

Exam on Dynamical Systems.
July 2, 2012

1. Find the general solution of the following equations.
a) $x' = \lambda x$, where $\lambda \in \mathbb{R}$ is a fixed parameter; b) $x' = tx$;
c) $x''' - x'' = 0$; d) $t^2 x'' - 3tx' + 3x = 0$.
2. Find the equilibria and study their stability for the pendulum equation $\ddot{\theta} + \frac{\nu}{m}\dot{\theta} + \frac{g}{l}\sin\theta = 0$ where all the parameters are positive real numbers.
3. a) Prove that the fixed point $(0, 0)$ of the difference system $x_{k+1} = \frac{3}{5}x_k + \frac{1}{5}y_k$, $y_{k+1} = \frac{1}{5}x_k + \frac{3}{5}y_k$ is asymptotically stable.
b) We consider the Fibonacci sequence $(x_k)_{k \geq 0}$ satisfying $x_{k+2} = x_{k+1} + x_k$ for any $k \geq 0$.
Find its general solution and study the stability of its fixed point 0.
4. (instead of the test seminar) Represent the phase portrait of the scalar differential equation $\dot{x} = 2x - x^2$ and of the planar differential system $\dot{x} = -x$, $\dot{y} = y$.