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Exercise Sheet no.11

Analysis for CS

GROUPWORK:

(G 28)

Study the improper integrability of the following functions on their domains and, in case they are improperly integrable, compute the corresponding improper integrals.

a) $f\colon (-1,1) \to \mathbb{R}$, $f(x) = \frac{1}{\sqrt{1-x^2}}$, b) $f\colon [1,\infty) \to \mathbb{R}$, $f(x) = \frac{1}{x(1+x)}$,

c) $f\colon (0,1] \to \mathbb{R}$, $f(x) = \ln x$, d) $f\colon [0,1) \to \mathbb{R}$, $f(x) = \frac{\arcsin x}{\sqrt{1-x^2}}$,

e) $f\colon (0,1] \to \mathbb{R}$, $f(x) = \frac{\ln x}{\sqrt{x}}$, f) $f\colon [e,\infty) \to \mathbb{R}$, $f(x) = \frac{1}{x(\ln x)^3}$,

g) $f\colon \left(\frac{1+\sqrt{3}}{2}, 2\right] \to \mathbb{R}$, $f(x) = \frac{1}{x\sqrt{2x^2-2x-1}}$, h) $f\colon [0,\infty) \to \mathbb{R}$, $f(x) = \frac{\pi}{2} - \operatorname{arctg} x$.

HOMEWORK:

(H 28) (To be delivered in the next exercise-class)

Study the improper integrability of the following functions on their domains and, in case they are improperly integrable, compute the corresponding improper integrals.

a) $f\colon (0,\infty) \to \mathbb{R}$, $f(x) = \frac{1}{4\sqrt{x}+\sqrt{x^3}}$, b) $f\colon (1,2] \to \mathbb{R}$, $f(x) = \frac{1}{x\ln x}$.

(H 29) (To be delivered in the next exercise-class)

Determine all local extrema, their type (minima or maxima) and the corresponding extreme values of the function $f\colon (0,\pi)\times(0,\pi) \to \mathbb{R}$ defined by $f(x,y) = \sin x + \sin y + \sin(x+y)$.

(H 30) (Train your brain)

Using the formula of Leibniz-Newton for definite integrals and the definition of improper integrals, prove the formula of Leibniz-Newton for improper integrals on intervals $[a,b)$, where $-\infty < a < b \leq \infty$.