Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

# Exam on Dynamical Systems
June 19, 2010
II

1. (3p) Integrate the following differential equations
(a) $y' = \dfrac{x - 2x^3}{y}$
(b) $y' = \dfrac{x - y^2}{y}$
and than find a first integral of each of it.

2. (2p) Let $\varphi : (-\varepsilon, \varepsilon) \to \mathbb{R}$ be some solution of the differential equation $y' = \dfrac{y - 1}{1 + x^2 + y}$ (the unknown is the function denoted $y$ of independent variable $x$, while $\varepsilon > 0$ is a positive constant). Decide whether or not the following situations are possible:
(a) $\varphi(0) = 1$ and $\varphi$ is a strictly increasing function;
(b) $\varphi(0) = 0$ and $\varphi$ is a strictly increasing function;
(c) $\varphi_1(0) = -2$ and $\varphi_1'(0) = 0$.

3. (1.5p) Find the flow of the planar linear differential system:

$$\dot{x} = x - y, \quad \dot{y} = 2x + 4y.$$