Rudi Linear Solver

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1 Quick Solver

The quick solver simply matches teams by meeting the constraints and the preferences of the participants.

1.1 Algebraic model

Indices

C	$c, cc \in \{a, m, d\}$	Dinner Courses
T	$t, tt \in \{t1, t2, t3, \ldots\}$	Teams ($T \mod 3 = 0, T \geq 9$)
G	$g, gg \in \{g1, g2, g3, \ldots\}$	Groups ($\lvert G\rvert = \lvert T\rvert \div 3$)

Parameters

$H_{t,c}$	Happiness of Team t preparing Course c

Binary Variables

$$Ass_{c,t,g} \qquad \begin{cases} 1 & \text{team } t \text{ is assigned to group } g \text{ in course } c \\ 0 & else \end{cases}$$

$$Ch_{c,t,g} \qquad \begin{cases} 1 & \text{team } t \text{ is chef for group } g \text{ in course } c \\ 0 & else \end{cases}$$

Objective

$$\max \left(\sum_{c}^{C} \sum_{t}^{T} \sum_{g}^{G} (Ch_{c,t,g} \cdot H_{c,t}) \right) \tag{1}$$

Constraints

$$\sum_{g}^{G} Ass_{c,t,g} = 1 \qquad \forall c, \forall t \tag{2}$$

$$\sum_{t}^{T} Ass_{c,t,g} = 3 \qquad \forall c, \forall g \tag{3}$$

$$Ass_{c,t,g} + Ass_{c,tt,g} + Ass_{cc,t,gg} + Ass_{cc,tt,gg} \leq 3 \qquad \forall c, \forall cc > c, \forall t, \forall tt > t, \forall g, \forall gg \tag{4}$$

$$\sum_{g}^{G} Ch_{c,t,g} \leq 1 \qquad \forall c, \forall t \tag{5}$$

$$\sum_{t}^{T} Ch_{c,t,g} = 1 \qquad \forall c, \forall g \tag{6}$$

$$Ch_{c,t,g} + Ch_{cc,t,gg} \leq 1 \qquad \forall c, \forall cc > c, \forall t, \forall g, \forall gg \tag{7}$$

$$Ch_{c,t,g} \leq Ass_{c,t,g} \qquad \forall c, \forall t, \forall g \tag{8}$$

1.2 Explanations

(1) Maximize the happiness of all teams.

(2) Assign each team to exactly one group per course.

(3) Assign exactly 3 teams to a group per course.

(4) Assign teams only once to be in the same group. In case they were to be in the same group again for another course the sum of their assignments would be 4.

(5) There can only be one chef in one group per course. Not every team has to be chef in a course.

(6) There is exactly one chef in one group per course.

(7) Set any team to be chef only once.

(8) Set a team to be chef in a group only if it is in that group.

2 Distance Solver

The distance solver optimizes the team assignments according to the distances teams have to walk between courses. The preferces of the teams are tolerated within a margin equivalent of the points assigned to the happiness $H_{c,t}$ walking. Due to the complexity (Traveling Salesman Problem) solving this model becomes very expensive. A different approach should be used for larger amount of participating teams.

2.1 Algebraic model - expanded

Indices

T $\quad t, tt, i, j, r \in \{t1, t2, t3, \ldots\}$ $\quad$ Teams ($T \mod 3 = 0, T \geq 9$)

Parameters

$D_{i,j}$ $\quad$ Distance between Team i's and Team j's homes.

Binary Variables

$$Arc_{c,i,j,t,g} \quad \begin{cases} 1 & \text{team } t \text{ uses route } (i,j) \text{ when assigned to group } g \text{ in course } c \\ 0 & else \end{cases}$$

Objective

$$\max \left(\sum_{c}^{C} \sum_{t,i,j}^{T} \sum_{g}^{G} (Ch_{c,t,g} \cdot H_{c,t} - Arc_{c,i,j,t,g} \cdot D_{i,j}) \right) \tag{9}$$

Constraints

$$\sum_{i}^{T} \sum_{j}^{T} Arc_{c,i,j,t,g} = Ass_{c,t,g} \qquad \forall c, \forall t, \forall g \tag{10}$$

$$\sum_{j}^{T} Arc_{c,i,j,t,g} = Ass_{c,t,g} \qquad c = a, \forall t, \forall g \tag{11}$$

$$Arc_{c,i,j,t,g} \leq Ch_{c,j,g} \qquad \forall c, \forall t, \forall g, \forall i, \forall j \tag{12}$$

$$\sum_{j}^{T} \sum_{g}^{G} Arc_{c,i,j,t,g} = \sum_{r}^{T} \sum_{g}^{G} Arc_{c-1,r,j,t,g} \qquad \forall c \geq m, \forall t, \forall i \neq j \tag{13}$$

2.2 Explanation

(9) Maximize the happiness of all teams while reducing the distances they have to walk between courses.

(10) Only teams that are assigned to a group can walk to it in a course.

(11) All teams start from their homes before the appetizer.

(12) Only chefs can be the destination of a route.

(13) The starting location of a route has to be the target location of the previous course.