Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - VII

1. (2.5p) We consider the difference equation

$$x_{k+2} + x_k = \cos \frac{k\pi}{2} \ .$$

a) Find a solution of the form $(x_k)_p = ak \cos \frac{k\pi}{2}$, with $a \in \mathbb{R}$. (Hint: we remind that $\cos(x + \pi) = -\cos x$ for any $x \in \mathbb{R}$)
b) Find its general solution.
c) Find the solution with $x_0 = x_1 = 0$ and describe its long-time behavior.

2. (2p) We consider the difference system

$$x_{k+1} = \frac{3}{5}x_k + \frac{1}{5}y_k, \quad y_{k+1} = \frac{1}{5}x_k + \frac{3}{5}y_k.$$

a) Study the stability of this linear system.
b) Find the general solution.

3. (1p) We consider the differential equation $x' = -2x + e^{3t}$.
a) Find a solution of the form $x_p = ae^{3t}$, with $a \in \mathbb{R}$.
b) Find its general solution.
c) Find the solution with $x(0) = 0$.