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Exercise Sheet no.1

Analysis for CS

GROUPWORK:

(G 4)

a) Let $S \subseteq \mathbb{R}$. Using the definition of the *lower* (respectively, *upper*) *bound* of S write down what it does mean that an element $x \in \mathbb{R}$ is not a lower (respectively, upper) bound of S.

b) Fill in the following table:

S	LB(S)	UB(S)	$\min S$	$\max S$	$\inf S$	$\sup S$
$\emptyset$						
$(-5,3) \cup [4,+\infty)$						
$(-2,4) \cup \{5\}$						
$(-\infty,0] \cup \{1,2\}$						
$(-2,3) \cap \mathbb{Z}$						
$\mathbb{N}$						
$(-2,\sqrt{3}) \cap \mathbb{Q}$						
$\{x \in \mathbb{R} \mid x^3 - x^2 - 6x \geq 0\}$						

c) Give an example of a subset S of $\mathbb{R}$ that satisfies simultaneously the following conditions: it is not an interval, it is unbounded below, it doesn't have a greatest element, and $\sup S = -1$.

(G 5) (Train your brain)

Let $S \subseteq \mathbb{R}$.

a) Prove that if $\mathrm{UB}(S) \neq \emptyset$, then $\mathrm{UB}(S)$ contains infinitely many elements.

b) Prove that if S has a greatest element, then $\max S = \sup S$.

c) Prove that S has at most one greatest element. (In other words, S cannot have two distinct greatest elements.)

d) Prove that S has at most one supremum. (In other words, S cannot have two distinct suprema.)

Homework:

(H 5) (To be delivered in the next exercise-class)

a) Fill in the following table:

A	LB(A)	UB(A)	$\min A$	$\max A$	$\inf A$	$\sup A$
$\mathbb{R}_+$						
$\mathbb{Q}^*$						
$[-2,1)\cup(2,\infty)$						
$(-\infty,-1)\cup(2,3)$						
$(-2,5)\cap\mathbb{N}$						
$\mathbb{Z}$						
$(-\infty,5]\cap\mathbb{Q}$						
$\left\{x\in\mathbb{R} \mid \frac{x+1}{x^2+1}<1\right\}$						

b) Give an example of a subset S of $\mathbb{R}$ that satisfies simultaneously the following conditions: it is not an interval, it is unbounded above, it doesn't have a least element, and $\inf S = 3$.

(H 6) (Train your brain)

Let $S \subseteq \mathbb{R}$.

a) Prove that if $\mathrm{LB}(S) \neq \emptyset$, then $\mathrm{LB}(S)$ contains infinitely many elements.

b) Prove that if S has a least element, then $\min S = \inf S$.

c) Prove that S has at most one least element. (In other words, S cannot have two distinct least elements.)

d) Prove that S has at most one infimum. (In other words, S cannot have two distinct infima.)

(H 7) (Train your brain)

Having the proof of **C2** in the first course as a model, prove **C4**.