bottom up

#op	op type	tested Module	Driver	Stub	Error
1	unit	D	D_D		on D
		C	D_C		on C, D $MUS_C(D)$, $MUN_C(D)$
		B	D_B		$MUS_B(C)$, $MUN_B(C)$
		E	D_E		
		F	D_F		
		A	D_A		

S5 Floyd's method for correctness

- entry point $x_{i,j}$ $\quad \overset{i}{P_i} \longrightarrow \overset{j}{P_j}$
- predicate

partial correctness

termination $x(m, v, a)$ $\underbrace{y(p)}$

1) search $(\underbrace{m, v, a}_{in}, \underbrace{p}_{out})$

$\varphi(x): v_1 \le a < v_m \wedge v_1 \le v_2 \le \dots \le v_m$

$\psi(x, z): 1 \le p < m \wedge v_p \le a < v_{p+1}$ $\quad , u_A := 1+m$

Subalg search $(\underline{m, v, a}, p)$ b $\quad \cdot A: \varphi(x)$

$s \leftarrow 1$, $d \leftarrow n$

while $(s < d)$ do $\quad$ —— $\quad B: v_s \le a < v_d$, $u_B = d - s$

$m \leftarrow (s+d)/2$

if $(a < v_m)$ then $d \leftarrow m$

else $s \leftarrow m+1$

$p \leftarrow s$

end Search. $\quad \cdot C: \psi(x, z)$ $\quad u_C := 0$

$R_{\alpha AB} ::= s < d$

$R_{\alpha BB_1} ::= (s < d) \wedge (a < v_m)$

$R_{\alpha BB_2} ::= (s < d) \wedge (a \geq v_m)$

$R_{\alpha BC} ::= \neg(s < d) \cong s \geq d \Rightarrow s = d$

$r_{\alpha AB} ::= (s, d) \leftarrow (1, m)$

$r_{\alpha BB_1} ::= (s, d) \leftarrow (s, m)$

$r_{\alpha BB_2} ::= (s, d) \leftarrow (m, d)$

$r_{\alpha BC} ::= (s) \leftarrow (s)$

$VC_{\alpha AB}$ $\forall x \forall y.\ v_1 \leq a < v_m \wedge v_1 \leq v_2 \leq .. \leq v_m \wedge s < d \rightarrow$
$\rightarrow (v_s \leq a < v_d)$ $/ (s, d) \leftarrow (1, m)$

$$\left.\begin{array}{l} s = 1,\ d = m \\ v_1 \leq a < v_m \end{array}\right\} \Rightarrow v_s \leq a < v_d$$

$VC_{\alpha BB_1}$ $\forall x \forall y.\ (v_s \leq a < v_d) \wedge (s < d) \wedge (a < v_m) \rightarrow v_s \leq a < v_d /$

$$\left.\begin{array}{l} v_s \leq a < v_d \\ s = s,\ d = m \end{array}\right\} \Rightarrow v_s \leq a < v_m$$

$r_{\alpha BB_1}$
$(s, d) \leftarrow (s, m)$

$VC_{\alpha BB_2}$ $\forall x \forall y\ (v_s \leq a < v_d) \wedge (s < d) \wedge (a \geq v_m)) \overset{?}{\rightarrow}$
$\rightarrow (v_s \leq a < v_d)$. $/ (s, d) \leq (m, d)$

$$\left.\begin{array}{l} s = m,\ d \\ v_s \leq a < v_d \end{array}\right\} \Rightarrow v_m \leq a < v_d$$

$VC_{\alpha BC}$. $\forall x \forall y.\ v_s \le a < v_d \wedge s = d \Rightarrow 1 \le p < m \wedge \cancel{v_p \le a < v_{p+1}}$

$\wedge\ v_p \le a < v_{p+1}\ /\ (p) \leftarrow (s)$.

$p = s$.

$v_p \le a < v_{p+1}$.

$v_s \le a < v_{s+1}$. $\rightarrow$ True

$TC_{\alpha AB}$ $\forall x \forall y.\ v_1 \le a \le v_m \wedge v_1 \le v_2 \le \dots \le v_m \wedge s < d \rightarrow$

$1+m > d-s\ /\ (s,d) \leftarrow (1,m)$. $\Leftrightarrow$

$1+m > m-1$ (True.

$TC_{\alpha BB_1}$ $\forall x \forall y.\ (v_s \le a < v_d) \wedge (s < d) \wedge (a < v_m) \rightarrow$

$\rightarrow d-s > d'-s'\ /\ (s',d') \leftarrow (s,m)$.

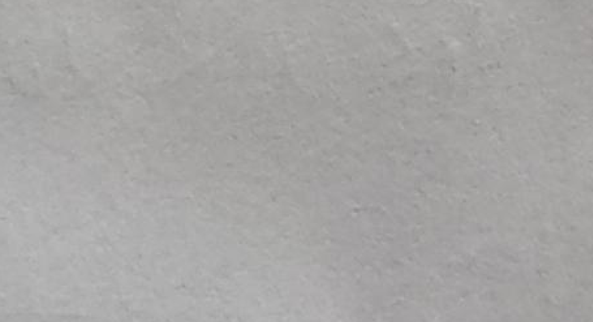
$$\left.\begin{array}{l} d-s > d'-s' \\ d' = m \end{array}\right\} \Rightarrow d-s > m-s \Rightarrow d > m \ \Big|\ \text{True}$$

$m = (s+d)/2$

$TC_{\alpha BB_2}$ $\forall x \forall y\ (v_s \le a < v_d) \wedge (s < d) \wedge (a \ge v_m) \rightarrow$

$\rightarrow d-s > d'-s'\ /\ (s',d') \leftarrow (m,d)$

$$\left.\begin{array}{l} d-s > d'-s' \\ s' = m \end{array}\right\} \Rightarrow d-s > d-m \Leftrightarrow s < m \ (T)$$



$TC_{\alpha BC}$ $\forall x \forall y\ v_s \le a < v_d \wedge s = d \rightarrow d-s > 0\ /\ p \leftarrow s$

$\Leftrightarrow 0 > 0$ False

2) gcd (m_1, m_2, d)

$\varphi(x) : m_1 \in N \wedge m_2 \in N.$

$\psi(x,z) := d = gcd(m_1, m_2).$

Subalg gcd (m_1, m_2, d) is

$d \leftarrow m_1 \quad , i \leftarrow m_2$

while $(d \neq i \ \& \ i > 0)$ do

if $(d > i)$ then $d \leftarrow d - i$

else $i \leftarrow i - d.$

$A : \varphi(x)$

$u_A : 2m_1 + 2m_2$

$B : (d, i) = (m_1, m_2).$

Remark $(a,b) = gcd(a,b)$

$u_B : d + i.$

$C \ \psi(x,z) \quad u_C := 0.$

$r\alpha_{AB}. \ (d,i) \leftarrow (m_1, m_2)$

$r\alpha_{BB_1} \ d \leftarrow d - i.$

$r\alpha_{BB_2} \quad i \leftarrow i - d$

$BC \quad \emptyset.$

$R_{\alpha_{AB}} \quad d \neq i \wedge i > 0$

$BB_1 \ d \neq i \wedge i > 0 \wedge d > i.$

$BB_2 \ d \neq i \wedge i > 0 \wedge d \leq i.$

$BC \ \neg(d \neq i \wedge i > 0) \equiv d = i \vee i \leq 0.$

$v_C\alpha_{AB} : \forall x \forall y \ m_1 \in N \wedge m_2 \in N. \wedge d \neq i \wedge i > 0 \rightarrow .(d, i) = (m_1, m_2) \Big/ r\alpha_{AB}$

$v_C\alpha_{BB_1} \ (d,i) = (m_1, m_2). \wedge d \neq i \wedge i > 0 \wedge d > i \rightarrow (d', i) = (m_1, m_2) \Big/ d' \leftarrow d - i$

$(d - i, i) = (m_1, m_2)$ true

$v_C\alpha_{BB_2}. (d, i) = (m_1, m_2) \wedge d \neq i \wedge i > 0 \wedge d \leq i. \rightarrow (d', i') = (m_1, m_2) \Big/ i' \leftarrow i - d$

$\rightarrow (d', i - d) = (m_1, m_1)$ true

$v_C\alpha_{BC}. (d,i) = (m_1, m_2) \wedge d = i \vee i \leq 0. \rightarrow. d = (m_1, m_2)$ true

$d = (d, d) = (m_1, m_2)$