

Exam on Dynamical Systems.
June 12, 2012

1. Find the general solution of the following equations.
a) $x' - 3t^2x = t^3$; b) $x' - 3t^2x = f(t)$ where $f \in C(\mathbb{R})$;
b) $x_{k+1} + 3x_k = 0$; c) $x_{k+1} + 3x_k = b$ where $b \in \mathbb{R}$.
2. a) Study the stability of the equilibria of the differential equation $\dot{x} = \frac{1}{2}(x^2 - a)$ where $a > 0$. Represent the phase portrait.
b) Study the stability of the fixed points of the difference equation $x_{k+1} = \frac{1}{2}(x_k + \frac{a}{x_k})$ where $a > 0$. Represent the stair-step diagram.
3. We consider the IVP $y' = 2xy^2 + x^3, x \in [0, 1], y(0) = 0$.
a) Describe the corresponding Euler's numerical algorithm.
b) Write the recurrence formula for the Picard sequence of successive approximations $(\varphi_n)_{n \geq 0}$. Starting with $\varphi_0(x) \equiv 0$ calculate $\varphi_1(x)$.