Seminar Test

1. Represent the phase portrait of the scalar dynamical system $\dot{x} = x(1-x^2)$. Find $\varphi(t,1)$ and justify. Specify the monotonicity of $\varphi(t,2)$ and, respectively, $\varphi(t,0.5)$.

2. Find the polar coordinates of the points whose cartesian coordinates are: $(1,0)$, $(0,1)$, $(-2,0)$ and $(0,-0.5)$, respectively.

3. We consider the planar system

$\dot{x} = -y + x(1-x^2-y^2),\ \dot{y} = x + y(1-x^2-y^2).$

a) Study the type and stability of the equilibrium point $(0,0)$ using the linearization method. There are other equilibria?

b) Transform the given system to polar coordinates.

c) What is the shape of the orbit corresponding to: $\varphi(t,1,0)$, $\varphi(t,0,1)$, $\varphi(t,-2,0)$ and $\varphi(t,0,-0.5)$, respectively? Justify.

d) What remarkable property has the function $\varphi(t,1,0)$?

Seminar Test

1. Let $c \geq 0$ be a parameter and consider the scalar dynamical system $\dot{x} = x(1-x) - cx$.

a) Find its equilibria and study their stability using the linearization method.

b) Represent its phase portrait.

c) When $x(t) > 0$ is considered to be the number of fish in some lake, and $c \geq 0$ to be the rate of fishing, try to predict the fate of the fish from the lake interpreting the theoretical result obtained at a) and b).

2. We consider the following linear planar system

$\dot{x} = -6x, \quad \dot{y} = -3y.$

a) Find its general solution and its flow.

b) Study the type and stability of its equilibrium point $(0,0)$.

c) Find the shape of the orbits in two ways: by using the definition of the orbit and then by using the differential equation of the orbits.

d) Represent its phase portrait.

Seminar Test

1. We consider the linear planar system $\dot{x} = -3x + y,\ \dot{y} = 3x - y$.

a) Find the eigenvalues and the determinant of the matrix of the system.

b) Find all the equilibria and represent their corresponding orbits in the phase space.

c) Find $\varphi(t, 0, 0)$, $\varphi(t, 1, 3)$ and $\varphi(t, 2, 6)$.

d) Find the shape of the orbits by using the differential equation of the orbits.

e) Find a first integral in $\mathbb{R}^2$ of this system.

f) Represent its phase portrait.

g) Find its general solution.

2. Find the equilibria and decide whether they are or not hyperbolic, for the nonlinear planar system $\ddot{y} + \dot{y} + y^3 = 0$.

Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2014/2015

Seminar Test

1. Represent the phase portrait of the scalar dynamical system $\dot{x} = -x^2 + 2x + 3$. Find $\varphi(t, -1)$ and justify. Specify the monotonicity of $\varphi(t, 2)$ and, respectively, $\varphi(t, 5)$.

2. We consider the following nonlinear planar systems

$\dot{x} = -x + xy, \quad \dot{y} = -2y + 3y^2.$

a) Find its equilibria and study their stability using the linearization method.

b) Find $\varphi(t, 0, 2/3)$, $\varphi(t, 4, 0)$ and $\varphi(t, 1, 2/3)$.

3. Give an example of a linear planar system with a center at the origin. Justify. What remarkable property have the solutions of such a system, i.e. how they change in time?

Seminar Test

1. Let $c \geq 0$ be a parameter and consider the scalar dynamical system $\dot{x} = x(1-x) - c$.

a) Find its equilibria and study their stability using the linearization method.

b) Represent its phase portrait.

c) When $x(t) \geq 0$ is considered to be the number of fish in some lake, and $c \geq 0$ to be the rate of fishing, try to predict the fate of the fish from the lake interpreting the theoretical result obtained at a) and b).

2. We consider the following linear planar system

$\dot{x} = -y, \quad \dot{y} = 4x.$

a) Find its general solution and its flow.

b) Study the type and stability of its equilibrium point $(0,0)$.

c) Find the shape of the orbits by using the differential equation of the orbits.

d) Represent its phase portrait.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = -x^3.$$

Find $\varphi(t,0)$ and determine the monotonicity of $\varphi(t,1)$ and $\varphi(t,-1)$.

2. Consider the following planar system

$$\dot{x} = -y - x(x^2 + y^2), \quad \dot{y} = x - y(x^2 + y^2). \tag{0.1}$$

a) Does system (0.2) have other equilibrium points besides $(0,0)$? Justify.

b) Use the system obtained by passing to polar coordinates to determine the shape of the orbits. Indicate the type and stability character of the equilibrium point $(0,0)$.

c) Apply the linearization method to system (0.2) in order to study the type and stability character of $(0,0)$. Is this a contradiction to the results obtained at the previous point? Justify.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = 2 - x^2.$$

Determine the stability character of the equilibrium points using the linearization method. Find $\varphi(t, \sqrt{2})$ and determine the monotonicity of $\varphi(t, -1.5)$, $\varphi(t, 0)$ and $\varphi(t, 2)$.

2. Let $a, b > 0$ be real parameters and consider the following planar system

$$\dot{x} = ax - by, \quad \dot{y} = -bx + ay.$$

a) Find the equilibrium points. Take into account that you may need to consider two distinct cases: $a \neq b$ and $a = b$.

b) For $a \neq b$, determine the type and stability character of the equilibrium points.

c) For $a = b$, find $\varphi(t, 1, 1)$ and $\varphi(t, -1, -1)$. Then find the shape of the orbits by using the differential equation of the orbits.

d) Find the general solution of the system.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = 1 - x^2.$$

Determine the stability character of the equilibrium points using the linearization method. Find $\varphi(t, 1)$ and determine the monotonicity of $\varphi(t, -2)$, $\varphi(t, 0)$ and $\varphi(t, 2)$.

2. Give an example of a liniar planar system that has an asymptotically stable node at the origin. Justify.

3. Consider the following planar system

$$\dot{x} = 2x - x^2, \quad \dot{y} = y - xy^2.$$

a) Find the equilibrium points.

b) Indicate the type and stability character of every equilibrium point.

c) Find $\varphi(t, 2, 1/2)$, $\varphi(t, 2, 0)$ and $\varphi(t, 0, 2)$.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = ax - 1, \quad \text{where } a \in \mathbb{R}^* \text{ is a parameter.}$$

Determine the stability character of the equilibrium point using the linearization method. Find $\varphi(t, 1/a)$ and determine the monotonicity of $\varphi(t, 1)$. Take into account that you may need to consider distinct cases depending on a.

2. Consider the following linear planar system

$$\dot{x} = 4y, \quad \dot{y} = -x.$$

a) Find its general solution and its flow.

b) Study the type and stability character of its equilibrium point $(0, 0)$.

c) Find the shape of the orbits in two ways: by using the definition of the orbit and then by using the differential equation of the orbits.

d) Represent its phase portrait.

e) What remarkable property have the solutions of this system?

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = -\frac{1}{5}x + x^2 - x^3.$$

Find $\varphi(t, 0)$, $\varphi(t, (5-\sqrt{5})/10)$ and determine the monotonicity of $\varphi(t, 1/4)$, $\varphi(t, 1/2)$ and $\varphi(t, 1)$.

2. Find the general solution of the following linear planar system

$$\dot{x} = 2x + y, \quad \dot{y} = x + 2y.$$

3. Consider the following planar system

$$\dot{x} = x - 2\,x\,y, \quad \dot{y} = x - y.$$

a) Find the equilibrium points.

b) Indicate the type and stability character of every equilibrium point.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = -\frac{1}{4}x + x^2 - x^3.$$

Find $\varphi(t, 0)$ and determine the monotonicity of $\varphi(t, 1/4)$, $\varphi(t, 1/2)$ and $\varphi(t, 1)$.

2. Consider the following planar system

$$\dot{x} = -y(x^2 + y^2), \quad \dot{y} = x(x^2 + y^2). \tag{0.2}$$

a) Does system (0.2) have other equilibrium points besides $(0, 0)$? Justify.

b) Decide whether the equilibrium point $(0, 0)$ is hyperbolic or not. If it is, apply the linearization method.

c) Find the shape of the orbits by passing to polar coordinates. Justify.

d) Find the shape of the orbits by using the differential equation of the orbits. Justify.

e) Represent the phase portrait of system (0.2).

f) What remarkable property have the solutions of system (0.2)?