Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
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1. (1.5p) Find the general solution of the differential equation:
(a) $t^2x'' - 3tx' + 3x = 0$, for $t \in (0, \infty)$;
(b) $t^2x'' + tx' + 4x = 0$, for $t \in (0, \infty)$.

2. (3p) We consider the differential equation:

$$y' = -\frac{x}{2y}.$$

(a) (True or False) "Through the point $(1, 1) \in \mathbb{R}^2$ passes one and only one integral curve of the given differential equation." Justify the answer.
(b) Find the maximal solution of the Initial Value Problem for the given differential equation with the condition $y(1) = 1$. Plot its graph.
(c) Represent the 3-level curve of the function $H : \mathbb{R}^2 \to \mathbb{R}$, $H(x, y) = x^2 + 2y^2$. What is the relation between this curve and the one plotted at (b)? What is the relation between H and the given differential equation?

3. (2p) Represent the phase portrait of:
(a) $\dot{x} = 4x - x^3$;
(b) $\dot{x} = 4x - x^3 + 1$;
(c) $\dot{x} = 4x - x^3 + 5$.