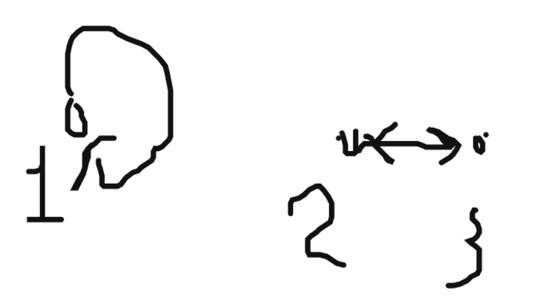
16.11.2025 Циклические группы, система порождающих

$M = \{1, 2, \dots, n\}$

$S_n = \{ f: M \to M \mid f$ - биекция $\}$ - подстановка n-й степ

$\sigma = \begin{pmatrix} 1 & 2 & \dots & n \\ \sigma(1) & \sigma(2) & & \sigma(n) \end{pmatrix}$

$|S_n| = n! = n(n-1)\dots 1$

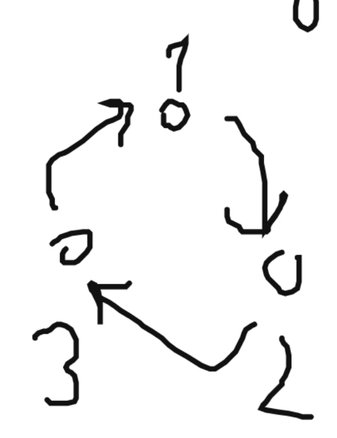


$n=3$ $\left\{ id = \sigma_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \sigma_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = (123) \right.$

$\sigma_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \sigma_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \sigma_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \Big\} = \{ \}$

(12)

~~(1)~~ (23) - цикл длины 2 - транспозиция



$= (132) = (13)(12)$ - чет

(13) - нечет

$\sigma_1\sigma_2(x) = \sigma_1(\sigma_2(x))$ - прав. умн.

$\sigma_1 \sigma_2 (x) = \sigma_2 \circ \sigma_1 (x) = \sigma_2(\sigma_1(x))$

$\sigma_2 \circ \sigma_5 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 \\ \downarrow & \downarrow & \downarrow \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = \sigma_6 \ne \sigma_5 \circ \sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = \sigma_4$

$\sigma_2 \sigma_5 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = \sigma_4$

~~(1)~~ $(23) \circ (123) = (13)$ ~~(2)~~

$o((i_1 i_2 \dots i_k)) = k$

$(123)^2 = (123)(123) = (132)$

$(123)^3 = (123)^2 (123) = (132)(123) = ~~(1)(2)(3)~~ = id$

$o((123)) = 3$

незав. циклы

$\sigma \in S_6$, $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 6 & 5 \end{pmatrix} = (12)(34)(56) = (56)(12)(34)$

$\sigma = \sigma_1 \sigma_2 \dots \sigma_s$, σ_i незав, $o(\sigma_i) = n_i$

$o(\sigma) = НОК(n_1, n_2, \dots, n_s)$

$o(\sigma) = НОК(2, 2, 2) = 2$

$\sigma^2 = id$

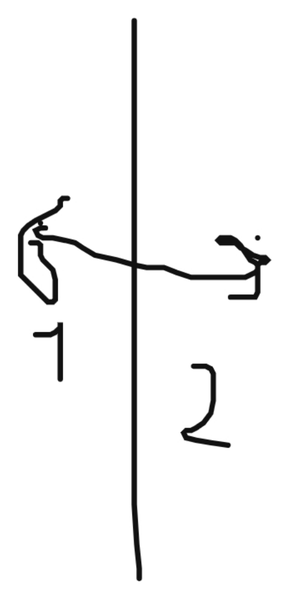
$\sigma^{133} = \sigma^{2 \cdot 66 + 1} = (\sigma^2)^{66} \sigma = id \cdot \sigma = \sigma$, $\sigma^2 = id$

$(ab)^{-1} = b^{-1} a^{-1}$

$S_6 = \{ \underbrace{id, (123), (132)}, \underbrace{\sigma_2, \sigma_3, \sigma_4} \}$

$\sigma_1\ \sigma_5\ \sigma_6$

знакопер. гр. четн. подст.



$S_n = A_n \sqcup S_n \setminus A_n$, $|A_n| = \frac{|S_n|}{2} = \frac{n!}{2}$

$\sigma_1 \sigma_2^{-1} \in A_n$ $\{ Ч \} \to \{ Н \}$, $\sigma \mapsto \sigma(ij)$ биекция

$H \ne \boxed{B_n}$

$id = (12)(12) = (12)(12)(12)(12) = (ij)(ij)$

Теор $\forall \sigma = \underbrace{(i_1 j_1) \dots (i_s j_{s+1})}_{m}$ - m трансп. $m = 2k \Rightarrow \sigma$ - четн.

$m = 2l+1 \Rightarrow \sigma$ нечетн

$(i_1 i_2 \dots i_s) = (i_1 i_2)(i_1 i_3) \dots (i_1 i_s) = (i_1 i_2 i_3 \dots i_s)$

$\underbrace{(\ )(\ )(\ ) \quad (\ )}$

$\emptyset \ne S \subseteq \langle G, \cdot \rangle$ подмн. $S = \{g_1, \dots, g_k\}$

$\langle S \rangle = \langle H = \{ g_1^{\alpha_1} \dots g_k^{\alpha_k} \mid \alpha_i \in \{\pm 1, 0\} \}, \cdot \rangle$ - подгруппа порожд $\langle S \rangle$

$S_n = \langle \{(ij) \mid i, j \in M\} \rangle$

$S_3 = \langle \{(12), (13), (23)\} \rangle = \langle (12), (123) \rangle$

$id = (12)(12)$

$(132) = (123)^{-1}$

$(13) = (12)(123)$

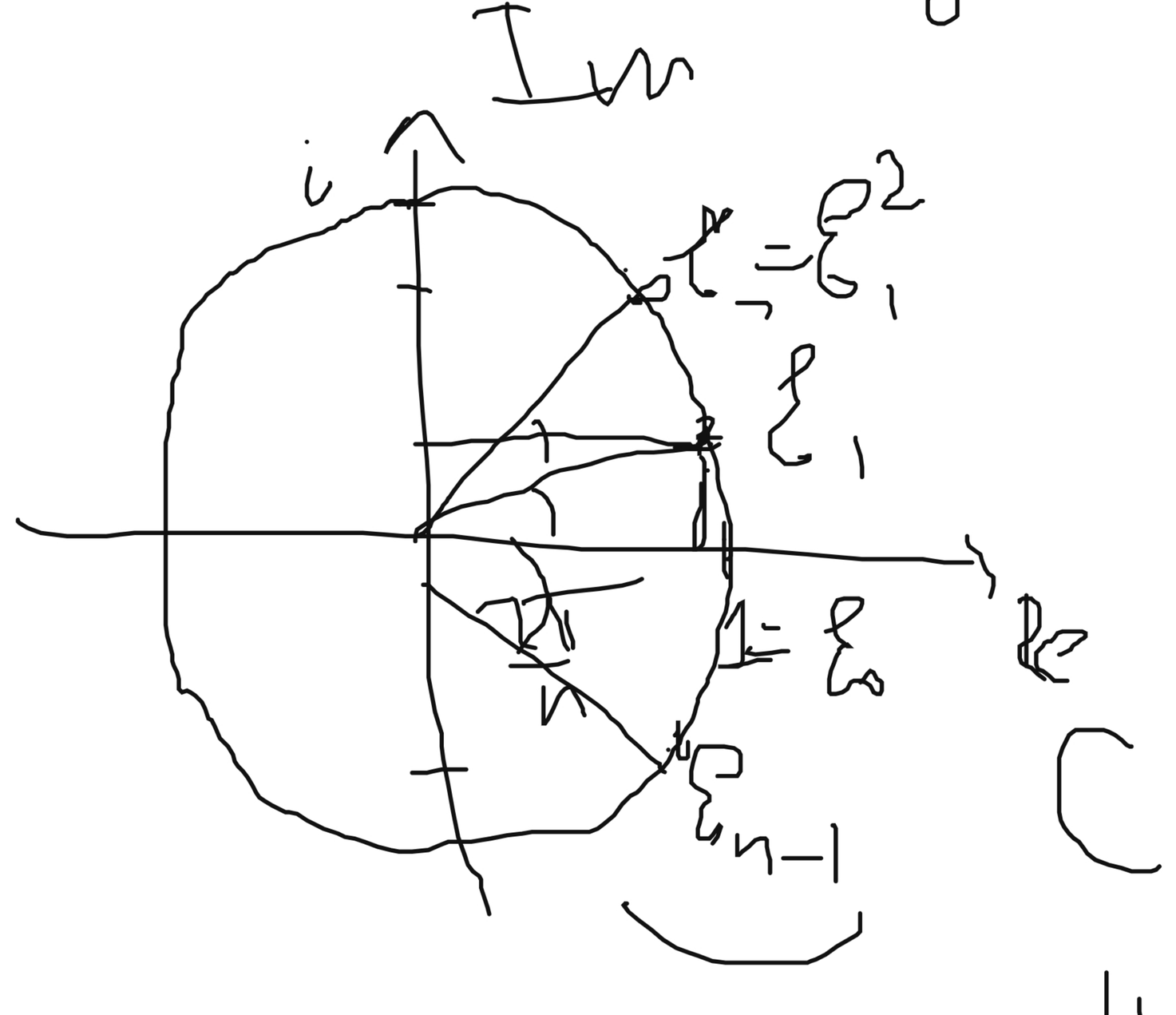
$(23) = (12)(123)^{-1} = (12)(132)$ ~~=~~

$S = \{g\}$ $\langle S \rangle = \langle g \rangle = \{ g^k \mid k \in \mathbb{Z} \}$ - цикл. гр

$\mathbb{Z} = \langle 1 \rangle = \{ n \cdot 1 \mid n \in \mathbb{Z} \}$ - бескон. цикл. гр

коммут. $\underbrace{1 + \dots + 1}$ $n>0$, $n=0$, $n<0$

$\langle A \rangle = \mathbb{Z}_n = \langle \bar{1} \rangle = \{ k \cdot \bar{1} \mid k = \overline{0, n-1} \}$ - кон



$C_n = \{ \varepsilon_k = \cos \frac{2\pi k}{n} + i \sin \frac{2\pi k}{n} \mid k = \overline{0, n-1} \}$, $\varepsilon_k \mapsto \bar{k}$

мнит. часть

$\langle \varepsilon_{n-1} \rangle = \langle \varepsilon_1 \rangle = \{ \varepsilon_0 = \varepsilon_1^0 = 1, \varepsilon_1^1 = \varepsilon_1, \varepsilon_2 = \varepsilon_1^2, \dots, \varepsilon_1^{n-1} \} = \langle \varepsilon_m \rangle$ - ~~первоо~~ образ. кор

$(m, n) = 1$