Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems
June 2014 - III

1. (1p) Find the linear homogeneous difference equation with constant coefficients, of minimal order, which has as solutions the two sequences

$$1,\ \frac{1}{2}\ ,\ \frac{1}{2^2}\ ,\ \frac{1}{2^3}\ ,\ \frac{1}{2^4}\ ,\ \frac{1}{2^5}\ ,\ \dots$$

and

$$1,\ -\frac{1}{2}\ ,\ \frac{1}{2^2}\ ,\ -\frac{1}{2^3}\ ,\ \frac{1}{2^4}\ ,\ -\frac{1}{2^5}\ ,\ \dots\ .$$

2. (1.5p) We consider the scalar difference equation

$$x_{k+1} = x_k + \lambda x_k(2 - x_k),$$

whose unknown is the sequence $(x_k)_{k\geq 0}$, and where $\lambda \in (0,1)$ is a parameter. Find its fixed points and study their stability. Discuss with respect to the parameter λ.

3. (3p) We consider the planar systems

$$(*) \left\{ \begin{array}{l} x' = -2x \\ y' = x - \sqrt{5}y \end{array} \right. \qquad \text{and} \qquad (**) \left\{ \begin{array}{l} x' = -2x \\ y' = x + 3x^2 - \sqrt{5}(y + y^3) \end{array} \right. .$$

a) Find the general solution of (*). For any $\eta = (\eta_1, \eta_2) \in \mathbb{R}^2$, find the solution, denoted $\varphi(t;\eta)$, of (*) satisfying $x(0) = \eta_1$, $y(0) = \eta_2$.

b) For any $\eta = (\eta_1, \eta_2) \in \mathbb{R}^2$, find $\lim\limits_{t\to\infty} \varphi(t;\eta)$.

c) For system (**), find its equilibria and study their stability.

d) For any $\eta = (\eta_1, \eta_2) \in \mathbb{R}^2$, denote by $\psi(t;\eta)$ the solution of (**) satisfying $x(0) = \eta_1$, $y(0) = \eta_2$. What can be deduced from c) about $\lim\limits_{t\to\infty} \psi(t;\eta)$?