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Exercise Sheet no.9

Analysis for CS

GROUPWORK:

(G 24)

Let $f\colon \mathbb{R}^* \times \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y,z) = \frac{z^2 e^y}{x}$. Determine all

a) first-order partial derivatives of f,

b) second-order partial derivatives of f.

(G 25)

For the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{x^4 - y^4}{2(x^4+y^4)}, & (x,y) \neq 0_2 \\ 0, & (x,y) = 0_2 \end{cases}$$

study

a) the partial differentiability with respect to both variables at 0_2,

b) the continuity at 0_2.

(G 26) (Train your brain)

Prove **P5** in the 9th lecture: If $M \subseteq \mathbb{R}^n$ then $\operatorname{int} M \subseteq M$ and $\operatorname{int} M \subseteq M'$.

HOMEWORK:

(H 23) (To be delivered in the next exercise-class)

a) Show that the function $f\colon \mathbb{R}^2 \to \mathbb{R}$, defined by

$$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^2}, & (x,y) \neq 0_2 \\ 0, & (x,y) = 0_2, \end{cases}$$

is partially differentiable with respect to both variables on $\mathbb{R}^2$, and determine both first-order partial derivatives of f.

b) Determine all first-order and all second-order partial derivatives of the function $f\colon \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = z\sin(x-y)$.

(H 24)

Determine the gradient of the function f at the point a in the following cases:

a) $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = e^{-x}\sin(x+2y)$, $a = \left(0, \frac{\pi}{4}\right)$,

b) $f\colon \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = (x-y)\cos \pi z$, $a = \left(1, 0, \frac{1}{2}\right)$.