

# **Virtual Machines**

## **Lecture 7-8**

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## **CoreJava Type System and Third Assignment**

# Overview

1. Discussion about the Second Assignment – CoreJava Dynamic Semantics – Questions on the Interpreter
2. Introduction in Static Semantics -- Type Systems
3. Third Assignment - CoreJava Type System

# Introduction in Static Semantics

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## Type Systems

# Static semantics

We use the simpler expression language to introduce the type system. For example the language contains Booleans, conjunction, and `if` expressions :

```
e ::= ... | b | e1 && e2  
    | if e1 then e2 else e3  
v ::= ... | b
```

We could get nonsensical expressions, e.g.,

```
5 + false  
if 5 then true else 0
```

Need *static semantics* (type checking) to rule those out...

# if expressions

**Syntax:**

**if e1 then e2 else e3**

**Type checking:**

**if e1 has type bool and e2 has type t and e3 has type t  
then if e1 then e2 else e3 has type t**

# Static semantics

Defined by a *judgement*:

$$\mathbb{T} \mid - e : t$$

- Read as in typing context  $\mathbb{T}$ , expression  $e$  has type  $t$
- Turnstile  $\mid -$  can be read as "proves" or "shows"
- You're already used to  $e : t$ , because Ocaml utop uses that notation
- *Typing context* is a dictionary mapping variable names to types
- The typing context is a new idea, but obviously needed to give types of variables in scope

# Static semantics

e.g.,

**`x:int |- x+2 : int`**

**`x:int, y:int |- x<y : bool`**

**`|- 5+2 : int`**

# Static semantics of expr. lang.

$T \vdash i : \text{int}$

$T \vdash b : \text{bool}$

$T, x:t \vdash x : t$



# Static semantics of expr. lang.

**T |- e1+ e2 : int**

**if T |- e1 : int**

**and T |- e2 : int**

**T |- e1 && e2 : bool**

**if T |- e1 : bool**

**and T |- e2 : bool**

# Static semantics of expr. lang.

$T \vdash \text{if } e1 \text{ then } e2 \text{ else } e3 : t$   
if  $T \vdash e1 : \text{bool}$   
and  $T \vdash e2 : t$   
and  $T \vdash e3 : t$

$T \vdash \text{let } x:t1 = e1 \text{ in } e2 : t2$   
if  $T \vdash e1 : t1$   
and  $T, x:t1 \vdash e2 : t2$

# Interpreter for expr. lang.

See `interp3-full.ml` code attached to this lecture

1. Type checks expression
2. Evaluates expression

# Purpose of type system

Ensure **type safety**: well-typed programs don't get *stuck*:

- haven't reached a value, and
- unable to evaluate further

Lemmas:

**Progress:** if  $e$  has type  $\tau$ , then either  $e$  is a value or  $e$  can take a step.

**Preservation:** if  $e$  has type  $\tau$ , and if  $e$  takes a step to  $e'$ , then  $e'$  has type  $\tau$ .

Type safety = progress + preservation

**Proving type safety is more difficult and therefore we ignore it in this course. Type safety MUST always be proved, since the compiler MUST be correct.**

# **Third Assignment**

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## **CoreJava Type System**

# Third Assignment – 25% of the final grade

Please implement in Ocaml a type checker for CoreJava language according to the CoreJava type system.

The type system ( or the static semantics) of CoreJava is described in the following slides

# CoreJava Type System

- In the following we present the type checking rules of all CoreJava.
- The presentation is not so formal as in the literature
- The judgements have the following form

<b>conditions to be met</b>		<b>(IF conds to be met)</b>
<b>-----</b>	<b>&lt;==&gt;</b>	<b>THEN</b>
<b>context  - type rule</b>		<b>for the given context</b>
		<b>The type<sup>15</sup> rule is true</b>

# CoreJava Type System

- The type system is presented top-down
- It consists of the following judgements for:
  - A well-typed program
  - A well-typed class declaration
  - Well-typed field declarations
  - A well-typed method declaration
  - A well-typed expression
  - Subtyping (you can also use this subtyping definition in the operational semantics where we defined not so rigorous)



# Well-typed program

**$\vdash \text{WellFoundedClasses}(P)$  and  $P = \text{clsD1}; \dots; \text{clsDn}$  and**

**For each class declaration  $\text{clsDi}$  we have:**

**$\vdash \text{methsOnce}(\text{clsDi})$  and  $\vdash \text{fieldsOnce}(\text{clsDi})$  and**

**$P \vdash \text{inheritanceOK}(\text{clsDi})$  and  $P \vdash \text{-def- clsDi}$**

**$\vdash P$**

- A program is well-typed if:
  - WellFoundedClasses: no duplicate definitions of the classes, no cycle in the class hierarchy and last class contains the main method
  - MethsOnce: no methods duplication in a class
  - FieldsOnce: no field duplication in a class
  - InheritanceOk: method overriding is sound
  - Each class is well typed

# Well-typed class declaration

**ClsD= class cn extends cn' {fldD1...fldDn # mthD1...mthDn} and**

**For each method declaration mthDi we have:**

**P, {this:cn} |-mth- mthDi**

**P |-def- clsD**

- A class is well typed if:
  - Each method from the class is well typed
  - {this:cn} denotes the initial type environment
  - A type environment is a dictionary containing mappings from the variable name to the type associated to that variable
  - Type environment is working as a stack where we continuously push new mappings

# Well-typed method declaration

$P, \{v_1:t_1, \dots, v_n:t_n\} + TE \vdash e : t \text{ and } P \vdash t <: tr$

---

$P, TE \vdash_{\text{meth}} tr \text{ mn}(t_1 v_1, \dots, t_n v_n) \{e\}$

- A method is well typed if:
  - The method body is well typed
  - TE denotes the type environment
  - $\{v_1:t_1, \dots, v_n:t_n\} + TE$  denotes the extension of a type environment TE with new mappings  $\{v_1:t_1, \dots, v_n:t_n\}$  corresponding to the formal parameters of the method
  - The judgement  $P, TE \vdash e:t$  says that the type of the expression  $e$  is  $t$  with respect to the program  $P$  and type environment  $TE$
  - The type of the method body must be a subtype of the declared return type of the method

# Subtyping Judgement

- In order to denote that a type  $t_1$  is a subtype of type  $t_2$  we used the following notation  $t_1 <: t_2$
- The rules of the subtyping relation are enumerated in the following
- If none of the following rule is applicable that means that  $t_1$  is not subtype of  $t_2$
- Note that in Lecture 6 we presented a draft implementation of the subtyping relation

# Subtyping Judgement

(inheritance rule)

Class  $cn1$  extends  $c2 \{...\}$  is a declared class in  $P$

---

$P \vdash cn1 <: cn2$

(reflexivity)

(transitivity)

$P \vdash t1 <: t2$  and  $P \vdash t2 <: t3$

---

---

$P \vdash t <: t$

---

$P \vdash t1 <: t3$

# Subtyping Judgement

**Cn is a declared class in P**

**cn is a declared class in P**

-----  
**P |- bot <: cn**

-----  
**P |- cn <: Object**

- **Note that the above 5 rules directly imply the followings:**
  - **int <:int ,**
  - **float<:float ,**
  - **void <:void**
  - **bool <: bool**

# Well-typed expressions

---

**P,TE |- null:bot**

---

**P,TE |- kint: int**

---

**P,TE |- kfloat:float**

---

**P,TE |- (): void**

---

**P,TE |- false:bool**

---

**P,TE |- true: bool**

# Well typed expressions

$TE = \{ \dots (v:t) \dots \}$

-----

$P, TE \vdash v : t$

- The type of the variable  $v$  is the declared type of the variable  $v$
- The declared type of a variable is stored in the type environment



# Well typed expressions

$P, TE \vdash v : cn$  and

( $cn$  is a declared class in  $P$ ) and

( $(f, t)$  is defined in  $fieldlist(P, cn)$ )

-----

$P, TE \vdash v.f : t$

- First we get the type of  $v$ , that type must be a class
- Second we get the type of the field  $f$

# fieldlist

---

**fieldlist(P, Object) = []**

**class cn1 extends cn2 {t1 f1;...;tn fn # ....}**

---

**fieldlist(P, cn1) = fieldlist(P, cn2) ++ [(f1, t1); ... (fn, tn)]**

- It computes all fields of a class

# Well typed expressions

**$P, TE \vdash v : t1$  and**

**$P, TE \vdash e : t2$  and**

**$P \vdash t2 \leq t1$**

-----

**$P, TE \vdash v = e : \text{void}$**

- The type  $t2$  of the expression  $e$  must be a subtype of the variable  $v$  type  $t1$

# Well typed expressions

$P, TE \vdash v.f : t_1$

$P, TE \vdash e : t_2$  and

$P \vdash t_2 <: t_1$

-----

$P, TE \vdash v.f = e : \text{void}$

# Well typed expressions

$P, \{v:t\} + TE \vdash e : t1$

-----

$P, TE \vdash \{(t \ v) \ e\} : t1$

$P, TE \vdash e : t1$

-----

$P, TE \vdash \{e\} : t1$

# Well typed expressions

$P, TE \vdash e1 : t1$  and

$P, TE \vdash e2 : t2$

-----

$P, TE \vdash e1;e2 : t2$

# Well typed expressions

$P, TE \vdash v : tv$  and  $P \vdash tv <: \text{bool}$  and

$P, TE \vdash \{e1\} : t1$  and  $P, TE \vdash \{e2\} : t2$  and

Find  $t$  such that

$P \vdash t1 <: t$  and  $P \vdash t2 <: t$  and

( $t$  is the least maximum type of  $t1$  and  $t2$ )

-----

$P, TE \vdash \text{if } v \text{ then } \{e1\} \text{ else } \{e2\} : t$

# Well typed expressions

(opint is either + or – or \* or /)

$P, TE \vdash e1 : t1$  and  $P \vdash t1 <: \text{int}$  and

$P, TE \vdash e2 : t2$  and  $P \vdash t2 <: \text{int}$

-----

$P, TE \vdash e1 \text{ opint } e2 : \text{int}$



# Well typed expressions

(opfloat is either +. or −. or \*. or /.)

$P, TE \vdash e1 : t1$  and  $P \vdash t1 <: \text{float}$  and

$P, TE \vdash e2 : t2$  and  $P \vdash t2 <: \text{float}$

-----

$P, TE \vdash e1 \text{ opfloat } e2 : \text{float}$

# Well typed expressions

(opbool is either && or ||) and

$P, TE \vdash e1 : t1$     and     $P \vdash t1 <: \text{bool}$  and

$P, TE \vdash e2 : t2$     and     $P \vdash t2 <: \text{bool}$

-----

$P, TE \vdash e1 \text{ opbool } e2 : \text{bool}$

$P, TE \vdash e : t$     and     $P \vdash t <: \text{bool}$

-----

$P, TE \vdash !e : \text{bool}$

# Well typed expressions

(opcmp is either < or <= or == or != or > or >=) and

$P, TE \vdash e1 : t1$  and  $P, TE \vdash e2 : t2$  and

$t1 <: t2$  and  $t2 <: t1$  and

(  $t1$  is not a declared class in  $P$ ) and

( $t2$  is not a declared class in  $P$ )

-----

$P, TE \vdash e1 \text{ opcmp } e2 : \text{bool}$

# Well typed expressions

(cn is a declared class in P) and

$P, TE \vdash v : t$  and

$(P \vdash cn <: t \text{ or } P \vdash t <: cn)$

-----

$P, TE \vdash (cn) v : cn$

# Well typed expressions

**(cn is a declared class in P) and  
P,TE |- v :t and (t <: cn or cn<:t)**

-----

**P,TE |- v instanceof cn : bool**

# Well typed expressions

**(cn is a declared class in P) and  
[(f1,t1),...,(fn,tn)]=fieldlist(P,cn) and  
P,TE |- v1 :t1' and ... and P,TE |- vn:tn' and  
P |- t1'<:t1 and ... and P |- tn'<:tn**

-----

**P,TE |- new cn(v1,...,vn) : cn**

# Well typed expressions

$P, TE \vdash v_0:t_0$  and ( $t_0$  is a declared class in  $P$ ) and  
( $P \vdash (tr \text{ mn}(t_1 \ v_1, \dots, t_n \ v_n)\{e\})$  is a declared method in  $t_0$ ) and  
 $P, TE \vdash v_1' :t_1'$  and ... and  $P, TE \vdash v_n':t_n'$  and  
 $P \vdash t_1'<:t_1$  and ... and  $P \vdash t_n'<:t_n$

-----

$P, TE \vdash v_0.mn(v_1', \dots, v_n') : tr$

# Well typed expressions

$P, TE \vdash v:t$     and    $P \vdash t <: \text{bool}$  and

$P, TE \vdash e : te$

-----

$P, TE \vdash \text{while } v \{e\} : \text{void}$



# Auxilliary rules

**clsD = class cn extends cn' {...# mthD1...mthDn}**

**For each i and j,  $0 \leq i \leq n$  and  $0 \leq j \leq n$  and  $i \neq j$**

**name(mthDi)  $\neq$  name(mthDj)**

-----

**| - methsOnce(clsD)**

- No method overloading/duplication in a class definition

# Auxilliary rules

**clsD = class cn extends cn' {fldD1...fldDn # ...}**

**For each i and j,  $0 \leq i \leq n$  and  $0 \leq j \leq n$  and  $i \neq j$**

**name(fldDi)  $\neq$  name(fldDj)**

-----

**| - fieldsOnce(clsD)**

- No field duplication in a class definition

# Auxilliary rules

$P = \text{clsD1}; \dots; \text{clsDn}$  and  $\text{clsDi} = \text{cni extends cni' \{...\}}$  and

$IR = \{(\text{cni}, \text{cni}') \mid 1 \leq i \leq n\}$  and  $ID = \{(\text{cni}, \text{cni}) \mid 1 \leq i \leq n\}$  and

$\text{TransitiveClosure}(IR) \cap ID = \{\}$  and

For all  $i, j$   $\text{cni} \neq \text{cnj}$  and

$\text{ClsDn} = \text{class Main extends cn' \{ \# void main() \{ e \} \}}$

---

**$\vdash \text{WellFoundedClasses}(P)$**

- no duplicate definitions of the classes, no cycle in the class hierarchy and last class contains the main method

# Transitive Closure

$$IR = \{(cni, cni') \mid 1 \leq i \leq n\}$$

**TransitiveClosure(IR) is computed as follows:**

- 1. TransitiveClosure(IR)=IR**
- 2. if (cn1,cn2) is in TransitiveClosure(IR) and (cn2,cn3) is in TransitiveClosure(IR) then the pair(cn1,cn3) is added to TransitiveClosure(IR)**
- 3. Step 2 is performed until no modification can be done to TransitiveClosure(IR)**

# Auxilliary rules

**clsD= class cn extends cn' {...# meth1...methn} and**

**For all  $1 \leq i \leq n$  if exists a method meth' such that**

**(meth' is a declared method in cn') and**

**name(methi) == name(meth') then**

**overridesOk(methi,meth')**

---

**P |- inheritanceOK(clsD)**

**meth1 = tr1 mn(t1 v1,...tn vn) {e1} and**

**Meth2 = tr2 mn(t1 v1,...tn vn) {e1} and tr1<:tr2**

---

**overridesOk(meth1,meth2)**

# Auxilliary rules

**$P = \text{clsD1} \dots \text{clsDi} \dots \text{clsDn}$  and**

**$\text{ClsDi} = \text{class cn extends cn'} \{ \dots \}$**

---

**cn is a declared class in P**

**$P \vdash (\text{tr mn}(t1 \ v1, \dots, tn \ vn)\{e\})$  is a directly declared method in cn**

---

**$P \vdash (\text{tr mn}(t1 \ v1, \dots, tn \ vn)\{e\})$  is a declared method in cn**

# Auxilliary rules

class cn extends cn' {...} and

(P  $\vdash$  (tr mn(t1 v1,..., tn vn){e})) is a declared method in cn') and

NOT (P  $\vdash$  (tr mn(t1 v1,..., tn vn){e})) is a directly declared method in cn)

---

P  $\vdash$  (tr mn(t1 v1,..., tn vn){e}) is a declared method in cn

Class cn extends cn' { ...#meth1...methi...methn}

Methi = tr mn(t1 v1,..., tn vn){e}

---

P  $\vdash$  (tr mn(t1 v1,..., tn vn){e}) is a directly declared method in cn

# Example

In the following we discuss the execution and the type checking for a simple program P written in CoreJava:

```
class A extends Object{  
  int f1;  
  #  
  int m1(int a, int b) { (int c)  
                        c=a+b;this.f1=this.f1+c;c};  
}
```



# Example

```
class B extends A{
```

```
  A f2;
```

```
  #
```

```
  A m2(A x, A y) {(A z) { (int n)
```

```
    n=x.m1(1,2)-y.m1(2,1);
```

```
    {(bool m) m=(x.f1-y.f1)>n;
```

```
    if m then {z=new A(m)} else {z=new A(n)}
```

```
  }
```

```
};this.f2=z;z
```

```
}
```

# Example

```
Class Main extends Object{ #  
  Void main(){ (B o1) o1=new B(0,null);  
    { (A o2) o2=new A(2);  
      { (A o3) o3=new A(3);  
        o2 =o1.m2(o2,o3)  
      }  
    }  
  }  
}
```

# TypeChecking Example

**| - WellFoundedClasses(P) and  $P = \text{clsA}; \text{clsB}; \text{clsMain}$  and**

**For each class declaration we have:**

**| -  $\text{methsOnce}(\text{clsDi})$  and | -  $\text{fieldsOnce}(\text{clsDi})$  and**

**$P$  | -  $\text{inheritanceOK}(\text{clsDi})$  and  $P$  | -  $\text{def- clsDi}$**

-----

**| - P**

# TypeChecking Example

**ClsA= class A extends Object {fldF1 # mthM1} and**

**P, {this:A} |-mth- mthM1**

---

**P |-def- clsA**

**ClsB= class B extends A {fldF2 # mthM2} and**

**P, {this:B} |-mth- mthM2**

---

**P |-def- clsB**

**ClsMain= class Main extends Object { # mthMain} and**

**P, {this:Main} |-mth- mthMain**

---

**P |-def- clsMain**

# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c=a+b : ?t1$  and

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1}=\text{this.f1}+c; c : ?t$

-----

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c=a+b; \text{this.f1}=\text{this.f1}+c; c : ? t$

-----

$P, \{a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \{ (\text{int } c) c=a+b; \text{this.f1}=\text{this.f1}+c; c \} : ? t$

and  $P \vdash ?t <: \text{int}$

-----

$P, \{\text{this}:A\} \vdash\text{-mth- } \text{int } m1(\text{int } a, \text{int } a) \{...\}$

# TypeChecking Example

**?t1'=int**

**{c:int} in {c:int;a:int;b:int;this:A}**

**P, {c:int;a:int;b:int;this:A} |- c:?t1'**

**P |- ?t1" <: ?t1' TRUE**

**?t11"=int    ?t12"=int**

**P, {c:int;a:int;b:int;this:A} |- a:?t11":int**

**P, {c:int;a:int;b:int;this:A} |- b:?t12":int**

**P |- ?t11<:int and P|-?t12:int TRUE**

**and -----**

**P, {c:int;a:int;b:int;this:A} |- a+b:int**

**?t1"=int**

**P, {c:int;a:int;b:int;this:A} |- c=a+b:void ?t1=void**

# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} = \text{this.f1} + c : ?t_2$

And

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c : ?t \quad ?t = \text{int}$

---

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} = \text{this.f1} + c; c : ?t \quad ?t = \text{int}$

# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c=a+b : ?t1$  and

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1}=\text{this.f1}+c; c : \text{int}$

-----

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c=a+b; \text{this.f1}=\text{this.f1}+c; c : \text{int}$

-----

$P, \{a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \{ (\text{int } c) c=a+b; \text{this.f1}=\text{this.f1}+c; c \} : \text{int}$

and  $P \vdash \text{int} <: \text{int}$

-----

$P, \{\text{this}:A\} \vdash \text{-mth- int m1(int a, int b) \{...\}}$



# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} + c : ?t22$  and

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} : ?t21$  and

$P \vdash ?t22 <: ?t21$

-----

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} = \text{this.f1} + c : \text{void} \quad ?t2 = \text{void}$

# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} : ?t221$        $?t221 = \text{int}$

and

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash c : ?t222$        $?t222 = \text{int}$

and

$P \vdash ?t221 <:\text{int}$     and     $P \vdash ?t222 <:\text{int}$       **BOTH TRUE**

-----

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} + c : ?t22$        $?t22 = \text{int}$

# TypeChecking Example

**P, {c:int;a:int;b:int;this:A}|- this: ?t21' and ?t21'=A**  
**(?t21' is a declared class in P) and TRUE**  
**( (f1,?t21) is defined in fieldlist(P,?t21')) fieldlist(P,A)={(f1,int)}**

-----

**P, {c:int;a:int;b:int;this:A} |-this.f1:?t21 ?t21=int**

# TypeChecking Example

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} + c : ?t22 \text{ and } ?t22 = \text{int}$

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} : ?t21 \text{ and } ?t21 = \text{int}$

$P \vdash ?t22 <: ?t21$  **TRUE**

-----

$P, \{c:\text{int}; a:\text{int}; b:\text{int}; \text{this}:A\} \vdash \text{this.f1} = \text{this.f1} + c : \text{void } ?t2 = \text{void}$

.....

# Execution Example

- We extend the configuration defined before in order to include the program P, such that a configuration is  $\langle P, H, V, \text{exp} \rangle$
- Execution starts with the main method of the Main class
- For brevity we present just a part of the execution

$\langle P, \{\}, \{\}, \{ (B \ o1) \ o1 = \text{new } B(0, \text{null}); \{ (A \ o2) \ o2 = \text{new } A(2);$   
 $\{ (A \ o3) \ o3 = \text{new } A(3); o2 = o1.m2(o2, o3) \} \} \rangle$

->

$\langle P, \{\}, \{(o1, (B, \text{null}))\}, \text{ret } (o1, \ o1 = \text{new } B(0, \text{null}); \{ (A \ o2) \ o2 = \text{new } A(2);$   
 $\{ (A \ o3) \ o3 = \text{new } A(3); o2 = o1.m2(o2, o3) \} \} \rangle$

# Execution Example

<P,{},{(o1,(B,null))},ret (o1, o1=new B(0,null); { (A o2) o2=new A(2);  
{ (A o3) o3=new A(3);o2 =o1.m2(o2,o3)}}) >

-->

<P,{(1,(B,[(f1,(int,0));(f2,(A,null))])),{(o1,(B,null))},ret (o1, o1=1; { (A  
o2) o2=new A(2); { (A o3) o3=new A(3);o2 =o1.m2(o2,o3)}}) >

-->

<P,{(1,(B,[(f1,(int,0));(f2,(A,null))])),{(o1,(B,1))},ret (o1, { (A o2)  
o2=new A(2); { (A o3) o3=new A(3);o2 =o1.m2(o2,o3)}}) >

-->

<P,{(1,(B,[(f1,(int,0));(f2,(A,null))])),{(o2,(A,null);(o1,(B,1))},ret (o1,  
ret(o2, o2=new A(2); { (A o3) o3=new A(3);o2 =o1.m2(o2,o3)}}) >

... --> ... -->

# Execution Example

**<P,{(3,(A,[(f1,(int,3))]);(2,(A,[(f1,(int,2))]);(1,(B,[(f1,(int,0));(f2,(A,null))])),{(o3,(A,3));(o2,(A,2));(o1,(B,1))},ret (o1, ret(o2, ret(o3,o2 =o1.m2(o2,o3))} ) >**

**-->**

**<P,{(3,(A,[(f1,(int,3))]);(2,(A,[(f1,(int,2))]);(1,(B,[(f1,(int,0));(f2,(A,null))])),{(x3,(A,3));(x2,(A,2));(x1,(B,1));(o3,(A,3));(o2,(A,2));(o1,(B,1))},ret (o1, ret(o2, ret(o3,o2 =ret(x1, ret(x2, ret(x3, { (A z) { (int n) n=x2.m1(1,2)-x3.m1(2,1); {(int m) m=x2.f1-x3.f2; if(m>n) then {z=new A(m)} else {z=new A(n)} } } ;x1.f2=z;z**

**))))} ) >**

.....