Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - IV

1. (1.5p) Find the general solution of the difference equation

$$x_{k+2} - 6x_{k+1} + 9x_k = 12k,$$

Hint: look for a particular solution of the form $(x_k)_p = ak + b$, with $a, b \in \mathbb{R}$.

2. (1p) How many solutions have each of the following problems:

$$\begin{array}{ll} \text{(i)} & x_{k+2} + k^2 x_k = 0, \quad x_0 = 0; \\ \text{(ii)} & x_{k+2} + k^2 x_k = 0, \quad x_0 = 0, \quad x_1 = 0; \\ \text{(iii)} & x_{k+2} + k^2 x_k = 0, \quad x_0 = 0, \quad x_1 = 0, \quad x_2 = 1? \end{array}$$

3. (1.5p) Find the general solution of each of the following differential equations and describe the long term behavior of such a solution:

$$\begin{array}{ll} \text{(i)} & x' = -5x; \\ \text{(ii)} & x' = -5x + 1; \\ \text{(iii)} & x'' + x' + x = 0. \end{array}$$

4. (1.5p) Find the fixed points and the 2–periodic points of the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 1 - 2x^2$.