$0^n 1^m$

Pumping Lemma (Bar-Hall Theorem)

J: Let L be a regular set. Then there exists $p \in N$ $(p>0)$ s.t. if $w \in L$,
$|w| > p$, then:

‧) $w = xyz \in L$ such that $0 < |y| \leq p$

‧) $xy^i z \in L$, $\forall i \geq 0$.

Proof: L is reg set $\Rightarrow \exists FA$ $M = (Q, \Sigma, \delta, q_0, F)$ s.t. $L(M) = L$.

$|Q| = p$.

$w \in L(M)$ $\qquad (q_0, w) \vdash^* (q_f, \varepsilon)$ $\quad q_f \in F \left\{ \Rightarrow \exists q_1 \in Q \quad \text{s.t } (q_0, xyz) \vdash^* \right.$

$|w| > p$ $\qquad\qquad\qquad\qquad\qquad\qquad \left. \vdash^* (q_1, yz) \vdash^* (q_1, z) \vdash^* (q_f, \varepsilon) \right.$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \uparrow$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad |y| > 0, |y| \leq p$

$i > 0.$ $(q_0, xy^i z) \vdash^* (q_1, y^i z) \vdash^* (q_1, y^{i-1} z) \vdash^* \dots \vdash^* (q_1, yz) \vdash^* (q_1, z) \vdash^* (q_f, \varepsilon)$

$\underline{xy^i z \in L, \forall i > 0}$

$(q_0, xz) \vdash^* (q_1, z) \vdash^* (q_f, \varepsilon)$

$\qquad xy^i z \in L, \forall i \geq 0.$

Context free grammars (Cfg)

$A \to \alpha \qquad A \in N$ and $\alpha \in (N \cup \Sigma)^*$

$G = (\{S\}, \{a, b\}, P, S)$ $\quad$ $P: S \to aSbS \mid aS \mid \varepsilon.$

$aaba \in L(G)$.

$S \underset{1}{\Rightarrow} aSbS \underset{p}{\Rightarrow} aaSbS \underset{p}{\Rightarrow} aabS \underset{1}{\Rightarrow} aabaS \underset{\varepsilon}{\Rightarrow} aaba$ (leftmost derivation)

$S \underset{1}{\Rightarrow} aSbS \underset{4}{\Rightarrow} aSbaS \underset{4}{\Rightarrow} aSba \underset{2}{\Rightarrow} aaSba \underset{4}{\Rightarrow} aaba$

Parse —

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Parse —