# Rejection Regions and P - Values

At the $\alpha \in (0,1)$ significance level, using the test statistic $TT$, with observed value $TT_0 = TT(\theta = \theta_0)$, to test

$$\mathrm{H}_0 : \theta \;=\; \theta_0, \quad \text{versus}$$

**a.** $\mathrm{H}_1 : \theta \;<\; \theta_0$ (left-tailed test):

$$RR = (-\infty, tt_\alpha), \quad P = P(TT < TT_0).$$

**b.** $\mathrm{H}_1 : \theta \;>\; \theta_0$ (right-tailed test):

$$RR = (tt_{1-\alpha}, \infty), \quad P = P(TT > TT_0).$$

**c.** $\mathrm{H}_1 : \theta \;\neq\; \theta_0$ (two-tailed test):

$$RR = (-\infty, tt_{\frac{\alpha}{2}}) \cup (tt_{1-\frac{\alpha}{2}}, \infty), \quad P = 2\min\{P(TT < TT_0), P(TT > TT_0)\} \Big( \stackrel{\text{sym}}{=} 2P(TT > |TT_0|) \Big).$$

**1.** For a population mean, $\theta = \mu$,

– large sample ($n > 30$) or normal underlying population and $\sigma$ known, $TT = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \in N(0,1)$;

– large sample ($n > 30$) or normal underlying population $TT = \dfrac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}} \in T(n-1)$.

**2.** For a population variance, $\theta = \sigma^2$, for a normal underlying population, $TT = \dfrac{(n-1)s^2}{\sigma^2} \in \chi^2(n-1)$.

**3.** For the difference of two population means, $\theta = \mu_1 - \mu_2$, for large samples ($n_1 + n_2 > 40$) or normal underlying populations and independent samples,

– $\sigma_1, \sigma_2$ known, $TT = \dfrac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \in N(0,1)$;

– $\sigma_1 = \sigma_2$, unknown, $TT = \dfrac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \in T(n_1 + n_2 - 2)$, where $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$;

– $\sigma_1 \neq \sigma_2$, unknown, $TT = \dfrac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \in T(n)$, with

$$\frac{1}{n} = \frac{c^2}{n_1 - 1} + \frac{(1-c)^2}{n_2 - 1} \quad \text{and} \quad c = \frac{\dfrac{s_1^2}{n_1}}{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}.$$

**4.** For the ratio of two population variances, $\theta = \dfrac{\sigma_1^2}{\sigma_2^2}$, for normal underlying populations and independent samples, $TT = \dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \in F(n_1 - 1, n_2 - 1)$.