

Exam on Dynamical Systems.  
June 11, 2008

1. Find the general solutions of the following differential equations:

$$x' = -x, \quad x' = 3x + 2 - 3t + e^{-3t}, \quad x'' - x' + 2x = 0, \quad x''' = 0.$$

2. We consider the differential equation

$$y' = \frac{1 - \sqrt[3]{y}}{1 - xy}$$

and three Initial Value Problems for it with the conditions:  $y(0) = 1$ ,  $y(1) = 1$  and  $y(0) = 0$ , respectively. Here the unknown function is  $y = y(x)$ .

- Are the above Initial Value Problems well-defined?
- If they are well-defined, decide whether or not the Local Existence and Uniqueness Theorem is applicable.
- If the Local Existence and Uniqueness Theorem is applicable, find the solution.

3. Find the differential equation of the family of planar curves described by  $x^2 + 9y^2 = c$ ,  $c \in \mathbb{R}$ . Find also a planar autonomous system whose trajectories are these curves.

4. We consider the logistic map  $f_\lambda : [0, 1] \rightarrow [0, 1]$   $f_\lambda(x) = \lambda x(1 - x)$ , where  $\lambda \in (0, 4)$  is a parameter. Find the fixed points of the logistic map and study their stability (discuss with respect to the parameter  $\lambda$ ).