Dr. Brigitte Breckner
Dr. Anca Grad

"Babeş–Bolyai" University, Cluj-Napoca
Faculty of Mathematics and Computer Science

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Exercise Sheet no.12

Analysis for CS

GROUPWORK:

(G 29)

Study the improper integrability of the following continuous functions, using the second comparison criteria for improper integrals.

a) $f\colon [1,\infty) \to \mathbb{R},\ f(x) = \frac{1}{x\sqrt{1+x^2}}$, b) $f\colon [0,\frac{\pi}{2}) \to \mathbb{R},\ f(x) = \frac{1}{\cos x}$,

c) $f\colon (0,\infty) \to \mathbb{R},\ f(x) = \left(\frac{\operatorname{arctg} x}{x}\right)^2$, d) $f\colon (1,\infty) \to \mathbb{R},\ f(x) = \frac{\ln x}{x\sqrt{x^2-1}}$,

e) $f\colon [0,1) \to \mathbb{R},\ f(x) = \frac{1}{\sqrt{(1-x^2)(1-a^2x^2)}}$, where $a \in (-1,1)$ is fixed.

(G 30)

Using the integral criterion, decide whether the following series are convergent or not:

a) $\displaystyle\sum_{n\geq 2} \frac{1}{n(\ln n)^2}$, b) $\displaystyle\sum_{n\geq 2} \frac{\ln n}{n^2}$, c) $\displaystyle\sum_{n\geq 1} \frac{1}{\sqrt{1+e^n}}$.

HINT for a) and b): Use the formula of Leibniz-Newton for improper integrals in order to study the improper integrability of the functions associated with the series given at a) and b).

HOMEWORK:

(H 31) (To be delivered in the next exercise-class)

Study the improper integrability of the following continuous functions, using the second comparison criteria for improper integrals.

a) $f\colon (0,\infty) \to \mathbb{R},\ f(x) = \frac{\operatorname{arctg} x}{x(1+x^2)}$,

b) $f\colon [x_0,\infty) \to \mathbb{R},\ f(x) = \frac{1}{\sqrt{x(x-a)(x-b)}}$, where $x_0 > a > b > 0$ are fixed.

(H 32) (To be delivered in the next exercise-class)

Determine all local extrema, their type (minima or maxima) and the corresponding extreme values of the function $f\colon \mathbb{R}^* \times \mathbb{R}^* \to \mathbb{R},\ f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.

(H 33) (Train your brain)

Having the proof of **Th2** as a model, prove **Th4** in the 12th lecture.