1 Proof

[882, 29 → 1056, resolution]

$$\frac{p(x_0, \sigma_0(x_0)) \quad \neg p(\sigma_3(a0), x_1)}{\square}$$

[20 → 29, cnf transformation]

$$\frac{(\forall x_1)\neg p(\sigma_3(a0), x_1)}{\neg p(\sigma_3(a0), x_1)}$$

[669, 32 → 882, resolution]

$$\frac{\neg p(z, x_1) \vee p(x_0, \sigma_0(x_0)) \quad p(z, z)}{p(x_0, \sigma_0(x_0))}$$

[31, 21 → 32, superposition]

$$\frac{p(add(x_1, x_1), x_1) \quad add(z, x_0) = x_0}{p(z, z)}$$

[1 → 21, cnf transformation]

$$\frac{(\forall x_0)add(z, x_0) = x_0}{add(z, x_0) = x_0}$$

[25 → 31, equality resolution]

$$\frac{p(x_0, x_1) \vee add(x_1, x_1) \neq x_0}{p(add(x_1, x_1), x_1)}$$

[13 → 25, cnf transformation]

$$\frac{(\forall x_0 \forall x_1)((\neg p(x_0, x_1) \vee add(x_1, x_1) = x_0 \vee s(add(x_1, x_1)) = x_0) \wedge ((add(x_1, x_1) \neq x_0 \wedge s(add(x_1, x_1)) \neq x_0) \vee p}{p(x_0, x_1) \vee add(x_1, x_1) \neq x_0}$$

[584, 28 → 669, resolution]

$$\frac{p(s(\sigma_1(a0)), \sigma_2(a0)) \quad \neg p(s(\sigma_1(a0)), x_3) \vee p(x_0, \sigma_0(x_0)) \vee \neg p(z, x_5)}{\neg p(z, x_1) \vee p(x_0, \sigma_0(x_0))}$$

[18 → 28, cnf transformation]

$$\frac{(\forall x_5)\neg p(z, x_5) \vee (p(\sigma_1(a0), \sigma_2(a0)) \wedge (\forall x_3)\neg p(s(\sigma_1(a0)), x_3)) \vee (\forall x_0)p(x_0, \sigma_0(x_0))}{\neg p(s(\sigma_1(a0)), x_3) \vee p(x_0, \sigma_0(x_0)) \vee \neg p(z, x_5)}$$

[30, 515 → 584, superposition]

$$\frac{p(s(add(x_1, x_1)), x_1) \quad \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}{p(s(\sigma_1(a0)), \sigma_2(a0))}$$

[454, 29 → 515, resolution]

$$\frac{\begin{array}{c} p(x_0, \sigma_0(x_0)) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0)) \\ \neg p(\sigma_3(a0), x_1) \end{array}}{\sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}$$

[221, 32 → 454, resolution]

$$\frac{\begin{array}{c} \neg p(z, x_1) \vee p(x_0, \sigma_0(x_0)) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0)) \\ p(z, z) \end{array}}{p(x_0, \sigma_0(x_0)) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}$$

[158, 28 → 221, resolution]

$$\frac{\begin{array}{c} p(s(\sigma_1(a0)), s(\sigma_2(a0))) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0)) \\ \neg p(s(\sigma_1(a0)), x_3) \vee p(x_0, \sigma_0(x_0)) \vee \neg p(z, x_5) \end{array}}{\neg p(z, x_1) \vee p(x_0, \sigma_0(x_0)) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}$$

[45, 64 → 158, superposition]

$$\frac{\begin{array}{c} p(s(s(add(x_1, x_1))), s(x_1)) \\ \sigma_1(a0) = s(add(\sigma_2(a0), \sigma_2(a0))) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0)) \end{array}}{p(s(\sigma_1(a0)), s(\sigma_2(a0))) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}$$

[24, 55 → 64, resolution]

$$\frac{\begin{array}{c} \neg p(x_0, x_1) \vee add(x_1, x_1) = x_0 \vee s(add(x_1, x_1)) = x_0 \\ p(\sigma_1(a0), \sigma_2(a0)) \end{array}}{\sigma_1(a0) = s(add(\sigma_2(a0), \sigma_2(a0))) \vee \sigma_1(a0) = add(\sigma_2(a0), \sigma_2(a0))}$$

[54, 29 → 55, resolution]

$$\frac{\begin{array}{c} p(x_0, \sigma_0(x_0)) \vee p(\sigma_1(a0), \sigma_2(a0)) \\ \neg p(\sigma_3(a0), x_1) \end{array}}{p(\sigma_1(a0), \sigma_2(a0))}$$

[27, 32 → 54, resolution]

$$\frac{\begin{array}{c} \neg p(z, x_5) \vee p(\sigma_1(a0), \sigma_2(a0)) \vee p(x_0, \sigma_0(x_0)) \\ p(z, z) \end{array}}{p(x_0, \sigma_0(x_0)) \vee p(\sigma_1(a0), \sigma_2(a0))}$$

[18 → 27, cnf transformation]

$$\frac{(\forall x_5)\neg p(z, x_5) \vee (p(\sigma_1(a0), \sigma_2(a0)) \wedge (\forall x_3)\neg p(s(\sigma_1(a0)), x_3)) \vee (\forall x_0) p(x_0, \sigma_0(x_0))}{\neg p(z, x_5) \vee p(\sigma_1(a0), \sigma_2(a0)) \vee p(x_0, \sigma_0(x_0))}$$

[13 → 24, cnf transformation]

$$\frac{(\forall x_0 \forall x_1)((\neg p(x_0, x_1) \vee add(x_1, x_1) = x_0 \vee s(add(x_1, x_1)) = x_0) \wedge ((add(x_1, x_1) \neq x_0 \wedge s(add(x_1, x_1)) \neq x_0) \vee p}{\neg p(x_0, x_1) \vee add(x_1, x_1) = x_0 \vee s(add(x_1, x_1)) = x_0}$$

[41, 38 → 45, forward demodulation]

$$\frac{\begin{array}{c} p(s(add(x_1, s(x_1))), s(x_1)) \\ add(x_3, s(x_2)) = s(add(x_2, x_3)) \end{array}}{p(s(s(add(x_1, x_1))), s(x_1))}$$

[22, 23 → 38, superposition]

$$\frac{\begin{array}{c} add(s(x_0), x_1) = s(add(x_0, x_1)) \\ add(x_0, x_1) = add(x_1, x_0) \end{array}}{add(x_3, s(x_2)) = s(add(x_2, x_3))}$$

[3 → 23, cnf transformation]

$$\frac{(\forall x_0 \forall x_1) add(x_0, x_1) = add(x_1, x_0)}{add(x_0, x_1) = add(x_1, x_0)}$$

[2 → 22, cnf transformation]

$$\frac{(\forall x_0 \forall x_1) add(s(x_0), x_1) = s(add(x_0, x_1))}{add(s(x_0), x_1) = s(add(x_0, x_1))}$$

[31, 22 → 41, superposition]

$$\frac{\begin{array}{c} p(add(x_1, x_1), x_1) \\ add(s(x_0), x_1) = s(add(x_0, x_1)) \end{array}}{p(s(add(x_1, s(x_1))), s(x_1))}$$

[26 → 30, equality resolution]

$$\frac{p(x_0, x_1) \vee s(add(x_1, x_1)) \neq x_0}{p(s(add(x_1, x_1)), x_1)}$$

[13 → 26, cnf transformation]

$$\frac{(\forall x_0 \forall x_1)((\neg p(x_0, x_1) \vee add(x_1, x_1) = x_0 \vee s(add(x_1, x_1)) = x_0) \wedge ((add(x_1, x_1) \neq x_0 \wedge s(add(x_1, x_1)) \neq x_0) \vee p}{p(x_0, x_1) \vee s(add(x_1, x_1)) \neq x_0}$$

[2, input]

$$(\forall x_0 \forall x_1) add(s(x_0), x_1) = s(add(x_0, x_1))$$

[3, input]

$$(\forall x_0 \forall x_1) add(x_0, x_1) = add(x_1, x_0)$$

[14, 17, 16, 15 → 18, skolemisation]

$$\frac{\begin{array}{c} (\forall x_5) \neg p(z, x_5) \vee (\exists x_2)((\exists x_4) p(x_2, x_4) \wedge (\forall x_3) \neg p(s(x_2), x_3)) \vee (\forall x_0)(\exists x_1) p(x_0, x_1) \\ (\exists x_4) p(\sigma_1(a0), x_4) \to p(\sigma_1(a0), \sigma_2(a0)) \\ (\exists x_2)((\exists x_4) p(x_2, x_4) \wedge (\forall x_3) \neg p(s(x_2), x_3)) \to ((\exists x_4) p(\sigma_1(a0), x_4) \wedge (\forall x_3) \neg p(s(\sigma_1(a0)), x_3)) \\ (\forall x_0)((\exists x_1) p(x_0, x_1) \to p(x_0, \sigma_0(x_0))) \end{array}}{(\forall x_5) \neg p(z, x_5) \vee (p(\sigma_1(a0), \sigma_2(a0)) \wedge (\forall x_3) \neg p(s(\sigma_1(a0)), x_3)) \vee (\forall x_0) p(x_0, \sigma_0(x_0))}$$

[15, choice axiom]

$$(\forall x_0)((\exists x_1) p(x_0, x_1) \to p(x_0, \sigma_0(x_0)))$$

[16, choice axiom]

$$(\exists x_2)((\exists x_4) p(x_2, x_4) \wedge (\forall x_3) \neg p(s(x_2), x_3)) \to ((\exists x_4) p(\sigma_1(a0), x_4) \wedge (\forall x_3) \neg p(s(\sigma_1(a0)), x_3))$$

[17, choice axiom]

$$(\exists x_4) p(\sigma_1(a0), x_4) \to p(\sigma_1(a0), \sigma_2(a0))$$

[10 → 14, rectify]

$$\frac{(\forall x_3)\neg p(z,x_3) \vee (\exists x_0)((\exists x_1)p(x_0,x_1) \wedge (\forall x_2)\neg p(s(x_0),x_2)) \vee (\forall x_4)(\exists x_5)p(x_4,x_5)}{(\forall x_5)\neg p(z,x_5) \vee (\exists x_2)((\exists x_4)p(x_2,x_4) \wedge (\forall x_3)\neg p(s(x_2),x_3)) \vee (\forall x_0)(\exists x_1)p(x_0,x_1)}$$

[9 → 10, flattening]

$$\frac{((\forall x_3)\neg p(z,x_3) \vee (\exists x_0)((\exists x_1)p(x_0,x_1) \wedge (\forall x_2)\neg p(s(x_0),x_2))) \vee (\forall x_4)(\exists x_5)p(x_4,x_5)}{(\forall x_3)\neg p(z,x_3) \vee (\exists x_0)((\exists x_1)p(x_0,x_1) \wedge (\forall x_2)\neg p(s(x_0),x_2)) \vee (\forall x_4)(\exists x_5)p(x_4,x_5)}$$

[8 → 9, ennf transformation]

$$\frac{((\exists x_3)p(z,x_3) \wedge (\forall x_0)((\exists x_1)p(x_0,x_1) \rightarrow (\exists x_2)p(s(x_0),x_2))) \rightarrow (\forall x_4)(\exists x_5)p(x_4,x_5)}{((\forall x_3)\neg p(z,x_3) \vee (\exists x_0)((\exists x_1)p(x_0,x_1) \wedge (\forall x_2)\neg p(s(x_0),x_2))) \vee (\forall x_4)(\exists x_5)p(x_4,x_5)}$$

[5 → 8, rectify]

$$\frac{((\exists x_1)p(z,x_1) \wedge (\forall x_0)((\exists x_1)p(x_0,x_1) \rightarrow (\exists x_1)p(s(x_0),x_1))) \rightarrow (\forall x_0)(\exists x_1)p(x_0,x_1)}{((\exists x_3)p(z,x_3) \wedge (\forall x_0)((\exists x_1)p(x_0,x_1) \rightarrow (\exists x_2)p(s(x_0),x_2))) \rightarrow (\forall x_4)(\exists x_5)p(x_4,x_5)}$$

[5, input]

$$((\exists x_1)p(z,x_1) \wedge (\forall x_0)((\exists x_1)p(x_0,x_1) \rightarrow (\exists x_1)p(s(x_0),x_1))) \rightarrow (\forall x_0)(\exists x_1)p(x_0,x_1)$$

[12 → 13, flattening]

$$\frac{(\forall x_0 \forall x_1)((\neg p(x_0,x_1) \vee (add(x_1,x_1) = x_0 \vee s(add(x_1,x_1)) = x_0)) \wedge ((add(x_1,x_1) \neq x_0 \wedge s(add(x_1,x_1)) \neq x_0) \vee}{(\forall x_0 \forall x_1)((\neg p(x_0,x_1) \vee add(x_1,x_1) = x_0 \vee s(add(x_1,x_1)) = x_0) \wedge ((add(x_1,x_1) \neq x_0 \wedge s(add(x_1,x_1)) \neq x_0) \vee}$$

[4 → 12, nnf transformation]

$$\frac{(\forall x_0 \forall x_1)(p(x_0,x_1) \leftrightarrow (add(x_1,x_1) = x_0 \vee s(add(x_1,x_1)) = x_0))}{(\forall x_0 \forall x_1)((\neg p(x_0,x_1) \vee (add(x_1,x_1) = x_0 \vee s(add(x_1,x_1)) = x_0)) \wedge ((add(x_1,x_1) \neq x_0 \wedge s(add(x_1,x_1)) \neq x_0) \vee}$$

[4, input]

$$(\forall x_0 \forall x_1)(p(x_0,x_1) \leftrightarrow (add(x_1,x_1) = x_0 \vee s(add(x_1,x_1)) = x_0))$$

[1, input]

$$(\forall x_0)add(z,x_0) = x_0$$

[11, 19 → 20, skolemisation]

$$\frac{\begin{array}{c}(\exists x_0)(\forall x_1)\neg p(x_0,x_1) \\ (\exists x_0)(\forall x_1)\neg p(x_0,x_1) \rightarrow (\forall x_1)\neg p(\sigma_3(a0),x_1)\end{array}}{(\forall x_1)\neg p(\sigma_3(a0),x_1)}$$

[19, choice axiom]

$$(\exists x_0)(\forall x_1)\neg p(x_0,x_1) \rightarrow (\forall x_1)\neg p(\sigma_3(a0),x_1)$$

[7 → 11, ennf transformation]

$$\frac{\neg(\forall x_0)(\exists x_1)p(x_0,x_1)}{(\exists x_0)(\forall x_1)\neg p(x_0,x_1)}$$

[6 → 7, negated conjecture]

$$\frac{(\forall x_0)(\exists x_1)p(x_0,x_1)}{\neg(\forall x_0)(\exists x_1)p(x_0,x_1)}$$

[6, input]

$$(\forall x_0)(\exists x_1)p(x_0,x_1)$$