

Exam on Dynamical Systems
June 27, 2009
II

1. (3.5p) We consider the differential system

$$\dot{x} = -x + 2y, \quad \dot{y} = -2x - y.$$

- (a) Study the type and stability of its equilibrium point $(0, 0)$.
- (b) Find its general solution.
- (c) Pass to polar coordinates.
- (d) Represent its phase portrait.

2. (1p) Write the statements of

- (a) the existence theorem (Peano)
 - (b) the existence and uniqueness theorem (Cauchy-Lipschitz)
- for a first order differential equation.

3. (2p) (a) Find the solution of the Initial Value Problem

$$y' = \frac{2y}{x}, \quad y(1) = \pi.$$

- (b) (True or False) "The solution of the previous IVP is a bounded function."