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Exercise Sheet no.13

Analysis for CS

GROUPWORK:

(G 31) (Integration over compact intervals)

Compute the following double and triple integrals:

a) $\int\int\int_A \frac{2z}{(x+y)^2} dxdydz$, where $A = [1,2] \times [2,3] \times [0,2]$,

b) $\int\int_A \frac{x}{(1+x^2+y^2)^{\frac{3}{2}}} dxdy$, where $A = [0, \sqrt{3}] \times [0,1]$,

c) $\int\int\int_A \frac{x^2 z^3}{1+y^2} dxdydz$, where $A = [0,1] \times [0,1] \times [0,1]$.

(G 32) (Integration over normal domains)

Let

$$M := \{(x,y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1,\ -x^2 \leq y \leq 1 + x^2\}.$$

a) Represent M in a Cartesian coordinate system.

b) Compute $\int\int_M (x^2 - 2y) dxdy$.

c) Is M a normal domain with respect to the x-axis?

HOMEWORK:

(H 34) (Integration over compact intervals)

Compute the following double and triple integrals:

a) $\int\int_A (xy + y^2) dxdy$, where $A = [0,1] \times [0,1]$,

b) $\int\int_A \min\{x,y\} dxdy$, where $A = [0,1] \times [0,2]$,

c) $\int\int\int_A \frac{1}{(x+y+z)^3} dxdydz$, where $A = [1,2] \times [1,2] \times [1,2]$.

(H 35) (Integration over normal domains)

Let M be the subset of $\mathbb{R}^2$ bounded by the triangle with vertices $(0,0)$, $(1,0)$ and $(\frac{1}{2}, \frac{1}{2})$.

a) Represent M in a Cartesian coordinate system.

b) Show that M is a normal domain with respect to the x-axis.

c) Compute $\int\int_M (x^2 + y^2) dxdy$.

(H 36) (Train your brain)

Show that the function $f\colon [0,1] \times [0,1] \to \mathbb{R}$, defined by

$$f(x) = \begin{cases} 1 \text{ if } x \in [0,1] \times [0,1] \cap \mathbb{Q} \times \mathbb{Q} \\ 0 \text{ else,} \end{cases}$$

is not Riemann integrable on $[0,1] \times [0,1]$.