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Exam on Dynamical Systems
June 18, 2010
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1. (2p) Find the general solution of the following differential equations

$$x' + ax = -at + 1, \quad x'' - ax' + (a-1)x = 0,$$

where $a \in \mathbb{R} \setminus \{0, 1\}$ is a real parameter. Here the unknown is the function denoted x of independent variable t.

2. (3p) Let φ_1, $\varphi_2 : \mathbb{R} \to \mathbb{R}$ be two distinct solutions of the differential equation $y' = \sqrt[3]{y - 1}$ (the unknown is the function denoted y of independent variable x). Decide whether or not the following situations are possible:
   (a) $\varphi_1(0) = 2$ and $\varphi_1'(0) = -1$;
   (b) $\varphi_1(0) = \varphi_2(0) = 2$;
   (c) $\varphi_1(0) = \varphi_2(0) = 1$ and $\varphi_1'(0) \neq \varphi_2'(0)$.

3. (1.5p) Represent the phase portrait of the scalar differential equations:
   (a) $\dot{x} = 2x - x^2$;
   (b) $\dot{x} = 1 + x + x^2$;
   (c) $\dot{x} = 1 + x + x^3$.