

Exam on Dynamical Systems.  
June 10, 2011

1. Find the general solution of the following differential equations
  - a) (1p)  $x' - 3x = 5t$ .
  - b) (1p)  $x' = y, \quad y' = -x - 2y$ .
  - c) (0.5p)  $y' = 2y/x$ .
2.
  - a) (0.5p) Write the statement of the existence and uniqueness theorem for first order nonlinear differential equations.
  - b) (0.5p) Prove that the Initial Value Problem  $y' = 1 - 4y^2, y(0) = 1/2$  has a unique solution and than name it.
  - c) (0.5p) Give an example of an Initial Value Problem for which existence and uniqueness theorem is not applicable. Justify.
3. (2p) We consider the nonlinear planar autonomous system

$$\dot{x} = -x + xy, \quad \dot{y} = -4y + 8xy.$$

Find its equilibria and study their stability.