④ $\frac{du}{dt} = u^q, \; t \in [0, 10]$

sol ex ⇒ $u(t) = e^t$ para $q = 1$

$u(t) = (t(1-q)+1)^{1/1-q}$ ⇒ $q < 1$

$t(1-q)+1 > 0$

① Ecuación separable

$$\int \frac{1}{u^q} du = \int 1 \, dt$$

② Separamos por casos ($q = 1$, $q \neq 1$)

· $q = 1$

$$\int \frac{1}{u} du = \int 1 dt$$

$\ln|u| + C = t$

$\ln|u| = t + C$ (constante de integración absorbe el -)

$|u| = e^{t+c}$

$|u| = e^t e^c$

$|u| = Ce^t$

$\Rightarrow u(t) = Ce^t$

· $q < 1$

$$\frac{du}{dt} = u^q$$

$\int x^n dx = \frac{x^{n+1}}{n+1}, \; n \neq 1$

$$\int \frac{1}{u^q} du = \int dt$$

$$\int u^{-q} du = t$$

$$\frac{u^{-q+1}}{-q+1} = C + t$$

$$\sqrt[-q+1]{u^{-q+1}} = ((C + t(-q+1))^{1/-q+1}$$

$$u(t) = \sqrt[1-q]{\underbrace{t(-q+1)+C}_{>0}}$$

Si asumimos $C = 1$ se cumple pero no tenemos condiciones iniciales para evaluar

5) $x^3y' = x^4y^2 - 2x^2y - 1$ sol p $\Rightarrow y_1 = x^{-2}$ $y_0 \Rightarrow y(\sqrt{2}) = 0$

Forma general de la ecuación de Riccati:

$$P_0(x)\frac{dy}{dx} + P_1(x)y = Q(x)y^2 + R(x)$$

1 Derivamos la solucion parcial

$y_1 = x^{-2}$

$y_1' = -2x^{-2-1}$

$= -2x^{-3}$

2 Sustituimos en la formula y_1, y_1'

$x^3(-2x^{-3}) = x^4(x^{-2})^2 - 2x^2(x^{-2}) - 1$

$-2x^{-3+3} = x^4x^{-4} - 2x^{2-2} - 1$

$-2 = \cancel{1} - 2 \cancel{-1}$

$-2 = -2$ (Verificamos la igualdad)

3 Cambio de variable y derivación

$y = y_1 + u^{-1} \Rightarrow y = x^{-2} + u^{-1}$

$y' = -2x^{-3} - 1u^{-1-1}u'$ $\quad \Rightarrow \frac{d}{dx} v^n = n v^{n-1} v'$

$= -2x^{-3} - u^{-2}u'$

4 Sustitución

$x^3(-2x^{-3} - u^{-2}u') = x^4(x^{-2}+u^{-1})^2 - 2x^2(x^{-2}+u^{-1}) - 1$

$-2x^{-3+3} - u^{-2}u'x^3 = x^4(x^{-4} + 2x^{-2}u^{-1} + u^{-2}) - 2x^{2-2} - 2x^2u^{-1} - 1$

$\cancel{-2} - u^{-2}u'x^3 = x^{4-4} + 2x^{4-2}u^{-1} + x^4u^{-2} \cancel{-2} - 2x^2u^{-1} - 1$

$-u^{-2}u'x^3 = \cancel{1} + \cancel{2x^2u^{-1}} + x^4u^{-2} - \cancel{2x^2u^{-1}} \cancel{-1}$

$-u^{-2}u'x^3 = x^4u^{-2}$

$u' = \frac{x^4u^{-2}}{-u^{-2}x^3} \Rightarrow u' = -x$

5 Resolvemos:

$\frac{du}{dx} = -x$

$\int du = \int -x\,dx$

$u = -\int x\,dx$

$u = -\frac{x^2}{2} + C$

6 Volvemos a la variable original

$y = x^{-2} + \left(-\frac{x^2}{2} + C\right)^{-1}$

7 Evaluamos valores iniciales

$0 = \frac{1}{(\sqrt{2})^2} + \frac{1}{C - \frac{(\sqrt{2})^2}{2}}$

$0 = \frac{1}{2} + \frac{1}{C - \frac{2}{2}}$

$0 = \frac{1}{2} + \frac{1}{C - 1}$

$-\frac{1}{2} = \frac{1}{C-1}$

$(C-1) = -2$

$C = -2+1$

$C = -1$

8 Reemplazamos:

$y = x^{-2} + \left(\frac{-x^2}{2} - 1\right)^{-1}$