

Exam on Dynamical Systems.
June 08, 2011

1. (0.5p) Find the solution of the initial value problem

$$y' = 2xy, \quad y(0) = -2.$$

2. (2.5p) We consider the differential equation $x'' + 2ax' + 4x = 0$, where $a > 0$ is a real parameter. Write the general solution and describe the long-term behavior of the solutions (for $t \in (0, \infty)$). Discuss with respect to the parameter a .

3. (1p) Write the statement of The Superposition Principle. Give an example.

4. We consider the scalar differential equation $\dot{x} = -x^2 + x + 2$.

- a) (1p) Represent its phase portrait.
b) (0.5p) Denote by $\varphi(t)$ its solution with $x(0) = 3$. Find $\lim_{t \rightarrow \infty} \varphi(t)$.
c) (0.5p) Write the Euler numerical formula with constant step size h for this differential equation.