FECHA D M A

sochagota

## 2.) Adams - Bashforth

(i) $y_{n+1} = y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2})$

(ii) $y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$

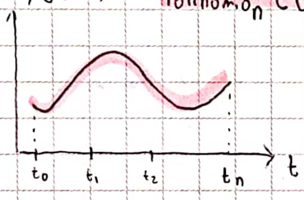
Forma general:

$$\frac{dy}{dt} = f(t, y) \quad \text{con } a \le t \le b$$

Valores:

| t | y | f |
|---|---|---|
| $t_0$ | $y_0$ | $f_0$ |
| $t_1$ | $y_1$ | $f_1$ |
| $t_2$ | $y_2$ | $f_2$ |
| ⋮ | ⋮ | ⋮ |
| $t_n$ | $y_n$ | $f_n$ |

Funcion: $f(t, y(t))$ Polinomio$_n$ $(t)$




$\Rightarrow f(t, y) \approx$ Polinomio de grado $n$ evaluado en $t$

$$\frac{dy}{dt} = P_n(t)$$

Resolvemos como una ecuación separable

$$\int_{y_{i-k}}^{y_{i+1}} dy = \int_{t_{i-k}}^{t_{i+1}} P_n(t)\, dt$$

Usamos: $P(x) = \int_{y_i}^{y_{i+1}} dy$

$$y\Big|_{y_{i-k}}^{y_{i+1}} = \int_{t_{i-k}}^{t_{i+1}} P_n(t)\, dt$$

$$(y_{i+1}) - (y_{i-k}) = \int_{t_{i-k}}^{t_{i+1}} P_n(t)\, dt$$

Cambio de variable:

$s = \frac{t_i - t_{i+1}}{h}$

$s = \frac{t_i}{h} - \cancel{\frac{t_{i+1}}{h}}$

$\frac{ds}{dt} = \frac{1}{h}$

$\Rightarrow dt = h\, ds$

$$y_{i+1} - y_i = \int_0^1 P_n(t)\, h\, ds$$

Ecuación general para métodos multipaso.

4 Pasos: (ii) $N = 3$, $K = 0$
↳ orden 3

$$P(x) = f(x_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2} + \binom{s+2}{3}\Delta^3 f_{i-3}$$

Polinomio de Newton

Evaluamos en multipaso

$$\Rightarrow y_{i+1} - y_i = \int_0^1 \left[f(t_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2} + \binom{s+2}{3}\Delta^3 f_{i-3}\right] h\, ds$$

$$= \int_0^1 \left[f(t_i) + \frac{s\Delta f_{i-1}}{1!} + \frac{(s+1)s\Delta^2 f_{i-2}}{2!} + \frac{(s+2)(s+1)s\Delta^3 f_{i-3}}{3!}\right] h\, ds$$

↳ Combinatoria de la forma:

$$\binom{s}{m} = \frac{s!}{m!(s-m)!} = \frac{s\cancel{(s-m)!}}{m!\cancel{(s-m)!}} = \frac{s}{m!}$$

Distribuimos y resolvemos las integrales:

$$= h\left[\int_0^1 f(t_i)\, ds + \int_0^1 \frac{s\Delta f_{i-1}}{1!} ds + \int_0^1 \frac{s(s+1)\Delta^2 f_{i-2}}{2!} ds + \int \frac{s(s+2)(s+1)\Delta^3 f_{i-3}}{3!} ds\right]$$

① $f(t_i)\, s\Big|_0^1 = f(t_i)1 - \cancel{f(t_i)0} = f(t_i)$

② $\frac{s^2}{2}\Big|_0^1 \Delta f_{i-1} = \frac{1}{2}\Delta f_{i-1} - \cancel{\frac{0}{2}\Delta f_{i-1}} = \frac{1}{2}\Delta f_{i-1}$

③ $\frac{\Delta^2 f_{i-2}}{2!}\int_0^1 s(s+1)\, ds =$ " " $\int_0^1 (s^2 + s)\, ds$

$=$ " " $\left[\left(\frac{s^3}{3} + \frac{s^2}{2}\right)\Big|_0^1\right] =$ " " $\left[\left(\frac{1}{3} + \frac{1}{2}\right) - \left(\cancel{\frac{0}{3}} - \cancel{\frac{0}{2}}\right)\right]$

$$= \frac{\Delta^2 f_{i-2}}{2!} \cdot \frac{5}{6} = \frac{5}{12}\Delta^2 f_{i-2}$$

④ $\frac{\Delta^3 f_{i-3}}{3!}\int_0^1 s(s+1)(s+2)\,ds = \frac{\Delta^3 f_{i-3}}{3!}\int_0^1 (s^2+s)(s+2)\,ds$

$\frac{\Delta^3 f_{i-3}}{3!}\int_0^1 (s^3+2s^2+s^2+2s)\,ds$

$= \left[\frac{s^4}{4}+\frac{2s^3}{3}+\frac{s^3}{3}+\frac{\cancel{2}s^2}{\cancel{2}}\right]\Big|_0^1$

$= \left[\frac{1}{4}+\frac{2}{3}+\frac{1}{3}+1\right]-\left[\frac{0}{4}+\frac{2\cdot 0}{3}+\frac{0}{3}+0\right]$

$\frac{\Delta^3 f_{i-3}}{6}\cdot\frac{9}{4} = \frac{3\Delta^3 f_{i-3}}{8}$

$\Rightarrow h[① + ② + ③ + ④]$

$h\left[f(t_i) + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2} + \frac{3}{8}\Delta^3 f_{i-3}\right]$

Retomando la forma original:

$y_{i+1} = y_i + h\left[f(t_i) + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2} + \frac{3}{8}\Delta^3 f_{i-3}\right]$

m.c.m:

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 12 | 8 | 2 |
| 1 | 1 | 6 | 4 | 2 |
| 1 | 0 | 3 | 2 | 2 |
| 1 | 0 | 3 | 1 | 3 |
| 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | |

m.c.m = 24

$y_{i+1} = y_i + h\left[\frac{24f(t_i) + 12\Delta f_{i-1} + 10\Delta^2 f_{i-2} + 9\Delta^3 f_{i-3}}{24}\right]$

$y_{i+1} = y_i + \frac{h}{24}\left[24f(t_i) + 12\Delta f_{i-1} + 10\Delta^2 f_{i-2} + 9\Delta^3 f_{i-3}\right]$

Reemplazamos los valores de $\Delta^n f_{i-n}$

$\Delta f = f_F - f_0$

| x | f | | | |
|---|---|---|---|---|
| $x_0$ | $f_{i-3}$ | $(f_{i-2}-f_{i-3})$ | $[(f_{i-1}-f_{i-2})-(f_{i-2}-f_{i-3})]$ | |
| $x_1$ | $f_{i-2}$ | $(f_{i-1}-f_{i-2})$ | | $[[(f_i - f_{i-1} - f_{i-1} + f_{i-2}) - (f_{i-1} - f_{i-2} - f_{i-2} + f_{i-3})]]$ |
| $x_2$ | $f_{i-1}$ | | $[(f_i - f_{i-1})-(f_{i-1}-f_{i-2})]$ | |
| $x_3$ | $f_i$ | $(f_i - f_{i-1})$ | | |
| | | $\Delta f_{i-1}$ | $\Delta^2 f_{i-2}$ | $\Delta^3 f_{i-3}$ |

$\Rightarrow \Delta f_{i-1} = (f_i - f_{i-1})$

$\Delta^2 f_{i-2} = f_i - f_{i-1} - f_{i-1} + f_{i-2}$
$= f_i - 2f_{i-1} + f_{i-2}$

$\Delta^3 f_{i-3} = f_i - f_{i-1} - f_{i-1} + f_{i-2} - f_{i-1} + f_{i-2} + f_{i-2} - f_{i-3}$
$= f_i - 3f_{i-1} + 3f_{i-2} - f_{i-3}$

$\Rightarrow y_{i+1} = y_i + \frac{h}{24}\left[24f(t_i) + 12(f_i - f_{i-1}) + 10(f_i - 2f_{i-1} + f_{i-2}) + 9(f_i - 3f_{i-1} + 3f_{i-2} - f_{i-3})\right]$

Expandiendo [ ]: $f(t_i) = f_i$

$24f(t_i) + 12f_i - 12f_{i-1} + 10f_i - 20f_{i-1} + 10f_{i-2} + 9f_i - 27f_{i-1} + 27f_{i-2} - 9f_{i-3}$

$55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}$

Reescribiendo la fórmula $i = n$

$$\boxed{y_{i+1} = y_i + \frac{h}{24}\left[55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}\right] \quad (ii)}$$

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# 3.) Adams - Moulton

(i) $y_{n+1} = y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1})$

(ii) $y_{n+1} = y_n + \frac{h}{24}(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2})$

(ii) 4 Pasos:

$$P(x) = f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1} + \binom{s+2}{3}\Delta^3 f_{i-2}$$

↳ Polinomio a integrar (Newton)

Valores: $\Rightarrow \int_{y_i}^{y_{i+1}} dy = \int_{t_i}^{t_{i+1}} P(x)\,dt$

| x | f |
|---|---|
| $x_{i-2}$ | $f_{i-2}$ |
| $x_{i-1}$ | $f_{i-1}$ |
| $x_i$ | $f_i$ |
| $x_{i+1}$ | $f_{i+1}$ |

$$y_{i+1} - y_i = \int_{-1}^{0} P_n(t)\,h\,ds$$

Cambio de variable:

$$\Rightarrow s = \frac{t - t_{i+1}}{h} = \frac{t}{h} - \cancel{\frac{t_{i+1}}{h}}^{0}$$

$$\frac{ds}{dt} = \frac{1}{h} \Rightarrow ds = \frac{dt}{h} \Rightarrow h\,ds = dt$$

Límites de integración:

$t$ — $h$ — $t_{i+1}$

$h$ = tamaño de paso $= t_{i+1} - t_i$

$$s = \frac{t - t_{i+1}}{h}$$

$$\lim_{t \to t_{i+1}} \frac{t_{i+1} - t_{i+1}}{h} = 0$$

$-h = t_i - t_{i+1}$

$$\Rightarrow y_{i+1} - y_i = \int_{-1}^{0} \left[ f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1} + \binom{s+2}{3}\Delta^3 f_{i-2} \right] h\,ds$$

$n! = n(n-1)!$

Evaluamos los combinatorios

$$\binom{s}{1} = \frac{s!}{1!(s-1)!} = \frac{s\cancel{(s-1)!}}{1!\cancel{(s-1)!}} = \frac{s}{1}$$

$$\binom{s+1}{2} = \frac{(s+1)!}{2!(s+1-2)!} = \frac{(s+1)(s+1-1)!}{2(s-1)!}$$

$$= \frac{(s+1)s!}{2(s-1)!} = \frac{(s+1)\cdot s\cancel{(s-1)!}}{2\cancel{(s-1)!}}$$

$$= \frac{(s+1)s}{2}$$

$$\binom{s+2}{3} = \frac{(s+2)!}{3!(s+2-3)!} = \frac{(s+2)(s+2-1)!}{6(s-1)!}$$

$$= \frac{(s+2)(s+1)!}{6(s-1)!} = \frac{(s+2)}{6(s-1)!}(s+1)(s+1-1)!$$

$$= \frac{(s+2)(s+1)s!}{6(s-1)!} = \frac{(s+2)(s+1)s\cancel{(s-1)!}}{6\cancel{(s-1)!}}$$

$$= \frac{(s+2)(s+1)s}{6}$$

$$\Rightarrow = \int_{-1}^{0} h\left[ f(t_{i+1}) + s\Delta f_i + \frac{s(s+1)}{2}\Delta^2 f_{i-1} + \frac{s(s+2)(s+1)}{6}\Delta^3 f_{i-2} \right] ds$$

Distribuimos y resolvemos las integrales

① $\int_{-1}^{0} f(t_{i+1})\,ds = f(t_{i+1})s\Big|_{-1}^{0}$

$= \cancel{f(t_{i+1})0} - f_i(t_{i+1})(-1) = f_i(t_{i+1})$

② $\int_{-1}^{0} s\Delta f_i\,ds = \frac{s^2}{2}\Delta f_i\Big|_{-1}^{0} = \cancel{\frac{0^2}{2}\Delta f_i} - \frac{(-1)^2}{2}\Delta f_i$

$= \frac{-1}{2}\Delta f_i$

(3) $$\int_{-1}^{0} \frac{s(s+1)}{2}\Delta^2 f_{i-1}\, ds = \int_{-1}^{0} s^2+s \,\frac{\Delta^2 f_{i-1}}{2}\, ds$$

$$= \frac{\Delta^2 f_{i-1}}{2}\left(\frac{s^3}{3}+\frac{s^2}{2}\right)\Big|_{-1}^{0} = \frac{\Delta^2 f_{i-1}}{2}\left[\left(\frac{0^3}{3}+\frac{0^2}{2}\right)-\left(\frac{(-1)^3}{3}+\frac{(-1)^2}{2}\right)\right]$$

$$= \frac{\Delta^2 f_{i-1}}{2}\left[-\left(\frac{-1}{3}+\frac{1}{2}\right)\right] = \text{" "}(-1/6)$$

$$= \frac{-\Delta^2 f_{i-1}}{12}$$

(4) $$\int_{-1}^{0} \frac{\Delta^3 f_{i-2}}{6} s(s+2)(s+1)\, ds = \frac{\Delta^3 f_{i-2}}{6}\int_{-1}^{0}(s^2+2s)(s+1)\, ds$$

$$= \text{" "}\int_{-1}^{0} s^3+s^2+2s^2+2s\, ds$$

$$= \text{" "}\left(\frac{s^4}{4}+\frac{\cancel{3}s^3}{\cancel{3}}+\frac{\cancel{2}s^2}{\cancel{2}}\right)\Big|_{-1}^{0}$$

$$= \text{" "}\left[\left(\cancel{\frac{0^4}{4}+0^3+0^2}\right)-\left(\frac{(-1)^4}{4}+(-1)^3+(-1)^2\right)\right]$$

$$= \frac{\Delta^3 f_{i-2}}{6}(-1/4) = \frac{-\Delta^3 f_{i-2}}{24}$$

$$\Rightarrow h\left[f_i(t+1)-\frac{1}{2}\Delta f_i - \frac{\Delta^2 f_{i-1}}{12} - \frac{\Delta^3 f_{i-2}}{24}\right]$$

Descomponemos $\Delta f_{i-\#}$ $\quad \Delta f = f_f - f_0$

| $x$ | $f$ |
|---|---|
| $x_{i-2}$ | $f_{i-2}$ |
| $x_{i-1}$ | $f_{i-1}$ |
| $x_i$ | $f_i$ |
| $x_{i+1}$ | $f_{i+1}$ |

$(f_{i-1}-f_{i-2})$

$(f_i - f_{i-1})$

$\underbrace{(f_{i+1}-f_i)}_{\Delta f_i}$

$[f_i - f_{i-1} - f_{i-1} + f_{i-2}]$

$\underbrace{[f_{i+1}-f_i-f_i+f_{i-1}]}_{\Delta^2 f_{i-1}}$

$\underbrace{[f_{i+1}-f_i-f_i+f_{i-1}-f_i+f_{i-1}+f_{i-1}-f_{i-2}]}_{\Delta^3 f_{i-2}}$

$$\Delta^2 f_{i-1} = f_{i+1} - 2f_i + f_{i-1}$$

$$\Delta^3 f_{i-2} = f_{i+1} - 2f_i + f_{i-1} - f_i + 2f_{i-1} - f_{i-2}$$

$$= f_{i+1} - 3f_i + 3f_{i-1} - f_{i-2}$$

$$\Rightarrow h\left[f_i(t+1) - \frac{1}{2}(f_{i+1}-f_i) - \frac{1}{12}(f_{i+1}-2f_i+f_{i-1}) - \frac{1}{24}(f_{i+1}-3f_i+3f_{i-1}-f_{i-2})\right]$$

mínimo común múltiplo

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 12 | 24 | 2 |
| 1 | 1 | 6 | 12 | 2 |
| 1 | 0 | 3 | 6 | 2 |
| 1 | 0 | 3 | 3 | 3 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | |

24

$$\Rightarrow \frac{h}{24}\left[24 f_i(t+1) - 12(f_{i+1}-f_i) - 2(f_{i+1}-2f_i+f_{i-1}) - f_{i+1} + 3f_i - 3f_{i-1} + f_{i-2}\right]$$

$\boxed{f_i(t+1) = f_{i+1}}$

$$-12f_{i+1} + 12f_i - 2f_{i+1} + 4f_i - 2f_{i-1} - f_{i+1} + 3f_i - 3f_{i-1} + f_{i-2} + 24f_{i+1}$$

$$9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2}$$

$\boxed{n=1}$

$$\Rightarrow y_{i+1} = y_i + \frac{h}{24}(9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2})$$

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## (i) 3 Pasos (Bashford):

$$y_{i+1} - y_i = \int_0^1 P_n(t)\,h\,ds$$

$$P_n(t) = f(x_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2}$$

$$= \int_0^1 h\left[f(t_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2}\right] ds$$

Distribuimos y resolvemos las integrales:

① $$\int_0^1 f(t_i)\,ds = f(t_i)\,s\Big|_0^1 = f(t_i)[1-0]$$

$$= f(t_i)$$

② $$\int_0^1 \binom{s}{1}\Delta f_{i-1}\,ds = \Delta f_{i-1}\cdot\frac{s^2}{2}\Big|_0^1 = \Delta f_{i-1}\left[\frac{1^2}{2}-\frac{0^2}{2}\right]$$

$$= \frac{\Delta f_{i-1}}{2}$$

③ $$\int_0^1 \Delta^2 f_{i-2}\binom{s+1}{2}ds = \Delta^2 f_{i-2}\int_0^1 \frac{s(s+1)}{2}\,ds$$

$$= \frac{\Delta^2 f_{i-2}}{2}\int_0^1 s^2+s\,ds = \frac{\Delta^2 f_{i-2}}{2}\left(\frac{s^3}{3}+\frac{s^2}{2}\right)\Big|_0^1$$

$$= \frac{\Delta^2 f_{i-2}}{2}\left[\left(\frac{1}{3}+\frac{1}{2}\right)-\cancel{\left(\frac{0}{3}+\frac{0}{2}\right)}\right] = \frac{\Delta^2 f_{i-2}}{2}\cdot\frac{5}{6}$$

$$= \frac{5\Delta^2 f_{i-2}}{12}$$

Formula general

$$\Rightarrow y_{i+1} = y_i + h\left[f(t_i) + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2}\right]$$

| x | f | | |
|---|---|---|---|
| $x_0$ | $f_{i-2}$ | | |
| | | $(f_{i-1} - f_{i-2})$ | |
| $x_1$ | $f_{i-1}$ | | $\underbrace{(f_i - f_{i-1} - f_{i-1} + f_{i-2})}_{\Delta^2 f_{i-2}}$ |
| | | $\underbrace{(f_i - f_{i-1})}_{\Delta f_{i-1}}$ | |
| $x_2$ | $f_i$ | | |

Reemplazando $\Delta^n f_{i-n}$ $\quad f(t_i) = f_i$

$$h\left[f_i + \frac{1}{2}(f_i - f_{i-1}) + \frac{5}{12}(f_i - f_{i-1} - f_{i-1} + f_{i-2})\right]$$

$$\Rightarrow h\left[f_i + \frac{1}{2}f_i - \frac{f_{i-1}}{2} + \frac{5f_i}{12} - \frac{5}{6}f_{i-1} + \frac{5f_{i-2}}{12}\right]$$

M.C.M.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 12 | 6 | 12 | 2 |
| 1 | 1 | 1 | 6 | 3 | 6 | 2 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | |

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$$\Rightarrow \frac{h}{12}\left[12f_i + 6f_i - 6f_{i-1} + 5f_i - 10f_{i-1} + 5f_{i-2}\right]$$

$$y_{i+1} = y_i + \frac{h}{12}\left[23f_i - 16f_{i-1} + 5f_{i-2}\right] \qquad i=n$$

## (i) 3 Pasos (Moulton):

$$y_{i+1} - y_i = \int_{-1}^0 P_n(t)\,h\,ds$$

$$P_n(t) = f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1}$$

$$y_{i+1} = y_i + h\int_{-1}^0 f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1}\,ds$$

Distribuimos y resolvemos las integrales

① $$\int_{-1}^0 f(t_{i+1})\,ds = f(t_{i+1})\,s\Big|_{-1}^0 = f(t_{i+1})[0-(-1)]$$

$$= f(t_{i+1})$$

② $\int_{-1}^{0} \binom{s}{1} \Delta f_i \, ds = \Delta f_i \int_{-1}^{0} s \, ds = \Delta f_i \frac{s^2}{2}\Big|_{-1}^{0}$

$= \Delta f_i \left[ \frac{0^2}{2} - \frac{(-1)^2}{2} \right] = \frac{-\Delta f_i}{2}$

③ $\int_{-1}^{0} \binom{s+1}{2} \Delta^2 f_{i-1} \, ds = \frac{\Delta^2 f_{i-1}}{2} \int_{-1}^{0} s(s+1) \, ds$

$\frac{\Delta^2 f_{i-1}}{2} \int_{-1}^{0} s^2 + s \, ds = \frac{\Delta^2 f_{i-1}}{2} \left( \frac{s^3}{3} + \frac{s^2}{2} \right)\Big|_{-1}^{0}$

$\frac{\Delta^2 f_{i-1}}{2} \left[ \cancel{\left( \frac{0^3}{3} + \frac{0^2}{2} \right)} - \left( \frac{(-1)^3}{3} + \frac{(-1)^2}{2} \right) \right]$

$\frac{\Delta^2 f_{i-1}}{2} \left( -\frac{1}{6} \right) = -\frac{\Delta^2 f_{i-1}}{12}$

$\Rightarrow y_{i+1} = y_i + h \left[ f_i(t+1) - \frac{\Delta f_i}{2} - \frac{\Delta^2 f_{i-1}}{12} \right]$

| $x$ | $f$ | | |
|---|---|---|---|
| $x_{i-1}$ | $f_{i-1}$ | $(f_i - f_{i-1})$ | |
| $x_i$ | $f_i$ | | $\underbrace{(f_{i+1} - f_i - f_i + f_{i-1})}_{\Delta^2 f_{i-1}}$ |
| $x_{i+1}$ | $f_{i+1}$ | $\underbrace{(f_{i+1} - f_i)}_{\Delta f_i}$ | |

$\boxed{f_i(t+1) = f_{i+1}}$

$y_{i+1} = y_i + h \left[ f_i(t+1) - \frac{1}{2}(f_{i+1} - f_i) - \frac{1}{12}(f_{i+1} - f_i 2 + f_{i-1}) \right]$

$\Rightarrow f_i(t+1) - \frac{f_{i+1}}{2} + \frac{f_i}{2} - \frac{f_{i+1}}{12} + \frac{2 f_i}{12} - \frac{f_{i-1}}{12}$

M.C.M.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 12 | 12 | 12 | 2 |
| 1 | 1 | 1 | 6 | 6 | 6 | 2 |
| 1 | 0 | 0 | 3 | 3 | 3 | 3 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | |

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$\Rightarrow \frac{1}{12} \left[ 12 f_{i+1} - 6 f_{i+1} + 6 f_i - f_{i+1} + 2 f_i - f_{i-1} \right]$

$\Rightarrow \frac{1}{12} [ 5 f_{i+1} + 8 f_i - f_{i-1} ]$

$\Rightarrow y_{i+1} = y_i + \frac{h}{12} \left[ 5 f_{i+1} + 8 f_i - f_{i-1} \right]$