Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - I

1. (1p) Find the linear homogeneous differential equation with constant coefficients, of minimal order, which has as solution the function

$$\cosh 3t.$$

(Hint: $\cosh x = \frac{e^x + e^{-x}}{2}$ for any $x \in \mathbb{R}$.)

2. (2p) We consider the scalar differential equation

$$\dot{x} = \lambda x(2 - x),$$

whose unknown is the function x of variable t, and where $\lambda \in \mathbb{R}^*$ is a fixed parameter. Find its equilibria, its orbits, depict its phase portrait, and study the stability of its equilibria. Discuss with respect to the parameter λ.

3. (2.5p) Let $f : (0, \infty) \to \mathbb{R}$,

$$f(x) = \frac{1}{2}\left(x + \frac{3}{x}\right).$$

Fix an arbitrary $x_0 \in (0, \infty)$ and consider the sequence $(x_k)_{k \geq 0}$ satisfying the recurrence

$$x_{k+1} = f(x_k), \text{ for any } k \geq 0.$$

a) Prove that $f(x) \in [\sqrt{3}, \infty)$ for any $x \in (0, \infty)$ and that $x_k \geq \sqrt{3}$ for any $k \geq 1$.

b) Prove that $x_{k+1} - x_k \leq 0$ for any $k \geq 1$.

b) Find the fixed points of f and study their stability.

c) Prove that the sequence $(x_k)_{k \geq 0}$ is convergent and $\lim_{k \to \infty} x_k = \sqrt{3}$.