# CS 5381

# Analysis of Algorithms

1. Select the correct answer (1 point for each question)
   1. Which of the following running time is :

a). b).

c). d). Both b) and c)

e). Both a) and b)

* 1. Which of the following running time is polynomial in

a). b).

c). d). Both b) and c)

e). Both a) and b)

* 1. Consider an undirected graph with vertices and edges, which is of the following conditions guarantee that is a tree:

a). does not contain any cycle and

b). does not contain any cycle and connected

c). is connected and

d). All of the above.

* 1. Consider the clustering algorithm (in Slides GreedyAlgorithmII, page 62). Suppose we are given a set of points in -dimensional Euclidean space, with the distance of any two points given by the standard -norm, i.e., . We want to partition points into clusters of maximum spacing, which of the following statement is true:

a). The clustering algorithm does not work anymore

b). The clustering algorithm only works for , i.e., when points are on the plane.

c). The clustering algorithm works, but its running time is exponential in , and is thus only in polynomial time when is a constant

d). The clustering algorithm works and runs in polynomial time for any input

1.5 Below is the Gale-Shapley algorithm.

Text

Description automatically generated

Which of the following statement is true:

a). Gale-Shapley algorithm does not necessarily finds a matching between hospitals and students

b). Gale-Shapley algorithm always finds a matching between hospitals and students, however, it may not be a stable matching

c). Gale-Shapley algorithm always finds a stable matching between hospitals and students, however, different execution of the algorithm may return different stable matchings as the while loop does not specify which hospital to choose.

d). Any execution of the above Gale-Shapley algorithm always finds the same stable matching between hospitals and students,

1.6 Consider the following graph

Chart

Description automatically generated

Which statement is true:

a). This is a bipartite graph and also a tree

b). This is a bipartite graph but not a tree

c). This is a tree but not a bipartite graph

d). This is not a tree and not a bipartite graph

1.7 Consider a graph whose vertices can be colored into three colors: red, blue and black. All the edges are either between a red vertex and a blue vertex, or a blue vertex and a black vertex. See below as an example.

Chart

Description automatically generated with medium confidence

Which of the following statement is true?

a). Such a graph can have both odd cycle and even cycle

b). Such a graph can have an even cycle but cannot have an odd cycle

c). Such a graph can have an odd cycle but cannot have an even cycle

d). Such a graph cannot have odd cycle nor even cycle

1.8 Consider the following graph and the cut

Chart

Description automatically generated

Which edge is in the cut set of :

a).

b).

c).

d).

1.9 Consider the following graph.

Chart

Description automatically generated

Which of the following edge set can belong to a spanning tree:

a).

b).

c).

d).

1.10 Consider the problem of finding the longest path in an edge-weighted graph . Recall Dijkstra's algorithm for finding the shortest path. Which of the following statement is true:

a). We can first change the edge weight in from to , then apply Dijkstra's algorithm. This gives the longest path in .

b). Dijkstra's algorithm does not work on negative weights. However, we can pick an arbitrary large number and change the edge weight in from to , then apply Dijkstra's algorithm. This gives the longest path in .

c). Dijkstra's algorithm does not work since we cannot transform longest path to shortest path by changing to .

d). Dijkstra's algorithm does not work in general, but if for all edges, then by changing to and run Dijkstra's algorithm we get the longest path.

1. (2 points) Find out a stable matching for the bipartite graph with the following preference list:

A:

B:

C:

X:

Y:

Z:

1. (2 points) Consider the following directed graph. Compute the shortest path from A to F

Chart, scatter chart

Description automatically generated

1. (3 points) Consider interval scheduling and interval partition problem (See slides GreedyAlgorithmI, page 9). We designed greedy algorithms for the two problems based on earliest-finish-time first and earliest-start-time first, respectively. What if we use the order of earliest-start-time first for the interval scheduling problem, and use the order of earliest-finish-time for interval partition? Can we still obtain optimal solution? State yes or no for each of the two problems. If the answer is yes, give an informal argument, if the answer is no, give a counter-example.
2. (3points) There are N workers. The i-th worker has a quality[i] and a minimum wage expectation wage[i]. Now we want to hire exactly K workers to form a paid group. When hiring a group of K workers, we must pay them according to the following rules:
3. Every worker in the paid group should be paid in the ratio of their quality compared to other workers in the paid group.
4. Every worker in the paid group must be paid at least their minimum wage expectation.

Q: Determine the least amount of money needed to form a paid group satisfying the above conditions.

Example 1: Input: quality = [10,20,5], wage = [70,50,30], K = 2

Output: 105.00000

Explanation: We pay 70 to 0-th worker and 35 to 2-th worker.

Example 2: Input: quality = [3,1,10,10,1], wage = [4,8,2,2,7], K = 3

Output: 30.66667

Explanation: We pay 4 to 0-th worker, 13.33333 to 2-th and 3-th workers separately.

Design an algorithm for the above problem which runs in polynomial time in the input. (Hint: consider the paid ratio of each worker in the solution for the example, do they always equal wage[i]/quality[i] for some i? Why?)