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Exercise Sheet no.10

Analysis for CS

GROUPWORK:

(G 27)

Determine all local extrema, their type (minima or maxima) and the corresponding extreme values of the following functions:

a) $f\colon \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = x^3 - 3x + y^2 + z^2$; b) $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^4 + y^4 - 4(x-y)^2$;

c) $f\colon \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = z^2(1+xy) + xy$; d) $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 + 3xy^2 - 15x - 12y$.

HOMEWORK:

(H 25) (To be delivered in the next exercise-class)

Determine all local extrema, their type (minima or maxima) and the corresponding extreme values of the following functions:

a) $f\colon \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = 2x^2 - xy + 2xz - y + y^3 + z^2$; b) $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^2 + y^2(1-x)^3$.

(H 26)

Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function defined in statement a) of exercise (H 23).

a) Show that f is twice partially differentiable with respect to (x,y) and (y,x) on $\mathbb{R}^2$, and determine the second-order partial derivatives $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$. Observe that

$$\frac{\partial^2 f}{\partial y \partial x}(0_2) \neq \frac{\partial^2 f}{\partial x \partial y}(0_2).$$

b) Show that the functions $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$ are not continuous at 0_2, thus $f \notin C^2(\mathbb{R}^2)$.

(H 27)

Let $M := \{(x,y) \in \mathbb{R}^2 \mid x + y > 0\}$ and let $f\colon M \to \mathbb{R}$ be defined by $f(x,y) = \frac{x^2+y^2}{x+y}$. Show that

$$x\frac{\partial f}{\partial x}(x,y) + y\frac{\partial f}{\partial y}(x,y) = f(x,y), \forall\ (x,y) \in M.$$