a.) $I_0 = \frac{1}{4} mr^2 + md^2$

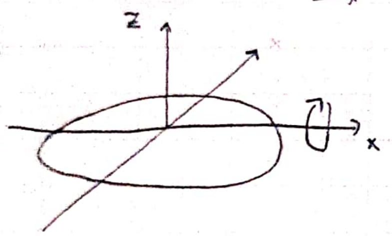
es el momento de inercia alrededor de la dirección azimutal.

$I_0 = I_{disco} + I_{cilindro}$

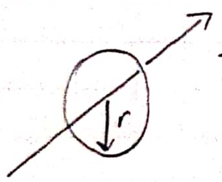
$I = \sum m_i r_i^2$ ⇒ Definición de momento de inercia

I disco:

$I_x = \frac{1}{4} m r^2$



I cilindro hueco

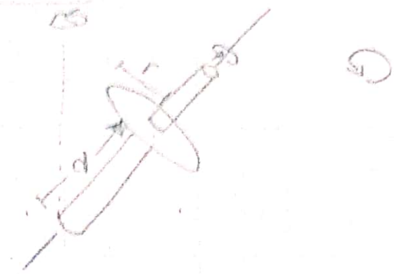


$I_c = mr^2$

$\Rightarrow \sum m_i r_i^2$

$= I_x + I_c$

$= \frac{1}{4} mr^2 + mr^2$



b.) $I_z = \frac{1}{2} mr^2$

$I_z = \frac{1}{2}\, 0.1 Kg \cdot (0.1)^2 m$

$= 5 \times 10^{-4}\ Kg\, m^2$

c.) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad q_i = \phi, \psi, \theta$

$E_k = \frac{1}{2} I_0 \dot{\phi}^2 + \frac{1}{2} I_z \dot{\psi}^2 + \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2)$

$E_p = mgr \operatorname{Sen}\theta$

$L = E_k - E_p$

$L = \frac{1}{2} I_0 \dot{\phi}^2 + \frac{1}{2} I_z \dot{\psi}^2 + \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2) - mgr \operatorname{Sen}\theta$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\psi}}\right) - \frac{\partial L}{\partial \psi} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \end{cases}$$