Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 09, 2009
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1. (1.5p)
(a) Find the general solution of the following differential equation

$$\varphi'' + \frac{9}{4}\varphi = 0.$$

(b) (True or False) "All the solutions of $\varphi'' + \frac{9}{4}\varphi = 0$ are periodic with a period $T = 4\pi$."

2. (1.5p) Let $I \subset \mathbb{R}$ be an open interval and $a, f : I \to \mathbb{R}$ be continuous functions. Write the general solution of the differential equations:
(a) $x' + a(t)x = 0$,
(b) $x' + a(t)x = f(t)$.

3. (0.5p) Write the Euler numerical formula for a first order differential equation.

4. (3p) We consider the differential system:

$$\dot{x} = -y + y^3, \quad \dot{y} = -x + x^3.$$

(a) Study the stability of the equilibrium point $(0, 0)$.
(b) Find a first integral.
(c) Find all the equilibria.