Seminar Test

1. Represent the phase portrait of the scalar dynamical system $\dot{x} = 1 - x^2$. Find $\varphi(t, 1)$ and justify. Specify the properties of the functions $\varphi(t, 2)$ and, respectively, $\varphi(t, 0.5)$.

2. Find the general solution of the linear system $\dot{x} = -y, \quad \dot{y} = x$.

3. We consider the planar system

$\dot{x} = -y + x(1 - x^2 - y^2),\ \dot{y} = x + y(1 - x^2 - y^2)$.

a) Study the type and stability of the equilibrium point $(0, 0)$ using the linearization method. There are other equilibria?

b) Verify that $\varphi(t, 1, 0) = (\cos t, \sin t)$ for all $t \in \mathbb{R}$. What is the shape of the corresponding orbit? Represent this orbit in the plane and insert the arrow.

Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2015/2016

Seminar Test

1. Let $0 < c < 1$ be a parameter and consider the scalar dynamical system $\dot{x} = x(1-x) - cx$.

a) Find its equilibria and study their stability using the linearization method.

b) Represent its phase portrait.

c) When $x(t) > 0$ is considered to be the number of fish in some lake, and $0 < c < 1$ to be the rate of fishing, try to predict the fate of the fish from the lake interpreting the theoretical result obtained at a) and b).

2. We consider the following linear planar system

$\dot{x} = -6x, \quad \dot{y} = -3y.$

a) Find its general solution and its flow.

b) Study the type and stability of this linear system.

c) Find the shape of the orbits in two ways: by using the definition of the orbit and then by using the differential equation of the orbits.

d) Represent its phase portrait.

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Seminar Test

1. We consider the linear planar system $\dot{x} = -3x + y,\ \dot{y} = y$.

a) Find the eigenvalues and the determinant of the matrix of the system. Specify the type and stability of this linear system.

b) Find its general solution.

c) Find the expression of its flow $\varphi(t, \eta_1, \eta_2)$.

d) Find $\varphi(t, 0, 0)$, $\varphi(t, 1, -2)$, $\varphi(t, 3, -6)$, $\varphi(t, -1, 2)$ and $\varphi(t, -3, 6)$.

e) Represent in the plane the orbits corresponding to the initial states $(0, 0)$, $(1, -2)$, $(3, -6)$, $(-1, 2)$ and $(-3, 6)$. Insert the arrows.

2. Find the equilibria and decide whether they are or not hyperbolic, for the nonlinear planar system $\ddot{y} + \dot{y} + y^3 = 0$.

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Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2015/2016

Seminar Test

1. Represent the phase portrait of the scalar dynamical system $\dot{y} = -2y + 3y^2$. Find $\varphi(t, 2/3)$ and $\varphi(t, 0)$ and justify. Specify the properties of the functions $\varphi(t, 2)$ and, respectively, $\varphi(t, 1/3)$.

2. We consider the following nonlinear planar systems

$\dot{x} = -x + xy, \quad \dot{y} = -2y + 3y^2.$

a) Find its equilibria and study their stability using the linearization method.

b) Find $\varphi(t, 0, 2/3)$, $\varphi(t, 4, 0)$ and $\varphi(t, 1, 2/3)$.

c) Represent in the phase plane the orbits corresponding to the initial values $(0, 2/3)$, $(4, 0)$ and $(1, 2/3)$.

Seminar Test

1. Let $c > 1/4$ be a parameter and consider the scalar dynamical system $\dot{x} = x(1 - x) - c$.

a) Represent its phase portrait.

b) When $x(t) \geq 0$ is considered to be the number of fish in some lake, and $c > 1/4$ to be the rate of fishing, try to predict the fate of the fish from the lake interpreting the theoretical result obtained at a).

2. We consider the following linear planar system

$\dot{x} = -y, \quad \dot{y} = 4x.$

a) Find its general solution and its flow.

b) Study its type and stability.

c) Find a first integral in $\mathbb{R}^2$.

d) Represent its phase portrait.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = -x^3.$$

Find $\varphi(t,0)$ and specify the properties of the functions $\varphi(t,1)$ and $\varphi(t,5)$.

2. Consider the following planar system

$$\dot{x} = -y - x(x^2 + y^2), \quad \dot{y} = x - y(x^2 + y^2).$$

a) Does this system have other equilibria besides $(0,0)$? Justify.

b)Prove that the equilibrium point $(0,0)$ is not hyperbolic.

c) Use the system obtained by passing to polar coordinates to determine the shape of the orbits. Indicate the type and stability character of the equilibrium point $(0,0)$.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = 2 - x^2.$$

Determine the stability character of the equilibrium points using the linearization method. Find $\varphi(t, \sqrt{2})$ and specify the properties of the functions $\varphi(t, -1.5)$, $\varphi(t, 0)$ and $\varphi(t, 2)$.

2. Let $a, b \geq 0$ be real parameters such that $a \neq b$ and consider the following planar system

$$\dot{x} = ax - by, \quad \dot{y} = bx + ay.$$

a) Find the determinant of the matrix of the system. There are values of a and b for which it is null?

b) Determine the type and stability character of this linear system. Discuss with respect to the parameters.

c) Find the general solution of the system.

Seminar Test

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = 2x - x^2.$$

Determine the stability character of the equilibria using the linearization method. Find $\varphi(t, 2)$, $\varphi(t, 0)$ and study the properties of the functions $\varphi(t, -2)$, $\varphi(t, 1)$ and $\varphi(t, 3)$. If there is an asymptotically stable equilibrium point, specify its basin of attraction.

2. Consider the following planar system

$$\dot{x} = 2x - x^2, \quad \dot{y} = y - xy^2.$$

a) Find the equilibria.

b) Indicate the type and stability character of every hyperbolic equilibrium point.

c) Find $\varphi(t, 2, 1/2)$, $\varphi(t, 2, 0)$ and $\varphi(t, 0, 2)$.

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Seminar Test

1. Find a first integral for the following linear planar system

$$\dot{x} = 2y, \quad \dot{y} = -5x.$$

2. Find the general solution of the following linear planar system

$$\dot{x} = 2x + y, \quad \dot{y} = x + 2y.$$

3. Consider the following planar system

$$\dot{x} = x - 2\,x\,y, \quad \dot{y} = x - y.$$

a) Find the equilibrium points.

b) Indicate the type and stability character of every hyperbolic equilibrium point.

Seminar Test

1. We consider the linear system $\dot{x} = -y, \quad \dot{y} = x$.

a) Find its general solution.

b) Find $\varphi(t, 1, 0)$, $\varphi(t, 2, 0)$ and $\varphi(t, 3, 0)$.

2. Consider the following planar system

$$\dot{x} = -y(x^2 + y^2), \quad \dot{y} = x(x^2 + y^2).$$

a) Does this system have other equilibria besides $(0, 0)$? Justify.

b) Decide whether the equilibrium point $(0, 0)$ is hyperbolic or not. If it is, apply the linearization method.

c) Verify that $\varphi(t, 1, 0) = (\cos t, \sin t)$, $\varphi(t, 2, 0) = (2 \cos 4t, 2 \sin 4t)$ for all $t \in \mathbb{R}$. Find $\varphi(t, 3, 0)$.

d) Find a first integral.

e) Represent its phase portrait.

f) What remarkable property have the solutions of this system?

1. Represent the phase portrait of the scalar dynamical system

$$\dot{x} = -\frac{1}{4}x + x^2 - x^3.$$

Find $\varphi(t, 0)$ and determine the monotonicity of $\varphi(t, 1/4)$, $\varphi(t, 1/2)$ and $\varphi(t, 1)$.

3. Give an example of a linear planar system with a center at the origin. Justify. What remarkable property have the solutions of such a system, i.e. how they change in time?

2. Give an example of a linear planar system that has an asymptotically stable node at the origin. Justify.