Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - V

1. (2.5p) We consider the planar differential system

$$x' = -4y, \quad y' = x .$$

a) Find its general solution.
b) Specify the type and stability of this linear system.
c) Represent its phase portrait. What type of curves are the orbits?

2. (1.5p) We consider the initial value problem

$$y' = 1 - xy^2, \quad y(0) = 0.$$

Write the Euler numerical formula for this IVP, in the interval $[0, 1]$, with constant stepsize $h = 0.02$. Compute the approximate values in $x = 0.02$ and, respectively $x = 0.04$. How many steps there are needed in order to calculate the approximate value in $x = 1$?

3. (1.5p) We consider the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x - \frac{1}{4}(x^2 - 2)$ and, given $x_0 \in \mathbb{R}$, consider the sequence $(x_k)_{k \geq 0}$ satisfying the recurrence

$$x_{k+1} = f(x_k) .$$

a) Find the fixed points of f, and study their stability.
b) Find $(x_k)_{k \geq 0}$ when $x_0 = \sqrt{2}$.
c) There exists some $x_0 \in \mathbb{R} \setminus \{\sqrt{2}\}$ such that $\lim_{k \to \infty} x_k = \sqrt{2}$?
d) There exists some $x_0 \in \mathbb{R}$ such that $\lim_{k \to \infty} x_k = 2$?