

Dynamical systems. Final exam 13-02-2005

1. Find the general solution of $y' = 3y + x^2$.

2. We consider the Initial Value Problem:

$$y' = \frac{1}{y - x^2} + 2x, \quad y(0) = -1.$$

a) Write the domain of the differential equation, denoted D_f , as $D_f = U_1 \cup U_2$, where U_1 and U_2 are open and connected.

b) Do the change of the variable $u = y - x^2$, where $u = u(x)$.

c) Find the maximal solution of this IVP.

3. We consider the differential equation $y' = \lambda + 2y - y^2$ where $\lambda \in \mathbb{R}$ is a parameter.

a) Find the equilibrium points and study their stability.

b) Write the Euler's numerical formula to find the approximate solution of this differential equation on the interval $[0, 1]$ that satisfies $y(0) = 0.5$.

4. We consider the scalar map $f : \mathbb{R} \rightarrow \mathbb{R}$ $f(x) = (1 + x)/2$. Find the fixed points of f . Draw the stair-step diagram starting with $x_0 = 3$.

5. Write the definition of the Wronskian of two C^1 functions.

6. Linear homogeneous second order differential equations with constant coefficients.