Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2014/2015

Seminar Test, March 24, 2015

1. Find the solution of the IVP

$$x'' + 4x' + 5x = 0\,, \quad x(0) = 1\,, \quad x'(0) = -2.$$

Represent this integral curve and describe its long term behavior.

2. Find all $a, b, c \in \mathbb{R}$ such that $x(t) = a\sin t + b\cos t + c\,e^t$ to be a solution of

$$x' + x = -3\sin t + 2\,e^t.$$

Find the general solution of this differential equation.

3. We consider the differential equation

$$x' + \frac{1}{t^2}x = 0\,, \quad t \in (-\infty, 0).$$

a) Check that $x = e^{1/t}$ is a solution of this d.e..

b) Find its general solution.

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Seminar Test, March 24, 2015

1. Consider the differential equation

$$x'' + 2x' + 2x = te^{-t}. \tag{0.1}$$

(i) Find a solution of the form $x_p(t) = (at + b)e^{-t}$ for (0.9).

(ii) Find the general solution of (0.9).

(iii) Find the solution of (0.9) that satisfies $x(0) = 0$ and $x'(0) = 3$.

2. Consider the differential equation

$$x' + 4tx = t. \tag{0.2}$$

(i) Find the general solution of (0.12).

(ii) Find the solution of (0.12) that satisfies $x(0) = 0$. Is this solution bounded? What about the other solutions of (0.12)? Justify.

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Seminar Test, March 24, 2015

In the following problems, the unknown is the function $x = x(t)$.

1. Consider the differential equation

$$x'' - 6x' + 9x = e^{3t}. \tag{0.3}$$

(i) Find a solution of the form $x_p(t) = at^2e^{3t}$ for (0.11).

(ii) Find the general solution of (0.11).

(iii) Find the solution of (0.11) that satisfies $x(0) = 0$ and $x'(0) = 3$.

2. Consider the differential equation

$$x' + \frac{2t}{1+t^2}x = 3. \tag{0.4}$$

(i) Find the general solution of (0.10).

(ii) Find the solution of (0.10) that satisfies $x(0) = 1$. Is this solution bounded? What about the other solutions of (0.10)? Justify.

Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2014/2015

Seminar Test, March 24, 2015

1. (i) Find a solution of the form $x_p(t) = (at + b)e^t$ for

$$x'' - 4x = te^t.$$

(ii) Find a solution of the form $x_p(t) = c\sin(2t) + d\cos(2t)$ for

$$x'' - 4x = \cos(2t).$$

(ii) Find the solution of

$$x'' - 4x = 9te^t + 8\cos(2t), \quad x(0) = 3, \quad x'(0) = 5.$$

2. Find the general solution of

$$x' + 2tx = 4t^3 e^{-t^2}.$$

3. Find the linear homogeneous differential equation of minimal order that has as solution the function $(t - 1)e^{t-1}$.

Seminar Test, March 24, 2015

In the following problems, the unknown is the function $x = x(t)$.

1. (i) Find a solution of the form $x_p(t) = ae^t$ for

$$x'' + 2x' + x = e^t.$$

(ii) Find a solution of the form $x_p(t) = bt^2e^{-t}$ for

$$x'' + 2x' + x = e^{-t}.$$

(ii) Find the solution of

$$x'' + 2x' + x = 4e^t + 2e^{-t}, \quad x(0) = 1, \quad x'(0) = 1.$$

2. Find the general solution of

$$x' - \frac{2}{t}x = t^2 \sin(2t) - 4t^3, \quad \text{where } t > 0.$$

3. Find the linear homogeneous differential equation of minimal order that has as solution the function $\sin(2t - 1)$.

Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2014/2015

Seminar Test, March 24, 2015

1. Consider the differential equation

$$x'' - 2x' + 5x = 5t^2 - 4t + 2. \tag{0.5}$$

(i) Find a solution of the form $x_p(t) = at^2 + bt + c$ for (0.9).

(ii) Find the general solution of (0.9).

(iii) Find the solution of (0.9) that satisfies $x(0) = x'(0) = 1$.

2. Consider the differential equation

$$x' + 2x\tan(t) = \cos^3(t), \quad \text{where } t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right). \tag{0.6}$$

(i) Find the general solution of (0.10).

(ii) Find the solution of (0.10) that satisfies $x(0) = 1$.

3. Find the linear homogenous differential equation of minimal order that has as solution the function $(2te^t)^2$.

Seminar Test, March 24, 2015

In the following problems, the unknown is the function $x = x(t)$.

1. Consider the differential equation

$$x'' + 2x' + x = 4te^t. \tag{0.7}$$

(i) Find a solution of the form $x_p(t) = (at + b)e^t$ for (0.9).

(ii) Find the general solution of (0.11).

(iii) Find the solution of (0.11) that satisfies $x(0) = x'(0) = 0$.

2. Consider the differential equation

$$x' + \frac{1}{\sqrt{t}}x = \frac{1}{2\sqrt{t}}, \quad \text{where } t > 0. \tag{0.8}$$

(i) Find the general solution of (0.12).

(ii) Find the solution of (0.12) that satisfies $x(1) = \frac{3}{2}$.

3. Find the linear homogenous differential equation of minimal order that has as solution the function $1 + t(1 + e^{-t})$.

Seminar Test, March 24, 2015

1. Consider the differential equation

$$x'' + x = \sin(t). \tag{0.9}$$

(i) Find a solution of the form $x_p(t) = at\cos(t)$ for (0.9).

(ii) Find the general solution of (0.9).

(iii) Find the solution of (0.9) that satisfies $x(0) = x'(0) = 0$.

2. Consider the differential equation

$$x' - \frac{1}{t}x = -1, \quad \text{where } t > 0. \tag{0.10}$$

(i) Find the general solution of (0.10).

(ii) Find the solution of (0.10) that satisfies $x(1) = 0$.

Seminar Test, March 24, 2015

In the following problems, the unknown is the function $x = x(t)$.

1. Consider the differential equation

$$x'' - x = 1. \tag{0.11}$$

(i) Find a constant solution of (0.11).

(ii) Find the general solution of (0.11).

(iii) Find the solution of (0.11) that satisfies $x(0) = 0$ and $\lim_{t\to\infty} |x(t)| < +\infty$.

2. Consider the differential equation

$$x' + \frac{2}{t}x = \frac{1}{t^2}, \quad \text{where } t > 0. \tag{0.12}$$

(i) Find the general solution of (0.12).

(ii) Find the solution of (0.12) that satisfies $x(1) = 0$.

Seminar Test, April 1, 2015

1. Find the solution of the IVP

$$x'' + 25x = 0, \quad x(0) = 0, \quad x'(0) = 1.$$

Represent this integral curve and describe its long-term behavior.

2. a) Find a particular solution for

$$x'' + 25x = 5\cos(5t)\,,$$

knowing that it has either the form $x_p(t) = a\,\cos(5t)$ or $x_p(t) = a\,t\,\cos(5t)$ or $x_p(t) = a\,t\,\sin(5t)$ or $x_p(t) = a\,e^{5t}$ or $x_p(t) = 5a\,t\,\sin(5t)$ (where the real coefficient a has to be determined).

b) Find a constant solution for

$$x'' + 25x = 5\,.$$

c) Find the general solution of the differential equation

$$x'' + 25x = 25 - 25\cos(5t)\,.$$

3. Find the general solution of $x' + \frac{1}{t}x = \frac{1}{t}e^{-2t+1}$ for $t \in (0, \infty)$. Justify the result in two ways.

Seminar Test, April 1, 2015

1. Find the solution of the IVP

$$x'' - 6x' + 9x = 0, \quad x(0) = 0, \quad x'(0) = 1.$$

2. a) Find a particular solution for

$$x'' - 6x' + 9x = -2e^{3t}\,,$$

knowing that it has either the form $x_p(t) = a\,e^{3t}$ or $x_p(t) = a\,t\,e^{3t}$ or $x_p(t) = a\,t^2\,e^{3t}$ or $x_p(t) = a\cos(3t)$ or $x_p(t) = a\,e^{-3t}$ (where the real coefficient a has to be determined).

b) Find a constant solution for

$$x'' - 6x' + 9x = 5\,.$$

c) Find the general solution of the differential equation

$$x'' - 6x' + 9x = 10 + 10e^{3t}\,.$$

3. Find the linear homogeneous differential equation of minimal order with constant coefficients that has as solution the function $-2te^{3t}$.

4. Find the general solution of $x' - x = e^{t-1}$ for $t \in (0, \infty)$. Justify the result in two ways.

Seminar Test, March 24, 2015

1. a) Find a particular solution of the form $x_p(t) = a\,t^2\,e^t$ (where the real coefficient a has to be determined) for

$$x'' - 2x' + x = e^t\,.$$

b) Find a constant solution for

$$x'' - 2x' + x = 5\,.$$

c) Find the general solution of the differential equation:

$$x'' - 2x' + x = 10 + 5e^t\,.$$

d) Find the solution of the IVP

$$x'' - 2x' + x = 5\,, \quad x(0) = 5\,, \quad x'(0) = 0.$$

2. Find the general solution of $x' + \frac{1}{t}x = -2$ for $t \in (0, \infty)$. Justify the result in two ways.

3. Find the linear homogeneous differential equation of minimal order that has as solution the function e^{-2t}.