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Exercise Sheet no.6

Analysis for CS

GROUPWORK:

(G 15)

1) Decide whether the following subsets of $\mathbb{R}$ are neighborhoods of 0 or not, and motivate your answer: a) $[-1,1)$, b) $\mathbb{Q}$, c) $\bigcap_{n\in\mathbb{N}^*}[-\frac{1}{n},\frac{1}{n}]$.

2) Let $A\subseteq\mathbb{R}$. Determine the set M of all reals x with the property that $A\in\mathcal{V}(x)$, where A is:
a) $A=[0,1]$, b) $A=(-\infty,-1)$, c) $A=(0,1]\cup[2,3]$, d) $A=\mathbb{R}$, e) $A=\mathbb{N}$.

3) Determine A', where A is: a) $A=\mathbb{Q}$, b) $A=(-\infty,1)\cup(2,\infty)$, c) $A=\mathbb{Z}$.

4) Finish the proof of **L1** in the 6th lecture.

(G 16)

Determine the one-sided limits of the function $f\colon D\to\mathbb{R}$ (with $D\subseteq\mathbb{R}$ the maximal domain of f) at $\alpha=1$, where

(1) $f(x)=e^{\frac{1}{x^2-1}}$, (2) $f(x)=e^{\frac{x^2-2}{x-1}}$, (3) $f(x)=e^{1+\frac{2}{|x-1|}}$, (4) $f(x)=\frac{|x|-1}{x-1}$.

(G 17)

Study the continuity of the following functions ($n\in\mathbb{N}$) and determine the type of their discontinuities:

$f\colon\mathbb{R}\to\mathbb{R}$, $f(x)=\lim\limits_{n\to\infty}\dfrac{e^{nx}}{1+e^{nx}}$, and $g\colon\mathbb{R}\setminus\{-1\}\to\mathbb{R}$, $g(x)=\lim\limits_{n\to\infty}\dfrac{x^n+x}{x^{2n}+1}$.

HOMEWORK:

(H 17)

Compute the following limits: (1) $\lim\limits_{x\to4}(-x^3+5x)$, (2) $\lim\limits_{x\to-\infty}(-x^3+2x)$, (3) $\lim\limits_{x\to-3}\dfrac{x^2-9}{(x+3)^2}$,

(4) $\lim\limits_{x\to\infty}\dfrac{3x^k+5}{8x^3-2}$, with $k\in\mathbb{N}$, (5) $\lim\limits_{x\to0}\dfrac{e^{2x}-1}{x}$, (6) $\lim\limits_{x\to0}\left(\dfrac{1+4x+x^2}{1+x}\right)^{\frac{1}{x}}$,

(7) $\lim\limits_{x\to1}\dfrac{x^2-1}{x^2+x-2}$, (8) $\lim\limits_{\substack{x\to1\\x>1}}\left(\dfrac{1}{1-x}-\dfrac{1}{x^3-1}\right)$, (9) $\lim\limits_{x\to\infty}(x-\sqrt{x^2-1})$, (10) $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+1}}$,

(11) $\lim\limits_{x\to-\infty}\dfrac{x}{\sqrt{x^2+1}}$, (12) $\lim\limits_{x\to1}\dfrac{x^3+x^2-x-1}{x^2-1}$, (13) $\lim\limits_{x\to0}\dfrac{1-\sqrt{1-x^2}}{x^2}$, (14) $\lim\limits_{x\to0}\dfrac{x^2}{|x|}$,

(15) $\lim\limits_{x\to\infty}\sqrt{x}(\sqrt{x+1}-\sqrt{x})$, (16) $\lim\limits_{x\to\infty}\dfrac{(-1)^{[x]}}{x}$, where $[x]$ denotes the largest integer not greater than x,

(17) $\lim\limits_{x\to-\infty}e^{\frac{|x|+1}{x-1}}$, (18) $\lim\limits_{x\to-\infty}\left(\dfrac{x^2+x+1}{x^2-x+1}\right)^{\sqrt{-x}}$, (19) $\lim\limits_{x\to0}\dfrac{\sqrt[3]{1+x}-1}{x}$.