Предсказание продаж товара

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```
In [4]:
         from clickhouse driver import Client
         import pandas as pd
         import csv
         import matplotlib.pyplot as plt
         import numpy as np
         from statsmodels.tsa.seasonal import seasonal decompose
         from statsmodels.tsa.stattools import adfuller
         import statsmodels.api as sm
         import statsmodels.tsa.api as tsa
         from statsmodels.graphics.tsaplots import plot acf, acf, plot pacf, pacf
         from statsmodels.tsa.stattools import acf, q_stat, adfuller
         from scipy.stats import probplot, moment
         import pmdarima.arima
         import warnings
         from statsmodels.tools.sm exceptions import ConvergenceWarning
         warnings.simplefilter('ignore', ConvergenceWarning)
         from catboost import CatBoostRegressor
         from datetime import date, datetime, timedelta
         from sklearn.metrics import make_scorer, mean_squared_error
         from sklearn.model selection import train test split
         from sklearn.model_selection import TimeSeriesSplit
         from joblib import Parallel, delayed
         %matplotlib inline
         plt.style.use('ggplot')
         from tensorflow import keras
In [5]:
         from google.colab import drive
         drive.mount('/content/drive')
        Mounted at /content/drive
In [6]:
         #PATH = 'Datasets/imprice/' # local
         PATH = '/content/drive/MyDrive/DataScience/imprice/' # colab
```

Постановка задачи

Требуется провести анализ данных и сделать предсказания продаж в штуках на последующую неделю для товара 7d185936-7a60-11eb-ba7f-4a6a34607ded.

Загрузка данных

Загрузим датасеты из файлов.

```
In [21]:
    products = pd.read_csv(PATH + 'products.csv')
    sales = pd.read_csv(PATH + 'sales.csv')
    stocks_items = pd.read_csv(PATH + 'stocks_items.csv')
    stocks_on_days = pd.read_csv(PATH + 'stocks_on_days.csv')
    warehouse = pd.read_csv(PATH + 'warehouse.csv')
```

Посмотрим на загруженные датасеты и предвариельно проанализируем данные, с которыми в дальнейшем буде работать.

Импортируем датасеты для товара 7d185936-7a60-11eb-ba7f-4a6a34607ded.

In [22]: products
Out[22]: item_id brand_id name

[22].				
	0	7d185936-7a60-11eb-ba7f- 4a6a34607ded	a2c5d335-815e-11eb-ba83- 4a6a34607ded	NaN

В таблице products мы ожидаемо получили одну строку с идентификатором бренда, соответствующим нашему товару. Для наших целий этот датасет не пригодится.

In [23]: sales.head(5)

Out[23]:		date	warehouse_id	item_id	order_number	price	quantity	amoun
	0	2021-01- 04 00:00:00	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	3547355649947603906	0.00	1	59.99
	1	2021-01- 04 00:00:00	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	4908248033917346488	0.00	1	59.99
	2	2021-01- 04 00:00:00	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1134184689814738432	0.00	1	59.99
	3	2021-01- 04 00:00:00	bb818041- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	765388806314934293	0.00	1	59.99
	4	2021-01- 04 00:00:00	d91c6b6b- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	15616877688297015768	59.99	1	59.99

В таблице **sales** содержатся данные о продажах. А именно:

- date дата продажи
- warehouse_id идентификатор склада,
- item_id идентификатор нашего товара,
- order_number номер заказа,
- price цена единицы товара,
- quantity количество единиц товара в заказе,
- amount стоимость заказа (цена, умноженная на количество единиц товара в заказе),
- sebes себестоимость единицы товара

In [24]:

stocks items

Out[24]:		stock_id	item_id	quantity
	0	4787ce96-7a69-11eb-ba7f- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	15
	1	126055b5-2c17-11ec-ba94- 8141c2516b9e	7d185936-7a60-11eb-ba7f- 4a6a34607ded	2
	2	01ffab2c-3c70-11ec-ba95- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	3
	3	708b9941-3c6f-11ec-ba95- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	6
	4	bb818041-3c6f-11ec-ba95- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	1
	5	e1627618-3c6f-11ec-ba95- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	5
	6	f6e80261-3c6f-11ec-ba95- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	-2
	7	d24bc91d-50f4-11ec-ba96- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	-2
	8	e523eaae-50f4-11ec-ba96- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	6
	9	ed86c36a-50f4-11ec-ba96- 4a6a34607ded	7d185936-7a60-11eb-ba7f- 4a6a34607ded	6

В таблице stocks_items содержится информация о балансе товара на складах.

- stock_id идентификатор склада,
- item_id идентификатор нашего товара,
- quantity количество единиц товара на складе

На двух складах баланс в штуках нашего товара отрицательный.

Предположительно, это может быть связано с логистическими особенностями, например, товар куплен с этого склада, но еще на него не поступил. Либо имеет место ошибка при подсчете изначального баланса или что-то в этом роде.

In [25]:

stocks_on_days.tail(5)

Out[25]:		date	stock_id	item_id	start_day	finish_day	
	4255	2022-01-09 00:00:00	e1627618-3c6f-11ec- ba95-4a6a34607ded	7d185936-7a60-11eb- ba7f-4a6a34607ded	5	5	
	4256	2022-01-09 00:00:00	f6e80261-3c6f-11ec- ba95-4a6a34607ded	7d185936-7a60-11eb- ba7f-4a6a34607ded	-2	-2	
	4257	2022-01-09 00:00:00	d24bc91d-50f4-11ec- ba96-4a6a34607ded	7d185936-7a60-11eb- ba7f-4a6a34607ded	-2	-2	
	4258	2022-01-09 00:00:00	e523eaae-50f4-11ec- ba96-4a6a34607ded	7d185936-7a60-11eb- ba7f-4a6a34607ded	6	6	
	4259	2022-01-09 00:00:00	ed86c36a-50f4-11ec- ba96-4a6a34607ded	7d185936-7a60-11eb- ba7f-4a6a34607ded	6	6	

В таблице **stocks_on_days** содержится информация об изменении баланса товара на складах.

- date дата изменения записи,
- stock_id идентификатор склада,
- start_day количество единиц товара на начало дня,
- finish_day количество единиц товара на конец дня

2022-01-09 на двух складах мы видим отрицательные балансы, соответствующие данным из предыдущей таблицы. Возможно, дальнейший анализ позволит предполжить причину.

In [26]:

warehouse

Out[26]:		id	name
	0	4787ce96-7a69-11eb-ba7f-4a6a34607ded	31
	1	126055b5-2c17-11ec-ba94-8141c2516b9e	29
	2	708b9941-3c6f-11ec-ba95-4a6a34607ded	30
	3	bb818041-3c6f-11ec-ba95-4a6a34607ded	36
	4	e1627618-3c6f-11ec-ba95-4a6a34607ded	32
	5	f6e80261-3c6f-11ec-ba95-4a6a34607ded	13
	6	01ffab2c-3c70-11ec-ba95-4a6a34607ded	43
	7	0ad8892d-3c70-11ec-ba95-4a6a34607ded	2
	8	d24bc91d-50f4-11ec-ba96-4a6a34607ded	10
	9	d91c6b6b-50f4-11ec-ba96-4a6a34607ded	17
	10	e523eaae-50f4-11ec-ba96-4a6a34607ded	11
	11	ed86c36a-50f4-11ec-ba96-4a6a34607ded	40
	12	6e1c3edd-50f7-11ec-ba96-4a6a34607ded	38
	13	77155156-50f7-11ec-ba96-4a6a34607ded	22

В таблице **warehouse** содержится информация о соответствии идентификатора склада имени склада.

- id идентификатор склада,
- name имя склада

Предобработка данных

Сначала проанализируем данные в каждой таблице отдельно. Нас в основном будут интересовать две таблицы: sales и stocks_on_days, т.к. в них есть информация по дням. Сразу преобразуем поле date в дату в обоих датасетах.

```
In [27]: sales.info()
```

```
<class 'pandas.core.frame.DataFrame'>
         RangeIndex: 7881 entries, 0 to 7880
         Data columns (total 8 columns):
                           Non-Null Count Dtype
             Column
                           -----
          0
             date
                           7881 non-null
                                           object
             warehouse_id 7881 non-null object
          1
                           7881 non-null
          2
             item_id
                                          object
          3
             order_number 7881 non-null
                                          uint64
          4
                           7881 non-null float64
             price
          5
                           7881 non-null int64
             quantity
                           7881 non-null float64
          6
             amount
          7
                           7881 non-null float64
         dtypes: float64(3), int64(1), object(3), uint64(1)
         memory usage: 492.7+ KB
In [28]:
         sales['date'] = pd.to datetime(sales['date'], format='%Y-%m-%d')
In [29]:
         stocks on days.info()
         <class 'pandas.core.frame.DataFrame'>
         RangeIndex: 4260 entries, 0 to 4259
         Data columns (total 5 columns):
             Column
                        Non-Null Count Dtype
          0
                         4260 non-null
             date
                                         object
             stock_id
          1
                         4260 non-null
                                         object
                         4260 non-null
          2
             item_id
                                         object
                         4260 non-null
          3
             start_day
                                         int64
             finish_day 4260 non-null
                                         int64
         dtypes: int64(2), object(3)
         memory usage: 166.5+ KB
        Преобразуем date в дату.
In [30]:
         stocks_on_days['date'] = pd.to_datetime(stocks_on_days['date'], format='%Y-
```

Таблица sales

```
In [31]: sales.info()
```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 7881 entries, 0 to 7880
Data columns (total 8 columns):

Non-Null Count Dtype Column 0 date 7881 non-null datetime64[ns] warehouse_id 7881 non-null object 1 7881 non-null 2 item id object 3 order_number 7881 non-null uint64 4 7881 non-null float64 price 5 7881 non-null int64 quantity 7881 non-null float64 6 amount 7 sebes 7881 non-null float64

dtypes: datetime64[ns](1), float64(3), int64(1), object(2), uint64(1) memory usage: 492.7 + KB

```
In [32]:
```

sales.describe()

ut[32]:		order_number	price	quantity	amount	sebes
	count	7.881000e+03	7881.000000	7881.000000	7881.000000	7881.000000
	mean	9.327293e+18	29.018286	1.215962	73.103140	62.887013
	std	5.398734e+18	30.190997	0.565779	34.144389	57.869118
	min	2.034849e+15	0.000000	1.000000	41.000000	46.810000
	25%	4.602076e+18	0.000000	1.000000	59.990000	53.500000
	50%	9.384644e+18	0.000000	1.000000	59.990000	53.500000
	75%	1.403695e+19	59.990000	1.000000	59.990000	53.500000
	max	1.844082e+19	72.890000	20.000000	1199.400000	428.000000

Усредненный пользователь заказывает 1.2 нашего товара. При этом 1.2 * 29 не равно 73. Видимо, с какой-то из этих колонок что-то не так. Медианное значение цены - 0, то есть в нашем датасете по меньшей мере половина строк, где товар бесплатен, что странно.

Так и есть - в нашем датасете 4092 записи, где товар стоит 0. Это скорее всего ошибка в данных. Хорошо, что для задачи прогнозирования продаж в штуках цена товара нам не понадобится. Однако мы можем приблизительно проследить, как менялась цена за единицу товара.

```
In [34]:
            sales[sales['price'] != 0].pivot table(index=('date'),\
                                             values=('price'), aggfunc='mean').resample('1W
                               .plot(grid=True, figsize=(10, 5),style='-');
           68
                                                                                            price
           67
           66
           65
           64
           63
           62
           61
                   Feb
                         Mar
                                      May
                                                           Aug
                                                                    Sep
                                                                                        Dec
                                                                          Oct
                                Apr
                                               Jun
                                                     Jul
                                                                                  Nov
                                                                                               Jan
2022
```

Итак, с ноября цена за единицу товара начинает расти.

Сгруппируем данные по номеру заказа и посмотрим суммарное количество единиц товара в одном заказе.

date

```
In [35]:
           sales.groupby('order_number')['quantity'].value_counts().sort_values(ascender)
Out[35]: order_number
                                quantity
          6731018479413670002
                                             5
          1672391227775584346
                                             5
          113212051826480409
                                1
                                             4
          2305051651363929932
                                1
                                             3
          9940791630401677462
                                             3
          6304597147929193647
                                1
                                             1
          6303587684150998541
                                2
                                             1
          6301763020603510346
                                2
                                             1
          6296363353016666437
                                1
                                             1
          9385861019613057384
                                1
          Name: quantity, Length: 7739, dtype: int64
```

Итак, одна строка в нашем датасете не соответствует одному заказу, т.к. есть строки с одинаковым номером заказа. Посмотрим на один из них.

```
In [36]: sales[sales['order_number'] == 6731018479413670002]
```

Out[36]:		date	warehouse_id	item_id	order_number	price	quantity	amount
	5323	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	41.99
	5324	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	59.99
	5325	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	41.99
	5326	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	41.99
	5327	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	41.99

В один день с одного и того же склада было добавлено в заказ 5 единиц товара. Причем поштучно. При этом мы изначально предполагали, что 'amount' есть произведение цены товара на количество товара - это явно прослеживалось по некоторым строкам, где 'quantity' больше 1. Однако здесь эта закономерность не работает в четырех из пяти случаев. Следовательно, можно предположить три варианта: первый - строки в датасете по какой-то причине задублированы, второй - счётчик количества товаров в одном заказе по какой-то причине не всегда работает, третий - количество в каждой строке это покупка вне зависимости от номера заказа.

Чтобы отбросить пару предположений, посмотрим, есть ли среди заказов такие, в которых количество в одной строке больше 1.

```
In [37]: sales[sales['quantity']>1].groupby('order_number')['quantity'].value_counts
```

In [38]:

```
Out[37]: order_number
                                 quantity
          11296194484965127955
                                              2
                                              2
          17309412162316737410
                                 2
          4049899956522145911
                                 4
                                              2
                                              2
          10232250504558250800
                                 2
          579778784968965584
                                 2
                                              2
                                 2
                                              2
          16799693829721819593
          2102220760535183504
                                              2
          10960844609808534839
                                 3
                                              2
                                              2
          11440118071441749957
                                 2
                                 2
                                              2
          2318083237555896309
                                              2
          13085452004830543073
                                 2
                                              2
          7110878873487874671
                                 2
                                 2
                                              2
          7102228416989677268
          9175776159446999722
                                              2
          2676459181329578499
                                 2
                                              2
                                              2
          10839405888077023675
                                 2
                                              2
          10574565064201526016
                                 2
          15109576344457757493
                                 2
                                              2
          6894026867048799798
                                 2
                                              2
                                              2
          18355528760472315420
                                 2
          1310799616417822319
                                              2
          17348045685869597356
                                              2
          5235063407714053795
                                 2
                                              2
                                              2
          4737158851813872577
                                 2
          13974361519919532404
                                 2
          12524739835036680597
                                 2
                                              1
                                 2
          12559081658821231518
                                              1
          12545640498402825074
                                              1
          12537836978842140941
          12530683771647003094
                                              1
          Name: quantity, dtype: int64
```

У нас есть несколько таких заказов. Посмотрим на некоторые из них.

sales[sales['order number'] == 4049899956522145911]

```
date warehouse_id
                                            item_id
                                                             order_number
                                                                            price quantity
                                                                                            amount
Out[38]:
                           126055b5-
                                          7d185936-
                 2021-
                           2c17-11ec-
                                          7a60-11eb-
           335
                                                     4049899956522145911
                                                                                             239.96
                                                                              0.0
                 01-15
                                               ba7f-
                               ba94-
                        8141c2516b9e 4a6a34607ded
                           126055b5-
                                          7d185936-
                 2021-
                           2c17-11ec-
                                          7a60-11eb-
           336
                                                     4049899956522145911
                                                                              0.0
                                                                                             239.96
                 01-15
                               ba94-
                                               ba7f-
                        8141c2516b9e 4a6a34607ded
```

```
In [39]: sales[sales['order_number'] == 10960844609808534839]
```

Out[39]:		date	warehouse_id	item_id	order_number	price	quantity	amou
	5272	2021- 06- 28	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	10960844609808534839	0.0	3	179.
	5273	2021- 06- 28	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	10960844609808534839	0.0	3	179.!

In [40]: sales[sales['order_number'] == 13974361519919532404]

Out[40]:		date	warehouse_id	item_id	order_number	price	quantity	amoun
	5613	2021- 07-13	d24bc91d- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	13974361519919532404	59.99	2	119.9
	5614	2021- 07-13	d24bc91d- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	13974361519919532404	59.99	2	119.9

Вряд ли проблема в счетчике количества единиц товара в одном заказе. Иначе мы могли бы увидеть для одного заказа нечто вроде двух строк с разным 'quantity'. Так же маловероятно, что пользователь сначала купил 4 единицы товара, а затем подумал, что ему не хватает, и заказал ещё 4 единицы. Поэтому логично всё же заключить, что записи задублированы. Возможно, логгирование завязано на не самое удачное событие. Например, на кнопку "Оплатить заказ", а не на подтверждение оплаты. Посмотрим, сколько всего у нас повторяющихся строк.

```
In [41]: sales.duplicated().sum()
```

Out[41]: 131

Удалим эти строки.

```
In [42]: sales = sales.drop_duplicates().reset_index(drop=True)
```

Рассмотрим оставшиеся заказы с большим количеством строк, чем 1.

```
In [43]: sales['order_number'].value_counts()[sales['order_number'].value_counts()
```

```
Out[43]: 785214136477173422
         16590744060919034086
                                  2
         6731018479413670002
         735175590017064594
                                  2
         1670133850376295321
                                  2
         1182413268119596500
                                  2
                                  2
         175523352856087190
         2918464380085550324
                                  2
         532907036741686853
         1128963427423366634
         14146929096356632958
         8129142197681254166
         6345548677911873275
         348314697940896314
         9586599714543904593
         9028684352588784015
                                  2
         Name: order_number, dtype: int64
```

Таких заказов немного, поэтому рассмотрим каждый индивидуально.

Out[44]:		date	warehouse_id	item_id	order_number	price	quantity	amou
	617	2021- 01- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6345548677911873275	0.00	2	119.
	618	2021- 01- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6345548677911873275	0.00	1	59.
	653	2021- 01- 24	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1670133850376295321	0.00	1	59.
	654	2021- 01- 24	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1670133850376295321	0.00	1	59.
	744	2021- 01- 26	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	175523352856087190	0.00	1	59.
	745	2021- 01- 26	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	175523352856087190	0.00	1	59.
	1756	2021- 02-21	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	9028684352588784015	59.99	1	59.
			ed86c36a-	7d185936-				

1757	2021- 02-21	50f4-11ec- ba96- 4a6a34607ded	7a60-11eb- ba7f- 4a6a34607ded	9028684352588784015	59.99	1	59.
1763	2021- 02- 22	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	9586599714543904593	0.00	1	59.
1764	2021- 02- 22	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	9586599714543904593	0.00	1	59.
1843	2021- 02- 24	d24bc91d- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1182413268119596500	59.99	1	59.
1844	2021- 02- 24	d24bc91d- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1182413268119596500	59.99	1	59.
2715	2021- 03- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	532907036741686853	0.00	2	119.
2716	2021- 03- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	532907036741686853	0.00	1	59.
3572	2021- 04- 26	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1128963427423366634	0.00	1	59.
3573	2021- 04- 26	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1128963427423366634	0.00	1	59.
4542	2021- 05- 30	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	14146929096356632958	59.99	2	119.
4625	2021- 06- 03	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	14146929096356632958	0.00	1	59.
4998	2021- 06- 19	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	735175590017064594	0.00	1	59.
4999	2021- 06- 19	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	735175590017064594	0.00	2	119.

5229	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	41.
5230	2021- 06- 29	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	6731018479413670002	59.99	1	59.
5454	2021- 07- 09	708b9941- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	348314697940896314	0.00	1	59.
5455	2021- 07- 09	708b9941- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	348314697940896314	0.00	1	59.
5579	2021- 07-17	6e1c3edd- 50f7-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	16590744060919034086	59.99	2	119.:
5580	2021- 07-17	6e1c3edd- 50f7-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	16590744060919034086	59.99	1	59.
5886	2021- 08- 03	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	2918464380085550324	0.00	1	64.
5887	2021- 08- 03	126055b5- 2c17-11ec- ba94- 8141c2516b9e	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	2918464380085550324	0.00	1	64.
6170	2021- 08- 24	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	8129142197681254166	59.99	1	59.
6171	2021- 08- 24	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	8129142197681254166	59.99	1	59.
7084	2021- 11-08	708b9941- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	785214136477173422	0.00	1	59.
7085	2021- 11-08	708b9941- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	785214136477173422	0.00	1	59.

Out[45

Рассмотрение показало, что возможны три варианта:

1. Разные даты, разные склады, разное количество.

5]:		date	warehouse_id	item_id	order_number	price	quantity	amou
	4542	2021- 05- 30	ed86c36a- 50f4-11ec- ba96- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	14146929096356632958	59.99	2	119.
	4625	2021- 06- 03	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	14146929096356632958	0.00	1	59.9

Если бы не половина датасета с нулевой ценой, можно было бы предположить возврат единицы товара покупателем и отгрузку нового товара со склада. Но мы не знаем этого. Также можно предположить, что 2021-05-30 товара на складе не хватило, и была отгрузка одной единицы товара 2021-06-03. Мы можем проверить, сколько единиц товара было на начало и конец дня на складе 2021-05-30, чтобы проверить эту версию.

Out[46]:		date	stock_id	stock_id item_id		finish_day
	1928	2021-	ed86c36a-50f4-11ec-	7d185936-7a60-11eb-	10	0
1928		05-30	ba96-4a6a34607ded	ba7f-4a6a34607ded	12	0

Похоже, версию с доотгрузкой можно отбросить. Однако, мы не знаем, могло ли произойти задвоение номера заказа от разных покупателей, когда они заказывали товар с разных складов. Оставим обе записи.

1. Одинаковые даты, одинаковые склады, разное количество:

```
In [47]: sales[sales['order_number'] == 532907036741686853]
```

Out[47]:		date	warehouse_id	item_id	order_number	price	quantity	amount
	2715	2021- 03- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	532907036741686853	0.0	2	119.66
	2716	2021- 03- 23	e1627618- 3c6f-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	532907036741686853	0.0	1	59.84

Мы не знаем, уменьшилось ли количество единиц в заказе или увеличилось, и следовательно мы не знаем, какие записи удалять и удалять ли. Оставим без изменения, т.к. их всё равно немного.

1. Одинаковые даты, одинаковые склады, одинаковое количество, разная стоимость с отличием на 0.01 (округление). Рассматриваем как дубль:

```
In [48]: sales[sales['order_number'] == 1128963427423366634]
```

Out[48]:		date	warehouse_id	item_id	order_number	price	quantity	amount
	3572	2021- 04- 26	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1128963427423366634	0.0	1	59.51
	3573	2021- 04- 26	01ffab2c- 3c70-11ec- ba95- 4a6a34607ded	7d185936- 7a60-11eb- ba7f- 4a6a34607ded	1128963427423366634	0.0	1	59.50

Индексы 653, 744, 1756, 1763, 1843, 3573, 5229, 5454, 5887, 6171, 7085 удаляем.

Посмотрим, сколько в датасете строк с разным количеством товара.

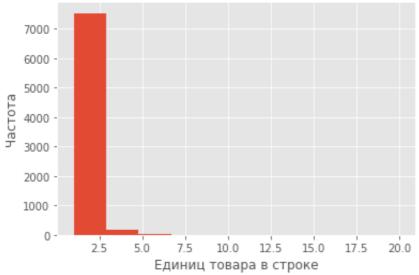
```
In [51]:
           sales['quantity'].value_counts()
                 6368
Out[51]: 1
                 1169
          2
                  149
          3
          4
                   35
          5
                    9
          6
                    5
                    2
          20
                    1
                    1
          Name: quantity, dtype: int64
         Чаще всего встречаются строки с одной единицей товара, но есть и случаи, когда
         их 9 или 20.
```

```
In [52]: sales.boxplot(column=['quantity']);
```



```
plt.title('Гистограмма количества товаров в строке датасета')
plt.xlabel('Единиц товара в строке')
plt.ylabel('Частота')
plt.hist(sales['quantity'], bins=10);
```

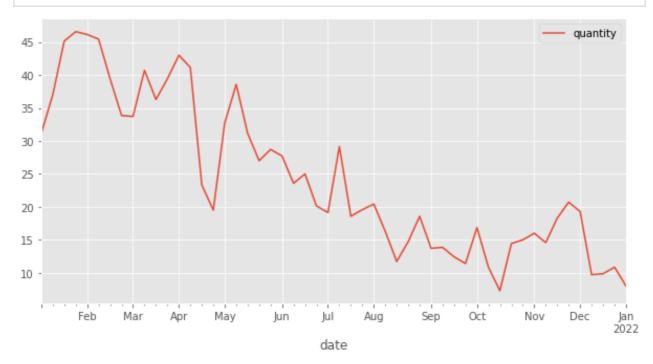


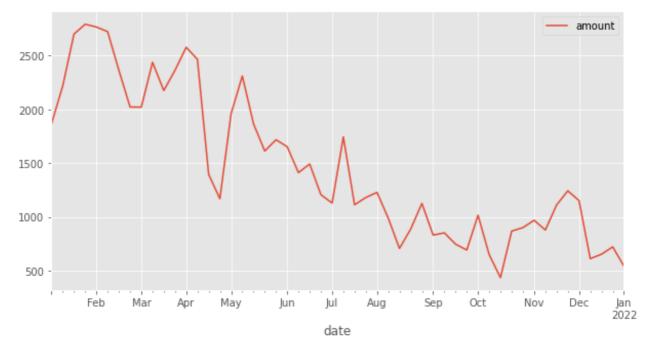


Очевидно, что количество единиц товара от 3 и более настолько редки, что их следует рассматривать, как выбросы. Оставим в датасете только те продажи, количество единиц товара в которых меньше 3.

```
In [54]: sales = sales[sales['quantity'] < 3]</pre>
```

Просуммируем количество проданных единиц товара по дням, затем ресемплируем по среднему за неделю, и построим график.





Судя по графикам очевидна зависимость между 'amount' и 'quantity'.

Таблица stocks_on_days

```
In [57]: stocks_on_days.columns

Out[57]: Index(['date', 'stock_id', 'item_id', 'start_day', 'finish_day'], dtype='ob ject')

В датасете sales наблюдения начинаются 2021-01-04:

In [58]: min(sales['date'])

Out[58]: Timestamp('2021-01-04 00:00:00')

Поэтому из stocks_on_days мы удалим все записи до этой даты.

In [59]: stocks_on_days = stocks_on_days[stocks_on_days['date'] >= '2021-01-04']

In [60]: stocks_on_days.info()
```

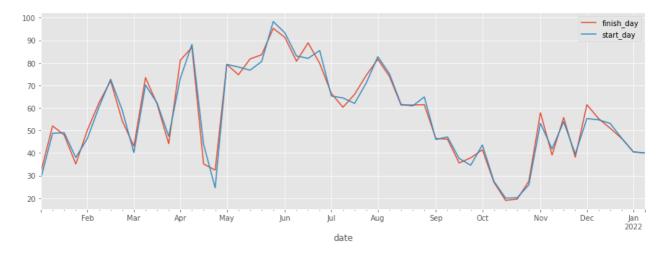
```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 3793 entries, 467 to 4259
Data columns (total 5 columns):
    Column Non-Null Count Dtype
 0
    date
               3793 non-null
                               datetime64[ns]
    stock_id 3793 non-null
 1
                               object
 2
    item_id
               3793 non-null
                               object
 3
    start_day
                3793 non-null
                              int64
    finish_day 3793 non-null
                               int64
dtypes: datetime64[ns](1), int64(2), object(2)
memory usage: 177.8+ KB
```

```
In [61]: stocks_on_days.describe()
```

Out[61]:		start_day	finish_day
	count	3793.000000	3793.000000
	mean	5.533087	5.557342
	std	5.474001	5.469993
	min	-2.000000	-2.000000
	25%	1.000000	1.000000
	50%	4.000000	4.000000
	75%	8.000000	8.000000
	max	45.000000	45.000000

Мы уже отмечали, что в датасете stocks_on_days присутствуют записи с отрицательным балансом. В среднем на складах присутствует около 5,5 единиц товара.

Построим график количества товара в среднем за неделю.

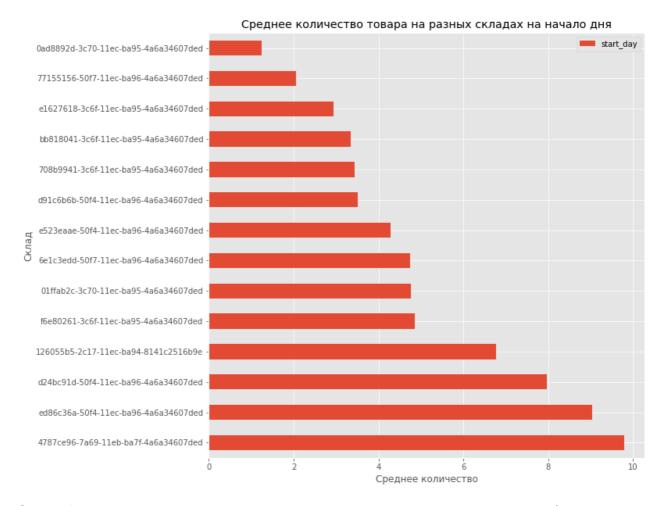


Товар появляется на складах в начале января 2021, в апреле наблюдается уменьшение количества единиц товара, в июне наблюдается пик количества, к октябрю товар раскупают быстрее, чем пополняют склады. Затем количество увеличивается и колеблется в диапозоне 40-60 шт. до конца наблюдений.

Посмотрим, какое количество товара в среднем лежит на разных складах.

In [64]:

grouped_barh(stocks_on_days, 'stock_id', 'start_day', 'mean', \
'Среднее количество товара на разных складах на начало дня',



Эта информация может косвенно говорить о том, насколько велик склад (или точка магазина), либо о популярности данного товара в разных районах/городах/регионах.

Исследовательский анализ данных

В дальнейшем мы скорее всего будем работать только с зависимостью количества продаж в день от времени. Но на данном этапе исследуем обе таблицы sales_q и stocks_on_days_q.

```
In [65]: sales_q = sales.drop(columns=['warehouse_id', 'item_id', 'order_number', ']
In [66]: sales_q = sales_q.groupby('date').sum()
    sales_q
```

Out[66]:		quantity
	date	
	2021-01-04	9
	2021-01-05	35
	2021-01-06	30
	2021-01-07	33
	2021-01-08	38
	•••	
	2021-12-26	11
	2021-12-27	7
	2021-12-28	6
	2021-12-29	11
	2021-12-30	8

353 rows × 1 columns

```
In [67]: stocks_on_days.groupby('date').sum()
```

Out[67]:	start_day	finish_	_day
----------	-----------	---------	------

date		
2021-01-04	0	20
2021-01-05	20	45
2021-01-06	45	23
2021-01-07	23	47
2021-01-08	47	20
		•••
2022-01-05	40	40
2022-01-06	40	40
2022-01-07	40	40
2022-01-08	40	40
2022-01-09	40	40

371 rows × 2 columns

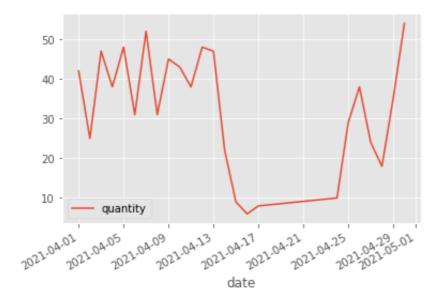
Так как графики 'start_day', 'finish_day' в stocks_on_days похожи, оставим для анализа только 'finish_day'. Таким образом мы получим информацию о количестве единиц товара, оставшихся на складах после завершения продаж (и поставок) в этот же день. Если бы мы оставили 'start_day', это бы по сути соответствовала результату продаж на предыдущий день.

Out[69]: finish_day

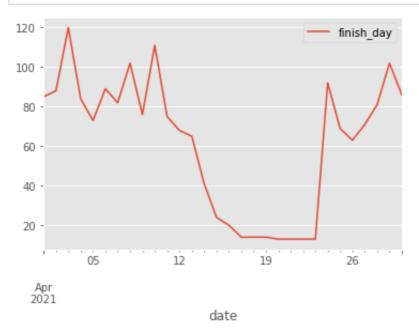
date	
2021-01-04	20
2021-01-05	45
2021-01-06	23
2021-01-07	47
2021-01-08	20
•••	
2022-01-05	40
2022-01-06	40
2022-01-07	40
2022-01-08	40
2022-01-09	40

371 rows × 1 columns

```
In [70]: sales_q.loc['2021-04'].plot();
```

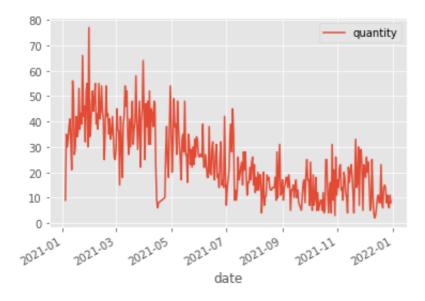


In [71]: stocks_on_days_q.loc['2021-04'].plot();

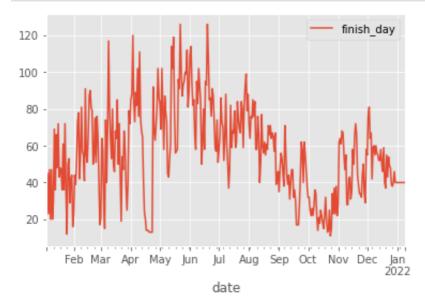


Можно предположить, что снижение количества продаж в апреле было связано с проблемами с поставками на склады.

```
In [72]: sales_q.plot();
```







Итак, у нас есть два временных ряда, но если попытаться разложить один из них ($sales_q$) на аддитивные компоненты, то возникает ошибка: ValueError: You must specify a period or x must be a pandas object with a DatetimeIndex with a freq not set to None.

Эту ошибку можно легко устранить, вручную задав частоту сезонной составляющей. Логично предположить, что когда работаешь с датасетом продаж, обычно должна присутствовать недельная сезонность. Попробуем сначала разложить на аддитивные компоненты временной ряд stocks_on_days_q.

```
In [74]: stocks_on_days_q = stocks_on_days_q.squeeze().dropna()

In [75]: components_sod = tsa.seasonal_decompose(stocks_on_days_q, model='additive')
```

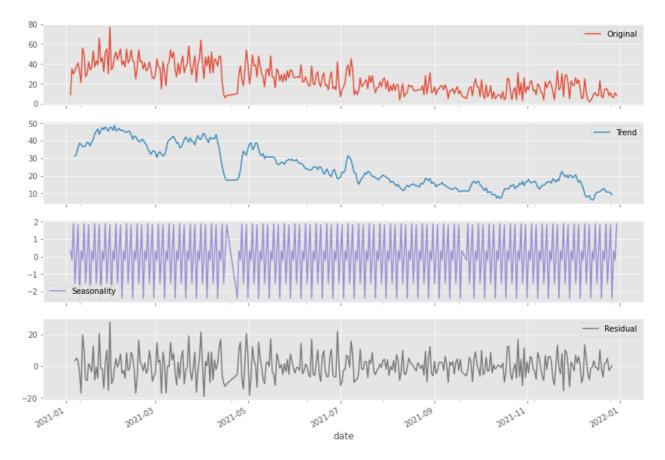


Код выполнился без ошибок и задания периода вручную. Посмотрим на первые несколько значений сезонной составляющей, чтобы оценить период.

```
In [77]: components_sod.seasonal.head(22)
```

```
Out[77]: date
         2021-01-04
                       -9.618561
         2021-01-05
                        4.634186
         2021-01-06
                       -1.904276
         2021-01-07
                        4.837742
         2021-01-08
                        0.356713
         2021-01-09
                      10.485834
         2021-01-10
                      -8.791638
         2021-01-11
                       -9.618561
         2021-01-12
                        4.634186
         2021-01-13
                       -1.904276
         2021-01-14
                        4.837742
                        0.356713
         2021-01-15
         2021-01-16
                       10.485834
         2021-01-17
                       -8.791638
         2021-01-18
                       -9.618561
         2021-01-19
                        4.634186
         2021-01-20
                       -1.904276
         2021-01-21
                        4.837742
         2021-01-22
                        0.356713
         2021-01-23
                      10.485834
         2021-01-24
                       -8.791638
         2021-01-25
                       -9.618561
         Name: seasonal, dtype: float64
```

Очевидно, период равен 7. Теперь вручную зададим период для временного ряда количества продаж.



Понятно, что эти ряды не стационарны, т.к. присутствуют и тренд, и сезонность. Убедимся в этом, проведя тест Дикки-Фуллера.

```
In [80]:
          def DF_test(series):
              Расширенный тест Дикки-Фуллера (augmented Dickey-Fuller), ADF
              но – образец временного ряда имеет единичный корень
              н1 - образец временного ряда не имеет единичных корней, ряд стационаре
              0.00
              X = series.values
              result = adfuller(X)
              print('ADF Statistic: %f' % result[0])
              print('p-value: %f' % result[1])
              print('Critical Values:')
              for key, value in result[4].items():
                  print('\t%s: %.3f' % (key, value))
              if result[0]> result[4]['5%']:
                  print('есть единичные корни, ряд не стационарен')
              else:
                  print('единичных корней нет, ряд стационарен')
In [81]:
          DF test(stocks on days q)
```

Для того, чтобы построить прогноз, нам нужно удовлетворить допущение о стационарности линейных моделей временных рядов. Воспользуемся общепринятыми преобразованиями.

Сначала применим логарифм (натуральный) к временным рядам для конвертирования регулярности экспоненциального роста в линейный тренд и стабилизации дисперсии.

```
In [83]: # проверка на нулевые значения
(sales_q == 0).any(), (stocks_on_days_q == 0).any()

Out[83]: (False, False)

In [84]: sales_log = np.log(sales_q)
stocks_on_days_log = np.log(stocks_on_days_q)
```

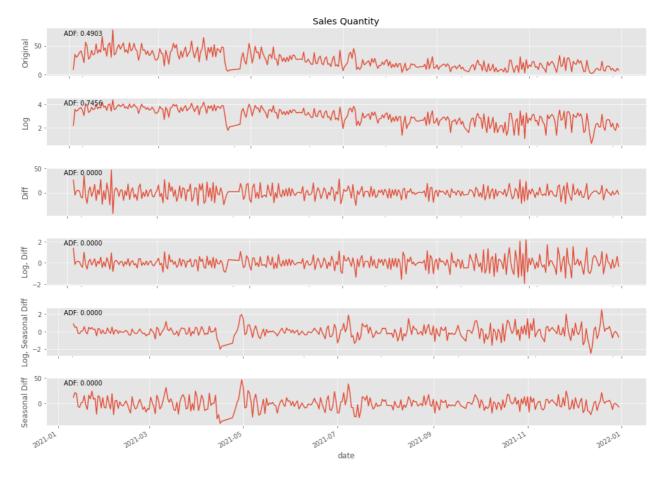
Во многих случаях для того, чтобы сделать ряд стационарным, детрендирования не достаточно. Вместо этого нам нужно преобразовать исходные данные во временной ряд попериодных и/или посезонных разниц. Другими словами, мы используем результат вычитания соседних точек данных или значений в сезонных сдвигах друг из друга.

```
sales_diff = sales_q.diff().dropna() # первые разности
sales_log_diff = sales_log.diff().dropna() # первые разности из логарифма
sales_log_diff_s = sales_log.diff(7).dropna() # сезонные разности из логари
sales_diff_s = sales_q.diff(7).dropna() # сезонные разности

stocks_on_days_diff = stocks_on_days_q.diff().dropna() # первые разности
stocks_on_days_log_diff = stocks_on_days_log.diff().dropna() # первые разности
stocks_on_days_log_diff_s = stocks_on_days_log.diff(7).dropna() # сезонные
```

Изобразим на графике сразу несколько вариантов преобразований.

```
In [86]:
         fig, axes = plt.subplots(nrows=6, ncols=1, figsize=(14,10))
          sales_q.plot(ax=axes[0], title='Sales Quantity')
          axes[0].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales_q.dropna())[1]:.4f]
          axes[0].set ylabel('Original')
          sales log.plot(ax=axes[1], sharex=axes[0])
          axes[1].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales log.dropna())[1]:...
          axes[1].set ylabel('Log')
          sales diff.plot(ax=axes[2], sharex=axes[0])
          axes[2].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales diff.dropna())[1]:
          axes[2].set ylabel('Diff')
          sales log diff.plot(ax=axes[3], sharex=axes[0])
          axes[3].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales_log_diff.dropna())
          axes[3].set ylabel('Log, Diff')
          sales log diff s.plot(ax=axes[4], sharex=axes[0])
          axes[4].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales log diff s.dropna(
          axes[4].set_ylabel('Log, Seasonal Diff')
          sales_diff_s.plot(ax=axes[5], sharex=axes[0])
          axes[5].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales_diff_s.dropna())[1
          axes[5].set ylabel('Seasonal Diff')
          #stocks on days q.plot(ax=axes[0][1], title='Stocks on Days Quantity')
          \#axes[0][1].text(x=.03, y=.85, s=f'ADF: \{tsa.adfuller(stocks on days q)[1]\}
          #axes[0][1].set ylabel('Original')
          #stocks on days log.plot(ax=axes[1][1], sharex=axes[0][1])
          \#axes[1][1].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(stocks on days log.d
          #axes[1][1].set ylabel('Log')
          #stocks on days diff.plot(ax=axes[2][1], sharex=axes[0][1])
          #axes[2][1].text(x=.83, y=.85, s=f'ADF: {tsa.adfuller(stocks on days diff.
          #axes[2][1].set ylabel('Diff')
          #stocks on days_log_diff.plot(ax=axes[3][1], sharex=axes[0][1])
          #axes[3][1].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(sales log diff.dropn
          #axes[3][1].set ylabel('Log, Diff')
          #stocks on days log diff s.plot(ax=axes[4][1], sharex=axes[0][1])
          #axes[4][1].text(x=.03, y=.85, s=f'ADF: {tsa.adfuller(stocks on days log d
          #axes[4][1].set ylabel('Log, Seasonal Diff')
          fig.tight layout()
          fig.align_ylabels(axes);
```



Остановимся на варианте "Diff". Ещё раз проведём тест Дикки-Фуллера.

```
In [87]: DF_test(sales_diff)
```

ADF Statistic: -10.756183

p-value: 0.000000 Critical Values: 1%: -3.450 5%: -2.870

10%: -2.571 единичных корней нет, ряд стационарен

Мы убедились, что наш ряд стационарен. Построим графики Q-Q и коррелограммы.

```
In [88]:
          def plot correlogram(x, lags=None, title=None):
              Строит Q-Q plot (от нормального распределения) и коррелограммы.
              Выводит некоторые статистики:
              Q-Stat: статистический показатель Льюнга-Бокса
              ADF: p-value расширенной проверки Дикки-Фуллера
              Mean: среднее
              SD: стандартное отклонение
              Skew: коэффициент асимметрии
              Kurtosis: ЭКСЦеСС
              lags = min(10, int(len(x)/5)) if lags is None else lags
              fig, axes = plt.subplots(nrows=2, ncols=2, figsize=(14, 8))
              x.plot(ax=axes[0][0])
              q p = np.max(q stat(acf(x, nlags=lags), len(x))[1])
              stats = f'Q-Stat: {np.max(q_p):>8.2f}\nADF: {adfuller(x)[1]:>11.2f}'
              axes[0][0].text(x=.02, y=.85, s=stats, transform=axes[0][0].transAxes)
              probplot(x, dist='norm', plot=axes[0][1])
              Skew - https://ru.wikipedia.org/wiki/KoЭффициент асимметрии
              Kurtosis - https://ru.wikipedia.org/wiki/Коэффициент_эксцесса
              mean, var, skew, kurtosis = moment(x, moment=[1, 2, 3, 4])
              s = f'Mean: \{mean:>12.2f\} \setminus \{np.sqrt(var):>16.2f\} \setminus \{skew:12.2f\}
              axes[0][1].text(x=.02, y=.75, s=s, transform=axes[0][1].transAxes)
              plot_acf(x=x, lags=lags, zero=False, ax=axes[1][0])
              plot pacf(x, lags=lags, zero=False, ax=axes[1][1])
              axes[1][0].set xlabel('Lag')
              axes[1][1].set xlabel('Lag')
              fig.suptitle(title, fontsize=20)
              fig.tight layout()
              fig.subplots_adjust(top=.9)
```

```
In [89]: plot_correlogram(sales_diff, lags=20, title='Sales (Diff)')
```

/usr/local/lib/python3.7/dist-packages/statsmodels/graphics/tsaplots.py:353 : FutureWarning: The default method 'yw' can produce PACF values outside of the [-1,1] interval. After 0.13, the default will change tounadjusted Yule-Walker ('ywm'). You can use this method now by setting method='ywm'. FutureWarning,



Мы видим, что точки на графике Q-Q Plot стелятся вдоль прямой, соответствующей нормальному распределению. То есть можем сделать вывод о том, что наши данные неплохо соответствуют теоретическим квантилям, отложенным по горизонтальной оси. Хвосты нашего распределения уходят в разные стороны от прямой на графике Q-Q, что говорит о более толстых хвостах нашего распределения по сравнению с нормальным.

Так как перед нами стоит задача прогнозирования продаж на неделю (или 7 дней) вперёд без каких-либо ещё вводных данных (например, известного количества товара, предполагаемого к поставке на определённый день), нам нужно строить модель, для которой на первый день из прогнозируемой недели будут все данные, необходимые для пргноза. Поэтому мы будем строить одномерную модель, то есть модель, целиком зависящую от значений продаж в прошлом.

Модель SARIMAX

Воспользуемся моделью SARIMA, построенной для ряда первых разностей. SARIMA(p,d,q)(P,D,Q,s) - Seasonal Autoregression Moving Average model. Итак, чтобы построить модель нам нужно определить следующие параметры:

- 1. Порядок (order)
 - р порядок авторегрессии тренда
 - d порядок разности тренда
 - q порядок скользящей средней тренда
- 2. Сезонный порядок(seasonal_order)
 - Р порядок авторегрессии для сезонного компонента
 - D порядок разности ряда для сезонного компонента
 - Q порядок скользящей средней для сезонного компонента
 - s количество временных шагов за один сезонный период

Есть различные способы определения коэффициентов. Можно пользоваться правилами подбора коэффициентов в зависимости от значимых лагов, а можно сделать подобие своего собственного Grid Search для подбора гиперпараметров.

Разработка Grid Search для модели SARIMAX

Для моделирования будем использовать модель SARIMA из библиотеки statsmodels Эта модель имеет следующие гиперпараметры:

- 1. order: кортеж p, d и q для моделирования тренда.
- 2. sesonal_order: кортеж параметров P, D, Q и m для моделирования сезонности
- 3. trend: параметр для управления моделью, принимающий одно из следующих значений:
 - 'n' no trend,
 - 'c constant,
 - 't' linear trend,
 - 'ct' constant with linear trend

Начнём с того, что определим функцию, которая будет принимать на вход конфигурацию модели и делать прогноз на один шаг вперёд.

```
In [245...
          # SARIMA-прогноз на шаг вперёд
          def sarima_forecast(history, config, plot_diagnostics=False):
              # определяем конфигурацию модели: [(p, d, q), (P, D, Q, s), 'n']
              order, sorder, trend = config
              # определяем модель
              model = tsa.statespace.SARIMAX(history,
                              order=order,
                              seasonal order=sorder,
                              trend=trend,
                              enforce stationarity=False,
                              enforce invertibility=False)
              # обучаем модель
              model fit = model.fit(disp=False)
              # делаем прогноз на шаг вперед
              prediction = model_fit.predict(len(history), # номер с которого начина
                                              len(history) # до какого номера предска.
                                             )
              if plot diagnostics:
                  model fit.plot diagnostics(figsize=(15, 12)) # диагностика остатко
              else:
                  return prediction[0]
```

Теперь напишем функции для реализации walk-forward валидации.

Сначала напишем свою функцию разбиения series_train_test_split(). В качестве параметров будем использовать список или массив данных временного ряда, а в качестве метки разбиения будем указывать количество шагов с конца временного ряда, которое отведём под тестовый набор.

```
In [246...
          # разбиваем датасет на train и test наборы
          def series train test split(data, n test):
              Функция разделяет список или массив на тренировочный и тестовый наборы
              n_test - КОЛИЧЕСТВО ВРЕМЕННЫХ ШАГОВ С КОНЦА, ИСПОЛЬЗУЕМЫХ В ТЕСТОВОМ (
              return data[:-n_test], data[-n_test:]
In [247...
          # проверка
          series_train_test_split(sales_diff.head(8), 3)
Out[247... (date
          2021-01-05
                        26.0
          2021-01-06
                       -5.0
                         3.0
          2021-01-07
          2021-01-08
                         5.0
          2021-01-09
                         3.0
          Name: quantity, dtype: float64, date
          2021-01-10
                      -9.0
          2021-01-11
                       -11.0
          2021-01-12
                        35.0
          Name: quantity, dtype: float64)
```

Будем использовать RMSE как метрику качества.

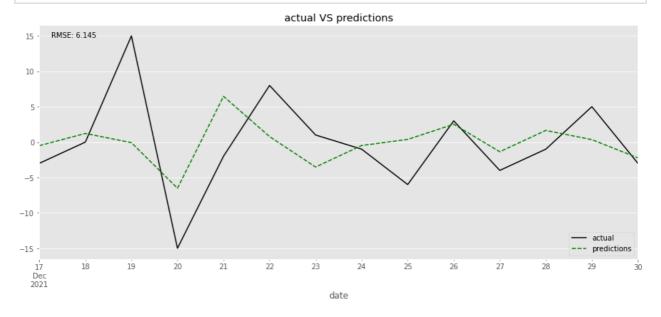
```
def rmse(y_true,y_pred):
    rmse = mean_squared_error(y_true, y_pred)**(0.5)
    return rmse
```

Теперь реализуем walk-forward валидацию.

```
In [249...
          # walk-forward validation
          def walk forward validation(data, # наш временной ряд
                                      n test, # КОЛИЧЕСТВО ШАГОВ С КОНЦА data, С КОТ
                                      model forecast, # имя функции прогноза
                                      config, # конфигурация модели SARIMA
                                      plot=False, # можем построить график сравнения
                                      plot resid=False # Γραφиκи остатков
                                     ):
              predictions = list()
              # разделяем датасет
              train, test = series train test split(data, n test)
              # помещаем train в список исторических данных history
              history = [x for x in train]
              # проходимся по каждому из значений в тестовом наборе
              for i in range(len(test)):
                  # обучаем модель на исторических данных и делаем прогноз на шаг вп
                  #prediction = sarima forecast(history, config)
                  prediction = model forecast(history, config)
                  # сохраняем предсказание в список предсказаний
                  predictions.append(prediction)
                  # действительное значение помещаем в историю для следующего цикла
                  history.append(test[i])
              # вычисляем ошибку предсказания сравнивая все одношаговые предсказания
              error = rmse(test, predictions)
              #print(len(predictions))
              #print(len(test))
              #print(len(train))
              #print(len(history))
              if plot:
                  # график для визуального сравнения предсказанных значений с действ
                  # (по умолчанию не строим)
                  plot result(n test, error, test, predictions)
              if plot resid:
                  # график остатков
                  model_forecast(test.asfreq('D'), config, True)
              else:
                  # по умолчанию возвращаем интересующую метрику качества
                  return error
```

In [250...

проверка работы функции walk_forward_validation(sales_diff, 14, sarima_forecast, [(1, 1, 1), (0, 0



Out[250... 6.145343339369119

Мы можем неоднократно вызывать walk_forward_validation() с различными списками конфигураций моделей.

Одна из возможных проблем заключается в том, что некоторые комбинации конфигураций модели могут не вызываться для модели и выдавать исключение, например, указывая некоторые, но не все аспекты сезонной структуры в данных.

Мы можем ловить исключения и игнорировать предупреждения во время поиска в сетке, обернув все вызовы walk_forward_validation() try-except блоком для игнорирования предупреждений. Мы также можем добавить поддержку отладки, чтобы отключить эти средства защиты в случае, если мы хотим увидеть, что на самом деле происходит. Наконец, если ошибка все же возникает, мы можем вернуть результат None, в противном случае мы сможем напечатать некоторую информацию о каждой оцениваемой модели. Это полезно при оценке большого количества моделей.

Функция score_model() ниже реализует это и возвращает кортеж (ключ и результат), где ключ является строковой версией протестированной конфигурации модели.

```
In [251...
          # оцениваем модель возвращаем None при возникновении ошибки
          def score model(data, n test, config, debug=False):
              result = None
              # конвертируем конфигурацию в строковый ключ
              key = str(config)
              # показываем все предупреждения, если debug=True
              if debug:
                  result = walk forward validation(data, n test, sarima forecast, col
              else:
                  # сбой во время проверки модели предполагает нестабильную конфигур
                      # во время поиска по сетке не показываем предупреждения
                      with catch warnings():
                          filterwarnings("ignore")
                          result = walk forward validation(data, n test, sarima fore
                  except:
                      error = None
              # проверяем на наличие интересующего результата
              if result is not None:
                  print(' > Model[%s] %.3f' % (key, result))
              return (key, result)
In [252...
          # проверка
          score_model(sales_diff, 8, [(4, 1, 1), (0, 0, 0, 0), 'n'], debug=False)[1]
```

```
http://localhost:8888/nbconvert/html/Projects/Продажи_товара_на_следующую_неделю.ipynb?download=false
```

> Model[[(4, 1, 1), (0, 0, 0, 0), 'n']] 3.004

Out[252... 3.0040426776882403

In [99]:

Далее нам нужен цикл для тестирования списка различных конфигураций моделей.

Это основная функция, которая управляет процессом поиска по сетке и вызывает функцию score_model() для каждой конфигурации модели.

Мы можем значительно ускорить процесс поиска по сетке, параллельно оценивая конфигурации моделей. Один из способов сделать это - использовать библиотеку Joblib.

Мы можем определить параллельный объект с количеством используемых ядер и установить его на количество ядер, обнаруженных в оборудовании.

```
from multiprocessing import cpu count
          from joblib import Parallel
          from joblib import delayed
          from warnings import catch warnings
          from warnings import filterwarnings
In [253...
          def grid search(data,
                          config list, # список всех конфигураций
                          n test,
                          parallel=False):
              scores = None
              if parallel:
                  # определяем количество используемых ядер
                  executor = Parallel(n jobs=cpu count(), backend='multiprocessing')
                  print(executor)
                  # создаём список задач для параллельного выполнения
                  tasks = (delayed(score model)(data, n test, config) for config in (
                  print(tasks)
                  # передаём задачи в executor
                  scores = executor(tasks)
              else:
                  scores = [score model(data, n test, config) for config in config 1
              # удаляем пустые результаты
              scores = [r for r in scores if r[1] != None]
              # сортируем кортежи в списке по баллу в порядке возрастания (сначала л
              scores.sort(key=lambda tup: tup[1])
              return scores
In [101...
          # проверка
          grid_search(sales_diff, [[(1, 0, 0), (0, 0, 0, 0), 'n'], [(1, 0, 0), (0, 5
          > Model[[(1, 0, 0), (0, 0, 0, 0), 'n']] 3.342
Out[101... [("[(1, 0, 0), (0, 0, 0, 0), 'n']", 3.342422124758878)]
```

Осталось только определить список конфигураций модели, которые можно попробовать для набора данных. Мы можем определить его в общем виде. Единственный параметр, который мы, возможно, захотим указать, это периодичность сезонного компонента в серии, если таковой существует. По умолчанию мы не будем считать сезонных компонентов.

Функция sarima_configs() ниже создаст список конфигураций моделей для оценки.

Конфигурации предполагают, что каждый из компонентов AR, MA и I для тренда и сезонности имеет низкий порядок, например, выключен (0) или в [1,2].

Теоретически, существует 2880 возможных конфигураций модели для оценки, но на практике многие из них не будут действительными и приведут к ошибке, которую мы поймаем и проигнорируем.

```
In [242...
          # Задаем конфигурации sarima
          def sarima configs(seasonal=[0]):
              models = list()
              # определяем параметры
              p params = [1, 2, 3, 4]
              d params = [0, 1]
              q params = [0, 1, 2]
              t_params = ['n','c','t','ct']
              P params = [0, 1, 2, 3, 4]
              D params = [0, 1]
              Q params = [0, 1, 2]
              s params = seasonal
              # сохраняем в список
              for p in p params:
                  for d in d_params:
                      for q in q params:
                          for t in t_params:
                              for P_ in P__params:
                                  for D in D params:
                                      for Q in Q params:
                                          for s in s_params:
                                              config = [(p,d,q), (P_,D_,Q_,s), t]
                                              models.append(config)
              return models
```

```
In [243... len(sarima_configs())
```

Out[243... 2880

Выбор лучшей конфигурации для модели SARIMAX

Теперь у нас есть структура для поиска гиперпараметров модели SARIMA.

Он является универсальным и будет работать для любых одномерных временных рядов, предоставленных в виде списка или массива NumPy.

```
In [254...
          %%time
          if __name__ == '__main__':
              # загружаем датасет
              print('Всего объектов в ряде:', sales_diff.shape[0])
              # размер выборки для walk-forward валидации
              n test = round(len(sales diff)*0.2)
              print('Объектов в тестовой выборке:', n test)
              # конфигурация модели
              config list = sarima configs()
              # grid search
              scores = grid_search(sales_diff, config_list, n_test)
              print('Лучшие модели:')
              # топ-3 конфигурации
              for config, error in scores[:3]:
                  print(config, error)
```

```
Всего объектов в ряде: 352
Объектов в тестовой выборке: 70
> Model[[(1, 0, 0), (0, 0, 0, 0), 'n']] 9.134
> Model[[(1, 0, 0), (0, 0, 0, 0), 'c']] 9.148
> Model[[(1, 0, 0), (0, 0, 0, 0), 't']] 9.177
> Model[[(1, 0, 0), (0, 0, 0, 0), 'ct']] 9.192
                                    'n']] 7.828
> Model[[(1, 0, 1), (0, 0, 0, 0),
> Model[[(1, 0, 1), (0, 0, 0, 0), 'c']] 8.311
> Model[[(1, 0, 1), (0, 0, 0, 0), 't']] 8.055
> Model[[(1, 0, 1), (0, 0, 0, 0), 'ct']] 8.412
> Model[[(1, 0, 2), (0, 0, 0, 0), 'n']] 8.243
> Model[[(1, 0, 2), (0, 0, 0, 0), 'c']] 8.478
> Model[[(1, 0, 2), (0, 0, 0, 0), 't']] 8.369
> Model[[(1, 0, 2), (0, 0, 0, 0), 'ct']] 8.527
> Model[[(1, 1, 0), (0, 0, 0, 0), 'n']] 13.599
> Model[[(1, 1, 0), (0, 0, 0, 0), 'c']] 13.622
> Model[[(1, 1, 0), (0, 0, 0, 0), 't']] 13.666
> Model[[(1, 1, 0), (0, 0, 0, 0), 'ct']] 13.687
> Model[[(1, 1, 1), (0, 0, 0, 0), 'n']] 9.146
> Model[[(1, 1, 1), (0, 0, 0, 0), 'c']] 9.190
> Model[[(1, 1, 1), (0, 0, 0, 0),
                                    't']] 9.441
> Model[[(1, 1, 1), (0, 0, 0, 0), 'ct']] 9.431
> Model[[(1, 1, 2), (0, 0, 0, 0),
                                    'n']] 8.263
> Model[[(1, 1, 2), (0, 0, 0, 0), 'c']] 8.172
> Model[[(1, 1, 2), (0, 0, 0, 0), 't']] 9.201
> Model[[(1, 1, 2), (0, 0, 0, 0), 'ct']] 9.431
> Model[[(2, 0, 0), (0, 0, 0, 0), 'n']] 8.749
> Model[[(2, 0, 0), (0, 0, 0, 0), 'c']] 8.761
> Model[[(2, 0, 0), (0, 0, 0, 0), 't']] 8.787
> Model[[(2, 0, 0), (0, 0, 0, 0), 'ct']] 8.802
> Model[[(2, 0, 1), (0, 0, 0, 0), 'n']] 7.837
> Model[[(2, 0, 1), (0, 0, 0, 0), 'c']] 8.372
> Model[[(2, 0, 1), (0, 0, 0, 0), 't']] 8.144
> Model[[(2, 0, 1), (0, 0, 0, 0), 'ct']] 8.373
> Model[[(2, 0, 2), (0, 0, 0, 0), 'n']] 8.281
> Model[[(2, 0, 2), (0, 0, 0, 0), 'c']] 8.736
```

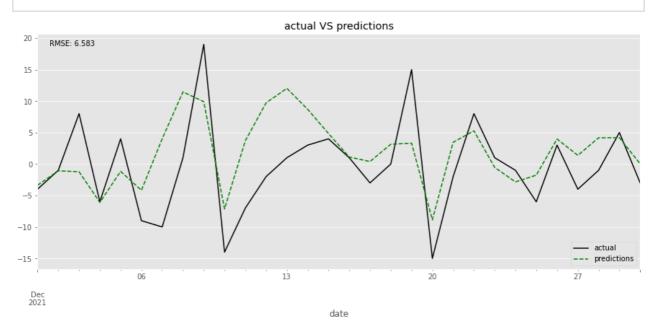
```
> Model[[(2, 0, 2), (0, 0, 0, 0), 't']] 8.556
                                   'ct']] 8.740
 Model[[(2, 0, 2), (0, 0, 0, 0),
> Model[[(2, 1, 0), (0, 0, 0, 0),
                                   'n']] 12.333
> Model[[(2, 1, 0), (0, 0, 0, 0),
                                   'c']] 12.354
> Model[[(2, 1, 0), (0, 0, 0, 0),
                                   't']] 12.394
                                   'ct']] 12.414
> Model[[(2, 1, 0), (0, 0, 0, 0),
                                   'n']] 8.761
> Model[[(2, 1, 1), (0, 0, 0, 0),
 Model[(2, 1, 1), (0, 0, 0, 0),
                                   'c'
                                      11 8.802
 Model[(2, 1, 1), (0, 0, 0, 0),
                                   't']] 9.228
 Model[[(2, 1, 1), (0, 0, 0, 0),
                                   'ct']] 9.225
>
 Model[[(2, 1, 2), (0, 0, 0, 0),
                                   'n']] 8.153
>
> Model[[(2, 1, 2), (0, 0, 0, 0), 'c']] 8.180
 Model[[(2, 1, 2), (0, 0, 0, 0),
                                   't'11 9.326
                                   'ct']] 9.485
> Model[[(2, 1, 2), (0, 0, 0, 0),
                                   'n']] 8.155
> Model[[(3, 0, 0), (0, 0, 0, 0),
                              0),
                                   'c'
 Model[[(3, 0, 0), (0,
                        0,0,
                                      11 8.164
> Model[[(3, 0, 0), (0, 0, 0, 0),
                                   't']] 8.187
> Model[[(3, 0, 0), (0, 0, 0, 0),
                                   'ct']] 8.200
> Model[[(3, 0, 1), (0, 0, 0, 0),
                                   'n']] 7.878
                                   'c']] 8.499
> Model[[(3, 0, 1), (0, 0, 0, 0),
> Model[[(3, 0, 1), (0, 0, 0, 0),
                                   't']] 8.162
                                   'ct']] 8.425
> Model[[(3, 0, 1), (0, 0, 0, 0),
                                   'n'11 8.298
 Model[[(3, 0,
                2), (0, 0, 0, 0),
>
 Model[[(3, 0, 2), (0, 0, 0, 0),
                                   'c'11 8.823
> Model[[(3, 0, 2), (0, 0, 0, 0),
                                   't']] 8.676
> Model[[(3, 0, 2), (0, 0, 0, 0),
                                  'ct']] 8.812
> Model[[(3, 1, 0), (0, 0, 0, 0), 'n']] 10.770
> Model[[(3, 1, 0), (0, 0, 0, 0),
                                  'c']] 10.787
> Model[[(3, 1, 0), (0, 0, 0, 0),
                                  't']] 10.822
                                   'ct']] 10.841
 Model[[(3, 1, 0), (0, 0, 0, 0),
 Model[[(3, 1, 1), (0, 0, 0, 0),
                                   'n']] 8.164
> Model[[(3, 1, 1), (0, 0, 0, 0),
                                   'c']] 8.200
> Model[[(3, 1, 1), (0, 0, 0, 0),
                                   't']] 8.418
> Model[[(3, 1, 1), (0, 0, 0, 0),
                                   'ct']] 8.325
> Model[[(3, 1, 2), (0, 0, 0, 0),
                                   'n']] 8.192
                                   'c']] 8.166
> Model[[(3, 1,
                2), (0, 0, 0, 0),
                                   't'
> Model[[(3, 1,
                2), (0, 0, 0, 0),
                                      11 8.778
 Model[[(3, 1, 2), (0, 0, 0, 0),
                                   'ct']] 8.753
> Model[[(4, 0, 0), (0, 0, 0, 0),
                                  'n']] 8.062
> Model[[(4, 0, 0), (0, 0, 0, 0),
                                  'c']] 8.069
> Model[[(4, 0, 0), (0, 0, 0, 0),
                                  't']] 8.093
> Model[[(4, 0, 0), (0, 0, 0, 0), 'ct']] 8.106
                                  'n']] 7.805
> Model[[(4, 0, 1), (0, 0, 0, 0),
> Model[[(4, 0, 1), (0, 0, 0, 0),
                                   'c'
                                      11 8.403
                                   't']] 8.111
 Model[[(4, 0, 1), (0, 0, 0, 0),
> Model[[(4, 0, 1), (0, 0, 0, 0),
                                   'ct']] 8.377
> Model[[(4, 0, 2), (0, 0, 0, 0),
                                   'n']] 8.278
> Model[[(4, 0, 2), (0, 0, 0, 0),
                                   'c']] 8.907
> Model[[(4, 0, 2), (0, 0, 0, 0),
                                   't']] 8.301
> Model[[(4, 0, 2), (0, 0, 0, 0),
                                   'ct']] 8.352
 Model[[(4, 1, 0), (0, 0, 0, 0),
                                   'n']] 10.382
 Model[[(4, 1, 0), (0, 0, 0, 0),
                                   'c']] 10.399
>
 Model[[(4, 1, 0), (0, 0, 0, 0),
                                   't']] 10.433
> Model[[(4, 1, 0), (0, 0, 0, 0), 'ct']] 10.453
> Model[[(4, 1, 1), (0, 0, 0, 0), 'n']] 8.069
> Model[[(4, 1, 1), (0, 0, 0, 0),
                                   'c'11 8.107
                                   't']] 8.595
> Model[[(4, 1, 1), (0, 0, 0,
                              0),
> Model[[(4, 1, 1), (0,
                              0),
                                   'ct']] 8.380
                       0,0,
                2), (0,
                                   'n']] 8.322
 Model[[(4, 1,
                        0,
                           0,
                              0),
                                   'c']] 8.356
 Model[[(4, 1, 2), (0, 0, 0, 0),
> Model[[(4, 1, 2), (0, 0, 0, 0),
                                   't']] 8.543
```

```
> Model[[(4, 1, 2), (0, 0, 0, 0), 'ct']] 8.643
Лучшие модели:
[(4, 0, 1), (0, 0, 0, 0), 'n'] 7.804742258221239
[(1, 0, 1), (0, 0, 0, 0), 'n'] 7.827625401830191
[(2, 0, 1), (0, 0, 0, 0), 'n'] 7.837023777784183
CPU times: user 26min 18s, sys: 19.5 s, total: 26min 37s
Wall time: 26min 22s
```

Итак, лучшая модель SARIMA имеет параметры [(4, 0, 1), (0, 0, 0, 0), 'n']. Оценим остатки модели. Для этого возьмём хотя бы 30 точек.



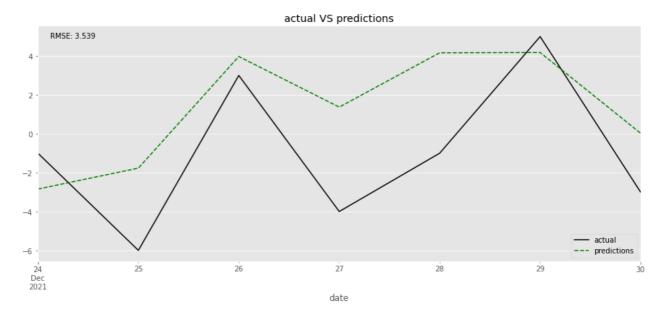




Out[105... 6.583204858236249

Судя по построенным графикам распределение остатков приближено к нормальному.

Так как нам в итоге нужно построить прогноз на неделю вперед, будем оценивать эту модель и сравнивать её по качеству с другими моделями метрикой RMSE за последнюю неделю.



Out[106... 3.5387752704318687

Мы построили модель SARIMA, теперь посмотрим, как распределены остатки модели. Судя по построенным графикам распределение остатков приближено к нормальному.

Градиентный бустинг CatBoost

Подготовка признаков

Мы можем реструктурировать последовательность временного ряда так, чтобы данные на входе в модель выглядели как задача обучения с учителем. Мы можем сделать это, используя предыдущие временные шаги в качестве входных переменных и используя следующий временной шаг в качестве выходной переменной. Попробуем добавить в качестве признаков также месяц, номер недели и день недели.

```
In [256... data = make_features(sales_diff, 'quantity', 7)
    data
```

Out[256	date	quantity	month	day	dayofweek	quantity_lag_1	quantity_lag_2	quantity_lag_3	q
	2021- 01- 05	26.0	1	5	1	NaN	NaN	NaN	
	2021- 01- 06	-5.0	1	6	2	26.0	NaN	NaN	
	2021- 01-07	3.0	1	7	3	-5.0	26.0	NaN	
	2021- 01- 08	5.0	1	8	4	3.0	-5.0	26.0	
	2021- 01- 09	3.0	1	9	5	5.0	3.0	-5.0	
	•••						•••		
	2021- 12-26	3.0	12	26	6	-6.0	-1.0	1.0	
	2021- 12-27	-4.0	12	27	0	3.0	-6.0	-1.0	
	2021- 12- 28	-1.0	12	28	1	-4.0	3.0	-6.0	
	2021- 12-29	5.0	12	29	2	-1.0	-4.0	3.0	
	2021- 12-	-3.0	12	30	3	5.0	-1.0	-4.0	

352 rows × 11 columns

30

Подготовка функций

Проверим написанную нами функцию разбиения на тренировочную и тестовую выборки применительно к новым данным.

```
# проверка
train, test = series_train_test_split(data, 70)
print(train.shape)
print(test.shape)
```

```
(282, 11)
(70, 11)
```

Изменим функцию walk-forward валидации с учётом формата входных данных.

```
In [258...
          # walk-forward validation for CatBoost
          def walk forward validation(data, n_test, target, config, plot=False):
              predictions = list()
              # разбиваем на train и test
              train, test = series_train_test_split(data, n_test)
              # помещаем train в историю
              history = train
              # проходимся по каждому из значений в тестовом наборе
              for i in range(len(test)):
                  # выделяем признаки и целевой признак
                  features test, target test = test.drop(columns=[target]).iloc[i],
                  # обучаем модель на истории и делаем предсказание на шаг вперёд
                  prediction = catboost forecast(history, features test, target, con
                  # сохраняем предсказание в список
                  predictions.append(prediction)
                  # добавляем действительное значение в историю
                  history.append(test.iloc[i])
                  # отслеживаем прогресс
                  print('>expected=%.1f, predicted=%.1f' % (target test, prediction)
              # вычисляем ошибку
              error = rmse(test[target], predictions)
              #return error, test[target], predictions
              if plot:
                  # график для визуального сравнения предсказанных значений с действ
                  # (по умолчанию не строим)
                  plot result(n test, error, test[target], predictions)
                  # график остатков
                  #model forecast(test.asfreq('D'), config, True)
              else:
                  # по умолчанию возвращаем интересующую метрику качества
                  return error
```

Напишем функцию прогноза на один шаг вперед, который будет делать лучшая выбранная по сетке параметров модель градиентного бустинга из библиотеки CatBoost.

```
In [259...
          # CatBoost-прогноз на шаг вперёд
          def catboost_forecast(train, features_test, target, config):
              # определяем конфигурацию
              learning rate, depth, 12 leaf reg, iterations, task type, verbose, rand
              # выделяем целевой признак из train
              features train, target train = train.drop(columns=[target]), train[target]
              # определяем модель
              model CBR = CatBoostRegressor(learning rate=learning rate,
                                             depth=depth,
                                             12_leaf_reg=12_leaf_reg,
                                             iterations=iterations,
                                             task_type=task_type,
                                             verbose=verbose,
                                             random_seed=random_seed,
                                             loss function='RMSE')
              # осуществляем grid search, на выходе получаем лучшую обученную модель
              model CBR.fit(features train, target train)
              # делаем прогноз на шаг вперед
              prediction = model CBR.predict(features test)
              return prediction
In [112...
          # проверка
          #catboost forecast(train, features_test.iloc[1], 'quantity', (0.01, 2, 1,
In [260...
          # оцениваем модель и возвращаем None при возникновении ошибки
          def score_model(data, n_test, target, config, debug=False):
              result = None
              # конвертируем конфигурацию в строковый ключ
              key = str(config)
              # показываем все предупреждения, если debug=True
              if debug:
                  result = walk_forward_validation(data, n_test, target, config)
              else:
                  # сбой во время проверки модели предполагает нестабильную конфигур
                      # во время поиска по сетке не показываем предупреждения
                      with catch warnings():
                          filterwarnings("ignore")
                          result = walk forward validation(data, n test, target, con
                  except:
                      error = None
              # проверяем на наличие интересующего результата
              if result is not None:
                  print(' > Model[%s] %.3f' % (key, result))
              return (key, result)
In [261...
          # проверка
          score model(data, 2, 'quantity', (0.01, 2, 1, 1000, 'CPU', 0, 12345), debug
```

```
>expected=5.0, predicted=3.6

>expected=-3.0, predicted=3.3

> Model[(0.01, 2, 1, 1000, 'CPU', 0, 12345)] 4.557

Out[261... ("(0.01, 2, 1, 1000, 'CPU', 0, 12345)", 4.556707122275873)
```

В библиотеке CatBoost есть штатная функция grid_search(), которая красиво визуализирует поиск по сетке, однако мы сделаем свою, чтобы затем можно было сравнить полученные результаты из разных типов моделей.

```
In [262...
          def grid search(data,
                          n test,
                          target,
                          config list, # список всех конфигураций
                          parallel=False):
              scores = None
              if parallel:
                  # определяем количество используемых ядер
                  #executor = Parallel(n jobs=cpu count()-1, backend='multiprocessin
                  scores = Parallel(n jobs=-1, backend='multiprocessing', verbose=1)
                  #print(executor)
                  # создаём список задач для параллельного выполнения
                  #tasks = (delayed(score model)(data, n test, target, config) for configuration
                  #print(tasks)
                  # передаём задачи в executor
                  #scores = executor(tasks)
              else:
                  scores = [score_model(data, n_test, target, config) for config in (
              # удаляем пустые результаты
              scores = [r for r in scores if r[1] != None]
              # сортируем кортежи в списке по баллу в порядке возрастания (сначала л
              scores.sort(key=lambda tup: tup[1])
              return scores
```

Создадим список всех конфигураций гиперпараметров.

```
In [263...
          # создаем набор конфигураций гиперпараметров
          def catboost_configs():
              models = list()
               # определяем гиперпараметры
               learning rate = [0.01, 0.03, 0.1, 0.2, 0.3]
               depth = [2, 4, 6, 8, 10]
               12 \text{ leaf reg} = [1, 3, 5, 7, 9]
               iterations = [1000]
               task_type = ['CPU']
              verbose = [0]
              random seed = [12345]
               # заполняем список конфигураций
               for lr in learning rate:
                   for d in depth:
                       for leaf in 12_leaf_reg:
                           for i in iterations:
                               for t in task type:
                                    for v in verbose:
                                        for r in random seed:
                                            config = lr,d,leaf,i,t,v,r
                                            models.append(config)
               return models
In [117...
          # проверка
          catboost configs()[1]
Out[117... (0.01, 2, 3, 1000, 'CPU', 0, 12345)
```

Выбор лучшей конфигурации гиперпараметров для модели CatBoost

```
In [264...
         if __name__ == '__main__':
              n test = 7
              target = 'quantity'
              config_list = catboost_configs()
              scores = grid search(data, n test, target, config list, parallel=False
              print('Лучшие модели:')
              # топ-3 конфигурации
              for config, error in scores[:3]:
                  print(config, error)
         >expected=-1.0, predicted=-2.2
         >expected=-6.0, predicted=0.3
         >expected=3.0, predicted=6.2
         >expected=-4.0, predicted=1.3
         >expected=-1.0, predicted=3.0
         >expected=5.0, predicted=3.7
         >expected=-3.0, predicted=3.1
          > Model[(0.01, 2, 1, 1000, 'CPU', 0, 12345)] 4.392
         >expected=-1.0, predicted=-2.1
         >expected=-6.0, predicted=0.4
         >expected=3.0, predicted=6.0
```

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>expected=-4.0, predicted=1.3
>expected=-1.0, predicted=2.8
>expected=5.0, predicted=3.3
>expected=-3.0, predicted=2.6
> Model[(0.01, 2, 3, 1000, 'CPU', 0, 12345)] 4.272
>expected=-1.0, predicted=-1.8
>expected=-6.0, predicted=0.3
>expected=3.0, predicted=5.3
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=2.7
>expected=5.0, predicted=2.8
>expected=-3.0, predicted=1.6
> Model[(0.01, 2, 5, 1000, 'CPU', 0, 12345)] 4.007
>expected=-1.0, predicted=-2.0
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=5.2
>expected=-4.0, predicted=0.7
>expected=-1.0, predicted=2.3
>expected=5.0, predicted=2.4
>expected=-3.0, predicted=1.3
> Model[(0.01, 2, 7, 1000, 'CPU', 0, 12345)] 3.803
>expected=-1.0, predicted=-1.8
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=5.1
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=2.2
>expected=5.0, predicted=2.3
>expected=-3.0, predicted=1.0
> Model[(0.01, 2, 9, 1000, 'CPU', 0, 12345)] 3.760
>expected=-1.0, predicted=-3.1
>expected=-6.0, predicted=1.5
>expected=3.0, predicted=6.1
>expected=-4.0, predicted=1.0
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=5.1
>expected=-3.0, predicted=4.3
> Model[(0.01, 4, 1, 1000, 'CPU', 0, 12345)] 4.888
>expected=-1.0, predicted=-3.0
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=7.3
>expected=-4.0, predicted=0.5
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=5.1
>expected=-3.0, predicted=4.6
> Model[(0.01, 4, 3, 1000, 'CPU', 0, 12345)] 4.913
>expected=-1.0, predicted=-3.5
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=6.8
>expected=-4.0, predicted=0.7
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=4.2
>expected=-3.0, predicted=3.5
> Model[(0.01, 4, 5, 1000, 'CPU', 0, 12345)] 4.699
>expected=-1.0, predicted=-2.8
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=0.4
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=4.3
>expected=-3.0, predicted=3.0
 > Model[(0.01, 4, 7, 1000, 'CPU', 0, 12345)] 4.364
```

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>expected=-1.0, predicted=-2.7
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=0.3
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=3.8
>expected=-3.0, predicted=2.4
> Model[(0.01, 4, 9, 1000, 'CPU', 0, 12345)] 4.243
>expected=-1.0, predicted=-5.2
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=6.6
>expected=-4.0, predicted=0.9
>expected=-1.0, predicted=2.8
>expected=5.0, predicted=6.4
>expected=-3.0, predicted=3.3
> Model[(0.01, 6, 1, 1000, 'CPU', 0, 12345)] 5.032
>expected=-1.0, predicted=-3.8
>expected=-6.0, predicted=2.6
>expected=3.0, predicted=7.0
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=5.4
>expected=-3.0, predicted=2.6
> Model[(0.01, 6, 3, 1000, 'CPU', 0, 12345)] 5.017
>expected=-1.0, predicted=-3.8
>expected=-6.0, predicted=2.0
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=1.4
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=4.6
>expected=-3.0, predicted=3.2
> Model[(0.01, 6, 5, 1000, 'CPU', 0, 12345)] 4.886
>expected=-1.0, predicted=-3.3
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=6.7
>expected=-4.0, predicted=0.6
>expected=-1.0, predicted=3.0
>expected=5.0, predicted=4.7
>expected=-3.0, predicted=2.6
> Model[(0.01, 6, 7, 1000, 'CPU', 0, 12345)] 4.709
>expected=-1.0, predicted=-3.1
>expected=-6.0, predicted=2.0
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=0.5
>expected=-1.0, predicted=3.4
>expected=5.0, predicted=3.6
>expected=-3.0, predicted=1.8
> Model[(0.01, 6, 9, 1000, 'CPU', 0, 12345)] 4.567
>expected=-1.0, predicted=-5.7
>expected=-6.0, predicted=1.6
>expected=3.0, predicted=6.6
>expected=-4.0, predicted=0.9
>expected=-1.0, predicted=4.1
>expected=5.0, predicted=5.9
>expected=-3.0, predicted=3.0
> Model[(0.01, 8, 1, 1000, 'CPU', 0, 12345)] 5.074
>expected=-1.0, predicted=-4.8
>expected=-6.0, predicted=1.6
>expected=3.0, predicted=6.4
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=2.7
```

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>expected=5.0, predicted=5.7
>expected=-3.0, predicted=2.5
> Model[(0.01, 8, 3, 1000, 'CPU', 0, 12345)] 4.648
>expected=-1.0, predicted=-3.8
>expected=-6.0, predicted=1.6
>expected=3.0, predicted=6.3
>expected=-4.0, predicted=0.6
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=4.7
>expected=-3.0, predicted=1.8
> Model[(0.01, 8, 5, 1000, 'CPU', 0, 12345)] 4.359
>expected=-1.0, predicted=-4.2
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=6.1
>expected=-4.0, predicted=0.5
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=4.5
>expected=-3.0, predicted=1.7
> Model[(0.01, 8, 7, 1000, 'CPU', 0, 12345)] 4.564
>expected=-1.0, predicted=-3.2
>expected=-6.0, predicted=2.1
>expected=3.0, predicted=5.9
>expected=-4.0, predicted=0.3
>expected=-1.0, predicted=3.1
>expected=5.0, predicted=4.2
>expected=-3.0, predicted=1.5
> Model[(0.01, 8, 9, 1000, 'CPU', 0, 12345)] 4.389
>expected=-1.0, predicted=-4.8
>expected=-6.0, predicted=1.4
>expected=3.0, predicted=6.1
>expected=-4.0, predicted=1.5
>expected=-1.0, predicted=2.7
>expected=5.0, predicted=5.6
>expected=-3.0, predicted=1.5
> Model[(0.01, 10, 1, 1000, 'CPU', 0, 12345)] 4.532
>expected=-1.0, predicted=-4.3
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=5.5
>expected=-4.0, predicted=1.4
>expected=-1.0, predicted=2.7
>expected=5.0, predicted=5.6
>expected=-3.0, predicted=1.4
> Model[(0.01, 10, 3, 1000, 'CPU', 0, 12345)] 4.279
>expected=-1.0, predicted=-3.8
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=5.8
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=3.0
>expected=5.0, predicted=4.1
>expected=-3.0, predicted=0.9
> Model[(0.01, 10, 5, 1000, 'CPU', 0, 12345)] 4.217
>expected=-1.0, predicted=-3.4
>expected=-6.0, predicted=1.1
>expected=3.0, predicted=5.4
>expected=-4.0, predicted=0.7
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=3.9
>expected=-3.0, predicted=0.9
> Model[(0.01, 10, 7, 1000, 'CPU', 0, 12345)] 4.017
>expected=-1.0, predicted=-2.9
>expected=-6.0, predicted=1.3
```

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>expected=3.0, predicted=4.8
>expected=-4.0, predicted=0.2
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=3.9
>expected=-3.0, predicted=0.7
> Model[(0.01, 10, 9, 1000, 'CPU', 0, 12345)] 3.871
>expected=-1.0, predicted=-3.7
>expected=-6.0, predicted=0.4
>expected=3.0, predicted=8.2
>expected=-4.0, predicted=3.3
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=5.3
>expected=-3.0, predicted=4.9
> Model[(0.03, 2, 1, 1000, 'CPU', 0, 12345)] 5.495
>expected=-1.0, predicted=-3.2
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=8.8
>expected=-4.0, predicted=3.5
>expected=-1.0, predicted=3.7
>expected=5.0, predicted=4.4
>expected=-3.0, predicted=5.1
> Model[(0.03, 2, 3, 1000, 'CPU', 0, 12345)] 5.611
>expected=-1.0, predicted=-3.2
>expected=-6.0, predicted=-0.1
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=3.1
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=4.2
>expected=-3.0, predicted=4.8
> Model[(0.03, 2, 5, 1000, 'CPU', 0, 12345)] 5.357
>expected=-1.0, predicted=-3.3
>expected=-6.0, predicted=0.0
>expected=3.0, predicted=8.7
>expected=-4.0, predicted=2.9
>expected=-1.0, predicted=3.4
>expected=5.0, predicted=3.9
>expected=-3.0, predicted=5.0
> Model[(0.03, 2, 7, 1000, 'CPU', 0, 12345)] 5.423
>expected=-1.0, predicted=-3.1
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=7.9
>expected=-4.0, predicted=2.9
>expected=-1.0, predicted=3.2
>expected=5.0, predicted=3.9
>expected=-3.0, predicted=4.7
> Model[(0.03, 2, 9, 1000, 'CPU', 0, 12345)] 5.225
>expected=-1.0, predicted=-7.1
>expected=-6.0, predicted=4.1
>expected=3.0, predicted=6.7
>expected=-4.0, predicted=2.2
>expected=-1.0, predicted=4.5
>expected=5.0, predicted=6.8
>expected=-3.0, predicted=6.2
> Model[(0.03, 4, 1, 1000, 'CPU', 0, 12345)] 6.650
>expected=-1.0, predicted=-6.1
>expected=-6.0, predicted=4.5
>expected=3.0, predicted=7.9
>expected=-4.0, predicted=2.5
>expected=-1.0, predicted=4.7
>expected=5.0, predicted=8.3
>expected=-3.0, predicted=6.7
```

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> Model[(0.03, 4, 3, 1000, 'CPU', 0, 12345)] 6.959
>expected=-1.0, predicted=-6.3
>expected=-6.0, predicted=1.9
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=2.0
>expected=-1.0, predicted=3.4
>expected=5.0, predicted=8.9
>expected=-3.0, predicted=8.0
> Model[(0.03, 4, 5, 1000, 'CPU', 0, 12345)] 6.647
>expected=-1.0, predicted=-6.2
>expected=-6.0, predicted=2.8
>expected=3.0, predicted=8.3
>expected=-4.0, predicted=2.6
>expected=-1.0, predicted=5.2
>expected=5.0, predicted=8.5
>expected=-3.0, predicted=7.2
> Model[(0.03, 4, 7, 1000, 'CPU', 0, 12345)] 6.884
>expected=-1.0, predicted=-6.4
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=8.0
>expected=-4.0, predicted=2.0
>expected=-1.0, predicted=4.0
>expected=5.0, predicted=7.8
>expected=-3.0, predicted=5.9
> Model[(0.03, 4, 9, 1000, 'CPU', 0, 12345)] 6.208
>expected=-1.0, predicted=-7.0
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=5.1
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=7.7
>expected=-3.0, predicted=5.3
> Model[(0.03, 6, 1, 1000, 'CPU', 0, 12345)] 5.737
>expected=-1.0, predicted=-7.3
>expected=-6.0, predicted=1.6
>expected=3.0, predicted=7.5
>expected=-4.0, predicted=2.2
>expected=-1.0, predicted=4.3
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=5.8
> Model[(0.03, 6, 3, 1000, 'CPU', 0, 12345)] 6.200
>expected=-1.0, predicted=-7.7
>expected=-6.0, predicted=3.1
>expected=3.0, predicted=7.8
>expected=-4.0, predicted=3.5
>expected=-1.0, predicted=3.7
>expected=5.0, predicted=7.8
>expected=-3.0, predicted=6.4
> Model[(0.03, 6, 5, 1000, 'CPU', 0, 12345)] 6.820
>expected=-1.0, predicted=-5.6
>expected=-6.0, predicted=2.9
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=2.4
>expected=-1.0, predicted=2.4
>expected=5.0, predicted=6.6
>expected=-3.0, predicted=5.0
> Model[(0.03, 6, 7, 1000, 'CPU', 0, 12345)] 5.730
>expected=-1.0, predicted=-6.1
>expected=-6.0, predicted=3.4
>expected=3.0, predicted=8.9
>expected=-4.0, predicted=3.0
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>expected=-1.0, predicted=2.8
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=4.3
> Model[(0.03, 6, 9, 1000, 'CPU', 0, 12345)] 6.235
>expected=-1.0, predicted=-5.3
>expected=-6.0, predicted=1.4
>expected=3.0, predicted=5.9
>expected=-4.0, predicted=1.8
>expected=-1.0, predicted=4.9
>expected=5.0, predicted=5.4
>expected=-3.0, predicted=2.5
> Model[(0.03, 8, 1, 1000, 'CPU', 0, 12345)] 5.073
>expected=-1.0, predicted=-5.4
>expected=-6.0, predicted=2.1
>expected=3.0, predicted=7.0
>expected=-4.0, predicted=2.0
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=4.0
> Model[(0.03, 8, 3, 1000, 'CPU', 0, 12345)] 5.395
>expected=-1.0, predicted=-5.1
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=8.1
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=3.2
>expected=5.0, predicted=7.4
>expected=-3.0, predicted=3.1
> Model[(0.03, 8, 5, 1000, 'CPU', 0, 12345)] 5.133
>expected=-1.0, predicted=-4.7
>expected=-6.0, predicted=2.8
>expected=3.0, predicted=6.5
>expected=-4.0, predicted=0.7
>expected=-1.0, predicted=4.5
>expected=5.0, predicted=8.5
>expected=-3.0, predicted=3.9
> Model[(0.03, 8, 7, 1000, 'CPU', 0, 12345)] 5.559
>expected=-1.0, predicted=-5.7
>expected=-6.0, predicted=2.7
>expected=3.0, predicted=8.2
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=4.0
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=4.4
> Model[(0.03, 8, 9, 1000, 'CPU', 0, 12345)] 5.828
>expected=-1.0, predicted=-3.4
>expected=-6.0, predicted=0.2
>expected=3.0, predicted=5.2
>expected=-4.0, predicted=2.8
>expected=-1.0, predicted=3.8
>expected=5.0, predicted=5.7
>expected=-3.0, predicted=2.5
> Model[(0.03, 10, 1, 1000, 'CPU', 0, 12345)] 4.614
>expected=-1.0, predicted=-5.6
>expected=-6.0, predicted=1.1
>expected=3.0, predicted=5.3
>expected=-4.0, predicted=0.9
>expected=-1.0, predicted=3.1
>expected=5.0, predicted=8.7
>expected=-3.0, predicted=3.3
> Model[(0.03, 10, 3, 1000, 'CPU', 0, 12345)] 4.937
>expected=-1.0, predicted=-3.9
```

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>expected=-6.0, predicted=1.4
>expected=3.0, predicted=5.6
>expected=-4.0, predicted=2.1
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=5.9
>expected=-3.0, predicted=4.5
> Model[(0.03, 10, 5, 1000, 'CPU', 0, 12345)] 5.040
>expected=-1.0, predicted=-4.4
>expected=-6.0, predicted=2.4
>expected=3.0, predicted=7.5
>expected=-4.0, predicted=1.4
>expected=-1.0, predicted=2.2
>expected=5.0, predicted=5.7
>expected=-3.0, predicted=2.7
> Model[(0.03, 10, 7, 1000, 'CPU', 0, 12345)] 4.994
>expected=-1.0, predicted=-5.0
>expected=-6.0, predicted=1.8
>expected=3.0, predicted=6.3
>expected=-4.0, predicted=2.1
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=6.0
>expected=-3.0, predicted=2.5
> Model[(0.03, 10, 9, 1000, 'CPU', 0, 12345)] 5.024
>expected=-1.0, predicted=-7.5
>expected=-6.0, predicted=1.5
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=5.9
>expected=-1.0, predicted=1.7
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=8.4
> Model[(0.1, 2, 1, 1000, 'CPU', 0, 12345)] 7.282
>expected=-1.0, predicted=-7.5
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=9.0
>expected=-4.0, predicted=5.1
>expected=-1.0, predicted=1.9
>expected=5.0, predicted=7.5
>expected=-3.0, predicted=9.7
> Model[(0.1, 2, 3, 1000, 'CPU', 0, 12345)] 7.289
>expected=-1.0, predicted=-8.0
>expected=-6.0, predicted=0.3
>expected=3.0, predicted=9.4
>expected=-4.0, predicted=4.5
>expected=-1.0, predicted=3.3
>expected=5.0, predicted=5.9
>expected=-3.0, predicted=8.6
> Model[(0.1, 2, 5, 1000, 'CPU', 0, 12345)] 7.128
>expected=-1.0, predicted=-9.0
>expected=-6.0, predicted=2.1
>expected=3.0, predicted=9.3
>expected=-4.0, predicted=5.0
>expected=-1.0, predicted=3.2
>expected=5.0, predicted=5.7
>expected=-3.0, predicted=9.3
> Model[(0.1, 2, 7, 1000, 'CPU', 0, 12345)] 7.739
>expected=-1.0, predicted=-7.9
>expected=-6.0, predicted=0.8
>expected=3.0, predicted=10.4
>expected=-4.0, predicted=5.0
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=5.3
```

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>expected=-3.0, predicted=9.4
> Model[(0.1, 2, 9, 1000, 'CPU', 0, 12345)] 7.520
>expected=-1.0, predicted=-7.4
>expected=-6.0, predicted=4.0
>expected=3.0, predicted=10.1
>expected=-4.0, predicted=4.6
>expected=-1.0, predicted=6.9
>expected=5.0, predicted=5.1
>expected=-3.0, predicted=7.6
> Model[(0.1, 4, 1, 1000, 'CPU', 0, 12345)] 7.932
>expected=-1.0, predicted=-8.3
>expected=-6.0, predicted=0.6
>expected=3.0, predicted=7.9
>expected=-4.0, predicted=1.8
>expected=-1.0, predicted=5.3
>expected=5.0, predicted=7.7
>expected=-3.0, predicted=8.1
> Model[(0.1, 4, 3, 1000, 'CPU', 0, 12345)] 6.812
>expected=-1.0, predicted=-8.2
>expected=-6.0, predicted=2.7
>expected=3.0, predicted=6.2
>expected=-4.0, predicted=3.5
>expected=-1.0, predicted=5.5
>expected=5.0, predicted=8.4
>expected=-3.0, predicted=6.8
> Model[(0.1, 4, 5, 1000, 'CPU', 0, 12345)] 7.043
>expected=-1.0, predicted=-9.4
>expected=-6.0, predicted=2.6
>expected=3.0, predicted=10.4
>expected=-4.0, predicted=3.5
>expected=-1.0, predicted=4.8
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=7.7
> Model (0.1, 4, 7, 1000, 'CPU', 0, 12345) ] 7.679
>expected=-1.0, predicted=-9.9
>expected=-6.0, predicted=3.6
>expected=3.0, predicted=9.7
>expected=-4.0, predicted=1.5
>expected=-1.0, predicted=4.4
>expected=5.0, predicted=8.1
>expected=-3.0, predicted=8.7
> Model[(0.1, 4, 9, 1000, 'CPU', 0, 12345)] 7.783
>expected=-1.0, predicted=-7.3
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=4.7
>expected=-4.0, predicted=1.9
>expected=-1.0, predicted=0.4
>expected=5.0, predicted=4.4
>expected=-3.0, predicted=4.2
> Model[(0.1, 6, 1, 1000, 'CPU', 0, 12345)] 5.328
>expected=-1.0, predicted=-8.6
>expected=-6.0, predicted=5.9
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=-0.4
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=6.2
> Model[(0.1, 6, 3, 1000, 'CPU', 0, 12345)] 7.023
>expected=-1.0, predicted=-7.7
>expected=-6.0, predicted=3.1
>expected=3.0, predicted=9.3
```

```
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=6.8
>expected=-3.0, predicted=5.6
> Model[(0.1, 6, 5, 1000, 'CPU', 0, 12345)] 6.333
>expected=-1.0, predicted=-8.0
>expected=-6.0, predicted=3.9
>expected=3.0, predicted=7.6
>expected=-4.0, predicted=1.6
>expected=-1.0, predicted=5.0
>expected=5.0, predicted=9.8
>expected=-3.0, predicted=5.5
> Model[(0.1, 6, 7, 1000, 'CPU', 0, 12345)] 6.880
>expected=-1.0, predicted=-5.9
>expected=-6.0, predicted=3.6
>expected=3.0, predicted=8.5
>expected=-4.0, predicted=1.3
>expected=-1.0, predicted=6.0
>expected=5.0, predicted=5.8
>expected=-3.0, predicted=6.3
> Model[(0.1, 6, 9, 1000, 'CPU', 0, 12345)] 6.668
>expected=-1.0, predicted=-6.5
>expected=-6.0, predicted=-0.1
>expected=3.0, predicted=6.4
>expected=-4.0, predicted=3.4
>expected=-1.0, predicted=1.3
>expected=5.0, predicted=8.0
>expected=-3.0, predicted=3.8
> Model[(0.1, 8, 1, 1000, 'CPU', 0, 12345)] 5.236
>expected=-1.0, predicted=-6.8
>expected=-6.0, predicted=2.1
>expected=3.0, predicted=7.4
>expected=-4.0, predicted=1.7
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=5.5
>expected=-3.0, predicted=4.0
> Model[(0.1, 8, 3, 1000, 'CPU', 0, 12345)] 5.525
>expected=-1.0, predicted=-6.3
>expected=-6.0, predicted=0.7
>expected=3.0, predicted=7.5
>expected=-4.0, predicted=3.0
>expected=-1.0, predicted=4.1
>expected=5.0, predicted=7.0
>expected=-3.0, predicted=1.6
> Model[(0.1, 8, 5, 1000, 'CPU', 0, 12345)] 5.262
>expected=-1.0, predicted=-7.0
>expected=-6.0, predicted=4.0
>expected=3.0, predicted=7.3
>expected=-4.0, predicted=0.9
>expected=-1.0, predicted=5.2
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=3.5
> Model[(0.1, 8, 7, 1000, 'CPU', 0, 12345)] 6.143
>expected=-1.0, predicted=-7.3
>expected=-6.0, predicted=2.1
>expected=3.0, predicted=7.0
>expected=-4.0, predicted=0.5
>expected=-1.0, predicted=6.0
>expected=5.0, predicted=9.0
>expected=-3.0, predicted=7.2
 > Model[(0.1, 8, 9, 1000, 'CPU', 0, 12345)] 6.667
```

```
>expected=-1.0, predicted=-2.6
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=8.0
>expected=-4.0, predicted=3.9
>expected=-1.0, predicted=4.4
>expected=5.0, predicted=6.5
>expected=-3.0, predicted=4.1
> Model[(0.1, 10, 1, 1000, 'CPU', 0, 12345)] 5.845
>expected=-1.0, predicted=-4.8
>expected=-6.0, predicted=1.6
>expected=3.0, predicted=6.2
>expected=-4.0, predicted=0.6
>expected=-1.0, predicted=3.3
>expected=5.0, predicted=6.8
>expected=-3.0, predicted=0.9
> Model[(0.1, 10, 3, 1000, 'CPU', 0, 12345)] 4.488
>expected=-1.0, predicted=-5.2
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=4.7
>expected=-4.0, predicted=0.3
>expected=-1.0, predicted=4.0
>expected=5.0, predicted=5.1
>expected=-3.0, predicted=2.8
> Model[(0.1, 10, 5, 1000, 'CPU', 0, 12345)] 4.582
>expected=-1.0, predicted=-4.5
>expected=-6.0, predicted=2.2
>expected=3.0, predicted=4.7
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=4.8
>expected=5.0, predicted=5.7
>expected=-3.0, predicted=-0.1
> Model[(0.1, 10, 7, 1000, 'CPU', 0, 12345)] 4.597
>expected=-1.0, predicted=-3.1
>expected=-6.0, predicted=2.0
>expected=3.0, predicted=5.8
>expected=-4.0, predicted=1.4
>expected=-1.0, predicted=3.2
>expected=5.0, predicted=7.1
>expected=-3.0, predicted=3.7
> Model[(0.1, 10, 9, 1000, 'CPU', 0, 12345)] 4.953
>expected=-1.0, predicted=-8.9
>expected=-6.0, predicted=3.0
>expected=3.0, predicted=8.7
>expected=-4.0, predicted=6.2
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=5.2
>expected=-3.0, predicted=9.9
> Model[(0.2, 2, 1, 1000, 'CPU', 0, 12345)] 8.127
>expected=-1.0, predicted=-12.2
>expected=-6.0, predicted=4.1
>expected=3.0, predicted=10.5
>expected=-4.0, predicted=5.0
>expected=-1.0, predicted=4.0
>expected=5.0, predicted=6.9
>expected=-3.0, predicted=12.5
> Model[(0.2, 2, 3, 1000, 'CPU', 0, 12345)] 9.519
>expected=-1.0, predicted=-9.4
>expected=-6.0, predicted=2.3
>expected=3.0, predicted=9.1
>expected=-4.0, predicted=3.6
>expected=-1.0, predicted=3.8
```

```
>expected=5.0, predicted=7.6
>expected=-3.0, predicted=11.4
> Model[(0.2, 2, 5, 1000, 'CPU', 0, 12345)] 8.199
>expected=-1.0, predicted=-11.9
>expected=-6.0, predicted=4.3
>expected=3.0, predicted=10.8
>expected=-4.0, predicted=4.9
>expected=-1.0, predicted=5.2
>expected=5.0, predicted=7.7
>expected=-3.0, predicted=10.7
> Model[(0.2, 2, 7, 1000, 'CPU', 0, 12345)] 9.244
>expected=-1.0, predicted=-11.1
>expected=-6.0, predicted=1.9
>expected=3.0, predicted=8.9
>expected=-4.0, predicted=3.4
>expected=-1.0, predicted=6.7
>expected=5.0, predicted=5.8
>expected=-3.0, predicted=8.7
> Model[(0.2, 2, 9, 1000, 'CPU', 0, 12345)] 8.010
>expected=-1.0, predicted=-5.1
>expected=-6.0, predicted=3.2
>expected=3.0, predicted=8.0
>expected=-4.0, predicted=2.5
>expected=-1.0, predicted=4.8
>expected=5.0, predicted=10.5
>expected=-3.0, predicted=7.2
> Model[(0.2, 4, 1, 1000, 'CPU', 0, 12345)] 6.926
>expected=-1.0, predicted=-8.2
>expected=-6.0, predicted=1.7
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=4.3
>expected=-1.0, predicted=4.8
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=6.3
> Model[(0.2, 4, 3, 1000, 'CPU', 0, 12345)] 6.910
>expected=-1.0, predicted=-9.8
>expected=-6.0, predicted=1.2
>expected=3.0, predicted=8.0
>expected=-4.0, predicted=4.9
>expected=-1.0, predicted=0.9
>expected=5.0, predicted=8.5
>expected=-3.0, predicted=8.5
> Model[(0.2, 4, 5, 1000, 'CPU', 0, 12345)] 7.385
>expected=-1.0, predicted=-11.1
>expected=-6.0, predicted=4.8
>expected=3.0, predicted=7.9
>expected=-4.0, predicted=3.4
>expected=-1.0, predicted=6.8
>expected=5.0, predicted=7.9
>expected=-3.0, predicted=7.9
> Model[(0.2, 4, 7, 1000, 'CPU', 0, 12345)] 8.333
>expected=-1.0, predicted=-12.8
>expected=-6.0, predicted=4.9
>expected=3.0, predicted=9.6
>expected=-4.0, predicted=4.6
>expected=-1.0, predicted=6.4
>expected=5.0, predicted=10.1
>expected=-3.0, predicted=8.7
> Model[(0.2, 4, 9, 1000, 'CPU', 0, 12345)] 9.192
>expected=-1.0, predicted=-8.4
>expected=-6.0, predicted=2.4
```

```
>expected=3.0, predicted=6.2
>expected=-4.0, predicted=4.5
>expected=-1.0, predicted=1.2
>expected=5.0, predicted=5.1
>expected=-3.0, predicted=3.5
> Model[(0.2, 6, 1, 1000, 'CPU', 0, 12345)] 6.031
>expected=-1.0, predicted=-9.2
>expected=-6.0, predicted=1.2
>expected=3.0, predicted=8.8
>expected=-4.0, predicted=4.7
>expected=-1.0, predicted=8.2
>expected=5.0, predicted=8.9
>expected=-3.0, predicted=4.5
> Model[(0.2, 6, 3, 1000, 'CPU', 0, 12345)] 7.410
>expected=-1.0, predicted=-5.4
>expected=-6.0, predicted=1.8
>expected=3.0, predicted=6.1
>expected=-4.0, predicted=4.3
>expected=-1.0, predicted=5.6
>expected=5.0, predicted=9.5
>expected=-3.0, predicted=5.1
> Model[(0.2, 6, 5, 1000, 'CPU', 0, 12345)] 6.411
>expected=-1.0, predicted=-3.3
>expected=-6.0, predicted=3.7
>expected=3.0, predicted=8.3
>expected=-4.0, predicted=3.5
>expected=-1.0, predicted=4.7
>expected=5.0, predicted=8.0
>expected=-3.0, predicted=6.8
> Model[(0.2, 6, 7, 1000, 'CPU', 0, 12345)] 6.787
>expected=-1.0, predicted=-8.1
>expected=-6.0, predicted=1.9
>expected=3.0, predicted=10.5
>expected=-4.0, predicted=0.3
>expected=-1.0, predicted=3.3
>expected=5.0, predicted=6.6
>expected=-3.0, predicted=6.2
> Model[(0.2, 6, 9, 1000, 'CPU', 0, 12345)] 6.465
>expected=-1.0, predicted=-8.6
>expected=-6.0, predicted=-1.8
>expected=3.0, predicted=5.0
>expected=-4.0, predicted=6.3
>expected=-1.0, predicted=1.9
>expected=5.0, predicted=5.6
>expected=-3.0, predicted=0.9
> Model[(0.2, 8, 1, 1000, 'CPU', 0, 12345)] 5.497
>expected=-1.0, predicted=-6.5
>expected=-6.0, predicted=-0.3
>expected=3.0, predicted=5.8
>expected=-4.0, predicted=1.1
>expected=-1.0, predicted=3.2
>expected=5.0, predicted=6.4
>expected=-3.0, predicted=2.7
> Model[(0.2, 8, 3, 1000, 'CPU', 0, 12345)] 4.608
>expected=-1.0, predicted=-4.9
>expected=-6.0, predicted=2.5
>expected=3.0, predicted=5.6
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=5.1
>expected=5.0, predicted=8.2
>expected=-3.0, predicted=5.3
```

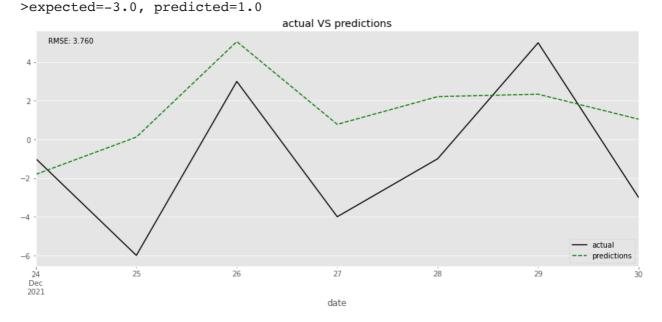
```
> Model[(0.2, 8, 5, 1000, 'CPU', 0, 12345)] 5.776
>expected=-1.0, predicted=-3.3
>expected=-6.0, predicted=-0.5
>expected=3.0, predicted=5.8
>expected=-4.0, predicted=2.2
>expected=-1.0, predicted=2.5
>expected=5.0, predicted=5.3
>expected=-3.0, predicted=3.0
> Model[(0.2, 8, 7, 1000, 'CPU', 0, 12345)] 4.336
>expected=-1.0, predicted=-6.4
>expected=-6.0, predicted=4.2
>expected=3.0, predicted=8.1
>expected=-4.0, predicted=-0.1
>expected=-1.0, predicted=4.4
>expected=5.0, predicted=10.8
>expected=-3.0, predicted=4.6
> Model[(0.2, 8, 9, 1000, 'CPU', 0, 12345)] 6.499
>expected=-1.0, predicted=2.2
>expected=-6.0, predicted=3.6
>expected=3.0, predicted=4.1
>expected=-4.0, predicted=2.3
>expected=-1.0, predicted=-0.4
>expected=5.0, predicted=5.6
>expected=-3.0, predicted=1.0
> Model[(0.2, 10, 1, 1000, 'CPU', 0, 12345)] 4.784
>expected=-1.0, predicted=-2.9
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=7.3
>expected=-4.0, predicted=4.3
>expected=-1.0, predicted=3.8
>expected=5.0, predicted=6.9
>expected=-3.0, predicted=0.5
> Model[(0.2, 10, 3, 1000, 'CPU', 0, 12345)] 5.062
>expected=-1.0, predicted=-5.5
>expected=-6.0, predicted=2.5
>expected=3.0, predicted=5.3
>expected=-4.0, predicted=0.8
>expected=-1.0, predicted=4.4
>expected=5.0, predicted=6.6
>expected=-3.0, predicted=1.9
> Model[(0.2, 10, 5, 1000, 'CPU', 0, 12345)] 5.039
>expected=-1.0, predicted=-3.4
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=8.6
>expected=-4.0, predicted=2.4
>expected=-1.0, predicted=4.3
>expected=5.0, predicted=6.7
>expected=-3.0, predicted=5.3
> Model[(0.2, 10, 7, 1000, 'CPU', 0, 12345)] 5.749
>expected=-1.0, predicted=-1.1
>expected=-6.0, predicted=1.1
>expected=3.0, predicted=5.1
>expected=-4.0, predicted=1.6
>expected=-1.0, predicted=4.7
>expected=5.0, predicted=6.1
>expected=-3.0, predicted=0.7
> Model[(0.2, 10, 9, 1000, 'CPU', 0, 12345)] 4.373
>expected=-1.0, predicted=-10.1
>expected=-6.0, predicted=4.5
>expected=3.0, predicted=10.9
>expected=-4.0, predicted=6.6
```

```
>expected=-1.0, predicted=6.1
>expected=5.0, predicted=10.4
>expected=-3.0, predicted=10.4
> Model[(0.3, 2, 1, 1000, 'CPU', 0, 12345)] 9.458
>expected=-1.0, predicted=-11.4
>expected=-6.0, predicted=1.0
>expected=3.0, predicted=9.3
>expected=-4.0, predicted=5.5
>expected=-1.0, predicted=3.5
>expected=5.0, predicted=11.0
>expected=-3.0, predicted=11.3
> Model[(0.3, 2, 3, 1000, 'CPU', 0, 12345)] 8.836
>expected=-1.0, predicted=-10.8
>expected=-6.0, predicted=2.6
>expected=3.0, predicted=10.5
>expected=-4.0, predicted=2.7
>expected=-1.0, predicted=4.2
>expected=5.0, predicted=6.4
>expected=-3.0, predicted=9.6
> Model[(0.3, 2, 5, 1000, 'CPU', 0, 12345)] 8.096
>expected=-1.0, predicted=-9.2
>expected=-6.0, predicted=1.2
>expected=3.0, predicted=10.3
>expected=-4.0, predicted=7.1
>expected=-1.0, predicted=1.5
>expected=5.0, predicted=4.7
>expected=-3.0, predicted=11.1
> Model[(0.3, 2, 7, 1000, 'CPU', 0, 12345)] 8.476
>expected=-1.0, predicted=-9.6
>expected=-6.0, predicted=2.5
>expected=3.0, predicted=10.9
>expected=-4.0, predicted=4.8
>expected=-1.0, predicted=4.8
>expected=5.0, predicted=7.3
>expected=-3.0, predicted=10.2
> Model[(0.3, 2, 9, 1000, 'CPU', 0, 12345)] 8.427
>expected=-1.0, predicted=-2.5
>expected=-6.0, predicted=3.0
>expected=3.0, predicted=13.2
>expected=-4.0, predicted=2.7
>expected=-1.0, predicted=13.8
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=10.0
> Model[(0.3, 4, 1, 1000, 'CPU', 0, 12345)] 9.465
>expected=-1.0, predicted=-5.9
>expected=-6.0, predicted=1.5
>expected=3.0, predicted=5.0
>expected=-4.0, predicted=2.1
>expected=-1.0, predicted=4.7
>expected=5.0, predicted=8.0
>expected=-3.0, predicted=4.8
> Model[(0.3, 4, 3, 1000, 'CPU', 0, 12345)] 5.649
>expected=-1.0, predicted=-10.0
>expected=-6.0, predicted=0.4
>expected=3.0, predicted=7.6
>expected=-4.0, predicted=-0.0
>expected=-1.0, predicted=3.4
>expected=5.0, predicted=7.0
>expected=-3.0, predicted=5.3
 > Model[(0.3, 4, 5, 1000, 'CPU', 0, 12345)] 6.004
>expected=-1.0, predicted=-9.8
```

```
>expected=-6.0, predicted=2.7
>expected=3.0, predicted=13.9
>expected=-4.0, predicted=1.3
>expected=-1.0, predicted=3.9
>expected=5.0, predicted=9.5
>expected=-3.0, predicted=6.4
> Model[(0.3, 4, 7, 1000, 'CPU', 0, 12345)] 7.847
>expected=-1.0, predicted=-9.7
>expected=-6.0, predicted=4.1
>expected=3.0, predicted=11.0
>expected=-4.0, predicted=1.8
>expected=-1.0, predicted=3.7
>expected=5.0, predicted=6.9
>expected=-3.0, predicted=7.2
> Model[(0.3, 4, 9, 1000, 'CPU', 0, 12345)] 7.601
>expected=-1.0, predicted=-4.8
>expected=-6.0, predicted=-3.1
>expected=3.0, predicted=9.3
>expected=-4.0, predicted=1.5
>expected=-1.0, predicted=0.8
>expected=5.0, predicted=6.2
>expected=-3.0, predicted=4.5
> Model[(0.3, 6, 1, 1000, 'CPU', 0, 12345)] 4.660
>expected=-1.0, predicted=-10.7
>expected=-6.0, predicted=1.5
>expected=3.0, predicted=6.9
>expected=-4.0, predicted=1.5
>expected=-1.0, predicted=0.5
>expected=5.0, predicted=6.6
>expected=-3.0, predicted=4.9
> Model[(0.3, 6, 3, 1000, 'CPU', 0, 12345)] 6.134
>expected=-1.0, predicted=-7.6
>expected=-6.0, predicted=2.0
>expected=3.0, predicted=6.6
>expected=-4.0, predicted=4.3
>expected=-1.0, predicted=2.9
>expected=5.0, predicted=7.2
>expected=-3.0, predicted=2.7
> Model[(0.3, 6, 5, 1000, 'CPU', 0, 12345)] 5.870
>expected=-1.0, predicted=-8.3
>expected=-6.0, predicted=2.8
>expected=3.0, predicted=8.9
>expected=-4.0, predicted=2.5
>expected=-1.0, predicted=6.1
>expected=5.0, predicted=7.0
>expected=-3.0, predicted=5.1
> Model[(0.3, 6, 7, 1000, 'CPU', 0, 12345)] 6.847
>expected=-1.0, predicted=-7.0
>expected=-6.0, predicted=3.3
>expected=3.0, predicted=7.6
>expected=-4.0, predicted=2.6
>expected=-1.0, predicted=2.6
>expected=5.0, predicted=6.7
>expected=-3.0, predicted=4.1
> Model[(0.3, 6, 9, 1000, 'CPU', 0, 12345)] 6.025
>expected=-1.0, predicted=-6.8
>expected=-6.0, predicted=-1.4
>expected=3.0, predicted=4.9
>expected=-4.0, predicted=4.8
>expected=-1.0, predicted=1.5
>expected=5.0, predicted=9.7
```

```
>expected=-3.0, predicted=0.3
> Model[(0.3, 8, 1, 1000, 'CPU', 0, 12345)] 4.999
>expected=-1.0, predicted=-7.8
>expected=-6.0, predicted=-0.8
>expected=3.0, predicted=8.3
>expected=-4.0, predicted=1.4
>expected=-1.0, predicted=0.1
>expected=5.0, predicted=7.6
>expected=-3.0, predicted=0.1
> Model[(0.3, 8, 3, 1000, 'CPU', 0, 12345)] 4.613
>expected=-1.0, predicted=-3.3
>expected=-6.0, predicted=2.6
>expected=3.0, predicted=4.4
>expected=-4.0, predicted=-1.8
>expected=-1.0, predicted=8.6
>expected=5.0, predicted=3.9
>expected=-3.0, predicted=2.6
> Model[(0.3, 8, 5, 1000, 'CPU', 0, 12345)] 5.496
>expected=-1.0, predicted=-5.8
>expected=-6.0, predicted=5.3
>expected=3.0, predicted=7.0
>expected=-4.0, predicted=1.7
>expected=-1.0, predicted=3.4
>expected=5.0, predicted=4.6
>expected=-3.0, predicted=2.3
> Model[(0.3, 8, 7, 1000, 'CPU', 0, 12345)] 5.919
>expected=-1.0, predicted=-2.9
>expected=-6.0, predicted=2.3
>expected=3.0, predicted=8.4
>expected=-4.0, predicted=2.3
>expected=-1.0, predicted=2.0
>expected=5.0, predicted=11.2
>expected=-3.0, predicted=5.0
> Model((0.3, 8, 9, 1000, 'CPU', 0, 12345)) 5.993
>expected=-1.0, predicted=-4.8
>expected=-6.0, predicted=0.9
>expected=3.0, predicted=4.6
>expected=-4.0, predicted=1.8
>expected=-1.0, predicted=4.6
>expected=5.0, predicted=10.1
>expected=-3.0, predicted=0.9
> Model[(0.3, 10, 1, 1000, 'CPU', 0, 12345)] 4.940
>expected=-1.0, predicted=-4.0
>expected=-6.0, predicted=0.1
>expected=3.0, predicted=1.1
>expected=-4.0, predicted=-0.5
>expected=-1.0, predicted=1.9
>expected=5.0, predicted=6.5
>expected=-3.0, predicted=3.2
> Model[(0.3, 10, 3, 1000, 'CPU', 0, 12345)] 3.985
>expected=-1.0, predicted=-6.9
>expected=-6.0, predicted=1.3
>expected=3.0, predicted=5.4
>expected=-4.0, predicted=0.2
>expected=-1.0, predicted=-3.1
>expected=5.0, predicted=8.6
>expected=-3.0, predicted=3.2
> Model[(0.3, 10, 5, 1000, 'CPU', 0, 12345)] 4.884
>expected=-1.0, predicted=-2.5
>expected=-6.0, predicted=3.5
>expected=3.0, predicted=1.4
```

```
>expected=-4.0, predicted=2.1
         >expected=-1.0, predicted=1.6
         >expected=5.0, predicted=6.1
         >expected=-3.0, predicted=3.7
          > Model[(0.3, 10, 7, 1000, 'CPU', 0, 12345)] 5.131
         >expected=-1.0, predicted=-3.4
         >expected=-6.0, predicted=3.6
         >expected=3.0, predicted=3.9
         >expected=-4.0, predicted=2.5
         >expected=-1.0, predicted=4.4
         >expected=5.0, predicted=5.3
         >expected=-3.0, predicted=2.7
          > Model[(0.3, 10, 9, 1000, 'CPU', 0, 12345)] 5.383
         Лучшие модели:
         (0.01, 2, 9, 1000, 'CPU', 0, 12345) 3.760407747333892
         (0.01, 2, 7, 1000, 'CPU', 0, 12345) 3.8028673083024542
         (0.01, 10, 9, 1000, 'CPU', 0, 12345) 3.8710005116083273
In [266...
          n test = 7
          target = 'quantity'
          config = (0.01, 2, 9, 1000, 'CPU', 0, 12345)
          walk forward_validation(data, n_test, target, config, plot=True)
         >expected=-1.0, predicted=-1.8
         >expected=-6.0, predicted=0.1
         >expected=3.0, predicted=5.1
         >expected=-4.0, predicted=0.8
         >expected=-1.0, predicted=2.2
         >expected=5.0, predicted=2.3
```



Мы построили модель CatBoost, однако в нашем случае она уступает модели SARIMA.

Нейронная сеть для временного ряда

Такие нейронные сети обладают рядом особенностей:

- Устойчивость к шуму. Нейронные сети устойчивы к шуму во входных данных и могут поддерживать обучение и прогнозирование при наличии отсутствующих значений.
- Нелинейность. Нейронные сети способны аппроксимировать произвольные нелинейные функции и учитывать линейные и нелинейные отношения.
- Многомерные входы. Можно указать произвольное количество входных функций.
- Многоэтапные прогнозы. Можно указать произвольное количество выходных значений, обеспечивающее поддержку многоступенчатого и даже многомерного прогнозирования.
- Фиксированные входы. Количество входных переменных с задержкой фиксировано так же, как и традиционные методы прогнозирования временных рядов.
- Выходные данные. Количество выходных переменных также фиксировано.

Рекуррентные нейронные сети, такие как сеть Long Short-Term Memory, добавляют явную обработку порядка между наблюдениями при изучении функции отображения от входов к выходам.

Добавление последовательности является новым измерением к аппроксимируемой функции. Вместо того, чтобы сопоставлять входы только с выходами, сеть способна со временем изучать функцию сопоставления входов с выходом.

В простейшем случае сеть показывает по одному наблюдению за раз из последовательности и может узнать, какие наблюдения она видела ранее, актуальны лм они, и насколько они актуальны для прогнозирования.

Если для традиционных методов прогнозирования временных рядов обязательно наличие стационарности ряда, то для нейронных сетей стационарность может не быть обязательным требованием для построения хорошей модели.

Vanilla LSTM

```
In [120...
```

```
from keras.models import Sequential
from keras.layers import LSTM
from keras.layers import Dense
```

На вход модель LSTM принимает последовательность прошлых наблюдений. Преобразуем датасет соответствующим образом.

Оставим в качестве признаков только лаги.

Out[121	quantity	quantity_lag_1	quantity_lag_2	quantity_lag_3	quantity_lag_4	quantity_la
---------	----------	----------------	----------------	----------------	----------------	-------------

date						
2021- 01- 05	26.0	NaN	NaN	NaN	NaN	I
2021- 01- 06	-5.0	26.0	NaN	NaN	NaN	I
2021- 01-07	3.0	-5.0	26.0	NaN	NaN	I
2021- 01- 08	5.0	3.0	-5.0	26.0	NaN	I
2021- 01- 09	3.0	5.0	3.0	-5.0	26.0	I
•••		•••				
2021- 12-26	3.0	-6.0	-1.0	1.0	8.0	
2021- 12-27	-4.0	3.0	-6.0	-1.0	1.0	
2021- 12- 28	-1.0	-4.0	3.0	-6.0	-1.0	
2021- 12-29	5.0	-1.0	-4.0	3.0	-6.0	
2021- 12- 30	-3.0	5.0	-1.0	-4.0	3.0	

352 rows × 8 columns

В отличие от моделей из библиотеки CatBoost, которые по умолчанию удаляют NaN значения, на вход в LSTM нам нужно подать уже очищенные значения.

```
In [122... data = data.dropna()
```

Изменим функцию walk_forward_validation() применительно к модели LSTM.

```
In [123...
```

```
# walk-forward validation for LSTM
def walk_forward_validation(data, n_test, target, config, plot=False):
    predictions = list()
    # разбиваем последовательность
    train, test = series train test split(data, n test)
    # сохраняем обучающую выборку в истории
    history = train
    # проходимся по каждому из значений в тестовой последовательности
    for i in range(len(test)):
        # выделяем признаки и целевой признак в тесте
        features test, target test = test.drop(columns=[target]).iloc[i],
        # обучаемся на истории и делаем предсказание на шаг вперёд
        prediction = lstm forecast(history, features test, target, config)
        # сохраняем предсказание в список
        predictions.append(prediction)
        # сохраняем действительное значение в историю
        history.append(test.iloc[i])
        # для отслеживания процесса на каждом шаге:
        print('>expected=%.1f, predicted=%.1f' % (target test, prediction)
    # вычисляем ошибку предсказаний
    error = rmse(test[target], predictions)
    #return error, test[target], predictions
    if plot:
        # график для визуального сравнения предсказанных значений с действ
        # (по умолчанию не строим)
        plot_result(n_test, error, test[target], predictions)
        return error, test[target], predictions
    else:
        # по умолчанию возвращаем интересующую метрику качества
        return error
```

Напишем функцию для одношагового прогноза при помощи LSTM.

```
In [124...
          config = (70, 'relu', 'adam', 'mse', 1, 270, 1)
          # lstm-прогноз на шаг вперёд
          def lstm_forecast(train,
                            features_test, # строка признаков для валидами
                            target, # имя колонки датафрейма целевого признака
                            config):
              # определяем конфигурацию
              units, activation, optimizer, loss, n features, epochs, verbose = conf
              # выделяем целевой признак из train
              features_train, target_train = train.drop(columns=[target]), train[target]
              # преобразуем датафреймы в объекты питру (нам нужен определённый форма
              features_train_arr = features_train.to_numpy()
              features_test_arr = features_test.to_numpy()
              target_train_arr = target_train.to_numpy()
              # Модель принимает тренировочные данные в таком формате: (samples, tim
              # Изменим форму входных данных.
              n steps = features train arr.shape[1]
              features train arr = features train arr.reshape((features train.shape[
              features test arr = features test arr.reshape((1, n steps, n features)
              # определяем модель
              model_vanilla_LSTM = Sequential()
              model_vanilla_LSTM.add(LSTM(units=units, activation=activation, input_s
              model vanilla LSTM.add(Dense(1))
              model vanilla LSTM.compile(optimizer='adam', loss='mse')
              # обучаем модель, на выходе получаем обученную модель с минимальной MS
              model_vanilla_LSTM.fit(features_train_arr, target_train_arr, epochs=epochs
              # делаем прогноз на шаг вперед
              prediction = model vanilla LSTM.predict(features test arr)
              #prediction = pd.DataFrame(prediction[0], columns=['prediction'], inde
              #prediction = prediction.reshape((features train.shape[0], n steps))
              return prediction[0]
```

Так как обучение нейросети займёт значительное время, а из гиперпараметров мы будем менять разве что количество нейронов в скрытом слое и количество эпох, то функцию grid_search() специально писать не будем.

```
In [125...
    error_stckd, val, pred_stckd = walk_forward_validation(data, 7, 'quantity'
   Epoch 1/270
   Epoch 2/270
   Epoch 3/270
   Epoch 4/270
   11/11 [============== ] - 0s 6ms/step - loss: 121.9947
   Epoch 5/270
   Epoch 6/270
   Epoch 7/270
   Epoch 8/270
   11/11 [=============] - Os 5ms/step - loss: 96.3748
   Epoch 9/270
```

```
Epoch 10/270
Epoch 11/270
Epoch 12/270
Epoch 13/270
Epoch 14/270
Epoch 15/270
Epoch 16/270
Epoch 17/270
Epoch 18/270
Epoch 19/270
Epoch 20/270
Epoch 21/270
Epoch 22/270
Epoch 23/270
Epoch 24/270
Epoch 25/270
Epoch 26/270
Epoch 27/270
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
Epoch 32/270
Epoch 33/270
Epoch 34/270
Epoch 35/270
Epoch 36/270
11/11 [============= ] - 0s 5ms/step - loss: 55.6348
Epoch 37/270
Epoch 38/270
Epoch 39/270
```

```
Epoch 40/270
Epoch 41/270
11/11 [============= ] - 0s 6ms/step - loss: 50.6574
Epoch 42/270
Epoch 43/270
Epoch 44/270
Epoch 45/270
Epoch 46/270
Epoch 47/270
Epoch 48/270
Epoch 49/270
Epoch 50/270
Epoch 51/270
Epoch 52/270
Epoch 53/270
Epoch 54/270
Epoch 55/270
11/11 [============= ] - Os 5ms/step - loss: 42.1760
Epoch 56/270
Epoch 57/270
Epoch 58/270
Epoch 59/270
Epoch 60/270
Epoch 61/270
Epoch 62/270
Epoch 63/270
Epoch 64/270
Epoch 65/270
Epoch 66/270
Epoch 67/270
Epoch 68/270
Epoch 69/270
Epoch 70/270
```

```
Epoch 71/270
11/11 [============= ] - 0s 5ms/step - loss: 23.6658
Epoch 72/270
Epoch 73/270
Epoch 74/270
Epoch 75/270
Epoch 76/270
Epoch 77/270
Epoch 78/270
11/11 [============= ] - Os 6ms/step - loss: 22.4330
Epoch 79/270
Epoch 80/270
Epoch 81/270
Epoch 82/270
Epoch 83/270
Epoch 84/270
Epoch 85/270
Epoch 86/270
Epoch 87/270
Epoch 88/270
Epoch 89/270
Epoch 90/270
Epoch 91/270
Epoch 92/270
Epoch 93/270
Epoch 94/270
Epoch 95/270
Epoch 96/270
Epoch 97/270
11/11 [============ ] - 0s 5ms/step - loss: 10.1109
Epoch 98/270
Epoch 99/270
Epoch 100/270
```

Fnoch	101/270									
		========		==]	_	0s	5ms/step	_	loss:	9.6695
	102/270			1		0~	F== /=+==		1	0 0425
	103/270	========		-=]	_	US	oms/step	-	loss:	8.8435
11/11	[======			==]	_	0s	5ms/step	-	loss:	9.2732
	104/270	========		1		٥٥	Ema/aton		1000.	0 0152
	105/270]	_	US	oms/step	_	TOSS:	0.0133
				==]	-	0s	5ms/step	-	loss:	7.8105
	106/270	========	========	== 1	_	0s	5mg/sten	_	1055:	7.7025
Epoch	107/270									
		=======		==]	-	0s	5ms/step	-	loss:	7.6829
	108/270	========		== 1	_	0s	6ms/step	_	loss:	7.9947
Epoch	109/270									
	[======================================	========	=======	==]	-	0s	5ms/step	-	loss:	7.5304
		========		==]	_	0s	6ms/step	_	loss:	7.5021
Epoch	111/270									
	[======================================	========		==]	-	0s	5ms/step	-	loss:	6.4878
				==]	_	0s	6ms/step	_	loss:	7.6778
	113/270			,		0 -	5 /		1	7 6174
	114/270	========		==]	_	US	5ms/step	_	Toss:	7.61/4
11/11	[======			==]	_	0s	5ms/step	-	loss:	7.8407
	115/270	========		1		٥٥	1mg /g+on		1000.	6 7010
	116/270]	_	US	4ms/step	_	1055;	0.7010
				==]	-	0s	4ms/step	-	loss:	7.5380
	117/270 [======	========	========	== 1	_	0s	5ms/step	_	loss:	7.4684
Epoch	118/270									
		========		==]	-	0s	6ms/step	-	loss:	6.2520
	119/270	========	========	==]	_	0s	5ms/step	_	loss:	5.6679
Epoch	120/270									
	[======================================	========		==]	-	0s	5ms/step	-	loss:	5.4801
11/11	[======			==]	_	0s	5ms/step	_	loss:	5.2674
	122/270	========		1		0 ~	Ema/aton		10000	4 6E06
	123/270]	_	US	oms/step	_	TOSS:	4.0380
		========		==]	-	0s	5ms/step	-	loss:	3.6173
	124/270			== 1	_	Λe	5mg/gten	_	1099.	3 5054
Epoch	125/270									
	-	========	=======	==]	-	0s	5ms/step	-	loss:	3.3579
	126/270	========		== 1	_	0s	5ms/step	_	loss:	3.6484
Epoch	127/270									
	[=====================================	========	=======	==]	-	0s	5ms/step	-	loss:	3.6577
		========		==]	_	0s	5ms/step	_	loss:	3.5619
Epoch	129/270									
	130/270	========	=======	==]	-	US	oms/step	-	TOSS:	3.8791
11/11	[======	=======		==]	-	0s	4ms/step	-	loss:	4.2394
Epoch	131/270									

```
Epoch 132/270
Epoch 133/270
11/11 [============= ] - Os 5ms/step - loss: 4.6814
Epoch 134/270
Epoch 135/270
Epoch 136/270
Epoch 137/270
Epoch 138/270
Epoch 139/270
Epoch 140/270
Epoch 141/270
11/11 [============= ] - 0s 5ms/step - loss: 2.5247
Epoch 142/270
Epoch 143/270
11/11 [============== ] - 0s 5ms/step - loss: 2.1797
Epoch 144/270
Epoch 145/270
Epoch 146/270
Epoch 147/270
Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
Epoch 156/270
Epoch 157/270
Epoch 158/270
Epoch 159/270
11/11 [=============] - 0s 4ms/step - loss: 1.7084
Epoch 160/270
Epoch 161/270
```

Encah	162/270							
_	162/270 [====================================	:=]	_	0s	5ms/step	_	loss:	2.2015
Epoch	163/270	_			_			
	[=====================================	=]	-	0s	5ms/step	-	loss:	2.1334
	[======================================	=]	_	0s	5ms/step	_	loss:	1.8343
Epoch	165/270							
	[=====================================	=]	-	0s	bms/step	_	loss:	1.6710
11/11	[======================================	=]	_	0s	6ms/step	_	loss:	1.5175
	167/270 [====================================	1		٥٩	Ema/aton		1000.	1 4007
	168/270	.— J	_	US	oms/scep	_	1055:	1.490/
		=]	-	0s	5ms/step	-	loss:	1.6539
	169/270 [====================================	:= 1	_	0s	5ms/step	_	loss:	1.7545
Epoch	170/270							
	[=====================================	=]	-	0s	5ms/step	-	loss:	1.7778
_	[======================================	:=]	_	0s	5ms/step	_	loss:	1.4865
Epoch	172/270							
	[=====================================	=]	-	0s	5ms/step	_	loss:	1.3338
11/11	[======================================	=]	_	0s	5ms/step	_	loss:	1.2012
	174/270 [====================================	= 1	_	Λα	5mg/gton		1000.	1 2679
Epoch	175/270							
	176/270	=]	-	0s	5ms/step	-	loss:	1.0546
	176/270 [====================================	:=]	_	0s	6ms/step	_	loss:	0.9938
Epoch	177/270							
	[=====================================	=]	-	0s	5ms/step	_	loss:	1.2345
11/11	[======================================	=]	_	0s	5ms/step	_	loss:	1.2018
-	179/270 [====================================	= 1	_	Λα	6mg/gton		1000	1 0084
Epoch	180/270							
	[=====================================	=]	-	0s	5ms/step	-	loss:	0.9694
	[======================================	:=]	_	0s	5ms/step	_	loss:	1.2091
	182/270	_		•			7	1 1001
	[=====================================	:= <u>J</u>	_	0s	4ms/step	_	loss:	1.1321
11/11	[======================================	=]	-	0s	5ms/step	_	loss:	0.9446
	184/270 [====================================	:= 1	_	Λs	5mg/sten	_	1055:	0.8419
Epoch	185/270							
	[=====================================	=]	-	0s	5ms/step	-	loss:	0.7889
	[======================================	:=]	_	0s	5ms/step	_	loss:	0.8105
	187/270			_	- / .			
	[=====================================	:=]	-	0s	6ms/step	_	loss:	0.6798
11/11	[======================================	=]	_	0s	5ms/step	_	loss:	0.6877
	189/270 [====================================	= 1	_	Λs	6mg/sten	_	1099.	0 6955
Epoch	190/270	_			_			
	[=====================================	=]	-	0s	5ms/step	-	loss:	0.6616
	[======================================	=]	_	0s	5ms/step	_	loss:	0.7610
	192/270							

```
Epoch 193/270
Epoch 194/270
Epoch 195/270
Epoch 196/270
Epoch 197/270
Epoch 198/270
Epoch 199/270
Epoch 200/270
11/11 [============== ] - 0s 5ms/step - loss: 1.0090
Epoch 201/270
Epoch 202/270
11/11 [============= ] - 0s 5ms/step - loss: 0.9945
Epoch 203/270
Epoch 204/270
Epoch 205/270
Epoch 206/270
Epoch 207/270
Epoch 208/270
Epoch 209/270
Epoch 210/270
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
Epoch 216/270
Epoch 217/270
Epoch 218/270
Epoch 219/270
Epoch 220/270
11/11 [============= ] - 0s 5ms/step - loss: 1.1059
Epoch 221/270
Epoch 222/270
```

```
Epoch 223/270
Epoch 224/270
11/11 [============= ] - Os 5ms/step - loss: 0.9994
Epoch 225/270
Epoch 226/270
Epoch 227/270
Epoch 228/270
Epoch 229/270
Epoch 230/270
Epoch 231/270
Epoch 232/270
Epoch 233/270
Epoch 234/270
Epoch 235/270
Epoch 236/270
Epoch 237/270
Epoch 238/270
11/11 [============] - 0s 5ms/step - loss: 0.6215
Epoch 239/270
Epoch 240/270
Epoch 241/270
Epoch 242/270
Epoch 243/270
Epoch 244/270
Epoch 245/270
11/11 [===========] - 0s 5ms/step - loss: 0.2898
Epoch 246/270
Epoch 247/270
Epoch 248/270
Epoch 249/270
Epoch 250/270
11/11 [============== ] - 0s 4ms/step - loss: 0.3071
Epoch 251/270
Epoch 252/270
Epoch 253/270
```

```
Epoch 254/270
11/11 [============= ] - Os 5ms/step - loss: 0.5004
Epoch 255/270
11/11 [============== ] - 0s 5ms/step - loss: 0.4004
Epoch 256/270
Epoch 257/270
Epoch 258/270
11/11 [============] - 0s 6ms/step - loss: 0.4253
Epoch 259/270
Epoch 260/270
Epoch 261/270
Epoch 262/270
Epoch 263/270
11/11 [============= ] - 0s 5ms/step - loss: 0.9808
Epoch 264/270
Epoch 265/270
11/11 [============= ] - 0s 5ms/step - loss: 1.0103
Epoch 266/270
Epoch 267/270
Epoch 268/270
Epoch 269/270
11/11 [============= ] - Os 5ms/step - loss: 0.9317
Epoch 270/270
>expected=-1.0, predicted=-11.4
Epoch 1/270
Epoch 2/270
Epoch 3/270
Epoch 4/270
Epoch 5/270
Epoch 6/270
Epoch 7/270
Epoch 8/270
Epoch 9/270
Epoch 10/270
Epoch 11/270
Epoch 12/270
Epoch 13/270
```

```
Epoch 14/270
Epoch 15/270
Epoch 16/270
Epoch 17/270
Epoch 18/270
Epoch 19/270
Epoch 20/270
Epoch 21/270
11/11 [============= ] - Os 6ms/step - loss: 76.0469
Epoch 22/270
Epoch 23/270
Epoch 24/270
Epoch 25/270
Epoch 26/270
11/11 [============== ] - 0s 5ms/step - loss: 70.9145
Epoch 27/270
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
Epoch 32/270
11/11 [=============] - Os 5ms/step - loss: 64.2529
Epoch 33/270
Epoch 34/270
Epoch 35/270
Epoch 36/270
Epoch 37/270
Epoch 38/270
Epoch 39/270
Epoch 40/270
Epoch 41/270
11/11 [============= ] - 0s 6ms/step - loss: 51.6227
Epoch 42/270
Epoch 43/270
```

```
Epoch 44/270
Epoch 45/270
Epoch 46/270
Epoch 47/270
Epoch 48/270
Epoch 49/270
Epoch 50/270
Epoch 51/270
Epoch 52/270
Epoch 53/270
Epoch 54/270
Epoch 55/270
Epoch 56/270
Epoch 57/270
Epoch 58/270
Epoch 59/270
11/11 [=============] - Os 7ms/step - loss: 35.0885
Epoch 60/270
Epoch 61/270
Epoch 62/270
Epoch 63/270
Epoch 64/270
Epoch 65/270
Epoch 66/270
Epoch 67/270
Epoch 68/270
Epoch 69/270
Epoch 70/270
Epoch 71/270
Epoch 72/270
Epoch 73/270
Epoch 74/270
```

```
Epoch 75/270
11/11 [============= ] - 0s 5ms/step - loss: 21.7424
Epoch 76/270
Epoch 77/270
Epoch 78/270
Epoch 79/270
Epoch 80/270
Epoch 81/270
Epoch 82/270
Epoch 83/270
Epoch 84/270
Epoch 85/270
Epoch 86/270
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Epoch 88/270
Epoch 89/270
Epoch 90/270
Epoch 91/270
Epoch 92/270
Epoch 93/270
Epoch 94/270
Epoch 95/270
Epoch 96/270
Epoch 97/270
Epoch 98/270
Epoch 99/270
11/11 [============] - 0s 5ms/step - loss: 9.3490
Epoch 100/270
Epoch 101/270
Epoch 102/270
Epoch 103/270
Epoch 104/270
```

Enoch	105/270							
	[========]	_	0s	6ms/step	_	loss:	8.2399
	106/270			0 ~	[ma/a+am		1	7 2750
	[=====================================	-======]	-	US	oms/step	_	loss:	7.2759
11/11	[========]	-	0s	5ms/step	_	loss:	6.3083
	108/270 [========	:======================================	_	Λς	5mg/gten	_	1099.	6 2939
Epoch	109/270							
	[=====================================]	-	0s	5ms/step	-	loss:	6.8045
	[=========	:=======]	_	0s	6ms/step	_	loss:	6.7804
Epoch	111/270							
	[=====================================	:=======]	-	0s	bms/step	_	loss:	7.5911
11/11	[========]	_	0s	6ms/step	_	loss:	7.4973
	113/270 [=========	1		٥٥	Emg/gton		1000.	7 0007
	114/270	J	_	05	Jiis/step	_	1055:	7.0097
	[======================================]	-	0s	5ms/step	-	loss:	5.8960
	115/270 [=========	:======================================	_	0s	5ms/step	_	loss:	5.1404
Epoch	116/270							
	[=====================================]	-	0s	5ms/step	-	loss:	4.9811
	[========	:======]	_	0s	6ms/step	_	loss:	4.5730
	118/270	_			_ ,			
	[=====================================	-=======]	-	0s	5ms/step	-	loss:	4.6545
11/11	[========]	-	0s	5ms/step	_	loss:	5.3582
	120/270 [=========	1		۸c	5mg/gton		1000	5 4086
Epoch	121/270							
	[======================================	:=======]	-	0s	5ms/step	-	loss:	4.7685
	122/270 [========	:======================================	_	0s	5ms/step	_	loss:	5.5480
Epoch	123/270							
	[=====================================	:=======]	-	0s	6ms/step	-	loss:	5.1087
11/11	[========]	-	0s	5ms/step	_	loss:	4.7227
	125/270 [=========	1		۸c	5mg/gton		logg•	5 4220
	126/270]	_	US	Jiis/scep	_	1055	3.4220
	[======================================	========]	-	0s	5ms/step	-	loss:	4.3288
	127/270 [========	:======================================	_	0s	5ms/step	_	loss:	3.7733
Epoch	128/270							
	[=====================================	:=======]	-	0s	5ms/step	-	loss:	3.2892
	[========]	_	0s	5ms/step	_	loss:	3.0319
	130/270 [=========	1		٥٩	Ema/aton		10000	2 0040
	131/270	J	_	US	oms/scep	_	TOSS:	2.9049
	[======================================]	-	0s	5ms/step	-	loss:	2.4921
	132/270 [=========	:======================================	_	0s	6ms/step	_	loss:	2.4724
Epoch	133/270							
	[=====================================]	-	0s	5ms/step	-	loss:	2.5362
	[=========]	_	0s	6ms/step	_	loss:	2.5688
Epoch	135/270							

```
Epoch 136/270
Epoch 137/270
Epoch 138/270
Epoch 139/270
Epoch 140/270
Epoch 141/270
Epoch 142/270
Epoch 143/270
Epoch 144/270
Epoch 145/270
Epoch 146/270
Epoch 147/270
11/11 [============= ] - 0s 5ms/step - loss: 2.3989
Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
Epoch 156/270
Epoch 157/270
Epoch 158/270
Epoch 159/270
Epoch 160/270
Epoch 161/270
Epoch 162/270
Epoch 163/270
11/11 [=============] - 0s 7ms/step - loss: 1.1605
Epoch 164/270
Epoch 165/270
```

Enogh	166/270							
	[======================================	======]	_	0s	5ms/step	_	loss:	1.0513
Epoch	167/270							
	[======================================	======]	-	0s	5ms/step	-	loss:	1.0656
	168/270	=======1	_	0s	6ms/step	_	loss:	0.9634
Epoch	169/270							
	[======================================	======]	-	0s	6ms/step	-	loss:	1.0667
	170/270 [====================================	=======1	_	0s	5ms/step	_	loss:	1.1367
Epoch	171/270							
	[======================================	======]	-	0s	5ms/step	-	loss:	1.0414
	172/270 [====================================	=======1	_	0s	5ms/step	_	loss:	1.1375
Epoch	173/270							
	[======================================	======]	-	0s	5ms/step	-	loss:	1.1384
	174/270 [====================================	=======1	_	0 =	5ms/sten	_	1088:	1.0495
Epoch	175/270							
	[======================================	======]	-	0s	5ms/step	-	loss:	1.1817
	176/270 [===========	=======1	_	0s	5ms/step	_	loss:	1.1678
Epoch	177/270							
	[=====================================	======]	-	0s	5ms/step	-	loss:	1.2634
	[======================================	======]	_	0s	7ms/step	_	loss:	0.9985
Epoch	179/270							
	[=====================================	======]	-	0s	5ms/step	-	loss:	0.8312
	[======================================	======]	_	0s	6ms/step	_	loss:	0.8420
Epoch	181/270							
	[=====================================	======]	-	0s	/ms/step	_	loss:	0.8948
	[======================================	======]	_	0s	8ms/step	_	loss:	0.8582
_	183/270			0 ~	/		1	0 0200
	[=====================================		_	US	oms/step	_	TOSS:	0.9309
11/11	[======================================	======]	-	0s	5ms/step	-	loss:	1.4930
	185/270 [====================================	=======1	_	Λe	5mg/gten		1000	1 6203
Epoch	186/270							
	[======================================	======]	-	0s	5ms/step	-	loss:	2.0648
	187/270 [====================================	=======1	_	0s	7ms/step	_	loss:	2.0803
Epoch	188/270							
	[=====================================	======]	-	0s	5ms/step	-	loss:	1.9618
	[======================================	======]	_	0s	6ms/step	_	loss:	2.1851
	190/270	_		_	_ ,			
	[=====================================	======]	-	0s	5ms/step	_	loss:	1.8274
	[======================================]	_	0s	5ms/step	_	loss:	1.4955
	192/270			^	7 / 1		,	1 7000
	[=====================================	======	_	US	/ms/step	_	TOSS:	1./083
11/11	[======================================	======]	_	0s	5ms/step	_	loss:	1.3971
	194/270 [====================================	=====1		Ωc	5mg/g+05		1000	1 36/12
Epoch	195/270							
11/11	[======================================	=====]	-	0s	5ms/step	-	loss:	1.3532
Epoch	196/270							

```
Epoch 197/270
Epoch 198/270
11/11 [============= ] - Os 6ms/step - loss: 1.6945
Epoch 199/270
Epoch 200/270
Epoch 201/270
Epoch 202/270
Epoch 203/270
Epoch 204/270
Epoch 205/270
Epoch 206/270
11/11 [============= ] - 0s 5ms/step - loss: 0.7521
Epoch 207/270
Epoch 208/270
Epoch 209/270
Epoch 210/270
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
Epoch 216/270
Epoch 217/270
Epoch 218/270
Epoch 219/270
Epoch 220/270
Epoch 221/270
11/11 [============== ] - 0s 7ms/step - loss: 0.4099
Epoch 222/270
Epoch 223/270
11/11 [============ ] - Os 6ms/step - loss: 0.3616
Epoch 224/270
11/11 [============== ] - 0s 6ms/step - loss: 0.3505
Epoch 225/270
Epoch 226/270
```

Fnoch	227/270							
		:=======]	_	0s	6ms/step	_	loss:	0.3164
Epoch	228/270							
]	-	0s	5ms/step	-	loss:	0.2592
	229/270 [======	:=======]	_	0s	5ms/step	_	loss:	0.2333
Epoch	230/270							
]	-	0s	6ms/step	-	loss:	0.2329
	231/270 [======	:========]	_	0s	6ms/step	_	loss:	0.2120
Epoch	232/270							
]	-	0s	7ms/step	-	loss:	0.2380
	233/270	:=======]	_	۸e	5mg/gten	_	1088.	0 2198
	234/270	J		UB	Jiib/ bccp		1055.	0.2190
]	-	0s	6ms/step	-	loss:	0.2450
	235/270	:========]		۸c	5mg/gton		1055	0 2504
	236/270	,	_	US	Jiiis/scep	_	TOSS.	0.2304
11/11	[======]	-	0s	6ms/step	_	loss:	0.3054
	237/270	:========]		٥٥	Ema/aton		1000.	0 2270
	238/270		_	US	Jiis/scep	_	1055:	0.3279
11/11	[======]	_	0s	6ms/step	_	loss:	0.3565
	239/270			0 ~	C== /=+==		1	0 2010
	240/270	:=======]	_	US	oms/step	_	ioss:	0.3910
_]	_	0s	5ms/step	_	loss:	0.4484
	241/270			•	- / .		-	
	242/270	:=======]	-	0s	5ms/step	-	loss:	0.4795
]	_	0s	6ms/step	_	loss:	0.3629
	243/270			•	<i>c</i>		-	
	244/270	:=======]	-	0s	6ms/step	-	loss:	0.3284
]	_	0s	5ms/step	_	loss:	0.4740
	245/270						-	
	246/270	:=======]	-	0s	5ms/step	-	loss:	0.5970
]	_	0s	5ms/step	_	loss:	0.6316
	247/270			•	<i>c</i>		-	0 6400
	248/270	:=======]	-	US	6ms/step	-	loss:	0.6433
]	_	0s	7ms/step	_	loss:	0.6276
	249/270			0			,	0 7100
	250/270	:=======]	-	US	6ms/step	-	loss:	0./133
]	_	0s	5ms/step	_	loss:	0.7797
	251/270			•	<i>c</i>		-	0 5006
	[====== 252/270	:=======]	-	US	6ms/step	-	loss:	0.7026
]	_	0s	5ms/step	_	loss:	0.7889
	253/270	_					-	
	[====== 254/270	:=======]	-	0s	5ms/step	-	loss:	0.6596
]	_	0s	5ms/step	_	loss:	0.7439
	255/270			•	.		,	0 0000
	[====== 256/270	:=======]	-	US	5ms/step	-	loss:	0.9389
		:=======]	_	0s	5ms/step	_	loss:	0.7090
Epoch	257/270							

```
Epoch 258/270
11/11 [============= ] - Os 6ms/step - loss: 0.6938
Epoch 259/270
Epoch 260/270
Epoch 261/270
Epoch 262/270
Epoch 263/270
Epoch 264/270
11/11 [============== ] - 0s 5ms/step - loss: 0.8644
Epoch 265/270
Epoch 266/270
Epoch 267/270
11/11 [============= ] - 0s 6ms/step - loss: 0.8817
Epoch 268/270
Epoch 269/270
Epoch 270/270
>expected=-6.0, predicted=-5.0
Epoch 1/270
Epoch 2/270
11/11 [============] - 0s 5ms/step - loss: 132.8432
Epoch 3/270
11/11 [============== ] - 0s 7ms/step - loss: 131.1074
Epoch 4/270
Epoch 5/270
Epoch 6/270
Epoch 7/270
Epoch 8/270
Epoch 9/270
Epoch 10/270
Epoch 11/270
Epoch 12/270
Epoch 13/270
Epoch 14/270
Epoch 15/270
Epoch 16/270
11/11 [============== ] - Os 6ms/step - loss: 77.2268
Epoch 17/270
```

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Epoch 18/270
Epoch 19/270
Epoch 20/270
Epoch 21/270
Epoch 22/270
Epoch 23/270
Epoch 24/270
Epoch 25/270
Epoch 26/270
Epoch 27/270
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
Epoch 32/270
Epoch 33/270
11/11 [============= ] - 0s 6ms/step - loss: 61.9689
Epoch 34/270
Epoch 35/270
Epoch 36/270
Epoch 37/270
Epoch 38/270
Epoch 39/270
Epoch 40/270
Epoch 41/270
Epoch 42/270
Epoch 43/270
Epoch 44/270
11/11 [============== ] - 0s 13ms/step - loss: 49.9284
Epoch 45/270
Epoch 46/270
Epoch 47/270
```

```
Epoch 48/270
Epoch 49/270
Epoch 50/270
Epoch 51/270
Epoch 52/270
11/11 [============== ] - 0s 10ms/step - loss: 43.9766
Epoch 53/270
Epoch 54/270
Epoch 55/270
Epoch 56/270
Epoch 57/270
Epoch 58/270
Epoch 59/270
Epoch 60/270
Epoch 61/270
Epoch 62/270
Epoch 63/270
11/11 [=============] - Os 7ms/step - loss: 30.6428
Epoch 64/270
Epoch 65/270
Epoch 66/270
Epoch 67/270
Epoch 68/270
Epoch 69/270
Epoch 70/270
Epoch 71/270
Epoch 72/270
Epoch 73/270
Epoch 74/270
Epoch 75/270
Epoch 76/270
Epoch 77/270
Epoch 78/270
```

```
Epoch 79/270
Epoch 80/270
11/11 [============] - Os 6ms/step - loss: 23.2560
Epoch 81/270
Epoch 82/270
Epoch 83/270
Epoch 84/270
Epoch 85/270
Epoch 86/270
Epoch 87/270
Epoch 88/270
Epoch 89/270
Epoch 90/270
Epoch 91/270
Epoch 92/270
Epoch 93/270
Epoch 94/270
Epoch 95/270
Epoch 96/270
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Epoch 99/270
Epoch 100/270
Epoch 101/270
Epoch 102/270
Epoch 103/270
Epoch 104/270
Epoch 105/270
Epoch 106/270
Epoch 107/270
Epoch 108/270
```

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Epoch 109/270
Epoch 110/270
Epoch 111/270
Epoch 112/270
Epoch 113/270
Epoch 114/270
11/11 [============ ] - Os 6ms/step - loss: 8.4429
Epoch 115/270
11/11 [============== ] - 0s 6ms/step - loss: 8.3210
Epoch 116/270
Epoch 117/270
Epoch 118/270
Epoch 119/270
11/11 [============= ] - 0s 6ms/step - loss: 8.2192
Epoch 120/270
Epoch 121/270
Epoch 122/270
Epoch 123/270
Epoch 124/270
11/11 [============] - 0s 7ms/step - loss: 7.1157
Epoch 125/270
Epoch 126/270
Epoch 127/270
Epoch 128/270
Epoch 129/270
Epoch 130/270
Epoch 131/270
Epoch 132/270
Epoch 133/270
Epoch 134/270
Epoch 135/270
Epoch 136/270
11/11 [============= ] - 0s 6ms/step - loss: 4.5640
Epoch 137/270
Epoch 138/270
Epoch 139/270
```

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Epoch 140/270
Epoch 141/270
Epoch 142/270
Epoch 143/270
Epoch 144/270
Epoch 145/270
Epoch 146/270
11/11 [============== ] - 0s 5ms/step - loss: 3.9984
Epoch 147/270
Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
11/11 [============= ] - Os 6ms/step - loss: 3.4019
Epoch 156/270
Epoch 157/270
Epoch 158/270
Epoch 159/270
Epoch 160/270
Epoch 161/270
Epoch 162/270
Epoch 163/270
Epoch 164/270
Epoch 165/270
Epoch 166/270
Epoch 167/270
11/11 [============= ] - 0s 7ms/step - loss: 1.9263
Epoch 168/270
Epoch 169/270
```

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Epoch 170/270
Epoch 171/270
Epoch 172/270
Epoch 173/270
Epoch 174/270
Epoch 175/270
Epoch 176/270
Epoch 177/270
Epoch 178/270
Epoch 179/270
Epoch 180/270
11/11 [============= ] - 0s 6ms/step - loss: 1.4292
Epoch 181/270
Epoch 182/270
Epoch 183/270
Epoch 184/270
Epoch 185/270
11/11 [============= ] - 0s 5ms/step - loss: 1.0366
Epoch 186/270
Epoch 187/270
Epoch 188/270
Epoch 189/270
Epoch 190/270
Epoch 191/270
Epoch 192/270
Epoch 193/270
Epoch 194/270
Epoch 195/270
Epoch 196/270
Epoch 197/270
Epoch 198/270
Epoch 199/270
11/11 [============] - 0s 5ms/step - loss: 1.2316
Epoch 200/270
```

```
Epoch 201/270
Epoch 202/270
11/11 [============= ] - Os 6ms/step - loss: 2.5099
Epoch 203/270
Epoch 204/270
Epoch 205/270
11/11 [================== ] - 0s 5ms/step - loss: 1.9734
Epoch 206/270
11/11 [============== ] - 0s 6ms/step - loss: 1.7425
Epoch 207/270
Epoch 208/270
Epoch 209/270
Epoch 210/270
11/11 [============== ] - 0s 5ms/step - loss: 1.3827
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
11/11 [=============] - Os 6ms/step - loss: 0.8742
Epoch 216/270
Epoch 217/270
Epoch 218/270
Epoch 219/270
Epoch 220/270
Epoch 221/270
Epoch 222/270
Epoch 223/270
Epoch 224/270
Epoch 225/270
Epoch 226/270
Epoch 227/270
11/11 [============ ] - Os 6ms/step - loss: 0.4456
Epoch 228/270
Epoch 229/270
Epoch 230/270
```

Enoch	231/270						
	[=======]	_	0s	6ms/step	_	loss:	0.4960
Epoch	232/270						
	[======]	-	0s	7ms/step	-	loss:	0.5790
	233/270 [=======]		Λσ	7mg/gton		1055.	0 6076
	234/270	_	US	/ms/scep	_	TOSS:	0.0070
11/11	[======]	_	0s	7ms/step	_	loss:	0.6382
	235/270			_ ,		_	
	[======] 236/270	-	0s	7ms/step	-	loss:	0.7777
	[=======]	_	0s	6ms/step	_	loss:	0.8155
Epoch	237/270						
	[=======]	-	0s	5ms/step	-	loss:	0.9056
	238/270 [=======]	_	۸e	7ms/sten	_	1088.	0 7155
	239/270		05	/mb/sccp		1055.	0.7133
	[======]	-	0s	6ms/step	_	loss:	0.7197
	240/270 [=======]		0 ~	7/		1	0 (227
	[=====================================	_	US	/ms/step	_	loss:	0.6337
	[=======]	_	0s	5ms/step	_	loss:	0.6004
	242/270			_ ,		-	
	[======] 243/270	-	0s	/ms/step	_	loss:	0.5069
	[=======]	_	0s	6ms/step	_	loss:	0.4943
Epoch	244/270						
	[======] 245/270	-	0s	7ms/step	-	loss:	0.5376
	[=========]	_	0s	6ms/step	_	loss:	0.3799
Epoch	246/270						
	[=======]	-	0s	6ms/step	-	loss:	0.4199
	247/270 [======]	_	۸e	6mg/sten	_	1088.	0 3572
	248/270		O D	ошь, всер		1000.	0.3372
	[=====]	-	0s	6ms/step	-	loss:	0.3217
	249/270 [=======]		٥٥	7mc/cton		logg•	0 2252
	250/270		05	/ms/scep	_	1055.	0.3332
	[======]	_	0s	7ms/step	_	loss:	0.4776
	251/270 [=======]		٥٩	6ma/aton		1000.	0 5071
	252/270	_	US	oms/scep	_	TOSS:	0.59/1
	[======]	_	0s	8ms/step	_	loss:	0.8953
	253/270		0	7 / 1		-	1 1527
	[======] 254/270	_	0s	/ms/step	_	loss:	1.153/
	[=======]	_	0s	6ms/step	_	loss:	1.1357
	255/270			_ ,		_	
	[=======] 256/270	-	0s	5ms/step	-	loss:	1.5135
	[========]	_	0s	7ms/step	_	loss:	1.9035
Epoch	257/270						
	[======================================	-	0s	7ms/step	-	loss:	2.3722
	258/270 [=======]	_	0s	5ms/step	_	loss:	2.5171
Epoch	259/270						
	[======================================	-	0s	6ms/step	-	loss:	3.1762
	260/270 [=======]	_	0.5	6ms/ster	_	1055:	2.3677
	261/270		0.5	J.m.D., B CCP			2.3077
-							

```
Epoch 262/270
11/11 [============= ] - Os 7ms/step - loss: 1.8006
Epoch 263/270
Epoch 264/270
Epoch 265/270
Epoch 266/270
Epoch 267/270
11/11 [============== ] - 0s 5ms/step - loss: 0.3616
Epoch 268/270
Epoch 269/270
Epoch 270/270
>expected=3.0, predicted=14.4
Epoch 1/270
Epoch 2/270
11/11 [============= ] - 0s 7ms/step - loss: 134.5323
Epoch 3/270
Epoch 4/270
11/11 [===========] - 0s 7ms/step - loss: 128.4761
Epoch 5/270
Epoch 6/270
11/11 [============] - 0s 7ms/step - loss: 108.0234
Epoch 7/270
Epoch 8/270
Epoch 9/270
Epoch 10/270
Epoch 11/270
Epoch 12/270
Epoch 13/270
Epoch 14/270
Epoch 15/270
Epoch 16/270
Epoch 17/270
Epoch 18/270
Epoch 19/270
Epoch 20/270
Epoch 21/270
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Epoch 22/270
11/11 [============= ] - 0s 6ms/step - loss: 69.6174
Epoch 23/270
Epoch 24/270
Epoch 25/270
Epoch 26/270
Epoch 27/270
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
Epoch 32/270
Epoch 33/270
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Epoch 36/270
Epoch 37/270
Epoch 38/270
Epoch 39/270
Epoch 40/270
Epoch 41/270
Epoch 42/270
Epoch 43/270
Epoch 44/270
Epoch 45/270
Epoch 46/270
Epoch 47/270
Epoch 48/270
11/11 [============= ] - 0s 7ms/step - loss: 45.8840
Epoch 49/270
11/11 [============= ] - 0s 7ms/step - loss: 42.8418
Epoch 50/270
Epoch 51/270
```

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Epoch 52/270
Epoch 53/270
Epoch 54/270
Epoch 55/270
Epoch 56/270
Epoch 57/270
Epoch 58/270
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Epoch 60/270
Epoch 61/270
Epoch 62/270
Epoch 63/270
Epoch 64/270
Epoch 65/270
Epoch 66/270
Epoch 67/270
11/11 [=============] - Os 7ms/step - loss: 28.7863
Epoch 68/270
Epoch 69/270
Epoch 70/270
Epoch 71/270
Epoch 72/270
Epoch 73/270
11/11 [============= ] - 0s 6ms/step - loss: 24.2874
Epoch 74/270
Epoch 75/270
Epoch 76/270
Epoch 77/270
Epoch 78/270
Epoch 79/270
Epoch 80/270
Epoch 81/270
Epoch 82/270
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Epoch 83/270
Epoch 84/270
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Epoch 110/270
Epoch 111/270
Epoch 112/270
```

Encah	112/270								
	113/270	========	======1	_	0s	7ms/step	_	loss:	6.9813
Epoch	114/270								
		========	======]	-	0s	6ms/step	-	loss:	6.0286
	115/270	========	.======1	_	۸e	7ms/sten	_	1088.	6 6751
	116/270		J		V D	/ mb/ bccp		1055.	0.0731
11/11	[=====	========	======]	_	0s	7ms/step	_	loss:	6.7093
	117/270		,		0 -	C / 1		1	F 0442
	118/270	=======	====== j	_	US	oms/step	_	loss:	5.9442
			======]	_	0s	6ms/step	_	loss:	5.6094
	119/270							_	
	[====== 120/270		======]	-	0s	7ms/step	-	loss:	4.8719
			======]	_	0s	5ms/step	_	loss:	4.7556
Epoch	121/270								
		========	======]	-	0s	7ms/step	-	loss:	4.5174
	122/270	========	:======1	_	0s	7ms/step	_	loss:	4.3491
Epoch	123/270								
		========	======]	-	0s	6ms/step	-	loss:	3.7790
	124/270	========	:======1	_	0s	7ms/step	_	loss:	3.5076
Epoch	125/270								
		========	======]	-	0s	6ms/step	-	loss:	3.1416
	126/270		:=======1	_	0 s	7ms/sten	_	1099:	3.1394
	127/270		J		O D	/шь/ всер		1000.	3.1371
			======]	-	0s	6ms/step	-	loss:	3.3935
	128/270		.======1	_	۸e	7ms/sten	_	1088.	3 2145
	129/270		J		O D	/шь/ всер		1000.	3.2113
		========	:======]	-	0s	8ms/step	-	loss:	3.2705
	130/270	========	:=======1	_	0 s	6ms/sten	_	1099:	3.2060
Epoch	131/270								
			:======]	-	0s	7ms/step	-	loss:	3.0964
	132/270	========	.======1	_	۸e	7ms/sten	_	1088.	2 8011
Epoch	133/270								
			======]	-	0s	7ms/step	-	loss:	3.2553
	134/270	========	=======================================		۸e	7mg/g+en	_	1000	3 3024
	135/270		J		0.5	/ mb/ bccp		1055.	3.3024
			======]	-	0s	6ms/step	-	loss:	3.2849
	136/270	========	=======================================		۸e	7mg/g+en	_	1000	3 2377
	137/270		J		0.5	/ mb/ bccp		1055.	3.2377
	-	========	======]	-	0s	6ms/step	-	loss:	3.5759
	138/270	========	1		٥٥	6mg/g+on		logg•	2 0501
	139/270			_	US	oms/scep	_	TOSS:	3.6301
11/11	[=====	========	======]	_	0s	5ms/step	-	loss:	3.2544
	140/270	========	1		٥٥	7mg/g+on		1000.	2 1107
	141/270		_]	_	UB	/ma/acep	_	TODD!	2.110/
11/11	[=====		======]	-	0s	6ms/step	-	loss:	2.5115
	142/270	========	====1		Ωc	2mg/g+0>		logge	2 5000
	143/270			_	UB	oma/acep	_	1022:	2.5000
_									

```
Epoch 144/270
Epoch 145/270
Epoch 146/270
Epoch 147/270
Epoch 148/270
Epoch 149/270
11/11 [============== ] - 0s 6ms/step - loss: 1.5211
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
Epoch 156/270
Epoch 157/270
Epoch 158/270
Epoch 159/270
11/11 [============== ] - Os 6ms/step - loss: 1.3521
Epoch 160/270
Epoch 161/270
Epoch 162/270
Epoch 163/270
Epoch 164/270
Epoch 165/270
Epoch 166/270
Epoch 167/270
Epoch 168/270
Epoch 169/270
Epoch 170/270
Epoch 171/270
11/11 [=============] - Os 6ms/step - loss: 1.4568
Epoch 172/270
Epoch 173/270
```

Enoch	174/270							
		:======================================	-	0s	7ms/step	_	loss:	1.2528
_	175/270			•	5 / .		-	1 0011
	176/270	:======================================	-	0s	/ms/step	-	loss:	1.2211
-			-	0s	7ms/step	_	loss:	1.0617
	177/270			•	7 / 1		,	1 2257
	178/270	:======================================	-	US	/ms/step	_	loss:	1.335/
11/11	[=====		-	0s	6ms/step	_	loss:	1.2634
	179/270	:======================================	ı	۸c	6mg/g+on		logg•	1 1102
	180/270		-	US	oms/scep	_	TOSS:	1.1192
			-	0s	6ms/step	-	loss:	1.1131
	181/270 [=====	:======================================	ı _	Λς	7ms/sten	_	1055:	1.0064
Epoch	182/270							
		:========:	-	0s	8ms/step	-	loss:	1.2104
	183/270 [=====	:======================================	ı –	0s	7ms/step	_	loss:	1.2838
Epoch	184/270							
	[====== 185/270	:=========:	-	0s	7ms/step	-	loss:	1.1845
		:======================================	-	0s	7ms/step	_	loss:	1.2132
	186/270			•	- / .		,	1 0705
	187/270		-	US	5ms/step	_	loss:	1.2/85
11/11	[======	:======::	-	0s	7ms/step	_	loss:	1.2957
	188/270	:======================================		٥a	6mg/g+on		1000.	1 2000
	189/270		_	US	oms/scep	_	1055	1.2009
		:======::	-	0s	6ms/step	-	loss:	1.3234
	190/270 [======	:======================================	١ _	0s	6ms/sten	_	loss:	1.3770
Epoch	191/270							
	[====== 192/270	:=========:	-	0s	6ms/step	-	loss:	1.7877
		:======================================	l –	0s	6ms/step	_	loss:	1.7471
Epoch	193/270							
	194/270	:======================================	-	0s	6ms/step	_	loss:	1.8/38
11/11	[=====	:======::	-	0s	6ms/step	_	loss:	1.2358
	195/270		ı	۸c	Oma/aton		logg•	1 0700
	196/270		_	US	oms/scep	_	1055	1.0790
		:======::	-	0s	6ms/step	-	loss:	1.0921
	197/270 [======	:======================================	ı –	0s	7ms/step	_	loss:	0.9614
Epoch	198/270							
	[====== 199/270	:=========:	-	0s	6ms/step	-	loss:	0.8419
		:======================================	ı –	0s	7ms/step	_	loss:	0.8112
	200/270				-			
	201/270	:======================================	-	0s	6ms/step	-	loss:	0./216
11/11	[=====	:========:	-	0s	7ms/step	-	loss:	0.7454
	202/270		ı	0.5	6mg/g+05		logge	0 6661
Epoch	203/270							
11/11	[=====	:======::	-	0s	7ms/step	-	loss:	0.5853
Epoch	204/270							

```
Epoch 205/270
Epoch 206/270
Epoch 207/270
Epoch 208/270
Epoch 209/270
Epoch 210/270
11/11 [============= ] - 0s 7ms/step - loss: 0.4926
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
Epoch 216/270
Epoch 217/270
Epoch 218/270
Epoch 219/270
Epoch 220/270
Epoch 221/270
Epoch 222/270
Epoch 223/270
Epoch 224/270
Epoch 225/270
Epoch 226/270
Epoch 227/270
Epoch 228/270
Epoch 229/270
Epoch 230/270
Epoch 231/270
Epoch 232/270
11/11 [============== ] - 0s 6ms/step - loss: 0.7744
Epoch 233/270
Epoch 234/270
```

Enoch	235/270						
	[========]	_	0s	7ms/step	_	loss:	0.9076
	236/270		•	7 / 1		,	1 0005
	[======] 237/270	-	US	/ms/step	_	loss:	1.2895
	[=======]	_	0s	6ms/step	_	loss:	1.2885
	238/270		0 -	C /		1	0.0100
	[=======] 239/270	-	US	6MS/Step	_	loss:	0.8199
11/11	[======]	-	0s	7ms/step	_	loss:	0.7493
	240/270 [=======]		٥٥	6mg/gton		logge	0 0500
	241/270	_	US	oms/step	_	1055:	0.6500
	[=====]	-	0s	7ms/step	-	loss:	1.0916
	242/270 [=======]	_	۸e	7ms/sten	_	10991	0 6854
Epoch	243/270						
	[=======]	-	0s	7ms/step	-	loss:	0.6412
	244/270 [=======]	_	0s	7ms/step	_	loss:	0.5032
Epoch	245/270						
	[======] 246/270	-	0s	6ms/step	-	loss:	0.3653
	[========]	_	0s	8ms/step	_	loss:	0.3905
Epoch	247/270						
	[======] 248/270	-	0s	7ms/step	-	loss:	0.3688
	[=======]	_	0s	6ms/step	_	loss:	0.4368
	249/270			-			
	[=======] 250/270	-	0s	8ms/step	-	loss:	0.4249
	[=======]	_	0s	6ms/step	_	loss:	0.4297
	251/270 [=======]		٥٩	7mg/g+on		10000	0 2602
	252/270	_	US	/ms/scep	_	TOSS:	0.3083
	[=====]	_	0s	6ms/step	_	loss:	0.3481
	253/270 [=======]	_	0s	6ms/sten	_	1055:	0.2982
Epoch	254/270						
	[======] 255/270	-	0s	7ms/step	-	loss:	0.2832
	[=========]	_	0s	6ms/step	_	loss:	0.2728
Epoch	256/270						
	[======] 257/270	-	0s	7ms/step	-	loss:	0.2718
	[=========]	_	0s	6ms/step	_	loss:	0.2821
	258/270 [=======]		٥٩	7mg/g+on		10000	0 2000
	259/270	_	US	/ms/scep	_	1055:	0.2900
	[======]	_	0s	8ms/step	_	loss:	0.3262
	260/270 [======]	_	۸e	8mg/sten	_	1099.	0 3523
Epoch	261/270						
	[======================================	-	0s	7ms/step	-	loss:	0.4228
	262/270 [=======]	_	0s	7ms/step	_	loss:	0.5093
Epoch	263/270						
	[======] 264/270	-	0s	9ms/step	-	loss:	0.7783
	[========]	_	0s	7ms/step	_	loss:	0.8116
Epoch	265/270						

```
Epoch 266/270
11/11 [============= ] - Os 9ms/step - loss: 0.7841
Epoch 267/270
11/11 [============= ] - 0s 12ms/step - loss: 0.9467
Epoch 268/270
Epoch 269/270
Epoch 270/270
11/11 [============= ] - Os 20ms/step - loss: 0.8635
>expected=-4.0, predicted=5.0
Epoch 1/270
Epoch 2/270
Epoch 3/270
Epoch 4/270
Epoch 5/270
Epoch 6/270
11/11 [============== ] - 0s 12ms/step - loss: 99.2565
Epoch 7/270
Epoch 8/270
11/11 [============] - 0s 12ms/step - loss: 92.0576
Epoch 9/270
Epoch 10/270
Epoch 11/270
11/11 [============== ] - 0s 10ms/step - loss: 83.2097
Epoch 12/270
Epoch 13/270
Epoch 14/270
Epoch 15/270
Epoch 16/270
Epoch 17/270
Epoch 18/270
Epoch 19/270
Epoch 20/270
11/11 [============== ] - 0s 12ms/step - loss: 70.6837
Epoch 21/270
11/11 [============== ] - 0s 12ms/step - loss: 68.8443
Epoch 22/270
11/11 [============== ] - 0s 11ms/step - loss: 67.6484
Epoch 23/270
Epoch 24/270
11/11 [============] - 0s 15ms/step - loss: 64.7102
Epoch 25/270
```

```
Epoch 26/270
11/11 [============= ] - 0s 13ms/step - loss: 63.6661
Epoch 27/270
11/11 [============== ] - 0s 15ms/step - loss: 62.0457
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
11/11 [============== ] - 0s 16ms/step - loss: 54.7359
Epoch 32/270
Epoch 33/270
11/11 [============== ] - 0s 15ms/step - loss: 54.9166
Epoch 34/270
11/11 [============== ] - 0s 15ms/step - loss: 51.7937
Epoch 35/270
Epoch 36/270
11/11 [============= ] - 0s 13ms/step - loss: 46.7240
Epoch 37/270
Epoch 38/270
Epoch 39/270
Epoch 40/270
Epoch 41/270
Epoch 42/270
Epoch 43/270
Epoch 44/270
Epoch 45/270
Epoch 46/270
11/11 [============= ] - 0s 16ms/step - loss: 37.5255
Epoch 47/270
Epoch 48/270
Epoch 49/270
Epoch 50/270
Epoch 51/270
11/11 [============== ] - 0s 13ms/step - loss: 32.9116
Epoch 52/270
11/11 [============] - 0s 12ms/step - loss: 31.4052
Epoch 53/270
Epoch 54/270
Epoch 55/270
```

```
Epoch 56/270
11/11 [============== ] - 0s 13ms/step - loss: 29.9676
Epoch 57/270
Epoch 58/270
Epoch 59/270
Epoch 60/270
11/11 [============== ] - 0s 16ms/step - loss: 26.3013
Epoch 61/270
Epoch 62/270
11/11 [============== ] - 0s 15ms/step - loss: 22.9683
Epoch 63/270
Epoch 64/270
Epoch 65/270
Epoch 66/270
Epoch 67/270
11/11 [============= ] - 0s 18ms/step - loss: 19.1240
Epoch 68/270
Epoch 69/270
Epoch 70/270
Epoch 71/270
11/11 [===========] - 0s 14ms/step - loss: 20.3869
Epoch 72/270
11/11 [============== ] - 0s 14ms/step - loss: 17.7225
Epoch 73/270
Epoch 74/270
Epoch 75/270
Epoch 76/270
Epoch 77/270
11/11 [============== ] - 0s 14ms/step - loss: 14.1671
Epoch 78/270
11/11 [===========] - 0s 10ms/step - loss: 13.5260
Epoch 79/270
Epoch 80/270
Epoch 81/270
11/11 [============== ] - 0s 17ms/step - loss: 16.2102
Epoch 82/270
11/11 [============== ] - 0s 12ms/step - loss: 18.2459
Epoch 83/270
Epoch 84/270
11/11 [=============== ] - 0s 16ms/step - loss: 14.5326
Epoch 85/270
Epoch 86/270
```

```
Epoch 87/270
11/11 [============= ] - 0s 16ms/step - loss: 14.2960
Epoch 88/270
11/11 [============== ] - 0s 16ms/step - loss: 13.9652
Epoch 89/270
Epoch 90/270
Epoch 91/270
Epoch 92/270
11/11 [============== ] - 0s 16ms/step - loss: 14.0113
Epoch 93/270
11/11 [============== ] - 0s 11ms/step - loss: 11.7602
Epoch 94/270
11/11 [============== ] - 0s 13ms/step - loss: 10.1869
Epoch 95/270
Epoch 96/270
Epoch 97/270
Epoch 98/270
Epoch 99/270
Epoch 100/270
Epoch 101/270
Epoch 102/270
Epoch 103/270
Epoch 104/270
Epoch 105/270
Epoch 106/270
Epoch 107/270
Epoch 108/270
Epoch 109/270
Epoch 110/270
Epoch 111/270
Epoch 112/270
Epoch 113/270
Epoch 114/270
Epoch 115/270
Epoch 116/270
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Epoch 117/270
Epoch 118/270
Epoch 119/270
Epoch 120/270
Epoch 121/270
Epoch 122/270
Epoch 123/270
Epoch 124/270
Epoch 125/270
Epoch 126/270
Epoch 127/270
Epoch 128/270
Epoch 129/270
Epoch 130/270
Epoch 131/270
Epoch 132/270
Epoch 133/270
Epoch 134/270
Epoch 135/270
Epoch 136/270
Epoch 137/270
Epoch 138/270
Epoch 139/270
Epoch 140/270
Epoch 141/270
Epoch 142/270
Epoch 143/270
Epoch 144/270
Epoch 145/270
Epoch 146/270
Epoch 147/270
```

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Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
11/11 [============== ] - 0s 9ms/step - loss: 2.5040
Epoch 156/270
Epoch 157/270
11/11 [============= ] - 0s 9ms/step - loss: 2.3286
Epoch 158/270
Epoch 159/270
11/11 [============= ] - 0s 8ms/step - loss: 2.6289
Epoch 160/270
Epoch 161/270
Epoch 162/270
Epoch 163/270
11/11 [============= ] - 0s 7ms/step - loss: 2.1208
Epoch 164/270
Epoch 165/270
11/11 [============] - Os 21ms/step - loss: 2.1799
Epoch 166/270
Epoch 167/270
Epoch 168/270
Epoch 169/270
Epoch 170/270
Epoch 171/270
Epoch 172/270
11/11 [=============] - 0s 8ms/step - loss: 1.5856
Epoch 173/270
11/11 [============== ] - 0s 9ms/step - loss: 1.7537
Epoch 174/270
Epoch 175/270
Epoch 176/270
Epoch 177/270
```

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Epoch 178/270
Epoch 179/270
Epoch 180/270
Epoch 181/270
Epoch 182/270
Epoch 183/270
11/11 [============== ] - 0s 9ms/step - loss: 1.9112
Epoch 184/270
Epoch 185/270
Epoch 186/270
Epoch 187/270
Epoch 188/270
Epoch 189/270
11/11 [============== ] - 0s 8ms/step - loss: 1.8455
Epoch 190/270
Epoch 191/270
Epoch 192/270
Epoch 193/270
Epoch 194/270
Epoch 195/270
Epoch 196/270
Epoch 197/270
Epoch 198/270
Epoch 199/270
11/11 [============ ] - 0s 9ms/step - loss: 1.5765
Epoch 200/270
Epoch 201/270
Epoch 202/270
Epoch 203/270
Epoch 204/270
11/11 [============= ] - 0s 9ms/step - loss: 1.7669
Epoch 205/270
11/11 [============== ] - 0s 9ms/step - loss: 1.7742
Epoch 206/270
Epoch 207/270
Epoch 208/270
```

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Epoch 209/270
11/11 [============= ] - 0s 13ms/step - loss: 0.9016
Epoch 210/270
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
Epoch 216/270
11/11 [============= ] - 0s 8ms/step - loss: 0.5145
Epoch 217/270
11/11 [============== ] - 0s 7ms/step - loss: 0.4971
Epoch 218/270
11/11 [============= ] - 0s 7ms/step - loss: 0.4084
Epoch 219/270
Epoch 220/270
Epoch 221/270
Epoch 222/270
Epoch 223/270
Epoch 224/270
Epoch 225/270
Epoch 226/270
Epoch 227/270
11/11 [===========] - 0s 8ms/step - loss: 0.4298
Epoch 228/270
Epoch 229/270
Epoch 230/270
Epoch 231/270
Epoch 232/270
Epoch 233/270
Epoch 234/270
Epoch 235/270
11/11 [============ ] - Os 6ms/step - loss: 0.5134
Epoch 236/270
Epoch 237/270
Epoch 238/270
```

```
Epoch 239/270
11/11 [============== ] - 0s 7ms/step - loss: 0.7012
Epoch 240/270
Epoch 241/270
Epoch 242/270
Epoch 243/270
Epoch 244/270
Epoch 245/270
Epoch 246/270
Epoch 247/270
Epoch 248/270
Epoch 249/270
11/11 [============= ] - 0s 8ms/step - loss: 0.9385
Epoch 250/270
11/11 [============== ] - 0s 7ms/step - loss: 1.0946
Epoch 251/270
Epoch 252/270
Epoch 253/270
Epoch 254/270
11/11 [=============] - Os 8ms/step - loss: 0.4418
Epoch 255/270
Epoch 256/270
Epoch 257/270
Epoch 258/270
Epoch 259/270
Epoch 260/270
Epoch 261/270
11/11 [============= ] - Os 7ms/step - loss: 0.6618
Epoch 262/270
Epoch 263/270
Epoch 264/270
Epoch 265/270
Epoch 266/270
11/11 [============== ] - 0s 7ms/step - loss: 0.6304
Epoch 267/270
Epoch 268/270
Epoch 269/270
```

```
Epoch 270/270
WARNING:tensorflow:5 out of the last 5 calls to <function Model.make predic
t function.<locals>.predict function at 0x7ff4279505f0> triggered tf.functi
on retracing. Tracing is expensive and the excessive number of tracings cou
ld be due to (1) creating @tf.function repeatedly in a loop, (2) passing te
nsors with different shapes, (3) passing Python objects instead of tensors.
For (1), please define your @tf.function outside of the loop. For (2), @tf.
function has experimental relax shapes=True option that relaxes argument sh
apes that can avoid unnecessary retracing. For (3), please refer to https:/
/www.tensorflow.org/guide/function#controlling retracing and https://www.te
nsorflow.org/api docs/python/tf/function for more details.
>expected=-1.0, predicted=2.9
Epoch 1/270
Epoch 2/270
Epoch 3/270
Epoch 4/270
11/11 [============= ] - 0s 7ms/step - loss: 122.8079
Epoch 5/270
11/11 [============== ] - 0s 7ms/step - loss: 112.9143
Epoch 6/270
11/11 [============= ] - 0s 7ms/step - loss: 113.4137
Epoch 7/270
11/11 [===========] - 0s 7ms/step - loss: 104.8689
Epoch 8/270
Epoch 9/270
Epoch 10/270
Epoch 11/270
Epoch 12/270
Epoch 13/270
Epoch 14/270
Epoch 15/270
Epoch 16/270
Epoch 17/270
Epoch 18/270
Epoch 19/270
Epoch 20/270
Epoch 21/270
Epoch 22/270
Epoch 23/270
Epoch 24/270
```

```
Epoch 25/270
11/11 [============= ] - 0s 7ms/step - loss: 67.7277
Epoch 26/270
Epoch 27/270
Epoch 28/270
Epoch 29/270
Epoch 30/270
Epoch 31/270
Epoch 32/270
Epoch 33/270
Epoch 34/270
Epoch 35/270
Epoch 36/270
Epoch 37/270
Epoch 38/270
Epoch 39/270
Epoch 40/270
Epoch 41/270
Epoch 42/270
Epoch 43/270
Epoch 44/270
Epoch 45/270
Epoch 46/270
Epoch 47/270
Epoch 48/270
Epoch 49/270
Epoch 50/270
Epoch 51/270
11/11 [============ ] - Os 7ms/step - loss: 41.9008
Epoch 52/270
11/11 [============= ] - 0s 8ms/step - loss: 36.7871
Epoch 53/270
Epoch 54/270
```

```
Epoch 55/270
Epoch 56/270
Epoch 57/270
Epoch 58/270
Epoch 59/270
Epoch 60/270
Epoch 61/270
Epoch 62/270
Epoch 63/270
Epoch 64/270
Epoch 65/270
11/11 [============= ] - 0s 13ms/step - loss: 24.7542
Epoch 66/270
11/11 [============== ] - 0s 11ms/step - loss: 24.8758
Epoch 67/270
11/11 [============= ] - 0s 15ms/step - loss: 22.9299
Epoch 68/270
Epoch 69/270
Epoch 70/270
11/11 [===========] - 0s 12ms/step - loss: 21.6780
Epoch 71/270
11/11 [============== ] - 0s 17ms/step - loss: 22.7037
Epoch 72/270
Epoch 73/270
Epoch 74/270
Epoch 75/270
Epoch 76/270
11/11 [============== ] - 0s 12ms/step - loss: 20.9014
Epoch 77/270
11/11 [============] - 0s 10ms/step - loss: 17.9945
Epoch 78/270
Epoch 79/270
Epoch 80/270
Epoch 81/270
11/11 [============= ] - 0s 11ms/step - loss: 15.4740
Epoch 82/270
Epoch 83/270
Epoch 84/270
Epoch 85/270
```

```
Epoch 86/270
Epoch 87/270
Epoch 88/270
11/11 [============== ] - 0s 10ms/step - loss: 10.7599
Epoch 89/270
Epoch 90/270
Epoch 91/270
Epoch 92/270
Epoch 93/270
Epoch 94/270
Epoch 95/270
Epoch 96/270
Epoch 97/270
11/11 [============= ] - 0s 8ms/step - loss: 9.2297
Epoch 98/270
Epoch 99/270
Epoch 100/270
Epoch 101/270
11/11 [============= ] - 0s 13ms/step - loss: 7.0796
Epoch 102/270
Epoch 103/270
Epoch 104/270
Epoch 105/270
Epoch 106/270
Epoch 107/270
Epoch 108/270
Epoch 109/270
Epoch 110/270
11/11 [============= ] - 0s 7ms/step - loss: 7.4874
Epoch 111/270
Epoch 112/270
Epoch 113/270
11/11 [============== ] - 0s 10ms/step - loss: 6.5476
Epoch 114/270
Epoch 115/270
```

```
Epoch 116/270
Epoch 117/270
Epoch 118/270
11/11 [============== ] - 0s 8ms/step - loss: 4.8541
Epoch 119/270
Epoch 120/270
Epoch 121/270
Epoch 122/270
Epoch 123/270
Epoch 124/270
Epoch 125/270
Epoch 126/270
Epoch 127/270
Epoch 128/270
Epoch 129/270
Epoch 130/270
Epoch 131/270
11/11 [============] - 0s 9ms/step - loss: 4.0858
Epoch 132/270
Epoch 133/270
Epoch 134/270
Epoch 135/270
Epoch 136/270
Epoch 137/270
Epoch 138/270
Epoch 139/270
Epoch 140/270
Epoch 141/270
Epoch 142/270
Epoch 143/270
Epoch 144/270
Epoch 145/270
Epoch 146/270
```

```
Epoch 147/270
11/11 [============= ] - Os 7ms/step - loss: 3.3368
Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
11/11 [============== ] - 0s 9ms/step - loss: 2.2317
Epoch 155/270
Epoch 156/270
11/11 [============== ] - 0s 7ms/step - loss: 2.1831
Epoch 157/270
Epoch 158/270
11/11 [============= ] - 0s 7ms/step - loss: 1.8444
Epoch 159/270
Epoch 160/270
Epoch 161/270
Epoch 162/270
11/11 [============= ] - Os 8ms/step - loss: 1.4444
Epoch 163/270
Epoch 164/270
Epoch 165/270
Epoch 166/270
Epoch 167/270
Epoch 168/270
Epoch 169/270
Epoch 170/270
Epoch 171/270
11/11 [============ ] - 0s 8ms/step - loss: 1.0283
Epoch 172/270
Epoch 173/270
11/11 [============== ] - 0s 8ms/step - loss: 0.9141
Epoch 174/270
11/11 [============ ] - 0s 13ms/step - loss: 0.9469
Epoch 175/270
Epoch 176/270
```

```
Epoch 177/270
Epoch 178/270
11/11 [============= ] - 0s 10ms/step - loss: 0.9989
Epoch 179/270
Epoch 180/270
Epoch 181/270
11/11 [============== ] - 0s 8ms/step - loss: 1.2588
Epoch 182/270
11/11 [=========== ] - 0s 7ms/step - loss: 1.3137
Epoch 183/270
Epoch 184/270
Epoch 185/270
Epoch 186/270
11/11 [============== ] - 0s 9ms/step - loss: 1.2687
Epoch 187/270
Epoch 188/270
Epoch 189/270
Epoch 190/270
Epoch 191/270
Epoch 192/270
11/11 [============= ] - Os 10ms/step - loss: 1.0961
Epoch 193/270
Epoch 194/270
Epoch 195/270
Epoch 196/270
Epoch 197/270
Epoch 198/270
Epoch 199/270
11/11 [============] - 0s 9ms/step - loss: 0.7447
Epoch 200/270
Epoch 201/270
Epoch 202/270
Epoch 203/270
Epoch 204/270
Epoch 205/270
Epoch 206/270
11/11 [============] - 0s 9ms/step - loss: 0.5528
Epoch 207/270
```

```
Epoch 208/270
11/11 [============= ] - 0s 12ms/step - loss: 0.5324
Epoch 209/270
Epoch 210/270
Epoch 211/270
Epoch 212/270
Epoch 213/270
Epoch 214/270
Epoch 215/270
11/11 [============== ] - 0s 8ms/step - loss: 0.5887
Epoch 216/270
Epoch 217/270
11/11 [============= ] - 0s 9ms/step - loss: 0.8593
Epoch 218/270
Epoch 219/270
Epoch 220/270
Epoch 221/270
Epoch 222/270
Epoch 223/270
11/11 [============= ] - Os 9ms/step - loss: 0.9093
Epoch 224/270
11/11 [============== ] - 0s 8ms/step - loss: 0.8241
Epoch 225/270
Epoch 226/270
Epoch 227/270
Epoch 228/270
Epoch 229/270
Epoch 230/270
Epoch 231/270
Epoch 232/270
Epoch 233/270
Epoch 234/270
11/11 [============ ] - 0s 16ms/step - loss: 0.3976
Epoch 235/270
Epoch 236/270
Epoch 237/270
```

```
Epoch 238/270
Epoch 239/270
11/11 [============= ] - Os 9ms/step - loss: 0.6137
Epoch 240/270
Epoch 241/270
Epoch 242/270
11/11 [============= ] - 0s 10ms/step - loss: 0.3629
Epoch 243/270
11/11 [============== ] - 0s 8ms/step - loss: 0.3036
Epoch 244/270
Epoch 245/270
Epoch 246/270
Epoch 247/270
Epoch 248/270
11/11 [============= ] - 0s 8ms/step - loss: 0.3560
Epoch 249/270
Epoch 250/270
Epoch 251/270
Epoch 252/270
Epoch 253/270
Epoch 254/270
Epoch 255/270
Epoch 256/270
Epoch 257/270
Epoch 258/270
Epoch 259/270
Epoch 260/270
11/11 [============= ] - 0s 12ms/step - loss: 0.5264
Epoch 261/270
Epoch 262/270
Epoch 263/270
Epoch 264/270
Epoch 265/270
Epoch 266/270
Epoch 267/270
Epoch 268/270
```

```
Epoch 269/270
11/11 [============= ] - 0s 15ms/step - loss: 0.2869
Epoch 270/270
WARNING: tensorflow: 6 out of the last 6 calls to <function Model.make predic
t_function.<locals>.predict_function at 0x7ff43243e950> triggered tf.functi
on retracing. Tracing is expensive and the excessive number of tracings cou
ld be due to (1) creating @tf.function repeatedly in a loop, (2) passing te
nsors with different shapes, (3) passing Python objects instead of tensors.
For (1), please define your @tf.function outside of the loop. For (2), @tf.
function has experimental relax shapes=True option that relaxes argument sh
apes that can avoid unnecessary retracing. For (3), please refer to https:/
/www.tensorflow.org/guide/function#controlling_retracing and https://www.te
nsorflow.org/api docs/python/tf/function for more details.
>expected=5.0, predicted=7.6
Epoch 1/270
Epoch 2/270
Epoch 3/270
11/11 [============= ] - 0s 6ms/step - loss: 131.1366
Epoch 4/270
Epoch 5/270
Epoch 6/270
11/11 [============] - 0s 7ms/step - loss: 113.3436
Epoch 7/270
Epoch 8/270
Epoch 9/270
Epoch 10/270
11/11 [============= ] - 0s 13ms/step - loss: 87.9093
Epoch 11/270
Epoch 12/270
Epoch 13/270
Epoch 14/270
11/11 [============== ] - 0s 14ms/step - loss: 86.1175
Epoch 15/270
Epoch 16/270
Epoch 17/270
11/11 [============] - 0s 18ms/step - loss: 78.8571
Epoch 18/270
11/11 [============= ] - 0s 15ms/step - loss: 79.9490
Epoch 19/270
Epoch 20/270
11/11 [============== ] - 0s 13ms/step - loss: 75.5078
Epoch 21/270
11/11 [============== ] - 0s 11ms/step - loss: 74.2676
Epoch 22/270
Epoch 23/270
```

```
Epoch 24/270
11/11 [============= ] - 0s 16ms/step - loss: 69.8818
Epoch 25/270
Epoch 26/270
11/11 [============== ] - 0s 16ms/step - loss: 68.9094
Epoch 27/270
Epoch 28/270
Epoch 29/270
11/11 [============= ] - 0s 13ms/step - loss: 66.2255
Epoch 30/270
11/11 [============== ] - 0s 16ms/step - loss: 64.4067
Epoch 31/270
11/11 [============== ] - 0s 12ms/step - loss: 62.4861
Epoch 32/270
11/11 [============== ] - 0s 17ms/step - loss: 60.8393
Epoch 33/270
11/11 [============= ] - 0s 20ms/step - loss: 59.4384
Epoch 34/270
11/11 [============== ] - 0s 14ms/step - loss: 59.3127
Epoch 35/270
Epoch 36/270
11/11 [=============== ] - 0s 19ms/step - loss: 61.9640
Epoch 37/270
Epoch 38/270
Epoch 39/270
11/11 [============= ] - 0s 13ms/step - loss: 53.4872
Epoch 40/270
Epoch 41/270
Epoch 42/270
Epoch 43/270
Epoch 44/270
Epoch 45/270
Epoch 46/270
Epoch 47/270
Epoch 48/270
Epoch 49/270
11/11 [============== ] - 0s 10ms/step - loss: 46.2353
Epoch 50/270
11/11 [============ ] - 0s 17ms/step - loss: 44.1586
Epoch 51/270
11/11 [============] - 0s 13ms/step - loss: 43.5262
Epoch 52/270
Epoch 53/270
```

```
Epoch 54/270
Epoch 55/270
11/11 [============= ] - 0s 15ms/step - loss: 39.7439
Epoch 56/270
Epoch 57/270
11/11 [============== ] - 0s 25ms/step - loss: 37.8810
Epoch 58/270
11/11 [============] - 0s 19ms/step - loss: 37.0440
Epoch 59/270
11/11 [============== ] - 0s 19ms/step - loss: 37.7727
Epoch 60/270
11/11 [============= ] - 0s 17ms/step - loss: 34.9896
Epoch 61/270
Epoch 62/270
Epoch 63/270
Epoch 64/270
11/11 [============= ] - 0s 19ms/step - loss: 32.9889
Epoch 65/270
11/11 [============== ] - 0s 22ms/step - loss: 30.9111
Epoch 66/270
11/11 [============= ] - 0s 14ms/step - loss: 34.5375
Epoch 67/270
Epoch 68/270
Epoch 69/270
11/11 [============] - 0s 10ms/step - loss: 27.0253
Epoch 70/270
Epoch 71/270
Epoch 72/270
Epoch 73/270
Epoch 74/270
Epoch 75/270
Epoch 76/270
11/11 [============= ] - Os 7ms/step - loss: 21.4198
Epoch 77/270
Epoch 78/270
Epoch 79/270
11/11 [============== ] - 0s 12ms/step - loss: 21.3418
Epoch 80/270
Epoch 81/270
11/11 [============== ] - 0s 14ms/step - loss: 17.4547
Epoch 82/270
Epoch 83/270
Epoch 84/270
```

```
Epoch 85/270
Epoch 86/270
Epoch 87/270
Epoch 88/270
11/11 [=============== ] - 0s 12ms/step - loss: 14.3360
Epoch 89/270
Epoch 90/270
11/11 [============== ] - 0s 12ms/step - loss: 12.9552
Epoch 91/270
11/11 [============== ] - 0s 10ms/step - loss: 15.2368
Epoch 92/270
11/11 [============== ] - 0s 12ms/step - loss: 13.5970
Epoch 93/270
11/11 [============== ] - 0s 10ms/step - loss: 13.0944
Epoch 94/270
Epoch 95/270
Epoch 96/270
Epoch 97/270
Epoch 98/270
Epoch 99/270
Epoch 100/270
11/11 [============= ] - 0s 12ms/step - loss: 11.9157
Epoch 101/270
Epoch 102/270
Epoch 103/270
Epoch 104/270
Epoch 105/270
Epoch 106/270
Epoch 107/270
Epoch 108/270
Epoch 109/270
Epoch 110/270
Epoch 111/270
Epoch 112/270
Epoch 113/270
Epoch 114/270
```

```
Epoch 115/270
Epoch 116/270
Epoch 117/270
Epoch 118/270
Epoch 119/270
Epoch 120/270
Epoch 121/270
Epoch 122/270
Epoch 123/270
Epoch 124/270
Epoch 125/270
Epoch 126/270
Epoch 127/270
Epoch 128/270
Epoch 129/270
Epoch 130/270
11/11 [============] - Os 8ms/step - loss: 4.3898
Epoch 131/270
Epoch 132/270
Epoch 133/270
Epoch 134/270
Epoch 135/270
Epoch 136/270
Epoch 137/270
Epoch 138/270
Epoch 139/270
Epoch 140/270
Epoch 141/270
Epoch 142/270
Epoch 143/270
Epoch 144/270
Epoch 145/270
```

```
Epoch 146/270
Epoch 147/270
Epoch 148/270
Epoch 149/270
Epoch 150/270
Epoch 151/270
Epoch 152/270
Epoch 153/270
Epoch 154/270
Epoch 155/270
11/11 [============= ] - 0s 7ms/step - loss: 3.3562
Epoch 156/270
Epoch 157/270
11/11 [============= ] - 0s 9ms/step - loss: 2.6810
Epoch 158/270
Epoch 159/270
Epoch 160/270
Epoch 161/270
11/11 [============= ] - 0s 13ms/step - loss: 1.4884
Epoch 162/270
Epoch 163/270
11/11 [============] - Os 12ms/step - loss: 1.0692
Epoch 164/270
Epoch 165/270
Epoch 166/270
Epoch 167/270
Epoch 168/270
Epoch 169/270
Epoch 170/270
11/11 [============] - 0s 9ms/step - loss: 1.3828
Epoch 171/270
11/11 [============== ] - 0s 8ms/step - loss: 1.5466
Epoch 172/270
11/11 [============== ] - 0s 9ms/step - loss: 1.5532
Epoch 173/270
11/11 [============= ] - 0s 11ms/step - loss: 1.4742
Epoch 174/270
Epoch 175/270
```

```
Epoch 176/270
Epoch 177/270
11/11 [============= ] - 0s 10ms/step - loss: 1.1180
Epoch 178/270
Epoch 179/270
Epoch 180/270
Epoch 181/270
Epoch 182/270
Epoch 183/270
Epoch 184/270
Epoch 185/270
Epoch 186/270
11/11 [============ ] - 0s 7ms/step - loss: 0.8349
Epoch 187/270
Epoch 188/270
Epoch 189/270
Epoch 190/270
Epoch 191/270
11/11 [============= ] - 0s 10ms/step - loss: 1.1850
Epoch 192/270
11/11 [============== ] - 0s 8ms/step - loss: 1.4661
Epoch 193/270
Epoch 194/270
Epoch 195/270
Epoch 196/270
Epoch 197/270
Epoch 198/270
Epoch 199/270
Epoch 200/270
Epoch 201/270
Epoch 202/270
Epoch 203/270
Epoch 204/270
Epoch 205/270
Epoch 206/270
```

```
Epoch 207/270
11/11 [============= ] - 0s 12ms/step - loss: 0.6209
Epoch 208/270
11/11 [============] - Os 11ms/step - loss: 0.5013
Epoch 209/270
Epoch 210/270
Epoch 211/270
Epoch 212/270
11/11 [============== ] - 0s 8ms/step - loss: 0.4944
Epoch 213/270
Epoch 214/270
Epoch 215/270
Epoch 216/270
11/11 [============= ] - 0s 9ms/step - loss: 0.7854
Epoch 217/270
Epoch 218/270
Epoch 219/270
Epoch 220/270
Epoch 221/270
Epoch 222/270
11/11 [============= ] - 0s 11ms/step - loss: 0.9490
Epoch 223/270
Epoch 224/270
Epoch 225/270
Epoch 226/270
Epoch 227/270
Epoch 228/270
Epoch 229/270
Epoch 230/270
Epoch 231/270
Epoch 232/270
Epoch 233/270
11/11 [============ ] - 0s 15ms/step - loss: 0.2750
Epoch 234/270
11/11 [============= ] - 0s 12ms/step - loss: 0.3431
Epoch 235/270
Epoch 236/270
```

```
Epoch 237/270
Epoch 238/270
11/11 [============= ] - 0s 14ms/step - loss: 0.5998
Epoch 239/270
11/11 [============== ] - 0s 9ms/step - loss: 0.9037
Epoch 240/270
Epoch 241/270
Epoch 242/270
Epoch 243/270
Epoch 244/270
Epoch 245/270
Epoch 246/270
Epoch 247/270
11/11 [============== ] - 0s 9ms/step - loss: 1.0716
Epoch 248/270
11/11 [============== ] - 0s 7ms/step - loss: 0.9381
Epoch 249/270
11/11 [============== ] - 0s 8ms/step - loss: 0.7890
Epoch 250/270
Epoch 251/270
Epoch 252/270
11/11 [===========] - 0s 8ms/step - loss: 0.5902
Epoch 253/270
Epoch 254/270
Epoch 255/270
Epoch 256/270
Epoch 257/270
Epoch 258/270
Epoch 259/270
11/11 [============] - 0s 7ms/step - loss: 1.1065
Epoch 260/270
Epoch 261/270
Epoch 262/270
Epoch 263/270
Epoch 264/270
Epoch 265/270
Epoch 266/270
Epoch 267/270
```

```
11/11 [==============] - 0s 7ms/step - loss: 0.4866

Epoch 268/270

11/11 [=============] - 0s 6ms/step - loss: 0.5507

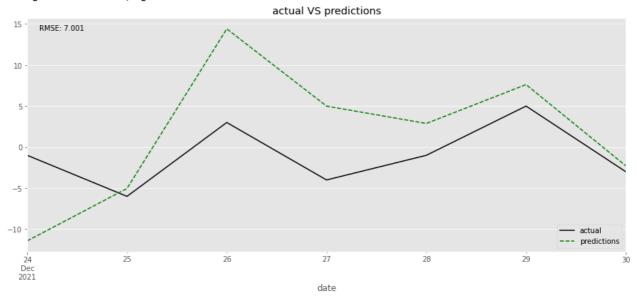
Epoch 269/270

11/11 [================] - 0s 8ms/step - loss: 0.5219

Epoch 270/270

11/11 [=================] - 0s 8ms/step - loss: 0.4380

>expected=-3.0, predicted=-2.3
```



В нашем случае мы определили модель с 70 единицами LSTM в скрытом слое и выходным слоем, который предсказывает одно числовое значение.

Обучилась модель с использованием эффективной версии стохастического градиентного спуска 'adam' и функцией потерь RMSE.

Stacked LSTM

Несколько скрытых слоев LSTM могут быть сложены друг на друга в так называемой модели Stacked LSTM.

Уровень LSTM требует трехмерного ввода, и LSTM по умолчанию будет возвращать двумерный вывод.

Мы можем решить эту проблему, установив аргумент return_sequences=True на слое. Это позволяет нам иметь 3D-выход со скрытого слоя LSTM в качестве входных данных на следующий.

Изменим функцию lstm_forecast задав модель Stacked LSTM.

```
In [126...
         config = (70, 'relu', 'adam', 'mse', 1, 170, 1)
         # lstm-прогноз на шаг вперёд
         def lstm_forecast(train,
                           features_test, # строка признаков для валидами
                           target, # имя колонки датафрейма целевого признака
                           config):
             # определяем конфигурацию
             units, activation, optimizer, loss, n features, epochs, verbose = conf
             # выделяем целевой признак из train
             features_train, target_train = train.drop(columns=[target]), train[target]
             # преобразуем датафреймы в объекты питру (нам нужен определённый форма
             features_train_arr = features_train.to_numpy()
             features_test_arr = features_test.to_numpy()
             target_train_arr = target_train.to_numpy()
             #target test arr = target test.to numpy()
             # Модель принимает тренировочные данные в таком формате: (samples, tim
             # Изменим форму входных данных.
             n steps = features train arr.shape[1]
             features_train_arr = features_train_arr.reshape((features_train.shape[
             features test arr = features test arr.reshape((1, n steps, n features)
             # определяем модель
             model stacked LSTM = Sequential()
             model stacked_LSTM.add(LSTM(20, activation='relu', return_sequences=Trut)
             model_stacked_LSTM.add(LSTM(50, activation='relu'))
             model stacked LSTM.add(Dense(1))
             model_stacked_LSTM.compile(optimizer='adam', loss='mse')
             # обучаем модель, на выходе получаем обученную модель с минимальной MS
             model stacked LSTM.fit(features train arr, target train arr, epochs=epochs
             # делаем прогноз на шаг вперед
             prediction = model stacked LSTM.predict(features test arr)
             #prediction = pd.DataFrame(prediction[0], columns=['prediction'], indel
             #prediction = prediction.reshape((features train.shape[0], n steps))
             return prediction[0]
In [127...
         error stckd, val, pred stckd = walk forward validation(data, 7, 'quantity'
         Epoch 1/170
```

```
Epoch 2/170
Epoch 3/170
Epoch 4/170
Epoch 5/170
11/11 [============== ] - 0s 8ms/step - loss: 104.8174
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
Epoch 10/170
```

```
Epoch 11/170
11/11 [============= ] - 0s 8ms/step - loss: 83.9873
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
Epoch 27/170
Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
Epoch 35/170
Epoch 36/170
Epoch 37/170
Epoch 38/170
Epoch 39/170
Epoch 40/170
```

```
Epoch 41/170
Epoch 42/170
11/11 [============= ] - 0s 7ms/step - loss: 52.5426
Epoch 43/170
Epoch 44/170
Epoch 45/170
Epoch 46/170
Epoch 47/170
Epoch 48/170
Epoch 49/170
Epoch 50/170
Epoch 51/170
Epoch 52/170
Epoch 53/170
11/11 [============== ] - 0s 12ms/step - loss: 47.7149
Epoch 54/170
Epoch 55/170
Epoch 56/170
11/11 [============] - 0s 12ms/step - loss: 47.7460
Epoch 57/170
Epoch 58/170
Epoch 59/170
Epoch 60/170
Epoch 61/170
Epoch 62/170
Epoch 63/170
Epoch 64/170
Epoch 65/170
Epoch 66/170
11/11 [============== ] - 0s 16ms/step - loss: 36.4318
Epoch 67/170
11/11 [============= ] - 0s 18ms/step - loss: 36.6710
Epoch 68/170
Epoch 69/170
Epoch 70/170
Epoch 71/170
```

```
Epoch 72/170
11/11 [============= ] - 0s 15ms/step - loss: 33.8846
Epoch 73/170
Epoch 74/170
11/11 [============= ] - 0s 12ms/step - loss: 31.7295
Epoch 75/170
Epoch 76/170
Epoch 77/170
11/11 [============== ] - 0s 11ms/step - loss: 30.8360
Epoch 78/170
Epoch 79/170
Epoch 80/170
11/11 [============== ] - 0s 12ms/step - loss: 28.8724
Epoch 81/170
11/11 [============= ] - 0s 13ms/step - loss: 30.5112
Epoch 82/170
11/11 [============= ] - 0s 14ms/step - loss: 28.1220
Epoch 83/170
11/11 [============= ] - 0s 12ms/step - loss: 24.5267
Epoch 84/170
11/11 [=============== ] - 0s 13ms/step - loss: 24.4230
Epoch 85/170
Epoch 86/170
Epoch 87/170
11/11 [============== ] - 0s 13ms/step - loss: 24.2008
Epoch 88/170
11/11 [============= ] - 0s 14ms/step - loss: 25.0220
Epoch 89/170
Epoch 90/170
Epoch 91/170
Epoch 92/170
11/11 [============= ] - 0s 17ms/step - loss: 20.8594
Epoch 93/170
11/11 [============== ] - 0s 16ms/step - loss: 19.5122
Epoch 94/170
Epoch 95/170
Epoch 96/170
Epoch 97/170
11/11 [============== ] - 0s 19ms/step - loss: 16.6953
Epoch 98/170
11/11 [============] - 0s 14ms/step - loss: 16.7195
Epoch 99/170
Epoch 100/170
11/11 [=============== ] - 0s 16ms/step - loss: 17.4386
Epoch 101/170
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Encah	102/170						
	102/170 [=======]	_	0s	21ms/step	_	loss:	16.7077
Epoch	103/170						
	[=====]	-	0s	20ms/step	-	loss:	15.9089
	104/170 [=======]		٥٠	16mg/g+on		1000.	14 0770
	105/170	_	US	10ms/step	_	1055:	14.0778
	[=======]	_	0s	15ms/step	_	loss:	12.7862
Epoch	106/170						
	[======] 107/170	-	0s	14ms/step	-	loss:	13.6738
	[========]	_	0s	13ms/step	_	loss:	12.2417
Epoch	108/170						
	[======]	-	0s	14ms/step	-	loss:	10.9211
	109/170 [=======]		۸c	16mg/g+an		1055	10 1047
	110/170		V S	1011137 5 CCP	_	1055.	10.1047
	[======]	-	0s	14ms/step	_	loss:	9.1998
	111/170 [=======]		0 ~	12/		1	0 0627
	112/170	_	US	13ms/step	_	loss:	8.963/
	[=======]	_	0s	14ms/step	_	loss:	8.3566
	113/170		•	10 / .			0.0650
	[======] 114/170	_	0s	13ms/step	_	loss:	9.0652
	[=======]	_	0s	14ms/step	_	loss:	8.9392
	115/170					_	
	[======] 116/170	-	0s	17ms/step	-	loss:	8.2536
	[=======]	_	0s	14ms/step	_	loss:	8.0427
Epoch	117/170						
	[======================================	-	0s	13ms/step	-	loss:	8.6756
	118/170 [=======]	_	0s	19ms/step	_	loss:	7.8276
Epoch	119/170						
	[======================================	-	0s	14ms/step	-	loss:	7.8392
	120/170 [======]	_	0s	17ms/step	_	loss:	9.5452
Epoch	121/170						
	[=======]	-	0s	14ms/step	-	loss:	8.9385
	122/170 [=======]	_	0 s	14ms/sten	_	1088:	8.1336
	123/170		O.D	I IMB/ BCCP		1000.	0.1330
	[======]	-	0s	13ms/step	-	loss:	7.4542
	124/170 [=======]		۸e	13mg/g+an		1000	7 1814
	125/170		V S	1311137 5 6 6 5	_	1055.	7.1014
	[========]	_	0s	16ms/step	_	loss:	6.6380
	126/170 [=======]		٥٥	15mg/g+on		logg.	6 0300
	127/170	_	US	13ms/scep	_	1055.	0.0300
11/11	[======]	-	0s	12ms/step	_	loss:	5.8143
	128/170		0 ~	15/		1	6 0475
	[======] 129/170	_	US	15ms/step	_	loss:	6.04/5
	[======]	_	0s	13ms/step	_	loss:	5.8102
	130/170		0	1 /		1	C 0000
	[=======] 131/170	-	US	14ms/step	-	TOSS:	0.0908
	[========]	_	0s	16ms/step	_	loss:	5.7849
Epoch	132/170						

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Epoch 133/170
Epoch 134/170
Epoch 135/170
Epoch 136/170
Epoch 137/170
11/11 [============= ] - 0s 13ms/step - loss: 5.9086
Epoch 138/170
Epoch 139/170
Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
Epoch 149/170
Epoch 150/170
11/11 [============ ] - 0s 13ms/step - loss: 5.5679
Epoch 151/170
Epoch 152/170
Epoch 153/170
Epoch 154/170
Epoch 155/170
Epoch 156/170
Epoch 157/170
Epoch 158/170
Epoch 159/170
Epoch 160/170
Epoch 161/170
Epoch 162/170
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Epoch 163/170
Epoch 164/170
Epoch 165/170
Epoch 166/170
Epoch 167/170
Epoch 168/170
Epoch 169/170
Epoch 170/170
>expected=-1.0, predicted=-7.1
Epoch 1/170
Epoch 2/170
11/11 [============= ] - 0s 8ms/step - loss: 135.7415
Epoch 3/170
Epoch 4/170
Epoch 5/170
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
Epoch 10/170
Epoch 11/170
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
11/11 [============= ] - 0s 8ms/step - loss: 80.1121
Epoch 21/170
Epoch 22/170
```

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Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
Epoch 27/170
Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
Epoch 35/170
Epoch 36/170
Epoch 37/170
Epoch 38/170
11/11 [============= ] - Os 8ms/step - loss: 56.1443
Epoch 39/170
Epoch 40/170
Epoch 41/170
Epoch 42/170
Epoch 43/170
Epoch 44/170
Epoch 45/170
Epoch 46/170
Epoch 47/170
Epoch 48/170
11/11 [============== ] - 0s 13ms/step - loss: 46.3915
Epoch 49/170
11/11 [============== ] - 0s 14ms/step - loss: 45.8001
Epoch 50/170
11/11 [============== ] - 0s 12ms/step - loss: 40.8364
Epoch 51/170
Epoch 52/170
Epoch 53/170
```

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Epoch 54/170
11/11 [============== ] - 0s 12ms/step - loss: 40.8885
Epoch 55/170
Epoch 56/170
Epoch 57/170
Epoch 58/170
Epoch 59/170
11/11 [============== ] - 0s 13ms/step - loss: 36.4497
Epoch 60/170
11/11 [============== ] - 0s 13ms/step - loss: 33.6993
Epoch 61/170
11/11 [============= ] - 0s 12ms/step - loss: 29.2639
Epoch 62/170
Epoch 63/170
11/11 [============= ] - 0s 14ms/step - loss: 28.8293
Epoch 64/170
Epoch 65/170
11/11 [============= ] - 0s 13ms/step - loss: 26.5974
Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
Epoch 71/170
Epoch 72/170
11/11 [============] - 0s 13ms/step - loss: 25.2075
Epoch 73/170
Epoch 74/170
11/11 [============== ] - 0s 12ms/step - loss: 22.4754
Epoch 75/170
Epoch 76/170
Epoch 77/170
Epoch 78/170
Epoch 79/170
Epoch 80/170
Epoch 81/170
Epoch 82/170
Epoch 83/170
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Epoch 84/170
Epoch 85/170
11/11 [============= ] - 0s 13ms/step - loss: 18.3531
Epoch 86/170
Epoch 87/170
11/11 [============== ] - 0s 12ms/step - loss: 17.6263
Epoch 88/170
11/11 [============== ] - 0s 13ms/step - loss: 18.7374
Epoch 89/170
11/11 [============== ] - 0s 13ms/step - loss: 18.9121
Epoch 90/170
11/11 [============== ] - 0s 13ms/step - loss: 19.8463
Epoch 91/170
Epoch 92/170
Epoch 93/170
Epoch 94/170
11/11 [============= ] - 0s 13ms/step - loss: 16.2369
Epoch 95/170
11/11 [============= ] - 0s 13ms/step - loss: 17.3129
Epoch 96/170
11/11 [============== ] - 0s 14ms/step - loss: 17.0284
Epoch 97/170
Epoch 98/170
Epoch 99/170
11/11 [============] - 0s 13ms/step - loss: 13.5776
Epoch 100/170
Epoch 101/170
Epoch 102/170
Epoch 103/170
Epoch 104/170
Epoch 105/170
Epoch 106/170
Epoch 107/170
Epoch 108/170
Epoch 109/170
Epoch 110/170
Epoch 111/170
Epoch 112/170
Epoch 113/170
11/11 [=============] - 0s 13ms/step - loss: 10.5034
Epoch 114/170
```

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Epoch 115/170
11/11 [============= ] - 0s 15ms/step - loss: 14.4328
Epoch 116/170
11/11 [===========] - 0s 14ms/step - loss: 17.6840
Epoch 117/170
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
Epoch 122/170
Epoch 123/170
Epoch 124/170
11/11 [============= ] - 0s 9ms/step - loss: 7.5517
Epoch 125/170
Epoch 126/170
Epoch 127/170
Epoch 128/170
Epoch 129/170
Epoch 130/170
11/11 [============= ] - 0s 7ms/step - loss: 8.7070
Epoch 131/170
Epoch 132/170
11/11 [============] - Os 8ms/step - loss: 7.3117
Epoch 133/170
Epoch 134/170
Epoch 135/170
Epoch 136/170
Epoch 137/170
Epoch 138/170
Epoch 139/170
11/11 [============= ] - 0s 8ms/step - loss: 5.4524
Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
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Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
11/11 [============== ] - 0s 8ms/step - loss: 4.9415
Epoch 149/170
Epoch 150/170
11/11 [============ ] - Os 8ms/step - loss: 4.8242
Epoch 151/170
11/11 [============= ] - 0s 9ms/step - loss: 4.7400
Epoch 152/170
Epoch 153/170
Epoch 154/170
Epoch 155/170
11/11 [============= ] - 0s 9ms/step - loss: 6.7903
Epoch 156/170
Epoch 157/170
11/11 [============== ] - 0s 8ms/step - loss: 4.6088
Epoch 158/170
Epoch 159/170
Epoch 160/170
11/11 [============] - Os 8ms/step - loss: 4.1179
Epoch 161/170
Epoch 162/170
Epoch 163/170
Epoch 164/170
Epoch 165/170
Epoch 166/170
Epoch 167/170
11/11 [============] - 0s 9ms/step - loss: 3.0660
Epoch 168/170
Epoch 169/170
Epoch 170/170
>expected=-6.0, predicted=-6.3
Epoch 1/170
Epoch 2/170
11/11 [============] - 0s 8ms/step - loss: 134.6905
Epoch 3/170
Epoch 4/170
```

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Epoch 5/170
11/11 [============== ] - 0s 8ms/step - loss: 106.2801
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
Epoch 10/170
Epoch 11/170
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
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Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
Epoch 35/170
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Epoch 36/170
11/11 [============= ] - 0s 8ms/step - loss: 59.7642
Epoch 37/170
Epoch 38/170
Epoch 39/170
Epoch 40/170
Epoch 41/170
Epoch 42/170
Epoch 43/170
Epoch 44/170
Epoch 45/170
Epoch 46/170
Epoch 47/170
Epoch 48/170
Epoch 49/170
Epoch 50/170
Epoch 51/170
Epoch 52/170
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Epoch 60/170
Epoch 61/170
Epoch 62/170
Epoch 63/170
Epoch 64/170
Epoch 65/170
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Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
Epoch 71/170
Epoch 72/170
Epoch 73/170
Epoch 74/170
Epoch 75/170
Epoch 76/170
Epoch 77/170
Epoch 78/170
Epoch 79/170
Epoch 80/170
Epoch 81/170
11/11 [=============] - Os 8ms/step - loss: 23.1917
Epoch 82/170
Epoch 83/170
Epoch 84/170
Epoch 85/170
Epoch 86/170
Epoch 87/170
Epoch 88/170
Epoch 89/170
Epoch 90/170
Epoch 91/170
Epoch 92/170
Epoch 93/170
Epoch 94/170
Epoch 95/170
Epoch 96/170
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Epoch 97/170
Epoch 98/170
Epoch 99/170
Epoch 100/170
Epoch 101/170
Epoch 102/170
Epoch 103/170
Epoch 104/170
Epoch 105/170
Epoch 106/170
Epoch 107/170
Epoch 108/170
Epoch 109/170
Epoch 110/170
Epoch 111/170
Epoch 112/170
Epoch 113/170
Epoch 114/170
Epoch 115/170
Epoch 116/170
Epoch 117/170
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
Epoch 122/170
11/11 [=============== ] - 0s 10ms/step - loss: 11.1654
Epoch 123/170
Epoch 124/170
Epoch 125/170
Epoch 126/170
```

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Epoch 127/170
Epoch 128/170
11/11 [============== ] - 0s 8ms/step - loss: 8.5744
Epoch 129/170
Epoch 130/170
Epoch 131/170
Epoch 132/170
11/11 [============ ] - Os 9ms/step - loss: 9.2161
Epoch 133/170
Epoch 134/170
Epoch 135/170
Epoch 136/170
Epoch 137/170
11/11 [============== ] - 0s 8ms/step - loss: 7.1745
Epoch 138/170
Epoch 139/170
Epoch 140/170
Epoch 141/170
Epoch 142/170
11/11 [============= ] - 0s 10ms/step - loss: 8.7184
Epoch 143/170
Epoch 144/170
Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
Epoch 149/170
Epoch 150/170
Epoch 151/170
Epoch 152/170
Epoch 153/170
Epoch 154/170
11/11 [============== ] - 0s 8ms/step - loss: 5.7396
Epoch 155/170
Epoch 156/170
Epoch 157/170
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Epoch 158/170
Epoch 159/170
Epoch 160/170
Epoch 161/170
Epoch 162/170
Epoch 163/170
Epoch 164/170
Epoch 165/170
11/11 [=============== ] - 0s 8ms/step - loss: 4.1152
Epoch 166/170
Epoch 167/170
Epoch 168/170
Epoch 169/170
Epoch 170/170
>expected=3.0, predicted=10.8
Epoch 1/170
Epoch 2/170
11/11 [============] - 0s 8ms/step - loss: 136.5268
Epoch 3/170
11/11 [============= ] - 0s 8ms/step - loss: 135.8970
Epoch 4/170
Epoch 5/170
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
Epoch 10/170
Epoch 11/170
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
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Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
Epoch 27/170
Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
11/11 [============= ] - 0s 17ms/step - loss: 70.5839
Epoch 34/170
Epoch 35/170
Epoch 36/170
Epoch 37/170
Epoch 38/170
Epoch 39/170
Epoch 40/170
Epoch 41/170
Epoch 42/170
11/11 [============= ] - Os 9ms/step - loss: 63.6893
Epoch 43/170
Epoch 44/170
Epoch 45/170
Epoch 46/170
Epoch 47/170
```

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Epoch 48/170
Epoch 49/170
11/11 [============= ] - 0s 8ms/step - loss: 57.3328
Epoch 50/170
Epoch 51/170
Epoch 52/170
11/11 [============= ] - 0s 10ms/step - loss: 55.9811
Epoch 53/170
Epoch 54/170
Epoch 55/170
Epoch 56/170
Epoch 57/170
Epoch 58/170
Epoch 59/170
Epoch 60/170
Epoch 61/170
Epoch 62/170
Epoch 63/170
11/11 [=============] - Os 8ms/step - loss: 44.1552
Epoch 64/170
Epoch 65/170
Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
11/11 [============== ] - Os 8ms/step - loss: 37.5118
Epoch 71/170
Epoch 72/170
Epoch 73/170
Epoch 74/170
Epoch 75/170
Epoch 76/170
Epoch 77/170
Epoch 78/170
```

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Epoch 79/170
Epoch 80/170
11/11 [============] - Os 9ms/step - loss: 49.6766
Epoch 81/170
Epoch 82/170
Epoch 83/170
Epoch 84/170
Epoch 85/170
Epoch 86/170
Epoch 87/170
Epoch 88/170
Epoch 89/170
Epoch 90/170
Epoch 91/170
Epoch 92/170
Epoch 93/170
Epoch 94/170
Epoch 95/170
Epoch 96/170
Epoch 97/170
Epoch 98/170
Epoch 99/170
Epoch 100/170
Epoch 101/170
Epoch 102/170
Epoch 103/170
Epoch 104/170
Epoch 105/170
Epoch 106/170
Epoch 107/170
Epoch 108/170
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Epoch 109/170
Epoch 110/170
Epoch 111/170
Epoch 112/170
Epoch 113/170
Epoch 114/170
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Epoch 116/170
Epoch 117/170
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
Epoch 122/170
Epoch 123/170
Epoch 124/170
11/11 [============= ] - Os 9ms/step - loss: 12.5227
Epoch 125/170
Epoch 126/170
Epoch 127/170
Epoch 128/170
Epoch 129/170
Epoch 130/170
Epoch 131/170
Epoch 132/170
Epoch 133/170
Epoch 134/170
Epoch 135/170
11/11 [============== ] - 0s 13ms/step - loss: 11.6851
Epoch 136/170
11/11 [============== ] - 0s 10ms/step - loss: 10.4650
Epoch 137/170
Epoch 138/170
Epoch 139/170
```

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Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
11/11 [============== ] - 0s 9ms/step - loss: 8.9365
Epoch 149/170
Epoch 150/170
Epoch 151/170
11/11 [============== ] - 0s 9ms/step - loss: 8.8945
Epoch 152/170
Epoch 153/170
11/11 [============== ] - 0s 8ms/step - loss: 7.4502
Epoch 154/170
Epoch 155/170
11/11 [============= ] - Os 9ms/step - loss: 7.1543
Epoch 156/170
Epoch 157/170
11/11 [============] - Os 8ms/step - loss: 6.5663
Epoch 158/170
Epoch 159/170
Epoch 160/170
Epoch 161/170
Epoch 162/170
Epoch 163/170
Epoch 164/170
Epoch 165/170
Epoch 166/170
Epoch 167/170
Epoch 168/170
Epoch 169/170
```

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Epoch 170/170
>expected=-4.0, predicted=2.4
Epoch 1/170
Epoch 2/170
Epoch 3/170
Epoch 4/170
Epoch 5/170
11/11 [============== ] - 0s 8ms/step - loss: 109.3459
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
Epoch 10/170
Epoch 11/170
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
11/11 [============= ] - Os 8ms/step - loss: 74.9012
Epoch 27/170
11/11 [============= ] - 0s 8ms/step - loss: 72.9584
Epoch 28/170
Epoch 29/170
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Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
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Epoch 39/170
Epoch 40/170
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Epoch 42/170
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Epoch 46/170
Epoch 47/170
Epoch 48/170
Epoch 49/170
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Epoch 51/170
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Epoch 53/170
Epoch 54/170
Epoch 55/170
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Epoch 57/170
Epoch 58/170
Epoch 59/170
Epoch 60/170
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Epoch 61/170
Epoch 62/170
Epoch 63/170
Epoch 64/170
Epoch 65/170
Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
Epoch 71/170
Epoch 72/170
11/11 [=============] - 0s 10ms/step - loss: 30.5707
Epoch 73/170
Epoch 74/170
Epoch 75/170
11/11 [=============] - Os 8ms/step - loss: 28.4913
Epoch 76/170
11/11 [============== ] - 0s 8ms/step - loss: 27.3700
Epoch 77/170
Epoch 78/170
Epoch 79/170
Epoch 80/170
Epoch 81/170
Epoch 82/170
Epoch 83/170
Epoch 84/170
Epoch 85/170
11/11 [============= ] - 0s 9ms/step - loss: 20.8029
Epoch 86/170
Epoch 87/170
Epoch 88/170
Epoch 89/170
Epoch 90/170
```

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Epoch 91/170
Epoch 92/170
11/11 [============= ] - 0s 9ms/step - loss: 16.6278
Epoch 93/170
Epoch 94/170
Epoch 95/170
Epoch 96/170
Epoch 97/170
Epoch 98/170
Epoch 99/170
Epoch 100/170
Epoch 101/170
11/11 [============= ] - 0s 10ms/step - loss: 13.3434
Epoch 102/170
Epoch 103/170
Epoch 104/170
Epoch 105/170
Epoch 106/170
11/11 [============] - 0s 10ms/step - loss: 13.0005
Epoch 107/170
Epoch 108/170
Epoch 109/170
Epoch 110/170
Epoch 111/170
Epoch 112/170
Epoch 113/170
Epoch 114/170
Epoch 115/170
Epoch 116/170
11/11 [============== ] - 0s 9ms/step - loss: 9.4617
Epoch 117/170
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
```

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Epoch 122/170
11/11 [============= ] - Os 9ms/step - loss: 8.0199
Epoch 123/170
11/11 [============= ] - 0s 8ms/step - loss: 7.2865
Epoch 124/170
Epoch 125/170
Epoch 126/170
Epoch 127/170
Epoch 128/170
11/11 [============== ] - 0s 9ms/step - loss: 6.6845
Epoch 129/170
Epoch 130/170
Epoch 131/170
Epoch 132/170
Epoch 133/170
11/11 [============== ] - 0s 9ms/step - loss: 5.8867
Epoch 134/170
Epoch 135/170
Epoch 136/170
Epoch 137/170
Epoch 138/170
Epoch 139/170
Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
11/11 [============== ] - 0s 9ms/step - loss: 4.9715
Epoch 149/170
Epoch 150/170
Epoch 151/170
```

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Epoch 152/170
Epoch 153/170
Epoch 154/170
Epoch 155/170
11/11 [============== ] - 0s 8ms/step - loss: 4.5503
Epoch 156/170
Epoch 157/170
Epoch 158/170
11/11 [============== ] - 0s 9ms/step - loss: 3.1580
Epoch 159/170
Epoch 160/170
Epoch 161/170
Epoch 162/170
11/11 [============= ] - 0s 9ms/step - loss: 3.4153
Epoch 163/170
Epoch 164/170
Epoch 165/170
Epoch 166/170
Epoch 167/170
11/11 [===========] - 0s 9ms/step - loss: 2.9502
Epoch 168/170
Epoch 169/170
Epoch 170/170
>expected=-1.0, predicted=11.7
Epoch 1/170
Epoch 2/170
Epoch 3/170
Epoch 4/170
Epoch 5/170
Epoch 6/170
Epoch 7/170
Epoch 8/170
Epoch 9/170
11/11 [============] - 0s 10ms/step - loss: 85.5527
Epoch 10/170
Epoch 11/170
```

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Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
Epoch 25/170
Epoch 26/170
Epoch 27/170
Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
Epoch 35/170
Epoch 36/170
Epoch 37/170
Epoch 38/170
Epoch 39/170
Epoch 40/170
Epoch 41/170
Epoch 42/170
```

```
Epoch 43/170
11/11 [============== ] - 0s 10ms/step - loss: 47.0459
Epoch 44/170
11/11 [============] - Os 9ms/step - loss: 47.9241
Epoch 45/170
11/11 [============] - Os 9ms/step - loss: 45.6059
Epoch 46/170
Epoch 47/170
11/11 [=============] - 0s 11ms/step - loss: 43.8740
Epoch 48/170
Epoch 49/170
Epoch 50/170
Epoch 51/170
Epoch 52/170
Epoch 53/170
Epoch 54/170
Epoch 55/170
Epoch 56/170
Epoch 57/170
Epoch 58/170
Epoch 59/170
Epoch 60/170
Epoch 61/170
Epoch 62/170
Epoch 63/170
Epoch 64/170
Epoch 65/170
Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
Epoch 71/170
Epoch 72/170
```

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Epoch 73/170
Epoch 74/170
Epoch 75/170
Epoch 76/170
Epoch 77/170
11/11 [============= ] - Os 9ms/step - loss: 17.9189
Epoch 78/170
Epoch 79/170
Epoch 80/170
Epoch 81/170
Epoch 82/170
Epoch 83/170
Epoch 84/170
Epoch 85/170
Epoch 86/170
Epoch 87/170
Epoch 88/170
11/11 [============= ] - Os 9ms/step - loss: 16.3723
Epoch 89/170
Epoch 90/170
Epoch 91/170
Epoch 92/170
Epoch 93/170
Epoch 94/170
11/11 [============= ] - 0s 9ms/step - loss: 13.0140
Epoch 95/170
Epoch 96/170
Epoch 97/170
Epoch 98/170
Epoch 99/170
Epoch 100/170
Epoch 101/170
Epoch 102/170
Epoch 103/170
```

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Epoch 104/170
11/11 [============= ] - 0s 12ms/step - loss: 10.3049
Epoch 105/170
Epoch 106/170
Epoch 107/170
Epoch 108/170
Epoch 109/170
Epoch 110/170
11/11 [============== ] - 0s 9ms/step - loss: 9.6317
Epoch 111/170
Epoch 112/170
11/11 [============== ] - 0s 10ms/step - loss: 11.9473
Epoch 113/170
Epoch 114/170
Epoch 115/170
Epoch 116/170
Epoch 117/170
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
Epoch 122/170
Epoch 123/170
Epoch 124/170
Epoch 125/170
Epoch 126/170
Epoch 127/170
Epoch 128/170
Epoch 129/170
Epoch 130/170
Epoch 131/170
Epoch 132/170
Epoch 133/170
```

```
Epoch 134/170
Epoch 135/170
11/11 [============= ] - 0s 9ms/step - loss: 4.1991
Epoch 136/170
Epoch 137/170
Epoch 138/170
Epoch 139/170
11/11 [========= ] - Os 9ms/step - loss: 4.1645
Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
11/11 [============= ] - 0s 9ms/step - loss: 3.8130
Epoch 145/170
Epoch 146/170
Epoch 147/170
Epoch 148/170
Epoch 149/170
11/11 [============= ] - 0s 10ms/step - loss: 3.9980
Epoch 150/170
Epoch 151/170
Epoch 152/170
Epoch 153/170
Epoch 154/170
Epoch 155/170
Epoch 156/170
Epoch 157/170
Epoch 158/170
Epoch 159/170
11/11 [============== ] - 0s 9ms/step - loss: 2.0485
Epoch 160/170
Epoch 161/170
Epoch 162/170
Epoch 163/170
Epoch 164/170
```

```
Epoch 165/170
Epoch 166/170
Epoch 167/170
Epoch 168/170
Epoch 169/170
11/11 [============ ] - 0s 9ms/step - loss: 2.2479
Epoch 170/170
>expected=5.0, predicted=12.9
Epoch 1/170
Epoch 2/170
Epoch 3/170
Epoch 4/170
11/11 [============= ] - 0s 9ms/step - loss: 112.7410
Epoch 5/170
11/11 [============== ] - 0s 9ms/step - loss: 101.2826
Epoch 6/170
Epoch 7/170
11/11 [============] - Os 9ms/step - loss: 91.9844
Epoch 8/170
Epoch 9/170
11/11 [============= ] - Os 9ms/step - loss: 88.1956
Epoch 10/170
Epoch 11/170
Epoch 12/170
Epoch 13/170
Epoch 14/170
Epoch 15/170
Epoch 16/170
Epoch 17/170
Epoch 18/170
Epoch 19/170
Epoch 20/170
Epoch 21/170
Epoch 22/170
Epoch 23/170
Epoch 24/170
```

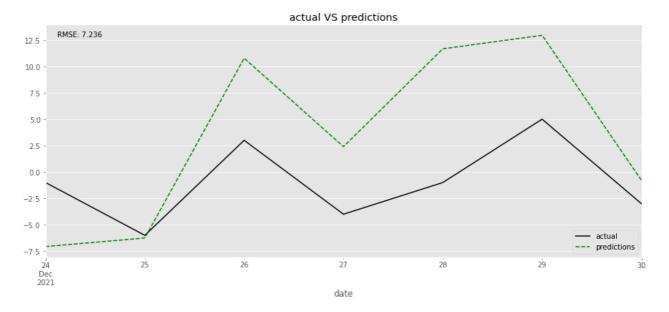
```
Epoch 25/170
11/11 [============= ] - 0s 9ms/step - loss: 67.4619
Epoch 26/170
Epoch 27/170
Epoch 28/170
Epoch 29/170
Epoch 30/170
Epoch 31/170
Epoch 32/170
Epoch 33/170
Epoch 34/170
Epoch 35/170
Epoch 36/170
11/11 [============= ] - 0s 10ms/step - loss: 54.9082
Epoch 37/170
Epoch 38/170
Epoch 39/170
Epoch 40/170
Epoch 41/170
Epoch 42/170
Epoch 43/170
Epoch 44/170
Epoch 45/170
Epoch 46/170
Epoch 47/170
Epoch 48/170
Epoch 49/170
Epoch 50/170
Epoch 51/170
11/11 [============= ] - Os 9ms/step - loss: 43.2396
Epoch 52/170
Epoch 53/170
Epoch 54/170
```

```
Epoch 55/170
Epoch 56/170
11/11 [============= ] - 0s 9ms/step - loss: 39.1970
Epoch 57/170
Epoch 58/170
Epoch 59/170
Epoch 60/170
Epoch 61/170
Epoch 62/170
Epoch 63/170
Epoch 64/170
Epoch 65/170
Epoch 66/170
Epoch 67/170
Epoch 68/170
Epoch 69/170
Epoch 70/170
11/11 [============] - 0s 10ms/step - loss: 28.8718
Epoch 71/170
Epoch 72/170
Epoch 73/170
Epoch 74/170
Epoch 75/170
Epoch 76/170
Epoch 77/170
11/11 [============= ] - Os 9ms/step - loss: 22.5468
Epoch 78/170
Epoch 79/170
Epoch 80/170
Epoch 81/170
Epoch 82/170
Epoch 83/170
Epoch 84/170
Epoch 85/170
```

```
Epoch 86/170
11/11 [============== ] - 0s 10ms/step - loss: 22.1420
Epoch 87/170
Epoch 88/170
Epoch 89/170
Epoch 90/170
11/11 [============] - 0s 10ms/step - loss: 16.6930
Epoch 91/170
Epoch 92/170
Epoch 93/170
Epoch 94/170
11/11 [============== ] - 0s 10ms/step - loss: 12.8837
Epoch 95/170
Epoch 96/170
Epoch 97/170
Epoch 98/170
Epoch 99/170
Epoch 100/170
Epoch 101/170
Epoch 102/170
Epoch 103/170
11/11 [===========] - 0s 10ms/step - loss: 10.2724
Epoch 104/170
Epoch 105/170
Epoch 106/170
11/11 [============== ] - 0s 11ms/step - loss: 10.7698
Epoch 107/170
Epoch 108/170
Epoch 109/170
Epoch 110/170
Epoch 111/170
Epoch 112/170
Epoch 113/170
Epoch 114/170
Epoch 115/170
```

```
Epoch 116/170
Epoch 117/170
11/11 [============= ] - 0s 10ms/step - loss: 7.9227
Epoch 118/170
Epoch 119/170
Epoch 120/170
Epoch 121/170
11/11 [============ ] - Os 9ms/step - loss: 6.0252
Epoch 122/170
Epoch 123/170
Epoch 124/170
Epoch 125/170
11/11 [============== ] - 0s 9ms/step - loss: 7.8318
Epoch 126/170
11/11 [============= ] - 0s 9ms/step - loss: 6.4395
Epoch 127/170
Epoch 128/170
Epoch 129/170
Epoch 130/170
Epoch 131/170
11/11 [============= ] - 0s 10ms/step - loss: 5.5508
Epoch 132/170
Epoch 133/170
Epoch 134/170
Epoch 135/170
Epoch 136/170
Epoch 137/170
Epoch 138/170
11/11 [============= ] - Os 8ms/step - loss: 3.9969
Epoch 139/170
Epoch 140/170
Epoch 141/170
Epoch 142/170
Epoch 143/170
Epoch 144/170
Epoch 145/170
Epoch 146/170
```

```
Epoch 147/170
Epoch 148/170
Epoch 149/170
11/11 [============= ] - Os 9ms/step - loss: 2.9751
Epoch 150/170
Epoch 151/170
11/11 [============= ] - 0s 9ms/step - loss: 4.0113
Epoch 152/170
Epoch 153/170
Epoch 154/170
Epoch 155/170
Epoch 156/170
11/11 [============= ] - 0s 9ms/step - loss: 2.4191
Epoch 157/170
11/11 [============= ] - Os 10ms/step - loss: 2.2592
Epoch 158/170
Epoch 159/170
Epoch 160/170
11/11 [============= ] - 0s 10ms/step - loss: 1.7658
Epoch 161/170
Epoch 162/170
Epoch 163/170
Epoch 164/170
11/11 [============] - Os 9ms/step - loss: 1.4819
Epoch 165/170
Epoch 166/170
Epoch 167/170
Epoch 168/170
Epoch 169/170
11/11 [============== ] - 0s 10ms/step - loss: 1.6394
Epoch 170/170
>expected=-3.0, predicted=-0.8
```



Модель всё ещё далека от идеала.

Многошаговая LSTM

Ранее мы пользовались расширяющимся окном для предсказания на каждом следующем шаге, то есть делали прогноз на один день вперед.

Теперь сделаем модель, которая будет делать прогноз сразу на несколько шагов (дней) вперед.

Напишем функцию split_sequence(), которая принимает на вход последовательность, количество последних нескольких шагов на входе и на выходе.

Здесь, в отличие от предыдущих моделей, в качестве целевого признака используется последовательность значений, а не одно значение. Так как мы пользуемся нейронной сетью, то в выходном слое просто укажем соответствующее количество нейронов.

```
In [128...
          # разбиваем последовательность
          def split_sequence(sequence, n_steps_in, n_steps_out):
              features, target = list(), list()
              for i in range(len(sequence)):
                  # находим конечный элемент
                  end ix = i + n steps in
                  out end ix = end ix + n steps out
                  # проверяем, не вышли ли мы за пределы последовательности
                  if out end ix > len(sequence):
                      break
                  # собираем входные и выходные последовательности
                  seq features, seq target = sequence[i:end ix], sequence[end ix:out
                  features.append(seq features)
                  target.append(seq_target)
              return np.array(features), np.array(target)
```

Пусть в качестве признаков у нас будут 28 последних дней, а предсказывать мы будем каждые семь следующих.

```
In [129...
         sales diff = sales q.diff().dropna()
In [224...
         # выбираем количество шагов
         n steps in, n steps out = 28, 7
         # разбиваем на признаки и цель
         features, target = split_sequence(sales_diff[:'2021-12-23'], n_steps_in, n_
         # посмотрим, как будут выглядеть данные
         for i in range(len(features)):
            print(features[i], target[i])
         [ 26. -5.
                          5.
                               3. -9. -11. 35.
                                                 -7. -22.
                                                                12.
                                                                   -8.
                     3.
                                                            3.
                                                                           2.
                              27. -24.
                                        4. -14.
                                                 20.
                                                       3. -25.
                                                                47. -43.
           17. -16.
                     6. -4.
                                                                           3.] [ 1
                     5.
                          6. -16.
             5. -8.
                3. 5.
                          3. -9. -11. 35. -7. -22.
                                                     3.
                                                           12.
         [ -5.
                                                               -8.
                                                                      2.
                                                                          17.
                6. -4. 27. -24. 4. -14. 20. 3. -25.
                                                           47. -43.
         5. -8.
                  5. \quad 6. \quad -16. \quad 4. \quad -6.
                5.
                                                                 2.
                     3. -9. -11. 35. -7. -22.
                                                 3. 12. -8.
            3.
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                              4. -14.
                    27. -24.
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                                             3. -25.
                                                     47. -43.
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               -4.
                                                                 3.
                                                                          5.] [ -
             5.
                  6. -16.
                           4.
                               -6.
                                    18.]
            5.
                3.
                    -9. -11. 35. -7. -22.
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                                                     -8.
                                                            2.
                                                                17. -16.
               27. -24.
                          4. -14. 20.
                                         3. -25.
                                                 47. -43.
                                                                10.
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                                                                          -8.] [
             6. -16.
                       4. -6. 18. -14.]
           3. -9. -11. 35. -7. -22.
                                        3. 12. -8.
                                                       2.
                                                           17. -16.
                                                                      6.
                                    3. -25. 47. -43.
                     4. -14.
                              20.
                                                       3.
                                                           10.
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           27. -24.
                                                                    -8.
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                     -6. 18. -14.
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         [-9. -11. 35. -7. -22.
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                4. -14.
                         20.
                               3. -25. 47. -43.
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             4. -6. 18. -14.
                                 3. 10.]
         [-11. 35. -7. -22.
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                                                                     27. -24.
            4. -14. 20.
                          3. -25. 47. -43.
                                             3. 10.
                                                       5.
                                                           -8.
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                            3. 10. -10.]
            -6. 18. -14.
         [ 35. -7. -22.
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            18. -14.
                       3.
                          10. -10. -3.]
         [-7.-22.
                     3. 12. -8.
                                    2. 17. -16.
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3. -25. 47. -43. 3. 10.
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       3. 10. -10. -3. -16.]
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[-22.
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-25. 47. -43.
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                                           3. -25.
                                                     47. -43.
                                                               3.
       5. -8. 5. 6. -16.
                             4. -6.
                                      18. -14.
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[6. -4. 27. -24. 4. -14. 20.
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[-25. 	47. -43. 	3. 	10.
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9. -1. 4. 16. -8. -1. -21.
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       3. -4. -7. -8. 13. 16. -19. 2. 14. -23.
                                                      21.
                                                          1. -8.] [-1
6. 18. -4. -1. 8. -6. 4.
```

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1. -10.
                    1. -5. 11. -2. -6. -2. -6.
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           -7. -8. 13. 16. -19.
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                4. -9.1
[ 8. 1. -10. 1. -5. 11. -2. -6. -2.
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                                                           -8.
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                                                               -6.] [
4. -9. -10. 1. 19. -14. -7.
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                                                           13.
                                                               16.
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\begin{bmatrix} -2. & -6. & 18. & -6. & 4. \end{bmatrix}
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                                                 4.
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9. -14. -7. -2. 1. 3. 4.]
[-6. 18. -6. 4.
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                                       -6.
                                            4.
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                                                                19.] [-1
4. -7. -2. 1. 3. 4. 1.]
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                3. -4. -7. -8.
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                                      16. -19.
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                                                          19. -14.] [-7
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                              8.
                                  -6.
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                              13. 16. -19.
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                                                14. -23.
      4.
           3. -4.
                    -7. -8.
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 -8. -16.
          18.
               -4 .
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                                                          -8. -16.
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               8. -6.
                         4. -9. -10.
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                                           19. -14.
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3. 4. 1. -3. 0. 15. -15.]
[-4. -7. -8. 13. 16. -19. 2. 14. -23. 21.
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                                                     -8. -16.
                                                               18.
 -4. -1. 8. -6. 4. -9. -10. 1. 19. -14.
                                                -7.
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                                                            1.
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       0. 15. -15. -2. 8.]
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                    2. 14. -23. 21.
[ -8. 13. 16. -19.
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                                                     18.
                                                          -4 .
            4. -9. -10. 1. 19. -14. -7. -2. 1.
      -6.
                                                     3.
                                                          4.
                                                                1.] [ -
3. 0. 15. -15. -2. 8.
                          1.]
```

Приведем входные данные к нужному формату.

Определим и обучим модель с одним скрытым слоем.

```
In [226...
     # определяем модель
     model_vector_LSTM1 = Sequential()
     model_vector_LSTM1.add(LSTM(50, activation='relu', input_shape=(n_steps_in
     model vector LSTM1.add(Dense(n steps out))
     model_vector_LSTM1.compile(optimizer='adam', loss='mse')
     # обучаем модель
     model vector LSTM1.fit(features, target, epochs=120, verbose=1)
     Epoch 1/120
     Epoch 2/120
     10/10 [=============== ] - 0s 12ms/step - loss: 117.4109
     Epoch 3/120
     10/10 [============== ] - 0s 13ms/step - loss: 116.6102
     Epoch 4/120
     Epoch 5/120
     Epoch 6/120
     10/10 [=============== ] - 0s 12ms/step - loss: 114.0004
     Epoch 7/120
     Epoch 8/120
     Epoch 9/120
     10/10 [=============== ] - 0s 13ms/step - loss: 113.3787
     Epoch 10/120
     Epoch 11/120
     Epoch 12/120
     10/10 [============== ] - 0s 13ms/step - loss: 110.9653
     Epoch 13/120
     10/10 [============= ] - Os 13ms/step - loss: 110.3367
     Epoch 14/120
     10/10 [============== ] - 0s 13ms/step - loss: 110.0408
     Epoch 15/120
     Epoch 16/120
     Epoch 17/120
     Epoch 18/120
     Epoch 19/120
     Epoch 20/120
     10/10 [============== ] - Os 12ms/step - loss: 102.4968
     Epoch 21/120
     10/10 [============== ] - 0s 13ms/step - loss: 101.3353
     Epoch 22/120
     10/10 [============= ] - 0s 12ms/step - loss: 99.3692
     Epoch 23/120
     10/10 [===============] - 0s 14ms/step - loss: 97.7048
     Epoch 24/120
     10/10 [=============== ] - 0s 12ms/step - loss: 95.8973
     Epoch 25/120
```

```
Epoch 26/120
10/10 [============ ] - 0s 12ms/step - loss: 93.8019
Epoch 27/120
10/10 [============== ] - 0s 14ms/step - loss: 95.2712
Epoch 28/120
10/10 [============== ] - 0s 12ms/step - loss: 94.1161
Epoch 29/120
Epoch 30/120
Epoch 31/120
10/10 [============ ] - 0s 14ms/step - loss: 83.6665
Epoch 32/120
10/10 [============= ] - 0s 13ms/step - loss: 81.7194
Epoch 33/120
10/10 [============= ] - 0s 12ms/step - loss: 78.0225
Epoch 34/120
10/10 [============= ] - 0s 13ms/step - loss: 76.7096
Epoch 35/120
Epoch 36/120
10/10 [============== ] - 0s 13ms/step - loss: 72.7344
Epoch 37/120
10/10 [============ ] - 0s 12ms/step - loss: 69.6232
Epoch 38/120
Epoch 39/120
Epoch 40/120
10/10 [============== ] - 0s 14ms/step - loss: 64.6706
Epoch 41/120
Epoch 42/120
10/10 [============= ] - 0s 13ms/step - loss: 56.3593
Epoch 43/120
10/10 [===========] - 0s 12ms/step - loss: 56.9381
Epoch 44/120
10/10 [============== ] - 0s 12ms/step - loss: 58.0235
Epoch 45/120
Epoch 46/120
Epoch 47/120
10/10 [============= ] - 0s 14ms/step - loss: 50.9025
Epoch 48/120
10/10 [============== ] - 0s 13ms/step - loss: 49.2141
Epoch 49/120
Epoch 50/120
10/10 [===============] - 0s 13ms/step - loss: 42.7039
Epoch 51/120
10/10 [============= ] - 0s 13ms/step - loss: 40.7957
Epoch 52/120
10/10 [============ ] - 0s 12ms/step - loss: 38.9471
Epoch 53/120
10/10 [============] - 0s 13ms/step - loss: 43.1451
Epoch 54/120
10/10 [============== ] - 0s 14ms/step - loss: 40.4229
Epoch 55/120
```

```
Epoch 56/120
10/10 [============== ] - 0s 13ms/step - loss: 37.9670
Epoch 57/120
10/10 [============= ] - 0s 13ms/step - loss: 34.6148
Epoch 58/120
10/10 [============== ] - 0s 12ms/step - loss: 32.0994
Epoch 59/120
10/10 [============== ] - 0s 13ms/step - loss: 30.3251
Epoch 60/120
10/10 [============== ] - 0s 13ms/step - loss: 28.7787
Epoch 61/120
10/10 [============== ] - 0s 14ms/step - loss: 28.3161
Epoch 62/120
10/10 [============= ] - 0s 13ms/step - loss: 29.9391
Epoch 63/120
Epoch 64/120
10/10 [===============] - 0s 12ms/step - loss: 27.6028
Epoch 65/120
Epoch 66/120
Epoch 67/120
10/10 [============= ] - 0s 13ms/step - loss: 24.0952
Epoch 68/120
10/10 [============= ] - 0s 12ms/step - loss: 23.3710
Epoch 69/120
Epoch 70/120
Epoch 71/120
10/10 [===========] - 0s 13ms/step - loss: 21.4159
Epoch 72/120
10/10 [============= ] - 0s 13ms/step - loss: 20.9838
Epoch 73/120
10/10 [============== ] - 0s 14ms/step - loss: 20.0904
Epoch 74/120
Epoch 75/120
Epoch 76/120
Epoch 77/120
10/10 [============== ] - 0s 13ms/step - loss: 18.4648
Epoch 78/120
10/10 [===========] - 0s 14ms/step - loss: 17.9766
Epoch 79/120
Epoch 80/120
10/10 [===============] - 0s 12ms/step - loss: 20.6201
Epoch 81/120
10/10 [============= ] - 0s 13ms/step - loss: 20.1444
Epoch 82/120
10/10 [============= ] - 0s 12ms/step - loss: 18.6531
Epoch 83/120
10/10 [============= ] - 0s 13ms/step - loss: 19.3739
Epoch 84/120
10/10 [============== ] - 0s 14ms/step - loss: 19.9813
Epoch 85/120
10/10 [=============== ] - 0s 12ms/step - loss: 17.0337
Epoch 86/120
```

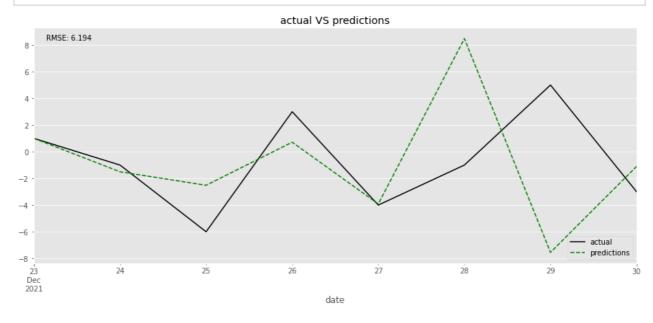
```
Epoch 87/120
Epoch 88/120
10/10 [===========] - 0s 13ms/step - loss: 14.2942
Epoch 89/120
10/10 [============== ] - 0s 13ms/step - loss: 14.4388
Epoch 90/120
10/10 [============== ] - 0s 13ms/step - loss: 13.0520
Epoch 91/120
10/10 [===============] - 0s 12ms/step - loss: 12.3095
Epoch 92/120
10/10 [============= ] - 0s 15ms/step - loss: 11.6519
Epoch 93/120
10/10 [============= ] - 0s 12ms/step - loss: 12.1931
Epoch 94/120
10/10 [============= ] - 0s 13ms/step - loss: 11.5654
Epoch 95/120
10/10 [============= ] - 0s 12ms/step - loss: 11.0613
Epoch 96/120
Epoch 97/120
Epoch 98/120
Epoch 99/120
Epoch 100/120
Epoch 101/120
Epoch 102/120
10/10 [============= ] - 0s 13ms/step - loss: 9.4551
Epoch 103/120
10/10 [============= ] - 0s 12ms/step - loss: 8.8518
Epoch 104/120
10/10 [=========== ] - 0s 13ms/step - loss: 7.7081
Epoch 105/120
10/10 [============= ] - 0s 12ms/step - loss: 7.9024
Epoch 106/120
Epoch 107/120
10/10 [============= ] - 0s 14ms/step - loss: 7.5121
Epoch 108/120
Epoch 109/120
Epoch 110/120
Epoch 111/120
Epoch 112/120
10/10 [============= ] - 0s 14ms/step - loss: 6.0639
Epoch 113/120
10/10 [============ ] - Os 13ms/step - loss: 6.1845
Epoch 114/120
10/10 [============= ] - 0s 14ms/step - loss: 6.3522
Epoch 115/120
Epoch 116/120
```

Определим признаки на входе.

```
features_input = np.array(sales_diff[:'2021-12-23'].tail(28))
features_input = features_input.reshape(1, n_steps_in, n_features)
```

Теперь вычислим RMSE на последней неделе и построим график.

```
prediction = model_vector_LSTM1.predict(features_input, verbose=0)
error = rmse(sales_diff['2021-12-24':], prediction[0])
```



Попробуем добавить ещё один слой.

```
In [136...
# определяем модель
model_vector_LSTM2 = Sequential()
model_vector_LSTM2.add(LSTM(30, activation='relu', return_sequences=True,
model_vector_LSTM2.add(LSTM(110, activation='relu'))
model_vector_LSTM2.add(Dense(n_steps_out))
model_vector_LSTM2.compile(optimizer='adam', loss='mse')
# обучаем модель
model_vector_LSTM2.fit(features, target, epochs=120, verbose=1)
```

```
Epoch 1/120
Epoch 2/120
10/10 [============== ] - 1s 50ms/step - loss: 117.8657
Epoch 3/120
Epoch 4/120
Epoch 5/120
10/10 [============== ] - 1s 52ms/step - loss: 117.7976
Epoch 6/120
10/10 [============== ] - 1s 54ms/step - loss: 117.7444
Epoch 7/120
Epoch 8/120
Epoch 9/120
Epoch 10/120
Epoch 11/120
Epoch 12/120
10/10 [============== ] - 1s 50ms/step - loss: 111.5622
Epoch 13/120
10/10 [============== ] - 1s 52ms/step - loss: 110.2635
Epoch 14/120
10/10 [============== ] - 0s 49ms/step - loss: 110.2715
Epoch 15/120
Epoch 16/120
10/10 [============== ] - 1s 51ms/step - loss: 104.7881
Epoch 17/120
Epoch 18/120
10/10 [============== ] - 1s 50ms/step - loss: 101.0126
Epoch 19/120
Epoch 20/120
Epoch 21/120
Epoch 22/120
10/10 [============= ] - 1s 51ms/step - loss: 86.5435
Epoch 23/120
10/10 [===========] - 1s 50ms/step - loss: 83.6029
Epoch 24/120
Epoch 25/120
Epoch 26/120
10/10 [============= ] - 0s 49ms/step - loss: 63.0773
Epoch 27/120
10/10 [============= ] - 1s 51ms/step - loss: 58.1042
Epoch 28/120
Epoch 29/120
Epoch 30/120
Epoch 31/120
```

```
Epoch 32/120
10/10 [============ ] - 0s 50ms/step - loss: 37.3055
Epoch 33/120
Epoch 34/120
Epoch 35/120
Epoch 36/120
10/10 [============] - 0s 49ms/step - loss: 19.1808
Epoch 37/120
10/10 [============== ] - 1s 53ms/step - loss: 17.1238
Epoch 38/120
10/10 [============= ] - 1s 51ms/step - loss: 14.3474
Epoch 39/120
10/10 [============= ] - 1s 52ms/step - loss: 12.4668
Epoch 40/120
10/10 [============== ] - 1s 50ms/step - loss: 10.1876
Epoch 41/120
10/10 [============ ] - 1s 51ms/step - loss: 8.5656
Epoch 42/120
Epoch 43/120
Epoch 44/120
Epoch 45/120
10/10 [============== ] - 1s 52ms/step - loss: 4.2996
Epoch 46/120
10/10 [============== ] - 0s 48ms/step - loss: 3.7847
Epoch 47/120
10/10 [============= ] - 1s 54ms/step - loss: 3.3046
Epoch 48/120
10/10 [============= ] - 0s 49ms/step - loss: 3.1634
Epoch 49/120
Epoch 50/120
Epoch 51/120
Epoch 52/120
10/10 [============== ] - 1s 51ms/step - loss: 1.7991
Epoch 53/120
Epoch 54/120
10/10 [============== ] - 1s 52ms/step - loss: 1.8356
Epoch 55/120
Epoch 56/120
10/10 [============== ] - 1s 51ms/step - loss: 1.4942
Epoch 57/120
Epoch 58/120
10/10 [=========== ] - 1s 51ms/step - loss: 1.3990
Epoch 59/120
10/10 [============= ] - 1s 51ms/step - loss: 1.2613
Epoch 60/120
Epoch 61/120
```

```
Epoch 62/120
10/10 [============== ] - 1s 51ms/step - loss: 1.0684
Epoch 63/120
10/10 [============ ] - 1s 53ms/step - loss: 0.9999
Epoch 64/120
Epoch 65/120
Epoch 66/120
10/10 [============= ] - 1s 50ms/step - loss: 0.7448
Epoch 67/120
10/10 [============= ] - 1s 50ms/step - loss: 0.6033
Epoch 68/120
10/10 [============= ] - 1s 52ms/step - loss: 0.6108
Epoch 69/120
Epoch 70/120
Epoch 71/120
Epoch 72/120
10/10 [============= ] - 1s 52ms/step - loss: 0.4125
Epoch 73/120
10/10 [============= ] - 1s 53ms/step - loss: 0.4298
Epoch 74/120
10/10 [============= ] - 1s 51ms/step - loss: 0.4253
Epoch 75/120
10/10 [============ ] - 1s 51ms/step - loss: 0.4598
Epoch 76/120
Epoch 77/120
10/10 [============] - 1s 53ms/step - loss: 0.4482
Epoch 78/120
Epoch 79/120
10/10 [============== ] - 1s 52ms/step - loss: 0.4135
Epoch 80/120
Epoch 81/120
Epoch 82/120
Epoch 83/120
10/10 [============= ] - 1s 53ms/step - loss: 0.3150
Epoch 84/120
10/10 [============= ] - 1s 50ms/step - loss: 0.2834
Epoch 85/120
Epoch 86/120
Epoch 87/120
Epoch 88/120
10/10 [============= ] - 1s 53ms/step - loss: 0.3116
Epoch 89/120
10/10 [============= ] - 1s 52ms/step - loss: 0.3140
Epoch 90/120
Epoch 91/120
Epoch 92/120
```

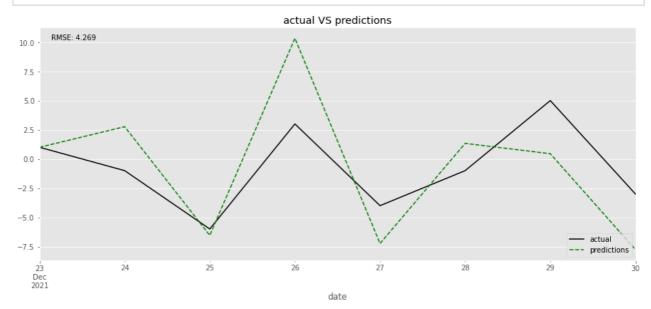
```
Epoch 93/120
10/10 [============= ] - 1s 52ms/step - loss: 0.2369
Epoch 94/120
Epoch 95/120
Epoch 96/120
Epoch 97/120
Epoch 98/120
Epoch 99/120
10/10 [============= ] - 1s 52ms/step - loss: 0.1535
Epoch 100/120
10/10 [============= ] - 1s 51ms/step - loss: 0.1648
Epoch 101/120
10/10 [============= ] - 1s 51ms/step - loss: 0.1786
Epoch 102/120
Epoch 103/120
Epoch 104/120
Epoch 105/120
Epoch 106/120
Epoch 107/120
10/10 [============== ] - 1s 52ms/step - loss: 0.1416
Epoch 108/120
10/10 [============= ] - 1s 54ms/step - loss: 0.1498
Epoch 109/120
10/10 [============= ] - 1s 52ms/step - loss: 0.1404
Epoch 110/120
Epoch 111/120
Epoch 112/120
Epoch 113/120
Epoch 114/120
Epoch 115/120
Epoch 116/120
10/10 [============= ] - 1s 53ms/step - loss: 0.1896
Epoch 117/120
10/10 [============= ] - 1s 52ms/step - loss: 0.1867
Epoch 118/120
10/10 [============= ] - 1s 54ms/step - loss: 0.1747
Epoch 119/120
Epoch 120/120
10/10 [============= ] - 1s 54ms/step - loss: 0.1537
```

Out[136... <keras.callbacks.History at 0x7ff4207c4750>

Вычислим RMSE и построим график.

```
In [137...
    prediction = model_vector_LSTM2.predict(features_input, verbose=0)
    error = rmse(sales_diff['2021-12-24':], prediction[0])
    error
```

Out[137... 4.269215704174214



Сделаем сеть сложнее.

10/10 [======

```
In [227...
         # определяем модель
         model_vector_LSTM3 = Sequential()
         model vector LSTM3.add(LSTM(30, activation='relu', return sequences=True,
         model vector LSTM3.add(LSTM(40, activation='relu', return sequences=True))
         model_vector_LSTM3.add(LSTM(70, activation='relu'))
         model vector_LSTM3.add(Dense(n_steps_out))
         model_vector_LSTM3.compile(optimizer='adam', loss='mse')
         # обучаем модель
         model_vector_LSTM3.fit(features, target, epochs=120, verbose=1)
        Epoch 1/120
                                    ======] - 60s 39ms/step - loss: 117.9580
        10/10 [=====
        Epoch 2/120
        10/10 [=====
                                 =======] - 1s 61ms/step - loss: 117.8687
        Epoch 3/120
        Epoch 4/120
                            ========= | - 1s 66ms/step - loss: 117.7844
        10/10 [=====
        Epoch 5/120
                              ========] - 1s 68ms/step - loss: 117.7048
        10/10 [=====
        Epoch 6/120
```

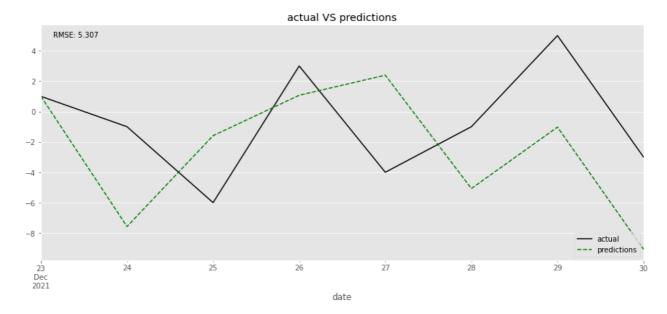
========] - 0s 49ms/step - loss: 117.5560

```
Epoch 7/120
Epoch 8/120
10/10 [============== ] - 0s 39ms/step - loss: 115.5281
Epoch 9/120
10/10 [=============== ] - 0s 39ms/step - loss: 113.7559
Epoch 10/120
10/10 [=============== ] - 0s 38ms/step - loss: 112.5152
Epoch 11/120
Epoch 12/120
10/10 [============== ] - 0s 37ms/step - loss: 109.8621
Epoch 13/120
10/10 [============= ] - 0s 40ms/step - loss: 109.6690
Epoch 14/120
Epoch 15/120
Epoch 16/120
Epoch 17/120
10/10 [============= ] - 0s 39ms/step - loss: 104.0506
Epoch 18/120
10/10 [============== ] - 0s 38ms/step - loss: 102.0928
Epoch 19/120
10/10 [============= ] - 0s 40ms/step - loss: 98.0574
Epoch 20/120
Epoch 21/120
Epoch 22/120
10/10 [===========] - 0s 38ms/step - loss: 85.2284
Epoch 23/120
10/10 [============= ] - 0s 39ms/step - loss: 80.2330
Epoch 24/120
Epoch 25/120
Epoch 26/120
Epoch 27/120
10/10 [============== ] - 0s 38ms/step - loss: 66.0530
Epoch 28/120
Epoch 29/120
10/10 [============] - 0s 39ms/step - loss: 52.4716
Epoch 30/120
Epoch 31/120
10/10 [============] - 0s 39ms/step - loss: 45.0525
Epoch 32/120
10/10 [============= ] - 0s 39ms/step - loss: 42.1776
Epoch 33/120
10/10 [============= ] - 0s 38ms/step - loss: 40.4676
Epoch 34/120
10/10 [============= ] - 0s 39ms/step - loss: 39.3530
Epoch 35/120
10/10 [=============== ] - 0s 41ms/step - loss: 35.9011
Epoch 36/120
10/10 [=============== ] - 0s 39ms/step - loss: 31.3137
Epoch 37/120
```

10/10	[======================================] -	0s	40ms/step	- loss	: 30.2402
	38/120	,	0~	20	1	. 26 0527
	[=====================================] -	US	38ms/step	- loss	: 26.0537
	[======================================] -	0s	39ms/step	- loss	: 23.2370
	40/120		•	20 / 1	1	01 0500
	[=====================================] -	0s	38ms/step	- loss	: 21.0588
	[======================================] -	0s	38ms/step	- loss	: 19.4086
	42/120		0 -	4.0	1	10 6250
	[=====================================] -	US	40ms/step	- IOSS	: 18.6250
10/10	[======================================] -	0s	38ms/step	- loss	: 17.4421
	44/120 [====================================	1	٥٥	30mg/g+op	logg	• 1/ 6262
	45/120] _	US	Johns, aceb	- 1055	. 14.0205
] –	0s	40ms/step	- loss	: 13.4950
	46/120 [====================================	1 _	Λs	39ms/sten	_ loss	• 12.8061
	47/120] _	0.5	37m3/5ccp	- 1055	. 12.0001
	[======================================] –	0s	39ms/step	- loss	: 12.4124
-	48/120 [====================================	1 -	0s	38ms/step	- loss	: 10.3351
Epoch	49/120	_		_		
	[=====================================] –	0s	38ms/step	- loss	: 10.3395
_	[======================================	1 -	0s	39ms/step	- loss	: 10.0022
Epoch	51/120	_		_		
	[=====================================] –	0s	39ms/step	- loss	: 9.6548
	[======================================] -	0s	39ms/step	- loss	: 9.2312
	53/120				-	
	[=====================================] -	0s	38ms/step	- loss	: 9.4/80
10/10	[======================================] -	0s	38ms/step	- loss	: 7.6084
	55/120 [====================================	,	0~	20/	1	
	56/120] -	US	39ms/step	- 1088	: 0.4102
10/10	[======================================] -	0s	38ms/step	- loss	: 5.9601
	57/120 [====================================	1 _	Λq	40mg/gten	_ 1099	• 5 3177
	58/120	1 _	VS	40ms/scep	- 1055	. 3.31//
	[======================================] –	0s	43ms/step	- loss	: 4.9244
	59/120 [====================================	1 –	0s	37ms/step	- loss	: 4.6874
Epoch	60/120					
	[=====================================] –	0s	39ms/step	- loss	: 4.9824
	[======================================] -	0s	40ms/step	- loss	: 4.3149
Epoch	62/120					
	[=====================================] –	0s	39ms/step	- loss	: 3.9703
	[======================================] -	0s	40ms/step	- loss	: 3.3336
	64/120		•	20 / 1	1	2 0276
	[=====================================] -	US	39ms/step	- loss	: 3.03/6
10/10	[======================================] –	0s	40ms/step	- loss	: 2.9737
	66/120 [====================================	1	0.5	38mg/g+05	_ logg	• 2 6272
	67/120	, –	UB	Jours' sceb	TOSS	• 4.03/4
10/10	[======================================] –	0s	38ms/step	- loss	: 2.7282

```
Epoch 68/120
Epoch 69/120
10/10 [============ ] - Os 39ms/step - loss: 2.5122
Epoch 70/120
Epoch 71/120
Epoch 72/120
10/10 [============== ] - 1s 59ms/step - loss: 2.1749
Epoch 73/120
Epoch 74/120
10/10 [============ ] - 1s 63ms/step - loss: 2.0725
Epoch 75/120
Epoch 76/120
Epoch 77/120
Epoch 78/120
10/10 [============= ] - 1s 64ms/step - loss: 1.4845
Epoch 79/120
Epoch 80/120
10/10 [============= ] - 1s 63ms/step - loss: 1.4213
Epoch 81/120
10/10 [============] - Os 47ms/step - loss: 1.4175
Epoch 82/120
10/10 [============= ] - 0s 39ms/step - loss: 1.2052
Epoch 83/120
10/10 [============ ] - Os 39ms/step - loss: 1.2091
Epoch 84/120
10/10 [============== ] - 0s 39ms/step - loss: 1.1364
Epoch 85/120
Epoch 86/120
Epoch 87/120
Epoch 88/120
10/10 [============= ] - 0s 39ms/step - loss: 1.1746
Epoch 89/120
10/10 [============= ] - 0s 39ms/step - loss: 1.1518
Epoch 90/120
10/10 [============== ] - 0s 40ms/step - loss: 1.0860
Epoch 91/120
Epoch 92/120
Epoch 93/120
Epoch 94/120
10/10 [============= ] - 0s 40ms/step - loss: 0.6478
Epoch 95/120
10/10 [============ ] - 0s 38ms/step - loss: 0.6337
Epoch 96/120
Epoch 97/120
Epoch 98/120
```

```
10/10 [============= ] - 0s 40ms/step - loss: 0.8320
     Epoch 99/120
     10/10 [============= ] - 1s 60ms/step - loss: 0.7686
    Epoch 100/120
     Epoch 101/120
     Epoch 102/120
     Epoch 103/120
     Epoch 104/120
     10/10 [============= ] - 1s 63ms/step - loss: 0.4644
     Epoch 105/120
     10/10 [============= ] - 1s 69ms/step - loss: 0.4825
     Epoch 106/120
     10/10 [============= ] - 1s 67ms/step - loss: 0.5356
    Epoch 107/120
     Epoch 108/120
     10/10 [============ ] - 0s 39ms/step - loss: 0.5431
     Epoch 109/120
     Epoch 110/120
     Epoch 111/120
     Epoch 112/120
     Epoch 113/120
     Epoch 114/120
     10/10 [============= ] - 0s 38ms/step - loss: 0.8958
    Epoch 115/120
     10/10 [============ ] - 0s 38ms/step - loss: 0.6970
    Epoch 116/120
     Epoch 117/120
     Epoch 118/120
     Epoch 119/120
     10/10 [============= ] - 1s 64ms/step - loss: 0.3911
    Epoch 120/120
    10/10 [============== ] - 1s 63ms/step - loss: 0.3344
Out[227... <keras.callbacks.History at 0x7ff3fe3c7850>
In [228...
     prediction = model vector LSTM3.predict(features input, verbose=0)
     error = rmse(sales diff['2021-12-24':], prediction[0])
     error
Out[228... 5.306878018027217
In [229...
     plot_result(8, error,
           sales diff['2021-12-23':],
           np.concatenate(([sales diff['2021-12-23']], prediction[0])))
```



Попробуем другое количество нейронов в скрытых слоях.

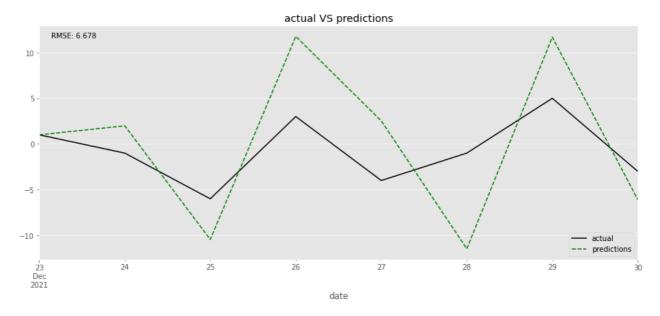
```
In [171...
       # определяем модель
       model vector LSTM4 = Sequential()
       model_vector_LSTM4.add(LSTM(28, activation='relu', return_sequences=True,
       model_vector_LSTM4.add(LSTM(35, activation='relu', return_sequences=True))
       model vector LSTM4.add(LSTM(77, activation='relu'))
       model_vector_LSTM4.add(Dense(n_steps_out))
       model vector LSTM4.compile(optimizer='adam', loss='mse')
       # обучаем модель
       model vector LSTM4.fit(features, target, epochs=120, verbose=1)
       Epoch 1/120
       10/10 [======
                       ========= ] - 5s 40ms/step - loss: 116.2020
       Epoch 2/120
       10/10 [=======
                     ========== ] - 0s 40ms/step - loss: 116.1203
       Epoch 3/120
                       ========= ] - 0s 38ms/step - loss: 116.1011
       10/10 [=======
       Epoch 4/120
       10/10 [============== ] - 0s 39ms/step - loss: 116.0937
       Epoch 5/120
       10/10 [============== ] - 0s 38ms/step - loss: 116.0711
       Epoch 6/120
       Epoch 7/120
       10/10 [================] - 0s 39ms/step - loss: 115.9756
       Epoch 8/120
       Epoch 9/120
       Epoch 10/120
       10/10 [======
                      ========= ] - 0s 38ms/step - loss: 115.4903
       Epoch 11/120
       10/10 [=======
                     ========== | - 0s 40ms/step - loss: 114.5592
       Epoch 12/120
       Epoch 13/120
       10/10 [=============== ] - 0s 39ms/step - loss: 111.8360
       Epoch 14/120
```

	[=====]	_	0s	40ms/step - loss:	109.5695
	15/120 [=======]	_	0 s	39ms/sten - loss:	108.5019
Epoch	16/120				
	[=======]	-	0s	39ms/step - loss:	107.0675
	17/120 [=======]	_	0s	39ms/step - loss:	104.4746
Epoch	18/120				
	[======] 19/120	-	0s	38ms/step - loss:	102.5650
	[========]	_	0s	39ms/step - loss:	104.1136
Epoch	20/120				
	[======] 21/120	-	0s	3/ms/step - loss:	99.96/0
10/10	[======]	_	0s	38ms/step - loss:	97.5387
	22/120 [=======]		٥٥	20mg/g+on logg.	05 4720
	23/120	_	US	39ms/scep - 10ss:	93.4739
	[======]	-	0s	38ms/step - loss:	95.3993
	24/120 [=======]	_	0s	39ms/step - loss:	91.4840
Epoch	25/120				
	[======] 26/120	-	0s	38ms/step - loss:	88.3904
10/10	[======]	_	0s	40ms/step - loss:	83.7082
	27/120 [======]		٥٥	20mg/g+on logg.	70 0705
	28/120	_	05	Johns/Step - 1055:	19.0193
	[======]	-	0s	39ms/step - loss:	73.8353
	29/120 [=======]	_	0s	40ms/step - loss:	70.5298
Epoch	30/120				
	[======] 31/120	-	0s	38ms/step - loss:	65.6685
	[========]	_	0s	37ms/step - loss:	58.8875
_	32/120 [=======]		٥٥	20mg/g+on logg.	E2 010E
	33/120	_	US	39ms/scep - 10ss:	JZ • 010J
	[========]	-	0s	38ms/step - loss:	52.8601
	34/120 [=======]	_	0s	37ms/step - loss:	46.5802
Epoch	35/120				
	[======] 36/120	-	0s	39ms/step - loss:	43.1144
10/10	[======]	_	0s	37ms/step - loss:	36.5833
	37/120 [======]	_	۸e	37mg/gten _ logg.	34 8358
Epoch	38/120				
	[=========]	-	0s	39ms/step - loss:	30.5373
	39/120 [=======]	_	0s	38ms/step - loss:	26.6086
Epoch	40/120				
	[======] 41/120	-	0s	39ms/step - loss:	28.0625
10/10	[======]	_	0s	38ms/step - loss:	28.7818
	42/120 [======]	_	Λe	38mg/g+en - logg.	23 0315
Epoch	43/120				
	[========]	-	0s	41ms/step - loss:	19.8876
	44/120 [=======]	_	0s	38ms/step - loss:	17.5934
			-	_	-

```
Epoch 45/120
10/10 [============== ] - 0s 39ms/step - loss: 17.0312
Epoch 46/120
10/10 [============== ] - 0s 37ms/step - loss: 16.4450
Epoch 47/120
Epoch 48/120
10/10 [============== ] - 0s 39ms/step - loss: 12.7773
Epoch 49/120
10/10 [============] - 0s 37ms/step - loss: 10.3710
Epoch 50/120
10/10 [============= ] - 0s 37ms/step - loss: 9.1595
Epoch 51/120
10/10 [============= ] - 0s 39ms/step - loss: 8.6687
Epoch 52/120
Epoch 53/120
Epoch 54/120
Epoch 55/120
10/10 [============ ] - 0s 38ms/step - loss: 5.6149
Epoch 56/120
10/10 [============= ] - 0s 38ms/step - loss: 5.0183
Epoch 57/120
10/10 [============= ] - 0s 38ms/step - loss: 4.6588
Epoch 58/120
10/10 [============= ] - 0s 38ms/step - loss: 4.5579
Epoch 59/120
Epoch 60/120
10/10 [============== ] - 1s 80ms/step - loss: 3.8518
Epoch 61/120
Epoch 62/120
Epoch 63/120
Epoch 64/120
Epoch 65/120
10/10 [============= ] - 0s 37ms/step - loss: 2.9877
Epoch 66/120
10/10 [============= ] - 0s 38ms/step - loss: 2.5778
Epoch 67/120
Epoch 68/120
Epoch 69/120
Epoch 70/120
Epoch 71/120
10/10 [============= ] - 0s 38ms/step - loss: 2.2927
Epoch 72/120
Epoch 73/120
Epoch 74/120
Epoch 75/120
```

10/10	[=====]	_	0s	38ms/step -	loss:	1.7363
	76/120 [=======]		٥٩	20mg/g+on	1000.	1 6746
	77/120	_	US	Joms/scep -	1055;	1.0/40
	[======]	-	0s	38ms/step -	loss:	1.5223
	78/120 [======]	_	0s	39ms/step -	loss:	1.5388
Epoch	79/120					
	[======] 80/120	-	0s	39ms/step -	loss:	1.5899
	[=======]	_	0s	40ms/step -	loss:	1.5360
	81/120		0 -	20/	1	1 4100
	[======] 82/120	-	US	38ms/step -	loss:	1.4189
10/10	[======]	-	0s	37ms/step -	loss:	1.2384
	83/120 [=======]	_	Λq	39ms/sten -	10881	1 1527
Epoch	84/120					
	[======================================	-	0s	38ms/step -	loss:	1.0600
	85/120 [=======]	_	0s	37ms/step -	loss:	1.0120
Epoch	86/120					
	[======] 87/120	-	0s	38ms/step -	loss:	1.0523
	[========]	_	0s	38ms/step -	loss:	0.9767
	88/120 [======]		٥٩	20mg/gton	1000	1 0210
	89/120	_	US	39MS/Step -	TOSS:	1.0218
	[======]	-	0s	38ms/step -	loss:	1.0077
	90/120	_	0s	37ms/step -	loss:	1.0024
Epoch	91/120			_		
	[======] 92/120	-	0s	38ms/step -	loss:	0.9058
	[=======]	_	0s	38ms/step -	loss:	0.8355
	93/120		0 -	20/	1	0 0614
	[======] 94/120	_	US	38ms/step -	loss:	0.8614
10/10	[======]	-	0s	37ms/step -	loss:	1.0171
	95/120 [=======]	_	0s	37ms/step -	loss:	0.9408
Epoch	96/120					
	[======] 97/120	-	0s	38ms/step -	loss:	0.8914
	[=======]	_	0s	38ms/step -	loss:	0.9055
	98/120		0 -	20/	1	0 0016
	[======] 99/120	_	US	38ms/step -	loss:	0.8216
10/10	[======]	-	0s	39ms/step -	loss:	0.8212
	100/120 [=======]	_	Λs	38ms/sten -	1055:	0.8751
Epoch	101/120					
	[=========]	-	0s	39ms/step -	loss:	0.8363
	102/120 [=======]	_	0s	38ms/step -	loss:	0.8568
Epoch	103/120					
	[======] 104/120	-	υs	3/ms/step -	loss:	U.8664
10/10	[======]	_	0s	39ms/step -	loss:	0.9883
	105/120 [=======]		0.5	37mg/g+0n	logg•	0 9107
10/10	[]	_	US	J/ma/acep -	TODD:	0.910/

```
Epoch 106/120
      Epoch 107/120
      10/10 [============= ] - 0s 39ms/step - loss: 0.7476
      Epoch 108/120
      Epoch 109/120
      Epoch 110/120
      10/10 [============= ] - 0s 38ms/step - loss: 0.6228
      Epoch 111/120
      Epoch 112/120
      10/10 [============ ] - 0s 39ms/step - loss: 0.6094
      Epoch 113/120
      Epoch 114/120
      10/10 [============== ] - 0s 38ms/step - loss: 0.5997
      Epoch 115/120
      10/10 [============= ] - 0s 38ms/step - loss: 0.5521
      Epoch 116/120
      10/10 [============ ] - 0s 38ms/step - loss: 0.5013
      Epoch 117/120
      10/10 [============= ] - 0s 40ms/step - loss: 0.5102
      Epoch 118/120
      Epoch 119/120
      10/10 [============== ] - 0s 37ms/step - loss: 0.5940
      Epoch 120/120
      10/10 [============== ] - 0s 38ms/step - loss: 0.6495
Out[171... <keras.callbacks.History at 0x7ff413056d10>
In [172...
      prediction = model_vector_LSTM4.predict(features_input, verbose=0)
      error = rmse(sales_diff['2021-12-24':], prediction[0])
      error
Out[172... 6.677857061958654
In [173...
      plot_result(8, error,
              sales diff['2021-12-23':],
              np.concatenate(([sales diff['2021-12-23']], prediction[0])))
```



```
In [189...
# определяем модель
model_vector_LSTM5 = Sequential()
model_vector_LSTM5.add(LSTM(14, activation='relu', return_sequences=True,
model_vector_LSTM5.add(LSTM(28, activation='relu', return_sequences=True))
model_vector_LSTM5.add(LSTM(70, activation='relu'))
model_vector_LSTM5.add(Dense(n_steps_out))
model_vector_LSTM5.compile(optimizer='adam', loss='mse')
# обучаем модель
model_vector_LSTM5.fit(features, target, epochs=120, verbose=1)
```

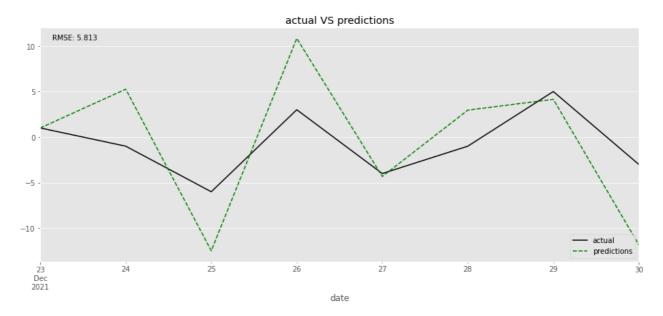
```
Epoch 1/120
                =======] - 4s 101ms/step - loss: 116.3946
10/10 [=====
Epoch 2/120
10/10 [=====
                ========] - 0s 47ms/step - loss: 116.1132
Epoch 3/120
             ========= ] - 0s 35ms/step - loss: 116.0882
10/10 [======
Epoch 4/120
10/10 [============== ] - 0s 37ms/step - loss: 116.0689
Epoch 5/120
Epoch 6/120
10/10 [======
             ======== ] - 0s 36ms/step - loss: 116.0309
Epoch 7/120
10/10 [======
           Epoch 8/120
10/10 [======
           ======== | - 0s 36ms/step - loss: 116.0042
Epoch 9/120
10/10 [================ ] - 0s 37ms/step - loss: 116.0049
Epoch 10/120
Epoch 11/120
           ========== | - Os 36ms/step - loss: 115.8479
10/10 [=======
Epoch 12/120
Epoch 13/120
10/10 [=============== ] - 0s 36ms/step - loss: 114.4861
Epoch 14/120
Epoch 15/120
```

```
10/10 [============== ] - 0s 37ms/step - loss: 112.8295
Epoch 16/120
10/10 [============= ] - 0s 35ms/step - loss: 111.5599
Epoch 17/120
10/10 [============== ] - 0s 36ms/step - loss: 109.8232
Epoch 18/120
10/10 [=============== ] - 0s 35ms/step - loss: 108.1963
Epoch 19/120
Epoch 20/120
Epoch 21/120
10/10 [============== ] - 0s 35ms/step - loss: 102.3773
Epoch 22/120
10/10 [============== ] - 0s 35ms/step - loss: 101.0084
Epoch 23/120
10/10 [============== ] - 0s 51ms/step - loss: 100.4267
Epoch 24/120
10/10 [============== ] - 0s 35ms/step - loss: 102.3352
Epoch 25/120
10/10 [============ ] - 0s 35ms/step - loss: 98.2563
Epoch 26/120
10/10 [============== ] - 0s 37ms/step - loss: 94.9799
Epoch 27/120
10/10 [============= ] - 0s 35ms/step - loss: 91.5185
Epoch 28/120
10/10 [============== ] - 0s 36ms/step - loss: 88.6491
Epoch 29/120
Epoch 30/120
10/10 [============== ] - 0s 36ms/step - loss: 83.0975
Epoch 31/120
10/10 [============ ] - 0s 35ms/step - loss: 79.7388
Epoch 32/120
Epoch 33/120
Epoch 34/120
10/10 [============== ] - 0s 37ms/step - loss: 69.9199
Epoch 35/120
Epoch 36/120
Epoch 37/120
10/10 [============= ] - 0s 36ms/step - loss: 61.1817
Epoch 38/120
Epoch 39/120
Epoch 40/120
10/10 [===============] - 0s 36ms/step - loss: 51.2740
Epoch 41/120
10/10 [============= ] - 0s 36ms/step - loss: 47.3602
Epoch 42/120
10/10 [============= ] - 0s 35ms/step - loss: 47.4027
Epoch 43/120
10/10 [============= ] - 0s 36ms/step - loss: 43.0608
Epoch 44/120
10/10 [============== ] - 0s 35ms/step - loss: 39.9905
Epoch 45/120
```

```
Epoch 46/120
10/10 [============== ] - 0s 37ms/step - loss: 32.4780
Epoch 47/120
10/10 [============ ] - 0s 36ms/step - loss: 29.2614
Epoch 48/120
10/10 [============== ] - 0s 37ms/step - loss: 26.0020
Epoch 49/120
10/10 [============== ] - 0s 36ms/step - loss: 27.1807
Epoch 50/120
10/10 [============] - 0s 35ms/step - loss: 27.2109
Epoch 51/120
10/10 [============= ] - 0s 36ms/step - loss: 24.7144
Epoch 52/120
10/10 [============ ] - 0s 35ms/step - loss: 20.6574
Epoch 53/120
Epoch 54/120
Epoch 55/120
10/10 [============== ] - 0s 36ms/step - loss: 14.8268
Epoch 56/120
Epoch 57/120
10/10 [============ ] - 0s 36ms/step - loss: 13.2235
Epoch 58/120
10/10 [============= ] - 0s 36ms/step - loss: 13.7518
Epoch 59/120
Epoch 60/120
Epoch 61/120
10/10 [============ ] - Os 35ms/step - loss: 9.3647
Epoch 62/120
Epoch 63/120
Epoch 64/120
Epoch 65/120
Epoch 66/120
10/10 [============= ] - 0s 35ms/step - loss: 5.4985
Epoch 67/120
10/10 [============ ] - 0s 35ms/step - loss: 4.9136
Epoch 68/120
10/10 [============= ] - 0s 36ms/step - loss: 4.5837
Epoch 69/120
10/10 [============== ] - 0s 36ms/step - loss: 4.2499
Epoch 70/120
Epoch 71/120
Epoch 72/120
10/10 [============= ] - 0s 34ms/step - loss: 3.5006
Epoch 73/120
10/10 [============= ] - 0s 35ms/step - loss: 3.4418
Epoch 74/120
Epoch 75/120
Epoch 76/120
```

	[=======]	-	0s	35ms/step - 1	oss:	2.7613
10/10	77/120	-	0s	36ms/step - 1	oss:	2.5754
	78/120 [=======]	_	0s	36ms/step - 1	oss:	2.4391
	79/120 [======]	_	0s	36ms/step - 1	loss:	2.2936
Epoch	80/120 [======]					
Epoch	81/120					
Epoch	[======] 82/120					
	[======] 83/120	-	0s	37ms/step - 1	.oss:	2.4388
	[=======] 84/120	-	0s	36ms/step - 1	.oss:	2.1180
10/10	[======]	-	0s	36ms/step - 1	oss:	2.0554
10/10	85/120 [======]	_	0s	36ms/step - 1	oss:	1.9726
	86/120 [=======]	_	0s	35ms/step - 1	Loss:	1.9965
Epoch	87/120 [======]					
Epoch	88/120 [=======]					
Epoch	89/120					
Epoch	[======] 90/120					
	[======] 91/120	-	0s	36ms/step - 1	oss:	1.4807
10/10	[=======] 92/120	-	0s	36ms/step - 1	.oss:	1.4167
10/10	[======]	-	0s	36ms/step - 1	.oss:	1.4708
10/10	93/120 [======]	_	0s	35ms/step - 1	oss:	1.4335
	94/120 [=======]	_	0s	35ms/step - 1	Loss:	1.3258
Epoch	95/120 [======]					
Epoch	96/120					
Epoch	[=======] 97/120					
	[======] 98/120	-	0s	37ms/step - 1	oss:	1.1334
	[======] 99/120	-	0s	35ms/step - 1	.oss:	1.1592
10/10	[=======] 100/120	-	0s	37ms/step - 1	.oss:	1.1860
10/10	[======]	-	0s	36ms/step - 1	oss:	1.0935
	101/120 [=======]	_	0s	37ms/step - 1	oss:	1.0751
	102/120 [======]	_	0s	38ms/step - 1	Loss:	1.0087
Epoch	103/120 [======]					
Epoch	104/120					
Epoch	[=======] 105/120					
	[=======] 106/120	-	0s	37ms/step - 1	oss:	0.9001
	[======]	-	0s	36ms/step - 1	oss:	0.8590

```
Epoch 107/120
      Epoch 108/120
      10/10 [============ ] - 0s 36ms/step - loss: 0.7274
      Epoch 109/120
      Epoch 110/120
      Epoch 111/120
      10/10 [============ ] - 0s 36ms/step - loss: 0.7059
      Epoch 112/120
      10/10 [============ ] - 0s 35ms/step - loss: 0.7210
      Epoch 113/120
      10/10 [============= ] - 0s 35ms/step - loss: 0.6432
      Epoch 114/120
      Epoch 115/120
      10/10 [============== ] - 0s 35ms/step - loss: 0.5776
      Epoch 116/120
      Epoch 117/120
      10/10 [============= ] - 0s 35ms/step - loss: 0.6211
      Epoch 118/120
      10/10 [============= ] - 0s 35ms/step - loss: 0.6477
      Epoch 119/120
      Epoch 120/120
      10/10 [============= ] - 0s 35ms/step - loss: 0.7071
Out[189... <keras.callbacks.History at 0x7ff410141950>
In [190...
      prediction = model_vector_LSTM5.predict(features_input, verbose=0)
      error = rmse(sales_diff['2021-12-24':], prediction[0])
      error
Out[190... 5.81318634524251
In [191...
      plot result(8, error,
              sales_diff['2021-12-23':],
              np.concatenate(([sales_diff['2021-12-23']], prediction[0])))
```



Лучшей из опробованных конфигураций оказалась конфигурация модели model_vector_LSTM2 .

ИТОГ

Мы построили несколько моделей, однако последний тип модели многошаговой LSTM отличается от предыдущих тем, что она возвращает прогноз на ближайшие 7 дней вперед. В предыдущих моделях чтобы повысить качество предсказаний, мы делали одношаговые прогнозы и обучали модель на каждом шаге с учётом последнего реального значения. Таким образом предсказание не далеко отклонялось от реальных данных. Поэтому модель SARIMA имеет лучшее качество в сравнении с остальными моделями.

Однако если мы проведем симуляцию, подставляя последние предсказанные значения на вход модели, то за несколько шагов SARIMA сведётся к предсказанию последним значением.

Поэтому более интересным является многошаговый прогноз LSTM. К тому же RMSE при валидации не слишком сильно уступает модели SARIMA, и выше, чем у градиентного бустинга и одношаговых LSTM-моделей.

Конечно, мы перебрали лишь некоторые модели LSTM. И если бы мы многократно запускали симуляцию одной и тоже конфигурации модели без фиксации random state, то можно было бы с большей уверенностью судить о качестве той или иной нейросети.

Обучим лучшую модель заново с учётом последней недели и сделаем прогноз.

```
In [230... n_steps_in, n_steps_out = 28, 7
features_, target_ = split_sequence(sales_diff, n_steps_in, n_steps_out)
```

```
In [231...
     # определяем модель
     model_vector_LSTM = Sequential()
     model_vector_LSTM.add(LSTM(30, activation='relu', return_sequences=True, in
     model vector LSTM.add(LSTM(110, activation='relu'))
     model vector LSTM.add(Dense(n steps out))
     model vector LSTM.compile(optimizer='adam', loss='mse')
     # обучаем модель
     model vector LSTM.fit(features , target , epochs=120, verbose=1)
    Epoch 1/120
    Epoch 2/120
    Epoch 3/120
    10/10 [============= ] - 0s 34ms/step - loss: 114.6666
    Epoch 4/120
    Epoch 5/120
    10/10 [============== ] - 0s 34ms/step - loss: 112.5899
    Epoch 6/120
    10/10 [============= ] - 0s 35ms/step - loss: 111.6240
    Epoch 7/120
    Epoch 8/120
    Epoch 9/120
    Epoch 10/120
    Epoch 11/120
    Epoch 12/120
    10/10 [============== ] - 0s 34ms/step - loss: 104.5637
    Epoch 13/120
    Epoch 14/120
    Epoch 15/120
    Epoch 16/120
    10/10 [============= ] - 0s 33ms/step - loss: 91.2400
    Epoch 17/120
    Epoch 18/120
    10/10 [=============== ] - 0s 35ms/step - loss: 84.2525
    Epoch 19/120
    10/10 [============== ] - 0s 33ms/step - loss: 77.0389
    Epoch 20/120
    10/10 [============== ] - 0s 33ms/step - loss: 71.5628
    Epoch 21/120
    10/10 [============= ] - 0s 35ms/step - loss: 75.4126
    Epoch 22/120
    Epoch 23/120
    10/10 [=============== ] - 0s 33ms/step - loss: 59.8477
    Epoch 24/120
    Epoch 25/120
```

```
Epoch 26/120
10/10 [============== ] - 0s 33ms/step - loss: 43.3999
Epoch 27/120
10/10 [============== ] - 0s 34ms/step - loss: 38.0629
Epoch 28/120
10/10 [============= ] - 0s 33ms/step - loss: 34.5020
Epoch 29/120
10/10 [============== ] - 0s 33ms/step - loss: 31.2430
Epoch 30/120
10/10 [===============] - 0s 34ms/step - loss: 27.2206
Epoch 31/120
10/10 [============ ] - 0s 33ms/step - loss: 23.9380
Epoch 32/120
10/10 [============= ] - 0s 34ms/step - loss: 20.0926
Epoch 33/120
Epoch 34/120
Epoch 35/120
10/10 [============== ] - 0s 33ms/step - loss: 14.7176
Epoch 36/120
Epoch 37/120
10/10 [============== ] - 0s 33ms/step - loss: 10.8718
Epoch 38/120
Epoch 39/120
10/10 [============ ] - Os 32ms/step - loss: 8.5725
Epoch 40/120
Epoch 41/120
Epoch 42/120
Epoch 43/120
10/10 [============= ] - 0s 32ms/step - loss: 5.3002
Epoch 44/120
Epoch 45/120
Epoch 46/120
10/10 [============= ] - 0s 33ms/step - loss: 3.5410
Epoch 47/120
Epoch 48/120
10/10 [============= ] - 0s 33ms/step - loss: 3.1475
Epoch 49/120
Epoch 50/120
Epoch 51/120
10/10 [============= ] - 0s 35ms/step - loss: 2.4342
Epoch 52/120
10/10 [============== ] - 0s 34ms/step - loss: 2.3043
Epoch 53/120
10/10 [============= ] - 0s 33ms/step - loss: 2.1700
Epoch 54/120
10/10 [=============== ] - 0s 33ms/step - loss: 2.1247
Epoch 55/120
Epoch 56/120
```

10/10	[=====]	_	0s	33ms/step -	loss:	2.0398
	57/120]		٥٥	2/mg/g+on	1000.	1 6561
	58/120		_	05	34ms/scep -	1055:	1.0301
]	-	0s	34ms/step -	loss:	1.5426
	59/120 [=====]	_	0s	34ms/step -	loss:	1.3707
Epoch	60/120						
	[====== 61/120]	-	0s	34ms/step -	loss:	1.3892
]	_	0s	34ms/step -	loss:	1.2964
	62/120]		٥٥	2/mg/g+on	1000.	1 2//7
	63/120		_	05	34ms/scep -	1055:	1.344/
		1	_	0s	34ms/step -	loss:	1.2816
	64/120 [=====]	_	0s	34ms/step -	loss:	1.0559
Epoch	65/120						
	[====== 66/120]	-	0s	34ms/step -	loss:	0.9683
10/10	[=====]	_	0s	33ms/step -	loss:	0.7891
	67/120]		Λσ	33mg/gten	1055	0 8444
Epoch	68/120						
		======]	-	0s	33ms/step -	loss:	0.8079
	69/120 [=====]	_	0s	33ms/step -	loss:	0.7483
Epoch	70/120						
	[====== 71/120]	-	0s	33ms/step -	loss:	0.6787
10/10	[=====]	_	0s	34ms/step -	loss:	0.5790
	72/120]	_	Λe	33mg/gten -	10881	0 5723
Epoch	73/120	_			_		
	[====== 74/120]	-	0s	33ms/step -	loss:	0.5698
]	_	0s	34ms/step -	loss:	0.4906
	75/120			0 -	24/	1	0 4505
	76/120]	_	US	34ms/step -	loss:	0.4595
]	-	0s	34ms/step -	loss:	0.4376
	77/120 [=====]	_	0s	33ms/step -	loss:	0.4249
Epoch	78/120						
	[====== 79/120]	-	0s	33ms/step -	loss:	0.4420
10/10	[=====]	_	0s	35ms/step -	loss:	0.3854
	80/120]	_	Λς	33mg/sten -	10991	0 4110
Epoch	81/120						
	[====== 82/120]	-	0s	34ms/step -	loss:	0.4146
]	_	0s	33ms/step -	loss:	0.4272
	83/120			0 -	22/	1	0 4450
	84/120]	-	US	ısıms/step −	TOSS:	0.4452
10/10	[=====]	-	0s	34ms/step -	loss:	0.4749
	85/120 [=====]	_	0s	34ms/step -	loss:	0.4550
Epoch	86/120	_			_		
10/10	[=====]	-	0s	34ms/step -	loss:	0.5445

Encah	07/120				
	87/120 [=======]	_	0s	33ms/step - loss: 0	0.5144
Epoch	88/120				
	[=======] 89/120	-	0s	34ms/step - loss: 0	0.4569
	[=========]	_	0s	33ms/step - loss: 0	0.4183
Epoch	90/120			_	
	[======] 91/120	-	0s	33ms/step - loss: 0	0.3752
	[========]	_	0s	35ms/step - loss: 0	0.3488
	92/120		•		
	[======] 93/120	-	0s	33ms/step - loss: (0.3552
10/10	[======]	_	0s	33ms/step - loss: 0	0.4297
	94/120 [=======]		٥٥	2/mg/g+on logg. (1002
	95/120	_	US	34ms/scep - 10ss: (7.4603
	[=====]	_	0s	34ms/step - loss: 0	0.4622
	96/120 [=======]	_	0s	33ms/step = loss: (0.4457
Epoch	97/120				
	[======] 98/120	-	0s	33ms/step - loss: 0	0.4226
	[=========]	_	0s	33ms/step - loss: 0	0.4261
Epoch	99/120				
	[=======] 100/120	-	0s	32ms/step - loss: (0.5144
10/10	[======]	_	0s	34ms/step - loss: 0	0.4760
	101/120 [=======]		٥٥	22mg/g+on logg. (1262
	102/120	_	US	33ms/scep - 10ss. (7.4202
	[======================================	-	0s	32ms/step - loss: 0	0.5867
	103/120 [=======]	_	0s	34ms/step - loss: 0	0.4936
-	104/120				
	[=======] 105/120	-	0s	33ms/step - loss: (0.42/2
10/10	[======]	-	0s	34ms/step - loss: 0	0.4115
	106/120 [=======]	_	Λe	34mg/sten _ loss. (1 3365
Epoch	107/120			_	
	[=========]	-	0s	34ms/step - loss: 0	0.2841
	108/120 [=======]	_	0s	33ms/step - loss: 0	0.2822
Epoch	109/120			_	
	[======] 110/120	-	0s	34ms/step - loss: 0	0.2684
10/10	[======]	_	0s	33ms/step - loss: 0	0.2501
	111/120 [=======]		0 a	24mg/gton logg. (2170
	112/120	_	US	34ms/step - 10ss: (J. 21/8
	[======]	-	0s	35ms/step - loss: 0	0.1791
	113/120 [=======]	_	0s	33ms/step = loss: (0.1508
Epoch	114/120			_	
	[======] 115/120	-	0s	33ms/step - loss: 0	0.1424
	[========]	_	0s	34ms/step - loss: 0	0.1268
	116/120		0 =	24mg/g+ 1 0	1260
	[======] 117/120	-	US	Jams/step - loss: (0.1300
-					

```
10/10 [============= ] - 0s 33ms/step - loss: 0.1305
       Epoch 118/120
       10/10 [============= ] - 0s 35ms/step - loss: 0.1401
       Epoch 119/120
       Epoch 120/120
       10/10 [============= ] - 0s 33ms/step - loss: 0.1384
Out[231... <keras.callbacks.History at 0x7ff406763f50>
In [232...
        features input = np.array(sales diff.tail(28))
        features_input_ = features_input_.reshape(1, n_steps_in, n_features)
In [233...
        forecast = model_vector_LSTM.predict(features_input_, verbose=0)
       Итак, прогноз разностей продаж на неделю вперёд исходя из данных о разностях
```

за последние 4 недели.

```
In [234...
         forecast[0]
Out[234... array([-2.7339063, 1.3911389, -1.0647684, 3.6626263, -5.723689,
                 4.9648724, -3.0692432], dtype=float32)
```

Данные о суммарном количестве продаж за день хранятся в сирии sales q.

```
In [235...
          sales q.tail()
Out[235... date
         2021-12-26
                      11
         2021-12-27
                        7
         2021-12-28
                        6
         2021-12-29
                       11
         2021-12-30
                        8
         Name: actual sales quantity, dtype: int64
```

Отталкиваясь от последнего значения, дополним этот временной ряд прогнозируемыми значениями.

```
In [236...
          sales forecast = []
          sales_forecast_idx = []
          last_value = sales_q.tail(1)[0]
          last_idx = sales_q.tail(1).index[0]
          sales forecast.append(last value) # для красоты графика
          sales_forecast_idx.append(last_idx) # для красоты графика
          for predicted diff in range(len(forecast[0])):
            predicted value = last value + forecast[0][predicted diff]
            predicted value idx = last idx + timedelta(days=1)
            sales forecast.append(round(predicted value))
            sales forecast idx.append(predicted value idx)
            last value = predicted value
            last_idx = predicted_value_idx
          sales_forecast = pd.DataFrame(data=sales_forecast, index=sales_forecast_id
```

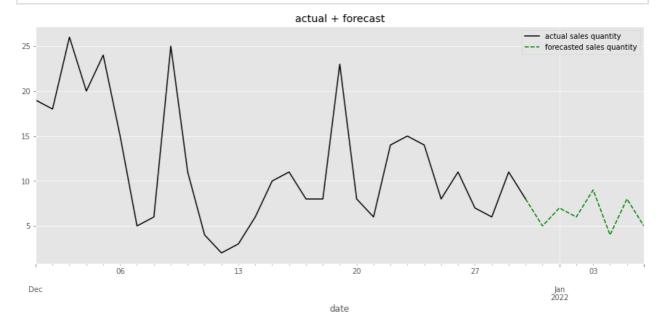
Out[237...

```
In [237... sales_forecast
```

forecasted sales quantit	f	tity
	021-12-30	8
	021-12-31	5
	022-01-01	7
)22-01-02	6
)22-01-03	9
)22-01-04	4
	22-01-05	8
	22-01-06	5

Построим итоговый график продаж, продолжаемый прогнозируемыми значениями.

```
sales_q.name = 'actual sales quantity'
ax = sales_q['12-01-2021':].plot(figsize=(15,6), color='black', title='actustion sales_forecast.plot(ax=ax, style='--', color='green')
ax.legend(loc='upper right')
plt.show();
```



Сохраним результат на диске.

```
sales_forecast.columns = ['quantities']
sales_forecast.to_csv(PATH+'sales_forecast.csv')
```