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Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 19, 2010
I

1. (3p) Integrate the following differential equations
(a) $y' = \dfrac{-x - x^3}{y}$
(b) $y' = \dfrac{-x - y^2}{y}$
and than find a first integral of each of it.

2. (2p) Let $\varphi : (-\varepsilon, \varepsilon) \to \mathbb{R}$ be some solution of the differential equation $y' = \dfrac{y-2}{1-x^2-y^2}$ (the unknown is the function denoted y of independent variable x, while $\varepsilon > 0$ is a positive constant). Decide whether or not the following situations are possible:
(a) $\varphi(0) = -2$ and φ is a strictly increasing function;
(b) $\varphi(0) = 2$ and φ is a strictly increasing function;
(c) $\varphi_1(0) = -2$ and $\varphi_1'(0) = 0$.

3. (1.5p) Find the flow of the planar linear differential system:

$$\dot{x} = x + y, \quad \dot{y} = -2x + 4y.$$