

Exam on Dynamical Systems.
July 09, 2011

1. (1.5p) Find the general solution of the following differential equation $x'' - x = e^{at}$. Discuss with respect to the real parameter a .
2. (0.5p) The classification of the singular point of a linear planar autonomous system.
3. We consider the differential equation $y' = -2x/y$.
 - a) (1p) Represent its direction field (hint: represent the 1, -1, 0, 2, -2 isoclines together with the corresponding directions).
 - b) (0.5p) Find a first integral.
 - c) (0.5p) Write the Euler numerical formula with constant step size h for this differential equation.
4. (2p) Study the stability of the equilibria at the positions $\theta = 0$, and $\theta = \pi$, respectively, of the differential equation $\ddot{\theta} + 4\dot{\theta} + \sin \theta = 0$.
5. (1p) (not compulsory) Find the general solution of the differential equation $t^2 x'' - 3tx' + 4x = 0$.