Exercises: Boolean functions and logical circuits

<u>Exercise 1</u>. For the following Boolean functions of 3 variables, given by their tables of values, write the corresponding *canonical disjunctive form* (CDF) and *canonical conjunctive form* (CCF). Using Veitch diagrams simplify the functions.

x	y	z	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	1	0	0	0
0	1	0	0	0	1	0	1	1	1	1
0	1	1	1	0	0	1	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0
1	0	1	0	1	0	0	1	0	1	1
1	1	0	0	0	0	1	1	1	0	1
1	1	1	1	1	0	1	0	0	1	0

<u>Exercise 2</u>. Simplify the following Boolean functions of 4 variables using Veitch diagrams.

1. $f_1(x_1,x_2,x_3,x_4)= x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4$
$\vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
2. $f_2(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4$
$\vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
3. $f_3(x_1,x_2,x_3,x_4)= \bar{x}_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4$
$\vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4$;
4. $f_4(x_1,x_2,x_3,x_4)= \bar{x}_1x_2\bar{x}_3x_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4$
$\vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
5. $f_5(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4$
$\vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4$;
6. $f_6(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4$
$\vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4$;
7. $f_7(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4$
$\vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4$;
8. $f_8(x_1,x_2,x_3,x_4)= x_1x_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4$
$\vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4$.

<u>Exercise 3</u>. Using Karnaugh diagrams simplify the following Boolean functions given in CDF (disjunction of minterms):

$f_1(x_1,x_2,x_3)= m_0 \vee m_3 \vee m_4 \vee m_5 \vee m_6 \vee m_7$; $f_2(x_1,x_2,x_3)= m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6 \vee m_7$;

$f_3(x_1,x_2,x_3)= m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_7$; $f_4(x_1,x_2,x_3)= m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6$;

$f_5(x_1,x_2,x_3)= m_0 \vee m_1 \vee m_2 \vee m_4 \vee m_6 \vee m_7$; $f_6(x_1,x_2,x_3)= m_0 \vee m_1 \vee m_3 \vee m_5 \vee m_6 \vee m_7$;

$f_7(x_1,x_2,x_3)= m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_7$; $f_8(x_1,x_2,x_3)= m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6$.

<u>Exercise 4</u>. Simplify the following Boolean functions using Karnaugh diagrams.

1. $f_1(x_1,x_2,x_3)= x_1(\bar{x}_2\uparrow x_3) \vee \bar{x}_1x_2$;
2. $f_2(x_1,x_2,x_3)= x_3(x_1\vee x_2) \vee (\bar{x}_2\downarrow x_3)$;
3. $f_3(x_1,x_2,x_3)= x_2(x_1\uparrow \bar{x}_3) \vee \bar{x}_2x_3$;
4. $f_4(x_1,x_2,x_3)= x_1(x_2\vee x_3) \vee (x_1\downarrow \bar{x}_3)$;
5. $f_5(x_1,x_2,x_3)= x_3(\bar{x}_1\uparrow x_2) \vee x_1\bar{x}_3$;
6. $f_6(x_1,x_2,x_3)= x_2(x_1\vee x_3) \vee (\bar{x}_1\downarrow x_2)$;

7. $f_7(x_1,x_2,x_3)= x_2(\bar{x}_1 \uparrow x_3) \vee x_1\bar{x}_2$;

8. $f_8(x_1,x_2,x_3)= x_3(x_1 \vee x_2) \vee (\bar{x}_1 \downarrow x_3)$.

<u>Exercise 5</u>. Using Quine's method simplify the following Boolean functions of 4 variables given by their values 1 :

$f_1(1,1,1,1)= f_1(1,1,0,1)= f_1(0,1,1,1)= f_1(1,1,0,0)= f_1(0,1,0,0)= f_1(0,0,0,0)= f_1(0,0,0,1)= f_1(0,0,1,1)=1$;

$f_2(1,1,0,1)= f_2(0,1,0,1)= f_2(0,1,0,0)= f_2(0,0,0,0)= f_2(0,0,1,0)= f_2(1,0,1,1)= f_2(1,0,0,1)= f_2(0,0,1,1)=1$;

$f_3(0,1,0,1)= f_3(0,1,0,0)= f_3(0,1,1,0)= f_3(1,0,1,0)= f_3(1,0,0,0)= f_3(0,0,1,0)= f_3(1,0,0,1)= f_3(0,0,0,1)=1$;

$f_4(0,1,0,1)= f_4(0,1,1,1)= f_4(1,1,1,0)= f_4(1,1,0,0)= f_4(0,1,1,0)= f_4(1,0,0,0)= f_4(0,0,0,0)= f_4(0,0,0,1)=1$;

$f_5(1,1,1,1)= f_5(0,1,0,1)= f_5(0,1,1,1)= f_5(1,1,1,0)= f_5(1,1,0,0)= f_5(1,0,0,0)= f_5(1,0,0,1)= f_5(0,0,0,1)=1$;

$f_6(1,1,0,1)= f_6(0,1,0,1)= f_6(0,1,1,1)= f_6(1,1,1,0)= f_6(0,1,1,0)= f_6(1,0,1,0)= f_6(1,0,1,1)= f_6(1,0,0,1)=1$;

$f_7(1,1,1,1)= f_7(1,1,0,1)= f_7(0,1,0,1)= f_7(0,1,0,0)= f_7(0,1,1,0)= f_7(0,0,1,0)= f_7(1,0,1,1)= f_7(0,0,1,1)=1$;

$f_8(1,1,1,1)= f_8(1,1,1,0)= f_8(1,1,0,0)= f_8(1,0,0,0)= f_8(0,0,0,0)= f_8(0,0,1,0)= f_8(1,0,1,1)= f_8(0,0,1,1)=1$.

<u>Exercise 6</u>. Using Quine's method simplify the following Boolean functions given by their values 0.

$f_1(0,1,0) = f_1(0,1,1) = f_1(1,0,1) = 0$; $f_2(0,0,0) = f_2(0,0,1) = f_2(1,1,1) = 0$;

$f_3(0,0,1) = f_3(0,1,0) = f_3(1,1,0) = 0$; $f_4(0,0,0) = f_4(0,1,1) = f_4(1,0,0) = 0$;

$f_5(0,0,0) = f_5(1,1,0) = f_5(1,1,1) = 0$; $f_6(0,1,0) = f_6(1,0,0) = f_6(1,0,1) = 0$;

$f_7(0,1,1) = f_7(1,0,0) = f_7(1,1,1) = 0$; $f_8(0,0,1) = f_8(1,0,1) = f_8(1,1,0) = 0$.

<u>Exercise 7</u>. Using Veitch-Karnaugh diagrams simplify the following Boolean functions of 4 variables.

1. $f_1(x_1,x_2,x_3,x_4)= x_1\bar{x}_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_4 \vee \bar{x}_1x_3 \vee x_3\bar{x}_4$;
2. $f_2(x_1,x_2,x_3,x_4)= x_1x_2 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_4 \vee \bar{x}_1x_3 \vee x_2x_3$;
3. $f_3(x_1,x_2,x_3,x_4)= x_1x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee x_3x_4$;
4. $f_4(x_1,x_2,x_3,x_4)= x_1\bar{x}_2 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_4 \vee \bar{x}_1x_3 \vee \bar{x}_2x_3$;
5. $f_5(x_1,x_2,x_3,x_4)= x_3\bar{x}_4 \vee x_1x_2x_3x_4 \vee \bar{x}_3x_2x_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1\;\bar{}\;\bar{x}_4$;
6. $f_6(x_1,x_2,x_3,x_4)= \bar{x}_1\;\bar{}\;\bar{x}_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2x_4 \vee x_1\bar{x}_3 \vee \bar{x}_3\bar{x}_4$;
7. $f_7(x_1,x_2,x_3,x_4)= \bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee x_2x_3x_4 \vee x_1x_3 \vee x_1\bar{x}_4$;
8. $f_8(x_1,x_2,x_3,x_4)= x_3x_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1x_4$.

<u>Exercise 8</u>. For each of the following Boolean functions do: draw the corresponding logical circuit using derived gates, simplify the function and draw the logical circuits associated to all simplified forms of the initial function using only basic gates.

1. $f_1(x, y, z) = x(y \oplus z) \vee y(x \oplus z) \vee x(\bar{y} \downarrow \bar{z}) \vee (x \downarrow y)\bar{z}$;
2. $f_2(x, y, z) = x(y \uparrow z) \vee \bar{x}(\bar{y} \oplus z) \vee y(\bar{x} \oplus \bar{z})$;
3. $f_3(x, y, z) = x(\bar{y} \oplus z) \vee y(\bar{x} \oplus z) \vee \bar{x}(\bar{y} \downarrow z) \vee (\bar{x} \downarrow y)z$;
4. $f_4(x, y, z) = \bar{x}(y \uparrow \bar{z}) \vee x(\bar{y} \oplus z) \vee \bar{y}(\bar{x} \oplus z)$;
5. $f_5(x, y, z) = \bar{x}(y \oplus \bar{z}) \vee \bar{y}(x \oplus z) \vee \bar{x}(y \downarrow z) \vee (\bar{x} \downarrow y)\bar{z}$;
6. $f_6(x, y, z) = x(\bar{y} \uparrow \bar{z}) \vee \bar{x}(y \oplus z) \vee \bar{y}(\bar{x} \oplus z)$;
7. $f_7(x, y, z) = x(y \oplus \bar{z}) \vee y(\bar{x} \oplus z) \vee x(y \downarrow z) \vee (x \downarrow y)\bar{z}$;
8. $f_8(x, y, z) = x(\bar{y} \uparrow z) \vee \bar{x}(\bar{y} \oplus z) \vee y(x \oplus \bar{z})$.

<u>Exercise 9</u>. Draw a logical circuit having 3 input wires and containing all basic and derived gates. Write the corresponding Boolean function, simplify it and then draw a simplified circuit equivalent with the initial circuit.

<u>Exercise 10</u>. Write a Boolean function of 4 variables given by its table of values, simplify it and draw the logical circuits corresponding to all its simplified forms.