TICKET

1. The semantics of propositional logic.
   Using a semantic proof method, prove that the syllogism rule is a tautology.

2. Check the distributivity property of the existential quantifier over implication using a syntactic proof method. The theorem of soundness and completeness of this method.

3. Draw a logical circuit having 3 inputs and containing all basic and derived gates. Write the corresponding Boolean function and simplify it. Draw the simplified circuit.

TICKET

1. Using lock resolution check if: $p \to q, \neg(q \to r) \to \neg p \vdash p \to r$.
   Resolution as a formal system.

2. Using a semantic proof method check if the formula: $(\forall x)A(x) \vee (\forall x)B(x)$ is a logical consequence of the formula: $(\forall x)(A(x) \wedge B(x))$. Theory.

3. Simplify the following Boolean function using Veitch diagram: $f(x_1, x_2, x_3) = x_1 x_3 \vee x_1 x_2 x_3 \vee \bar{x}_1 x_3 \vee \bar{x}_1 \bar{x}_2 x_3$. Implement the logical circuits corresponding to the initial form of *f* and to all simplified forms of *f*.

TICKET

1. The formal (axiomatic) system of propositional logic. What is a theorem?
   Using a syntactic and direct method prove that the second axiom of propositional logic is a theorem.

2. Using linear resolution check if the following set of formulas is inconsistent.
   $S = \{p(x) \wedge q(x) \vee r(x), \neg q(y) \vee r(y), r(a) \wedge \neg p(a)\}$. Theory.

3. Using Quine's method simplify the Boolean function:
   $f(x_1, x_2, x_3, x_4) = x_1 \bar{x}_2 x_3 \vee x_1 x_2 x_3 \bar{x}_4 \vee x_1 x_2 \bar{x}_3 x_4 \vee x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \vee \bar{x}_1 x_2 x_3 \bar{x}_4$.
   Implement the logical circuit associated to a simplified form of *f*.

TICKET

1. Write all the models and anti-models of the formula: V= $((p \wedge \neg r) \to q) \to \neg p \wedge \neg q \wedge r$.
   Theory.

2. Evaluate the formula U= $((\forall x)p(x) \to (\forall x)q(x)) \to (\forall x)(p(x) \wedge q(x))$ in 2 interpretations:
   one with a finite domain and the other with an infinite domain. How many possible interpretations has U? Is predicate logic decidable? Argue your answer.

3. Definitions for: minterm, maxterm, central monom , maximal monom, factorization.
   Examples of 4 minterms and 4 maxterms of 3 variables: expressions, notations and tables of values. Draw the logical circuit associated to the Boolean function:
   $f(x_1, x_2, x_3, x_4) = m_1 \vee m_{13} \vee m_8 \vee m_5$.