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Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 18, 2010
II

1. (2p) Find the general solution of the following differential equations

$$x' + x = -2e^t, \quad x'' = \frac{2}{t}x'.$$

Here the unknown is the function denoted x of independent variable t.

2. (3p) Let $\varphi_1,\ \varphi_2 : \mathbb{R} \to \mathbb{R}$ be two distinct solutions of the differential equation $y' = \sqrt[3]{y+2}$ (the unknown is the function denoted y of independent variable x). Decide whether or not the following situations are possible:
(a) $\varphi_1(0) = 1$ and $\varphi_1'(0) = -1$;
(b) $\varphi_1(0) = \varphi_2(0) = 1$;
(c) $\varphi_1(0) = \varphi_2(0) = -2$ and $\varphi_1'(0) \neq \varphi_2'(0)$.

3. (1.5p) Decide the type and stability of the equilibrium point $(0, 0)$ of the differential systems:
(a) $\dot{x} = 2x, \quad \dot{y} = -x - 3y$;
(b) $\dot{x} = 2x + y, \quad \dot{y} = -x + 3y$.
Here the unknowns are the functions denoted x and, respectively, y, of independent variable t.