

Dynamical systems. Final exam 06-09-2007

1. Find the solution of each of the following Initial Value Problems:

a) $x' = x$, $x(0) = 1$.

b) $x' = y$, $y' = x$, $x(0) = 2$, $y(0) = -2$.

c) $t^2x'' + tx' - x = 0$, $x(1) = 1$, $x'(1) = -1$.

2. Galileo's pendulum.

a) Deduce its differential equation.

b) Find the general solution of the linearized equation $\theta'' + \omega^2\theta = 0$, where $\omega = g/L > 0$ (g the gravitational constant and L the length of the rod). Interpret the result.

3. Represent the phase portrait and find a first integral for the planar system:

$$\dot{x} = -y, \quad \dot{y} = x.$$

4. The statement of the Local Existence and Uniqueness Theorem for a first order scalar Initial Value Problem.

5. Determine the equilibria and study their stability for the planar system:

$$\dot{x} = x - 2xy, \quad \dot{y} = x^2 - 2y.$$

6. (*instead of the point from the seminar partial exam*) Find the value of the real parameter b for which the given equation is exact and then find a first integral using the value of b :

$$(2xy^2 + bx^2y)dx + x^2(x + 2y)dy = 0.$$