

Rules of calculation for limits

$$\begin{aligned}
 x + \infty &= \infty + x = \infty, \quad \forall x \in \mathbb{R}, \\
 x + (-\infty) &= (-\infty) + x = -\infty, \quad \forall x \in \mathbb{R}, \\
 \infty + \infty &= \infty, \quad (-\infty) + (-\infty) = -\infty, \\
 x \cdot \infty &= \infty \cdot x = \begin{cases} \infty, & \text{if } x \in (0, \infty) \\ -\infty, & \text{if } x \in (-\infty, 0), \end{cases} \\
 x \cdot (-\infty) &= (-\infty) \cdot x = \begin{cases} -\infty, & \text{if } x \in (0, \infty) \\ \infty, & \text{if } x \in (-\infty, 0), \end{cases} \\
 \infty \cdot \infty &= \infty, \quad (-\infty) \cdot (-\infty) = \infty, \quad \infty \cdot (-\infty) = (-\infty) \cdot \infty = -\infty, \\
 \frac{x}{\infty} &= \frac{x}{-\infty} = 0, \quad \forall x \in \mathbb{R}, \\
 \frac{1}{0+} &= \infty, \quad \frac{1}{0-} = -\infty, \\
 x^\infty &= \begin{cases} \infty, & \text{if } x \in (1, \infty) \\ 0, & \text{if } x \in [0, 1), \end{cases} \\
 x^{-\infty} &= \begin{cases} 0, & \text{if } x \in (1, \infty) \\ \infty, & \text{if } x \in (0, 1), \end{cases} \\
 (\infty)^x &= \begin{cases} \infty, & \text{if } x \in (0, \infty) \\ 0, & \text{if } x \in (-\infty, 0), \end{cases} \\
 \infty^\infty &= \infty, \quad \infty^{-\infty} = 0.
 \end{aligned}$$

Not defined

$$\begin{aligned}
 &\infty + (-\infty), \quad (-\infty) + \infty, \\
 &0 \cdot \infty, \quad \infty \cdot 0, \quad 0 \cdot (-\infty), \quad (-\infty) \cdot 0, \\
 &\frac{\infty}{\infty}, \quad \frac{-\infty}{-\infty}, \quad \frac{\infty}{-\infty}, \quad \frac{-\infty}{\infty}, \\
 &1^\infty, \quad 0^0, \quad \infty^0, \quad 1^{-\infty}.
 \end{aligned}$$