Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems.
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1. (3p) Specify the type and stability of the equilibrium point $(0,0)$ of the differential system:

$$\dot{x} = 3x + 2y, \quad \dot{y} = 4x + y.$$

Find the general solution of the above differential system.

2. (2.5p) Represent the phase portrait of the scalar differential equation

$$\dot{x} = 2x\left(3 - \frac{x}{100}\right).$$

What remarkable property has the solution of the above equation with the initial value $x(0) = 100$? (*Hint: "read" its phase portrait*)
Find the general solution of the above equation.

3. (1p) Find a first integral of $(x^2 - 5xy^2)dx + (y^3 - 5x^2y + 3)dy = 0$.

4. (1p, not compulsory) Determine the equilibria and study the stability of one of them for the following planar differential system:

$$\dot{x} = -2x + y^2, \quad \dot{y} = y - 2xy.$$