Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - VIII

1. (1.5p) Find the general solution of
(i) $x' + 4x = 0$; (ii) $x' + tx = 0$; (iii) $x'' + 2x' + x = 0$.
Here the unknown is the function x of variable t.

2. (1p) We consider the differential equation

$$y' = 1 - \frac{x}{y^2} \ .$$

Compute the slope of its direction field in the points $(0, 1)$ and, respectively, $(1, 1)$. What type of curve is the 1-isocline, respectively, the 0-isocline?

3. (1p) Find (directly) the solution of

$$x_{k+2} + x_{k+1} + x_k = 0, \quad x_0 = 0, \quad x_1 = 1 \ .$$

Describe its long-term behavior.

4. (2p) We consider the IVP $x' = -200x, \quad x(0) = 1$.
a) Find the solution and its limit as $t \to \infty$.
b) Write the Euler's numerical formula with constant step-size h.
c) Find a range of values for the step-size h such that the solution $(x_k)_{k\geq 0}$ of the difference equation found at b) satisfies $\lim\limits_{k\to\infty} x_k = 0$.