

Exam on Dynamical Systems  
June 22, 2013

1. We consider the equation  $x'' - x = te^{-2t}$ .
  - a) Find a particular solution of the form  $x_p(t) = (at + b)e^{-2t}$ , where  $a, b \in \mathbb{R}$ .
  - b) Find the general solution.
  - c) Find the solution that satisfies the initial conditions  $x(0) = 0$ ,  $x'(0) = 0$ .
  
2. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x - (x^2 - 2)/4$ .
  - a) Find the fixed points of  $f$  and study their stability.
  - b) What can we say about the solution of  $x_{k+1} = f(x_k)$  with  $x_0 = \sqrt{2}$  and, respectively, with  $x_0 \in \mathbb{R}$  such that  $|x_0 - \sqrt{2}|$  is sufficiently small?
  
3. Find the general solution of  $dy/dx = -2y/x$ .