Lab 8

Quadrature formulas (2)

Simpson's formula for double integral

Consider the integral $I = \int_a^b \int_c^d f(x,y)dydx$. Let $m, n \in \mathbb{N}$ and the equidistant points $x_0, ..., x_{2m}$ in $[a,b]$, with step $h = \frac{b-a}{2m}$, respectively $y_0, ..., y_{2n}$ in $[c,d]$, with step $k = \frac{d-c}{2n}$.

We apply the repeated Simpson's formula to the integral $\int_c^d f(x,y)dy$ and then to the integral $\int_a^b \int_c^d f(x,y)dydx$.

Algorithm:

INPUT: a,b,c,d,m,n

OUTPUT: the approximant J of the integral I

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h=(b-a)/(2*n);
j1=0; j2=0; j3=0
for i=0,1,...,2*n
        Let x=a+i*h;
         hx=(d-c)/(2*m);
         k1=f(x,c)+f(x,d);
         k2=0;
         k3=0;
         for j=1,2,...,2*m-1
                 y=c+j*hx;
                 z=f(x,y);
                 if j is even do k2=k2+z;
                      else k3=k3+z;
                end{if}
         end{for}
        l=(k1+2*k2+4*k3)*hx/3;
         if (i==0)| (i==2*n) do j1=j1+l;
              else if i is even do j2=j2+l;
                      else j3=j3+l;
                      end{if}
            end{if}
end
J=(j1+2*j2+4*j3)*h/3
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Problems:

1. Use Romberg's algorithm for trapezium and Simpson's formulas to approximate the integral
$$\int_0^1 \frac{2}{1+x^2}dx,$$
with precision $\varepsilon = 10^{-5}$.

2. Plot the graph of $f : [1,3] \to \mathbb{R}$, $f(x) = \frac{100}{x^2}\sin\frac{10}{x}$. Use an adaptive quadrature algorithm for Simpson's formula to approximate the integral
$$\int_1^3 f(x)dx,$$
with precision $\varepsilon = 10^{-4}$. Compare the obtained result with the one obtained applying repeated Simpson formula for $n = 50$ and 100. (The exact value is -1.4260247818.)

3. Use the repeated rectangle formula, for $n = 150$ and 500, to evaluate the integral
$$\int_1^{1.5} e^{-x^2}dx.$$
(*Answer:* 0.1094)

4. The volume of a solid is given by $\int_{0.1}^{0.5}\int_{0.01}^{0.25} e^{\frac{y}{x}}dydx$. Approximate this volume applying Simpson's algorithm for double integrals for $m = n = 10$. (Result: 0.178571)