

Exam on Dynamical Systems.
September 1, 2008

1. We consider the following differential system:

$$\begin{cases} x' = y \\ y' = -6x + 5y. \end{cases}$$

- a) Find its general solution.
- b) Study the type and stability of its equilibrium point.

2. Find the solution of the following Initial Value Problem:

$$y' = -\frac{2x}{y}, \quad y(0) = 3.$$

3. We consider the Initial Value Problem (IVP):

$$y' = \frac{y+x}{y-2x}, \quad y(0) = 1.$$

- a) Find the domain of definition of the given differential equation and show that the point $(0, 1)$ belongs to it.
- b) Find the set of all existence and uniqueness points of the given differential equation and show that the point $(0, 1)$ belongs to it.
- c) Show that the solution of this IVP is strictly increasing in a small neighborhood of $x = 0$.
- d) Write the Euler numerical formula for this IVP.
- e) Write the Runge-Kutta numerical formula for this IVP.
- f) Write the recurrence formula for the Picard sequence of successive approximations for this IVP.

4. (instead of the partial exam) Find the solution of the following Initial Value Problem:

$$y' = -\frac{y}{2x} + x, \quad y(1) = 0.$$