Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems.
August 31, 2009
II

1. (3p) Specify the type and stability of the equilibrium point $(0,0)$ of the differential system:

$$\dot{x} = 5x - 7y, \quad \dot{y} = 2x - 4y.$$

Find the general solution of the above differential system.

2. (2.5p) Represent the phase portrait of the scalar differential equation

$$\dot{x} = 3x\left(-2 + \frac{x}{100}\right).$$

What remarkable property has the solution of the above equation with the initial value $x(0) = 100$? (*Hint: "read" its phase portrait*)
Find the general solution of the above equation.

3. (1p) Find a first integral of $(x^3 - 3xy^2 + 2)dx + (y^2 - 3x^2y)dy = 0$.

4. (1p, not compulsory) Determine the equilibria and study the stability of one of them for the following planar differential system:

$$\dot{x} = x - 2xy, \quad \dot{y} = x^2 - 2y.$$