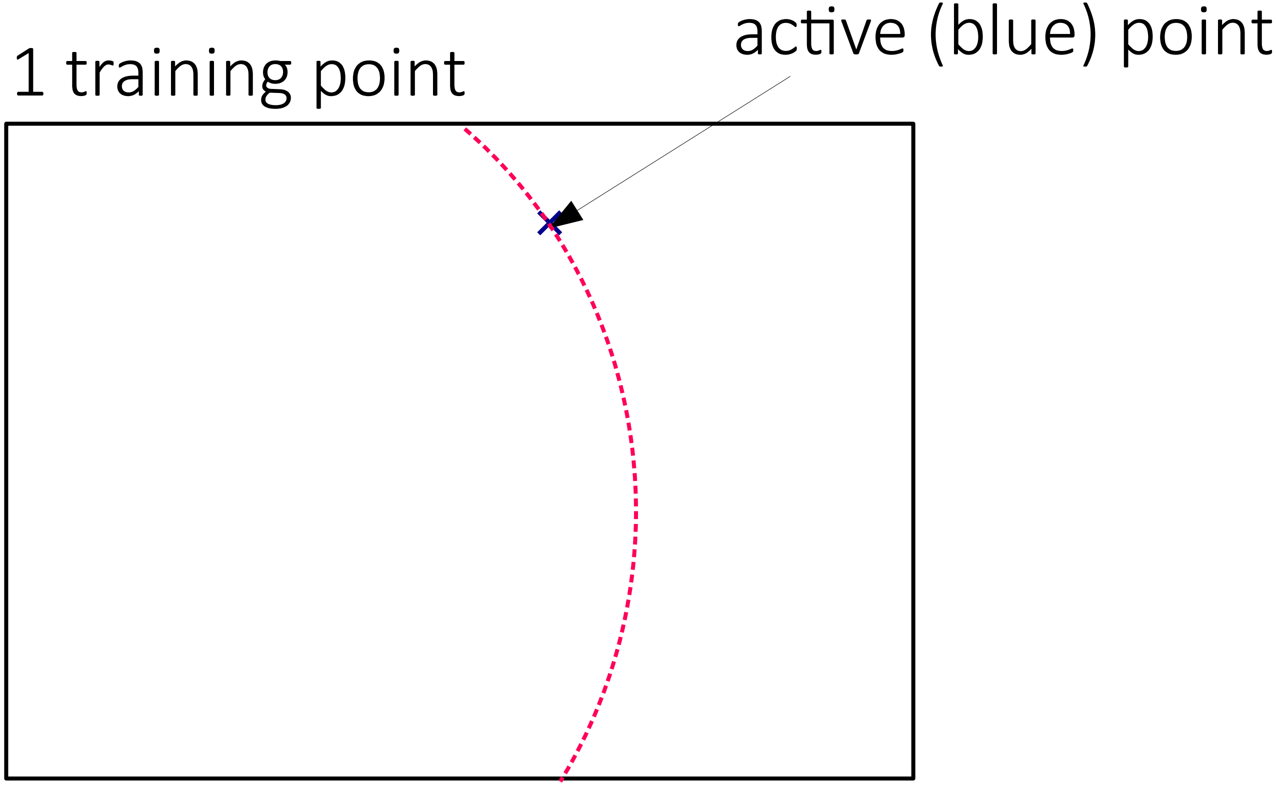

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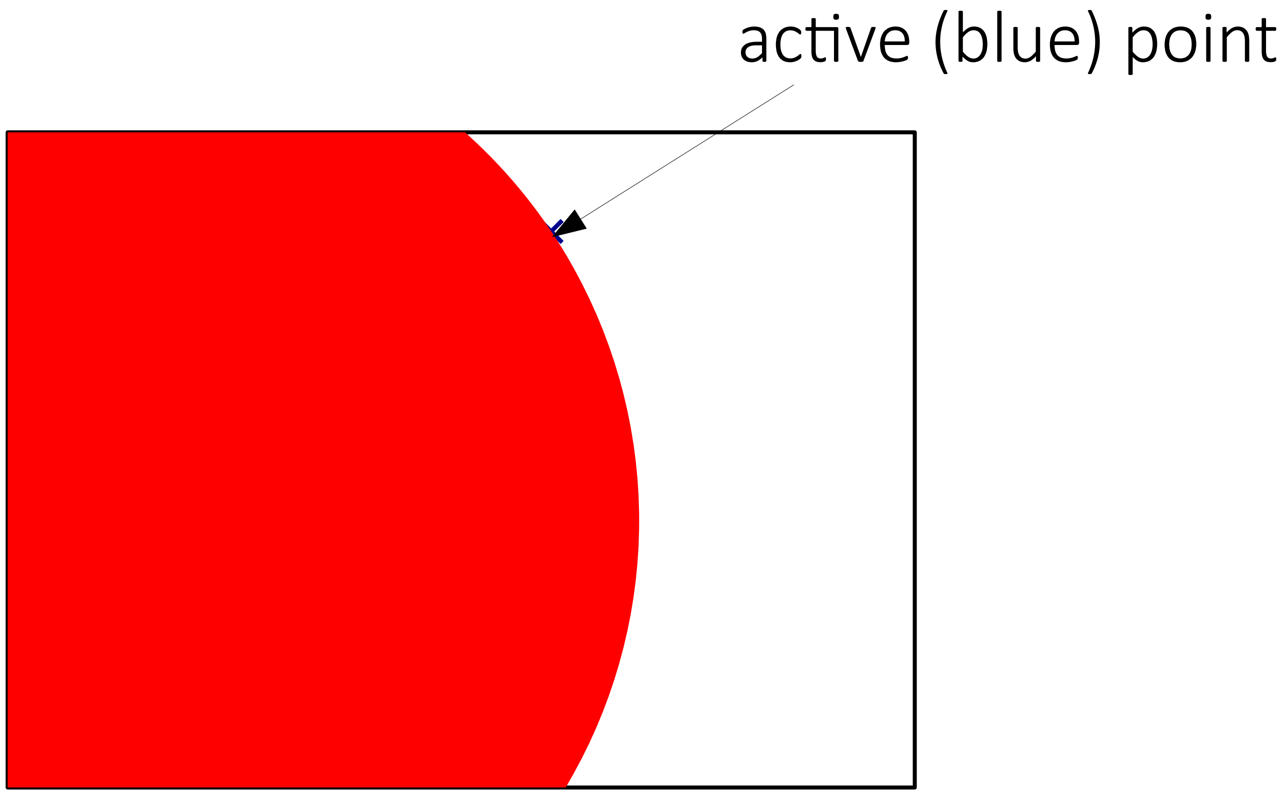
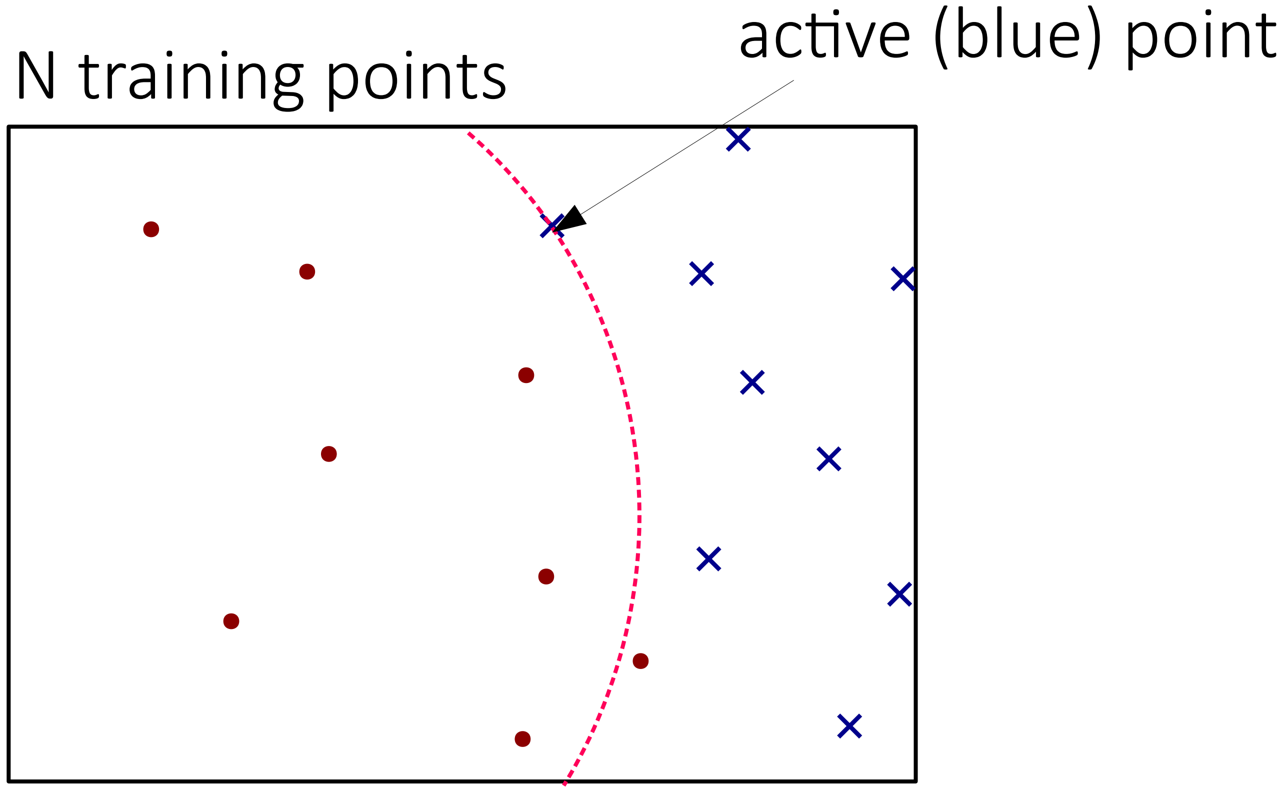
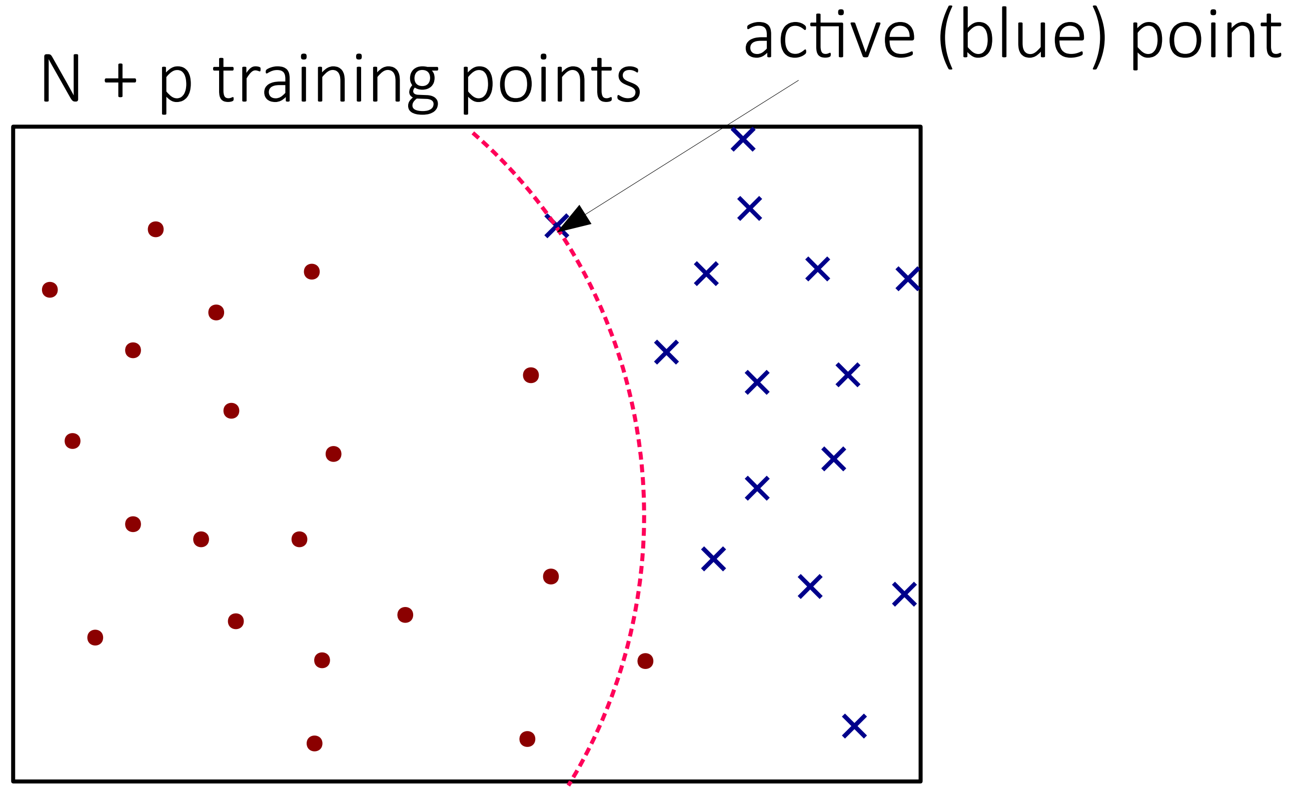


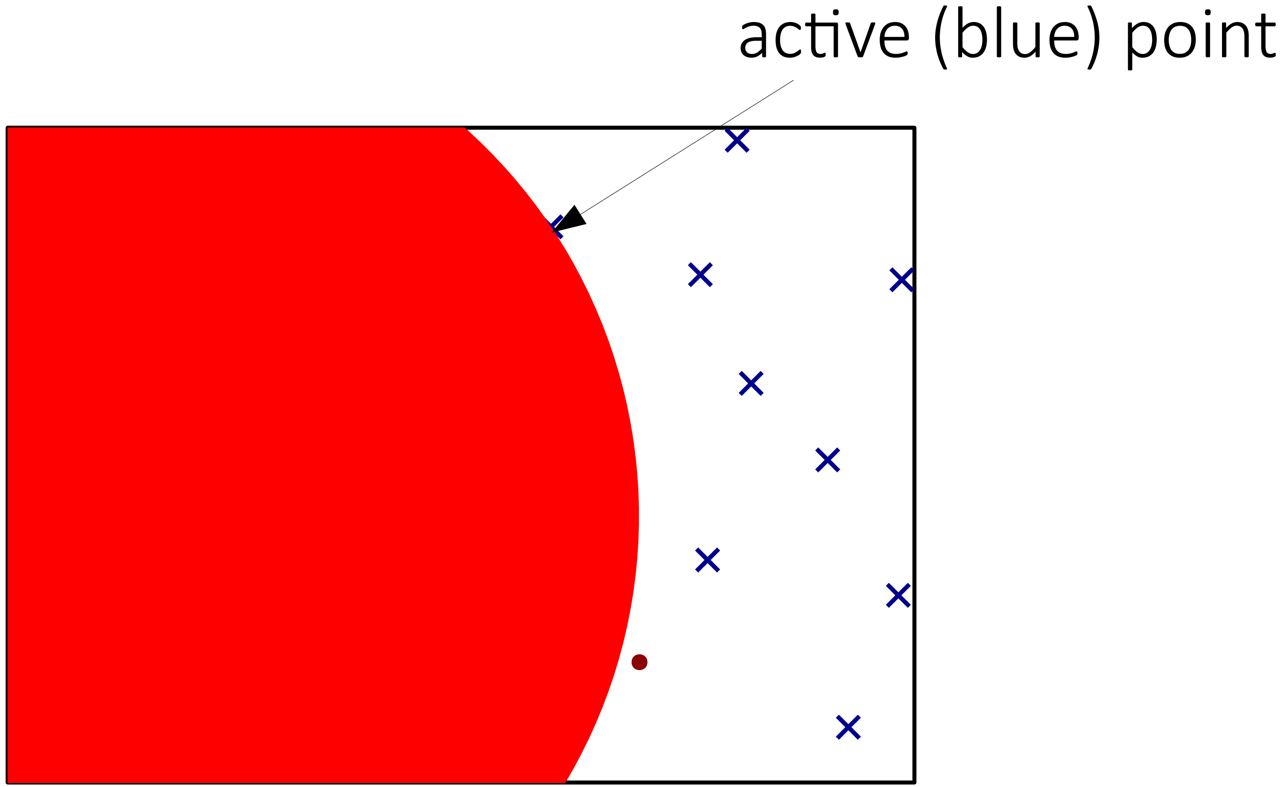
A taste of the theory



A taste of the theory



(2) Stability in the case of confirmation



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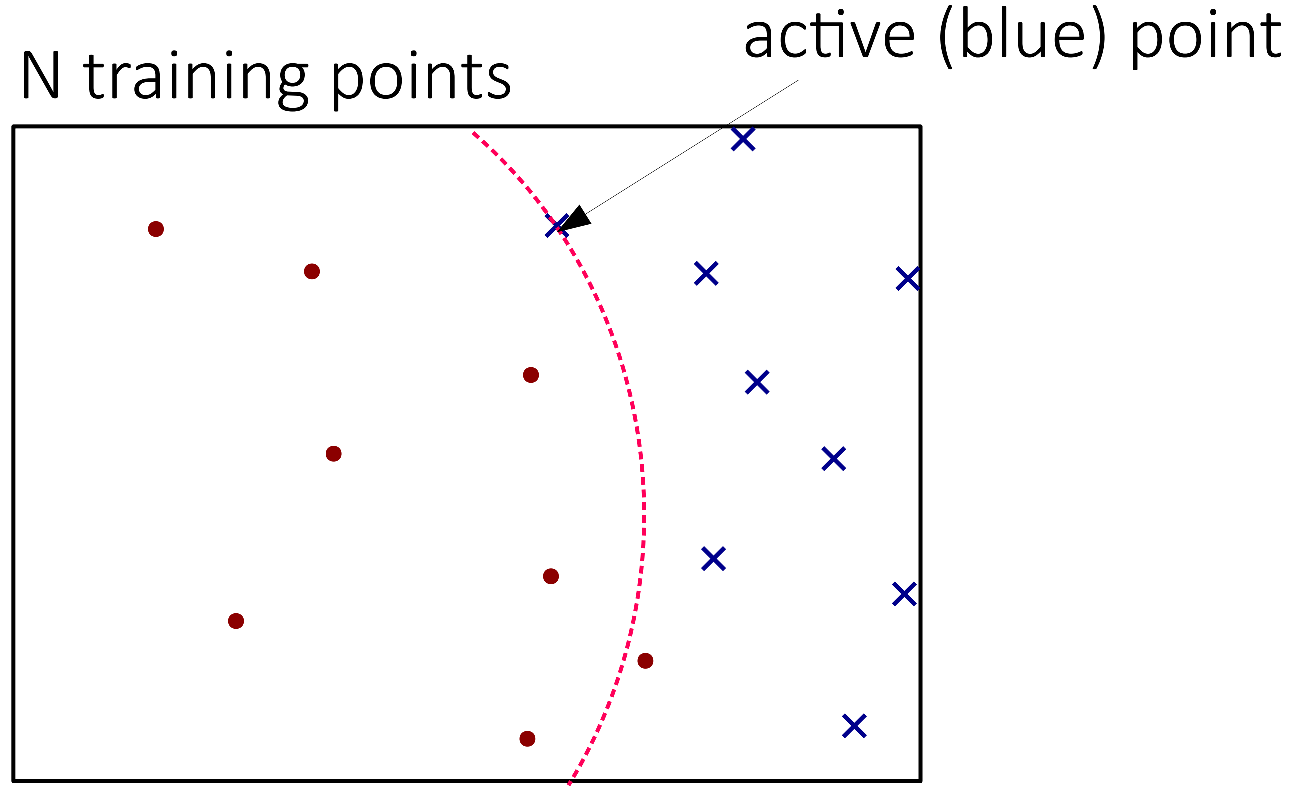
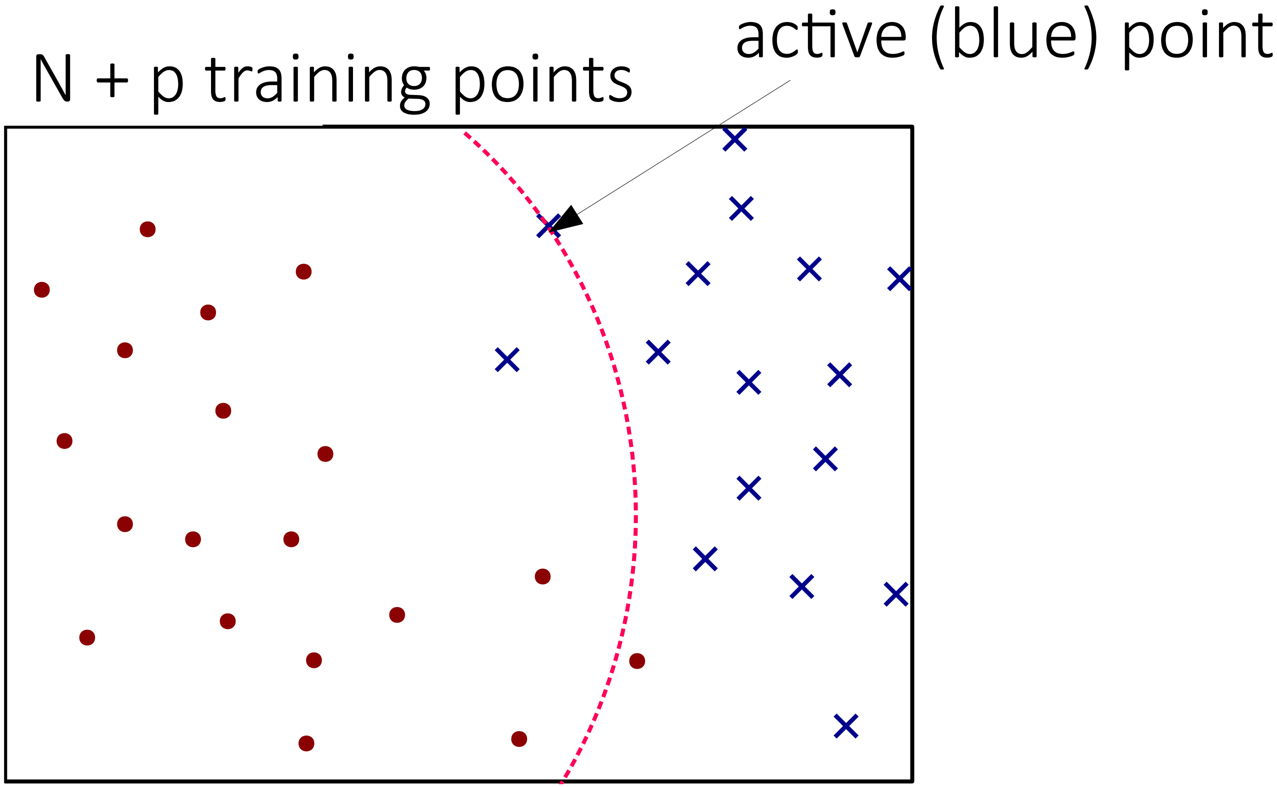
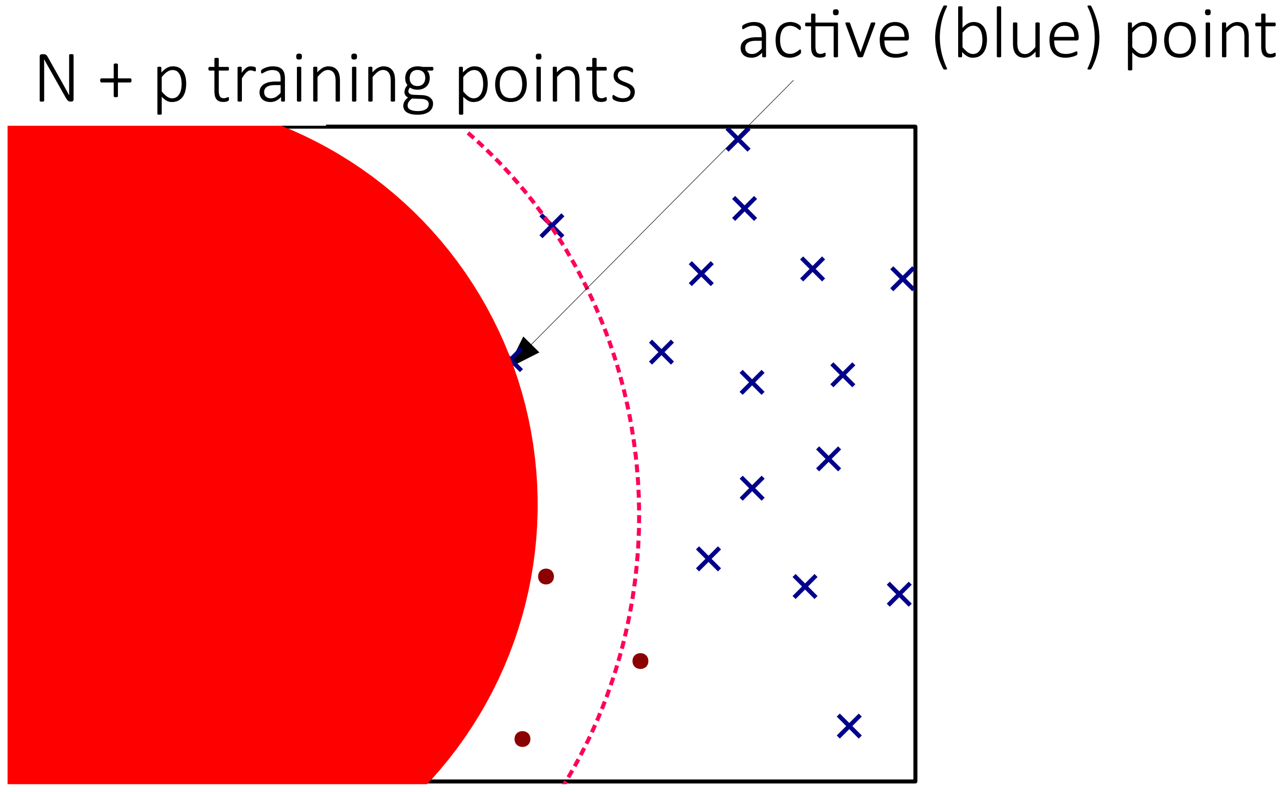


active (blue) point
N training points

A taste of the theory



(3) Responsiveness to contradiction



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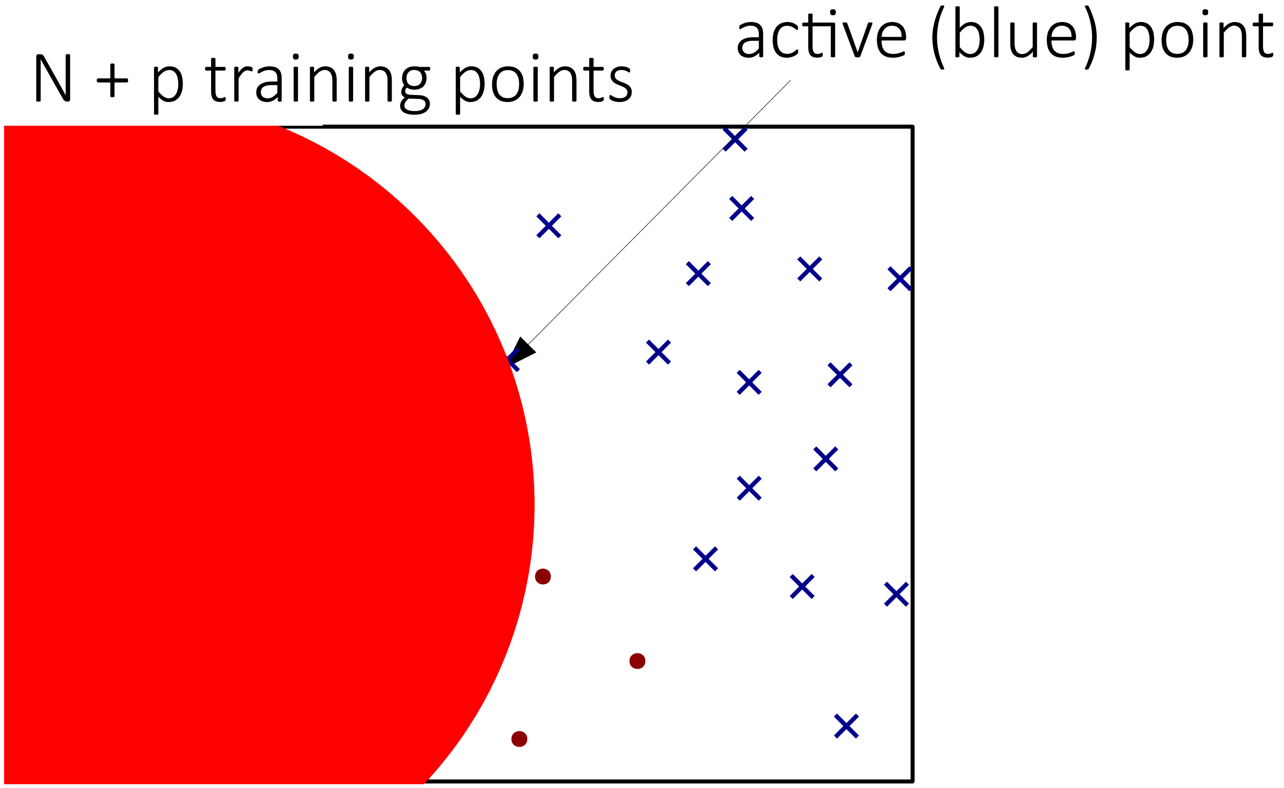

Thanks to (1),(2) and (3):

Take N+p points

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$$\Longrightarrow \quad PE(\hat{y}_N(x)) \sim \mathrm{Beta}[1, N]$$

Thank you!

Are you interested in knowing more?
Write to us!

algo.care@unibs.it

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Bibliography about GEM

GEM-BALLS

- "A new classification algorithm with guaranteed sensitivity and specificity for medical applications"
  A. Carè, F.A. Ramponi, M.C. Campi
  IEEE Control Systems Letters. 2(3):393-398 [doi: 10.1109/LCSYS.2018.2840427], 2018

The original GEM

- "Classification with Guaranteed Probability of Error"
  M.C. Campi
  Machine Learning, 80:63-84, 2010.

More about the VF application

- "Ventricular defibrillation: Classification with GEM and a roadmap for future investigations"
  F. Baronio, M. Baronio, M.C. Campi, A. Carè, S. Garatti, G. Perone
  2017 IEEE 56th Annual Conference on Decision and Control (CDC), 2718-2723

Bibliography about GEM

New lines of research

- "Novel bounds on the probability of misclassification in majority voting: leveraging the majority size"
  A.T.J.R. Cobbenhagen, A. Carè, M.C. Campi, F.A. Ramponi, D.J. Antunes, W.P.M.H. Heemels
  IEEE Control Systems Letters, vol. 5, no. 5, pp. 1513-1518, 2020.

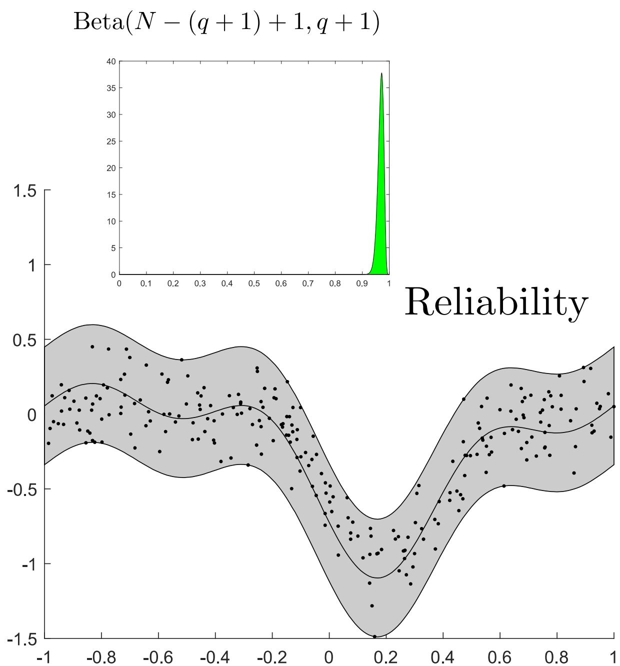
- "A study on majority-voting classifiers with guarantees on the probability of error"
  by A. Carè, M.C. Campi, F.A. Ramponi, S. Garatti, A.T.J.R. Cobbenhagen
  IFAC World Congress 2020.

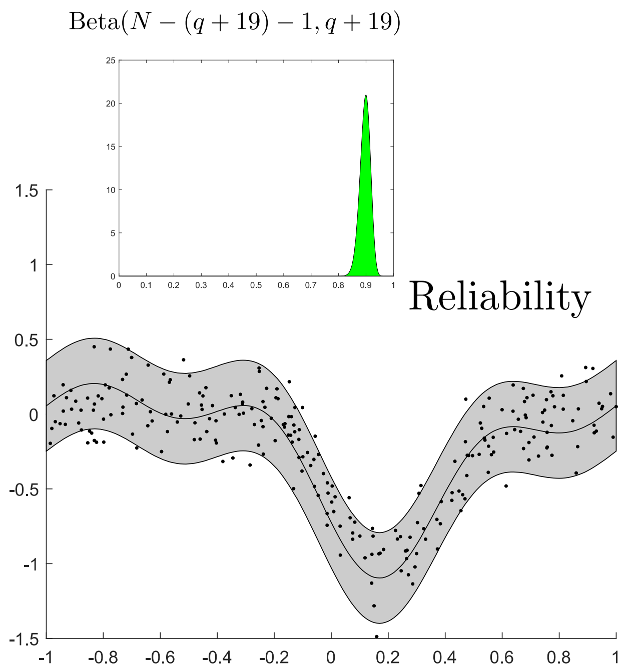
- "Consensus and Reliability: The Case of Two Binary Classifiers"
  A.T.J.R. Cobbenhagen. A. Carè, M.C. Campi, F.A. Ramponi, W.P.M.H. Heemels
  8th IFAC Workshop on Distributed Estimation and Control in Networked Systems
  Sept. 16-17, 2019, Chicago, IL, USA.

Bibliography beyond GEM

Regression (L-infinity)

- "On a class of Interval Predictor Models with universal reliability,"
  S. Garatti, M.C. Campi, A. Carè
  Automatica. 110, 108542 [doi: 10.1016/j.automatica.2019.108542], 2019.





Bibliography beyond GEM

Other decision schemes with strong generalization properties

- “Scenario optimization with relaxation: a new tool for design and application to machine learning problems”
  M.C. Campi, S. Garatti
  2020 59th IEEE Conference on Decision and Control (CDC), 2463-2468

- Book:
  “Introduction to the scenario approach”
  M.C. Campi, S. Garatti.
  SIAM, 2018

- Forthcoming review paper:
  “Data-driven decision-making and the scenario approach“

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