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Exercise Sheet no.7

Analysis for CS

Groupwork:

(G 18)

Let $f\colon [\frac{1}{2}, 3] \to \mathbb{R}$, $f(x) = \sin(\sqrt{x})$. Write down:

a) Taylor's polynomial $T_2(x, 1)$,

b) the remainder term $R_2(x, 1)$, for $x \in [\frac{1}{2}, 3] \setminus \{1\}$, according to Taylor's formula.

(G 19)

Consider the trigonometric functions sin, cos: $\mathbb{R} \to \mathbb{R}$.

a) Determine $\sin^{(n)}$ and $\cos^{(n)}$, for every $n \in \mathbb{N}$.

b) Write down the Taylor polynomials $T_n(x, 0)$, for every $n \in \mathbb{N}$, of these two functions.

c) Show that both sin and cos may be expanded as Taylor series around 0 on $\mathbb{R}$, and find the corresponding Taylor series expansions.

(G 20)

Determine the following higher order derivatives:

a) $(e^{3x})^{(n)}$, $n \in \mathbb{N}$, b) $(x^2 \sin 2x)^{(100)}$, c) $((x^3 + 2x - 1)e^{2x})^{(n)}$, $n \in \mathbb{N}$.

Homework:

(H 18)

a) Show that the following equalities hold true for every $n \in \mathbb{N}$ and every $x \in \mathbb{R}$

$$\sin^{(n)}(x) = \sin\left(x + n\frac{\pi}{2}\right), \quad \cos^{(n)}(x) = \cos\left(x + n\frac{\pi}{2}\right).$$

b) Determine the higher order derivatives $(e^x \sin x)^{(n)}$ and $(e^{-2x} \cos x)^{(n)}$, $n \in \mathbb{N}$.

(H 19)

Let α, $\beta > 0$. Compute the following limits:

a) $\lim\limits_{x\to\infty} \dfrac{e^{\alpha x}}{x}$, b) $\lim\limits_{x\to\infty} \dfrac{e^{\alpha x}}{x^\beta}$, c) $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^\alpha}$, d) $\lim\limits_{x\to\infty} \dfrac{(\ln x)^\beta}{x^\alpha}$, e) $\lim\limits_{\substack{x\to 0\\ x>0}} x^\alpha \ln x$, f) $\lim\limits_{\substack{x\to 0\\ x>0}} x^x$.