Algebra Theory

Notion	Definition/Method to prove
Semigroup	Composition Law
Monoid	Composition Law and Associativity
Group	Composition Law, Associativity and Neutral element
Abelian Group	Group and commutativity
Equivalence Relation	Reflexivity, transitivity and symmetry
Partition	$\forall i \in I \cup A_i = A \wedge \forall i, j \in I\ A_i \cap A_j = \varnothing$
Ring	$(R,+,\cdot) \Leftrightarrow$ $R,+¿$ $¿$ A. Group, $(R,\cdot)$ semigroup and Distributive Law x(y+z)=xy+xz and (y+z)x=yx+zx
Unitary Ring	Ring and $(R,\cdot)$ - monoid
Division Ring	Ring and $(R,\cdot)$ - group
Field	Ring and $(R,\cdot)$ - abelian group
Integral domain	$\forall x, y \in R, xy = 0 \Leftrightarrow x = 0 \vee y = 0$
Subgroup	$H \Leftrightarrow H \neq \varnothing \wedge \forall x, y \in H, x \cdot y \in H \wedge \forall x \in H, x^{-1} \in H$
Subring	$A \Leftrightarrow A \neq \varnothing \wedge 0 \in A \wedge \forall x, y \in A, x - y \in A \wedge \forall x, y \in A, x \cdot y \in A$
Subfield	$A \Leftrightarrow card(A) \geq 2 \wedge 0, 1 \in A \wedge ¿$ $\forall x, y \in A, x - y \in A \wedge \forall x, y \in A, x \cdot y^{-1} \in A$
Homomorphism	For groups: $f(x \cdot y) = f(x) \cdot f(y)$

	For rings: $f(x \cdot y) = f(x) \cdot f(y)$ and $f(x+y) = f(x) + f(y)$
K-vector space	$k(v1+v2) = k \cdot v1 + k \cdot v2 \wedge (k1+k2) \cdot v = k1 \cdot v + k2 \cdot v \wedge ¿$ $(k1 \cdot k1) \cdot v = k1 \cdot (k2 \cdot v) \wedge 1 \cdot v = v$
Subspace	$S \underset{\Leftrightarrow}{\square} S \neq \varnothing \wedge \forall v1, v2 \in S, v1+v2 \in S \wedge \forall k \in K, \forall v \in S, k \cdot v \in S$
Linear map	$f(v1+v2) = f(v1) + f(v2) \wedge f(k \cdot v) = k \cdot f(v)$
Izomorphism	Bijective linear map
Endomorphism	Linear map with $V = V'$
Automorphism	Bijective endomorphism
Linear independece	$k1 \cdot v1 + \ldots + kn \cdot vn = 0 \underset{\Leftrightarrow}{\square} k1 = k2 = \ldots = kn = 0$
Basis	Linear independence and Generator
Generator	$\langle x \rangle = \cap \{S \leq_K V, x \leq S\}$
Image of f	$Imf = \{f(x) \in R \vee x \in R\}$
Kernel of f	$Kerf = \{x \in R \vee f(x) = 0\}$