Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, An: 2015/2016

Seminar Test, March 22, 2016

1. a) Find a particular solution of the form $x_p(t) = a\,t^2\,e^t$ (where the real coefficient a has to be determined) for

$$x'' - 2x' + x = e^t\,.$$

b) Find a constant solution for

$$x'' - 2x' + x = 5\,.$$

c) Find the general solution of the differential equation:

$$x'' - 2x' + x = 10 + 5e^t\,.$$

d) Find the solution of the IVP

$$x'' - 2x' + x = 5\,, \quad x(0) = 5\,, \quad x'(0) = 0.$$

2. We consider the differential equation

$$t^2x'' + 2tx' - 2x = 0, \quad t \in (0, \infty).$$

a) Find solutions of the form $x(t) = t^r$ where $r \in \mathbb{R}$ has to be determined.

b) Specify its type and find its general solution.

c) Find the solution of the IVP

$$t^2x'' + 2tx' - 2x = 0, \quad x(1) = 0, \quad x'(1) = 1.$$

Seminar Test, March 22, 2016

1. Let $\alpha > 0$ and $\varphi(t, \alpha)$ be the solution of the IVP

$$x'' - 4x = e^{\alpha t}, \quad x(0) = x'(0) = 0.$$

(i) When $\alpha \neq 2$ find a solution of the form $x_p(t) = ae^{\alpha t}$ for $x'' - 4x = e^{\alpha t}$. (Here you have to determine the real coefficient a.)

(ii) Find a solution of the form $x_p(t) = ate^{2t}$ for $x'' - 4x = e^{2t}$.

(iii) Find $\varphi(t, \alpha)$ for any $\alpha > 0$.

(iv) Prove that $\lim_{\alpha \to 2} \varphi(t, \alpha) = \varphi(t, 2)$ for each $t \in \mathbb{R}$.

2. Let $\alpha \in \mathbb{R}$. Describe the long-term behavior of the function $x(t) = e^{\alpha t} \cos 2t$, $t \in \mathbb{R}$. Discuss with respect to α.

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Seminar Test, March 22, 2016

1. Let $\gamma > 0$ and $\omega > 0$. Decide if the following statement is true.

"All the solutions of $x'' + \gamma x' + \omega^2 x = 0$ satisfy $\lim_{t\to\infty} x(t) = 0$."

2. a) Find a solution of the form $x(t) = a\cos 3t + b\sin 3t$ (where the real coefficients a and b have to be determined) for

$$x'' + 4x = -2\sin 3t.$$

b) Find the solution of the IVP

$$x'' + 4x = -2\sin 3t, \quad x(0) = 0, \quad x'(0) = 0.$$

Seminar Test, March 22, 2016

1. Find $\gamma \in \mathbb{R}$ such that all the solutions of the following differential equation are periodic

$$x'' + \gamma x + 9x = 0.$$

2. Let $\lambda \in \mathbb{R}$. Find the general solution of the differential equations

a) $tx' + x = 0$;

b) $tx' + x = -3$.

3. Decide if the following statement is true.

"The following BVP has at least a solution

$$x'' + 9x = 0, \quad x(0) = 0, \quad x(\pi) = 9."$$

Seminar Test, March 22, 2016

1. a) Find a solution of the form $x(t) = t(a\cos t + b\sin t)$ for $x'' + x = \cos t$.

b) Find the solution of the IVP

$$x'' + x = \cos t, \quad x(0) = 0, \quad x'(0) = 2.$$

c) Describe the long-term behavior of the function found at b).

2. Find a linear homogeneous differential equation with constant coefficients for which the following functions are solutions

$$t \text{ and } e^t.$$

Write the general solution of the equation that you found.

Seminar Test, March 22, 2016

1. We consider the differential equation

$$t^2x'' - 3tx' + 3x = 0, \quad t \in (0, \infty).$$

a) Find solutions of the form $x(t) = t^r$, where $r \in \mathbb{R}$ has to be determined.

b) Specify its type and find its general solution.

c) Find the solution of the IVP

$$t^2x'' - 3tx' + 3x = 0, \quad x(1) = 0, \quad x'(1) = 1.$$

2. Find a linear homogeneous differential equation with constant coefficients for which the following functions are solutions

$$t \text{ and } \sin 3t.$$

Write the general solution of the equation that you found.

3. Find the solution of the IVP

$$x' + 3x = -2, \quad x(0) = 0, \quad x'(0) = 0.$$

Seminar Test, March 29, 2016

We use the notation

$$\mathcal{L}(x) = x'' + 25x.$$

1. Find the solution of the IVP

$$\mathcal{L}(x) = 0, \quad x(0) = 0, \quad x'(0) = 1.$$

Represent this integral curve and describe its long-term behavior.

2. a) Let $\varphi_1(t) = t\,\cos(5t)$ and $\varphi_2(t) = t\,\sin(5t)$ for all $t \in \mathbb{R}$. Compute $\mathcal{L}(5)$, $\mathcal{L}(\varphi_1)$ and $\mathcal{L}(\varphi_2)$.

b) Find a constant solution for

$$\mathcal{L}(x) = 5\,.$$

c) Find the general solution of the differential equation

$$\mathcal{L}(x) = 25 - 25\sin(5t)\,.$$

3. Find the general solution of $x' + \frac{1}{t}x = \frac{1}{t}e^{-2t+1}$ for $t \in (0, \infty)$.

Seminar Test, March 29, 2016

1. Find the solution of the IVP

$$x'' + 4x' + 5x = 0\,, \quad x(0) = 1\,, \quad x'(0) = -2.$$

Represent this integral curve and describe its long term behavior.

2. Find all $a, b, c \in \mathbb{R}$ such that $x(t) = a\sin t + b\cos t + c\,e^t$ to be a solution of

$$x' + x = -3\sin t + 2\,e^t.$$

Find the general solution of this differential equation.

3. We consider the differential equation

$$x' + \frac{1}{t^2}x = 0\,, \quad t \in (-\infty, 0).$$

a) Check that $x = e^{1/t}$ is a solution of this d.e..

b) Find the solution of the IVP $x' + \frac{1}{t^2}x = 0, \quad x(-1) = 1.$

c) Find the general solution of $x' + \frac{1}{t^2}x = 1 + \frac{1}{t}$, $t \in (-\infty, 0)$.

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Seminar Test, March 29, 2016

1. Let $\lambda \in \mathbb{R}$ be a fixed parameter. Find the solution of the IVP:

$$x' + \lambda x = 0,\ x(0) = -1.$$

2. Find the linear homogeneous differential equation with constant coefficients and of minimal order that has as solution the following function. Then write also the general solution of the differential equation that you found.

$$1 + 2te^{-3t}$$

3. a) Find a particular solution of the form $x_p(t) = a \sin t + b \cos t$ for

$$x'' - 5x' + 6x = 5 \cos t$$

b) Find a particular solution of the form $x_p(t) = at + b$ for

$$x'' - 5x' + 6x = t - 1$$

c) Find the general solution of the differential equation:

$$x'' - 5x' + 6x = 5 \cos t + t - 1.$$

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Seminar Test, March 29, 2016

1. Let $0 < \gamma \leq 1$ be a fixed parameter and let us consider the equation $x'' + 2\gamma x' + x = 0$. Prove that every solution is bounded on $(0, \infty)$. Find γ such that every solution is periodic. What is the period in this case?

2. a) Find a particular solution for

$$x'' - 6x' + 9x = -2e^{3t}\,,$$

knowing that it has the form $x_p(t) = a\,t^2\,e^{3t}$ (where the real coefficient a has to be determined).

b) Find a constant solution for

$$x'' - 6x' + 9x = 5\,.$$

c) Find the general solution of the differential equation

$$x'' - 6x' + 9x = 10 + 10e^{3t}\,.$$

3. Find the general solution of $x' - x = e^{t-1}$ for $t \in (0, \infty)$.