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Exercise Sheet no.5

Analysis for CS

GROUPWORK:

(G 13)

Decide whether the following series are convergent or not, indicating in each case the criterion you are using.

a) $\displaystyle\sum_{n\geq 0} \frac{3^n}{4^n+5^n}$, b) $\displaystyle\sum_{n\geq 1} \frac{1}{(2n)^\alpha}$, where $\alpha \in \mathbb{R}$, c) $\displaystyle\sum_{n\geq 1} \frac{\sqrt{n+1}-\sqrt{n}}{n^{\frac{3}{4}}}$,

d) $\displaystyle\sum_{n\geq 1} \left(\frac{n}{n+1}\right)^{n^2}$, e) $\displaystyle\sum_{n\geq 1} \frac{x^n}{n^p}$, where $x>0$ and $p\in\mathbb{R}$, f) $\displaystyle\sum_{n\geq 1} \sin\frac{1}{n}$.

HINT for f): If $(x_n)_{n\in\mathbb{N}}$ is a sequence of nonzero reals converging to 0, then $\displaystyle\lim_{n\to\infty} \frac{\sin x_n}{x_n} = 1$.

(G 14)

Study the convergence and the absolute convergence of the following series.

a) $\displaystyle\sum_{n\geq 1} (-1)^n \left(e - \left(1+\frac{1}{n}\right)^n\right)$, b) $\displaystyle\sum_{n\geq 1} \sin\frac{x}{n}$, where $x\in\mathbb{R}$.

HINT for a): For the absolute convergence of the series use the fact that $(*)\ \displaystyle\lim_{n\to\infty} n\left(e-\left(1+\frac{1}{n}\right)^n\right) = \frac{e}{2}$.

HOMEWORK:

(H 15) (To be delivered in the next exercise-class)

1) Decide whether the following series are convergent or not, indicating in each case the criterion you are using.

a) $\displaystyle\sum_{n\geq 0} \frac{1}{(2n+1)^\alpha}$, where $\alpha\in\mathbb{R}$, b) $\displaystyle\sum_{n\geq 1} \frac{1}{\sqrt{n(n+1)}}$, c) $\displaystyle\sum_{n\geq 1} \sin^3\frac{1}{n}$,

d) $\displaystyle\sum_{n\geq 2} \frac{1}{(\ln n)^n}$, e) $\displaystyle\sum_{n\geq 1} n^4 e^{-n^2}$.

2) Study the convergence and the absolute convergence of the series $\displaystyle\sum_{n\geq 1} \frac{(-1)^{n+1}}{n^\alpha}$, where $\alpha\in\mathbb{R}$.

(H 16)

Consider the series $\displaystyle\sum_{n\geq 1} n!\left(\frac{x}{n}\right)^n$, where $x>0$. Using the equality $(*)$ given in the hint for exercise (G 14), determine the set of those values of x for which the series is convergent.