Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 10, 2009
II

1. (2.5p) Describe the oscillations of a spring-mass system governed by the differential equation

$$x'' + x = cos(\omega t)$$

and subjected to the initial conditions $x(0) = x'(0) = 0$. Here $\omega > 0$.

2. (2p) Find the general solution of the differential equation:

$$y' - \frac{x}{2(x^2 - 1)}y = \frac{x}{2y} \quad \text{for} \quad x \in (1, \infty).$$

3. (0.5p) Find the differential equation of the family of planar curves $y = ax^2$, $a \in \mathbb{R}$.

4. (1.5p) We consider the Initial Value Problem

$$y' = -y, \quad y(0) = 2.$$

(a) Write the Volterra integral equation equivalent.
(b) Write the recurrence formula for the Picard sequence of succesive approximations, denoted $(\varphi_n)_{n \geq 0}$.
(c) Starting with $\varphi_0(x) = 2$ for all $x \in \mathbb{R}$, calculate $\varphi_1(x)$ and $\varphi_2(x)$.