MATH 2360-111, Linear Algebra

Review 2

October 28, 2019

1. If A and B are 2×2 matrices, $\det(A) = 10$, $\det(B) = -5$, then

   $\det(AB) =$ _____,
   $\det(3A) =$ _____,
   $\det(A^T) =$ _____,
   $\det(B^{-1}) =$ _____,
   $\det(B^2) =$ _____.

2.
   - If the determinant of a 4×4 matrix A_1 is $\det(A_1) = -3$, and the matrix C_1 is obtained from A_1 by swapping the first and second rows, then $\det(C_1) =$ ___.

     Answer: 3

   - If the determinant of a 3×3 matrix A_2 is $\det(A_2) = 8$, and the matrix B_2 is obtained from A_2 by multiplying the first row by 4, then $\det(B_2) =$ ___.

     Answer: $4 * 8 = 32$

   - If the determinant of a 4×4 matrix A_3 is $\det(A_3) = 5$, and the matrix D_3 is obtained from A_3 by adding 9 times the third row to the second, then $\det(D_3) =$ ___.

     Answer: 5

3. Determine if the subset of $\mathbb{R}^3$ consisting of vectors of the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where at most one of a,b and c is non-zero, is a subspace.
Select YES or NO for each statement. Explain your answer.
    (a) This set is a subspace.
    (b) This set is closed under scalar multiplication.
    (c) This set is closed under addition.
    (d) This set contains the zero vector.

4. Determine whether the following sets form subspaces of $\mathbb{R}^3$. Explain your answers.

   (a) $\{(x_1, x_2, x_3)^T$ such that $x_1 + x_3 = 1\}$ ☐ YES ☐ NO

   (b) $\{(x_1, x_2, x_3)^T$ such that $x_1 = x_2 = x_3\}$ ☐ YES ☐ NO

   Answer:

   a) NO

   $$\alpha \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha x_1 + \alpha x_3 \neq 1$$

   b) YES

   $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1 + y_1) = (x_2 + y_2) = (x_3 + y_3)$$

   $$\alpha \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha x_1 = \alpha x_2 = \alpha x_3$$

   $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is in the set.

5. Determine whether the following is a spanning set for $\mathbb{R}^3$. Explain your answer.
   $\{(-2, 1, 0)^T, (2, -1, 1)^T, (2, 1, 3)^T\}$ ☐ YES ☐ NO

6. Let $\mathbf{u} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 8 \\ 22 \end{bmatrix}$.

   We want to determine by inspection (with minimal computation) if the set $\{\mathbf{u}, \mathbf{v},\}$ is linearly dependent or independent.

7. Find a basis for the space of 2×2 lower triangular matrices.

   Answer:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

8. Find the coordinate representation of $\begin{bmatrix} -3 \\ 3 \\ 9 \end{bmatrix}$ in the following ordered basis $\mathcal{B}$ of $\mathbb{R}^3$.

$$\mathcal{B} = \left(\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right).$$

9. Let $A = \begin{bmatrix} \text{-9} & \text{-6} \\ 18 & 12 \\ 3 & 2 \end{bmatrix}$.

   Find a basis and the dimension for the column space of A (you should basically need no computation to solve this problem. Explain your answer).

10. Consider the following two ordered bases of $\mathbb{R}^2$:

$$\begin{aligned} \mathcal{B} &= \left(\begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right), \\ \mathcal{C} &= \left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right). \end{aligned}$$

Find the transition matrix from the basis $\mathcal{B}$ to the basis $\mathcal{C}$.

11. Find a basis of the row space of the matrix

$$A = \begin{bmatrix} -4 & 0 & 2 & 2 \\ -4 & 4 & -1 & -1 \\ 0 & -4 & 3 & 3 \end{bmatrix}.$$

Answer:

$$\begin{pmatrix} -4 & 0 & 2 & 2 \\ -4 & 4 & -1 & -1 \\ 0 & -4 & 3 & 3 \end{pmatrix} \to \begin{pmatrix} -4 & 0 & 2 & 2 \\ 0 & 4 & -3 & -3 \\ 0 & -4 & 3 & 3 \end{pmatrix} \to \begin{pmatrix} -4 & 0 & 2 & 2 \\ 0 & 4 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -2 & 0 & 1 & 1 \\ 0 & 4 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Basis; $\begin{pmatrix} 2 \\ 0 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ -3 \\ -3 \end{pmatrix}$