Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems.
June 11, 2012

1. Represent the phase portrait of $\dot{x} = (N - x)x - c$ where $N > 0$ is a fixed constant related with the birth rate of some population of fishes in a lake. Discuss with respect to the parameter $c \geq 0$ that represents the fishing rate. Interpret the results.

2. We consider the linear differential system $\dot{x} = y, \quad \dot{y} = -4x$.
a) Show that all its solutions are periodic with the same principal period.
b) Represent its phase portrait.
c) Find a real function $H(x, y)$ that takes constant value on each orbit.

3. Study the long term behavior of the solution of the IVP $x' = -120x, \quad x(0) = x_0$, (where $x_0 > 0$) and of the solution of the corresponding difference equation obtained by Euler's numerical formula. What is the largest safe stepsize in this numerical integration?