COURSE 6

Relational Algebra

Relational Query Languages

- *Query languages:* Allow manipulation and retrieval of data from a database.
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic.
  - Allows for much optimization.
- Query Languages **!=** programming languages!
  - QLs not expected to be "Turing complete".
  - QLs not intended to be used for complex calculations.
  - QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

- Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:
  - *Relational Algebra*: More operational, very useful for representing execution plans.
  - *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-operational, *declarative*)

Preliminaries

- A query is applied to *relation instances*, and the result of a query is also a relation instance.
  - *Schemas* of input relations for a query are fixed (but query will run regardless of instance!)
  - The schema for the *result* of a given query is also fixed! Determined by definition of query language constructs.
- Positional vs. named-field notation:
  - Positional notation easier for formal definitions, named-field notation more readable.
  - Both used in SQL

Relational Algebra

- Basic operations:
  - *Projection* (π) Deletes unwanted columns from relation.
  - *Selection* (σ) Selects a subset of rows from relation.
  - *Cross-product* (X) Allows us to combine two relations.
  - *Set-difference* (-) Tuples in reln. 1, but not in reln. 2.
  - *Union* ($\cup$) Tuples in reln. 1 and in reln. 2.
- Additional operations:
  - Intersection, *join*, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, operations can be *composed*! (Algebra is "closed".)

Projection

- $L = (a_1, \ldots, a_n)$ is a list of attributes (i.e. *a list of columns*) of the relation R
- Keeping vertical slices of a relation according to L

$$\pi_L(R) = \{ t \mid t_1 \in R \wedge \\ t.a_1 = t_1.a_1 \wedge \\ \ldots \wedge \\ t.a_n = t_1.a_n \}$$

Projection (cont.)

$$\pi_{\text{cid, grade}}(\text{Enrolled})$$

$\pi_{\text{cid, grade}}($

sid	***cid***	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB1	10
1234	DB2	9
1236	DB1	7
1237	DB2	9`
1237	DB1	5
1237	Alg1	10

$) =$

cid	*grade*
Alg1	9
Alg1	10
DB1	10
DB2	9
DB1	7
DB1	5

Projection (cont.)

Is $\pi_{\text{cid, grade}}$(Enrolled) equivalent to

```
SELECT cid, grade FROM Enrolled
```
?

NO! Relational algebra works with sets => no duplicates.

```
SELECT DISTINCT cid, grade
FROM Enrolled
```

Selection

- Selecting the t-uples of a relation R verifying a condition *c* (*selection predicate*).

$$\sigma_{c}(R) = \{ t \mid t \in R \wedge c \}$$

$$\sigma_{grade > 8}(\text{Enrolled}) = \{t \mid t \in \text{Enrolled} \wedge grade>8 \}$$

$\sigma_{grade > 8}($

sid	***cid***	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB2	9
1236	DB1	7
1237	DB1	5
1237	Alg1	6

$) =$

sid	*cid*	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB2	9

Selection (cont.)

$$\sigma_{\text{grade} > 8}(\text{Enrolled})$$

```
SELECT DISTINCT *
FROM Enrolled
WHERE grade > 8
```

Confusing terms

Projection

$\pi_{\text{attr1, attr2}}(\text{Relation})$

```
SELECT DISTINCT attr₁, attr₂
FROM  Relation
WHERE c
```

Selection $\sigma_{\text{c}}(\text{Relation})$

Selection Condition

- **Term Op Term** is a condition
  - where **Term** is an attribute name
  - or **Term** is a constant
  - **Op** is one of <, >, =, ≠ etc.

- **(C1 $\wedge$ C2)**, **(C1 $\vee$ C2)**, **($\neg$ C1)** are conditions where C1 and C2 are conditions

Composability

The result of an expression is a relation

$$\pi_{cid,\ grade}(\sigma_{grade > 8}(\text{Enrolled}))$$

$\pi_{cid,\ grade}(\sigma_{grade > 8}($

sid	*cid*	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB1	10
1234	DB2	9
1236	DB1	7
1237	DB2	9`
1237	DB1	5
1237	Alg1	10

$)) =$

cid	*grade*
Alg1	9
Alg1	10
DB1	10
DB2	9

Composability (cont.)

$$\pi_{\text{cid, grade}}(\sigma_{\text{grade} > 8}(\text{Enrolled}))$$

```
SELECT DISTINCT cid, grade
FROM  Enrolled
WHERE grade > 8
```

$$\sigma_{\text{grade} > 8}(\pi_{\text{cid, grade}}(\text{Enrolled}))$$

What is the equivalent SQL query?

Can we always exchange the order of σ and π?

Union, Intersection, Set-difference

- $R_1 \cup R_2 = \{ t \mid t \in R_1 \vee t \in R_2 \}$
- $R_1 \cap R_2 = \{ t \mid t \in R_1 \wedge t \in R_2 \}$
- $R_1 - R_2 = \{ t \mid t \in R_1 \wedge t \notin R_2 \}$

The relations R_1 and R_2 must be union compatible:

- same number of attributes (same *arity*)

- corresponding attributes have *compatible* domains and the *same name*

Set operations and equivalent SQL statements

$R_1 \cup R_2$

```
SELECT DISTINCT *
FROM R1
UNION
SELECT DISTINCT *
FROM R2
```

$R_1 \cap R_2$

```
SELECT DISTINCT *
FROM R1
INTERSECT
SELECT DISTINCT *
FROM R2
```

$R_1 - R_2$

```
SELECT DISTINCT *
FROM R1
EXCEPT
SELECT DISTINCT *
FROM R2
```

Are all operators essential?

$$R_1 \cap R_2 = ((R_1 \cup R_2) - (R_1 - R_2)) - (R_2 - R_1)$$

Compute all tuples
belonging to R_1 or R_2

Remove the ones
that
belong only to R_1

Remove the ones
that
belong only to R_2

Cartesian Product

- Combining two relations

$$R_1(a_1, \ldots, a_n) \text{ and } R_2(b_1, \ldots, b_m)$$

$$R_1 \text{ X } R_2 = \{ t \mid t_1 \in R_1 \wedge t_2 \in R_2$$
$$\wedge\ t.a_1 = t_1.a_1 \ldots \wedge\ t.a_n = t_1.a_n$$
$$\wedge\ t.b_1 = t_2.b_1 \ldots \wedge\ t.b_m = t_2.b_m\}$$

```
SELECT DISTINCT *
FROM R1, R2
```

θ-Join

- Combining two relations R_1 and R_2 on a condition c

$$R_1 \otimes_c R_2 = \sigma_c (R_1 \times R_2)$$

$$\text{Students} \otimes_{\text{Students.sid=Enrolled.sid}} \text{Enrolled}$$

```
SELECT DISTINCT *
FROM Students,Enrolled
WHERE Students.sid =
Enrolled.sid
```

```
SELECT DISTINCT *
FROM Students
INNER JOIN Enrolled ON
Students.sid=Enrolled.sid
```

The Equi-Join

- Combines two relations on a condition composed only of equalities of attributes of the first and second relation and projects only one of the redundant attributes (since they are equal)

$$R_1 \otimes_{E(c)} R_2$$

Courses

cid	*cname*
Alg1	Algorithms1
DB1	Databases1
DB2	Databases2

$\otimes_{E(\text{Courses.cid} = \text{Enrolled.cid})}$

Enrolled

sid	*cid*	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB1	10
1234	DB2	9
1236	DB1	7

=

cname	*sid*	*cid*	*grad*
Algorithms1	1234	Alg1	9
Algorithms1	1235	Alg1	10
Databases1	1234	DB1	10
Databases2	1234	DB2	9
Databases1	1236	DB1	7

The Natural Join

- Combines two relations on the equality of the attributes with the same names and projects only one of the redundant attributes

$$R_1 \otimes R_2$$

Courses

cid	*cname*
Alg1	Algorithms1
DB1	Databases1
DB2	Databases2

$\otimes$

Enrolled

sid	*cid*	*grade*
1234	Alg1	9
1235	Alg1	10
1234	DB1	10
1234	DB2	9
1236	DB1	7

=

cname	*sid*	*cid*	*grad*
Algorithms1	1234	Alg1	9
Algorithms1	1235	Alg1	10
Databases1	1234	DB1	10
Databases2	1234	DB2	9
Databases1	1236	DB1	7

Division

- Not supported as a primitive operator, but useful
- Let R_1 have 2 fields, x and y; R_2 have only field y:

  $R_1 / R_2 = \{ \langle x \rangle \mid \exists \langle x,y \rangle \in R_1 \ \ \forall \langle y \rangle \in R_2 \}$

  i.e., ***R_1 / R_2* contains all *x* tuples such that for <u>*every*</u> *y* tuple in *R_2*, there is an *xy* tuple in *R_1*.**

  Or: If the set of y values associated with an x value in R_1 contains all y values in R_2, the x value is in R_1/R_2.
- In general, x and y can be any lists of fields; y is the list of fields in R_2, and $x \cup y$ is the list of fields of R_1.

Expressing R_1/R_2 Using Basic Operators

- Division is not essential op; just a useful shorthand.
  - (Also true of joins, but joins are so common that systems implement joins specially.)
- *Idea*: For R_1/R_2, compute all x values that are not `disqualified' by some y value in R_2.
  - x value is *disqualified* if by attaching y value from R_2, we obtain an xy tuple that is not in R_1.

Disqualified x values: $\pi_x((\pi_x(R_1) \times R_2) - R_1)$

$$R_1/R_2 = \pi_x(R_1) - \text{all disqualified values}$$

Renaming

- If attributes or relations have the same name (for instance when joining a relation with itself) it may be convenient to rename one

$$\rho(R' \,(N_1 \rightarrow N'_1, N_2 \rightarrow N'_2\,), R)$$

alternative notation: $\rho_{R'\,(N'1,\, N'2\,)}(R)$,

- The new relation R′ has the same instance has R, its schema has attribute N'_i instead of attribute N_i

Renaming (cont.)

$$\rho(\text{Courses2}\ (\text{cid} \rightarrow \text{code},\ \text{cname} \rightarrow \text{description}),\ \text{Courses})$$

Courses

cid	*cname*	*credits*
Alg1	Algorithms1	7
DB1	Databases1	6
DB2	Databases2	6

→

Courses2

code	*description*	*credits*
Alg1	Algorithms1	7
DB1	Databases1	6
DB2	Databases2	6

```
SELECT cid as code,
       cname as description,
       credits
FROM Courses Courses2
```

Assignment Operation

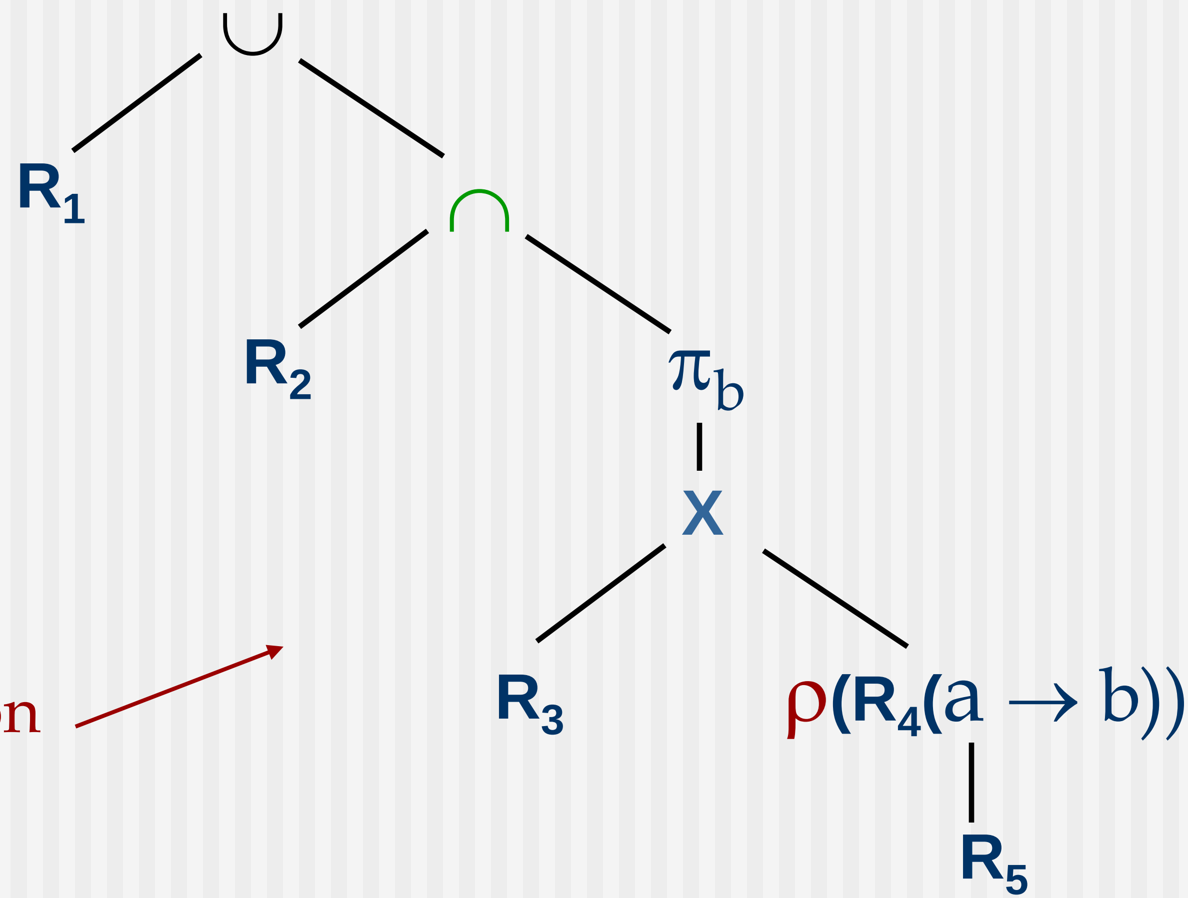
- The assignment operation $\leftarrow$ provides a convenient way to express complex queries.
  - Assignment must always be made to a temporary relation variable

$$Temp \leftarrow \pi_x(R_1 \; X \; R_2)$$

- The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.
- May use variable in subsequent expressions.
  - result $\leftarrow$ Temp – R_3

Complex Expression

$$R_1 \cup (R_2 \cap \pi_b (R_3 \times \rho(R_4(a \rightarrow b), R_5)))$$



Execution plan

Find names of students enrolled at 'BD1'

Solution 1: $\pi_{name}((\sigma_{cid='BD1'}(Enrolled)) \otimes Students)$

Solution 2: $\rho\ (Temp_1, \sigma_{cid='BD1'}(Enrolled))$

$\rho\ (Temp_2, Temp_1 \otimes Students)$

$\pi_{name}(\ Temp_2)$

Solution 3: $\pi_{name}(\ \sigma_{cid='BD1'}(Enrolled \otimes Students)\ )$

Find names of students enrolled at a 5 credits course

- Information about course credits only available in Courses; so need an extra join:

$$\pi_{name}\,(\,(\sigma_{credits=5}(Courses)) \otimes Enrolled \otimes Students\,)$$

- A more efficient solution:

$$\pi_{name}\,(\,\pi_{sid}(\pi_{cid}(\sigma_{credits=5}(Courses)) \otimes Enrolled) \otimes Students\,)$$

A query optimizer can find this, given the first solution!

Find students enrolled at a 4 or 5 credits course

- Can identify all 4 or 5 credits courses, then find students who're enrolled in one of these courses:

$$\rho\ (\mathit{TempCourses}, (\sigma_{credits=4 \vee credits=5}(\text{Courses})))$$

$$\pi_{name}\ (\mathit{TempCourses} \otimes \text{Enrolled} \otimes \text{Students}\ )$$

- Can also define TempCourses using union!

- What happens if $\vee$ is replaced by $\wedge$ in this query?

Find students enrolled at a 5 and 4 credits course

- Previous approach won't work! Must identify students who're enrolled at 4 credits courses, students who're enrolled at 5 credits courses, then find the intersection (note that *sid* is a key for Students):

$$\rho\ (\mathit{Temp4}, \pi_{\text{sid}}(\sigma_{\text{credits=4}}(\text{Courses}) \otimes \text{Enrolled}))$$

$$\rho\ (\mathit{Temp5}, \pi_{\text{sid}}(\sigma_{\text{credits=5}}(\text{Courses}) \otimes \text{Enrolled}))$$

$$\pi_{\text{name}}((\mathit{Temp4} \cap \mathit{Temp5}) \otimes \text{Students})$$

Find names of students enrolled at all courses

- Uses division; schemas of the input relations must be carefully chosen:

$$\rho\ (\mathit{TempSIDs},\ \pi_{\text{sid, cid}}(\text{Enrolled}\ )\ /\ \pi_{\text{cid}}\ (\text{Courses}))$$

$$\pi_{\text{name}}(\mathit{TempSIDs} \otimes \text{Students})$$

Extended Relational Algebra Operations

- Generalized Projection
- Aggregate Functions
- Outer Join
- Database modification

Generalized projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\pi_{F1,\ F2,\dots,\ Fn}\ (R)$$

- *R* is any relational-algebra expression
- Each of F_1, F_2, …, F_n are are arithmetic expressions involving constants and attributes in the schema of *R*.

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Aggregate operation** in relational algebra

$$_{G_1, G_2, \ldots, G_n}\vartheta_{F_1(A_1), F_2(A_2), \ldots, F_n(A_n)}(R)$$

- *R* is any relational-algebra expression
  - $G_1, G_2 \ldots, G_n$ is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

Aggregate Operation – Example

Relation *R*:

A	B	C
α	α	7
α	β	7
β	β	3
β	β	10

$g_{\textbf{sum(C)}}$ (R)

sum(C)
27

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

Outer Join

- An extension of the join operation that avoids loss of information.
  - Left Outer Join ⟕
  - Right Outer Join ⟖
  - Full Outer Join ⟗
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist
  - All comparisons involving *null* are (roughly speaking) **false** by definition.

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion $R \leftarrow R - E$
  - Insertion $R \leftarrow R \cup E$
  - Updating $R \leftarrow \pi_{F1, F2, \ldots, Fn}(R)$

- All these operations are expressed using the assignment operator.