Dr. Brigitte Breckner
Dr. Anca Grad

"Babeş–Bolyai" University, Cluj-Napoca
Faculty of Mathematics and Computer Science

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Exercise Sheet no.14

Analysis for CS

GROUPWORK:

(G 33) (Integration over normal domains)

Let $\emptyset \neq M \subseteq \mathbb{R}^2$ be bounded and let $f\colon M \to \mathbb{R}$ be continuous. Represent M in a Cartesian coordinate system and compute $I := \int\int_M f(x,y)dxdy$ in the following cases:

a) $M = \{(x,y) \in \mathbb{R}^2 \mid\ 0 \leq y \leq 1,\ -1 \leq x \leq y\}$, $f(x,y) = xy - y^3$,

b) $M =$ the domain in the first quadrant which lies between the line $y = x$ and the parabola $y = x^2$, $f(x,y) = xy$,

c) $M = \{(x,y) \in \mathbb{R}^2 \mid\ 0 \leq x \leq 1,\ 0 \leq y \leq x\}$, $f(x,y) = y + \sin(\pi x^2)$.

(G 34) (Improper integrals)

Using the formula of Leibniz-Newton for improper integrals, study the improper integrability of the following functions on their domains and, in case they are improperly integrable, compute the corresponding improper integrals.

a) $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = e^{-2x}$,

b) $f\colon [2,\infty) \to \mathbb{R}$, $f(x) = \frac{1}{x(\ln x)^\alpha}$, where $\alpha \in \mathbb{R}$ is a parameter.

(G 35) (Limits of real-valued functions of several variables)

1) Show that, in each of the following cases, the function $f\colon \mathbb{R}^2 \setminus \{0_2\} \to \mathbb{R}$ does not have a limit at 0_2:

a) $f(x,y) = \frac{y^2}{x^2+y^2}$, b) $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$.

2) Show that the function $g\colon \mathbb{R}^2 \setminus \{0_2\} \to \mathbb{R}$, defined by $g(x,y) = \frac{xy^3}{x^2+y^2}$, has a limit at 0_2 and determine this limit.

(G 36) (Pythagoras' theorem in $\mathbb{R}^n$)

Let x, $y \in \mathbb{R}^n$ be two orthogonal vectors, i.e., $\langle x, y\rangle = 0$. Prove that then the equality

$$||x+y||^2 = ||x||^2 + ||y||^2$$

holds true.