Dynamical systems. Final exam 26-01-2005

1. We consider the system $\dot{x} = x\,,\quad \dot{y} = 1 + y\,.$ Write its general solution and represent its phase portrait.

2. Represent the phase portrait of $\dot{x} = 2x - \sin x\,.$

3. Write the general solution of $y'' - a^2 y = e^{bx}$, where $a > 0$ and $b \in \mathbb{R}$ are parameters.

4. a) Verify that $y_1 = x$ and $y_2 = e^{-2x}$ are solutions of
$(2x+1)y'' + 4xy' - 4y = 0\,.$
b) Find the maximal solution of the Initial Value Problem:
$(2x+1)y'' + 4xy' - 4y = (2x+1)^2\,,\quad y(0) = 1\,,\quad y'(0) = 0.$

5. Write the definitions for a fixed point of a scalar map for an asymptotically stable fixed point.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map and $\eta\,, \eta^* \in \mathbb{R}$ be such that $f^n(\eta) \to \eta^*$ as $n \to \infty$. Prove that η^* is a fixed point of f.

7. Let $\eta \in \mathbb{R}$ be such that $|\eta|$ is sufficiently small. Study the convergence of the sequence given by the recurrence

$$x_{n+1} = \frac{1}{2}x_n - 3x_n^3 \quad , n \geq 0,\ x_0 = \eta.$$