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Exercise Sheet no.2

Analysis for CS

GROUPWORK:

(G 6)

Using the rules of calculation for limits, compute the limit of the sequences having the general term defined as follows:

a) $\frac{5^n+1}{7^n+1}$, b) $\frac{4^n+(-2)^n}{4^{n-1}+2}$, c) $\left(\sin\frac{\pi}{10}\right)^n$, d) $\sqrt{9n^2+2n+1}-3n$, e) $\left(5+\frac{1-2n^3}{3n^4+2}\right)^2$,

f) $\sqrt{n^2+3}-\sqrt{n^3+1}$, g) $\left(\frac{n^3+5n+1}{n^2-1}\right)^{\frac{1-5n^4}{6n^4+1}}$, h) $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\ldots\left(1-\frac{1}{n}\right)$.

(G 7)

Study the monotonicity, the boundedness properties and the convergence of the sequence $(x_n)_{n\in\mathbb{N}^*}$ in each of the following cases, also motivating your statements:

a) $x_n=\frac{2^n+3^n}{5^n}$, b) $x_n=\frac{(-1)^n}{n}$, c) $x_n=\frac{2^n}{n!}$, d) $x_n=\frac{n}{n^2+1}$.

(G 8) (Train your brain)

Treat cases 3 and 4 in the proof of **Th2** in the second course.

HOMEWORK:

(H 8) (To be delivered in the next exercise-class)

1) Using the rules of calculation for limits, compute the limit of the sequences having the general term defined as follows:

a) $\frac{3^n}{4^n}$, b) $\frac{2^n+(-2)^n}{3^n}$, c) $\frac{5-n^3}{n^2+1}$, d) $\left(2+\frac{4^n+(-5)^n}{7^n+1}\right)^{2n^3-n^2}$, e) $\frac{1+2+\cdots+n}{n^2}$,

f) $\left(\frac{n^3+4n+1}{2n^3+5}\right)^{\frac{-2n^4+1}{n^4+3n+1}}$, g) $(\cos(-2013))^n$, h) $\left(\frac{n^5+3n+1}{2n^5-n^4+3}\right)^{\frac{3n-n^4}{n^3+1}}$.

2) Study the monotonicity, the boundedness properties and the convergence of the sequence $(x_n)_{n\in\mathbb{N}}$, where $x_n=\sqrt{n+1}-\sqrt{n}$, and motivate your statements.

(H 9)

Study the convergence of the sequence with general term defined as follows

$$a_n=\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\ldots\left(1-\frac{1}{n^2}\right),\ n\geq 2.$$

In case the sequence is convergent, determine its limit.

(H 10) (Train your brain)

Using the density property of the set $\mathbb{Q}$ (see **Th5** in the first course), prove that every real number is the limit of a sequence of rational numbers. Moreover, prove that this sequence can be chosen to be increasing (or decreasing).