

Exam on Dynamical Systems
June 2014 - VI

1. (2p) We consider the differential equation

$$x'' + 9x = \cos 3t.$$

- a) Find a solution of the form $x_p = t(a \cos 3t + b \sin 3t)$, with $a, b \in \mathbb{R}$.
- b) Find its general solution.
- c) Describe the motion of a spring-mass system governed by this equation.

2. (2p) We consider the differential system

$$x' = x - y, \quad y' = x + y.$$

- a) Find the type and stability of this linear system.
- b) Pass to polar coordinates, i.e. find the differential system in the unknowns $(\rho(t), \theta(t))$ when

$$x(t) = \rho(t) \cos \theta(t), \quad y(t) = \rho(t) \sin \theta(t).$$

- c) What type of curves are the orbits?

3. (0.75p) We consider the difference equation $x_{k+1} = -2x_k + 3^k$.

- a) Find a solution of the form $x_k = a3^k$, with $a \in \mathbb{R}$.
- b) Find its general solution.
- c) Find the solution with $x_0 = 0$.

4. (0.75p) Write the definition of a fixed point and, respectively, of a 2-periodic point for some map $f : \mathbb{R} \rightarrow \mathbb{R}$.