Hash function

- the universe of keys is a subset of

  $N = \{0,1,2, \ldots\}$
- if the keys are not natural numbers - interpret them as natural numbers

Example:

a character string

consider successive ASCII codes

Method for Creating Hash Function

maps the universe *U* of keys

into the slots of a ***hash table*** *T*

1. *The division method.*
2. *The multiplication method.*
3. *Universal hashing.*

Building hash function: division method

$h(k) = k \bmod m$

0-based arrays

experiments =>

good values for m are

prime not too close to exact powers of 2

Building hash function: multiplication method

The multiplication method

h(k) = floor(m * frac(k * A))

where

m - hash table size

A - constant in the range 0< A <1

Remark:

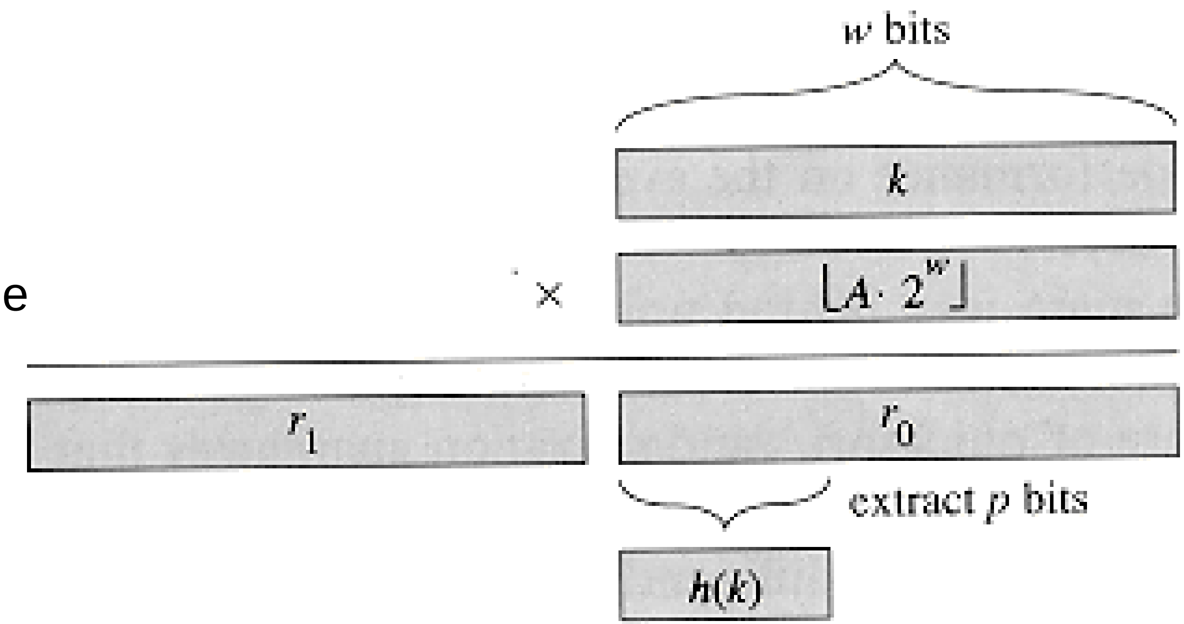
- the value of *m* is not critical

Easy implementation:

restrict $A = s/2^w$,

w=machine word size

$m = 2^p$ for some integer p



Building hash function: multiplication method

The multiplication method

good value for A (experimental)

$$A \approx \frac{\sqrt{5}-1}{2} \approx 0.6180339887$$

Donald Knuth, *The Art of Computer Programming* , **1968**

Numeric example:

k = 123456

m = 10000

A = 0.6180339887

$h(k)$ = floor(41.151…) = 41

k = 50

$h(k)$ = 9016

Hash function

	Multiplication Method	Division Method
m	1000	1000
A	0.618033988749895	
key	*h(key)= floor(m * frac(key * A))*	*h(key) = key mod* 1000
123456	4	456
123459	858	459
123496	725	496
123956	21	956
129456	208	456
193456	383	456
923456	195	456

the value of *m* is not critical

Building hash function: universal hashing

Universal hashing: refers to selecting a hash function at random from a family of hash func. with a certain property

universal class of hash functions

Let H be a finite collection of

hash functions that map : U ➔ {0,1, . . . , m - 1}

Such a collection is said to be **universal**

if for each pair of distinct keys x, y $\in$ U,

the nr. of hash functions for which h(x) = h(y) is at most **|H|/m**

With a function h chosen uniformly at random from H, the chance of a collision between x and y, where x <> y, is less than 1/m.

$$P(h(x) = h(y)) \leq \frac{1}{m}$$

Building hash function: universal hashing

Example:

m – the size of hash table, prime

key x: decompose a key x into r *bytes*

$$x = \langle x_1, x_2, \ldots, x_r \rangle$$

with: $x_i \le m$

hash function:

$$h_a(x) = \sum_{i=1}^{r} a_i * x_i \mod m$$

$\langle a_1, a_2, \ldots, a_r \rangle$ is a fixed sequence of random numbers

$a_i \in \{0, \ldots, m-1\}$

universal class of hash functions

$$H = \bigcup_a h_a$$

union taken over all possible ***a***-s

- m^r members
- can be shown to be universal

Building hash function: universal hashing

Universal hashing: refers to selecting a hash function at random from a family of hash func. with a certain property

Useful for algorithms that need multiple hash functions

ex.: rehashing

the data structure needs to be rebuilt

if too many collisions occur

Hash function

- perfect hash function
  - injective: maps distinct elements with no collisions
  - it is too expensive to compute it for every input

➔ build a hash function to minimize collisions

good hash function

In practice:

- use **heuristic** information to create a hash function that is likely to perform well

Choose between:

- simple and fast, but have a high number of collisions;
- more complex functions, with better quality, but take more time to calculate

Good hash function

A **good hash function** satisfies

the assumption of **simple uniform hashing**

- a key x is equally likely to hash to any of the *m* slots

  $P(h(x)=j) = 1/m \quad , \text{for any } j=0,\ldots,m-1$

- each bucket is equally likely to be occupied

- probability that two keys map to the same slot is 1/m

$$P(h(x) = h(y)) = \frac{1}{m}$$

x, y - independent random variable

Good hash function

<u>Need</u>: qualitative information about P

uniform distributed keys

<u>Example</u>:

- keys are random real numbers independently and uniformly distributed in the range [0,1).

  $h(k) = [k * \boldsymbol{m}]$

  satisfies the simple uniform hashing property

- keys are random integers independently and uniformly distributed in the range 0 to N−1

  where N much larger than m

  $h(k) = k \bmod \boldsymbol{m}$

  satisfies the simple uniform hashing property

Hash function

Need: qualitative information about P

not uniformly distributed keys

Special-purpose hash function

- exceptionally good for a specific kind of data

  no performance on data with different distribution

Example (1)

input data: file names such as FILE0000.CHK, FILE0001.CHK, FILE0002.CHK, etc., with mostly sequential numbers.

- extracts the numeric part **k** of the file name ***fn***

  *h(**fn**) = numeric_part(**fn**) mod m*

Hash function

<u>Example (2)</u>

input data: text in any natural language

has highly non-uniform distributions of characters, and character pairs, very characteristic of the language

- string
- variable length data

it is prudent to use a hash function that depends on all characters of the string—and depends on each character in a different way

```
Example of hash function:
Function HashMultiplicative(strKey) {
    hash = INITIAL_VALUE;
    for i = 1, length(strKey) do
      hash = M * hash + strKey [i]
    endfor
    return hash % TABLE_SIZE;
}
```

D. Bernstein, `comp.lang.c`, (1991 ?) INITIAL_VALUE = 5381, M = 33

B. Kernighan, D. Ritchie, *The C Programming Language*, 1978 INITIAL_VALUE = 0, M = 31

Hash function

Example (3)

input data: an unchanging dictionary
(text in a natural language)

If the dictionary is unchanging, you might want to consider perfect hashing;

- for a given dataset you can guarantee that there will be no collisions

Hash function

<u>Example (4)</u>

assume

input data: three-letter words

formed with any of a set of char extended ASCII code

perfect hashing

- h(str) = ASCIIcode(str[0]) * 256^2
  + ASCIIcode(str[1]) * 256^1
  + ASCIIcode(str[2])

- ASCIIcode(str[i]) : values from range 0..255
- hash table of size 3^{256} **?!**

Hash table and hash function in programming languages

Java

HashMap

– Hash table based implementation of the Map interface

HashSet

– implements the Set interface, backed by a hash table

Hash in programming languages

Java Object

- public int hashCode()

As much as is reasonably practical, the hashCode method defined by class Object does return distinct integers for distinct objects. (This is typically implemented by converting the internal address of the object into an integer, but this implementation technique is not required by the JavaTM programming language.)

- public boolean equals(Object obj)

  if two objects are equal then they must return same hash code

  - that is compared by equal() of that class

Hash in programming languages

The java.lang.String hash function

Given: s of java.lang.String

h(s)= s[0]*31^(n-1) + s[1]*31^(n-2) + ... + s[n-1]

- uses arithmetic int

where s[i] is the ith character of the string,

n is the length of the string

^ indicates exponentiation.

(The hash value of the empty string is zero.)

Hash tables in programming languages

- STL map: Associative key-value pair held in balanced binary tree structure
  – usually a red-black tree

New in C++ 11

- unordered_map

Some implementations

- hash_map was a common extension provided by many library implementations