Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - II

1. (1p) Find the general solution of the differential equation

$$x^2u'' - 6xu' + 10u = 0,$$

whose unknown is the function u of variable x. Hint: look for solutions of the form $u = x^r$, with $r \in \mathbb{R}$.

2. (2p) a) Write the general form of a second order linear differential equation. Formulate the Initial Value Problem for these type of equations, and write the statement of the Existence and Uniqueness Theorem for it.

b) How many solutions have each of the following problems:

$$\begin{array}{ll} \text{(i)} & x'' + t^2x = 0, \quad x(0) = 0; \\ \text{(ii)} & x'' + t^2x = 0, \quad x(0) = 0, \quad x'(0) = 0; \\ \text{(iii)} & x'' + t^2x = 0, \quad x(0) = 0, \quad x'(0) = 0, \quad x''(0) = 1? \end{array}$$

3. (2.5p) Find the solution of each of the following difference equations and describe its long term behavior:

$$\begin{array}{ll} \text{(i)} & x_{k+1} = \frac{1}{5}x_k, \quad x_0 = 2; \\ \text{(ii)} & x_{k+1} = \frac{1}{5}x_k + 1, \quad x_0 = \frac{5}{4}; \\ \text{(iii)} & x_{k+1} = \frac{1}{5}x_k + 1, \quad x_0 = 2; \\ \text{(iv)} & x_{k+2} = x_{k+1} + x_k, \quad x_0 = 0, \quad x_1 = 1. \end{array}$$