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Exam on Dynamical Systems.
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1. Let $\alpha \in \mathbb{R}$. We consider the differential equation $x'' + \alpha x' + 9x = 0$.

(a) Find the general solution when $\alpha = 4$ and $\alpha = 0$, respectively.

(b) Find α such that all the solutions are periodic. What is the period in this case? Does it depend on α?

2. We consider the Initial Value Problem $x' + \alpha(t)x = f(t)$, where $\alpha, f \in C(\mathbb{R})$.

(a) Find the solution when $\alpha(t) = 2t$ and $f(t) = 3e^{-t^2}$.

(b) Find the solution (eventually only an integral representation of it) when $\alpha(t) = 2t$ and $f(t) = 1$.

(c) Write an integral representation of the solution of this IVP for arbitrary α and f.

3. Find a first integral for the differential equation

$$(5x - 2xy)dx + (3y^2 - x^2)dy = 0\,.$$

4. Represent the phase portrait of the following differential equation:

$$\dot{x} = \frac{1}{2}x(1 - x)\,.$$

5. We consider the map $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{2}x(1 - x)$. Find its fixed points and study their stability. For what type of values of $\eta \in \mathbb{R}$ we can deduce from the above study that the sequence $(x_n)_{n\geq 0}$ given by the recurrence $x_{n+1} = \frac{1}{2}x_n(1 - x_n)\,, n \geq 0, \quad x_0 = \eta$ is convergent?