Practical theory

Chapter 1

f - inj $\Leftrightarrow f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

f surj $\Leftrightarrow \forall y \in B, \exists x \in A \Rightarrow f(x) = y$

Unitary ring

- ring
- $\exists$ $i \in R$ id. el in the semigroup $(R, \cdot)$

Ring

$(R, +, \cdot)$
- $(R, +)$ - group
- operation (ex. $\forall f, g \in R \Rightarrow f \cdot g \in R$) lege de comp.
- distributivity $b(a+c) = ba + bc$
  $(a+c)b = ab + cb$

Division ring

- $(R, +)$ - abelian group
- $(R^*, \cdot)$ - group, $R \setminus \{0\} = R^*$ (id. el)
- distributivity

Field

- commutative division ring

Subgroup

$A \subseteq G$ subgroup if

$A \neq \Phi$

$\forall x_1, x_2 \in A \Rightarrow x_1 \cdot x_2 \in A$

$\forall x \in A \Rightarrow x^{-1} \in A$

$\Big\}$ $x \cdot x_2^{-1} \in A$

Subring

$A \subseteq R$ subring if

$A \neq \Phi$

$\forall x_1, x_2 \in A \Rightarrow x_1 - x_2 \in A$

$\forall x_1, x_2 \in A \Rightarrow x_1 \cdot x_2 \in A$

Group homomorphism

$f(x_1 \cdot x_2) = f(x_1) \cdot f(x_2)$

Ring homomorphism

$f(x_1 \cdot x_2) = f(x_1) \cdot f(x_2)$

$f(x_1 + x_2) = f(x_1) + f(x_2)$

Chapter 2

Vector space

$(K,+,\cdot)$ field 1) $(V,+)$ ab. group
$K \times V \to V$

2) i) $(\alpha+\beta)x = \alpha x + \beta x$

ii) $\alpha(x+y) = \alpha x + \alpha y$

iii) $\alpha(\beta x) = (\alpha\beta)x$

iv) $1 \cdot x = x$

Subspace

$A \leq_K V$

i) $A \neq \emptyset$

ii) $\forall x, y \in A \Rightarrow x+y \in A$

iii) $\forall \alpha \in K, \forall x \in A \Rightarrow \alpha x \in A$

generated subspaces

$\langle X \rangle = \{\alpha_1 x_1 + \dots + \alpha_n x_n\}$

Direct sum

$S \oplus T = V$

$\begin{cases} S \cap T = \{0\} \\ S+T = V \end{cases}$

Linear map

f - linear map:

1. $f(x_1 + x_2) = f(x_1) + f(x_2)$
2. $f(\alpha x) = \alpha f(x)$

Isomorphism - bijective lin. map

Endomorphism - lin. map with $V = V$

Automorphism - bijective endomorphism

$\operatorname{Ker} f = \{x \in V \mid f(x) = 0\}$

$\operatorname{Im} f = \{z \mid f(x) = z\}$

Linear independence

v_1, v_2, v_3 lin. indep $\Rightarrow \det A \neq 0$

$\alpha v_1 + \beta v_2 + \gamma v_3 \Rightarrow \alpha = \beta = \gamma = 0$

lin dep $\Rightarrow \det A = 0$

Basis

v_1, v_2, v_3 ...

Eingen Value / Vector

$x \in V$ eingenvector if $(f - \lambda\, I_V)(x) = 0$

$P_f(\lambda) = (f - \lambda\, I)$

Valori proprii

1. $P_f(\lambda) = [f] - \lambda\, I_V$

2. $|P_f(\lambda)| = 0$ sol gasite sunt λ (val proprii)

3. $(f - \lambda_i I_V) \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

Chapter 3

Sist liniar → compatibil
- omogen
  - banală
  - nebanală
    - $m = n \Rightarrow \Delta = 0$
    - $m < n$, rang $< n$
- neomogen $\xrightarrow{\text{k.r.C}}$
  - $r = n$ (det)
  - $r < n$ (nedet)
- neomogen → Rouché: rang A = rang $\bar{A}$
  - $\Delta p \neq 0$
  - $\Delta c = 0$
  - $r = n$ (det)
  - $r < n$ (nedet)

Sist liniar → incompatibil
- $\Delta p \neq 0$, $\exists\, \Delta c = 0$
- rang $A \neq$ rang $\bar{A}$

The matrix of a linear map

V sp vect peste k

$B = (v_1, \dots, v_n)$

$V = k_1 v_1 + \dots + k_n v_n$

$$[V]_B = \begin{pmatrix} k_1 \\ \vdots \\ k_n \end{pmatrix}$$

$B = (v_1, \dots, v_n)$

$B' = (v_1', \dots v_n')$

$f : V \to V'$

$f(v_1) = \alpha_1 v_1' + \dots + \alpha_n v_n'$

$f(v_n) = \alpha_1^m v_1' + \dots + \alpha_n^m v_n'$

$$[f]_{BB'} = \begin{vmatrix} \alpha_1^1 & \dots & \alpha_1^m \\ \vdots & & \\ \alpha_n^1 & & \alpha_n^m \end{vmatrix}$$

$[\alpha f]_B = \alpha [f]_B$

$[f+g]_B = [f]_B + [g]_B$

$[f \circ g]_{BB''} = [f]_{BB'} [g]_{B'B''}$

$E = (e_1, e_2, e_3)$

$e_1 = (1, 0, 0)$

$e_2 = (0, 1, 0)$

$e_3 = (0, 0, 1)$

$T_{BB'} = (T_{B'B})^{-1}$

$[v]_{B'} = T_{B'B} \cdot [v]_B$